Dislocation-Mediated Quantum Melting

Aron Beekman

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“Chiral Matter and Topology” workshop, NTU, December 7th 2018

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- JSPS Kakenhi Grant-in-Aid for Early-Career Scientists (grant no. 18K13502)
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Figure 3 Schematic view of the local stripe order in the various phases discussed in the text. Here, we have assumed that the stripes maintain their integrity throughout, although in reality they must certainly become less and less well defined as the system becomes increasingly quantum, until eventually they are not the correct variables for describing the important correlations in the system. Heavy lines represent liquid-like stripes, along which the electrons can flow, whereas the filled circles represent pinned, density-wave order along the stripes. The stripes are shown executing more or less harmonic oscillations in the smectic phase. Two dislocations, which play an essential role in the smectic-to-nematic phase transition, are shown in the view of the nematic phase.
Dislocations

Topological defect associated with translational order

Topological charge: Burgers vector $B^\alpha$
Outline

- Classical dislocations
  - restricted motion
  - interdependence with disclinations
- Dislocation condensation = quantum melting
  - duality
  - deconfinement of disclinations
- Recent developments
  - critical properties of dislocation condensation
  - relation to fractons
  - superfluids without $U(1)$ breaking
Dislocation motion

(a) initial dislocation  
(b) glide motion  
(c) climb motion

climb motion involves the addition/removal of (interstitial) particles and is suppressed ↔ particle number conservation

Glide constraint:
“dislocations can only move in the direction of their Burgers vector”
Dislocations and disclinations

- dislocation: Burgers vector $B^a$, torsion
- disclination: Frank scalar $\Omega$, curvature

(d) dislocation  (e) disclination  (f) Volterra construction
Interdependence of dislocations and disclinations

(g) atoms
(h) disclination
(i) stack of dislocations
(j) disclination pair
(k) two disclination pairs
Defect-mediated melting

Nobel Prize in Physics 2016 citation:
“for theoretical discoveries of topological phase transitions ...”

Berezinskii–Kosterlitz–Thouless melting

Berezinskii 1970-71; Kosterlitz Thouless 1972-73
Nobel Prize in Physics 2016 citation:
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Berezinskii–Kosterlitz–Thouless melting

- in 2D, no true long-range order
- higher dimensions: order–disorder defect-unbinding phase transition
- 2+1D superfluid–Bose-Mott insulator quantum phase transition
• $D$-dim. quantum field theory $\leftrightarrow D + 1$-dim. statistical physics, e.g. 2D superfluid–insulator QPT is in the 3D $XY$ universality class.
• Time axis is the additional dimension. Statistical physics of *worldlines*.
Two-dimensional classical melting

- Unbinding of dislocations = loss of translational order

  Berezinskii 1970-71; Kosterlitz Thouless 1972-73
Two-dimensional classical melting

- Unbinding of dislocations = loss of translational order
  Berezinskii 1970-71; Kosterlitz Thouless 1972-73

- Two types of topological defects

(a) dislocation – translational

(b) disclination – rotational
Two-dimensional classical melting

- Unbinding of dislocations = loss of translational order
  Berezinskii 1970-71; Kosterlitz Thouless 1972-73

- Two types of topological defects

(a) dislocation – translational
(b) disclination – rotational

Nelson Halperin 1978-79; Young 1979
Two-dimensional classical melting

- Why is the ordinary solid-to-liquid transition first order?
Two-dimensional classical melting

- Why is the ordinary solid-to-liquid transition first order?
- Simultaneous unbinding

Kleinert 1983
Two-dimensional classical melting

- Why is the ordinary solid-to-liquid transition first order?
- Simultaneous unbinding

Towards quantum melting, zero-temperature phase transition

Kleinert 1983
Two-dimensional quantum melting

(a) 2D bound pairs

(b) 2D unbound
Two-dimensional quantum melting

(a) 2D bound pairs
(b) 2D unbound
(c) 2+1D bound loops
(d) 2D unbound worldlines
2+1D dislocation-mediated quantum melting

- ‘Statistical physics’/quantum partition sum of dislocation worldlines
- Role of inverse temperature is played by temporal correlations
- 2D quantum corresponds to 3D classical
- Proliferation of dislocation lines 3D classical

