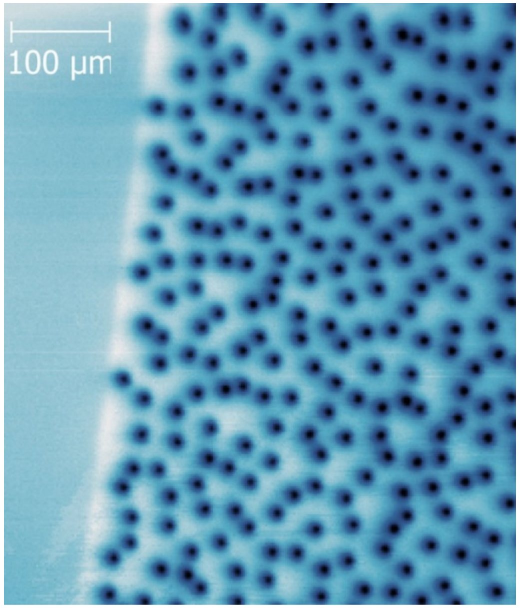
Non-Abelian vortex in lattice gauge theory

Arata Yamamoto (U. Tokyo)

AY, PTEP 2018, 103B03 (2018)

Introduction

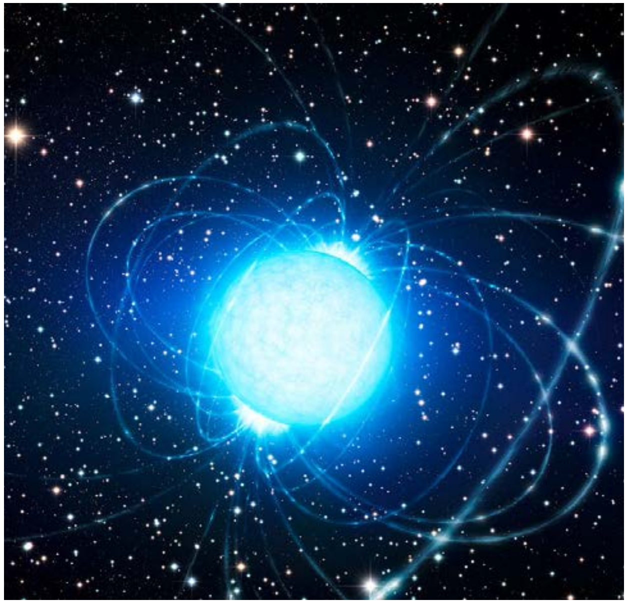
condensed matter



Wells, Pan, Wang, Fedoseev, Hilgenkamp (2015)

superconductors

QCD



https://en.wikipedia.org/wiki/Magnetar

neutron stars

Introduction

condensed matter

Cooper pair of electrons

condensation

U(1)

QCD

Cooper pair of quarks

"diquark"

color-flavor locking

$SU(3)_{C\text{-}F} \times U(1)/Z_3$

Introduction

condensed matter	QCD
Abelian vortex	non-Abelian vortex
$Q = 1, 2, 3 \cdots$	$Q = \frac{1}{3}, \frac{2}{3}, 1 \cdots$

Introduction

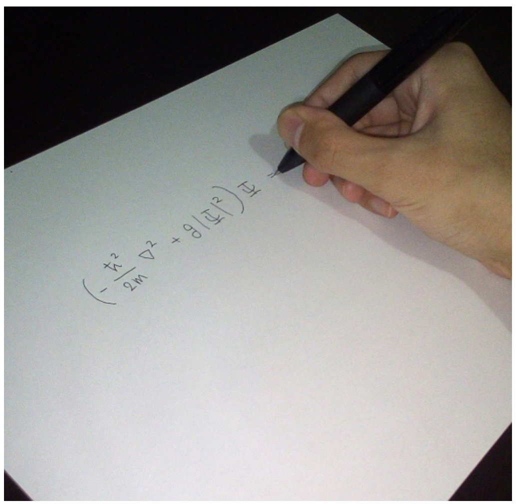
condensed matter	QCD
Abelian vortex	non-Abelian vortex
$Q = 1, 2, 3 \cdots$	$Q = \frac{1}{3}, \frac{2}{3}, 1 \cdots$

e.g. $\langle \phi \rangle \propto \begin{pmatrix} e^{i\theta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Introduction

semi-classical level

full-quantum level



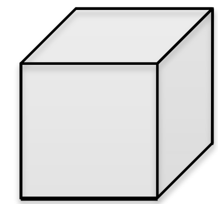
http://jcahpc.jp/pr/pr-20171115.html

mean-field analysis

lattice gauge theory

Setup

$$Z = \int \mathcal{D}\phi \mathcal{D}\phi^* \mathcal{D}A \; e^{-S}$$



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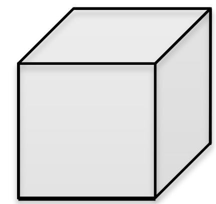
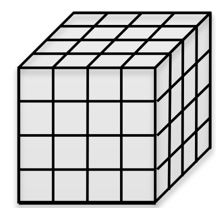


↓ discretize

$$Z = \int D\phi D\phi^* DA \; e^{-S_{\mathrm{lattice}}}$$



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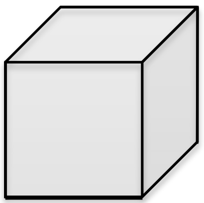

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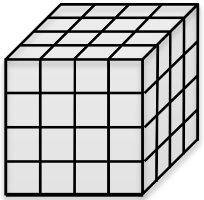
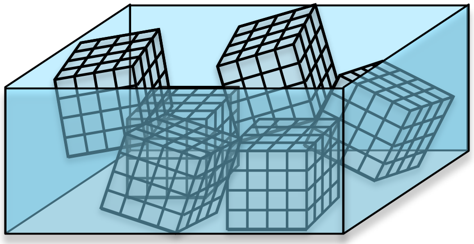


