RIKEN interdisciplinary
Theoretical & Mathematical
Sciences

Topology and Chiral Physics in Atomic, Molecular, and Optical systems

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RIKEN iTHEMS, Japan

@ Workshop on “Recent Developments in Chiral Matter and Topology”, Dec 8, 2018

Topology in various systems

In this conference, we cover:

Which Physical Review?

- Nuclear physics / High energy physics — ✓ Physical Review C & Physical Review D
- Solid-state physics — ✓ Physical Review B
- Atomic, molecular and optical (AMO) physics — ✓ Physical Review A

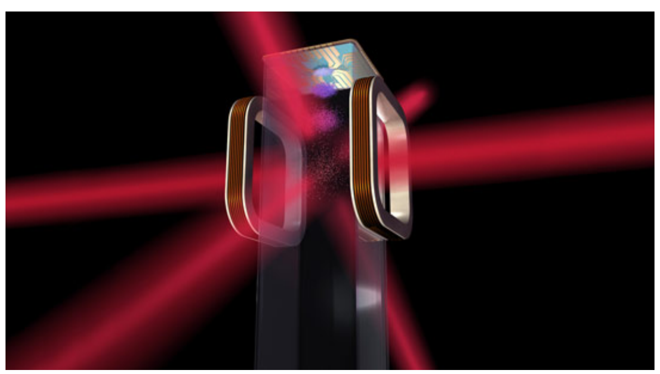
How about Physical Review E ? ・・・Topological soft matter, Topological origami

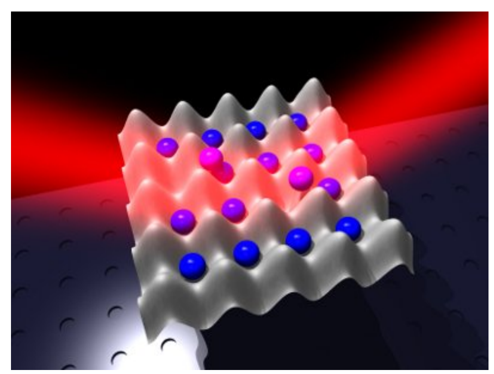
What is AMO?

AMO physics studies atoms, molecules, and light using laser

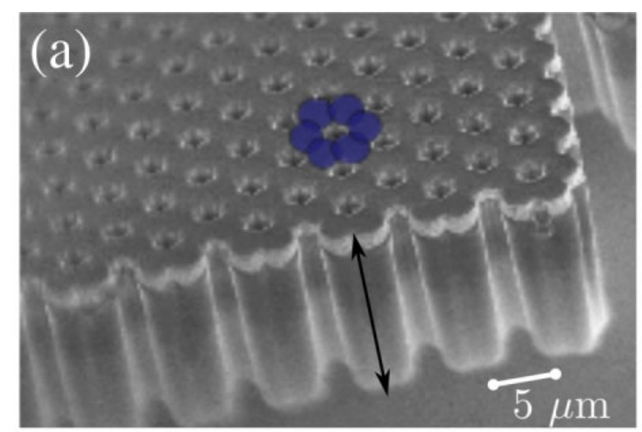
High controllability of system parameters allows one to realize various Hamiltonians

- ultracold atoms

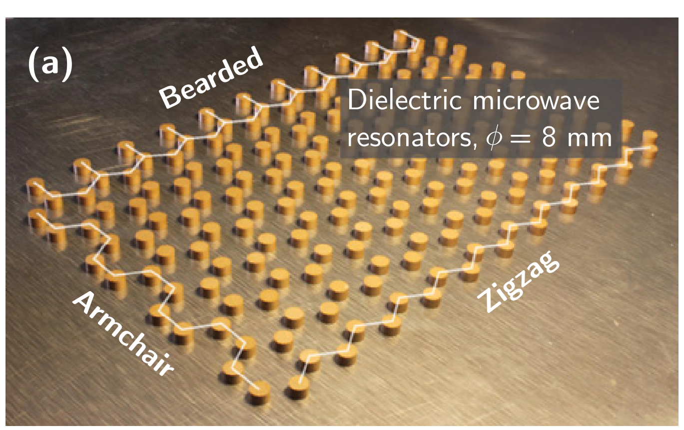


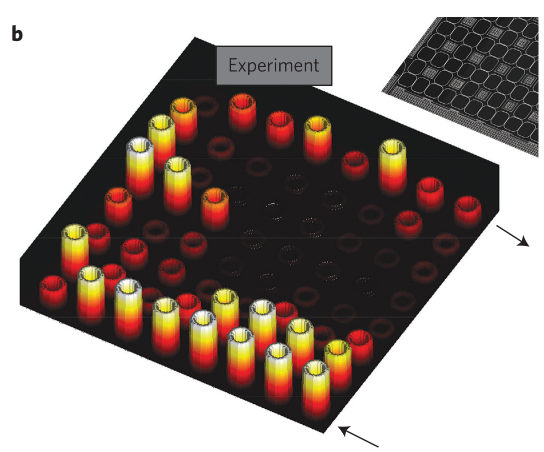


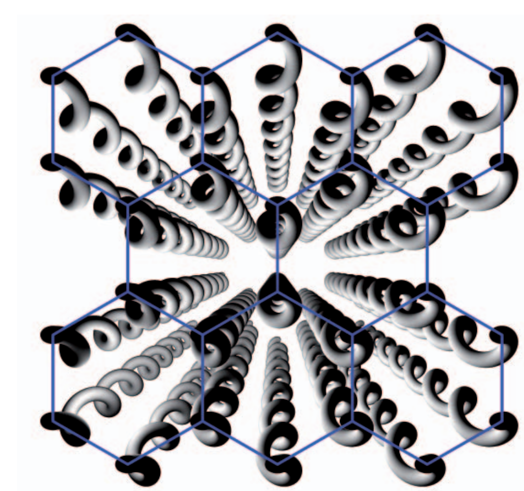
- exciton-polaritons



- photonics







Outline

1. Topological physics in ultracold atomic gases
2. Topological physics in photonics
3. Synthetic dimensions and higher dimensional topological effects

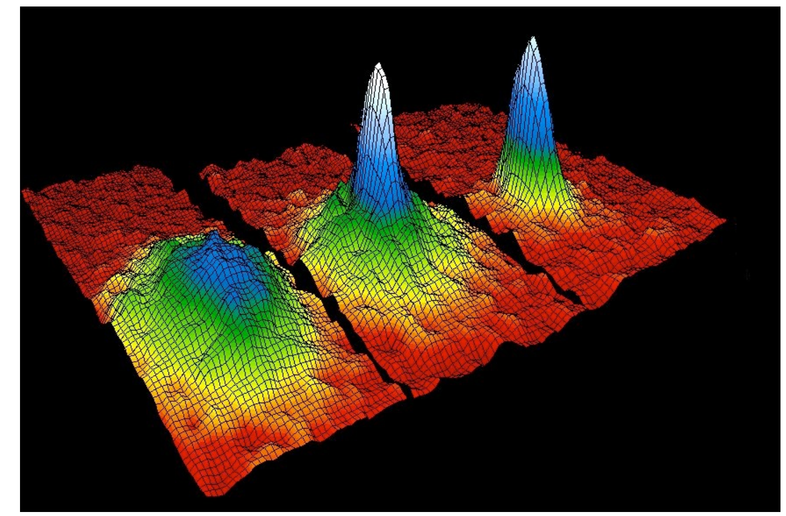
Quantum simulation - ultracold atoms

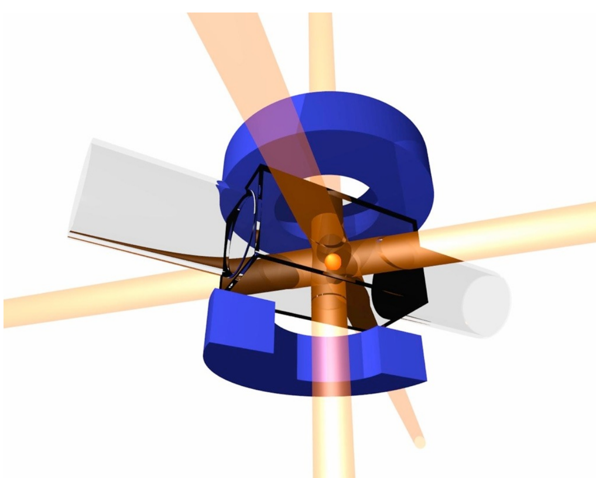
1995 : Realization of BEC - Colorado, Rice, MIT

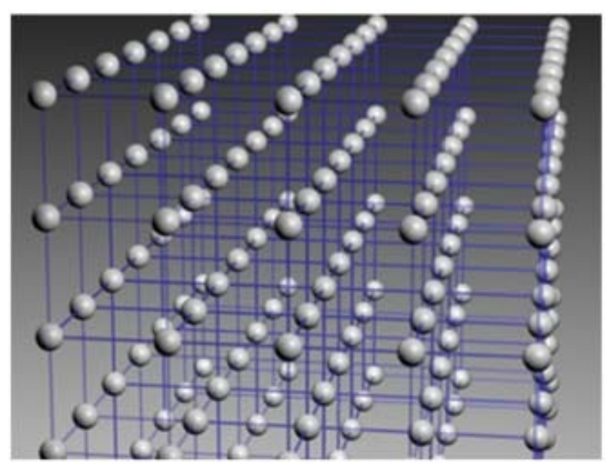
1999 : Realization of degenerate Fermi gas - Colorado

