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Chiral Matter & Topology
Dec 7, 2018
Taipei, Taiwan

Chiral Plasmas: from Cosmology to Technology

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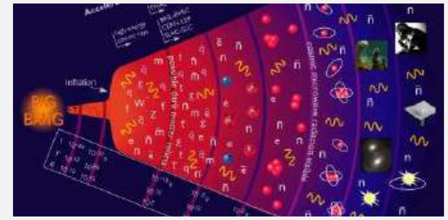
Workshop on Recent Developments in Chiral Matter and Topology

December 6 ~ 9, 2018
National Taiwan University, Taipei, Taiwan

Chiral forms of matter

- **Early Universe**, e.g.,

  [Boyarsky, Frohlich, Ruchayskiy, Phys.Rev.Lett. 108, 031301 (2012)]

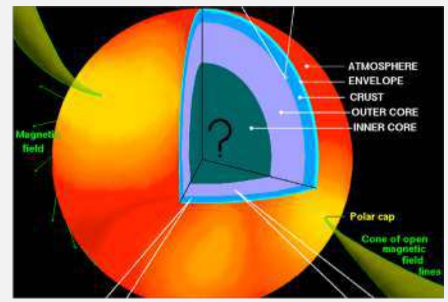


- **Heavy-ion collisions**, e.g.,

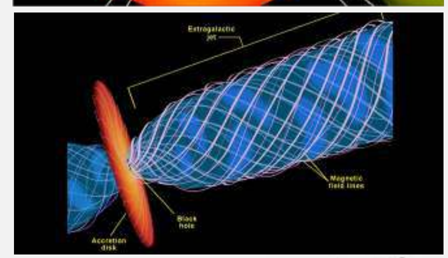
  [Kharzeev, Liao, Voloshin, Wang, Prog.Part.Nucl.Phys. 88, 1 (2016)]

- **Super-dense matter in compact stars**, e.g.,

  [Yamamoto, Phys.Rev. D93, 065017 (2016)]



- **Ultra-relativistic jets from black holes**

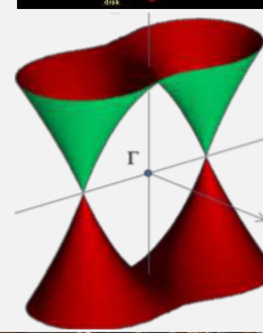


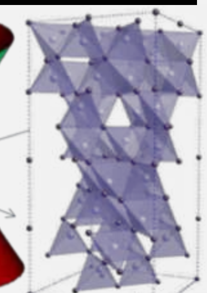
- **Superfluid ^{3}He-A**, e.g.,

  [Volovik, JETP Lett. 105, 34 (2017)]

- **Dirac/Weyl (semi-)metals**, e.g.,

  [Li et. al. Nature Phys. 12, 550 (2016)]





Anomalous chiral matter

- Matter made of chiral fermions may allow $n_\mathrm{L} \neq n_\mathrm{R}$

- The chiral charge (n_R - n_L), unlike the electric charge ($n_\mathrm{R} + n_\mathrm{L}$), is **not** conserved

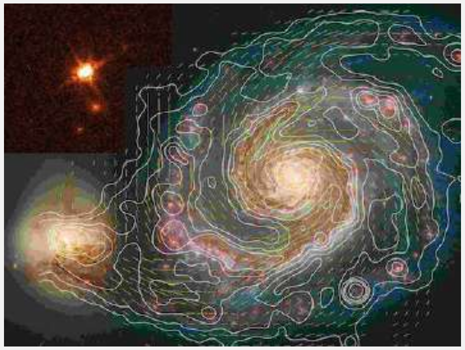
$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla}\cdot\vec{\boldsymbol{j}} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla}\cdot\vec{\boldsymbol{j}}_5 = \frac{e^2\vec{\boldsymbol{E}}\cdot\vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral anomaly can have *macroscopic* implications in chiral matter

Anomalous plasmas elsewhere

- Inverse magnetic cascade may produce seeds of helical magnetic fields in the early Universe



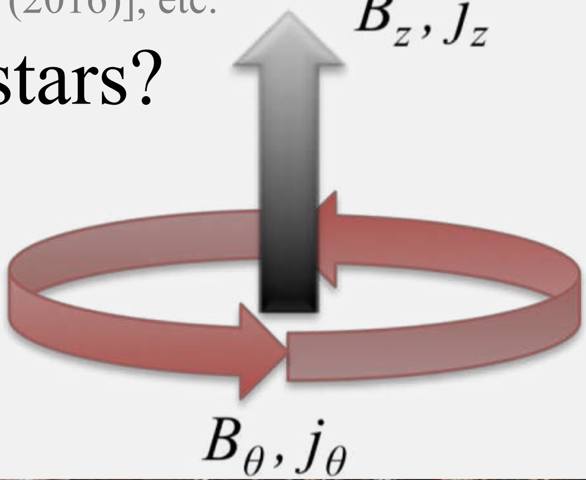
[Vilenkin, Phys. Rev. D22, 3080 (1980)],
[Joyce & Shaposhnikov, astro-ph/9703005],
[Giovannini & Shaposhnikov, hep-ph/9710234]

- Eigenmodes of long wavelength and fixed helicity grow:

$$\frac{dB_k}{dt} \propto \frac{ck}{\sigma}\left(4\pi C_5 \mu_5 - ck\right) B_k$$

[Boyarsky et al., PRL **108**, 031301 (2012)], [Tashiro et al., PRD **86**, 105033 (2012)],
[Manuel et al., PRD **92**, 074018 (2015)], [Hirono et al., PRD **92**, 125031 (2015)],
[Buividovich et al., PRD **94**, 025009 (2016)], [Gorbar et al., PRD **94**, 103528 (2016)], etc.

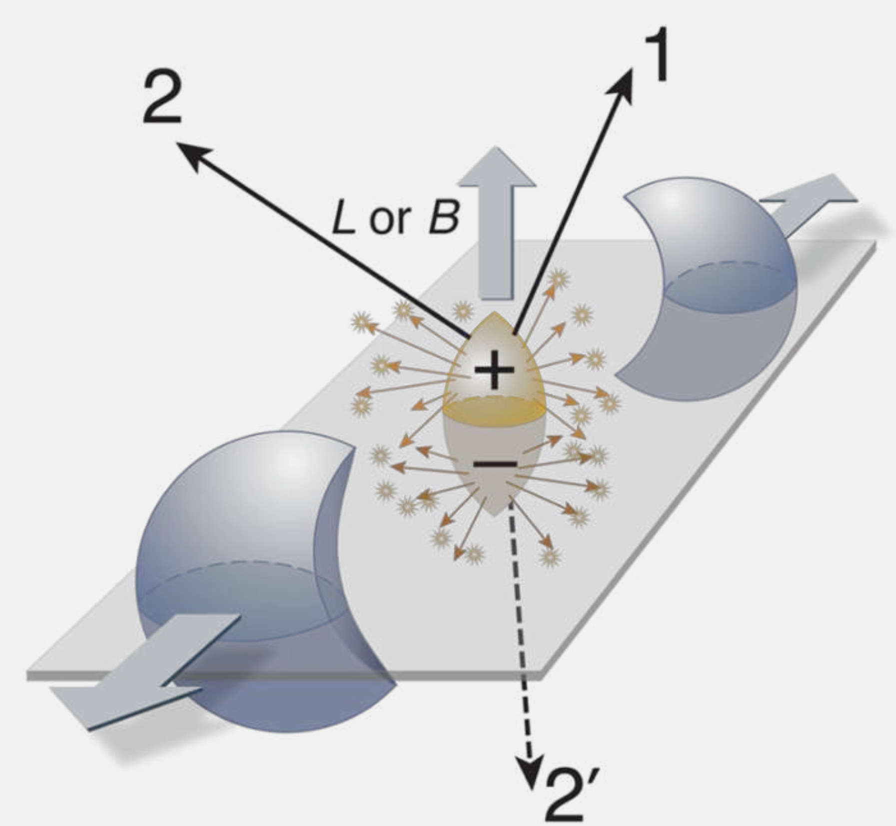
- Strong helical magnetic field in compact stars?



[Ohnishi, Yamamoto, arXiv:1402.4760]
[Yamamoto, Phys. Rev. D **93**, 065017 (2016)]
[Dvornikov, J. Exp. Theor. Phys. **123** 967 (2016)]

- Perhaps, chirality flipping is too strong…

[Grabowska, Kaplan, Reddy, Phys. Rev. D **91**, 085035 (2015)]



https://physics.aps.org/articles/v2/104

ANOMALOUS EFFECTS IN HIC

[Miransky & Shovkovy, Phys. Rep. **576**, 1 (2015)]
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)]
[Hattori & Huang, Nucl. Sci. Tech. **28**, 26 (2017)]

Chiral Magnetic Effect ($\mu_5 \neq 0$)

