Lattice Study of **Energy-Momentum Tensor** with Gradient Flow: Thermodynamics, Correlations, and Stress

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In Collaboration with:
FlowQCD Collab.(R. Yanagihara, T. Iritani, Asakawa, Hatsuda, Suzuki)
WHOT Collab.(Taniguchi, Kanaya, Ejiri, Suzuki, Umeda, Shirogane, …)

Workshop of Recent Developments in QCD and QFT, Nov. 10, 2017, Taipei

Energy-Momentum Tensor

One of the most fundamental quantities in physics

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\partial_{\mu} T_{\mu\nu} = 0$$

Energy-Momentum Tensor

One of the **most fundamental** quantities in physics

energy momentum

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pressure

stress

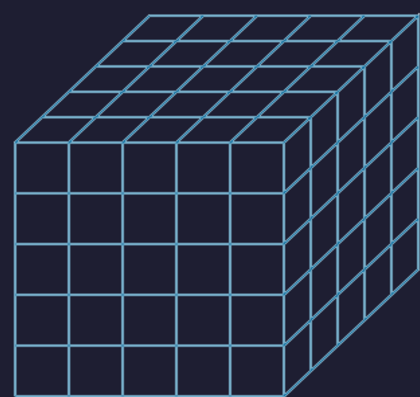
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\partial_\mu T_{\mu\nu} = 0$$

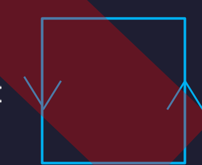
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy

due to high dimensionality and etc.

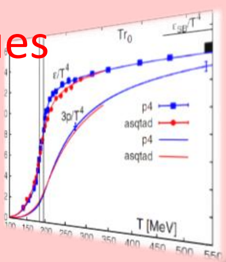
$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

energy (T_{00}), momentum (T_{01}, T_{02}, T_{03}), stress (T_{ij}), pressure (diagonal T_{11}, T_{22}, T_{33})

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

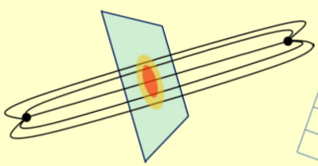
viscosity, specific heat, ...

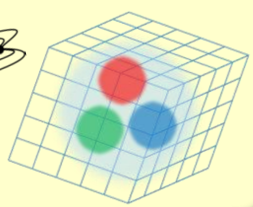
$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$

$c_V \sim \langle \delta T_{00}^2 \rangle$

Hadron Structure

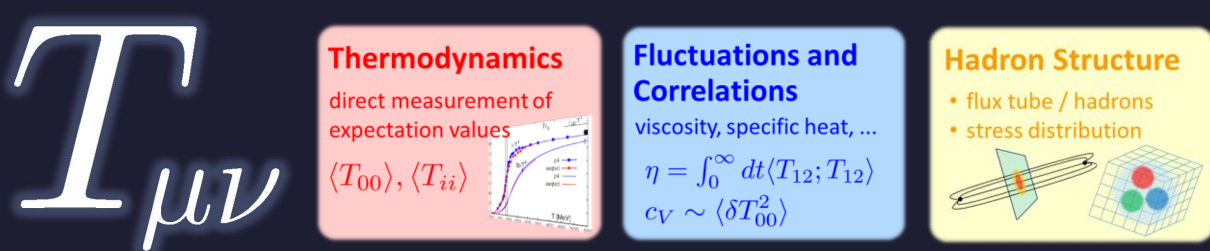
- flux tube / hadrons
- stress distribution





Contents

$T_{\mu\nu}$



(Yang-Mills) Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

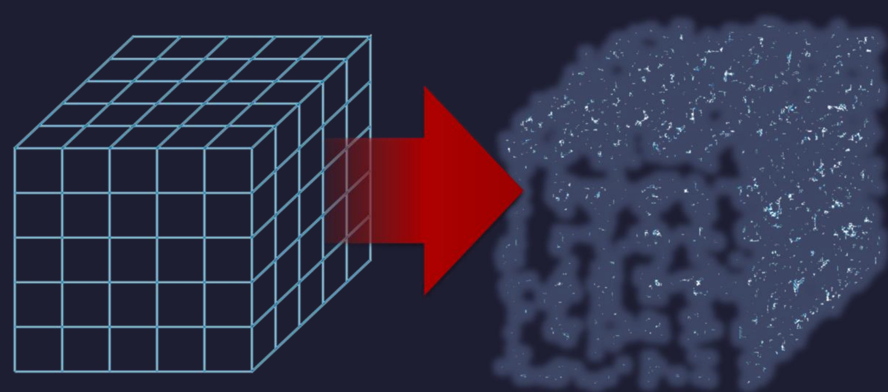
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing



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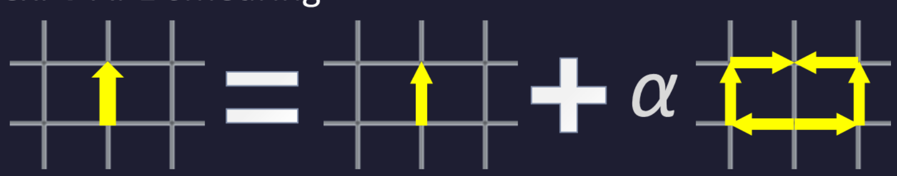
ex. ：APE smearing



(YM) Gradient Flow: Properties

- All (composite) operators are finite at t>0 Luscher,Weisz,2011

  ⇒ All operators are renormalized at t>0
  Safe a→0 limit

- Quite effective in reducing statistical error.
- Flowed field ≠ original field
  - Gradient flow is not an approximation method.

- Applications
  - scale setting
  - topological charge/susceptibility
  - Structure Func. / PDF

見小利則大事不成

Miss the wood for the trees

小利を見ればすなわち大事成らず

孔子

confucius

（論語、子路13）

見小利則大事不成

UV fluctuations　　Physics

Miss the wood for the trees

小利を見ればすなわち大事成らず

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confucius

（論語、子路13）

Gradient Flow for Fermions

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

- Not "gradient" flow but a "diffusion" equation.
- Flow for gauge field is independent of fermion field.

- Divergence in field renormalization of fermions.
- All observables becomes finite once Z(t) is determined.

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

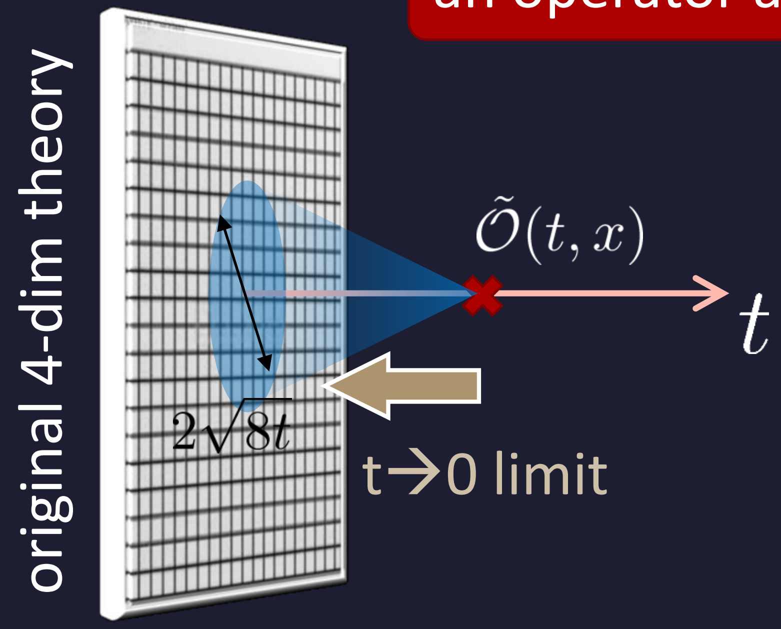
Small Flow-time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

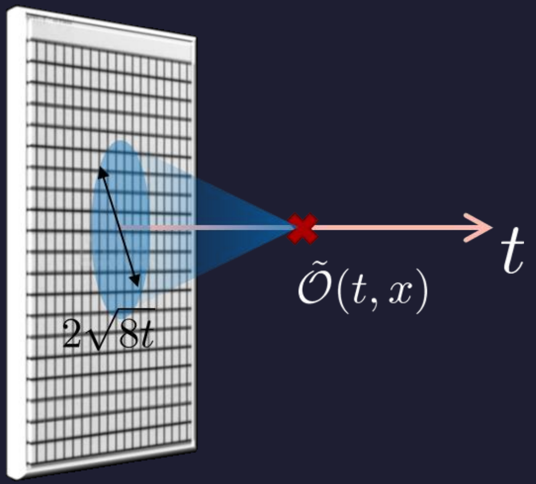
an operator at t>0

remormalized operators of original theory



Constructing EMT

Suzuki, 2013
DelDebbio,Patella,Rago,2013



$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

- **gauge-invariant dimension 4 operators**

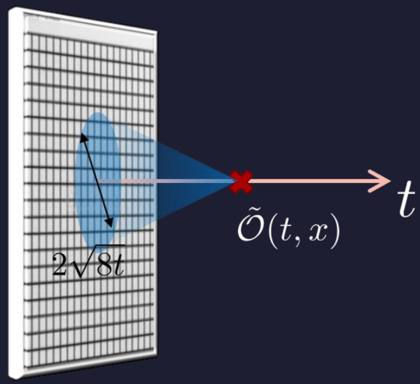
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs.

$$\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

Constructing EMT 2

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$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

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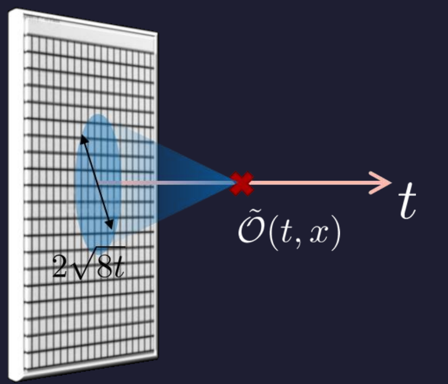


Suzuki coeffs. $\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

EMT with Fermions

Makino, Suzuki, 2014

$$T_{\mu\nu}(t,x) = c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big)$$
$$+ c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big)$$
$$+ c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big)$$

$$T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\nu}(t,x)$$

$$\tilde{O}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{O}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{O}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),$$

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

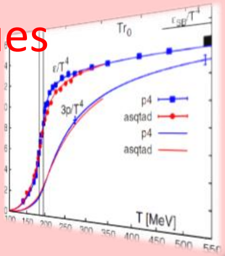
$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right],$$
$$c_2(t) = \frac{1}{(4\pi)^2}\frac{33}{16},$$
$$c_3(t) = \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\},$$
$$c_4(t) = \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2,$$
$$c^f_5(t) = -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\}$$

Contents

Thermodynamics

direct measurement of expectation values

$\langle T_{00}\rangle, \langle T_{ii}\rangle$



$T_{\mu\nu}$

Thermodynamics

direct measurement of expectation values

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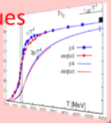


Fluctuations and Correlations

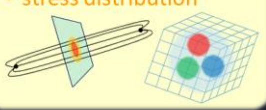
viscosity, specific heat, ...