↓ Monte Carlo

$$Z = \sum_{\text{ensemble}} e^{-S_{\text{lattice}}}$$



Setup

non-Abelian Higgs model

ϕ_i :$SU(3)_F \times SU(3)_C \times U(1)$ scalar field

A_μ^α :dynamical $SU(3)_C$ gauge field

A_μ^0 :external $U(1)$ gauge field

Setup

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$$S = \int d^d x \left[\frac{1}{4} F_{\mu\nu}^\alpha F_{\mu\nu}^\alpha + (D_\mu \phi_i)^\dagger (D_\mu \phi_i) + V[\phi] \right]$$

$SU(3)_C$ field strength

scalar field potential

$SU(3)_C \times U(1)$ covariant derivative

Setup

non-Abelian Higgs model

ϕ_i :anti-triplet diquarks

A_μ^α :dynamical gluons

A_μ^0 :external magnetic field in neutron stars

$$S = \int d^d x \left[\frac{1}{4} F_{\mu\nu}^\alpha F_{\mu\nu}^\alpha + (D_\mu \phi_i)^\dagger (D_\mu \phi_i) + V[\phi] \right]$$

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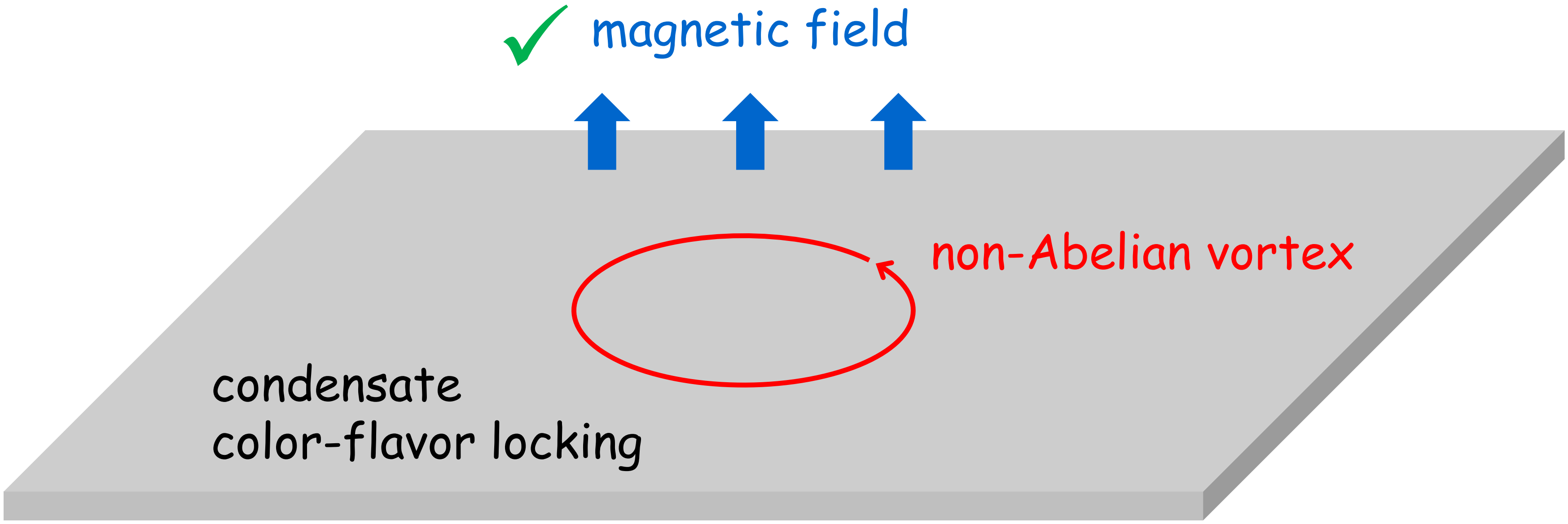
Phase structure



Phase structure

$$V[\phi] = -m^2 \phi_i^\dagger \phi_i + \lambda(\phi_i^\dagger T^0 \phi_i)^2 + \nu(\phi_i^\dagger T^\alpha \phi_i)^2$$

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↓

~~U(1)~~

condensate

$$\Delta \overset{?}{=} \left\{ \lim_{|x-y|\to\infty} \langle \phi_i^\dagger(x)\phi_i(y)\rangle \right\}^{1/2}$$

gauge dependent

Phase structure

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$$\Delta = \left\{ \lim_{|x-y|\to\infty} \langle H^\dagger(x) H(y) \rangle \right\}^{1/2}$$

$$H(x) = \frac{1}{3!}\epsilon_{ijk}\epsilon_{abc}\phi_{ai}(x)\phi_{bj}(x)\phi_{ck}(x)$$

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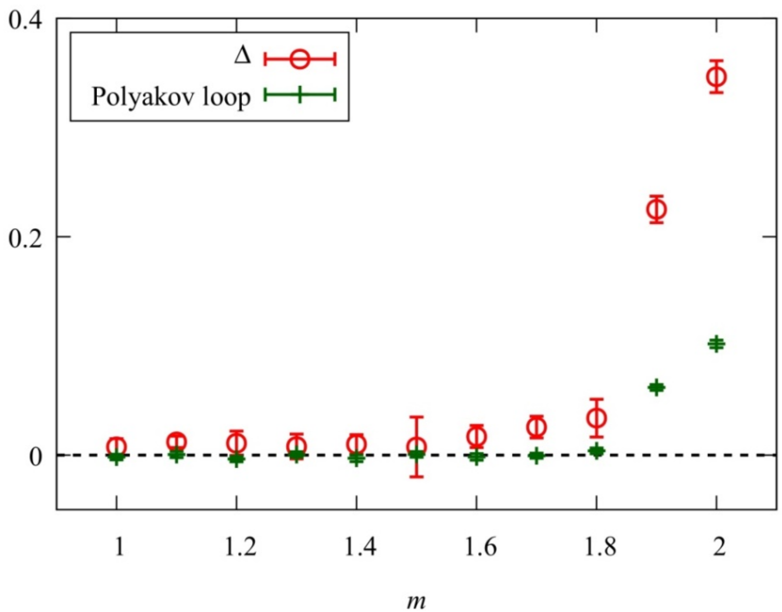

