Quantum Response of (Helical) Majorana Fermions in Topological Superconductors

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Kobayashi-Yamakage-Tanaka-MS, arXiv:1812.01857(today)

In collaboration with

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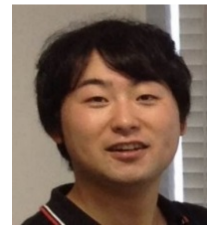
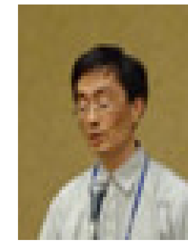
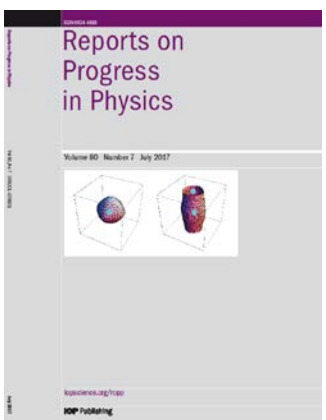






A review paper on topological SCs with Yoichi Ando

MS, Ando, Rep. Prog. Phys. 80, 076501 (17)

Outline

1. Motivation

2. Anisotropic magnetic response of helical MFs

MS-Fujimoto, PR B79, 094504 (2009)
Mizushima-MS-Machida, PRL, 109, 165301 (2012)

3. Majorana multipole response of helical MFs

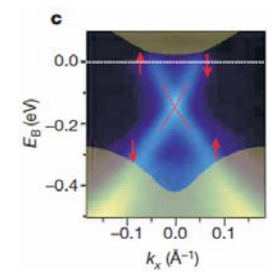
Kobayashi-Yamakage-Tanaka-MS, arXiv:1812.01857

Motivation

MFs were originally proposed as elementary particles, but now we know that they can be emergent excitations in electron or atomic systems.

Majorana Fermions in S-wave SC

- Dirac fermion + s-wave condensate — MS(03), Fu-Kane (08)



Hsieh et al

- S-wave superconducting state with Rashba SO + Zeeman field — MS-Takahashi-Fujimoto (09), J. Sau et al (10)

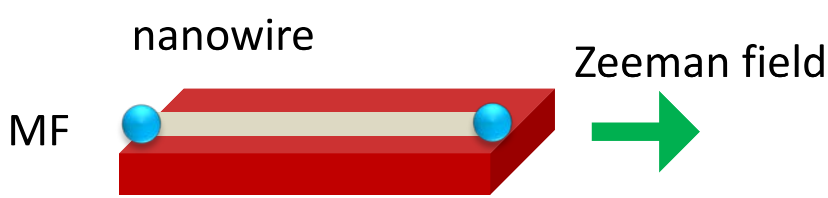

PRL 103, 020401 (2009) PHYSICAL REVIEW LETTERS week e[nding] 10 JULY

Non-Abelian Topological Order in *s*-Wave Superfluids of Ultracold Fermionic Atoms

Masatoshi Sato,[1] Yoshiro Takahashi,[2] and Satoshi Fujimoto[2]

$$B_{\rm Zeeman} > \sqrt{\Delta^2 + \mu^2}$$

Condition for MF



Lutchyn et al (10), Oreg et al (10)

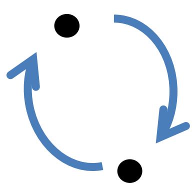
These emergent MFs in condensed matter physics share some properties with elementary Majorana particles in high energy physics

- Both obey the Dirac equation with self-charge-conjugation condition

$$\mathcal{H}(\boldsymbol{k}) = \sum_{i=1}^{d} \gamma_i k_i \qquad\qquad \psi(\boldsymbol{k}) = \mathcal{C}\psi(-\boldsymbol{k})$$

charge-conjugation

- Zero modes exhibit non-Abalian anyon statistics



However, there is an essential difference between them

CPT theorem

- CPT is a fundamental symmetry of relativistic QFT

  C: charge conjugation P: parity (inversion) T: time-reversal

- Any reasonable relativistic QFT is invariant under CPT

- Elementary MFs should respect CPT inv. since they should respect Lorentz inv.
- This means that **elementary MFs are self-conjugate under CPT,** not merely under C

➡ **This fundamental invariance of elementary MFs gives a strong constraint in electromagnetic responses**

Electromagnetic response of elementary MFs

Kayser-Goldhaber (83)

