Particle productions from chiral matter

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Quark gluon plasma as chiral matter

Quarks are almost massless Dirac fermion

\[ m_q/T \sim 0.03 \]
Two topics

Electric conductivity in a magnetic field

Dilution production in a magnetic field and vorticity
Motivation

Strong magnetic field and rotation in heavy ion collisions
Vorticity in HIC

Lambda polarization

\[ \omega = (9 \pm 1) \times 10^{21} \text{s}^{-1} \]

\( \sim 3 \text{ MeV} \)

AMPT simulation

Jiang, Z. W. Lin and J. Liao
Erratum: [Phys. Rev. C 95, no. 4, 049904 (2017)]

\[ \langle |\omega_y| \rangle (\text{fm}^{-1}) \]

\[ \sim 20 \text{ MeV} \]
Chiral effect in QGP

\[ J = \xi_B B + \xi_\omega \omega \]

Chiral magnetic effect
Kharzeev, McLerran, Warringa ('08)
Fukushima, Kharzeev, Warringa ('08)

\[ \xi_B \sim \mu_5 \]

Chiral vortical effect
Son, Surowka ('09),
Landsteiner, Megias, Pena-Benitez (11)

\[ \xi_\omega \sim \mu_5 \mu \]

\[ J_5 = \zeta_B B + \zeta_\omega \omega \]

Chiral separation effect
Son, Zhitnitsky ('04)
Metlitski, Zhitnitsky ('05)

related to chiral anomaly
\[ \partial_\mu j_5^\mu = CE \cdot B \]
Electric conductivity in $B$


Current $J^i = \sigma^{ij} E^j + \cdots$

$\sigma^{ij} = \sigma_H \epsilon^{ijk} \hat{B}^k + \sigma_{\parallel} \hat{B}^i \hat{B}^j + \sigma_{\perp} (\delta^{ij} - \hat{B}^i \hat{B}^j)$

- Hall
- longitudinal
- perpendicular

Related to CME
Strong B, chiral limit

Chiral anomaly

\[ \partial_\mu j_5^{\mu} = CE \cdot B \rightarrow n_5 \propto tE \cdot B \]

CME

\[ j_{\text{CME}} \sim \mu_5 B \sim n_5 B \]

implies \( \sigma_\parallel \rightarrow \infty \)

in cond-mat

Interaction with phonon or impurity

\[ t \rightarrow \tau_R \]

\[ \sigma_\parallel \propto \tau_R B^2 + \text{Ohmic term} \]

in QCD

explicit breaking by \( m_q \)
chiral magnetic effect in cond-mat.


resistance: $\rho = \frac{1}{\frac{\tau}{\chi} (CB)^2 + \sigma_{\text{Ohm}}}$
Conductivity of QCD in strong B

\[ \sqrt{eB} \gg T \]

\[ \mu = 0 \]

\[ \sigma_{||}/T \sim \frac{eB}{m_q^2 g^2} \]

**Strong B**
- Effective 1+1 dynamics
- +chiral symmetry

**suppression of interactions**
Electric conductivity in B

Current: \( \mathbf{J}^i = \sigma^{ij} \mathbf{E}^j + \cdots \)

\[ \sigma^{ij} = \sigma_H \epsilon^{ijk} \mathbf{\hat{B}}^k + \sigma_\parallel \mathbf{\hat{B}}^i \mathbf{\hat{B}}^j + \sigma_\perp (\delta^{ij} - \mathbf{\hat{B}}^i \mathbf{\hat{B}}^j) \]

- Hall
- Longitudinal
- Perpendicular

One-loop

Resume

Two loop

\( \sigma_H = \frac{n_e}{B} \)

\( \frac{\sigma_\parallel}{T} \sim \frac{1}{g^n} F(T^2/eB) \)

\( \frac{\sigma_\perp}{T} \sim \frac{g^2T^2}{|eB|} \)
Scattering v.s. radiation

Leading contribution to conductivity

$\sim g^4$

Usually suppress by kinematics

$\sim g^2$

The situation is different in B!

ex) Syncrotron radiation
Longitudinal conductivity $\sigma_{||}$

Infinitely diagrams contribute to the conductivity

which is generated by

Solving linearized Boltzmann Eq.

