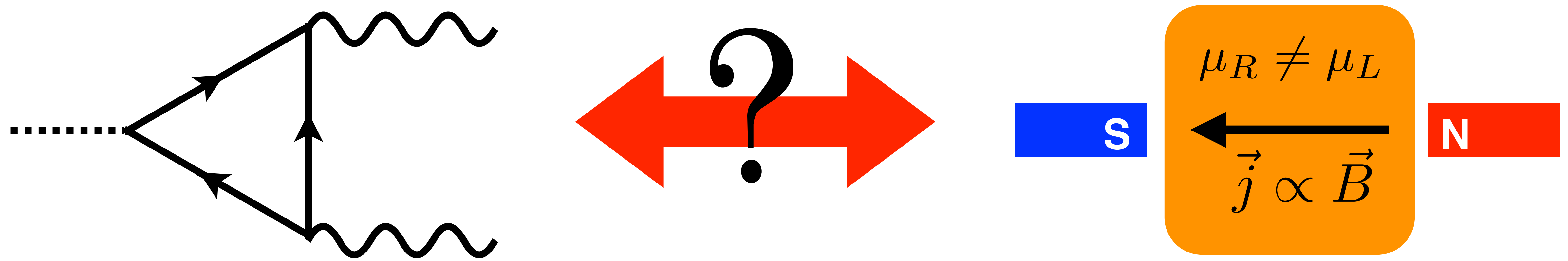
Talk by T. Hirano Talk by Y. Hidaka

Hydrodynamics, current algebra, and quantum anomaly

Talk by D. Kharzeev



國立臺灣大學

Masaru Hongo (RIKEN, iTHES)

Workshop of Recent developments in QCD and Quantum field theories, 2017 Nov. 11th

Outline

MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?

APPROACH:

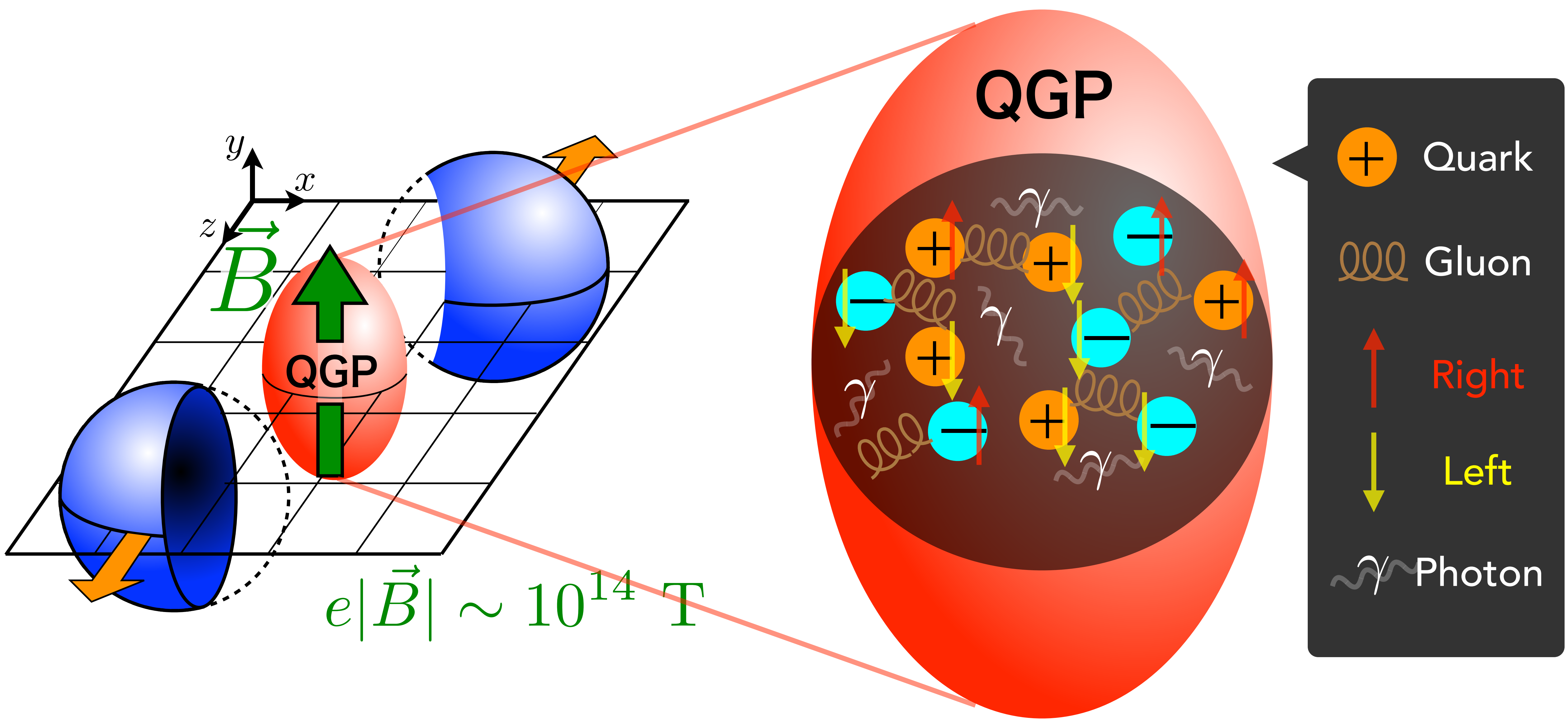
Mori's method as a generalization of current algebra