Kleinert 1980s
• ‘Statistical physics’/quantum partition sum of dislocation worldlines
• Role of inverse temperature is played by temporal correlations
• 2D quantum corresponds to 3D classical
• Proliferation of dislocation lines 3D classical

Kleinert 1980s

• Time direction manifestly different from space directions
• Condensation of dislocations = proliferation of worldlines 2+1D quantum

Zaanen Nussinov Mukhin 2004; Cvetkovic Zaanen 2006; AJB et al. 2017

Essence: the dislocation condensate is described by a collective complex field

\[ \Psi^a(x), \quad a = x, y, \quad |\Psi^a|^2 \sim \text{density of ‘worldline tangle’} \quad (1) \]

Ginzburg–Landau-type action

\[ \mathcal{L}_{GL} = \sum_{a=x,y} \left( \frac{1}{2} \alpha_a |\Psi^a|^2 + \frac{1}{4} \beta_a |\Psi^a|^4 \right) + \frac{1}{2} \gamma |\Psi^x|^2 |\Psi^y|^2 \quad (2) \]
• Duality mapping, analogous to vortex–boson / Abelian-Higgs duality
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- Phonons are gauge bosons or stress photons
- Dislocations are shear stress charges
- A solid is a stress vacuum or Coulomb gas of stress charges
Dual gauge field theory of defect-mediated melting

- Duality mapping, analogous to vortex–boson / Abelian-Higgs duality
- Phonons are gauge bosons or stress photons
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- A solid is a stress vacuum or Coulomb gas of stress charges

- An hexatic is a stress superconductor
- Dual Meissner effect: shear stress is expelled from the liquid crystal
Dual stress effective action

Classical stress energy

\[ E_{\text{solid}} = \frac{1}{2} \sigma_m^a \left( C_{mnb}^{-1} \right) \sigma_n^b \]

elastic moduli

Quantum stress Lagrangian

\[ L_{\text{solid}} = \frac{1}{2\rho} (\sigma^a_\tau)^2 + \frac{1}{2} \sigma_m^a C_{mnb}^{-1} \sigma_n^b \]
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Stress is conserved

\[ \partial_\tau \sigma^a_\tau + \partial_m \sigma^a_m = \partial_\mu \sigma^a_\mu = 0 \]

Dual stress gauge field

\[ \sigma^a_\mu = \epsilon_{\mu \nu \lambda} \partial_\nu b^a_\lambda, \quad a = x, y \]
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\[ L_{\text{solid}} = \frac{1}{2} (\epsilon_{\mu\kappa\lambda} \partial_\kappa b^a_\lambda) C_{\mu\nu ab}^{-1} (\epsilon_{\nu\rho\sigma} \partial_\rho b^a_\sigma), \]
Dual stress effective action

Classical stress energy

\[ E_{\text{solid}} = \frac{1}{2} \sigma_m \left( \begin{array}{cc} \sigma^a_m & C^{-1}_{mnab} \sigma^b_n \end{array} \right) \]

quantitative moduli

Quantum stress Lagrangian

\[ L_{\text{solid}} = \frac{1}{2\rho} (\sigma^a_\tau)^2 + \frac{1}{2} \sigma^a_m C^{-1}_{mnab} \sigma^b_n \]

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\[ L_{\text{GL}} = \sum_{a=x,y} \left( \frac{1}{2} \alpha_a |\Psi^a|^2 + \frac{1}{4} \beta_a |\Psi^a|^4 \right) + \frac{1}{2} \gamma |\Psi^x|^2 |\Psi^y|^2, \]

\[ L_{\text{coupling}} = \frac{1}{2} \sum_{a=x,y} |(\partial_\mu - ib^a_\mu - i\lambda \epsilon_{\tau \mu a})\Psi^a|^2. \]
1. Phonons are gauge bosons
2. The disordered solid is a stress superconductor
3. The disordered solid is a real superfluid (longitudinal response)
4. Rotational Goldstone mode deconfines in qu. hexatic (transverse response)
5. Transverse phonon becomes gapped shear mode in quantum hexatic
6. The gapped shear mode is detectable by finite-momentum spectroscopy