1998, 2004 : Feshbach resonance - MIT, Colorado

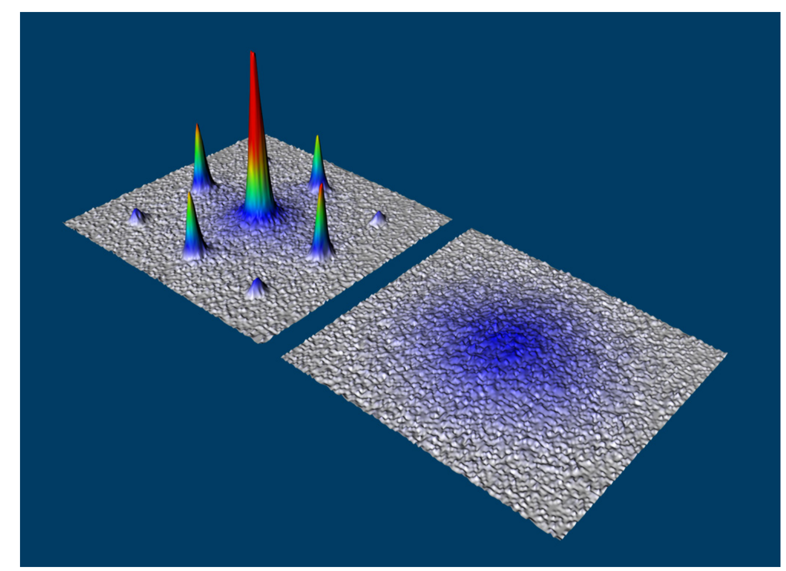
2002 : Superfluid - Mott insulator transition - Max-Planck







[Bloch, Nat. Phys. **1**, 23 (2005)]



[Bloch group website @ MPQ, Munich]

Extreme controllability of the system:

Can choose bosons, fermions, or both

Can change interaction, underlying confinement (trap, lattice, box, etc…), spins, number of species, density, temperature, dimensionality, etc…

Quantum simulation of topological models?

How can one simulate topologically nontrivial models?

How can one simulate quantum Hall effect?

How can one simulate an effect of a magnetic field at all?

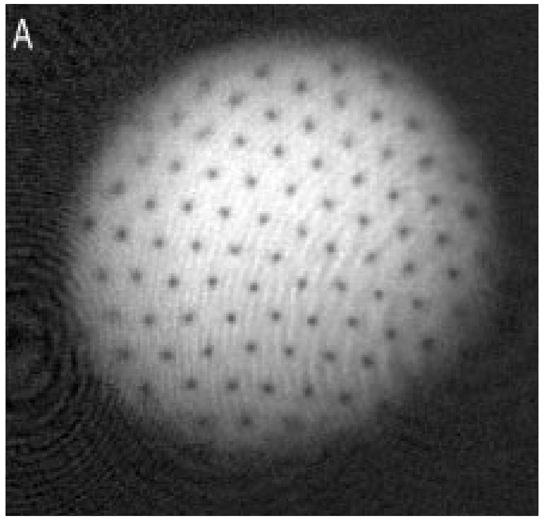
$$\frac{p^2}{2m} \longrightarrow \frac{(\mathbf{p} - e\mathbf{A})^2}{2m} \quad ?$$

Example: rotate the system

$$\frac{p^2}{2m} - \mathbf{\Omega} \cdot \mathbf{L} = \frac{1}{2m}\left(\mathbf{p} - m\mathbf{\Omega} \times \mathbf{r}\right)^2 - \frac{1}{2}m\left(\mathbf{\Omega} \times \mathbf{r}\right)^2$$

($m\mathbf{\Omega} \times \mathbf{r}$) corresponds to $e\mathbf{A}$

Equivalent to having an effective magnetic field $\mathbf{B} = \nabla \times \mathbf{A} = \frac{2m}{e}\mathbf{\Omega}$



Artificial magnetic field

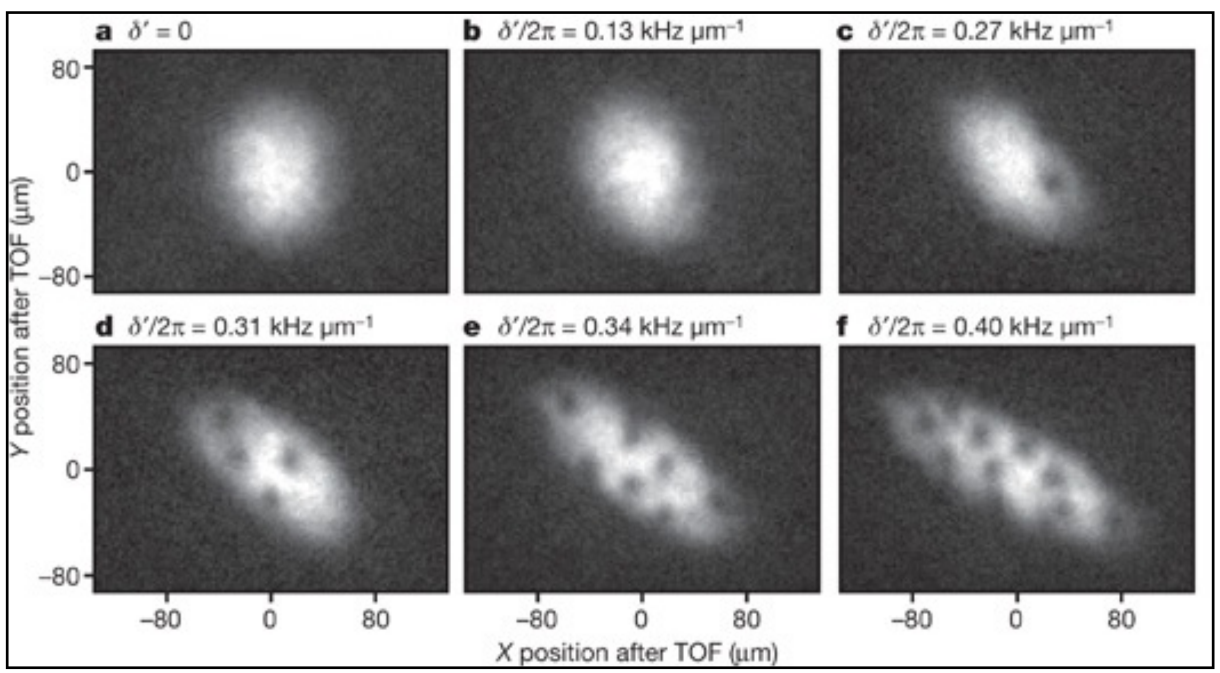
Consider when the internal degrees of freedom of an atom $|\chi(\mathbf{r})\rangle$ depends on position

The total state is $\psi(\mathbf{r},t)|\chi(\mathbf{r})\rangle$ where $\psi(\mathbf{r},t)$ is the center-of-mass wavefunction

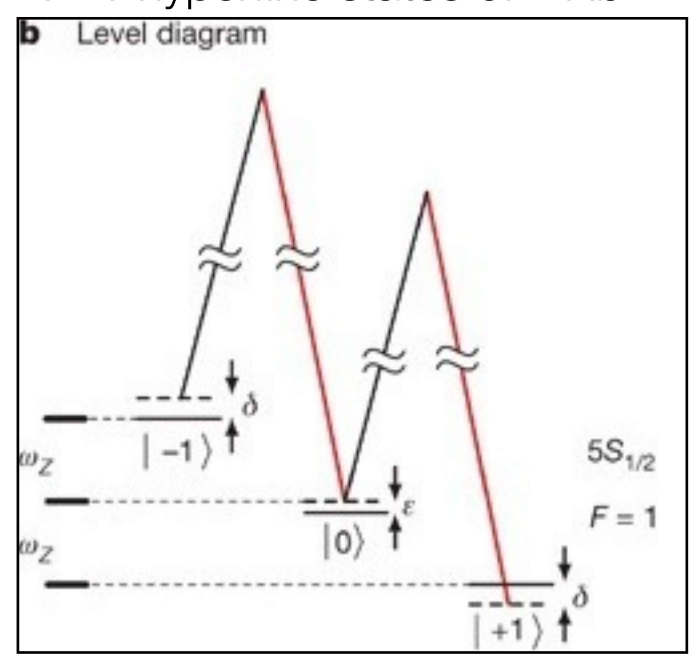
Assuming that the center-of-mass motion is adiabatic enough so that one stays in $|\chi(\mathbf{r})\rangle$

$$i\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \frac{1}{2m}\left[-i\nabla - \underbrace{i\langle\chi(\mathbf{r})|\nabla|\chi((r))\rangle}_{\equiv \mathcal{A}(\mathbf{r})}\right]^2 \psi(\mathbf{r},t) + \tilde{V}(\mathbf{r})\psi(\mathbf{r},t)$$

$\equiv \mathcal{A}(\mathbf{r})$: Berry connection ← Acts as an artificial magnetic field



F=1 hyperfine states of ^{87}Rb



Lin, Compton, Jiménez-García, Porto, and Spielman (NIST), Nature **462**, 628 (2009)