General form of one-particle EM-coupling for spin-1/2 relativistic fermions

$$(q = p_f - p_i)$$

$$\langle p_f | j_\mu A^\mu | p_i \rangle = i\bar{u}_{p_f} \left[F\gamma_\mu + M\sigma_{\mu\nu}q^\nu + iE\sigma_{\mu\nu}q^\nu\gamma_5 + G(q^2\gamma_\nu - \not{q}q_\nu)\gamma_5 \right] u_{p_i} A^\mu$$

electric charge — magnetic dipole $\propto \boldsymbol{B}$ — electric dipole $\propto \boldsymbol{E}$ — toroidal moment

self-conjugation condition under CPT

$$\langle p_f | j_\mu^{\mathrm{MF}} A^\mu | p_i \rangle = i\bar{u}_{p_f} \left[0 + 0 + 0 + G(q^2\gamma_\nu - \not{q}q_\nu)\gamma_5 \right] u_{p_i} A^\mu$$

- Charge neutral condition forMFs (F=0)
- Electro-magnetic dipole momenta of MFs vanish (M=E=0)

Elementary MFs only show moderate EM responses

However, emergent MFs are not subject to such a strong constraint.

- Emergent MFs only have approximate Lorentz invariance.
- They are self-conjugate just under C, not under CPT.

◆ A different scheme is needed to describe EM responses of emergent MFs

◆ In this talk, I will present a general theory of EM responses of emergent MFs in time-reversal invariant TSCs

Majorana multipole response in topological superconductors

Xiong-Yamakage-Kobayashi-MS-Tanaka, Crystal 2017, 7, 58
Kobayashi-Yamakage-Tanaka-MS, arXiv:1812.01857

Anisotropic magnetic response of MF

Helical Majorana fermions in TRI topological SCs show peculiar anisotropic magnetic response.

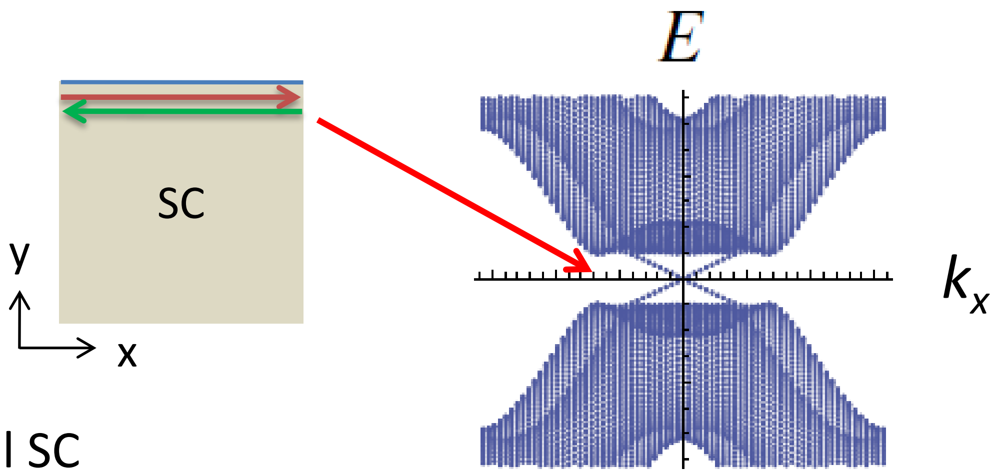
MS-Fujimoto (09)
Chung-Zhang (09)

2dim p-wave Rashba noncentrosymmetric SC

$$\Delta(\boldsymbol{k}) = i\boldsymbol{d}(\boldsymbol{k})\cdot\boldsymbol{\sigma}\sigma_y$$