$\eta = \int_0^\infty dt \langle T_{12}; T_{12}\rangle$

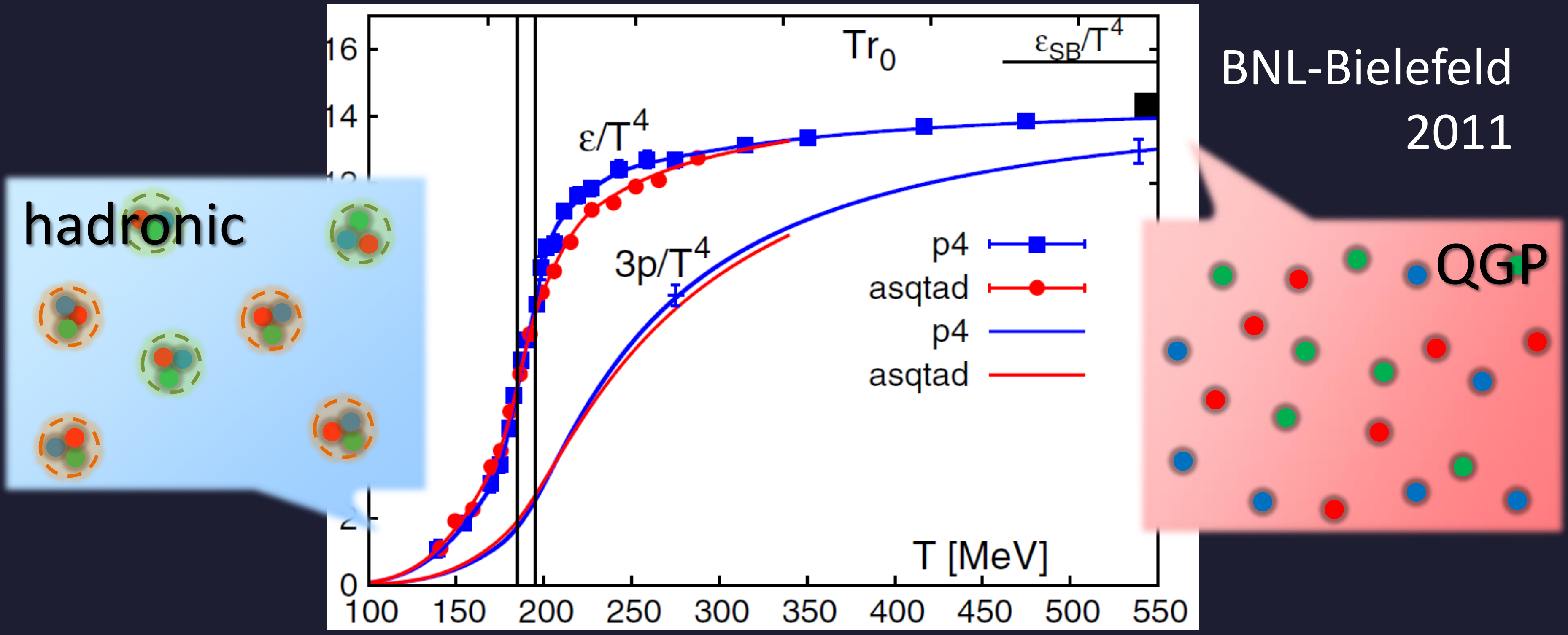
$c_V \sim \langle \delta T_{00}^2\rangle$

Hadron Structure

- flux tube / hadrons
- stress distribution



QCD EoS
(Energy Density, Pressure)



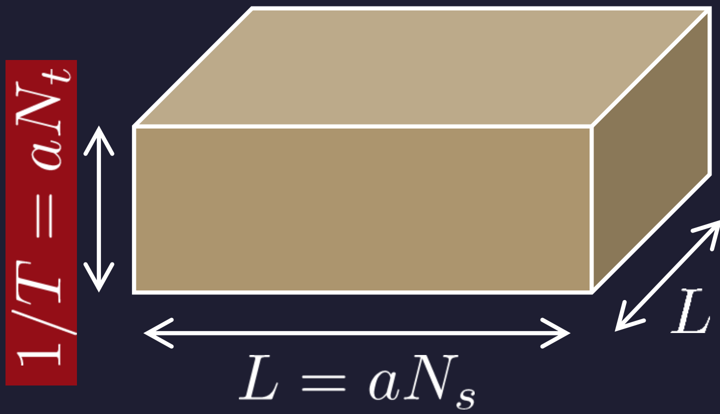
BNL-Bielefeld
2011

- Rapid increase of ε/T^4 around T=150-200 MeV
- Crossover transition
- Low T: hadron resonance gas model / High T: perturbative QCD

QCD Thermodynamics

Statistical Mechanics

$$\varepsilon = \frac{T^2}{V}\frac{\partial \ln Z}{\partial T} \qquad p = T\frac{\partial \ln Z}{\partial V}$$



Changing lattice spacing a ⇒ 1/T and V change

$$\begin{cases} \dfrac{\partial \ln Z}{\partial a} \sim \varepsilon - 3p \\ \dfrac{\partial \ln Z}{\partial a} = \dfrac{\partial \beta}{\partial a}\dfrac{\partial \ln Z}{\partial \beta} \sim \dfrac{\partial \beta}{\partial a}\langle S \rangle \\ \beta = 2N_c/g^2 \end{cases}$$

$\dfrac{\partial \beta}{\partial a}$, $\langle S \rangle$

⇓

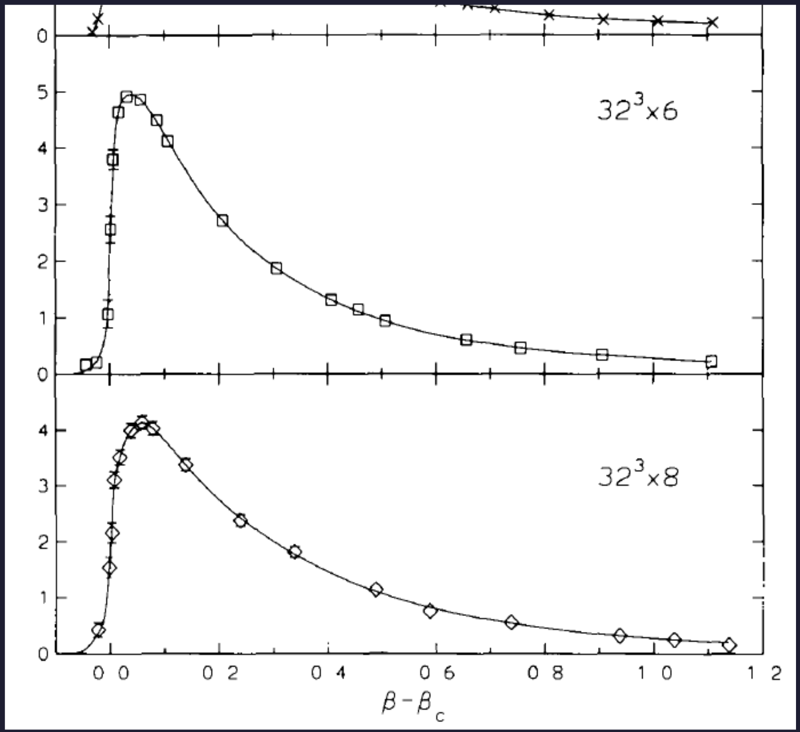
e-3p

Integral Method

$$T\frac{\partial(p/T^4)}{\partial T} = \frac{\varepsilon - 3p}{T^4}$$

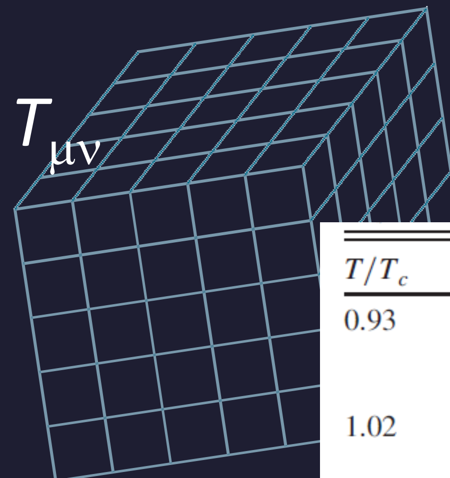
$$\frac{p}{T^4} = \int_{T_0}^{T} dT \frac{\varepsilon - 3p}{T^5}$$

Boyd+ 1996



- measurements of e-3p for many T
- vacuum subtraction for each T
- information on beta function

Numerical Simulation



MK+ (FlowQCD),
PRD**94**, 114512 (2016)

- ☐ Expectation values of $T_{\mu\nu}$
- ☐ SU(3) YM theory
- ☐ Wilson gauge action
- ☐ Parameters:
  - N_t = 12, 16, 20-24
  - aspect ratio $5.3<N_s/N_t<8$
  - 1500~2000 configurations

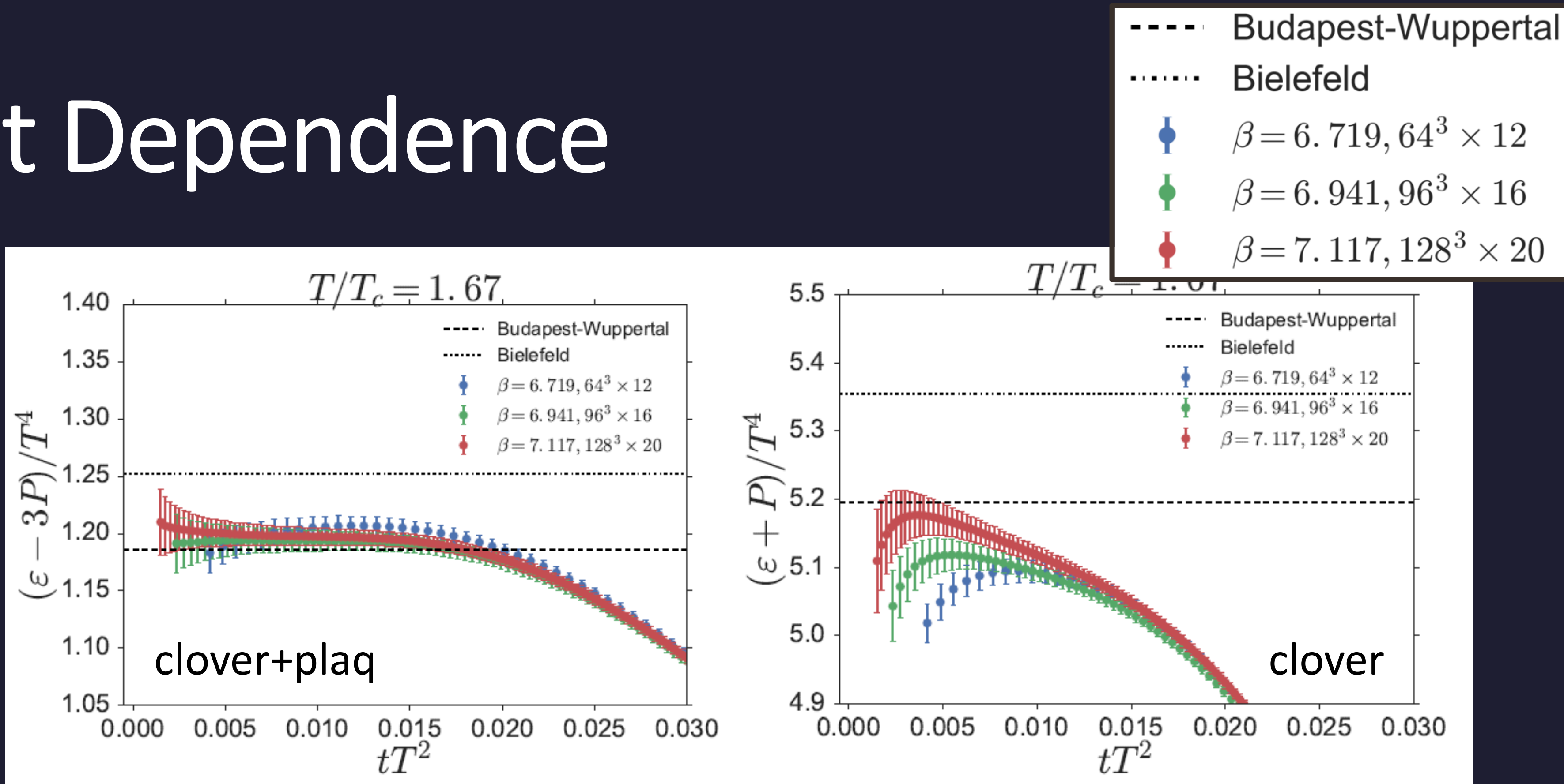
- ☐ Scale from gradient flow

$$\to aT_c \text{ and } a\Lambda_{\text{MS}}$$

FlowQCD 1503.06516

T/T_c	β	N_s	N_τ	Configurations
0.93	6.287	64	12	2125
	6.495	96	16	1645
	6.800	128	24	2040
1.02	6.349	64	12	2000
	6.559	96	16	1600
	6.800	128	22	2290
1.12	6.418	64	12	1875
	6.631	96	16	1580
	6.800	128	20	2000
1.40	6.582	64	12	2080
	6.800	128	16	900
	7.117	128	24	2000
1.68	6.719	64	12	2000
	6.941	96	16	1680
	7.117	128	20	2000
2.10	6.891	64	12	2250
	7.117	128	16	840
	7.296	128	20	2040
2.31	7.200	96	16	1490
	7.376	128	20	2020
	7.519	128	24	1970
2.69	7.086	64	12	2000
	7.317	96	16	1560
	7.500	128	20	2040