Anomalous commutation:

RESULT:

Chiral Magnetic Effect in operator formalism:

QGP as Chiral fluid

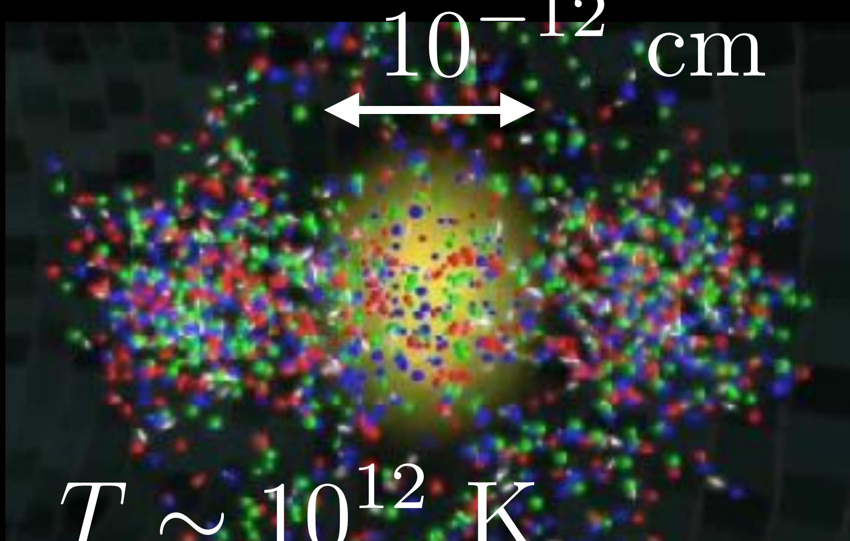


- Existence of extremely strong magnetic field
- Chirality drastically affect hydrodynamic transport

Hydrodynamics is

- Effective theory for macroscopic dynamics
- Universal description, not depending on details
- Only conserved quantity ~ ~~symmetry~~ of system