Disclination deconfinement

- Displacement field $u^a(x)$
- Rotation field $\omega^{ab} = \partial_a u^b(x) - \partial_b u^a(x)$

<table>
<thead>
<tr>
<th></th>
<th>solid</th>
<th>hexatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagrangian stress</td>
<td>$u^a(\partial_t^2 + \nabla^2)u^a$</td>
<td>$u^a(\partial_t^2 + \nabla^2 +</td>
</tr>
<tr>
<td>rotation</td>
<td>$\omega^{ab}\nabla^2(\partial_t^2 + \nabla^2)\omega^{ab}$</td>
<td>$\ldots + \omega^{ab}</td>
</tr>
<tr>
<td>propagator stress</td>
<td>$\frac{1}{\omega^2+q^2}$</td>
<td>$\frac{1}{\omega^2+q^2+</td>
</tr>
<tr>
<td>rotation</td>
<td>$\frac{1}{q^2(\omega^2+q^2)}$</td>
<td>$\ldots + \frac{</td>
</tr>
<tr>
<td>static limit stress</td>
<td>$\frac{1}{q^2}$</td>
<td>$\frac{1}{q^2+</td>
</tr>
<tr>
<td>rotation</td>
<td>$\frac{1}{q^4}$</td>
<td>$\ldots + \frac{</td>
</tr>
</tbody>
</table>

For the same reason, rotational Nambu–Goldstone modes are absent in solid, but present in quantum hexatic.
Helium monolayers on ZYX exfoliated graphite
S. Nakamura et al. PRB 94, 180501(R) (2016)
Helium monolayer experiments

Helium monolayers on ZYX exfoliated graphite
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- anomaly in specific heat: BKT-like defect-unbinding transition
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Effective field theory (Ginzburg–Landau)

\[ \mathcal{L}_{\text{solid}} = \frac{1}{2} (\epsilon_{\mu\kappa\lambda} \partial_\kappa b^a_\lambda) C^{-1}_{\mu\nu a b} (\epsilon_{\nu\rho\sigma} \partial_\rho b^a_\sigma), \]

\[ \mathcal{L}_{\text{GL}} = \sum_{a = x, y} \left( \frac{1}{2} \alpha_a |\Psi^a|^2 + \frac{1}{4} \beta_a |\Psi^a|^4 \right) + \frac{1}{2} \gamma |\Psi^x|^2 |\Psi^y|^2, \]

\[ \mathcal{L}_{\text{coupling}} = \frac{1}{2} \sum_{a = x, y} |(\partial_\mu - ib^a_\mu - i\lambda \epsilon_\tau a) \Psi^a|^2. \]

Simplified to

\[ \mathcal{L}_{\text{solid}} = \frac{1}{2} (\nabla \times b^x)^2 + \frac{1}{2} (\nabla \times b^y)^2, \]

\[ \mathcal{L}_{\text{GL}} = \frac{1}{2} m^2 (|\Psi^x|^2 + |\Psi^y|^2) + \frac{1}{4} \lambda (|\Psi^x|^4 + |\Psi^y|^4) + \frac{1}{2} g |\Psi^x|^2 |\Psi^y|^2, \]

\[ \mathcal{L}_{\text{coupling}} = \frac{1}{2} |(\nabla - i e b^x) \Psi^x|^2 + \frac{1}{2} |(\nabla - i e b^x) \Psi^y|^2 \]

• Only \( m^2 \) and \( \lambda^2 \): \( O(2) \) Wilson-Fisher theory
• with \( e \): charged \( O(2) \), Abelian-Higgs model
• with \( g \): \( O(2) \times O(2) \), two-component BEC
Critical properties of solid-to-hexatic quantum melting in $d = 3$

Abelian-Higgs $d = 3$

shown with FRG down to $N = 2$:
G. Fejos & T. Hatsuda

Two-component BEC $d = 3$

$\varepsilon$-expansion
Ceccarelli et al.
PRA 92, 024513 (2016) 93, 033647 (2017)

Work in progress (with Gergely Fejos):
- FRG for charged $O(2) \times O(2)$ in $d = 3$, charged fixed points
- Influence of stress gauge field dynamics
- Influence of glide constraint
- Quantum critical exponent for specific heat
Fractons: objects/particles with spatially restricted dynamics