Artificial magnetic fields on lattice

In the presence of a periodic potential, when the lattice is sufficiently deep

$$\frac{p^2}{2m} + V(\mathbf{r}) \quad \Rightarrow \quad -J \sum_{<i,j>} c_j^\dagger c_i + h.c.$$

tight-binding model

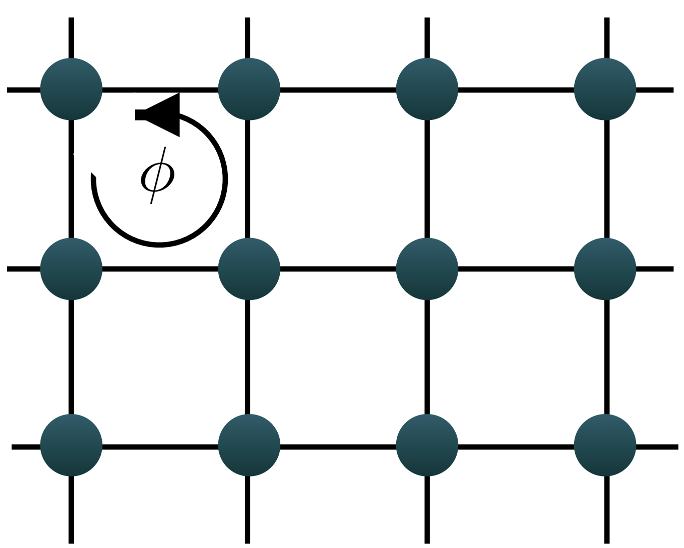
In the presence of a magnetic field

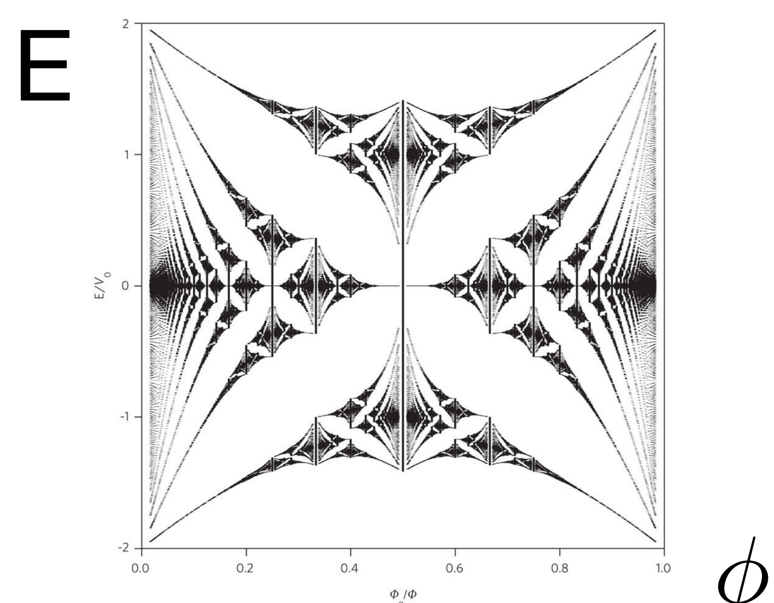
Peierls phase

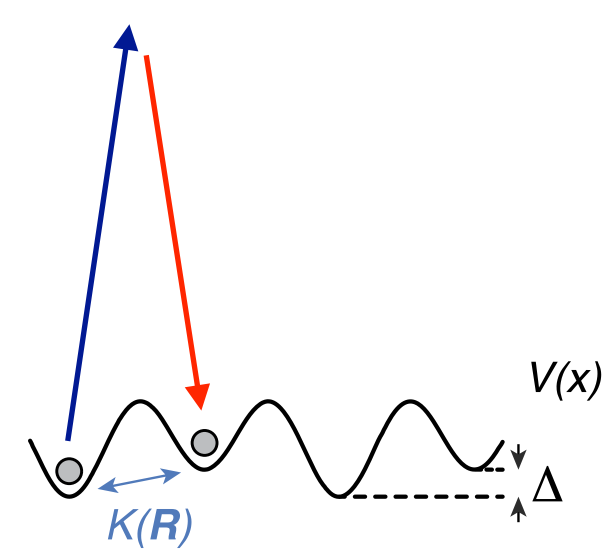
$$\frac{(\mathbf{p} - \mathbf{A})^2}{2m} + V(\mathbf{r}) \quad \Rightarrow \quad -J \sum_{<i,j>} e^{i \int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathbf{A} \cdot d\mathbf{r}} c_j^\dagger c_i + h.c.$$

Magnetic field appears as the Peierls phase in tight-binding models

Harper-Hofstadter model:

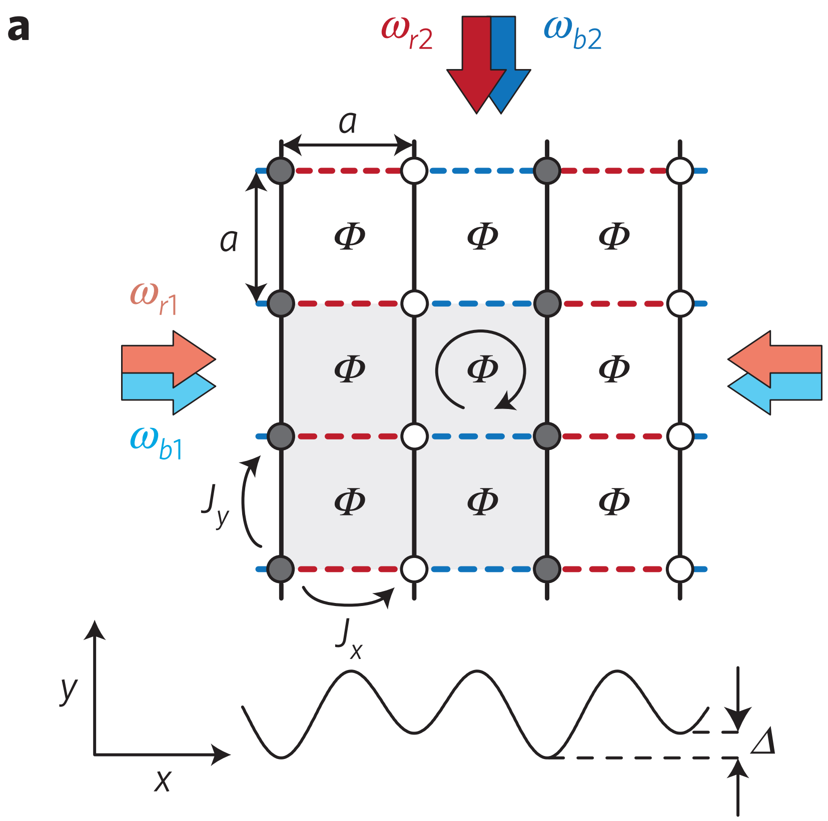






Topological lattices in ultracold atoms

- Harper-Hofstadter model



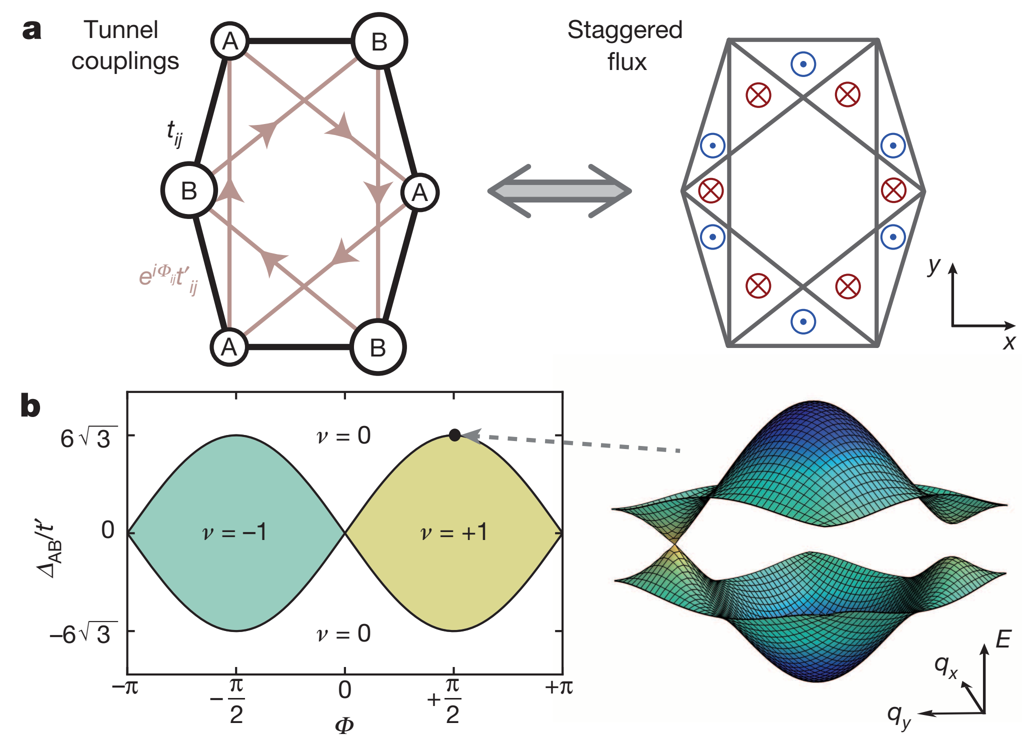
Bloch group @ Munich

Aidelsburger *et al*., PRL **111**, 185301 (2013)
Aidelsburger *et al*., Nature Physics **11**, 162 (2015)

Ketterle group @ MIT

Miyake *et al*., PRL **111**, 185302 (2013)
Kennedy *et al*., Nature Physics **11**, 859 (2015)

- Haldane model



Esslinger group @ ETH

Jotzu *et al*., Nature **515**, 237 (2014)

Topological physics with ultracold gases

- Measurement of Chern number

  Aidelsburger *et al*. (Munich), Nature Physics **11**, 162 (2015).

- Measurement of Zak phase, Berry phase

  Atala et al. (Munich), Nature Physics **9**, 795 (2013); Duca *et al*. (Munich), Science **347**, 288 (2015)

- Detection of chiral edge state

  Mancini *et al* (Florence)., Science **349**, 1510 (2015); Stuhl et al (Maryland)., Science **349**, 1514 (2015).