- Dirac equation @ $B \neq 0$

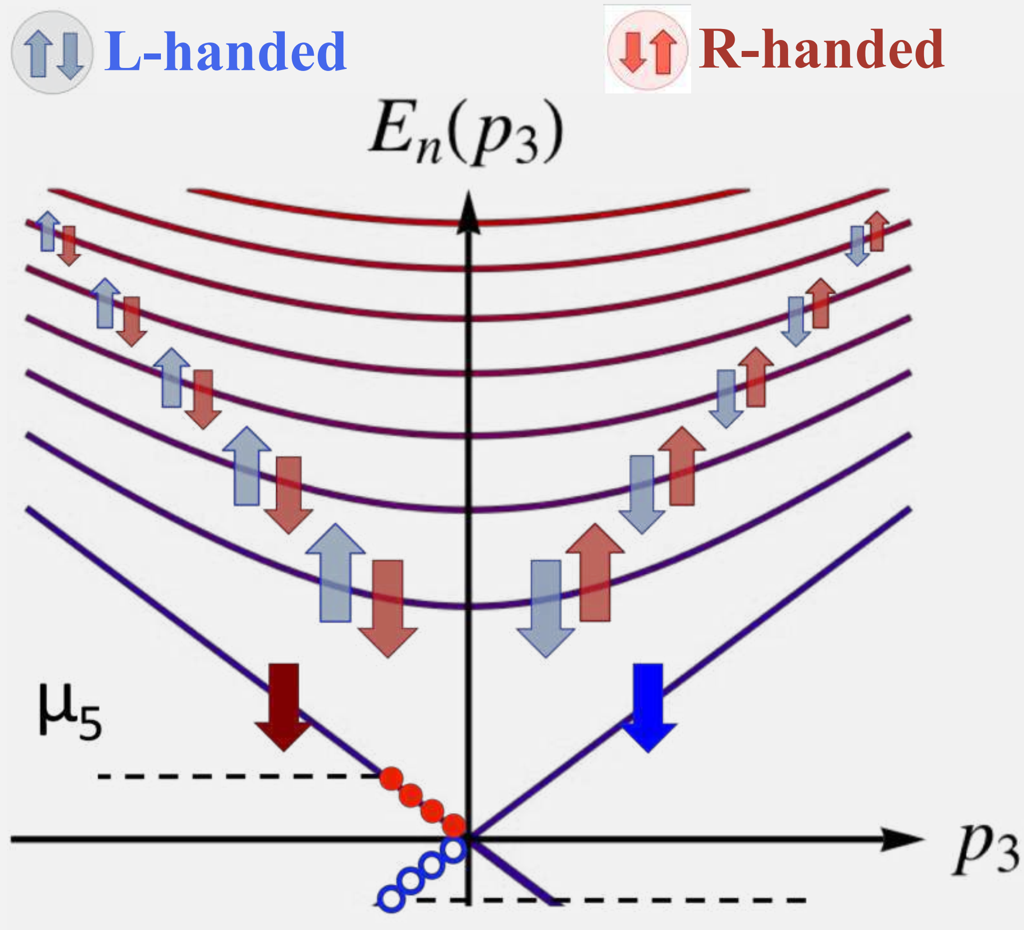
$$\left[i\gamma^0\partial_0 - i\vec{\gamma}\cdot\left(\vec{\nabla} + ie\vec{A}\right)\right]\Psi = 0$$

Energy spectrum

$$E_n^{(3+1)}(p_3) = \pm\sqrt{2n|eB| + p_3^2}$$

Spin polarized LLL ($s=\downarrow$):

- R-handed electrons $p_3<0$
- L-handed positrons $p_3>0$

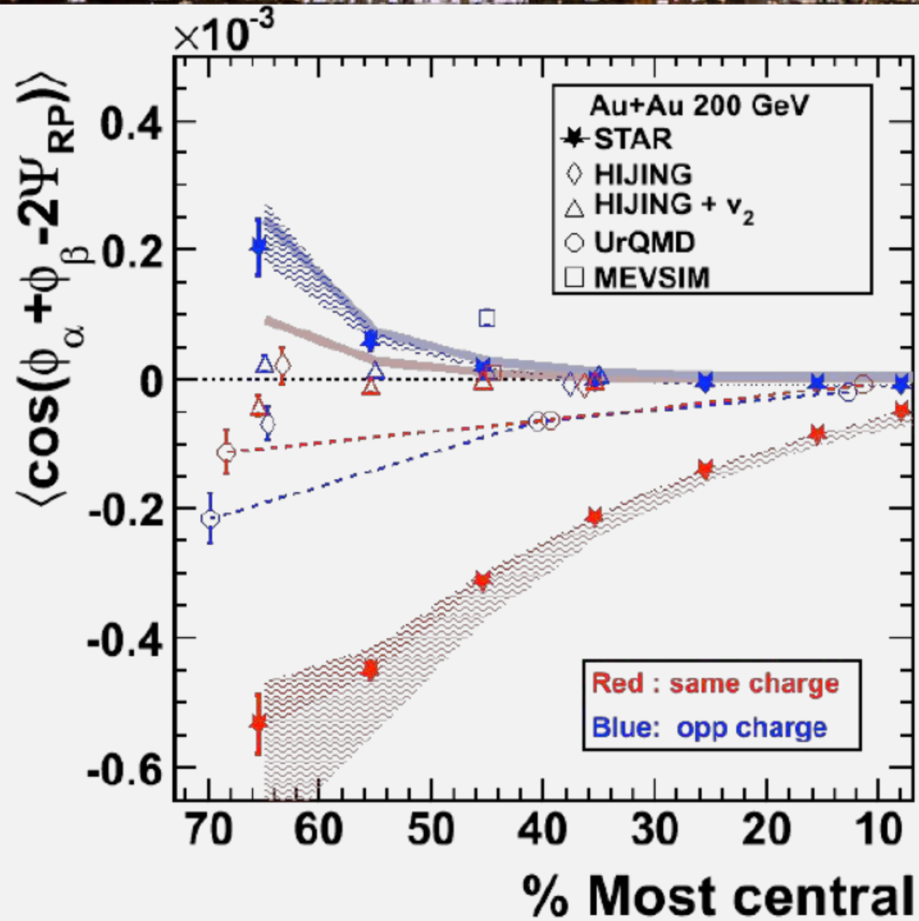


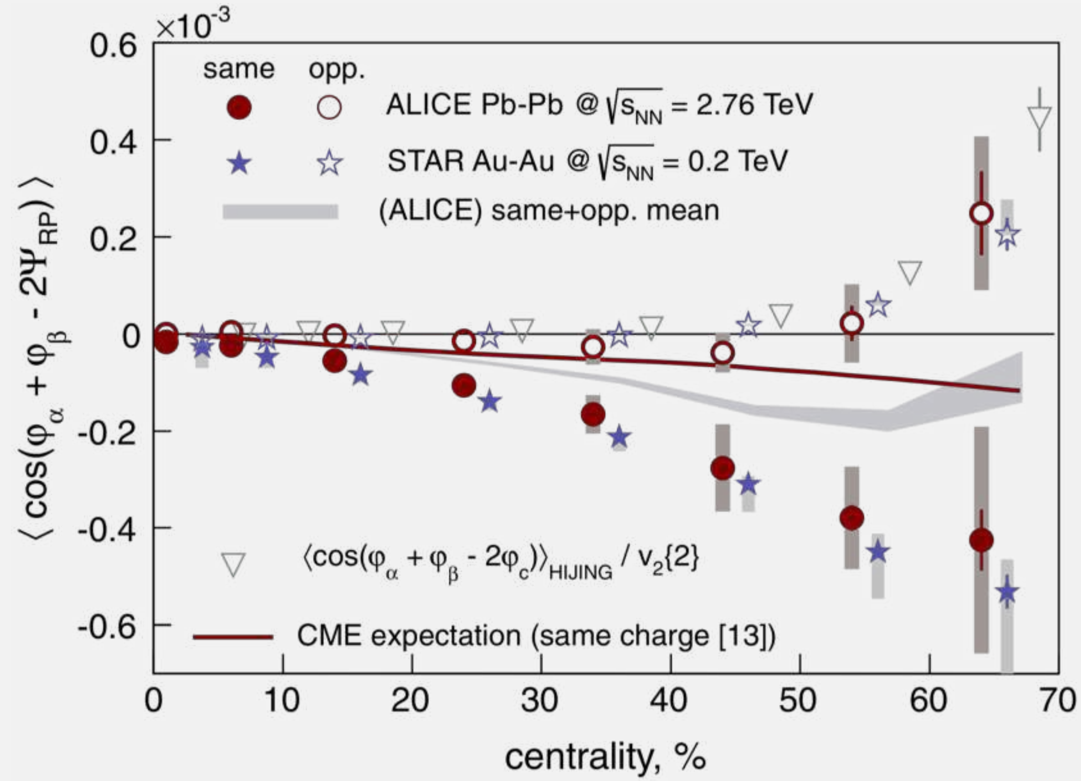
Topological fluctuations could induce nonzero chiral charge ($\mu_5 \neq 0$)

CME current: $$\langle \vec{j} \rangle = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

CME: Experimental evidence





Correlations of same & opposite charge particles:

$$\begin{cases} \langle\cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\mp} - 2\Psi_{\mathrm{RP}})\rangle \\ \langle\cos(\varphi_\alpha^{\pm} + \varphi_\beta^{\pm} - 2\Psi_{\mathrm{RP}})\rangle \end{cases}$$

[Abelev et al. (STAR), PRL **103**, 251601 (2009)]
[Abelev et al. (STAR), PRC **81**, 054908 (2010)]
[Abelev et al. (ALICE), PRL **110**, 012301 (2013)]
[Adamczyk et al. (STAR), PRC **88**, 064911 (2013)]

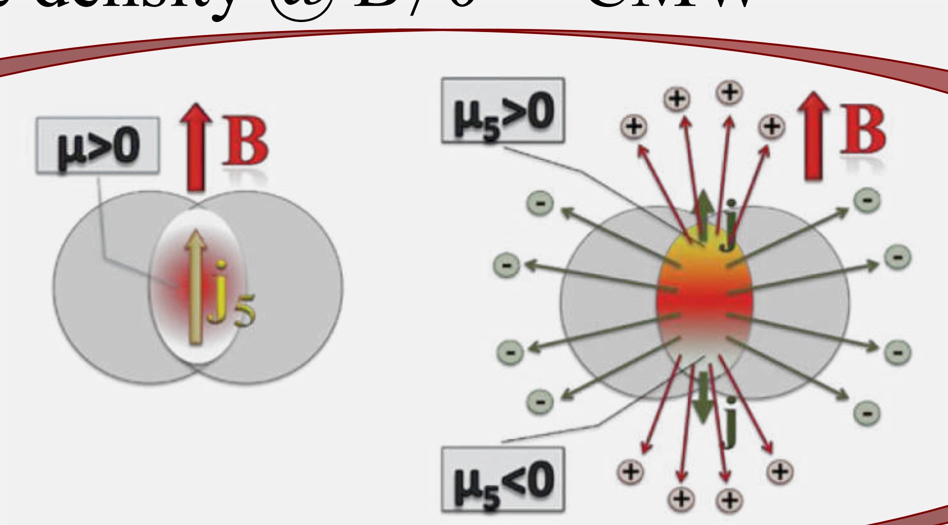
LARGE BACKGROUND EFFECTS!