$$\boldsymbol{d}(\boldsymbol{k}) \propto \lambda_{\rm SO}(-k_y, k_x, 0)$$

Helical Majorana fermion



Non-trivial Z_2 topological number

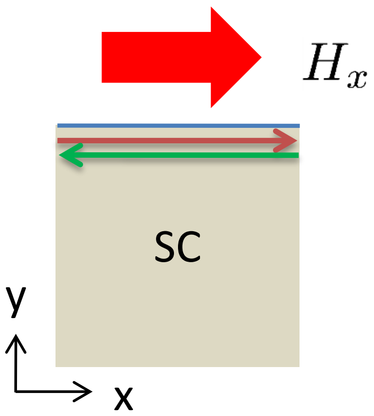
2dim time-reversal invariant helical SC

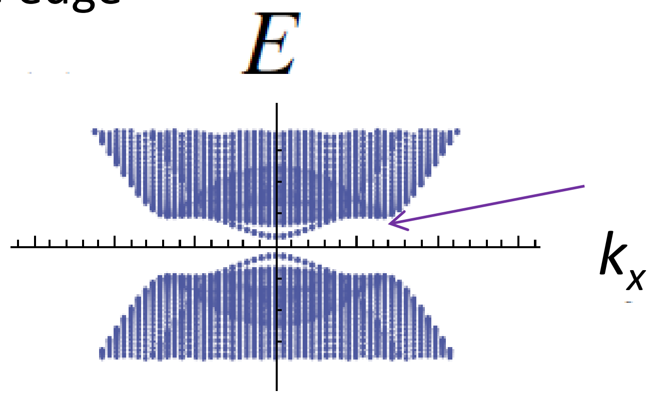
MS-Fujimoto (09), Y. Tanaka et al (09)

Under Zeeman fields, the helical MF shows anisotropic response.

MS-Fujimoto (09)

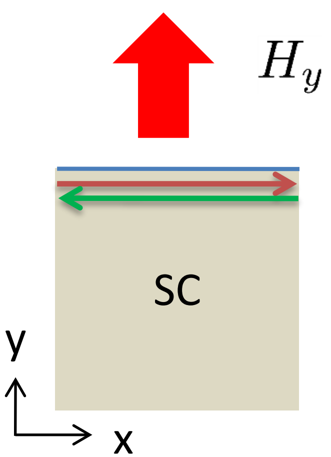
- Zeeman field **along** the edge

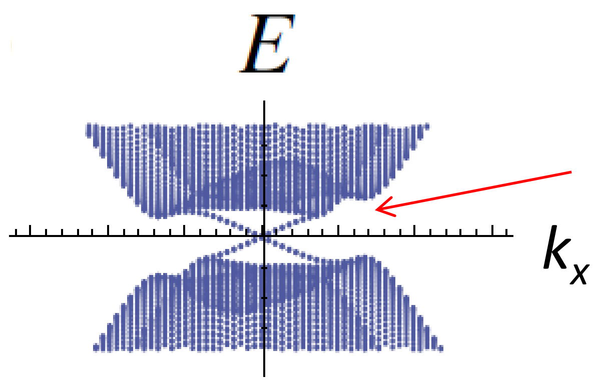




Gap opens due to TR breaking

- Zeeman field **normal** to edge



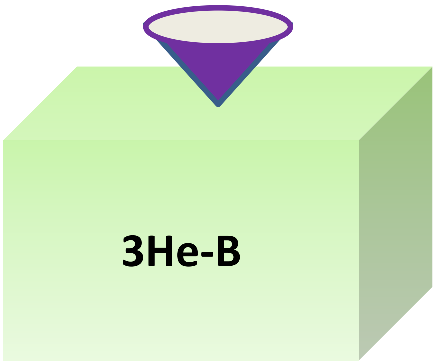


No gap opens in spite of TR breaking

A similar anisotropic magnetic response has been reported in 3dim time-reversal invariant SCs

Chung-Zhang(09)
Shindou-Furusaki-Nagaosa(10)

Helical Majorana surface state in ^{3}He-B



$$\begin{bmatrix} \hat{\psi}_{\rightarrow} \\ \hat{\psi}_{\leftarrow} \\ \hat{\psi}^{\dagger}_{\rightarrow} \\ \hat{\psi}^{\dagger}_{\leftarrow} \end{bmatrix} = \sum_{\boldsymbol{k}_{\parallel}} (\hat{\gamma}_{\boldsymbol{k}} e^{i\boldsymbol{k}_{\parallel}\cdot\boldsymbol{r}_{\parallel}} + \hat{\gamma}^{\dagger}_{\boldsymbol{k}} e^{-i\boldsymbol{k}_{\parallel}\cdot\boldsymbol{r}_{\parallel}}) \begin{bmatrix} \cos\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \\ \cos\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \end{bmatrix}$$

$$\times u_{\boldsymbol{k}} e^{\Delta z/\hbar v_{\mathrm{F}}} \sin(\sqrt{k_{\mathrm{F}}^2 - k_{\parallel}^2}\, z)$$