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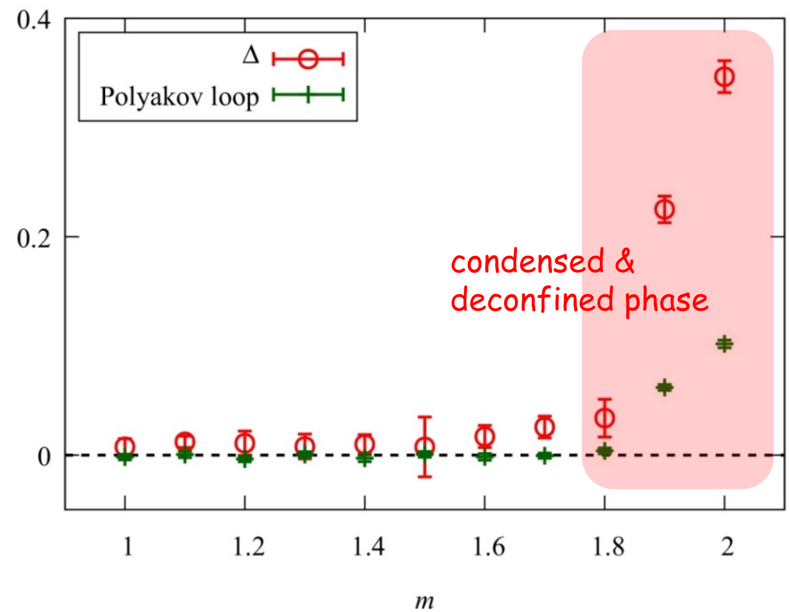

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~~U(1)~~ ~~$SU(3)_{C\text{-}F}$~~

color-flavor locking

color / flavor

$$\begin{pmatrix} \langle\phi_{ru}\rangle & \langle\phi_{rd}\rangle & \langle\phi_{rs}\rangle \\ \langle\phi_{gu}\rangle & \langle\phi_{gd}\rangle & \langle\phi_{gs}\rangle \\ \langle\phi_{bu}\rangle & \langle\phi_{bd}\rangle & \langle\phi_{bs}\rangle \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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~~U(1)~~ ~~SU(3)$_{C\text{-}F}$~~

color-flavor locking

$$\Gamma_{ij} = \left\{ \lim_{|x-y|\to\infty} \langle G_{ij}^\dagger(x) G_{ij}(y)\rangle \right\}^{1/2}$$

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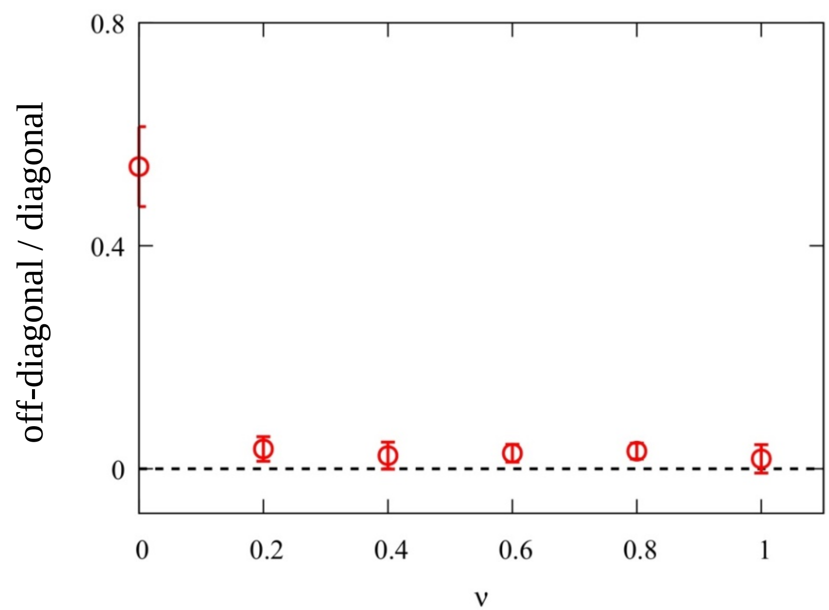