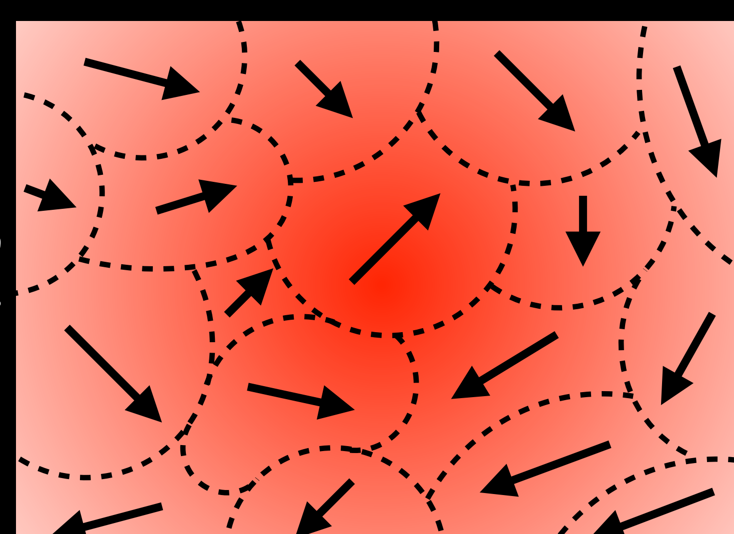
Quark-Gluon Plasma



http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

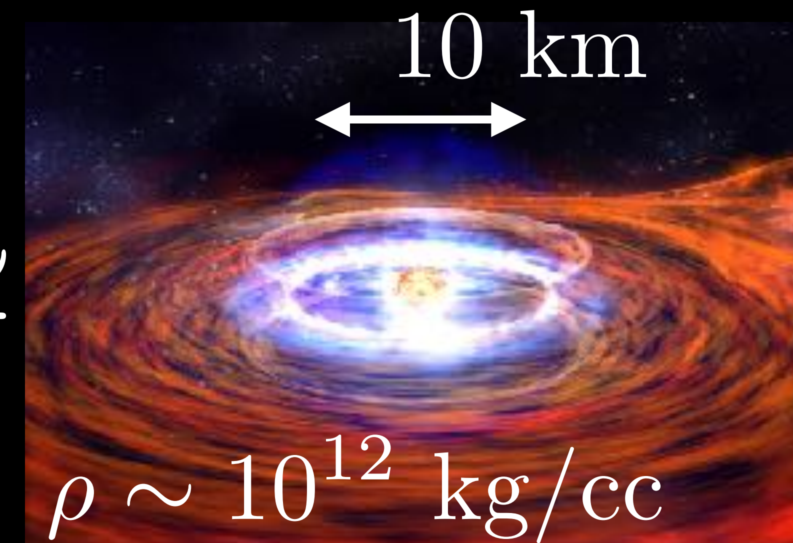
$\simeq$

Hydro: $\{\beta(x),\ \vec{v}(x)\}$



$\simeq$

Neutron Star

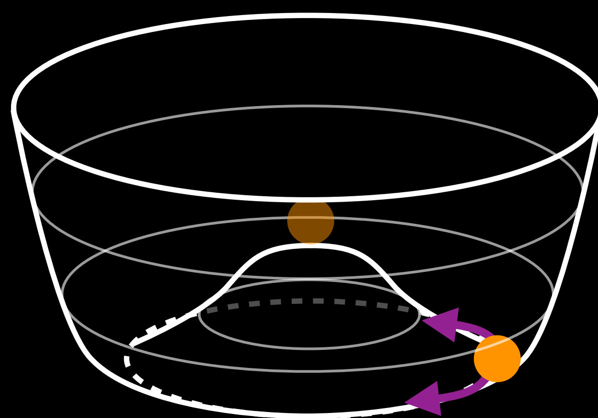


http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

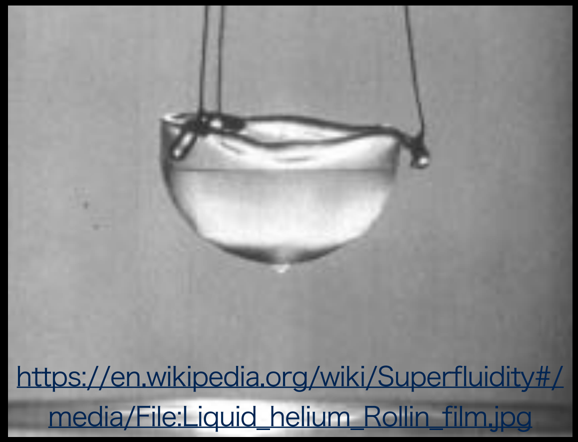
Symmetry breaking and Hydro

- Spontaneous symmetry breaking

Micro : Selecting vacuum

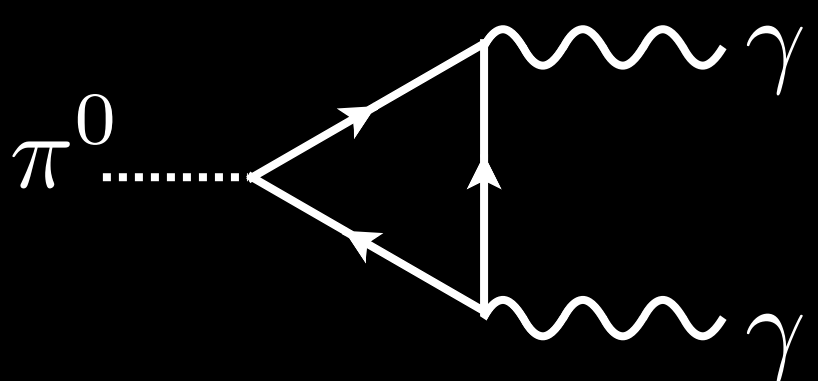


Macro : Superfluid