Spin density op.

$$\hat{I}_x = (\hat{\psi}^{\dagger}_{\rightarrow}\hat{\psi}_{\rightarrow} - \hat{\psi}^{\dagger}_{\leftarrow}\hat{\psi}_{\leftarrow})/2 = 0 \qquad \hat{I}_y = (\hat{\psi}^{\dagger}_{\rightarrow}\hat{\psi}_{\leftarrow} + \hat{\psi}^{\dagger}_{\leftarrow}\hat{\psi}_{\rightarrow})/2 = 0$$

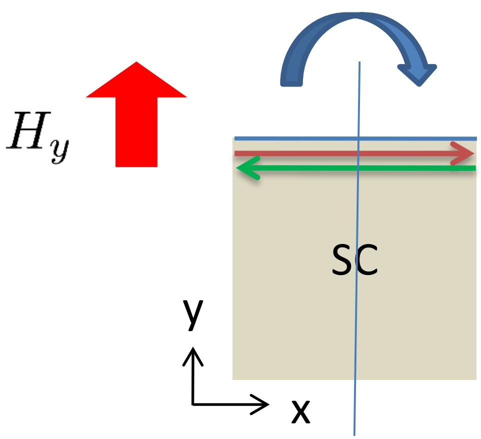
$$\hat{I}_z = -i\hat{\psi}^{\dagger}_{\rightarrow}\hat{\psi}_{\leftarrow} \neq 0$$

- MF behaves like **Ising spin (=magnetic dipole)**
- MF does **not couple to magnetic fields parallel to the surface**

These anisotropic behaviors can be explained by crystalline sym.

Mizushima-MS-Machida (12) Shiozaki-MS (14)

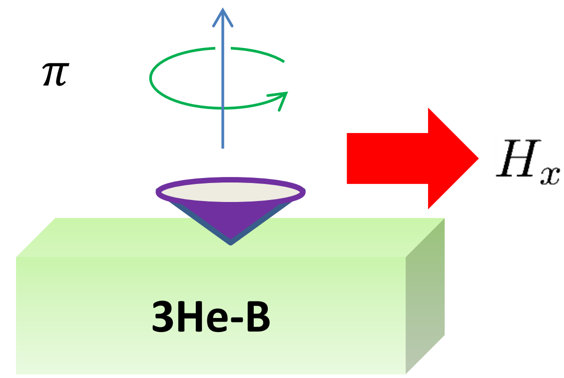
Rashba SC



TRS, mirror reflection

magnetic mirror reflection
(TRS+mirror reflection)

3He-B



TRS, rotation

magnetic two-fold rotation
(TRS+two-fold rotation)

- TRS can remain partially as magnetic symmetry.
- The remaining anti-unitary symmetry may stabilize gapless helical MFs under magnetic fields

Actually, one can define top. # by using these magnetic symmetries

BdG Hamiltonian

symmetric momentum under mirror/C_2-rot.

$$w_{\mathrm{M1D}} = \frac{i}{4\pi}\int dk_\perp \mathrm{tr}\left[\Gamma_{\mathrm{M}}\mathcal{H}^{-1}(k_\perp, \boldsymbol{k}_\parallel^0)\partial_{k_\perp}\mathcal{H}(k_\perp, \boldsymbol{k}_\parallel^0)\right]$$

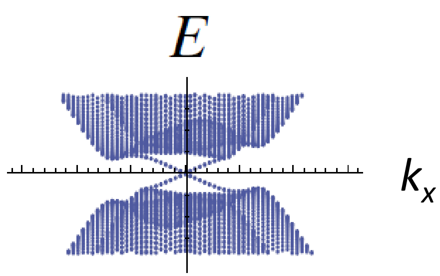
$$\Gamma_{\mathrm{M}} = \mathcal{U}\mathcal{T}\mathcal{C}$$

PHS

magnetic mirror/ C_2-rot.