Phase structure

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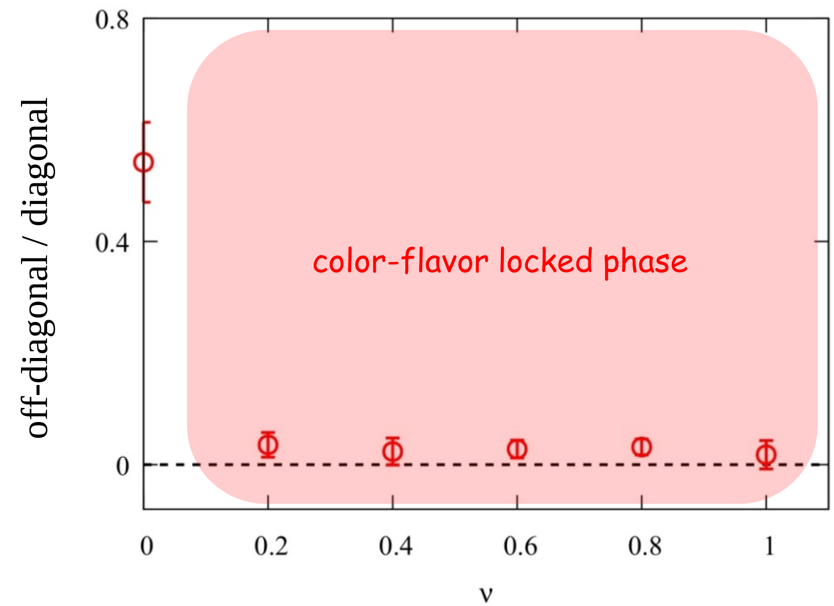
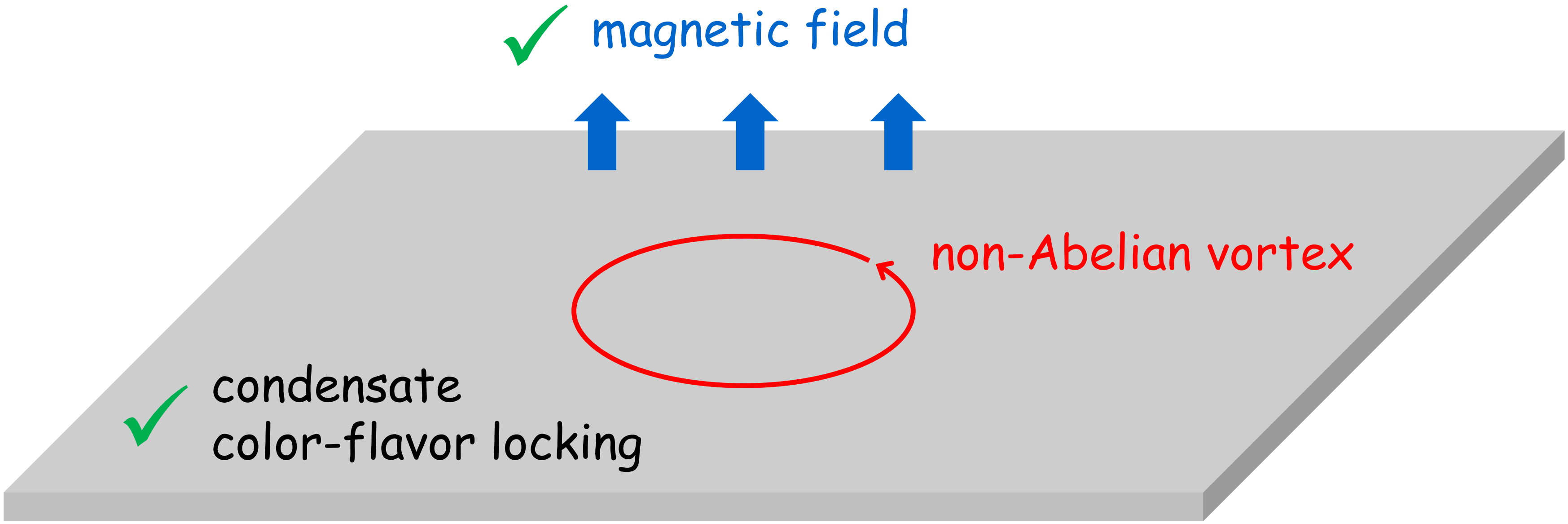


Non-Abelian vortex



Non-Abelian vortex

phase variable

$$\theta(x) \stackrel{?}{=} \arg\left[\phi_i(x)\right]$$

gauge dependent

Non-Abelian vortex

phase variable Alford, Baym, Fukushima, Hatsuda, Tachibana (2018)

$$\theta(x) = \frac{1}{3}\arg\left[H(x)\right]$$

$$H(x) = \frac{1}{3!}\epsilon_{ijk}\epsilon_{abc}\phi_{ai}(x)\phi_{bj}(x)\phi_{ck}(x)$$

Non-Abelian vortex

phase variable   Alford, Baym, Fukushima, Hatsuda, Tachibana (2018)

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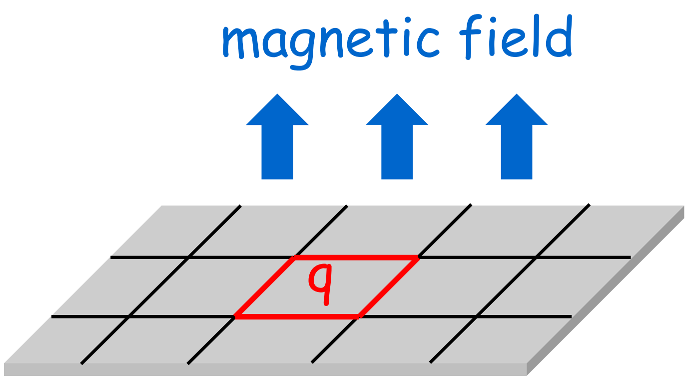
vortex number density

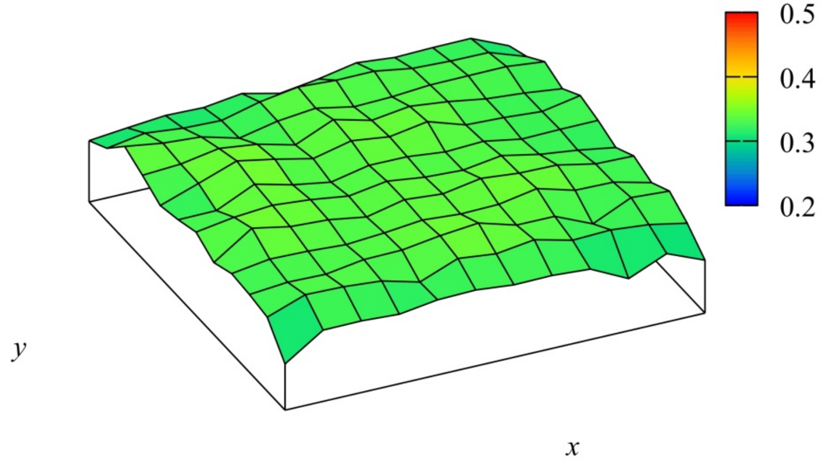
$$q = \frac{1}{2\pi}\oint \delta\theta(x)dx$$