[Belmont & Nagle, PRC **96**, 024901 (2017)], [ALICE Collaboration, Phys. Lett. B**777**, 151 (2018)]

Chiral Magnetic Wave

- Nonzero charge density @ B≠0 → CMW

$$\left\langle \vec{j}_5 \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu$$

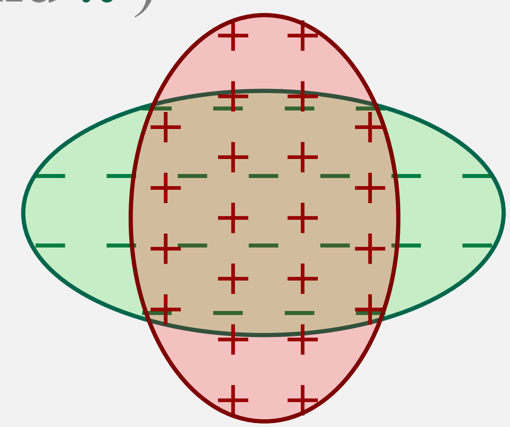


$$\left\langle \vec{j} \right\rangle = \frac{e\vec{B}}{2\pi^2}\mu_5$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83**, 085003 (2011)]

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of in π^+ and π^-)

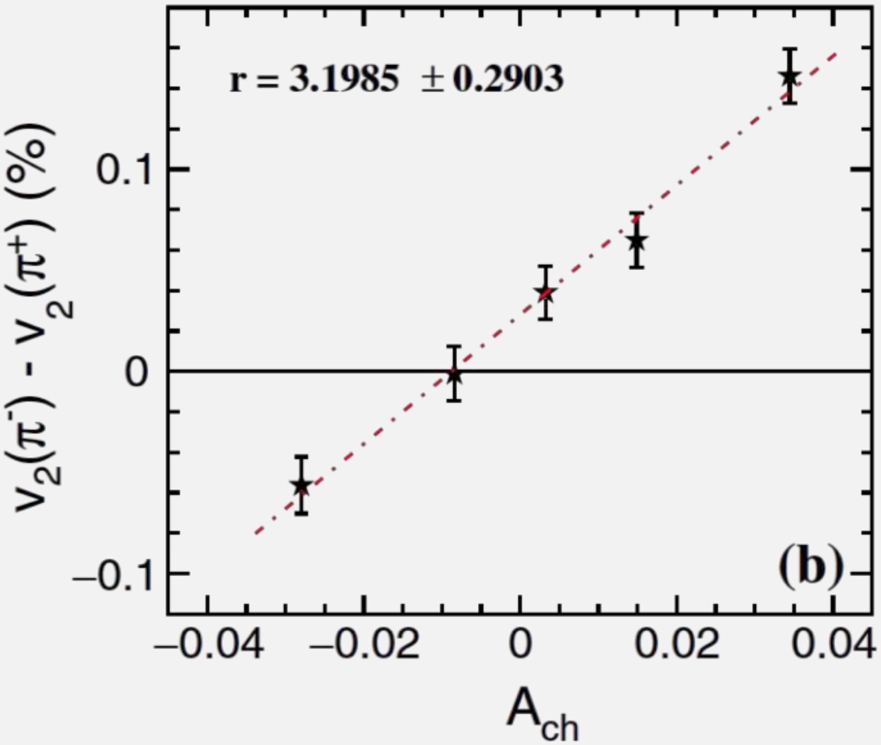
$$\frac{dN_\pm}{d\phi} \approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]$$

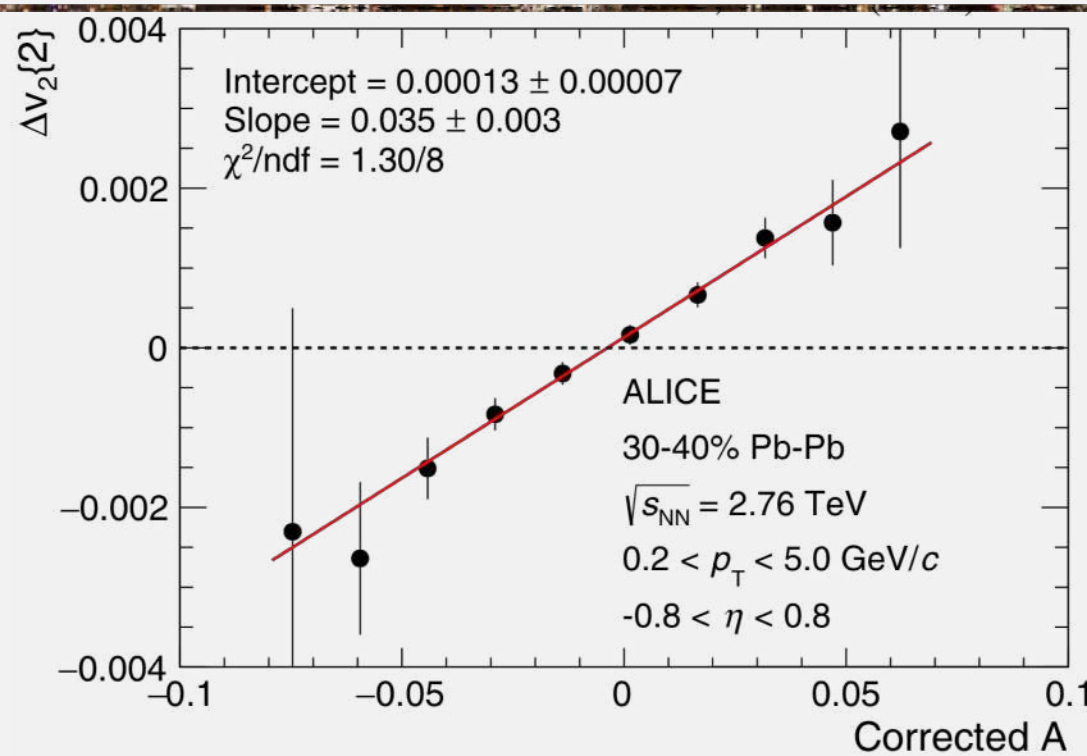


where $A_\pm$ is the charge asymmetry

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

CMW: Experimental evidence





[Ke (for STAR) J. Phys. Conf. Series **389**, 012035 (2012)]

[Adamczyk et al. (STAR), Phys. Rev. Lett. **114**, 252302 (2015)]

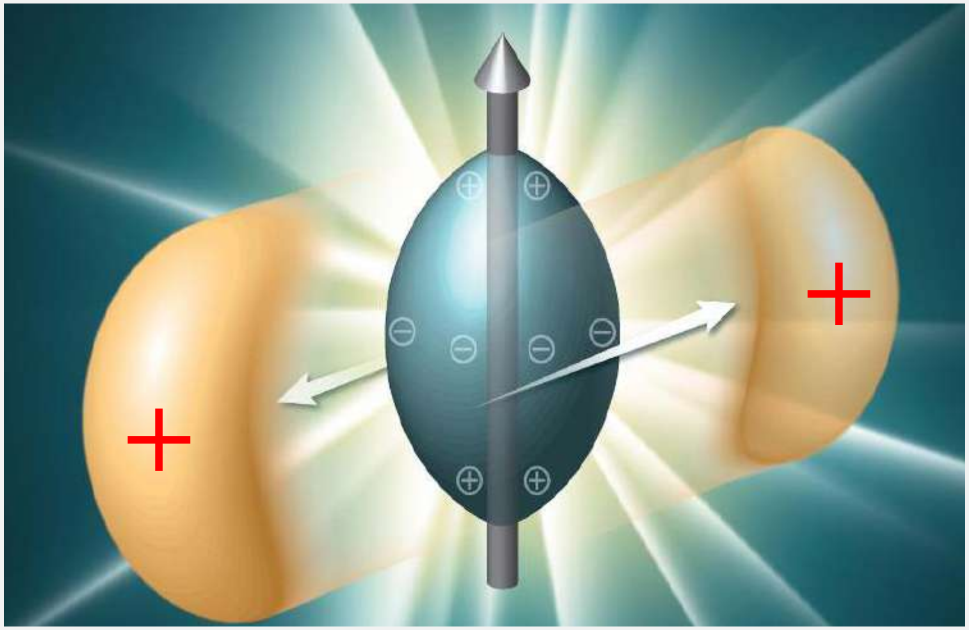
[Adam et al. (ALICE), Phys. Rev. C **93**, 044903 (2016)]

Higher harmonics of particle correlations indicate a possible strong background

BACKGROUND EFFECTS MAY DOMINATE OVER THE SIGNAL!