Hidaka, Kunihiro, Phys. Rev. D83 (2011) 076004
Fukushima, YH (2018)
Just complicated

\[ X(n, n', \xi) = g^2 N_c C_F |q_f B| \frac{e^{-\xi n} \xi^{n-n}}{2\pi} \sum_{l,s,c} \int \frac{d^2 p_\perp}{(2\pi)^2} |\mathcal{M}_{p+p'\rightarrow k}|^2 = X(n, n', \xi) \]

\[ F(n, n', \xi) = \left[ L^{(n'-n)}_n(\xi) \right]^2 + \frac{n'}{n} \left[ L^{(n'-n)}_{n-1}(\xi) \right]^2 \]

Laguerre Polynomials

\[ \xi = \frac{(\epsilon_{fn} + \epsilon_{fn'})^2 - (p_z + p_z')^2}{2|q_f B|} \]
B dependence

non-monotonic behavior

Degrees of freedom $\sim \frac{eB}{2\pi}$

v.S.

Higher Landau level is suppressed by

Boltzmann factor: $\exp(-\sqrt{eBn/T})$
B dependence

FIG. 2: Magnetoresistance in field parallel to current ($B$) at various temperatures. For clarity, the resistivity curves were shifted by 1.5 m$\Omega$cm (150 K), 0.9 m$\Omega$cm (100 K), 0.2 m$\Omega$cm (70 K) and 0.02 m$\Omega$cm (5 K). (b) MR at 20K (red symbols) fitted with the CME curve (blue line); inset: temperature dependence of the fitting parameter $a(T)$ in units of $\text{S/cm}^2$. Observed resistivity can be fitted with a simple quadratic term (Supplementary materials, Fig. S1). This term is treated as a background and subtracted from the parallel field component for all MR curves recorded at $T \gtrsim 100$ K.

A negative MR is observed for $T \gtrsim 100$ K, increasing in magnitude as temperature decreases. We found that the magnetic field dependence of the negative MR can be nicely fitted with the CME contribution to the electrical conductivity, given by $\text{CME} = \sigma_0 + a(T)B^2$, where $\sigma_0$ represents the zero field conductivity. The fitting is illustrated in Fig. 2(b) for $T = 20$K, with excellent agreement between data and the CME fitting curve. At 4 Tesla, the CME conductivity is about the same as the zero-field conductivity. At 9T, the CME contribution increases by $\sim 400\%$, resulting in a negative MR that is much stronger than any conventional one reported at an equivalent magnetic field in a non-magnetic material.

At very low field, the data show a small cusp-like feature. The origin of this feature is not completely understood, but it probably indicates some form of anti-localization coming from the perpendicular ($B_{\perp}$) component. Inset in Fig. 2(b) shows the temperature dependence of the fitting parameter $a(T)$, which decreases with temperature faster than $1/T$, again consistent with the CME.


Similar behavior although physical processes are different
Dilepton production

\[ \omega \]

\[ \theta_q \]

\[ B \]

\[ \theta_q \]
Chiral kinetic equation (CKE)

\[(\partial_t + \dot{x} \cdot \nabla_x + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}})f = C[f]\]

Equations of motion
\[
\dot{x} = \dot{\mathbf{p}} + \mathbf{p} \times \Omega
\]
\[
\dot{\mathbf{p}} = \dot{x} \times \mathbf{B} + \mathbf{E}
\]

Berry curvature
\[
\Omega = \nabla_p \times \mathbf{a} = \frac{\mathbf{p}}{2p^2}
\]

Anomaly
\[
\partial_\mu j^\mu = \frac{1}{4\pi^2} E \cdot B
\]
Covariant version of Chiral kinetic equation (CKE)

\[
\Delta_\mu S^{<\mu} = \sum_\mu S^{>\mu} - \sum_\mu S^{<\mu} \quad \Delta_\mu = \partial_\mu + F_{\nu\mu} \partial_{p_\nu}
\]

\[
S^{<\mu} = 2\pi\epsilon(p \cdot n) \left[ \delta(p^2)(p^\mu + S_{n}^{\mu\nu} \mathcal{D}_\nu) + p_\nu \tilde{F}^{\mu\nu} \delta'(p^2) \right]f
\]

spin: \[ S_{n}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha n_\beta}{p \cdot n} \quad \mathcal{D}_\mu f = \Delta_\mu f + \sum_\mu f - \sum_\mu \bar{f}
\]
(local) Equilibrium