t Dependence

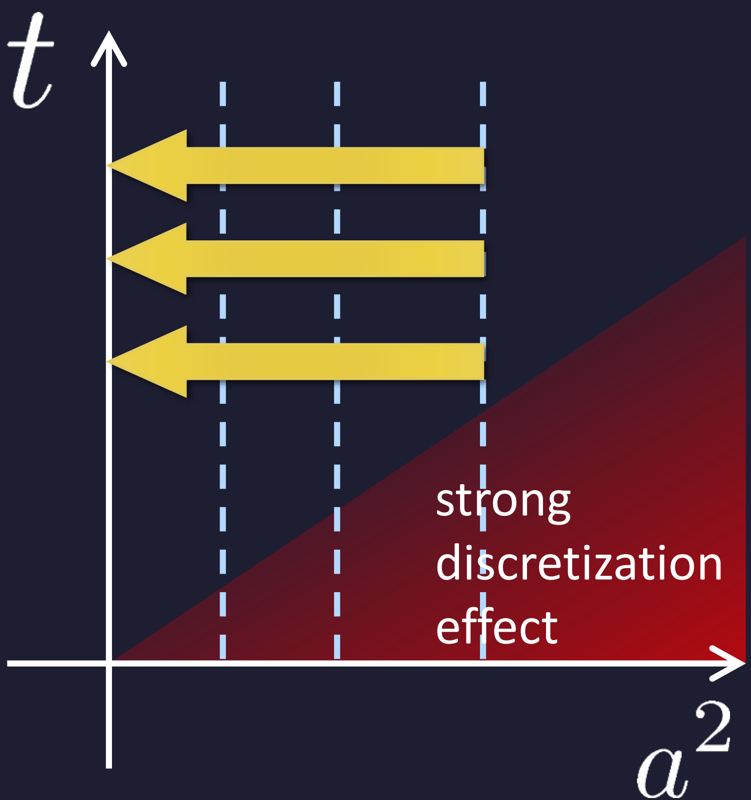


$\sqrt{8t} < a$: strong discretization effect

$\sqrt{8t} > 1/(2T)$: over smeared

$a < \sqrt{8t} < 1/(2T)$: Linear t dependence

Double Extrapolation

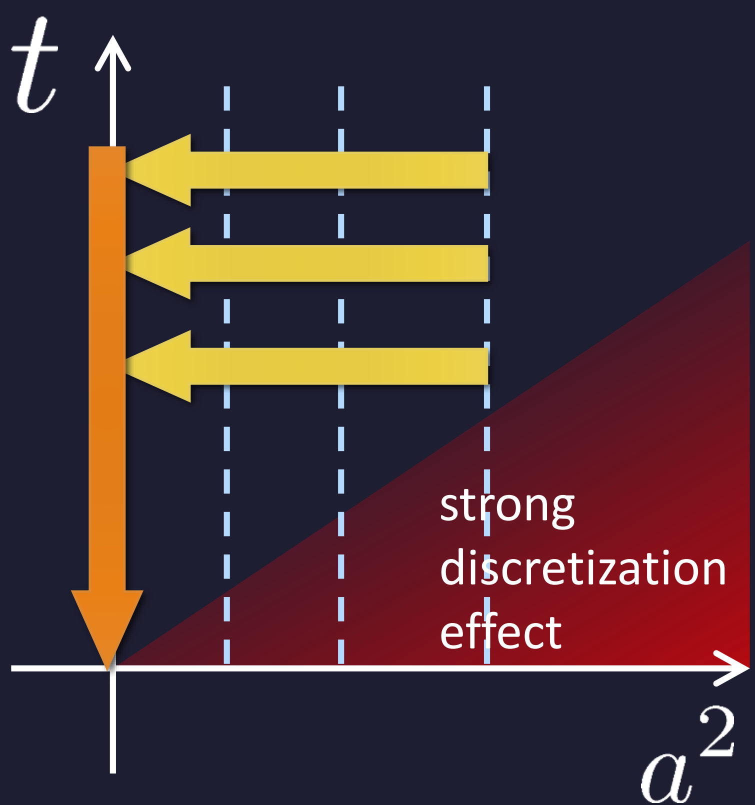


Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Note: FlowQCD, 2014: continuum extrapolation only
WHOT-QCD, 2016: small t limit only

Double Extrapolation



Continuum extrapolation

$$\langle T_{\mu\nu}(t)\rangle_{\rm cont} = \langle T_{\mu\nu}(t)\rangle_{\rm lat} + C(t)a^2$$

Small t extrapolation

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

Note: FlowQCD, 2014: continuum extrapolation only
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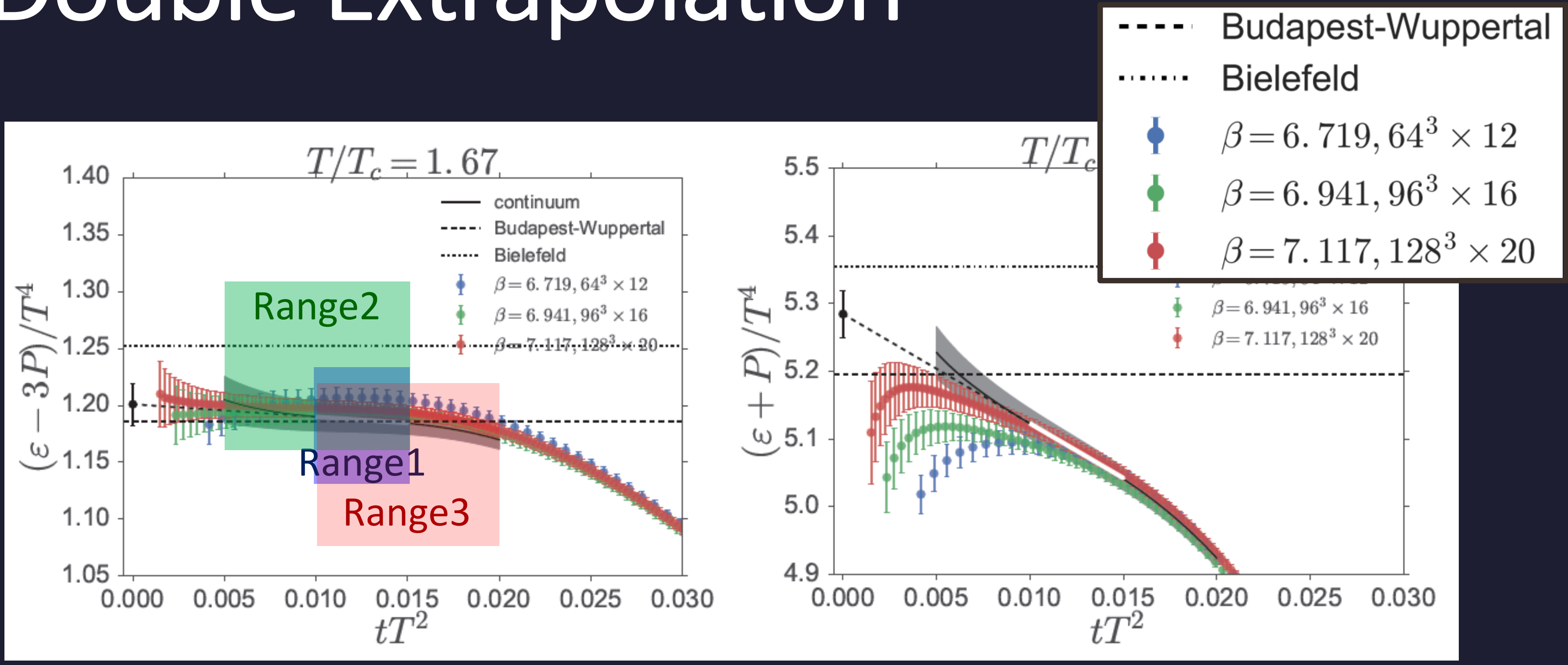
Double Extrapolation



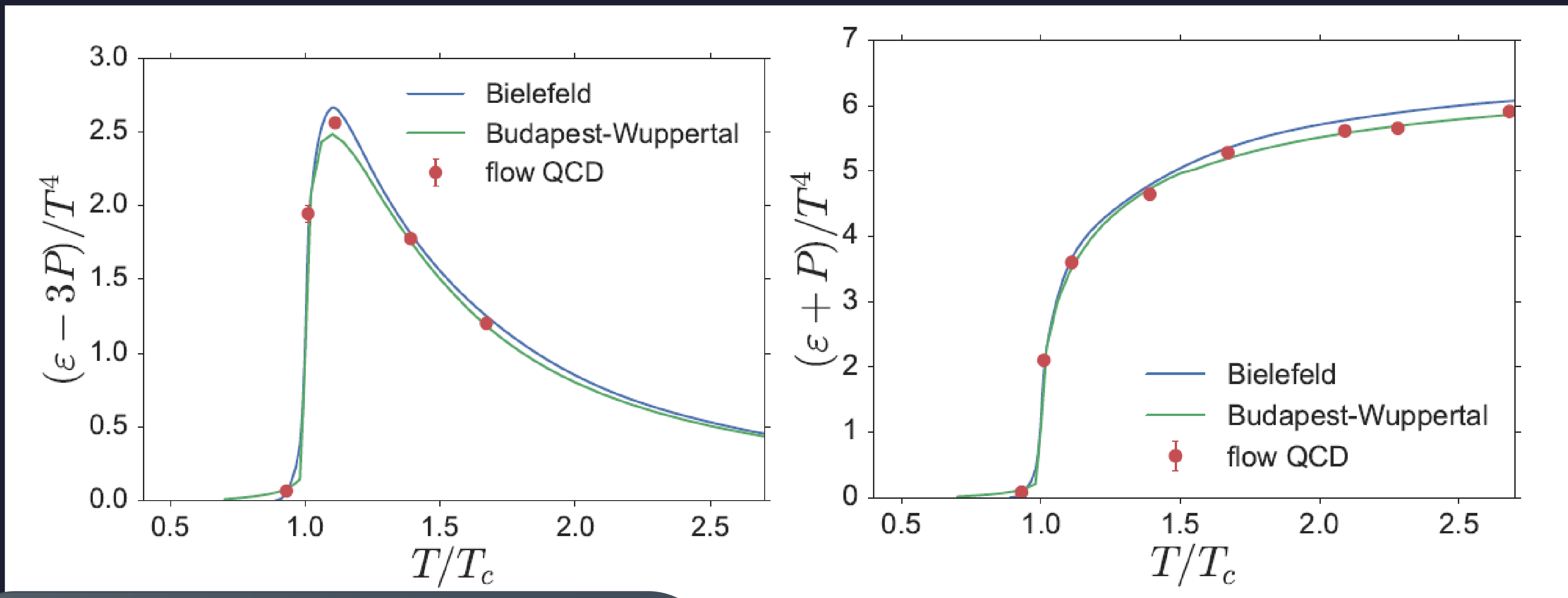
Black line: continuum extrapolated

- Fitting ranges:
  - range-1: $0.01 < tT^2 < 0.015$
  - range-2: $0.005 < tT^2 < 0.015$
  - range-3: $0.01 < tT^2 < 0.02$

Systematic error from the choice of fitting range ≈ statistical error

T Dependence

FlowQCD, PRD, 2016



Error includes

- statistical error
- choice of t range for t→0 limit
- uncertainty in $a\Lambda_{MS}$

total error <1.5% for $T>1.1T_c$

- ☐ Excellent agreement with integral method
- ☐ High accuracy only with ~2000 confs.