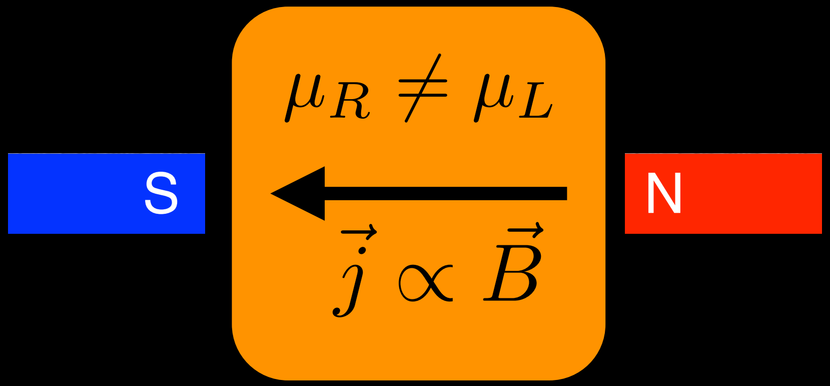
- Symmetry breaking by quantum anomaly

Micro : π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Macro : Anomalous transport



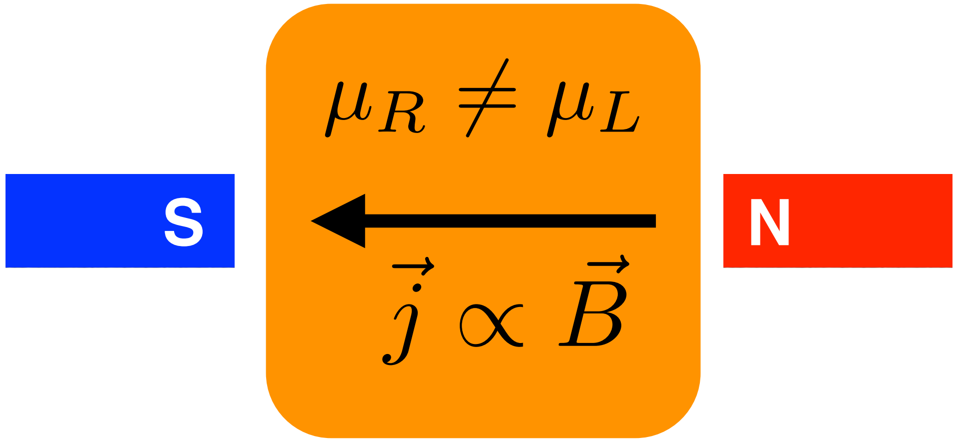
[Erdmenger et al. (2008), Son-Surowka (2009)]

Parity-violating chiral transport

[Fukushima et al. (2008), Vilenkin (1980)]

◆Chiral Magnetic Effect (CME)

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$



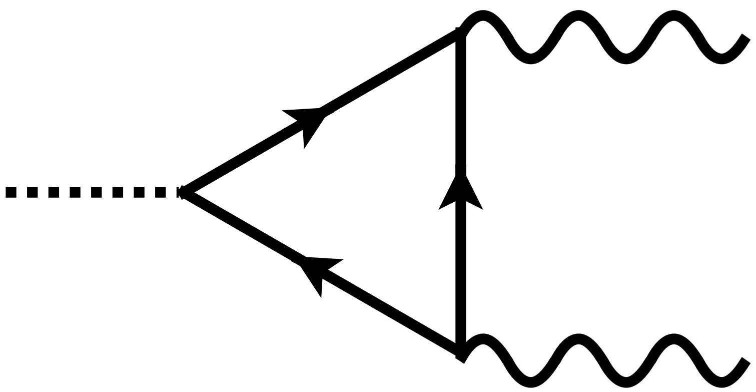
[Erdmenger et al. (2008), Son-Surowka (2009)]