[CMS Collaboration, arXiv:1708.08901]

$B \approx 10^{18}$ G



FRESH LOOK AT CHIRAL MAGNETIC WAVE

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

Fresh look at CMW

- Simple 1-flavor model (**k** ∥ **B**):

$$k_0\delta n - kB\delta\sigma_B + i\frac{\tau}{3}k^2\delta n - \frac{1}{e}\sigma_E k\delta E_z = 0$$

$$k_0\delta n_5 - kB\delta\sigma_B^5 + i\frac{\tau}{3}k^2\delta n_5 - i\frac{e^2}{2\pi^2}B\delta E_z = 0$$

$$k\delta E_z + ie\delta n = 0$$

- The dispersion of the CMW mode:

$$k_0^{(\pm)} = -i\frac{\sigma_E}{2} \pm i\frac{\sigma_E}{2}\sqrt{1-\left(\frac{3eB}{\pi^2T^2\sigma_E}\right)^2\left(k^2+\frac{e^2T^2}{3}\right)} - i\frac{\tau}{3}k^2$$

- This is a completely diffusive mode when

$$\frac{3eB}{\pi^2T^2\sigma_E}\sqrt{k^2+\frac{e^2T^2}{3}} < 1$$

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

Naïve analysis ($\sigma_E \to 0$)

- If Gauss's law is ignored, the CMW is non-diffusive, i.e.,

$$k_0^{(\pm)} = \pm \frac{3eB}{2\pi^2 T^2} \sqrt{k^2 + \frac{e^2 T^2}{3}} - i\frac{\tau}{3}k^2, \quad \text{when} \quad \sigma_E \to 0.$$

- In the limit $k \to 0$, there is only a small dissipation due to the charge diffusion
- The CMW is gapped when $\sigma_E \to 0$
- Nonzero gap is from the anomaly term $\propto \delta E_z$

$$-i\frac{e^2}{2\pi^2} B\delta E_z = -i\frac{e^2 B}{2\pi^2}\left(\frac{-ie\delta n}{k}\right)$$

CMW in HIC

- Model with two light u- and d-quarks:

$$k_0 \delta n_f - \frac{e q_f B k}{2\pi^2 \chi_{f,5}} \delta n_{f,5} + i D_f k^2 \delta n_f - \frac{1}{e q_f} \sigma_{E,f} k \delta E_z = 0$$

$$k_0 \delta n_{f,5} - \frac{e q_f B k}{2\pi^2 \chi_f} \delta n_f + i D_f k^2 \delta n_{f,5} - i \frac{e^2 q_f^2}{2\pi^2} B \delta E_z = 0$$

$$k \delta E_z + i e \sum_f q_f \delta n_f = 0$$

where $f = u, d$, and $q_u = {}^2/_3$, $q_d = -{}^1/_3$

χ_f, D_f and $\sigma_{E,f}$ are susceptibilities, diffusion coefficients and electrical conductivities for each quark flavor, respectively

Non-perturbative regime

Near-critical strongly coupled quark-gluon plasma

$$\sigma_E = \sum_f \sigma_{E,f} = c_\sigma C^\ell_{\text{em}} T$$

$$\chi_f = c_\chi \chi_f^{(SB)}$$

$$D_f = \frac{c_D}{2\pi T}$$

$$\boxed{C^\ell_{\text{em}} = \left({}^5\!/_9\right) 4\pi\alpha_{\text{em}} \approx 0.051}$$

Lattice data

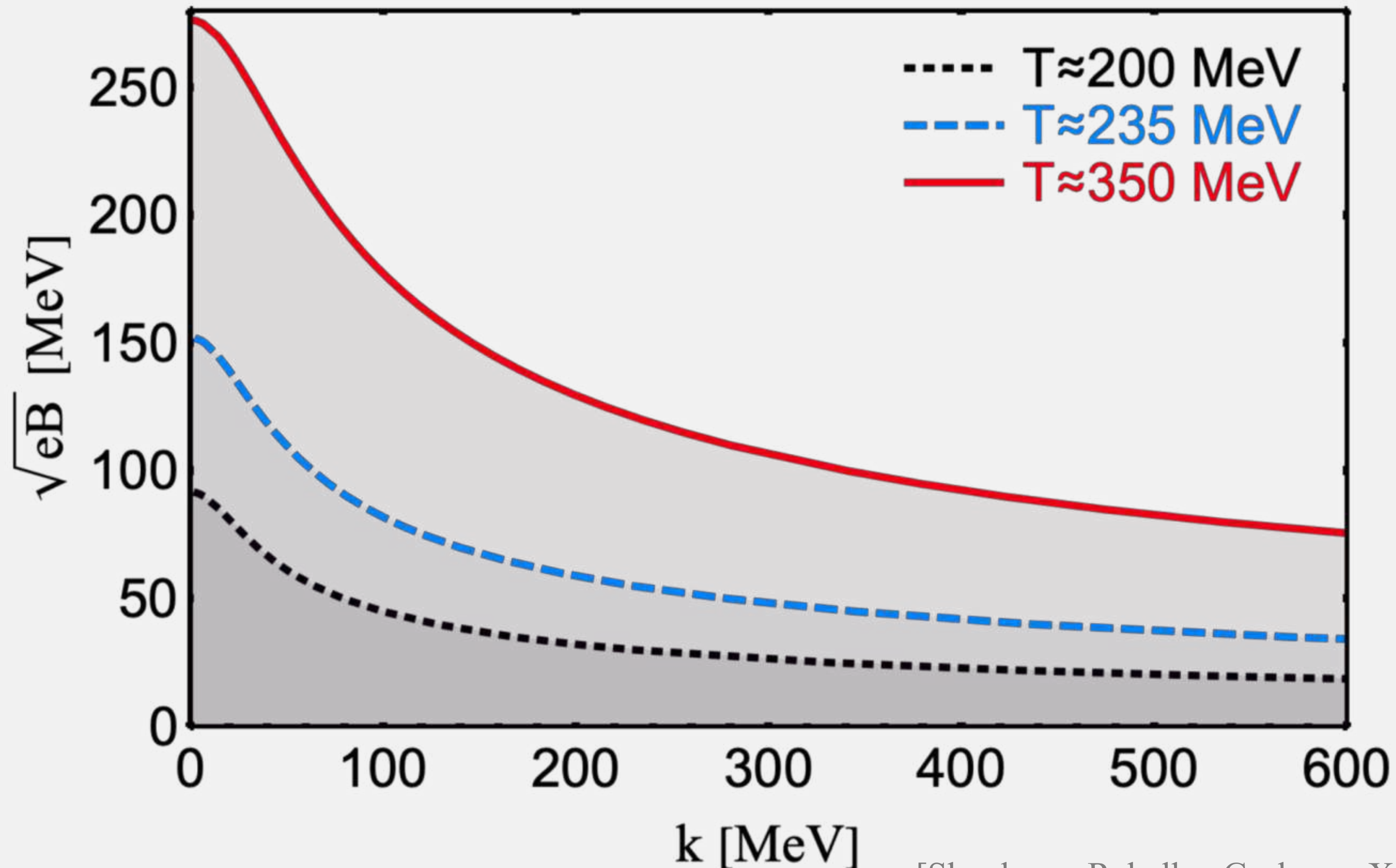
[Aarts et. al., JHEP 1502, 186 (2015)]

	c_σ	c_χ	c_D
T=200 MeV	0.11	0.80	0.78
T=235 MeV	0.21	0.89	1.37
T=350 MeV	0.32	0.87	1.85

Results

Two sets of modes: $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

CMW is completely diffusive at small eB & k:

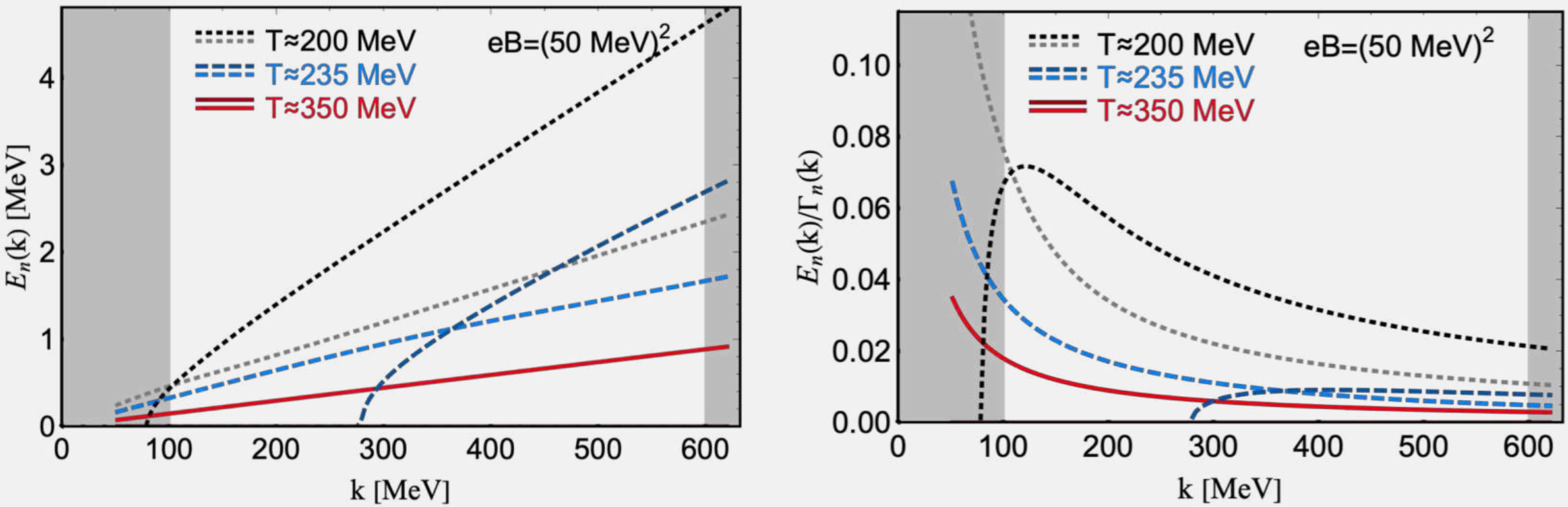


[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

Moderately strong B-field

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



Allowed range of wave vectors:

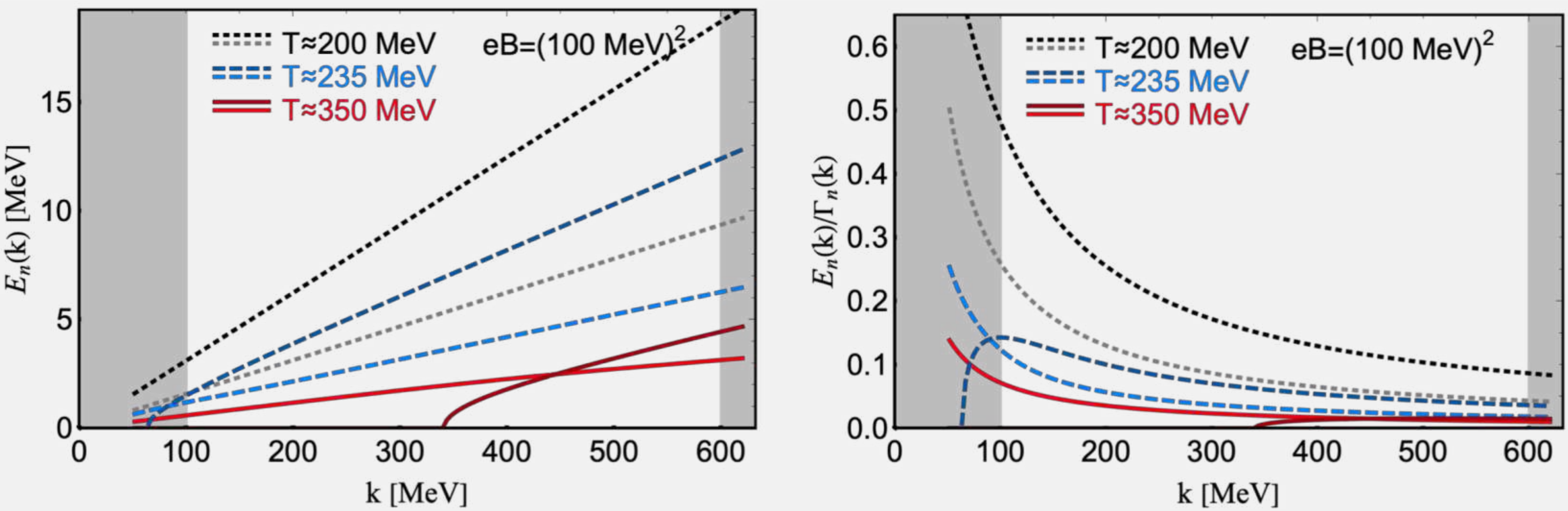
$$(50\ \text{MeV})\ 100\ \text{MeV} \lesssim k \lesssim 600\ \text{MeV}$$

Wavelengths: $2\ \text{fm} \lesssim \lambda_k \lesssim 12\ \text{fm}\ (24\ \text{fm})$

Strong B-field

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



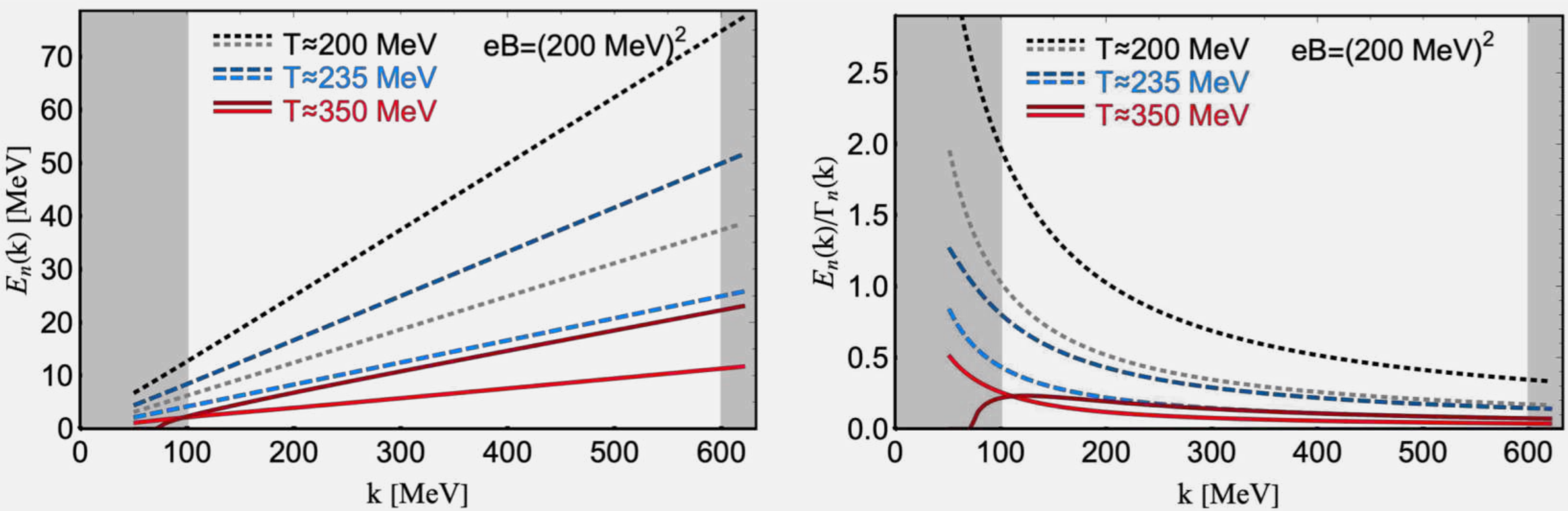
Even for such strong B-field, the CMW is strongly overdampled

Charge diffusion $iD_f k^2$ plays a big role $(k \gtrsim {}^{2\pi}/_{R})$

Very strong B-field

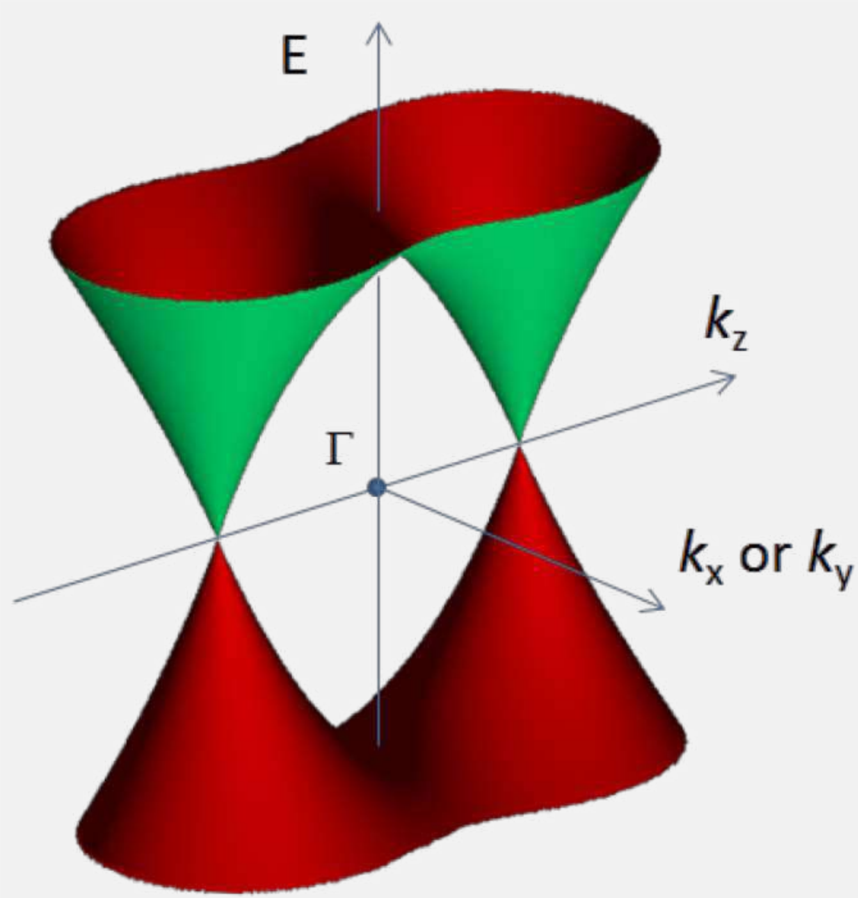
[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

$$k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$$



The CMW may become a propagating mode only at extremely strong B-field, $eB \gtrsim (200\ \text{MeV})^2$

In realistic heavy-ion collisions, it is overdampled



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

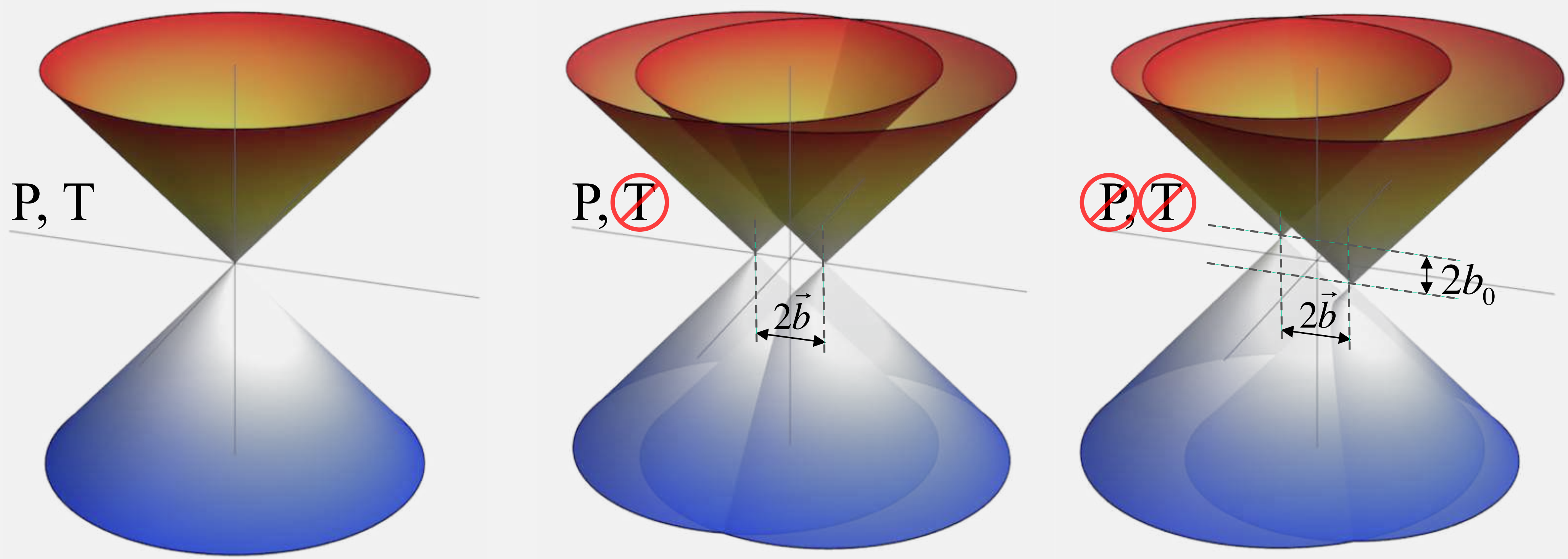
DIRAC & WEYL MATERIALS

Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

$$H = \int d^3\mathbf{r}\, \bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \overset{\cancel{\mathrm{T}}}{\left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5} + \overset{\cancel{\mathrm{P}}}{b_0\gamma^0\gamma^5}\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) **Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Strain in Weyl materials