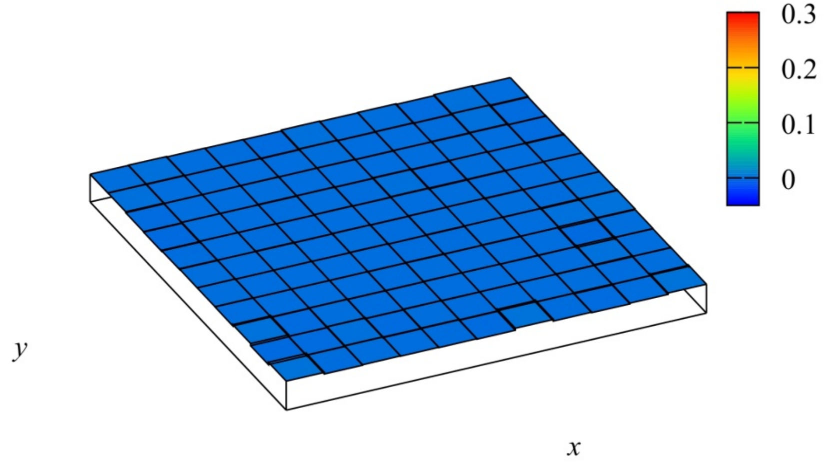
Non-Abelian vortex

$$eB = 0$$

condensate



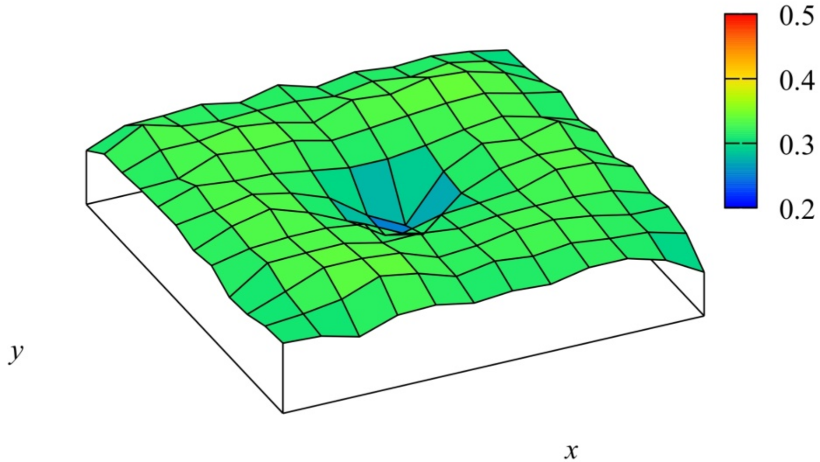
vortex number density



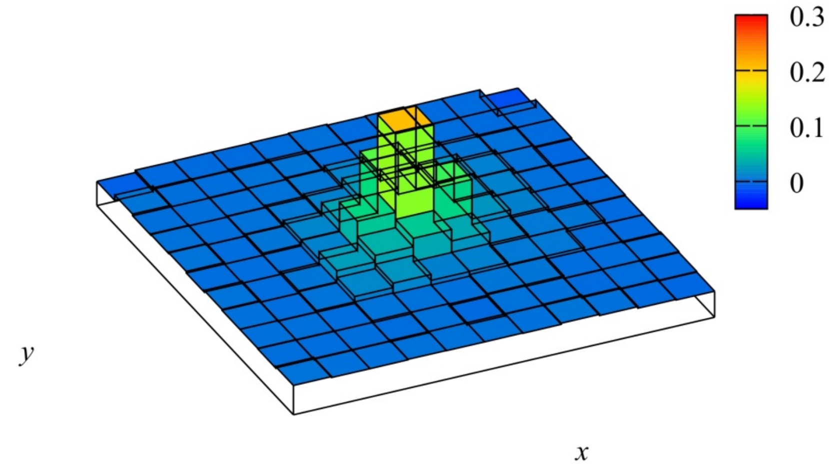
Non-Abelian vortex

$$eB = 0.06$$

condensate



vortex number density

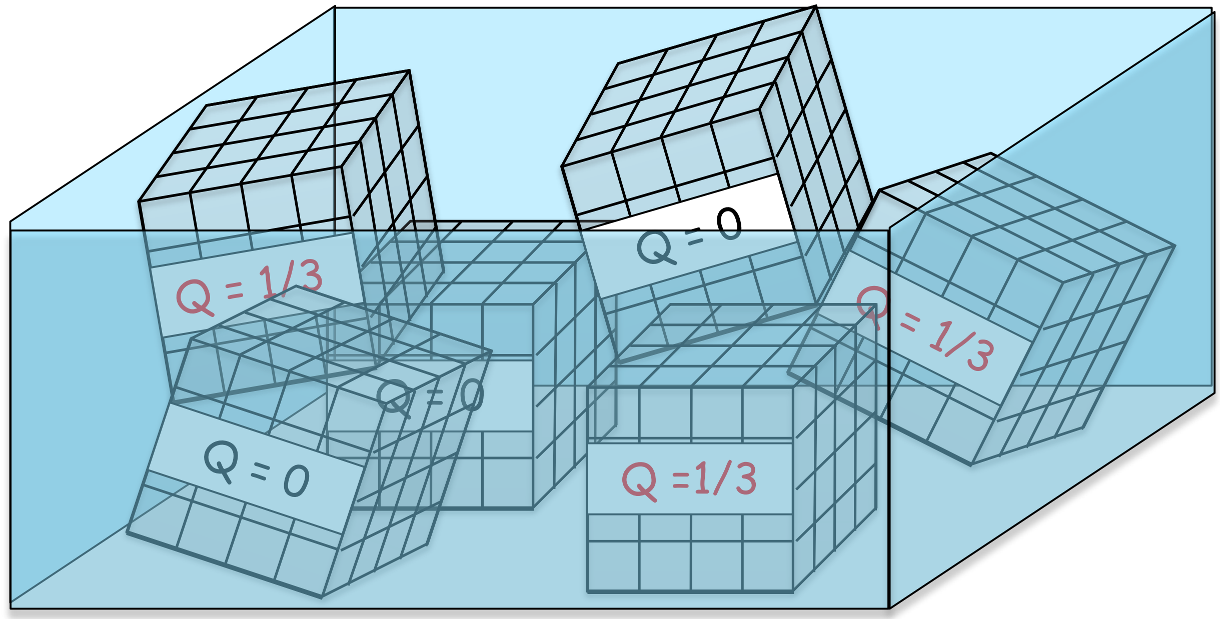


$$\langle Q \rangle = \sum_{\text{plaquette}} \langle q \rangle \simeq \frac{1}{3}$$