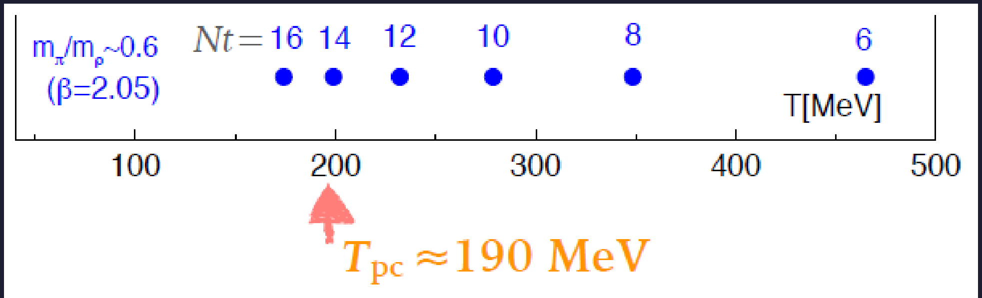
N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD),
PR**D96**, 014509 (2017)

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- m_{PS}/m_V ≈0.63 / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, ... , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)

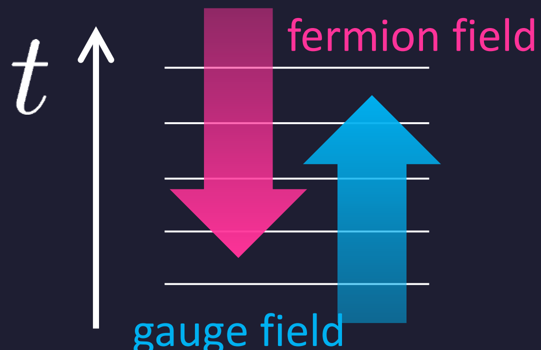


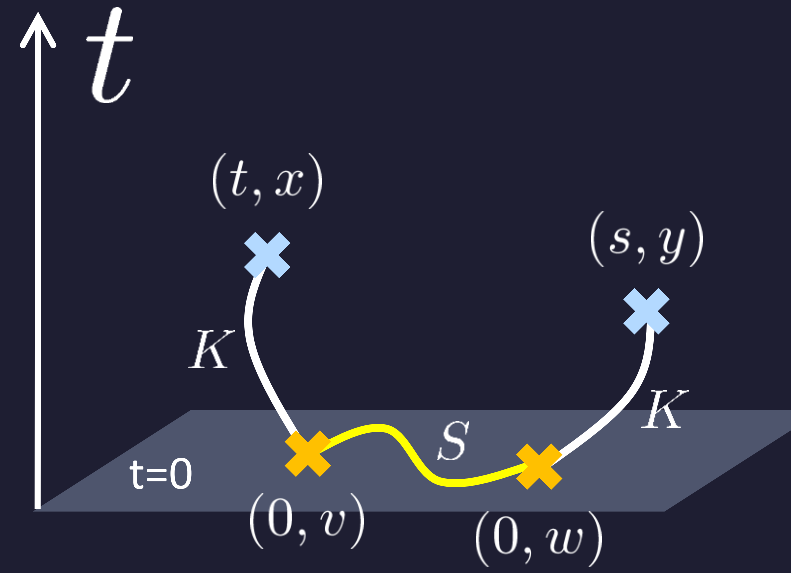
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^\dagger$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

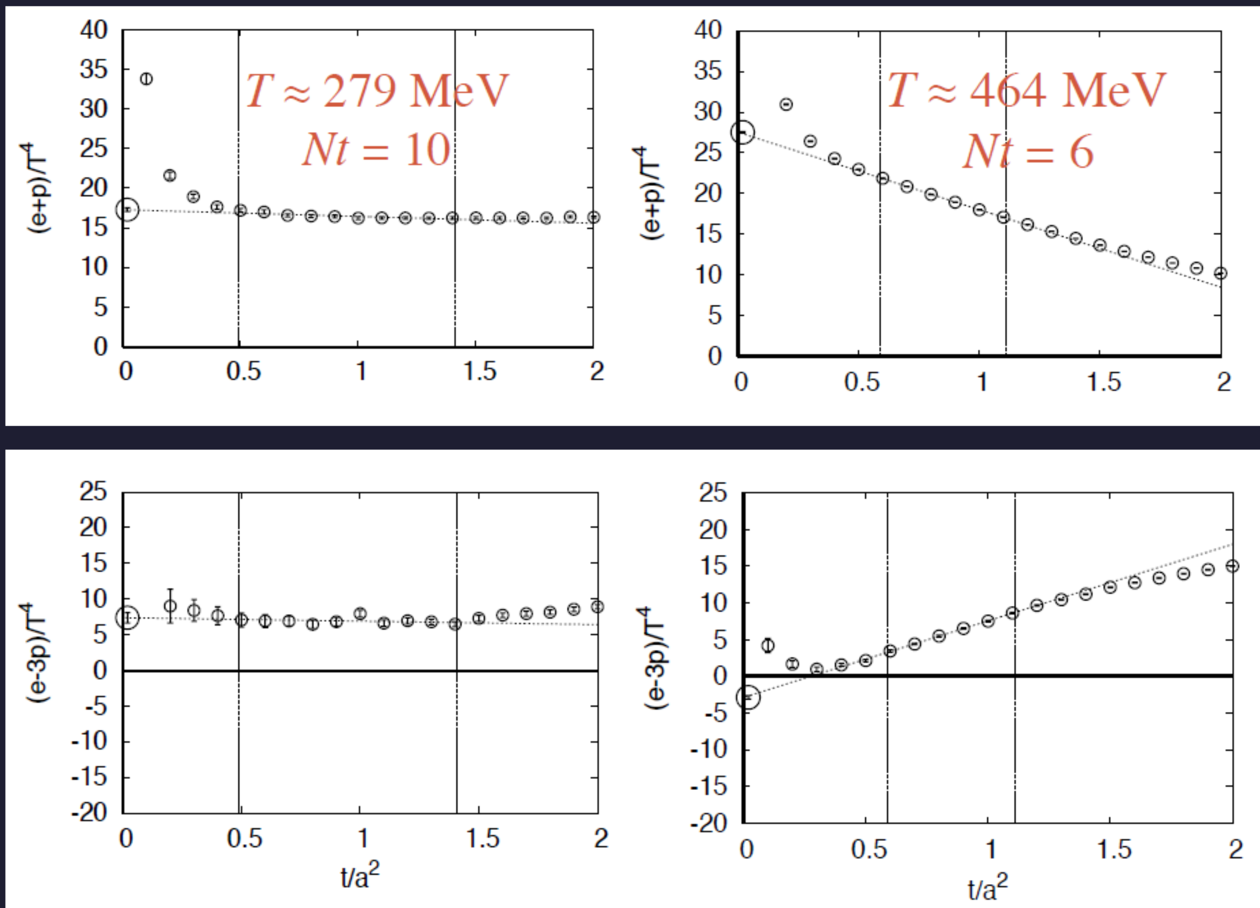
- propagator of flow equation
- Inverse propagator is needed

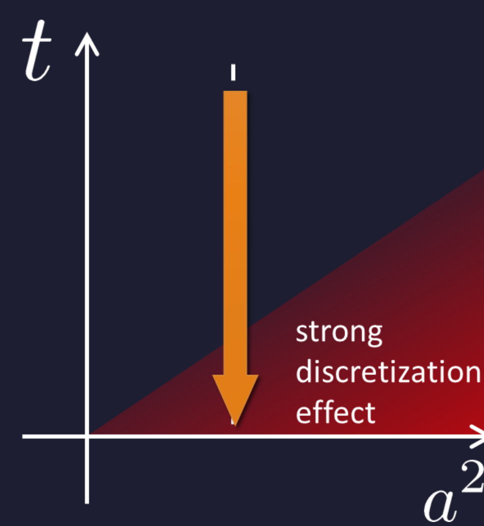




t→0 Extrapolation

Taniguchi+ (WHOT-QCD), PR**D96**, 014509 (2017)



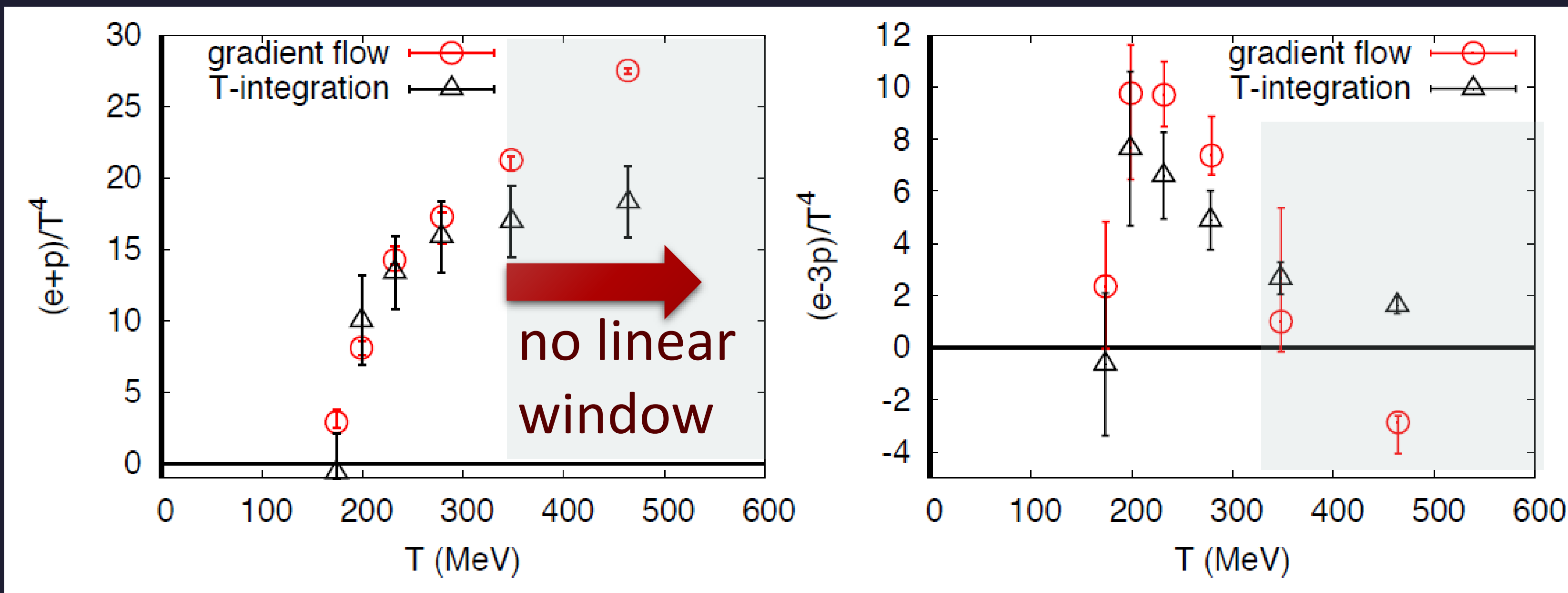


$$\begin{cases} \dfrac{t}{a^2} \geq 1 \\ \sqrt{8t} \leq \dfrac{1}{2T} \end{cases}$$

- ”linear window” for Nt>6
- Checked: fit range, a^2/t term

N_f=2+1 Thermodynamics

Taniguchi+ (WHOT-QCD),
PR**D96**, 014509 (2017)



- ☐ Agreement with integral method except for N_t=4, 6
- ☐ No stable extrapolation for N_t=4, 6
- ☐ Suppression of statistical error