- Measurement of Berry curvature

  Li *et al*. (Munich), Science **352**, 1094 (2016); Fläschner, *et al*. (Hamburg), Science **352**, 1091 (2016).

- Realization of Su-Schrieffer-Heeger model

  Meier *et al*. (Urbana), Nature communications **7**, 13986 (2016).

- Topological charge pumping

  Nakajima, *et al*. (Kyoto), Nature Physics **12**, 296 (2016); Lohse *et al*. (Munich), Nature Physics **12**, 350 (2016).

- Observation of quantized circular dichroism

  Asteria *et al*. (Hamburg), arXiv:1805.11077.

Review: Cooper, Dalibard, & Spielman, "Topological Bands for Ultracold Atoms," arXiv:1803.00249

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1. Topological physics in ultracold atomic gases
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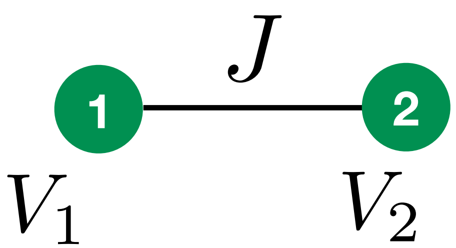
Band structure can be classically realized

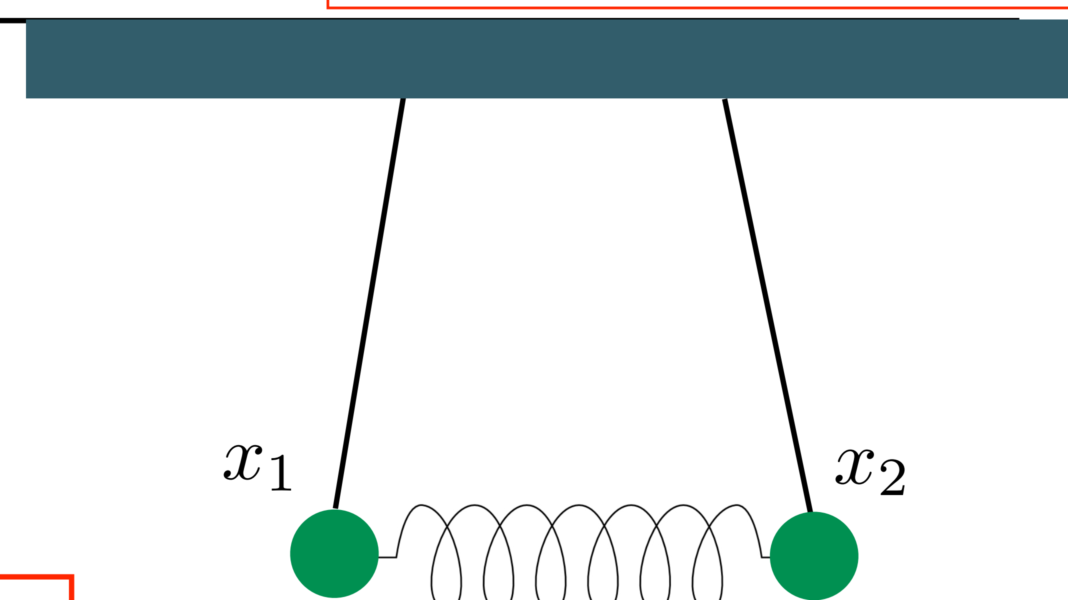
Tight-binding model can be realized classically. For example, consider a two-site model



$$\hat{H} = J\hat{c}_2^\dagger \hat{c}_1 + J\hat{c}_1^\dagger \hat{c}_2 + V_1 \hat{c}_1^\dagger \hat{c}_1 + V_2 \hat{c}_2^\dagger \hat{c}_2$$
$$= \begin{pmatrix} \hat{c}_1^\dagger & \hat{c}_2^\dagger \end{pmatrix} \begin{pmatrix} V_1 & J \\ J & V_2 \end{pmatrix} \begin{pmatrix} \hat{c}_1 \\ \hat{c}_2 \end{pmatrix}$$

Eigen-energies are determined by the eigenvalues of this matrix

Consider two pendula coupled via a spring



$$m\frac{d^2 x_1}{dt^2} = -m\omega_1^2 x_1 + \kappa(x_2 - x_1)$$
$$m\frac{d^2 x_2}{dt^2} = -m\omega_2^2 x_2 + \kappa(x_1 - x_2)$$

$$\frac{d^2}{dt^2}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\omega_1^2 - \kappa/m & \kappa/m \\ \kappa/m & -\omega_2^2 - \kappa/m \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Eigenfrequencies are determined by the eigenvalues of this matrix

Optical resonators and tight-binding model

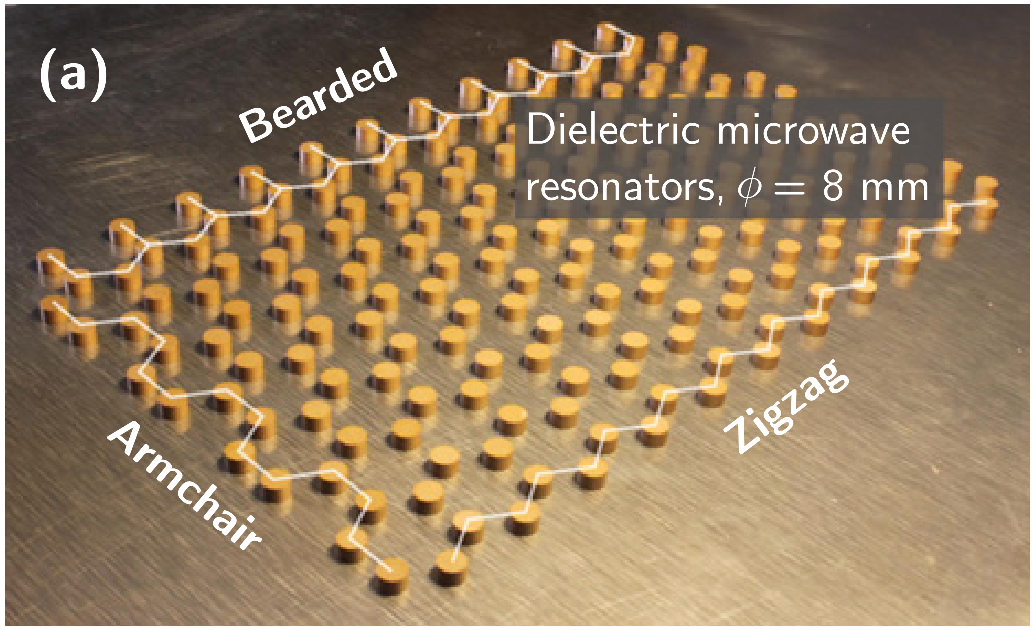
Tight-binding models naturally appear in photonic resonators

Assume each resonator hosts localized mode $\mathbf{E}_0(\mathbf{r})$

Align resonators in positions $\mathbf{R}_i$ to form a lattice

The total electromagnetic field can be written as

$$\mathbf{E}(\mathbf{r},t) = \sum_{\mathbf{R}_i} a_i(t)\mathbf{E}_0(\mathbf{r}-\mathbf{R}_i)$$



The coefficients $a_i(t)$ evolve in time with suitable coupling constants:

$$i\frac{\partial a_i(t)}{\partial t} = -\sum_{\mathbf{R}_j} t_{ij} a_j(t)$$

This is exactly the Heisenberg equation of motion of "quantum mechanical" tight-binding model