- Strains in the low-energy effective Weyl Hamiltonian

$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

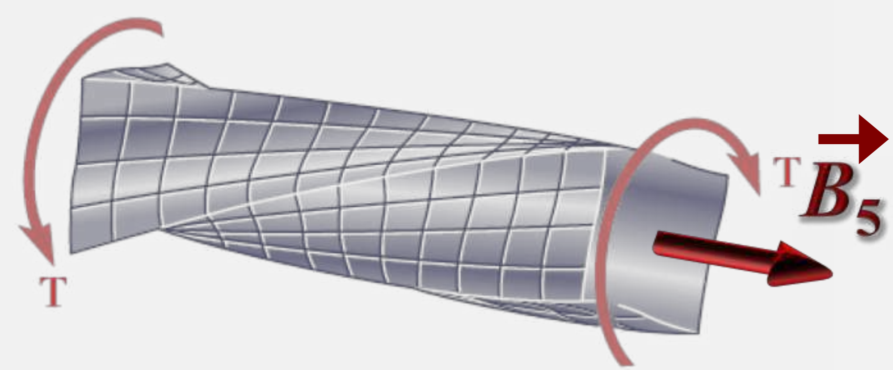
where the chiral gauge fields are

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

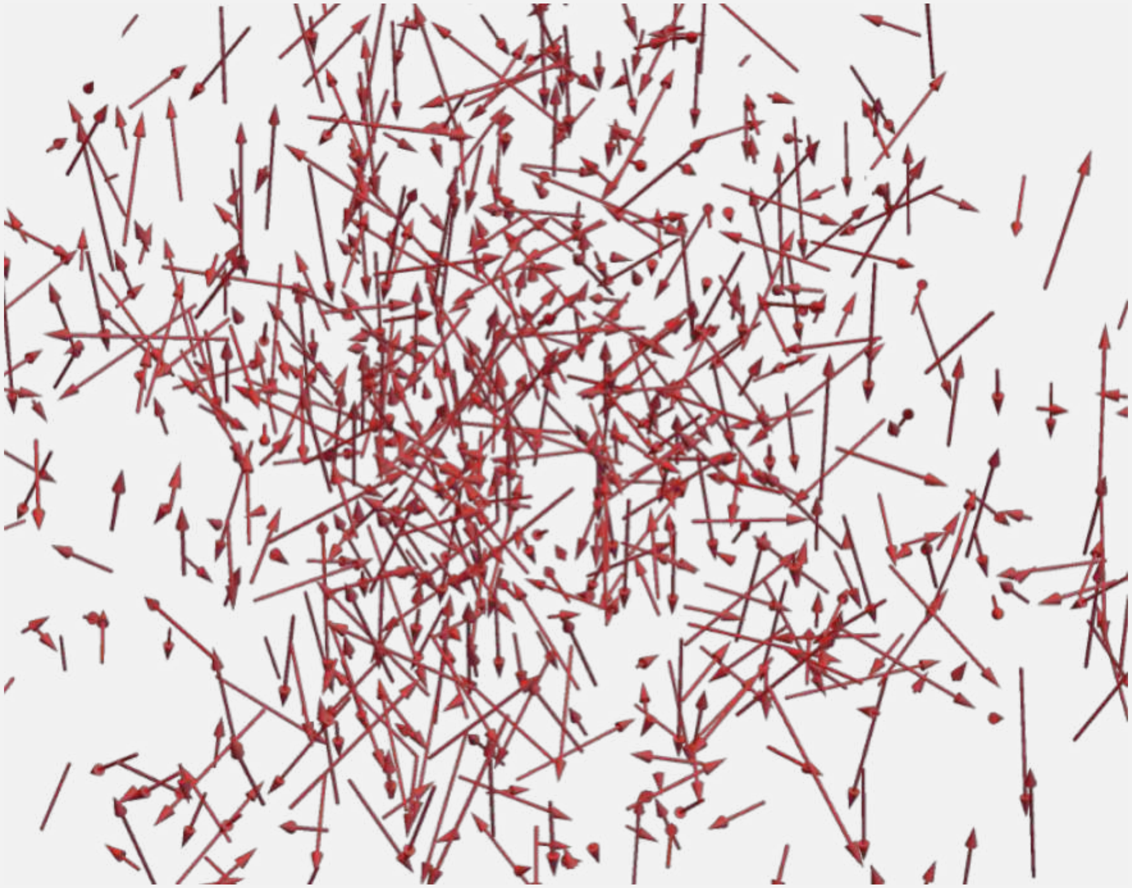
$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. PRL **115**, 177202 (2015)]
[Pikulin, Chen, Franz, PRX **6**, 041021 (2016)]
[Grushin, Venderbos,Vishwanath, Ilan, PRX **6**, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, PRB **94**, 241405 (2016)]



leading to the pseudo-EM fields

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0-\partial_t\vec{A}_5$$



CONSISTENT CHIRAL KINETIC THEORY

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral kinetic theory

- Kinetic equation:

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\boldsymbol{\Omega}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{p}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)}$$
$$+\frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\boldsymbol{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\boldsymbol{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\boldsymbol{\nabla}_{\mathbf{r}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\boldsymbol{\nabla}_{\mathbf{r}}\epsilon_{\mathbf{p}}$, $\mathbf{v} = \boldsymbol{\nabla}_{\mathbf{p}}\epsilon_{\mathbf{p}}$, and

$$\epsilon_{\mathbf{p}} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\boldsymbol{\Omega}_\lambda)\right]$$

and $\boldsymbol{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Current and chiral anomaly

- The definitions of density and current are

$$\rho_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)\right] f_\lambda,$$

$$\mathbf{j}_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[\mathbf{v}+\frac{e}{c}(\mathbf{v}\cdot\mathbf{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda\times\mathbf{\Omega}_\lambda)\right] f_\lambda$$

$$+e\mathbf{\nabla}\times\int \frac{d^3p}{(2\pi\hbar)^3} f_\lambda \epsilon_{\mathbf{p}}\mathbf{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial\rho_5}{\partial t} + \mathbf{\nabla}\cdot\mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B}) + (\mathbf{E}_5\cdot\mathbf{B}_5)\right] \quad ✔$$

$$\frac{\partial\rho}{\partial t} + \mathbf{\nabla}\cdot\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\left[(\mathbf{E}\cdot\mathbf{B}_5) + (\mathbf{E}_5\cdot\mathbf{B})\right] \quad ✘$$

Consistent definition of current

- Bardeen-Zumino (Chern-Simons) term is needed,

$$\delta j^{\mu} = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda} A^5_{\nu} F_{\rho\lambda}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRL **118**, 127601 (2017)]