Physical mass: Kanaya+ (WHOT-QCD), 1710.10015

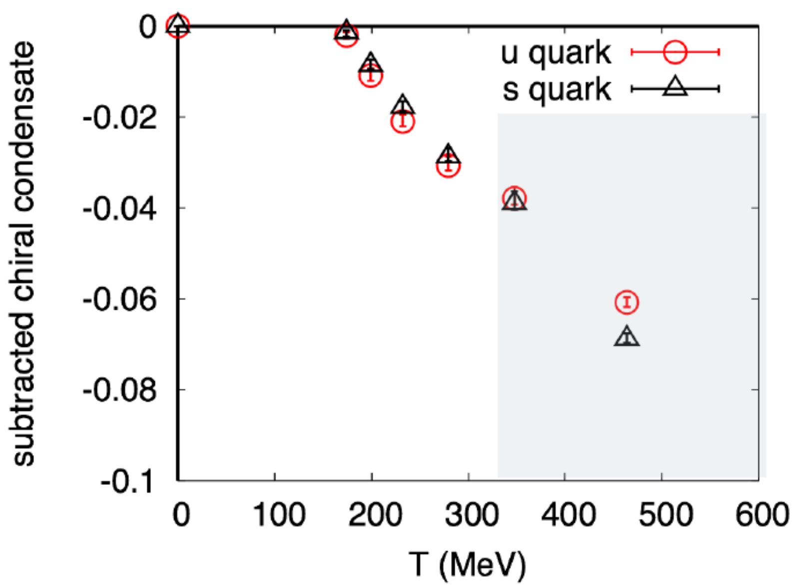
Chiral Condensate / Suceptibility

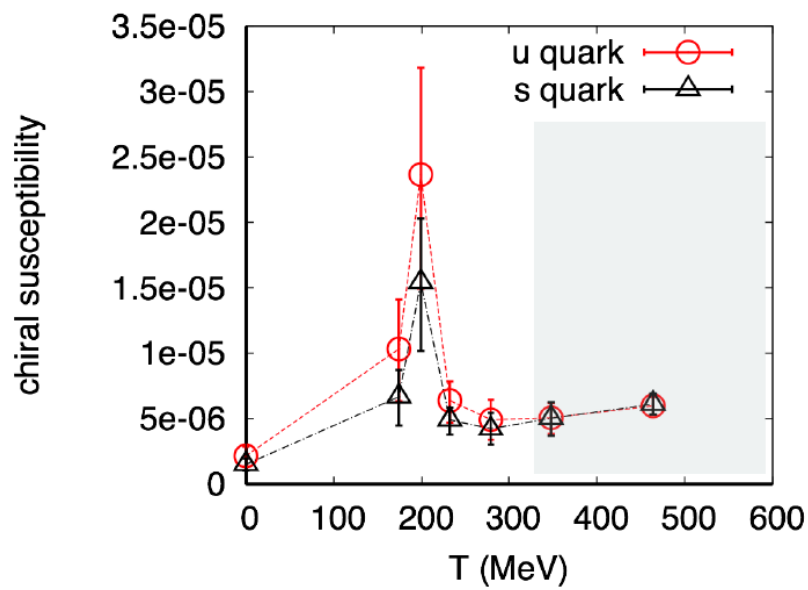
Taniguchi+ (WHOT-QCD),
PR**D96**, 014509 (2017)

Subtracted condensate

$$\langle\bar{\psi}\psi\rangle - \langle\bar{\psi}\psi\rangle_{T=0}$$

Chiral susceptibility





- Chiral condensate decreases for T>Tc.
- Chiral susceptibility has a sharp peak around T=Tc.

Contents

1. Constructing EMT on the lattice
2. Thermodynamics
3. Correlation Function
4. Stress distribution in $q\bar{q}$ system
5. Summary

Fluctuations and Correlations

viscosity, specific heat, ...

$$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$$

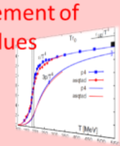
$$c_V \sim \langle \delta T_{00}^2 \rangle$$

$$T_{\mu\nu}$$

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

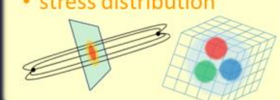
viscosity, specific heat, ...

$\eta = \int_0^\infty dt \langle T_{12}; T_{12} \rangle$

$c_V \sim \langle \delta T_{00}^2 \rangle$

Hadron Structure

- flux tube / hadrons
- stress distribution



Why EMT Correlation Func.?

- Kubo Formula: T_{12} correlator ←→shear viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x, -i\tau) T_{12}(0,t) \rangle$$

  - Hydrodynamics describes long range behavior of $T_{\mu\nu}$

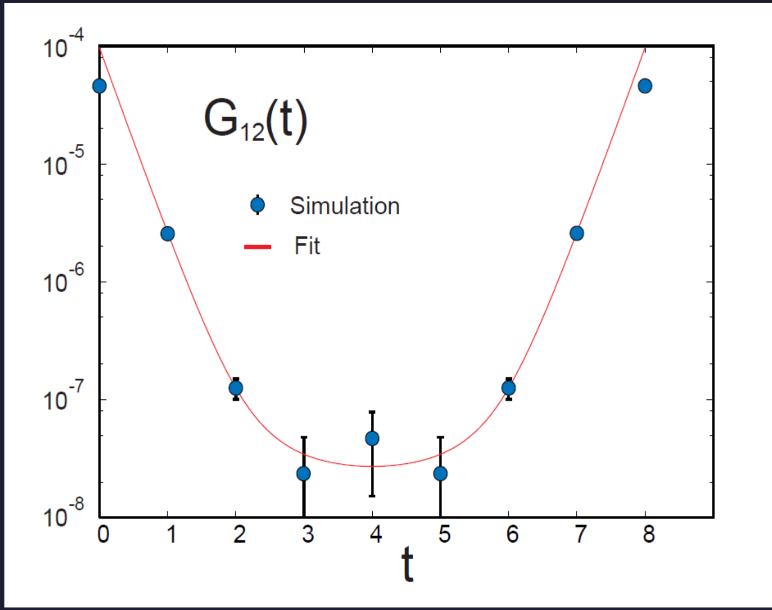
- Energy fluctuation ←→specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

EMT Correlator: Extremely Noisy...

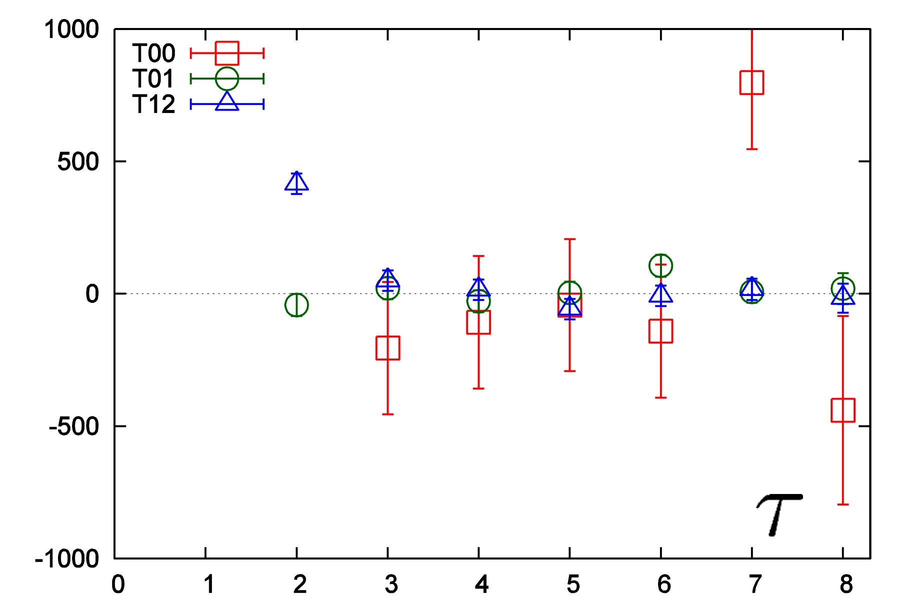
With naïve EMT operators

$$\langle T_{12}(\tau) T_{12}(0) \rangle$$



Nakamura, Sakai, PRL,2005
N_t=8
improved action
~10^6 configurations

$$\langle T_{\mu\nu}(\tau) T_{\mu\nu}(0) \rangle$$



Nt=16
standard action
5x10^4 configurations

... no signal

Conservation Law

$$\bar{T}_{\mu\nu} = \int d^3x T_{\mu\nu}(x)$$

$$\frac{\partial}{\partial\tau}\bar{T}_{00} = 0$$

$$\frac{\partial}{\partial\tau}\bar{T}_{01} = 0$$

$$\frac{\partial}{\partial\tau}\langle\bar{T}_{00}(\tau)\bar{T}_{00}(0)\rangle = 0$$

$$\frac{\partial}{\partial\tau}\langle\bar{T}_{00}(\tau)\bar{T}_{11}(0)\rangle = 0$$

$$\frac{\partial}{\partial\tau}\langle\bar{T}_{01}(\tau)\bar{T}_{01}(0)\rangle = 0$$

$$(\tau \neq 0)$$

$$\langle\bar{T}_{0\mu}(\tau)\bar{T}_{\alpha\beta}(0)\rangle$$

τ independent constant

Linear Response Relations

$$c_V = \frac{d}{dT}\langle E\rangle = \frac{\langle \bar{T}_{00}^2\rangle}{VT^2}$$

Specific heat

$$s = \frac{d}{dT}P = \frac{\langle \bar{T}_{11}\bar{T}_{00}\rangle}{VT^2}$$

entropy density

$$\varepsilon + p = \frac{\langle \bar{T}_{01}^2\rangle}{VT}$$

enthalpy density

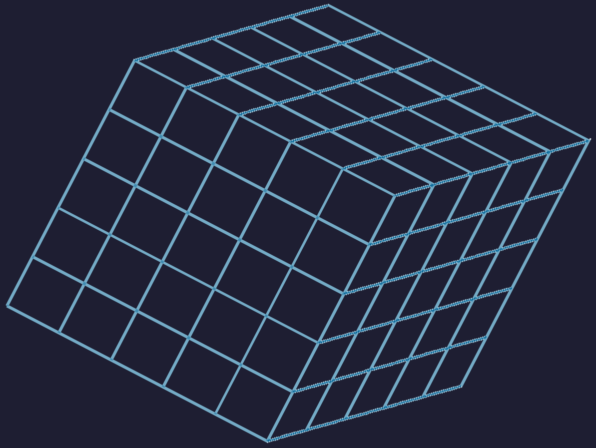
Giusti, Meyer, 2011
Minami, Hidaka, 2012

Derivation

$$\langle \hat{O}\rangle = \frac{1}{Z}\mathrm{Tr}\left[\hat{O}e^{-\beta\hat{H}}\right] \Rightarrow \frac{d}{d\beta}\langle \hat{O}\rangle = -\langle \delta\hat{O}\delta\hat{H}\rangle$$

Numerical Simulation

FlowQCD, arXiv:1708.01415



- SU(3) pure gauge
- Wilson gauge action / clover operator
- Ns/Nt=4
- Statistics: 18-20x10^4

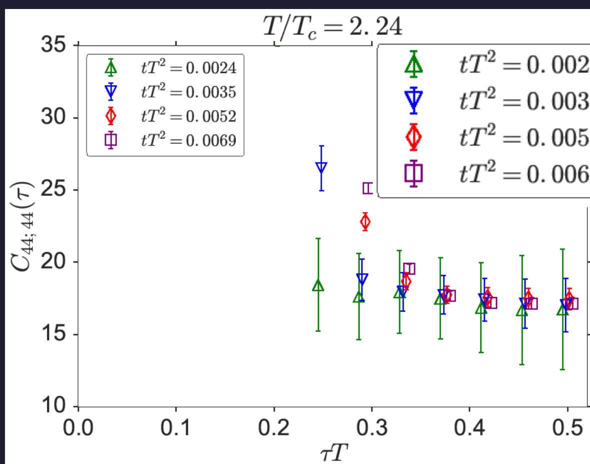
β	$T=1.66T_c$	$T=2.22T_c$
48^3x12	6.719	6.943
64^3x16	6.941	7.170
96^3x24	7.265	7.500