$$\hat{H} = -\sum_{i,j} t_{ij}\hat{a}_i^\dagger \hat{a}_j$$

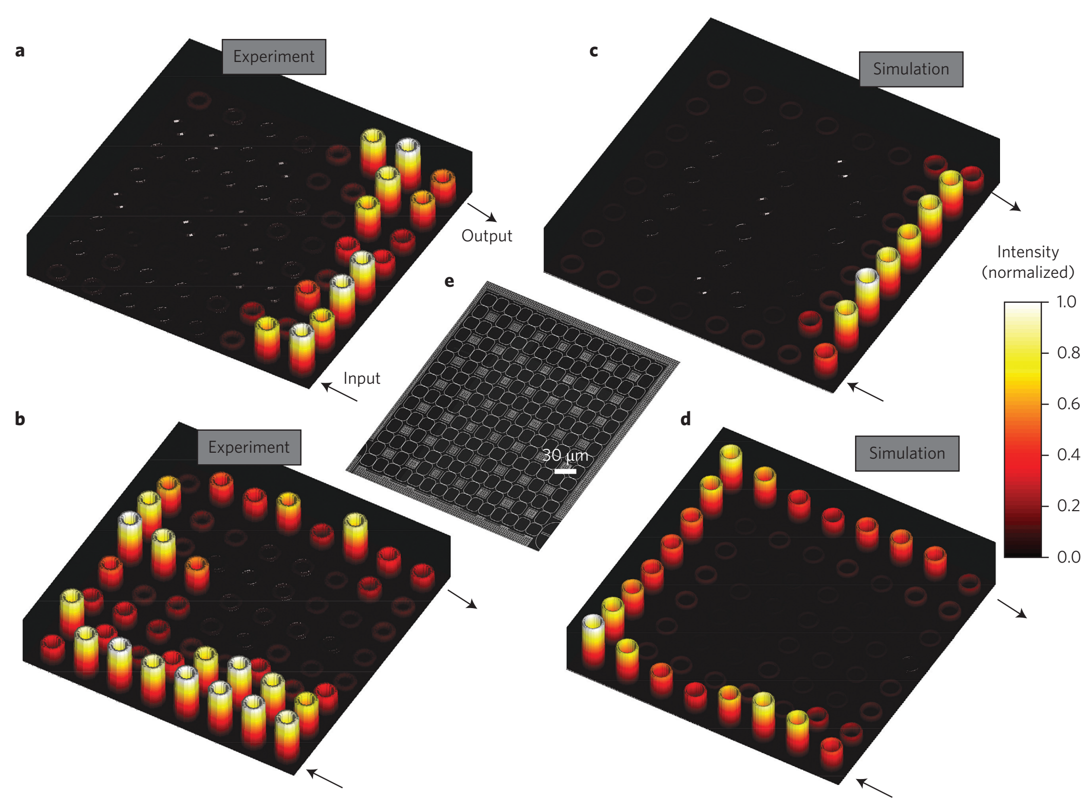
Harper-Hofstadter model with light

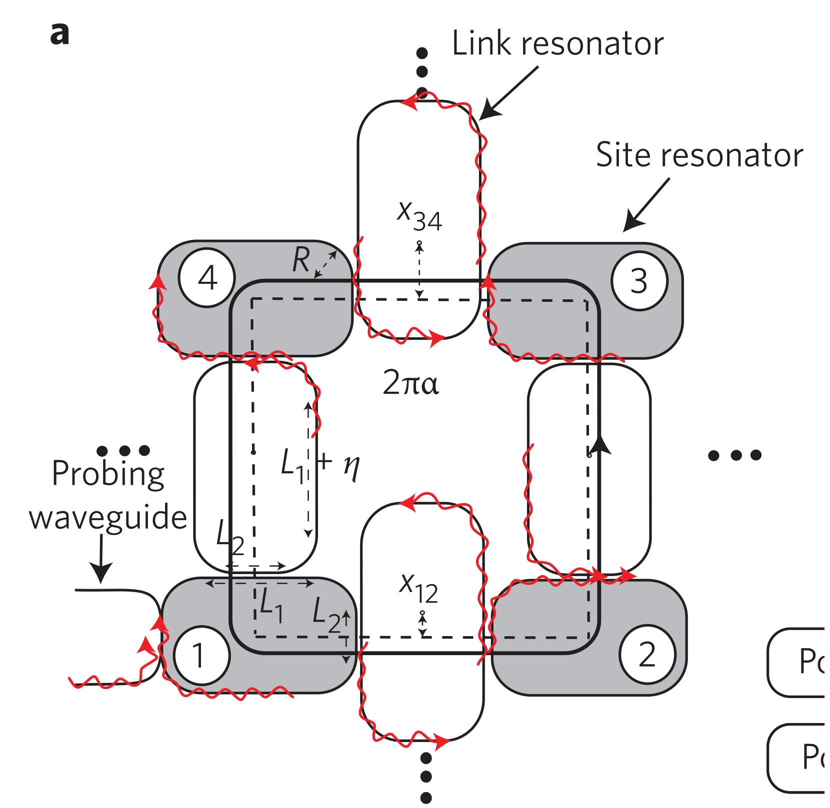
Hafezi, et al. (JQI), Nature Photonics **7**, 907 (2011).

nature photonics
ARTICLES
PUBLISHED ONLINE: 20 OCTOBER 2013 | DOI: 10.1038/NPHOTON.2013.274

Imaging topological edge states in silicon photonics

M. Hafezi*, S. Mittal, J. Fan, A. Migdall and J. M. Taylor



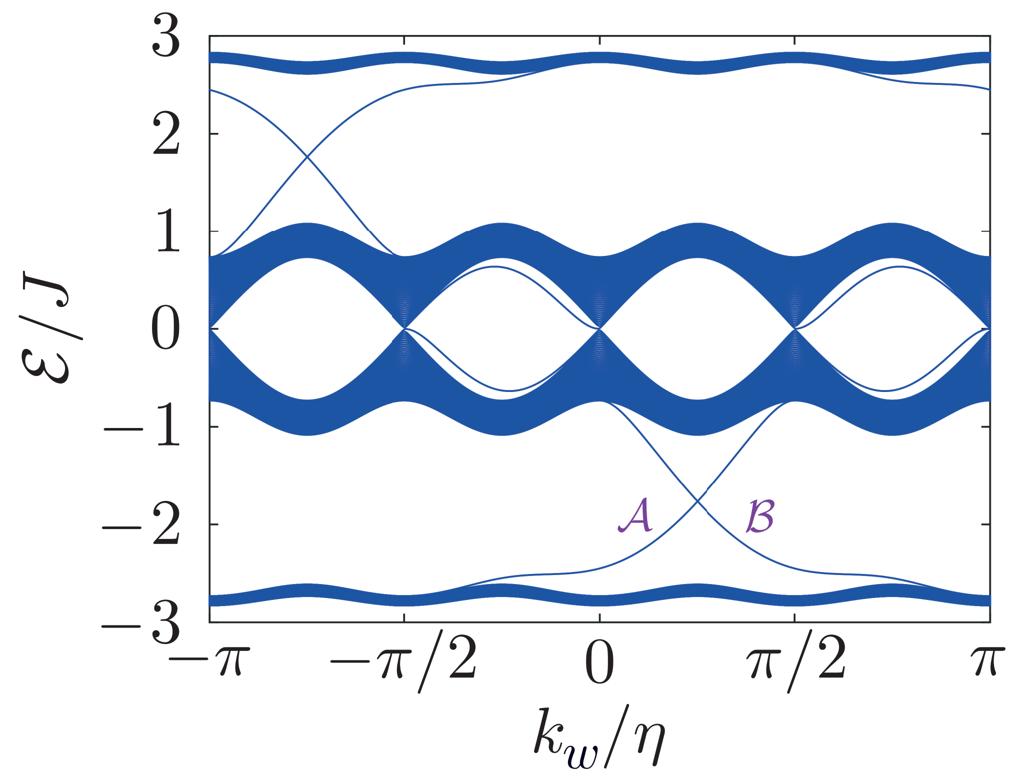


Quantum Hall effect with drive and dissipation

Since photonic systems have dissipation, (Hall) current is usually not a good quantity to look at.

Instead, one can look at the **steady-state** reached as a result of drive and dissipation

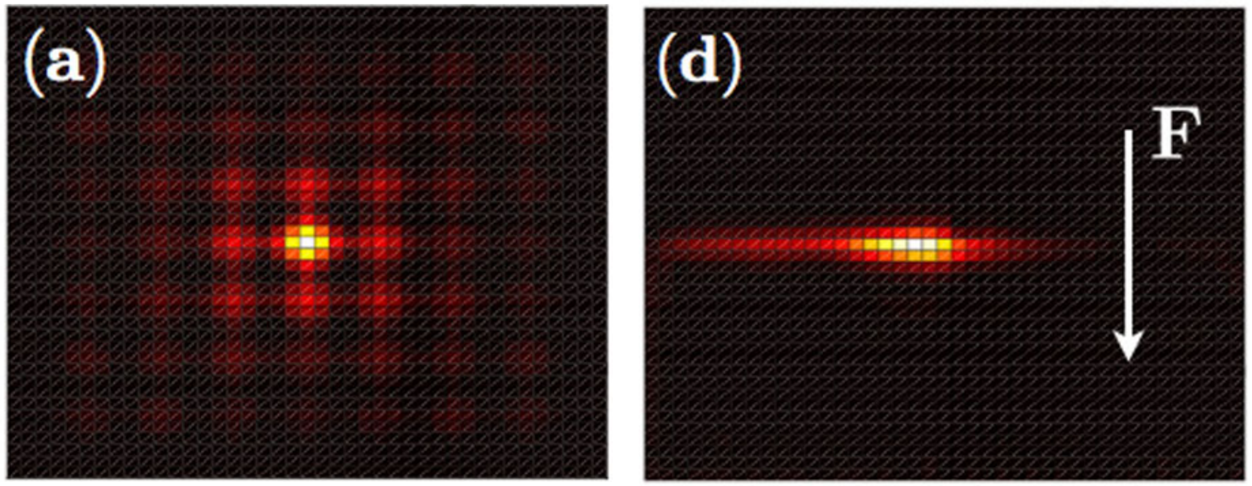
Harper-Hofstadter model (ϕ = 1/4)



Chern number External force

$$\langle x \rangle \approx \frac{2\pi \mathcal{C}_1 F}{A_{\mathrm{BZ}} \gamma}$$

Brillouin zone area Loss



TO & Carusotto, PRL **112**, 133902 (2014)

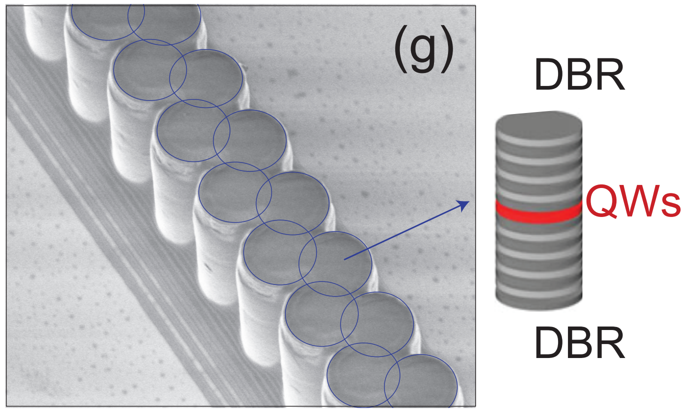
Topological laser

St-Jean, et al. (Marcoussis), Nature Photonics **11**, 651 (2017)