Current: $J^\mu = 2 \int \frac{d^4 p}{(2\pi)^4} S^{<\mu}(p, X)$

$J = nu + \sigma_B B + \sigma_\omega \omega$

CME  CVE
Dissipative current

CKE with relaxation time approximation

Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy ('16)
Chen, Ishii, Pu, Yamamoto ('16)
YH, Pu, Yang ('17)

\[ \delta J = C_1 E \times \nabla \mu + \underbrace{C_2 E \times \nabla T + C_3 \nabla \mu \times \nabla T}_{\text{correction}} \]

\[ C_i \sim \tau_R \]

\[ \nabla \mu, \nabla T \]

low-T  \quad  high-T
Dissipative current

Shear and bulk correction

\[ \delta J^i = C_4 \pi^{ij} B_j + C_5 \pi^{ij} \omega_j \\
+ C_6 (\nabla \cdot u) B^i + C_7 (\nabla \cdot u) \omega^i \]
Dilepton production

Gongyo, YH, Tachibana ('18)

Photon polarization function

\[ \Pi^{<\mu\nu}(X, q) = \int d^4s e^{iq\cdot s} \langle j^\nu(X - s/2) j^\mu(X + s/2) \rangle \]

Dilepton production rate

\[ \frac{d\Gamma}{d^4q} = -\frac{\alpha}{24\pi^4} \Pi_{\mu}^{<\mu}(q, X) \]
Di-lepton production in $\omega$

\[
\frac{d\Gamma}{d^4q} = \frac{d\Gamma_0}{d^4q} + \frac{d\Gamma_\omega}{d^4q}
\]

with

\[
\frac{d\Gamma_\omega}{d^4q} = (\Omega_\gamma \cdot \omega) C_\omega(q)
\]

\[
C_\omega \sim \mu_5 \quad \Omega_\gamma = \frac{\hat{q}}{|q|^2}
\]

Gongyo, YH, Tachibana (’18)

\[
\text{angle dependence}
\]

\[
\begin{align*}
T &= 200 \text{ MeV} \\
\mu_5 &= 20 \text{ MeV} \\
\omega &= 10 \text{ MeV} \\
q_0 &= 4 \text{ GeV} \\
|q| &= 2 \text{ GeV}
\end{align*}
\]
Di-lepton production in $B$

\[ \frac{d\Gamma}{d^4q} = \frac{d\Gamma_0}{d^4q} + \frac{d\Gamma_B}{d^4q} \]

with \[ \frac{d\Gamma_B}{d^4q} = (\Omega_{\gamma} \cdot B) C_B(q) \]

\[ C_\omega \sim \mu_5 \mu \]

\[ \Omega_{\gamma} = \frac{\hat{q}}{|q|^2} \]

$B$ dependence

$B = 10 \text{ MeV}$

$q_0 = 4 \text{ GeV}$

$|q| = 2 \text{ GeV}$

$T = 200 \text{ MeV}$

$\mu_5 = 20 \text{ MeV}$

$\mu = 20 \text{ MeV}$
Summary

Electric conductivity

Particle production: Novel chiral effects:

\[
\frac{d\Gamma_\omega}{d^4q} = (\omega \cdot \Omega_\gamma)C_\omega \quad \frac{d\Gamma_B}{d^4q} = (B \cdot \Omega_\gamma)C_B
\]

\[
\Omega_\gamma = \frac{\hat{q}}{|q|} \quad C_\omega \sim \mu_5 \quad C_B \sim \mu_5\mu
\]

\[T = 200\text{MeV} \quad \mu = 0 \quad m_\omega = 3\text{MeV} \quad m_\gamma = 5\text{MeV}\]

\[\rho(T) \text{ vs. } eB/T^2\]

\[\rho(T) \text{ vs. } B\]

Backup
Large B behavior

\[ g = 2 \quad T = 200\text{MeV} \quad m_u = 3\text{MeV}, m_d = 5\text{MeV} \]

\[
\begin{align*}
\sigma || / T & \quad 0 & \quad 50 & \quad 100 & \quad 150 & \quad 200 & \quad 250 & \quad 300 & \quad 350 \\
eB / T^2 & \quad 0 & \quad 1000 & \quad 2000 & \quad 3000 & \quad 4000 & \quad 5000 \\
\end{align*}
\]

- Lowest Landau level
- Including higher levels
For small current mass, higher Landau levels are important.
\[
\frac{d\Gamma}{d^4q} = (\Omega_\gamma \cdot \omega)C(q)
\]