For Rashba SC

$$\mathcal{U} = \mathcal{M}_y$$

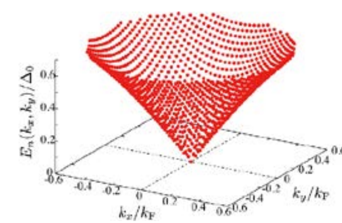


$$w_{\mathrm{M1D}} = 2$$

spin-degeneracy

For 3He-B

$$\mathcal{U} = C_2$$



$$w_{\mathrm{M1D}} = 2$$

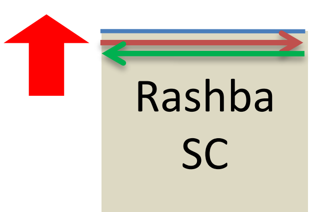
spin-degeneracy

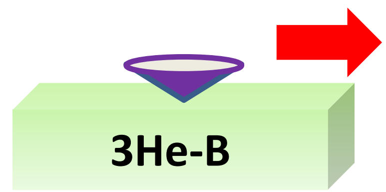
Therefore, the magnetic winding # naturally explain why helical MFs can stay gapless even under magnetic fields

Question

1. How can we know magnetic response more systematically?

2. What determines the details of anisotropic behavior?





Similar but different anisotropic behavior

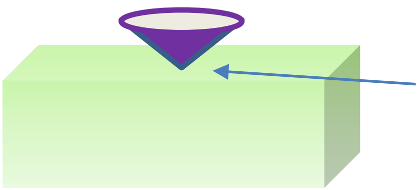
To address these questions, we develop a general theory of quantum response of MFs

Basic idea

Use the generalized index theorem to evaluate quantum operator

Generalized index theorem

MS-Tanaka-Yada-Yokoyama PRB (11)、
Xiong-Yamakage-Kobayashi-MS-Tanaka(17)



$|u_0^{(a)}\rangle$

Gapless MF is an eigenstate of $\Gamma_{\rm M}$

For $w_{\rm M1D} > 0$ $\quad \Gamma_{\rm M}|u_0^{(a)}\rangle = |u_0^{(a)}\rangle$

For $w_{\rm M1D} < 0$ $\quad \Gamma_{\rm M}|u_0^{(a)}\rangle = -|u_0^{(a)}\rangle$

$$n = |w_{\rm M1D}| \equiv \left| \frac{i}{4\pi} \int dk_\perp {\rm tr}[\Gamma_{\rm M} \mathcal{H}^{-1}(k_\perp, \boldsymbol{k}_\parallel) \partial_{k_\perp} \mathcal{H}(k_\perp, \boldsymbol{k}_\parallel)] \right|$$

Spin Structure
= Magnetic Response

How to evaluate quantum operator

Nambu base

Quantum op. $$\hat{O} = \hat{c}^\dagger_\sigma(x) O_{\sigma,\sigma'} \hat{c}_{\sigma'}(x) = \frac{1}{2}\hat{\Psi}^\dagger(x)\mathcal{O}\hat{\Psi}(x)$$

hermitian

$$\Psi(x) = (\hat{c}_\sigma(x), \hat{c}^\dagger_\sigma(x))^t \qquad \mathcal{O} = \begin{pmatrix} O & 0 \\ 0 & -O^T \end{pmatrix}$$

First, perform mode expansion of quantum field,

$$\Psi(x) = \sum_a \hat{\gamma}^{(a)} |u_0^{(a)}\rangle + \cdots, \qquad \{\hat{\gamma}^{(a)}, \hat{\gamma}^{(b)}\} = 2\delta^{ab}$$

gapless MF

Substituting this, we can extract the contribution of gapless MFs as

$$\hat{O}_{\rm MF} = \frac{1}{2}\sum_{ab} \hat{\gamma}^{(a)}\hat{\gamma}^{(b)} \langle u_0^{(a)}|\mathcal{O}|u_0^{(b)}\rangle$$

$$\hat{O}_{\rm MF} = \frac{1}{4i}\sum_{ab} \hat{\gamma}^{(a)}\hat{\gamma}^{(b)} {\rm tr}\left[\mathcal{O}\rho^{(ab)}\right] \qquad \rho^{(ab)} = i\left(|u_0^{(b)}\rangle\langle u_0^{(a)}| - |u_0^{(a)}\rangle\langle u_0^{(b)}|\right)$$

Using this expression, we derive symmetry constraints for gap function and quantum operator with nonzero value $O_{\rm MF}$

First, sym. of the gap fun. should be selected to obtain nonzero w_{M1D},

1d wind. # for mag. C_2 rot.