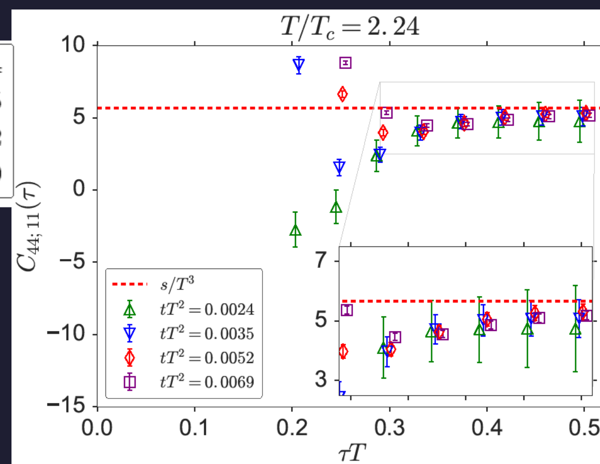
on Bluegene/Q @KEK

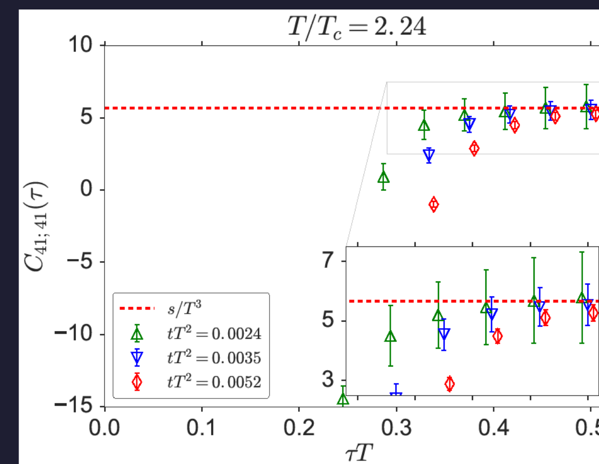
Euclidean Correlator @T=2.24Tc

FlowQCD, arXiv:1708.01415

$$\langle \bar{T}_{44}(\tau)\bar{T}_{44}(0)\rangle \qquad \langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle \qquad \langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle$$





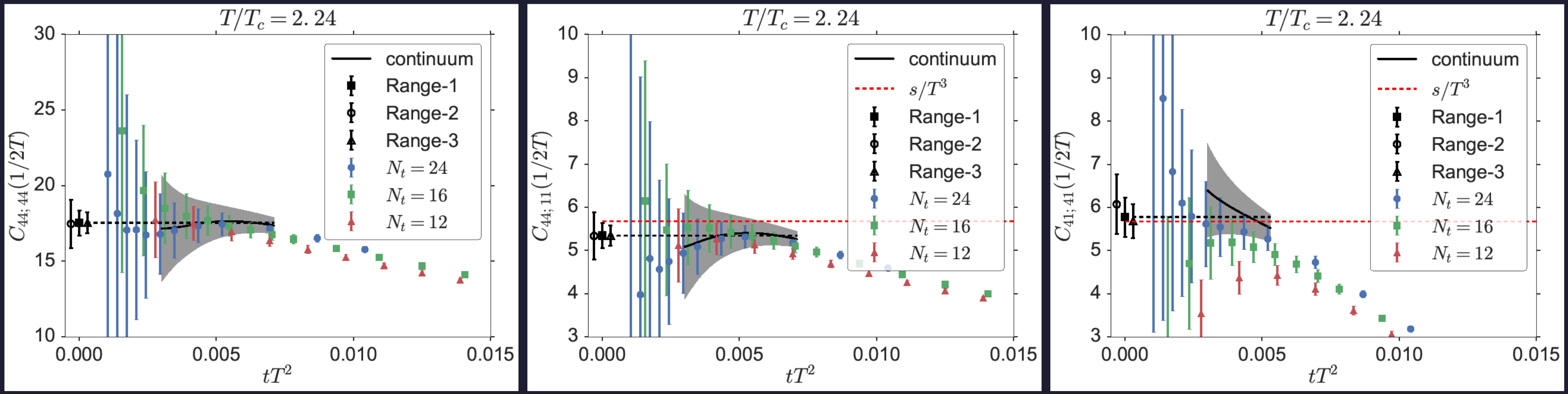


- ☐ τ-independent plateau in all channels ➔ conservation law
- ☐ small τ region: artificial enhancement due to overlap of operators
- ☐ linear response relations for 4411, 4141 channels

$$\frac{s}{T^3} = \frac{\langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle}{VT^5} = \frac{\langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle}{VT^5}$$

Mid-Point Correlator @T=2.24Tc

$\langle T_{44}(\tau)T_{44}(0)\rangle$ $\langle T_{44}(\tau)T_{11}(0)\rangle$ $\langle T_{41}(\tau)T_{41}(0)\rangle$



- (44;11), (41;41) channels : confirmation of LRR
- (44;44) channel: new measurement of c_V

c_V/T^3

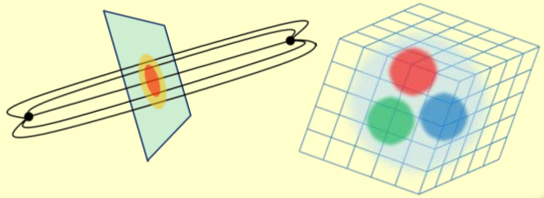
T/T_c	$C_{44;44}(\tau_m)$	Ref.[19]	Ref.[11]	ideal gas
1.68	$17.7(8)(^{+2.1}_{-0.4})$	$22.8(7)^*$	17.7	21.06
2.24	$17.5(0.8)(^{+0}_{-0.1})$	$17.9(7)^{**}$	18.2	21.06

2+1 QCD:
Taniguchi+ (WHOT-QCD), 1711.02262

Contents

Hadron Structure

- flux tube / hadrons
- stress distribution

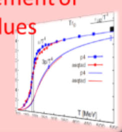


$T_{\mu\nu}$

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

viscosity, specific heat, ...

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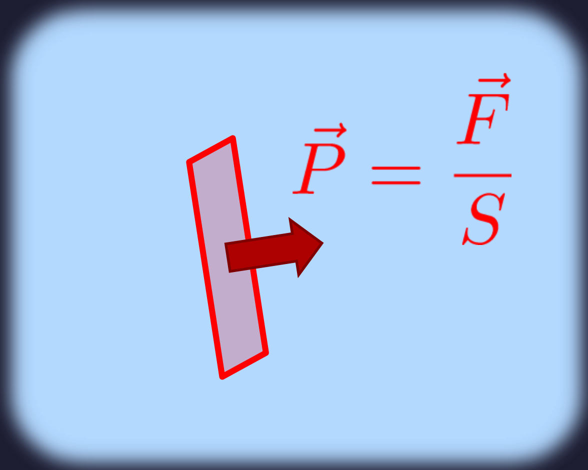
Hadron Structure

- flux tube / hadrons
- stress distribution

Stress

Pressure

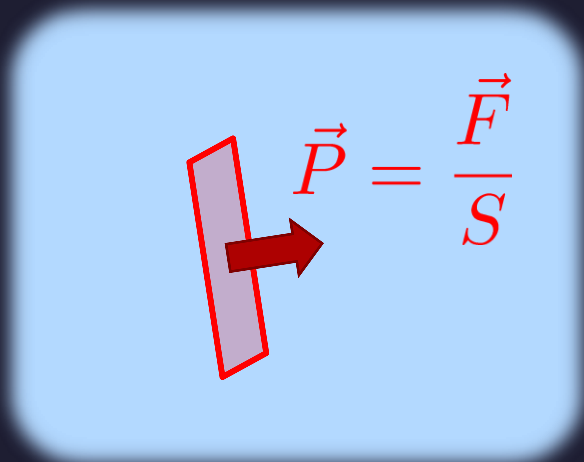


force on a surface
per unit area

$$\vec{P} = P\vec{n}$$

Stress

Pressure

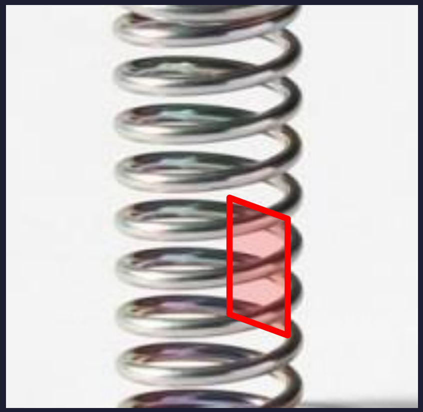


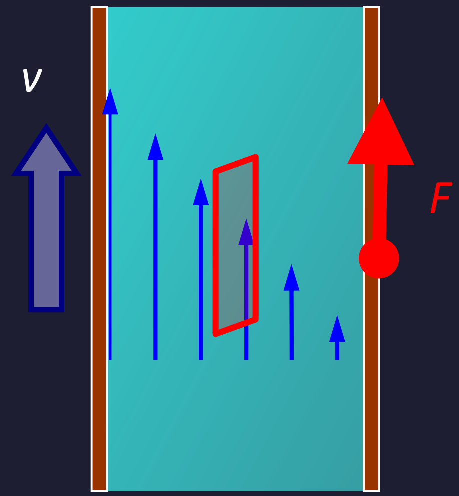
force on a surface
per unit area

$$\vec{P} = P\vec{n}$$

Stress

Generally, F and n are not parallel



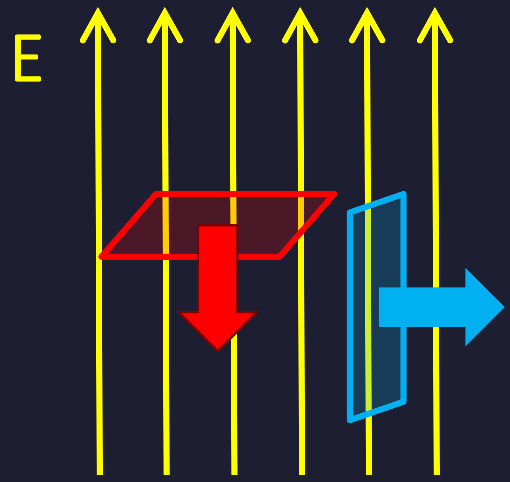


Stress Tensor

$$P_i = T_{ij} n_j$$

Maxwell Stress

$$T_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$$



$$\vec{E} = (E, 0, 0)$$

$$T = \frac{1}{2}\begin{bmatrix} E^2 & 0 & 0 \\ 0 & -E^2 & 0 \\ 0 & 0 & -E^2 \end{bmatrix}$$

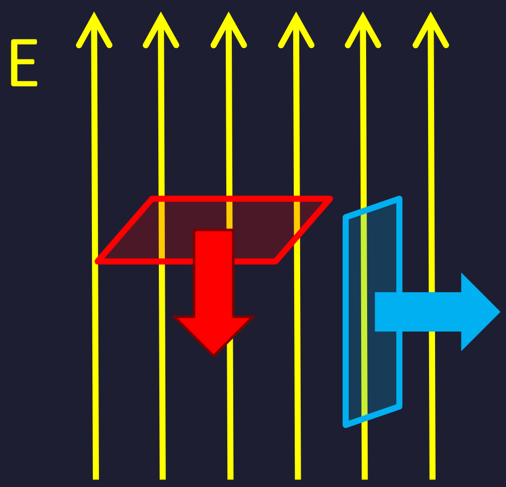
Parallel to field: Attractive

Vertical to field: Repulsive

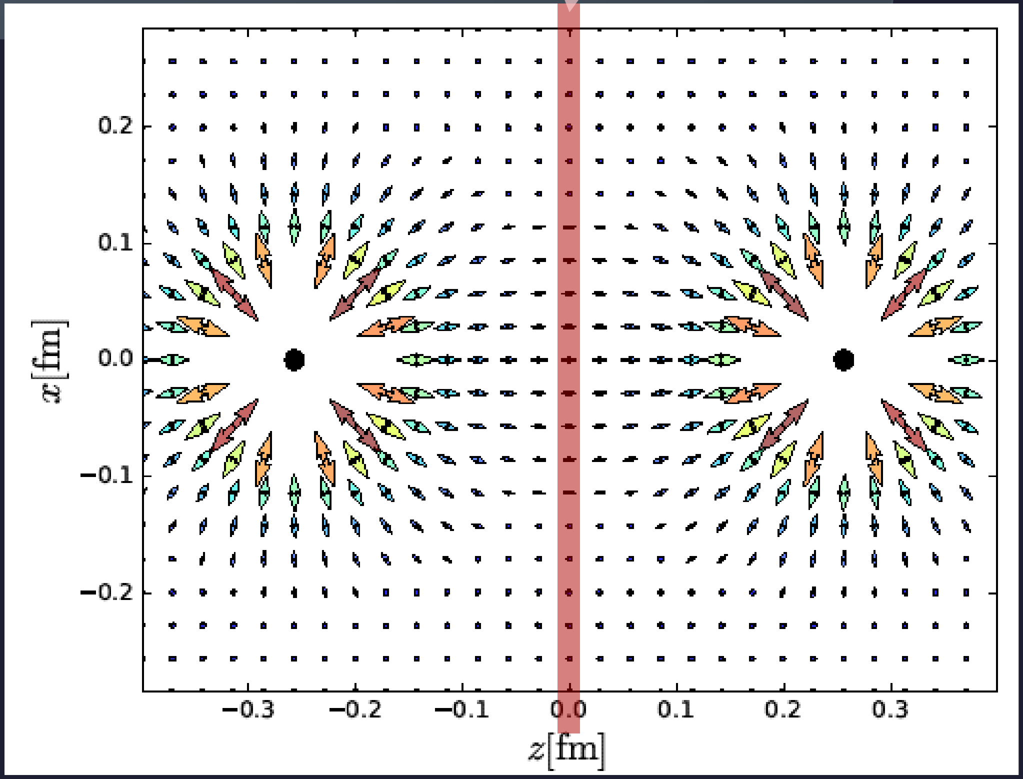
Maxwell Stress

$$\int_S dxdyP_z =$$ Coulomb force

$$T_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



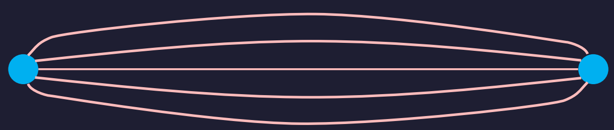
Parallel to field: Attractive
Vertical to field: Repulsive