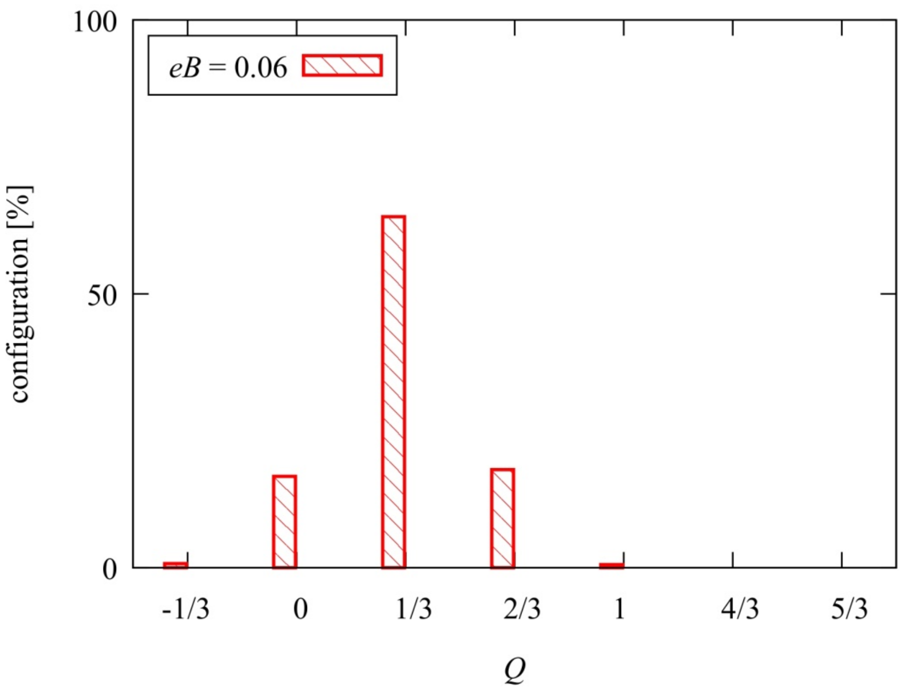
Non-Abelian vortex

Monte Carlo

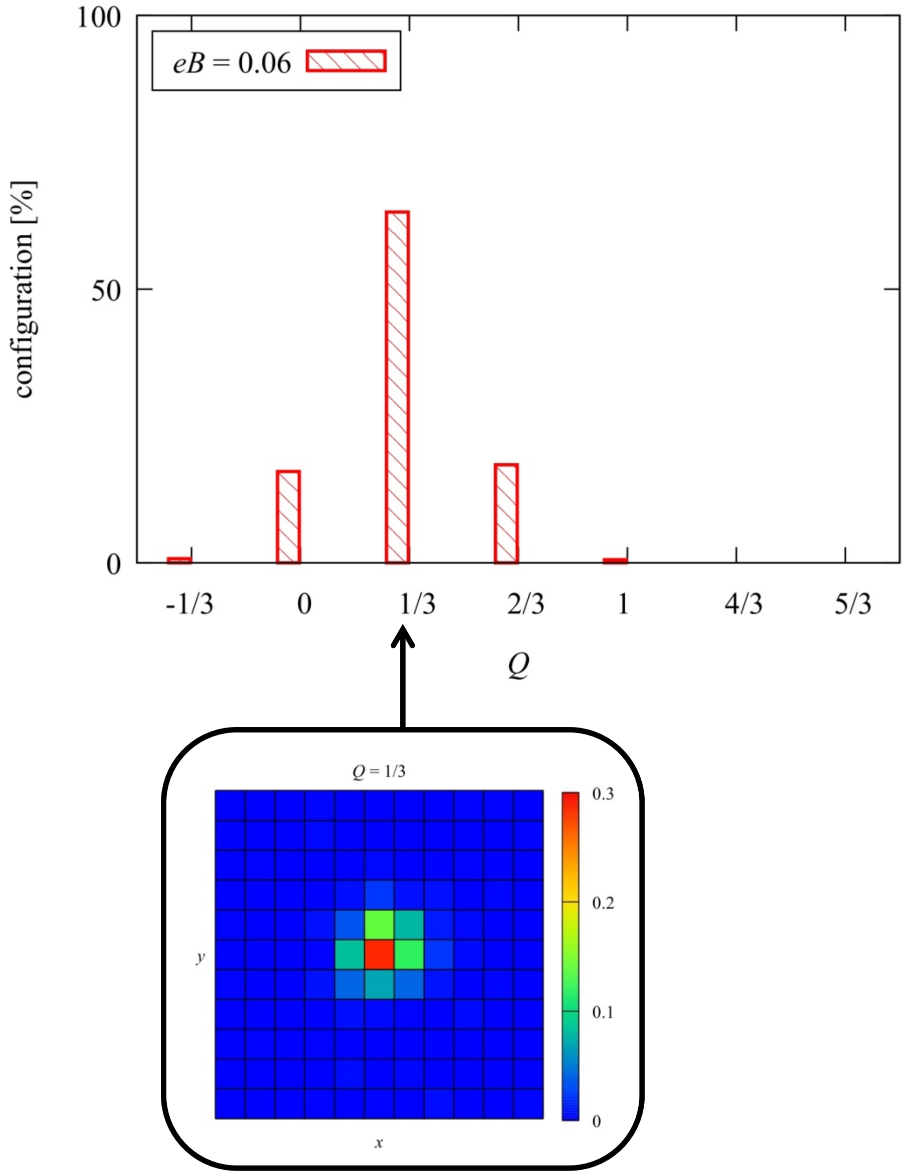
$Z =$



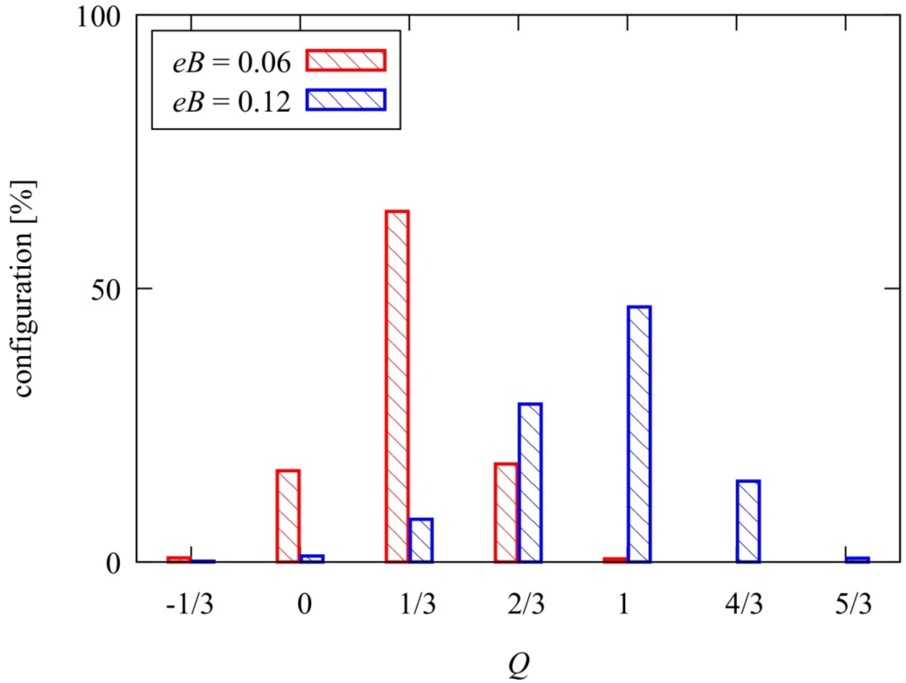
Non-Abelian vortex



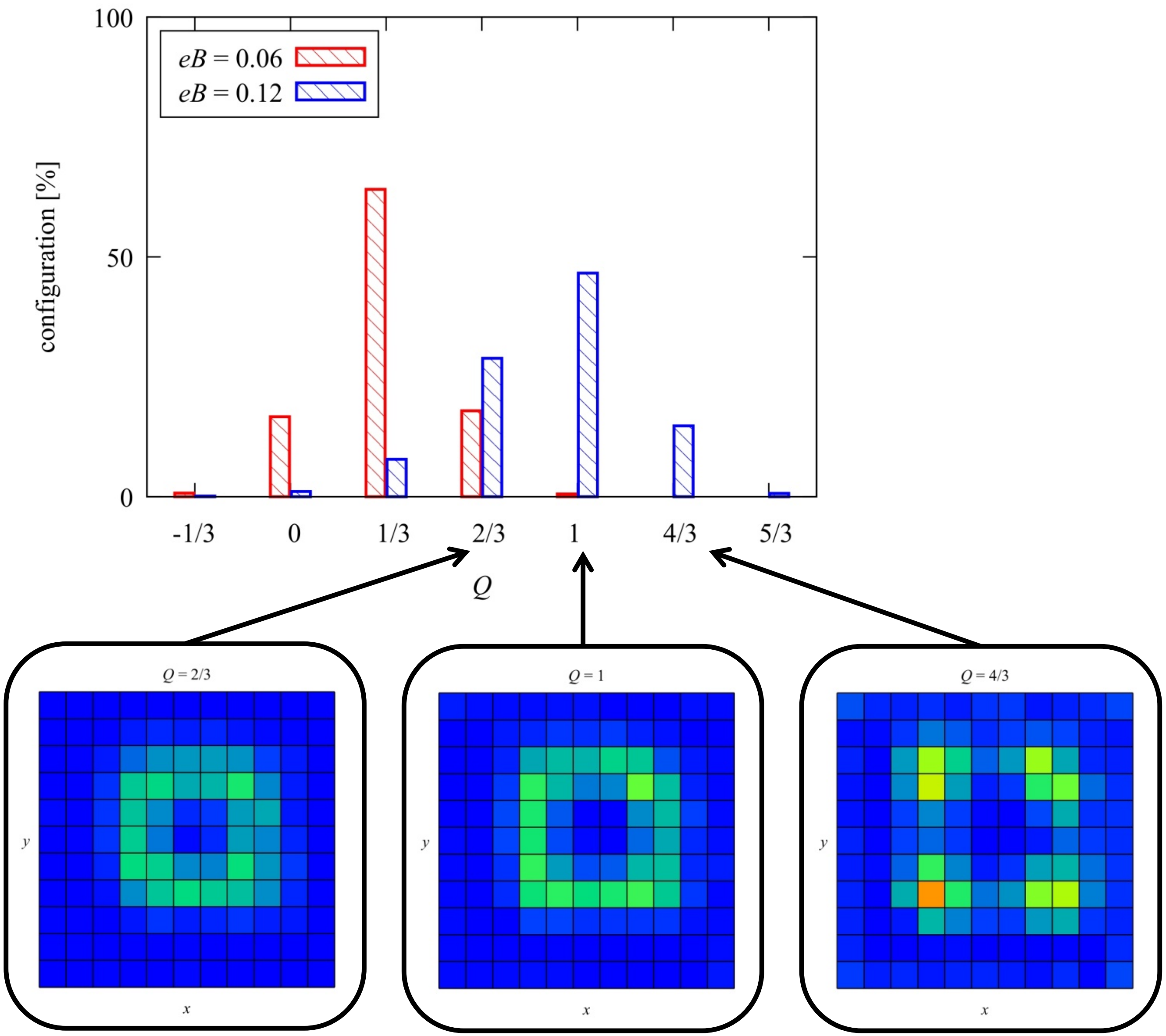
Non-Abelian vortex



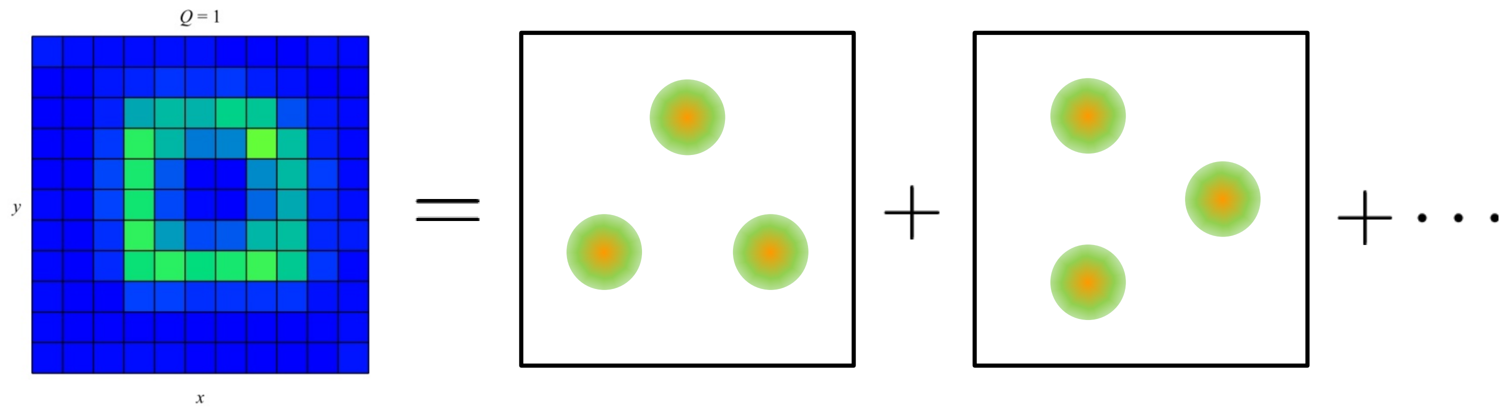