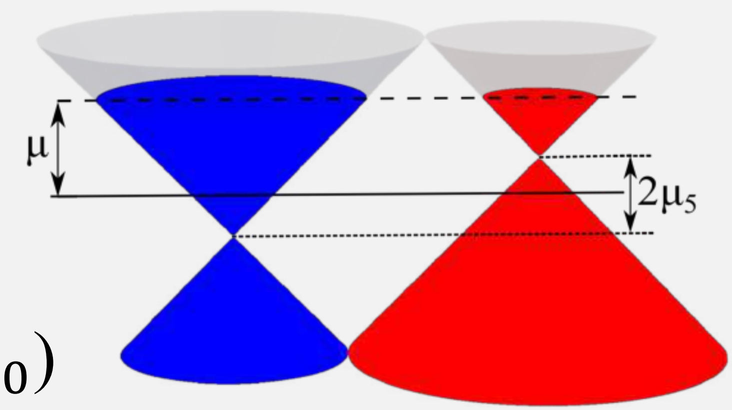
or

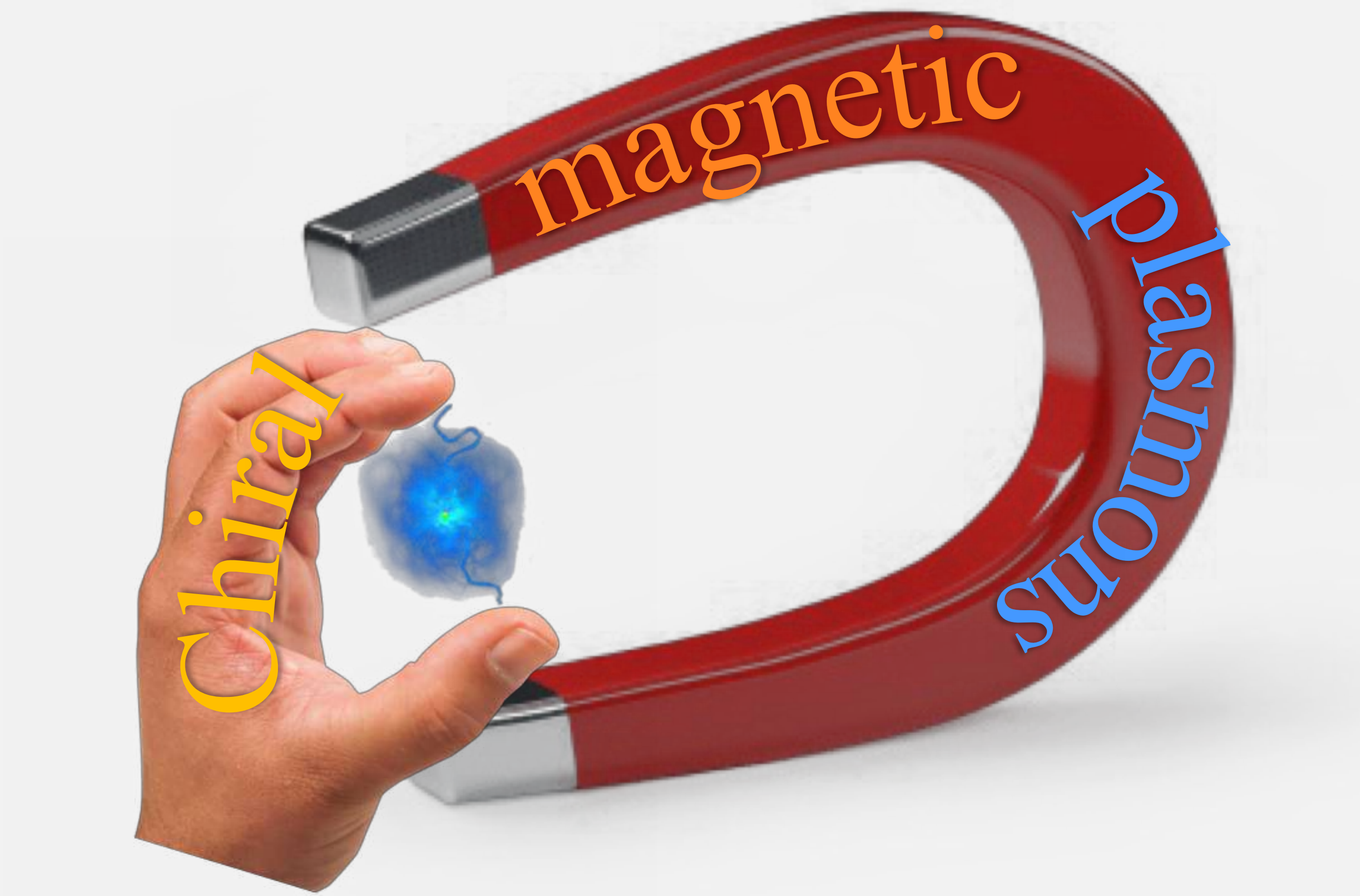
$$\delta\rho = -\frac{e^3}{2\pi^2\hbar^2 c^2}(\mathbf{b}\cdot\mathbf{B})$$

$$\delta\mathbf{j} = -\frac{e^3}{2\pi^2\hbar^2 c} b_0\mathbf{B} + \frac{e^3}{2\pi^2\hbar^2 c}[\mathbf{b}\times\mathbf{E}]$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **96**, 085130 (2017)]

- Its role and implications:
  - Electric charge is conserved ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)





CHIRAL MAGNETIC PLASMONS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\rm tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

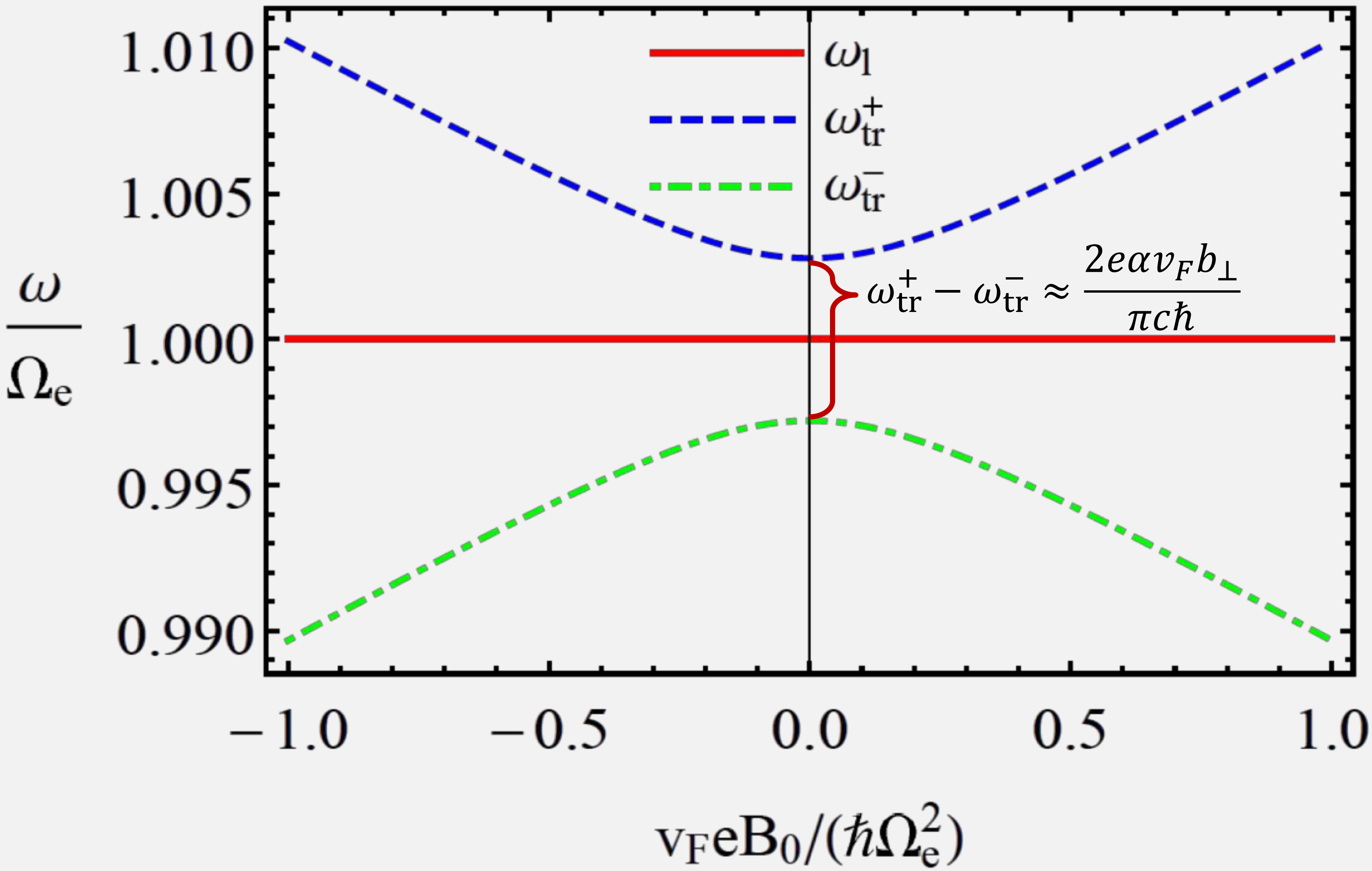
and

$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\left\{9\hbar^2 b_\perp^2 + \left[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5) - 3\hbar b_\parallel - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right)\right]^2\right\}^{1/2}$$

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

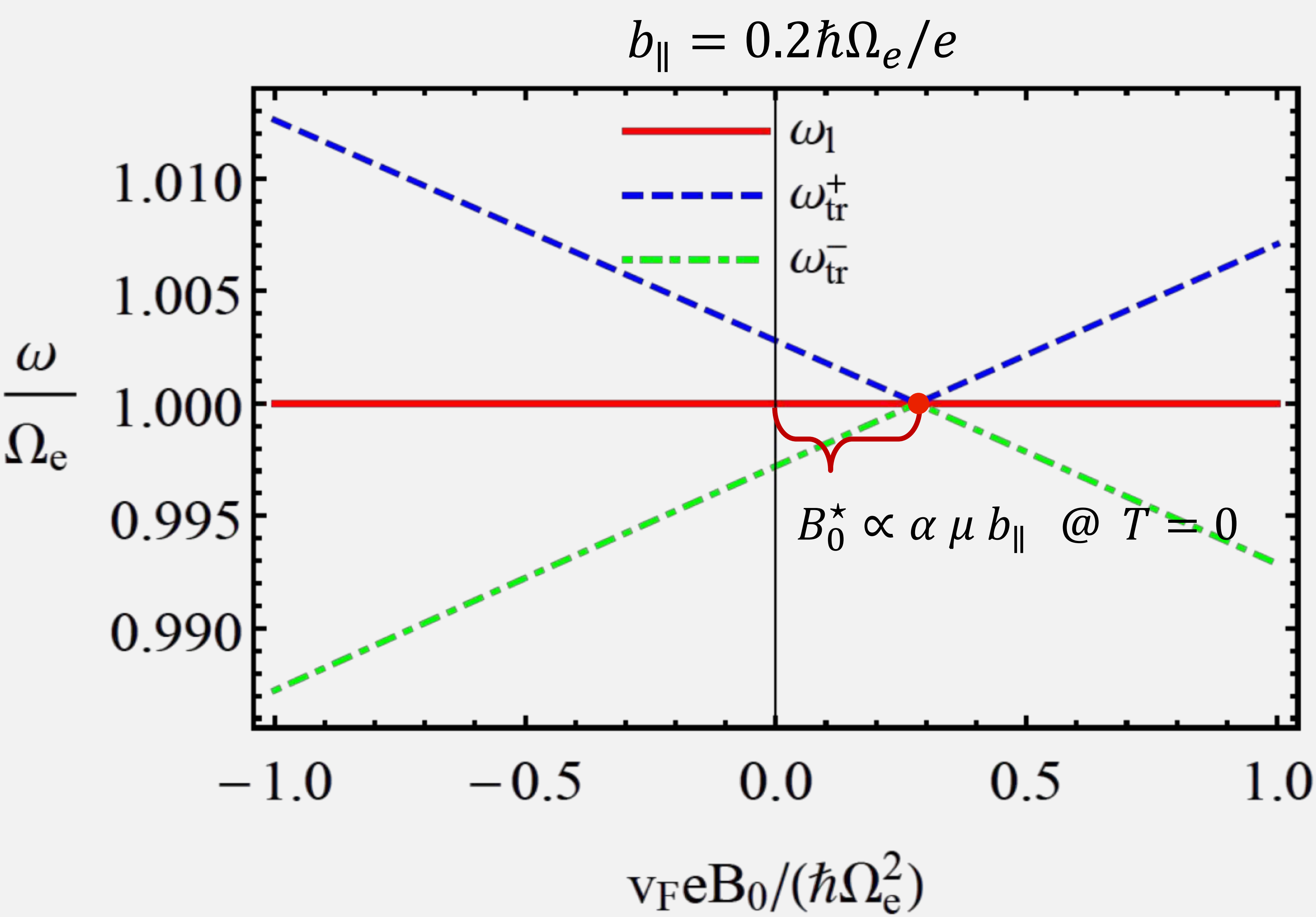
Plasmon frequencies, $\vec{B} \perp \vec{b}$