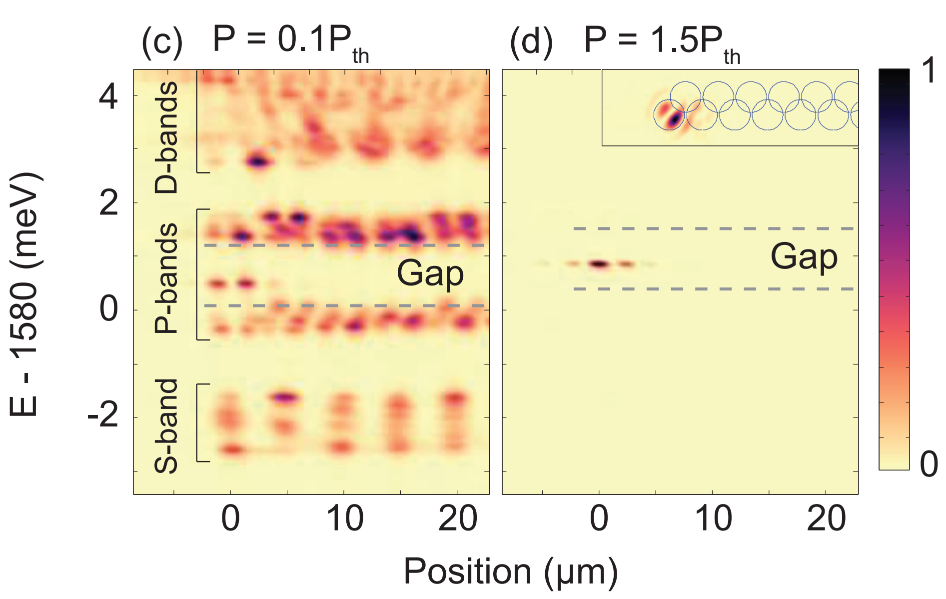
nature photonics
ARTICLES
DOI: 10.1038/s41566-017-0006-2

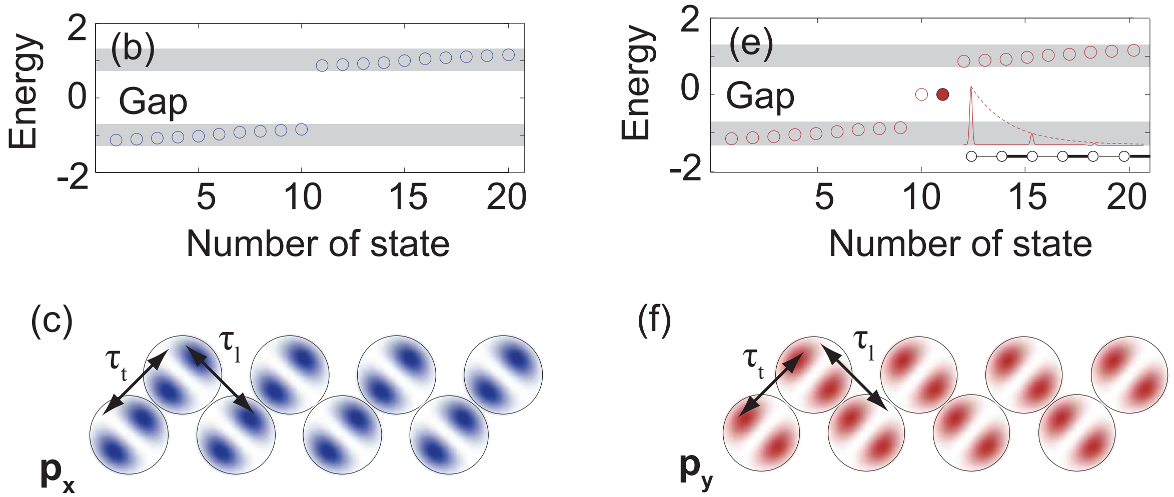
Lasing in topological edge states of a one-dimensional lattice

P. St-Jean[1]*, V. Goblot[1], E. Galopin[1], A. Lemaître[1], T. Ozawa[2], L. Le Gratiet[1], I. Sagnes[1], J. Bloch[1] and A. Amo[1]



- Lasing in the topological edge state of SSH model in exciton-polariton micropillars





2D Topological laser

Bahari, et al. (UC San Diego), Science **358**, 636 (2017)

Science

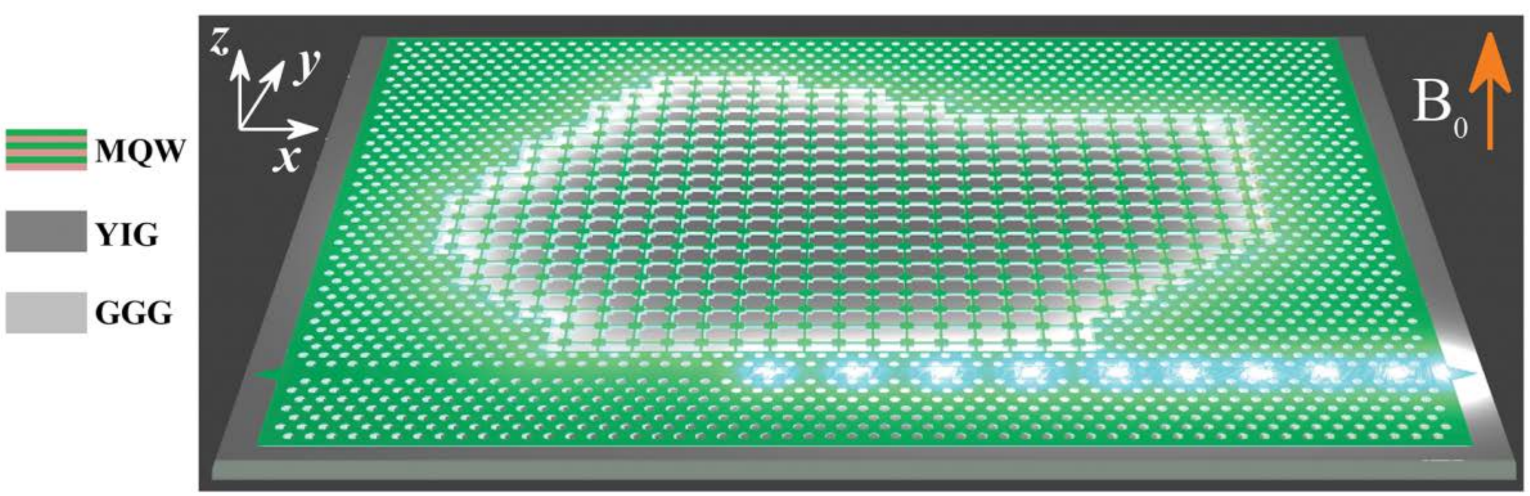
REPORTS

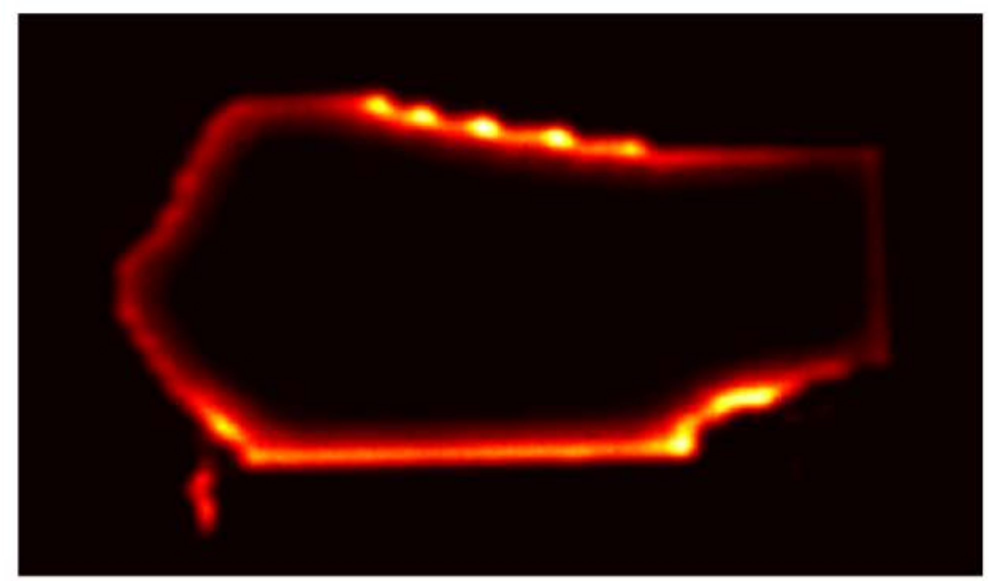
Cite as: B. Bahari *et al.*, *Science* 10.1126/science.aao4551 (2017).

Nonreciprocal lasing in topological cavities of arbitrary geometries

Babak Bahari, Abdoulaye Ndao, Felipe Vallini, Abdelkrim El Amili, Yeshaiahu Fainman, Boubacar Kanté*

Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093, USA.