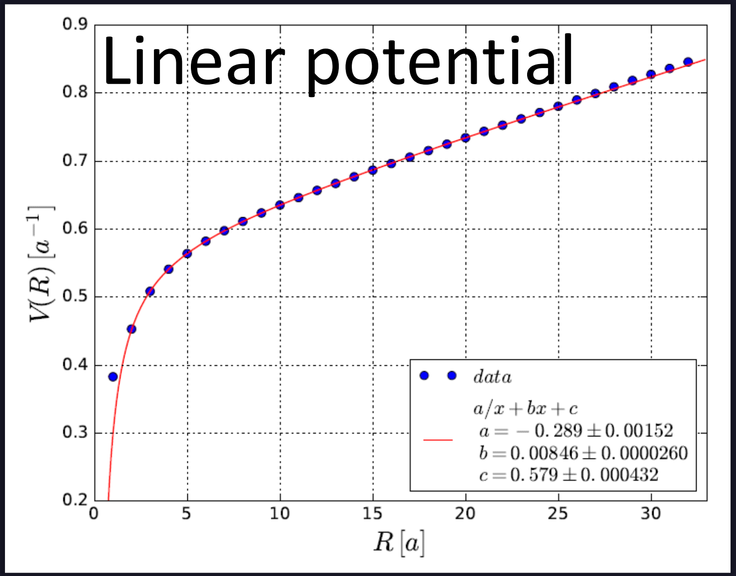


attractive eigenvector
(≈Line of electric field)

q-qbar System in YM Theory
= Flux Tube Formation

Color electric field is confined into a flux tube





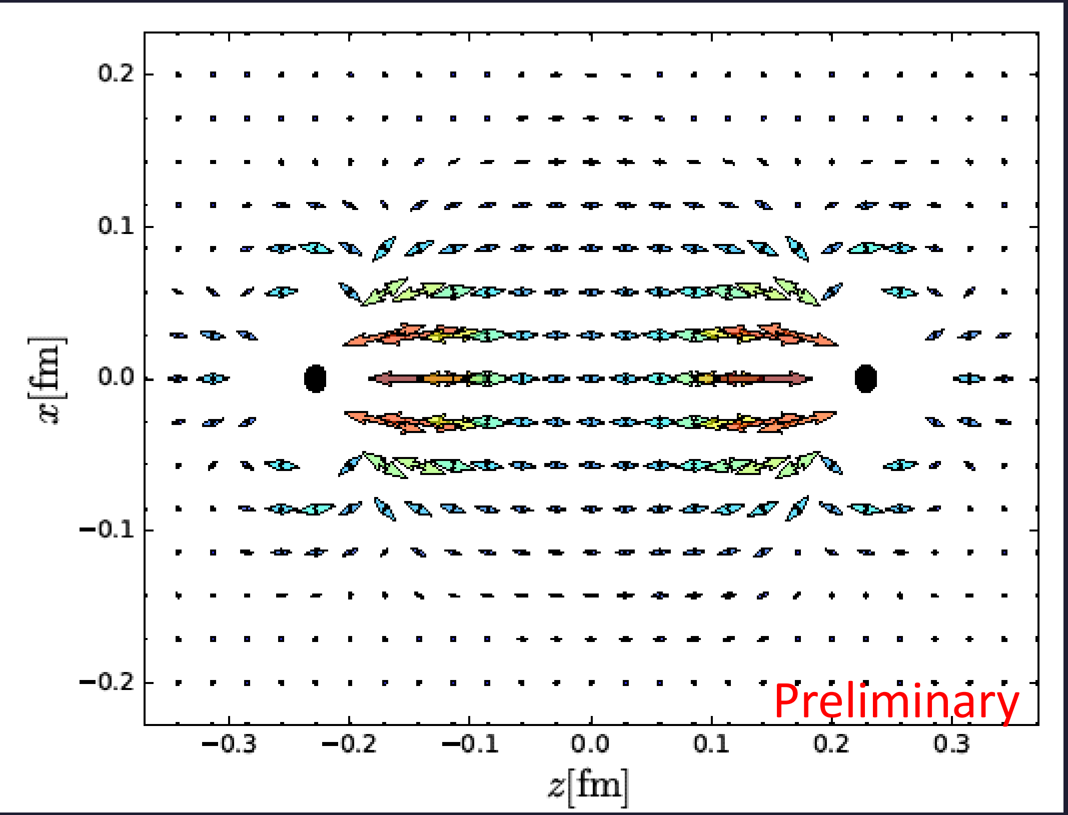
Previous Studies

- action density
- color electric field
- (color electric field)^2

This Study: stress

- gauge invariant!
- definite physical meaning!
- establish action thr. medium

Stress Distribution in $q\bar{q}$ System



attractive eigenvector

R. Yanagihara+ (FlowQCD)
to appear soon!

beta=6.819 (a=0.029fm)
R/a=16
t→0 limit
No continuum limit

First visualization of distortion in space due to color charges

Stress Distribution in $q\bar{q}$ System

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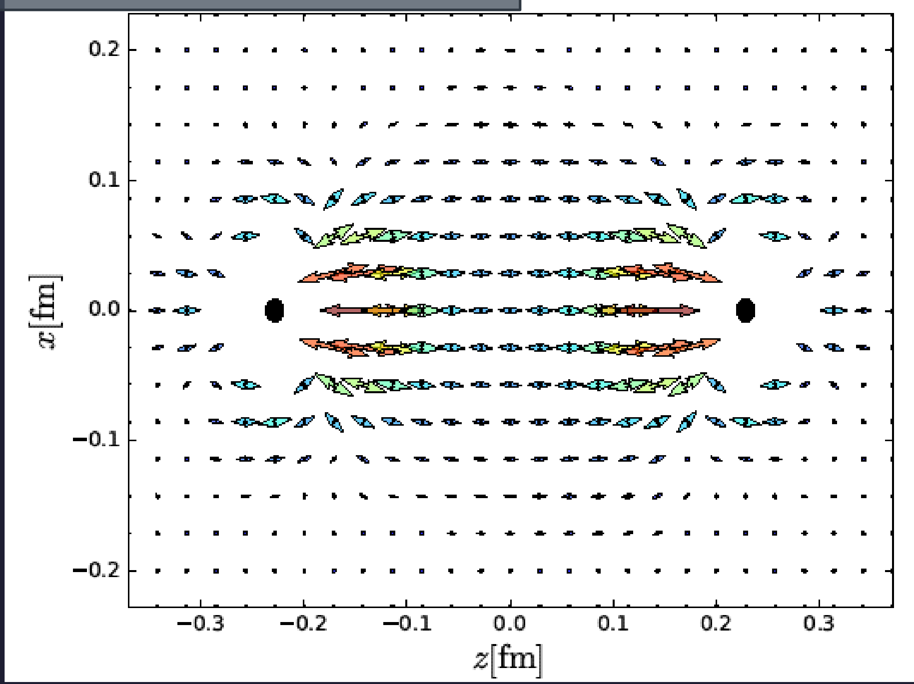
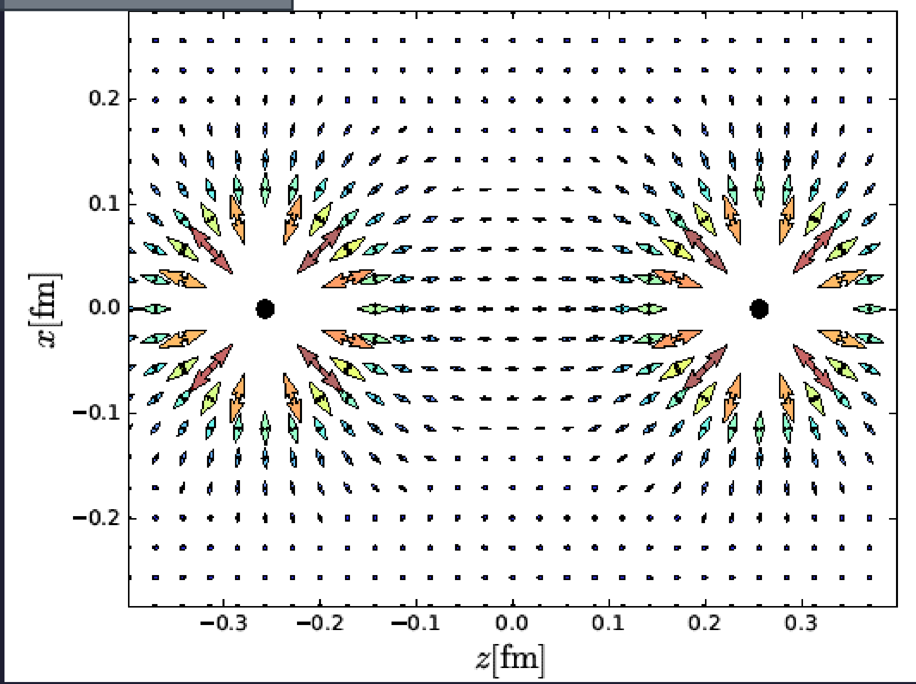
Yang-Mills SU(3)



Maxwell



Stress Distribution in $q\bar{q}$ System

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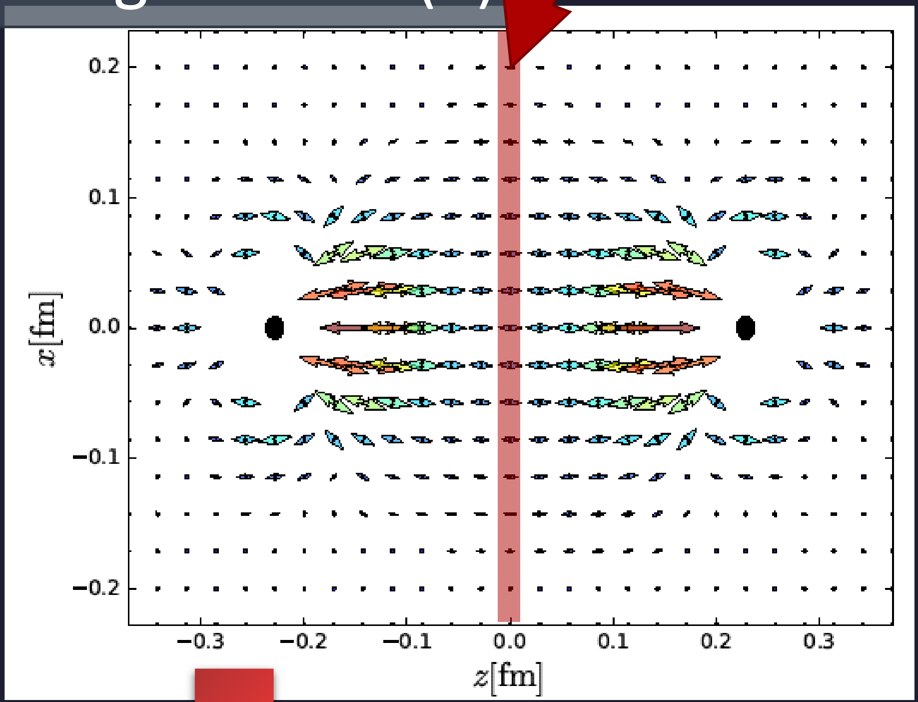
qq force?