$$w_{\mathrm{M1D}} = \frac{i}{4\pi}\int dk_\perp \mathrm{tr}\left[\Gamma_{\mathrm{M}}\mathcal{H}^{-1}\partial_{k_\perp}\mathcal{H}\right]$$

even under C_2 (A rep)

$$C_2\Delta(\boldsymbol{k})C_2^t = \Delta(C_2\boldsymbol{k})$$

For Nambu space

$$\tilde{C}_2 = \begin{pmatrix} C_2 & 0 \\ 0 & C \end{pmatrix}$$

Particle and hole b

the same way

$$\tilde{C}_2\mathcal{C} = \mathcal{C}\tilde{C}_2$$

compatible with PHS $\mathcal{C}$ **stable MF**

$$w_{\mathrm{M1D}} \neq 0$$

odd under C_2 (B rep)

$$C_2\Delta(\boldsymbol{k})C_2^t = -\Delta(C_2\boldsymbol{k})$$

For Nambu space

n a different

manner

$$\tilde{C}_2\mathcal{C} = -\mathcal{C}\tilde{C}_2$$

incompatible with PHS **unstable MF**

$$w_{\mathrm{M1D}} = 0$$

helical MFs protected by mag. C_2

Second, the operator O should be the same representation as $\rho^{(ab)}$

$$\hat{O}_{\mathrm{MF}} = \frac{1}{4i}\sum_{ab}\hat{\gamma}^{(a)}\hat{\gamma}^{(b)}\mathrm{tr}\left[\mathcal{O}\rho^{(ab)}\right] \qquad \rho^{(ab)} = i\left(|u_0^{(b)}\rangle\langle u_0^{(a)}| - |u_0^{(a)}\rangle\langle u_0^{(b)}|\right)$$

For instance, from the index theorem

$$\Gamma_{\mathrm{M}}|u_0^{(a)}\rangle = |u_0^{(a)}\rangle \;\Rightarrow\; \Gamma_{\mathrm{M}}\rho^{(ab)}\Gamma_{\mathrm{M}}^{-1} = \rho^{(ab)}$$

Thus, O should be even under magnetic CS

$$\Gamma\mathcal{O}\Gamma_{\mathrm{M}}^{-1} = \mathcal{O}$$

In Nambu space

$$\mathcal{O} = \begin{pmatrix} O & 0 \\ 0 & -O^T \end{pmatrix} = \begin{pmatrix} O & 0 \\ 0 & -O^* \end{pmatrix} \;\Rightarrow\; \mathcal{C}\mathcal{O}\mathcal{C}^{-1} = -\mathcal{O}$$

Thus, for magnetic operator $\mathcal{T}\mathcal{O}\mathcal{T}^{-1} = -\mathcal{O}$, using the definition $\Gamma_{\mathrm{M}} = \tilde{C}_2\mathcal{T}\mathcal{C}$, we have

$$\tilde{C}_2\mathcal{O}\tilde{C}_2^{-1} = \mathcal{O}$$

even under C_2 (A rep)

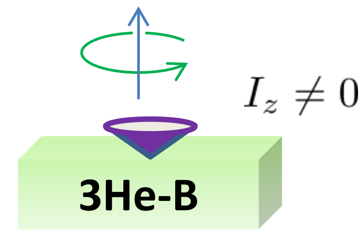
In this manner, we complete list of gap functions with nonzero w_{M1D} and the corresponding mag. multipoles for all surface point groups

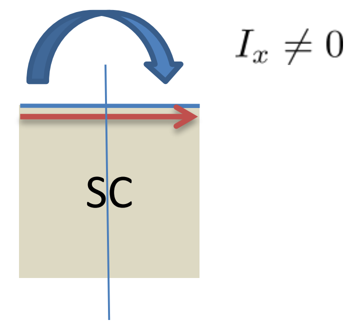
Kobayashi-Yamakage-Tanaka-MS (18)