Topological physics with photons

✓Detection of chiral edge state

✓Landau levels of photons

✓Measurement of Zak phase & Berry curvature

✓Realization of anomalous Floquet topological insulators

✓Realization of Su-Schrieffer-Heeger model

✓Observation of three-dimensional Weyl dispersion

✓Topological charge pumping

- Bosons instead of fermions
- More control on realizing various Hamiltonians
- Photons are lossy; sometimes one needs non-Hermitian Hamiltonians

Review: Ozawa *et al*. “Topological Photonics,” arXiv:1802.04173

Outline

1. Topological physics in ultracold atomic gases
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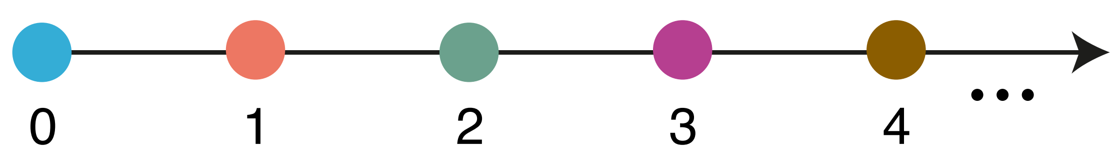
Synthetic dimensions

Simulate higher-dimensions by regarding internal degrees of freedom as dimensions

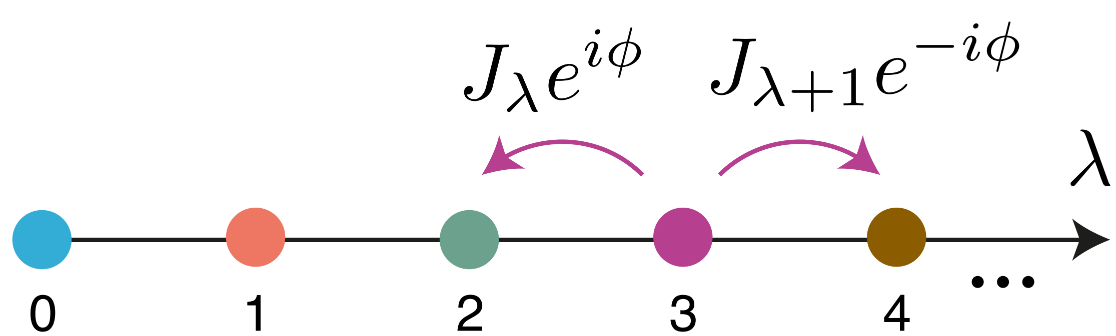
- Choose degrees of freedom you want to use as synthetic dimensions

Example : Hyperfine degrees of freedom of ultracold atoms

Modes of photons in resonators



- Induce hopping (kinetic energy) along the synthetic dimension



Celi *et al.*, PRL **112**, 043001 (2014)

Experimental realization in ultracold gases

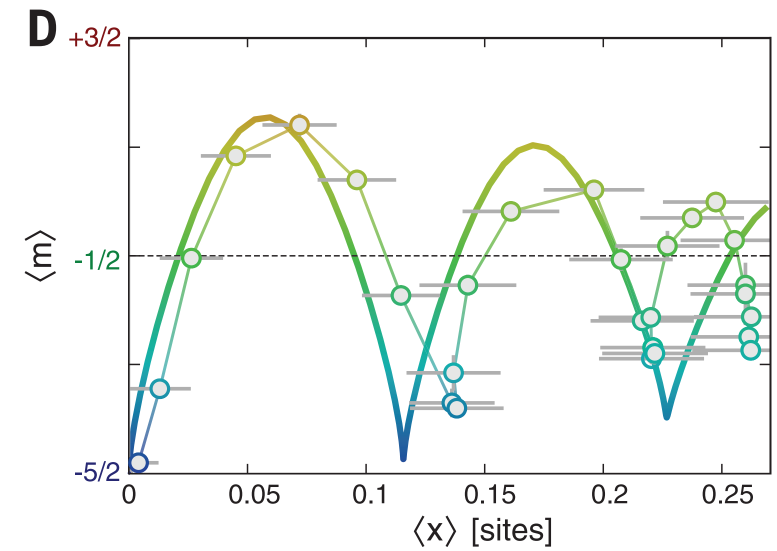
Florence (Fallani & Inguscio) - ^{173}Yb (fermion)

Mancini et al., Science **349**, 1510 (2015); Livi, et al., PRL **117**, 220401 (2016)

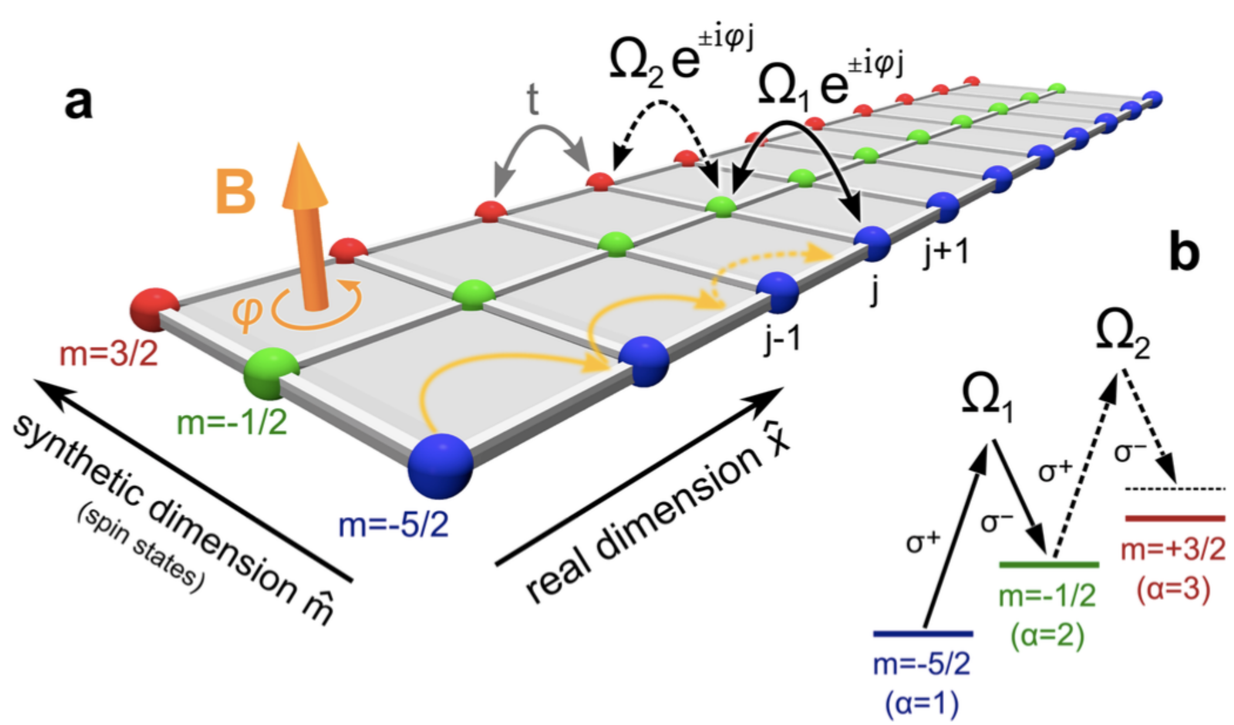
Maryland (Spielman) - ^{87}Rb (boson)

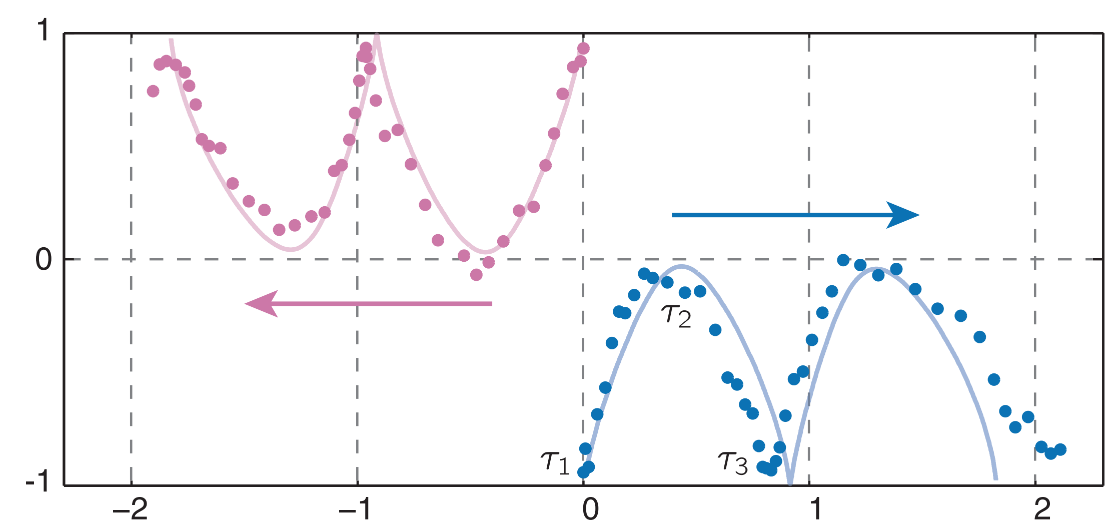
Stuhl et al., Science **349**, 1514 (2015)

- Use hyperfine degrees of freedom as a synthetic dimension
- Three sites along the synthetic direction
- Chiral propagation of edge states observed



Florence





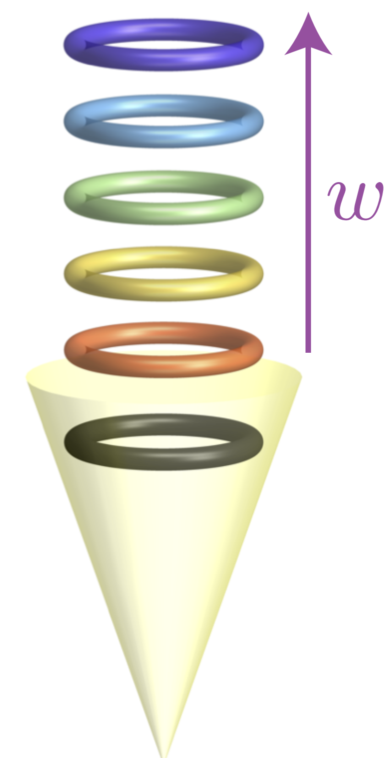
Maryland

Synthetic dimensions with photons

Synthetic dimensions with photons in a ring resonator

- Use different angular momentum modes as a synthetic dimension
- Couple modes via external modulations of refractive index