Fracton-Elasticity Duality

Motivated by recent studies of fractons, we demonstrate that elasticity theory of a two-dimensional quantum crystal is dual to a fracton tensor gauge theory, providing a concrete manifestation of the fracton phenomenon in an ordinary solid. The topological defects of elasticity theory map onto charges of the tensor gauge theory, with disclinations and dislocations corresponding to fractons and dipoles, respectively. The transverse and longitudinal phonons of crystals map onto the two gapless gauge modes of the gauge theory. The restricted dynamics of fractons matches with constraints on the mobility of lattice defects. The duality leads to numerous predictions for phases and phase transitions of the fracton system, such as the existence of gauge theory counterparts to the (commensurate) crystal, supersolid, hexatic, and isotropic fluid phases of elasticity theory. Extensions of this duality to generalized elasticity theories provide a route to the discovery of new fracton models. As a further consequence, the duality implies that fracton phases are relevant to the study of interacting topological crystalline insulators.

DOI: 10.1103/PhysRevLett.120.195301

FIG 1  (a) The Fracton-Elasticity Dictionary: Excitations and...
Duality in 2 + 1D quantum elasticity: superconductivity and quantum nematic order

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† Theoretical Physics Department, Moscow Institute for Steel and Alloys, Moscow 119992, Russian Federation

Received 18 September 2003


a superfluid is an elastic medium having lost its rigidity against shear stresses due to the presence of a dual dislocation condensate, to arrive at a complete characterization. We emphasize that both viewpoints are equally correct. All one can discern in the scaling limit is the universal definition, as given by Landau:

a superfluid is a state of matter characterized by a low energy spectrum which is exhausted by a propagating, massless compression mode.

Since both ‘states’ are hydrodynamically indistinguishable they are actually the same state: by general principle it has to be possible to adiabatically continue the Bose-gas superfluid into our ‘order’ superfluid. These are just limiting ‘microscopic’ descriptions of the same entity. Among others, this also implies a ‘don’t worry’ theorem regarding the problem that interstitials cannot be incorporated in the field theory. Because of continuity, it has to be that these can be ‘smoothly’ inserted in the theory, driving the superfluid away from the order-asymptote which is not a singular limit.

To complete the argument we still have to demonstrate that in this irrotational fluid a genuine Meissner effect should occur when electromagnetic fields are coupled in.
Duality in 2 + 1D quantum elasticity: superconductivity and quantum nematic order

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“Superfluid Goldstone mode arises only when $U(1)$ particle conservation symmetry is broken, i.e. when the glide constraint is relaxed.”

Pretko & Radzihovsky arXiv:1808.05616

Kumar & Potter arXiv:1808.05621


Nicolis, Brauner, Watanabe, Hidaka, ...
Summary

- Condensation of topological defects as symmetry-restoring phase transition
- Topological defects in solids study case for restricted mobility
  - dislocations: glide constraint
  - disclination: confinement
- Possibly novel critical behaviour
- Nature of superfluid sound

Collaborators:

Jan Zaanen  Leiden  Robert-Jan Slager  Dresden
Jaakko Nissinen  Aalto  Vladimir Cvetkovic
Kai Wu  Zohar Nussinov  St. Louis
Ke Liu  Munich  Gergely Fejos  Keio U
Assumptions and limitations

- Zero temperature
- Ginzburg–Landau → only near the phase transition
- London limit, phase fluctuations only
- Maximal crystalline correlations (collective physics only)
- No interstitials
- No disclinations
- Bosons only but 4-He and 3-He experiments similar
- Isotropic solid only
3D quantum liquid crystals

- phonons are now two-form gauge fields $b_{\mu\nu}$
- quantum versions of columnar, smectic and nematic liquid crystals

<table>
<thead>
<tr>
<th>Goldstone modes</th>
<th>solid</th>
<th>columnar</th>
<th>smectic</th>
<th>nematic</th>
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<tbody>
<tr>
<td>phonons</td>
<td>2+1D</td>
<td>2 / 0</td>
<td>1 / 0</td>
<td>0 / 1</td>
</tr>
<tr>
<td>rotational</td>
<td>3+1D</td>
<td>3 / 0</td>
<td>2 / 0</td>
<td>1 / 1</td>
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