Yang-Mills SU(3)



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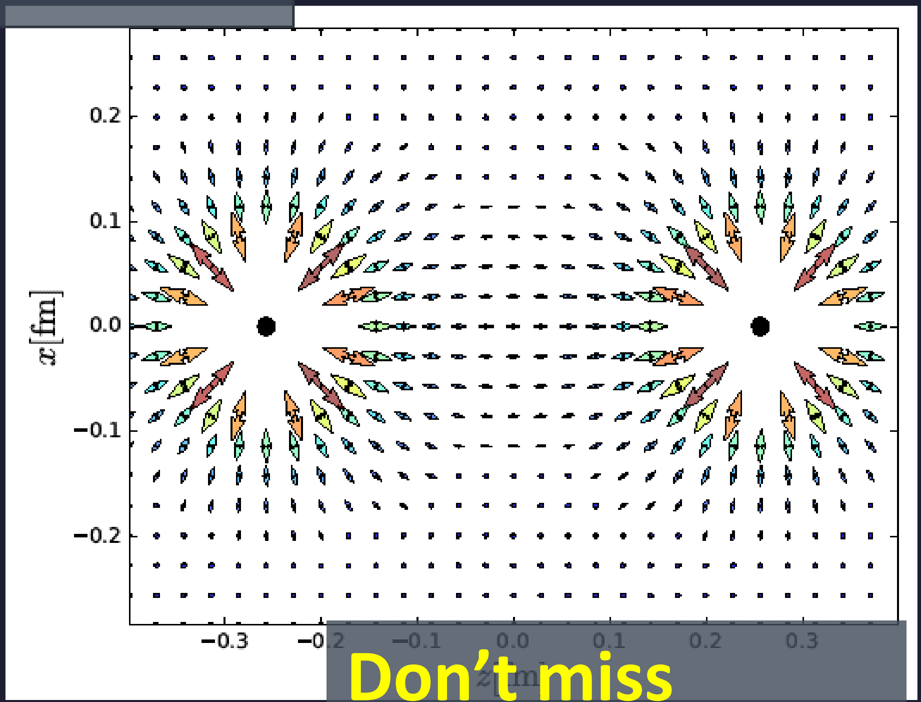


New insights into physics
of confinement

Don't miss Yanagihara's talk this afternoon

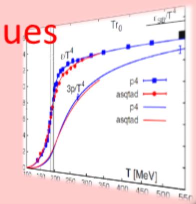
Summary

- EMT operator on the lattice is now available!
  - Correctly renormalized operator
  - Statistical error is suppressed thanks to gradient flow.
- The operator is applied to various analyses:

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



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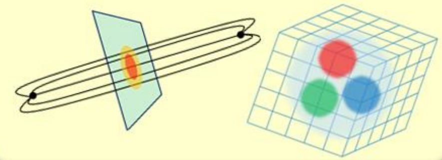


$T_{\mu\nu}$

Many future studies

- transport coefficient
- EM/stress distribution in hadrons
- Flux tube: T dependence

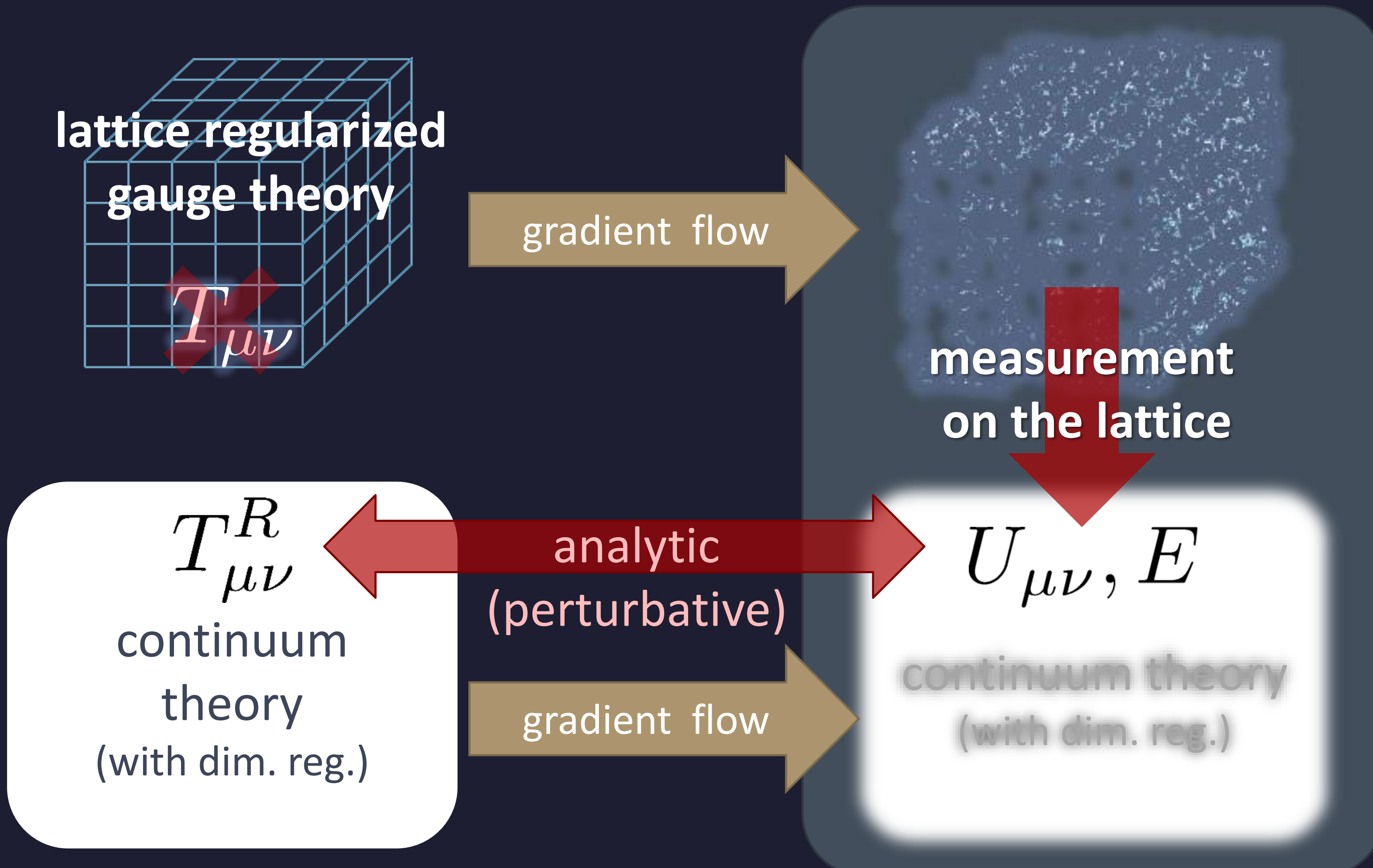
EMT (Naïve Construction)

$$T_{\mu\nu}(x) = Z_1\left(F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma}\right) + Z_2\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma} + Z_3\delta_{\mu\nu}F_{\mu\rho}F_{\nu\rho}$$

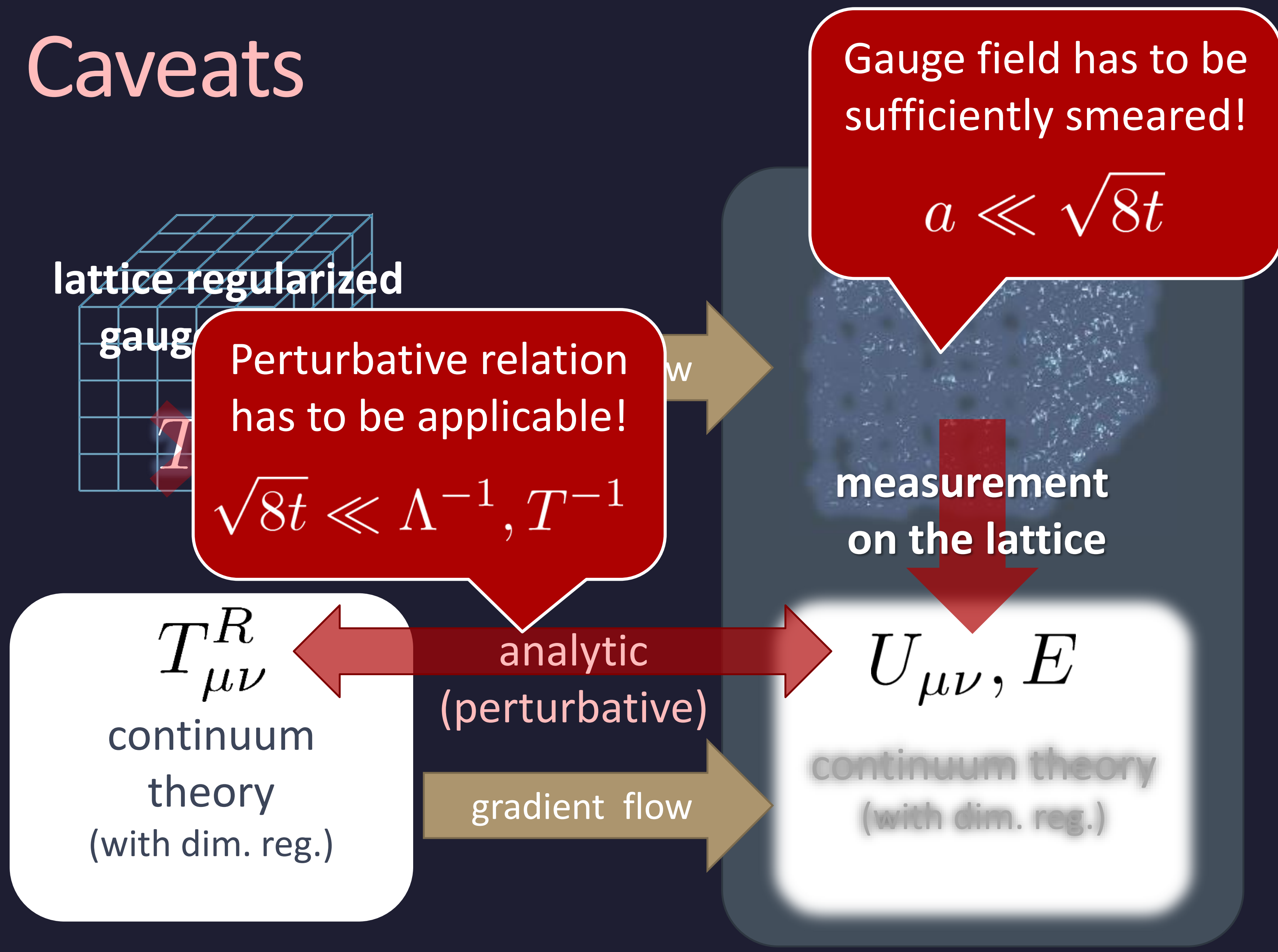
- Z_1, Z_2, Z_3 have to be determined non-perturbatively.
- Accurate determinations of Z_1, Z_3: Giusti, Pepe, 2014-; BW, 2016

- So far, only for pure gauge theory
  - multi-level algorithm

Gradient Flow Method



Caveats



Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized
Perturbative relation has to be applicable!
$\sqrt{8t} \ll \Lambda^{-1}, T^{-1}$
measurement on the lattice
$T^{R}_{\mu\nu}$
continuum theory
(with dim. reg.)
analytic
(perturbative)
$U_{\mu\nu}, E$
continuum theory
(with dim. reg.)
gradient flow

Caveats

lattice regularized

gaug

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analytic

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$$U_{\mu\nu}, E$$

continuum theory

(with dim. reg.)

$$a \ll \sqrt{8t} \ll T^{-1}$$

Topological Charge