  [cf. Yuan, *et al.*, Opt. Lett. **41**, 741 (2016)])

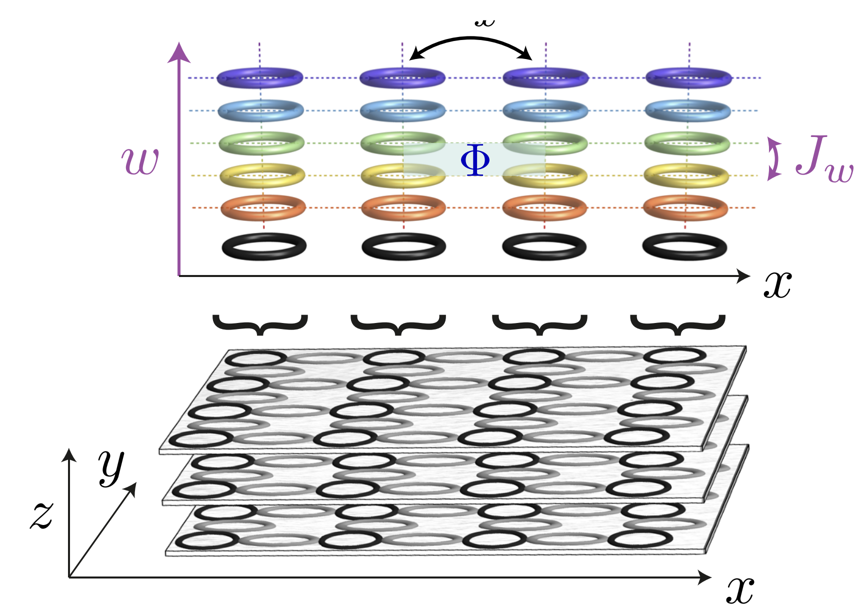


The resulting single-site effective Hamiltonian:

$$H = -\sum_{w} \mathcal{J} e^{i\theta} b^{\dagger}_{w+1} b_w + h.c.$$

- 1D tight-binding Hamiltonian with hopping phases -

Spatially aligning resonators, one can build up to 4D Hamiltonian

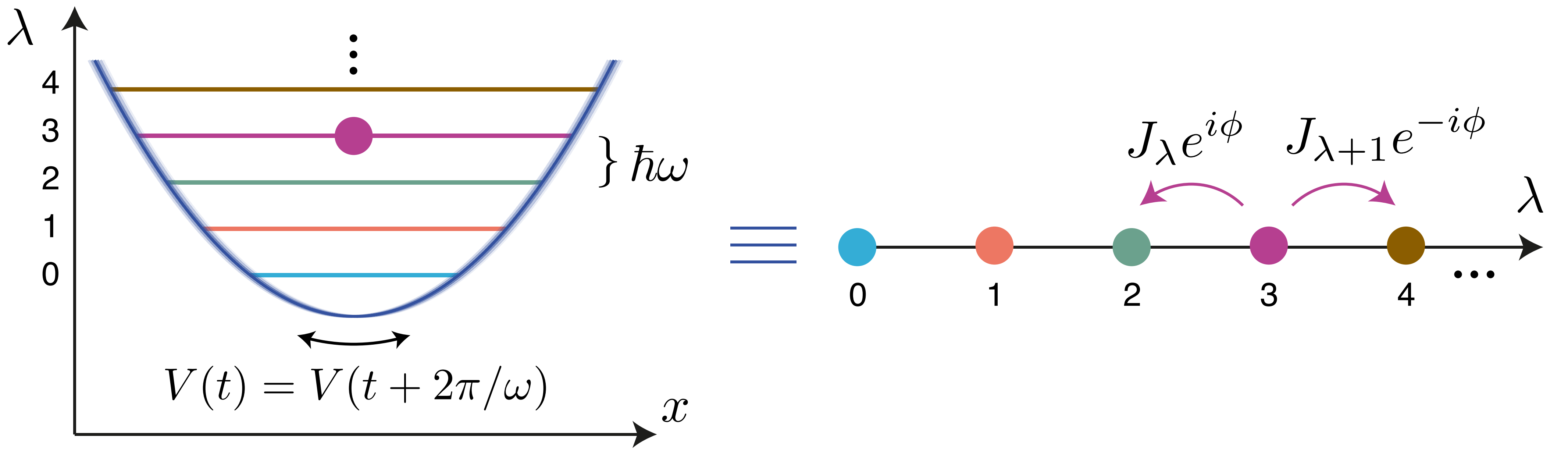


TO, Price, Goldman, Zilberberg, Carusotto, PRA **93**, 043827 (2016)

TO & Carusotto, PRL **118**, 013601 (2017)

Price, TO, & Goldman, PRA **95**, 023607 (2017)

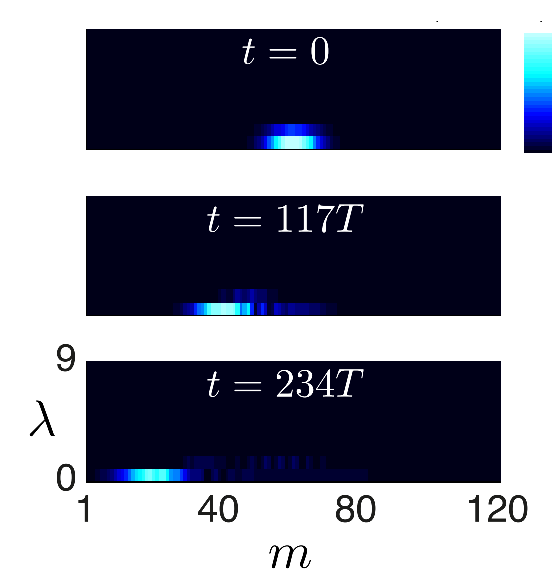
Harmonic potential eigenstates as synthetic dimensions



- Hopping among different states can be introduced by shaking the lattice
- In principle, one can simulate up to 6D (3 real dimensions + 3 harmonic potential directions)

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 = \sum_{\lambda=0}^{\infty} \omega\lambda |\lambda\rangle\langle\lambda|$$

$$H = H_0 + V(t) \to \sum_{\lambda} \kappa \sqrt{\frac{\lambda}{8m\omega}} \left(|\lambda - 1\rangle\langle\lambda| e^{i\phi} + h.c.\right)$$



Price, TO, & Goldman, PRA **95**, 023607 (2017)

cf. Lustig, *et al*., arXiv:1807.01983 for photonic realization

Four-dimensional quantum Hall effect

Quantum Hall effect occurs in any even dimensions (2, 4, 6, etc…), and characterized by the n-th Chern number: $\mathcal{C}_n$

cf. Sugawa et al., Science **360**, 1429 (2018)

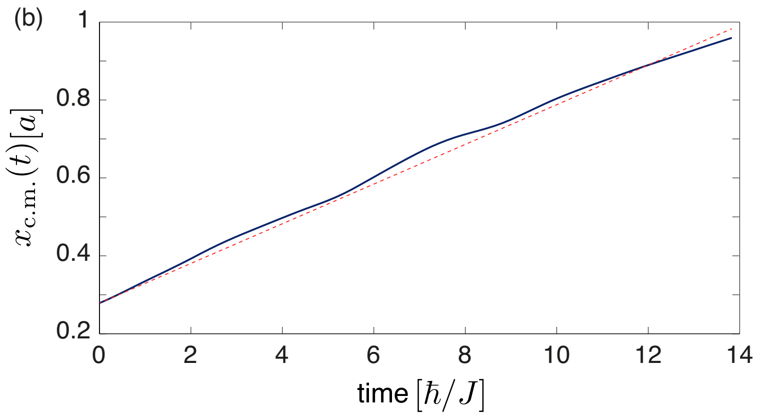
2D: $j^y = -\frac{e^2}{h}\mathcal{C}_1 E_x$

4D: $j^w = -\frac{e^3}{h^2}\mathcal{C}_2 E_x B_{yz}$

4D quantum Hall effect can be explored with synthetic dimensions

$$\mathcal{H} = -J \sum_{x,y,z,w} \left(c^\dagger_{\mathbf{r}+\hat{e}_x} c_\mathbf{r} + c^\dagger_{\mathbf{r}+\hat{e}_y} c_\mathbf{r} + e^{iB_{xz}x} c^\dagger_{\mathbf{r}+\hat{e}_z} c_\mathbf{r} + e^{iB_{yw}y} c^\dagger_{\mathbf{r}+\hat{e}_w} c_\mathbf{r} + \text{h.c.} \right)$$

Simulated wavepacket dynamics in the above Hamiltonian to look for 4D quantum Hall effect



Extracted 2nd Chern number = -0.98

Cold atom: Price, Zilberberg, TO, Carusotto, Goldman, PRL **115**, 195303 (2015) ; PRB **93**, 245113 (2016)

Photonics: TO, Price, Goldman, Zilberberg, Carusotto, PRA **93**, 043827 (2016)

cf. Charge pumping: Lohse et al. (Munich), Nature **553**, 55 (2018); Zilberberg et al. (Penn State), Nature **553**, 59 (2018)

Summary

- Atomic, molecular, and optical systems provide powerful platforms to explore topological physics
- Ultracold gases are good for exploring many-particle and quantum properties
- Photons are good for exploring single-particle non-equilibrium properties
- They are both often bosons and often lossy

- Interaction effects?
- Non-Hermitian topological physics?
- Higher dimensional topology?

Review: Cooper, Dalibard, & Spielman, "Topological Bands for Ultracold Atoms," arXiv:1803.00249
Review: Ozawa *et al*. "Topological Photonics," arXiv:1802.04173
Both accepted for publication in Rev. Mod. Phys.