$b_\perp = 0.2\hbar\Omega_e/e$

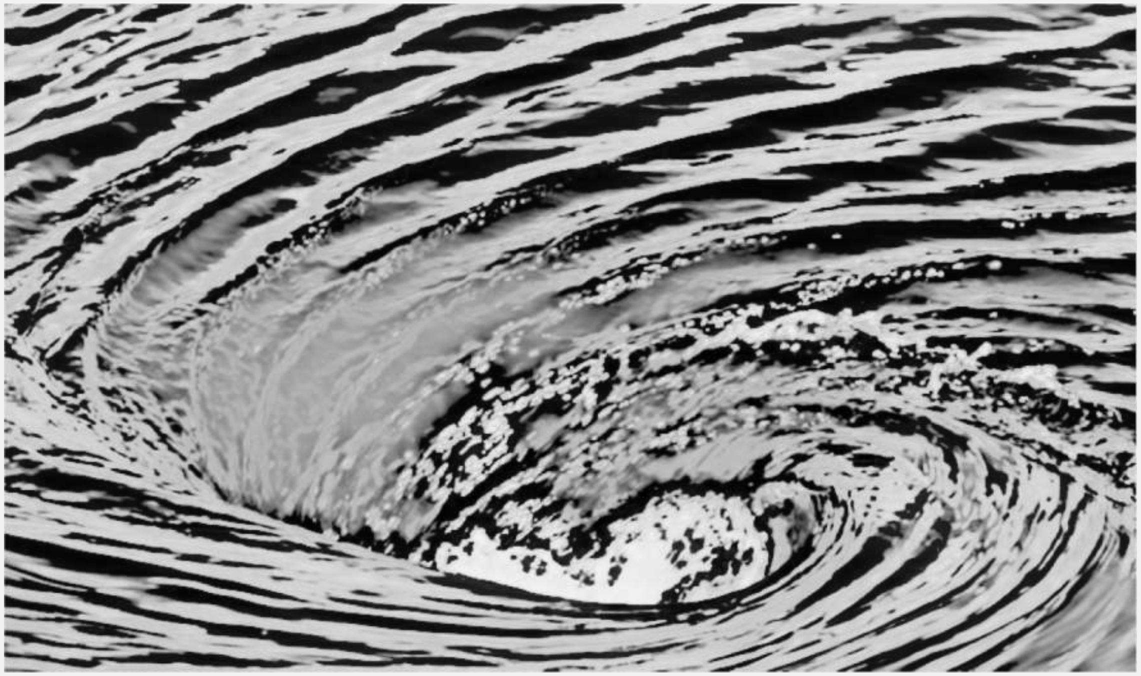


[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmon frequencies, $\vec{B} \parallel \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]



ELECTRON TRANSPORT IN HYDRODYNAMIC REGIME

Hydrodynamics in Weyl metals

The Euler equation for electron fluid: [Gurzhi, JETP 17, 521 (1963)]

$$\frac{1}{v_F}\partial_t\left(\frac{\epsilon+P}{v_F}\mathbf{u}+\sigma^{(\epsilon,B)}\mathbf{B}\right)=-en\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)+\frac{\sigma^{(B)}(\mathbf{E}\cdot\mathbf{B})}{3v_F^2}\mathbf{u}-\frac{\epsilon+P}{\tau v_F^2}\mathbf{u}+O(\nabla_{\mathbf{r}})$$

[Gorbar, Miransky, Shovkovy, Sukhachov, PRB **97**, 121105(R) (2018)]

The energy conservation

$$\partial_t\epsilon=-\mathbf{E}\cdot(en\mathbf{u}-\sigma^{(B)}\mathbf{B})+O(\nabla_{\mathbf{r}})$$

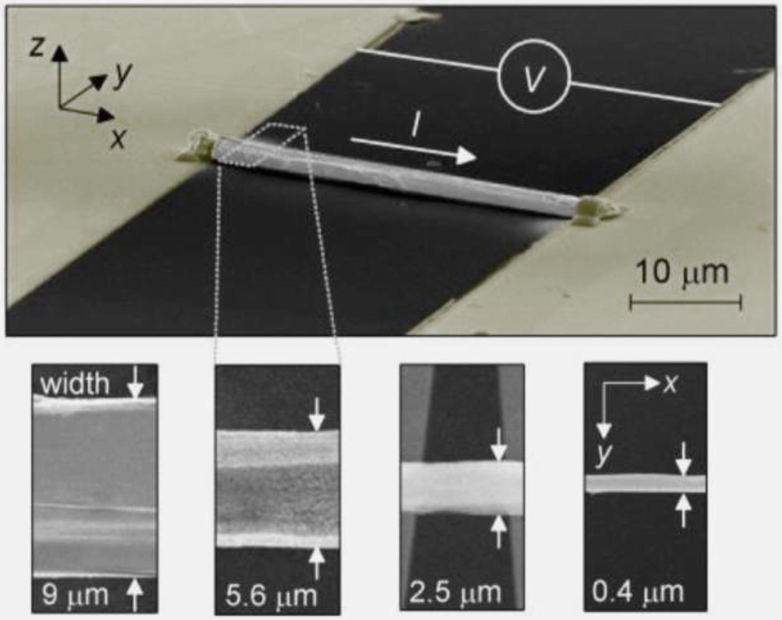
+ Maxwell equations with the Chern-Simons currents

$$\rho_{\mathrm{CS}}=-\frac{e^3(\mathbf{b}\cdot\mathbf{B})}{2\pi^2\hbar^2c^2}$$

$$\mathbf{J}_{\mathrm{CS}}=-\frac{e^3b_0\mathbf{B}}{2\pi^2\hbar^2c}+\frac{e^3\,[\mathbf{b}\times\mathbf{E}]}{2\pi^2\hbar^2c}$$

Experimental evidence in tungsten diphosphide (WP_2)

[Gooth et al., Nature Commun. **9**, 4093 (2018)]



Rich spectrum of hydro modes

- Magneto-acoustic wave ($\rho = 0$):

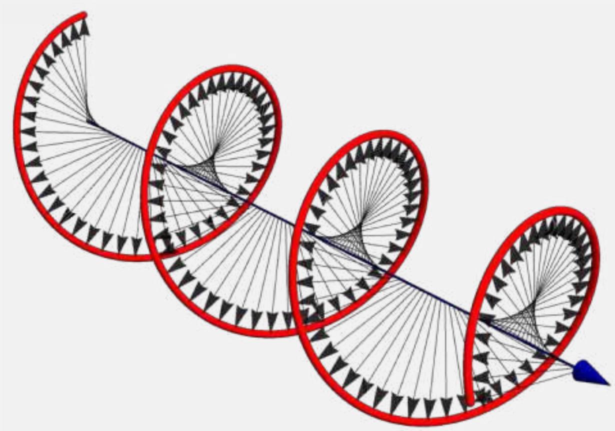
$$\omega_{\mathrm{s},\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau}\sqrt{1 - 4\tau^2 v_F^2 \frac{|\mathbf{k}|^2 w_0 - \sigma^{(\epsilon,u)}\left[2|\mathbf{k}|^2 B_0^2 - (\mathbf{k}\cdot\mathbf{B}_0)\right]}{3w_0}}$$

- *Gapped* chiral magnetic wave ($\rho = 0$):

$$\omega_{\mathrm{gCMW},\pm} = \pm\frac{eB_0\sqrt{3v_F^3\left(4\pi e^2 T_0^2 + 3\varepsilon_e \hbar^3 v_F^3 k_\parallel^2\right)}}{2\pi^2 T_0^2 c\sqrt{\varepsilon_e \hbar}}$$

- Helicons ($\rho \neq 0$):

$$\omega_{\mathrm{h},\pm} \approx \mp\frac{ck_\parallel^2 B_0}{4\pi\mu_m \rho_0} - \frac{i}{\tau}\frac{c^2 k_\parallel^2 w_0}{4\pi\mu_m v_F^2 \rho_0^2} + O(k_\parallel^3)$$



- New anomalous Hall waves at $\vec{b} \neq 0$, etc.

[Sukhachov, Gorbar, Shovkovy, Miransky, J. Phys. Cond. Matt. **30**, 275601 (2018)]

Some lessons (for high energy)

- Propagating (not overdamped) hydrodynamic modes in relativistic **charged** chiral plasma are
  - Sound and Alfven waves @ high temperature
  - Plasmons and helicons @ high density

[Rybalka, Gorbar, Shovkovy, arXiv:1807.07608]

- **Dynamical electromagnetism** plays a crucial role
  - Electrical conductivity leads to screening of charge fluctuations
  - Dynamical anomalous production of chirality plays a big role
  - Charge diffusion is substantial, unless $k \to 0$
- **Chiral magnetic waves** are strongly overdamped in near-critical quark-gluon plasma created in heavy-ion collisions

[Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]

Summary

- Anomalous *macroscopic* effects are expected in many forms of chiral plasmas
- Experimental search for anomalous signatures in high-energy physics is *extremely difficult*
- Low-energy *chiral fermions* can be realized in Dirac/Weyl materials
- Fundamental anomalous physics could be tested in *table-top experiments*
- Non-dissipative anomalous effects could be valuable in *applied* research