PG	Δ_Γ	basis of Δ_Γ $(\times e^{-i\pi J_y})$	$\mathcal{U}$	$\mathcal{O}_\Gamma$	basis of $\mathcal{O}_\Gamma$
C_2, C_4, C_6	A	$\boldsymbol{k}\cdot\boldsymbol{J}$	C_2	A	J_z
C_3	–	–	–	–	–
C_s	A	$k_xJ_z, k_xJ_y, k_yJ_x, k_zJ_x$	$\sigma_v(yz)$	A	J_x
C_{2v}	A_2	k_zJ_z	C_2	A_2	J_z
	B_1	k_xJ_z, k_zJ_x	$\sigma_v(yz)$	B_1	J_x
	B_2	k_yJ_z, k_zJ_y	$\sigma_v(xz)$	B_2	J_y
C_{3v}	A_1	$k_z(J_x^3 - J_xJ_yJ_y - J_yJ_xJ_y - J_yJ_yJ_x)$	$\sigma_v(yz)$	A_1	$J_x^3 - J_xJ_yJ_y - J_yJ_xJ_y - J_yJ_yJ_x$
C_{4v}	A_2	k_zJ_z	C_2	A_2	J_z
C_{6v}	A_2	k_zJ_z	C_2	A_2	J_z
	B_1	$k_z(J_x^3 - J_xJ_yJ_y - J_yJ_xJ_y - J_yJ_yJ_x)$	$\sigma_v(yz)$	B_1	$J_x^3 - J_xJ_yJ_y - J_yJ_xJ_y - J_yJ_yJ_x$
	B_2	$k_z(J_y^3 - J_yJ_xJ_x - J_xJ_yJ_x - J_xJ_xJ_y)$	$\sigma_d(xz)$	B_2	$J_y^3 - J_yJ_xJ_x - J_xJ_yJ_x - J_xJ_xJ_y$

high spin Cooper pairs

magnetic octupole

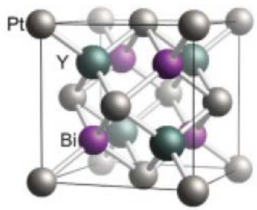




Our theory predicts magnetic octupole response in high spin TSC !!

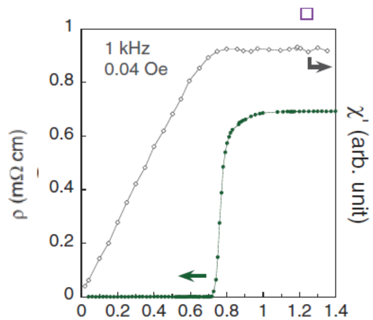
Application to half-Heusler SCs

YPtBi



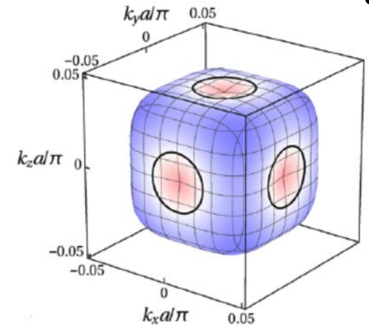
Butch et al (11)

experiment



T_c=0.7K

theory



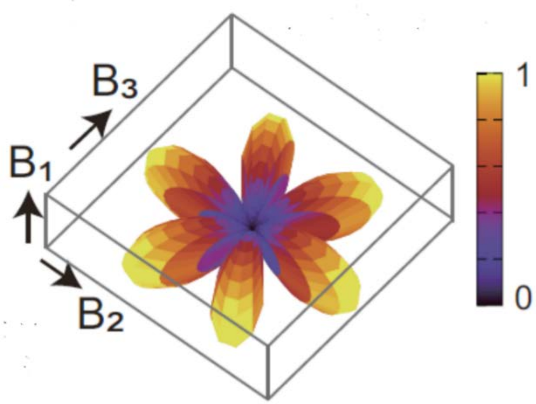
Brydon et al (16)

proposed gap fn.

$$\Delta(\boldsymbol{k}) \propto \left[\eta \mathbf{1}_{4\times 4} + \sum_i k_i (J_{i+1} J_i J_{i+1} - J_{i+2} J_i J_{i+2})\right] e^{-iJ_y \pi}$$

high-spin Cooper pair

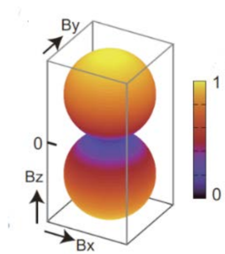
Our result for mag resp of MFs on [111]



$$\Delta E \propto |B_2|(B_2^2 - 3B_3^2)$$

octupole response

c.f) ^{3}He-B



Summary

1. In contrast to elementary Majorana particles, emergent MFs may exhibit richer magnetic structures.

2. We find a one-to-one correspondence between symmetry of Cooper pairs and rep. of magnetic response, which provides a novel way to identify unconventional SC.

3. Detection of magnetic octupole response of MFs is a direct evidence of high spin topological superconductivity.