Non-Abelian vortex

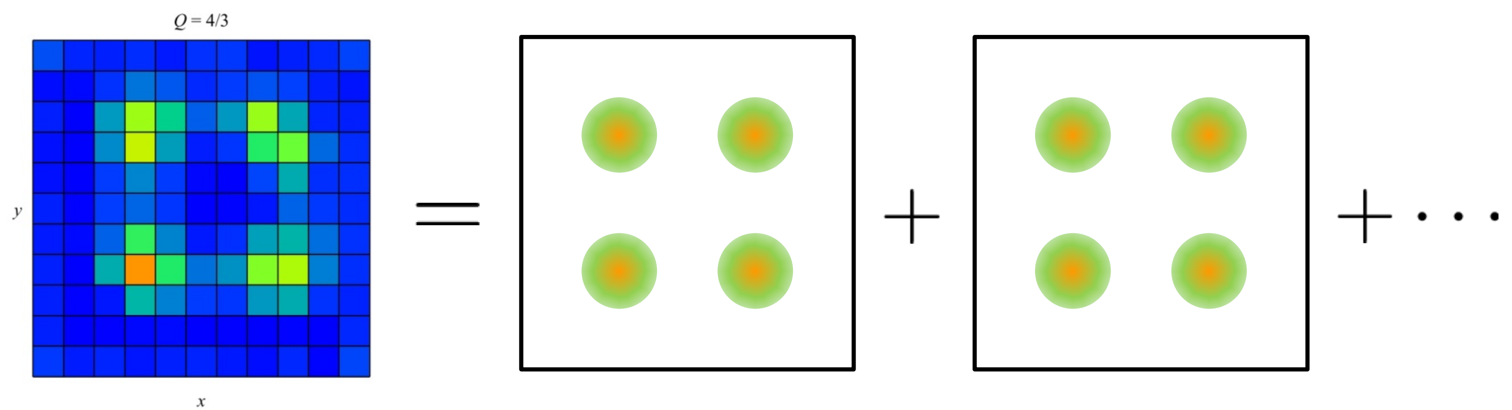


Non-Abelian vortex



Non-Abelian vortex





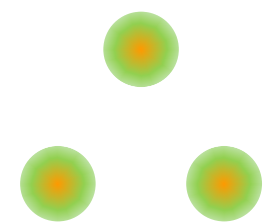
Non-Abelian vortex

vortex-vortex interaction is repulsive

one Abelian vortex

three non-Abelian vortices

decay →



$Q = 1$

$Q = 1$

cf. mean-field analysis Nakano, Nitta, Matsuura (2008) Alford, Mallavarapu, Vachaspati, Windisch (2016)

Summary

I studied non-Abelian vortices in lattice gauge theory.

- ✓ used gauge-invariant operators
- ✓ obtained color-flavor locked phase
- ✓ obtained vortex distributions
- ✓ ultimate goal: lattice QCD at high density