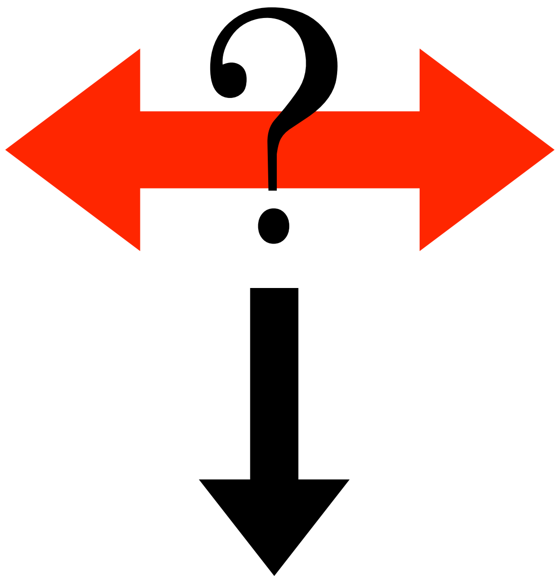
◆ Chiral Vortical Effect (CVE)

$$\vec{J} = \frac{\mu\mu_5}{2\pi^2}\vec{\omega}$$

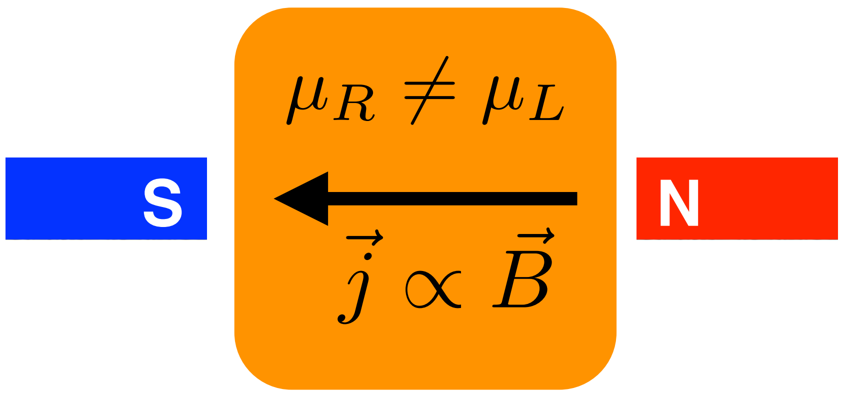
Anomaly and chiral transport

Quantum anomaly





CME



Can we understand this based on current algebra?

Problem.

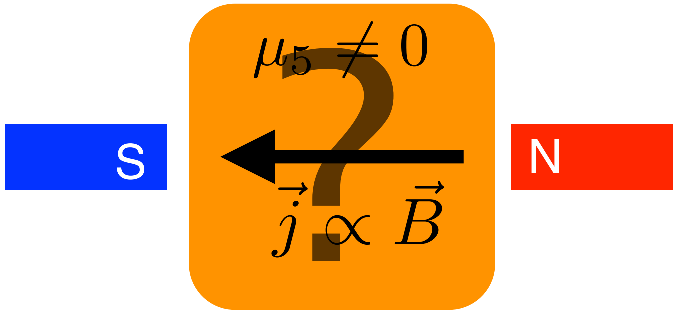
Not vacuum physics as is the case for QCD!

$\rightarrow$ We have to generalize current algebra for $T \neq 0, \mu \neq 0$

Outline

MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra

Anomalous commutation:

RESULT:

Chiral Magnetic Effect in operator formalism:

Review: Current algebra for QCD

◆ **Current algebra for** $SU(N)_R \times SU(N)_L$

$$\left[\hat{Q}_{L,a}, \hat{J}^{\mu}_{L,b}(x)\right] = if^{c}{}_{ab}\hat{J}^{\mu}_{L,c}(x), \quad \left[\hat{Q}_{L,a}, \hat{J}^{\mu}_{R,b}(x)\right] = 0$$

$$\left[\hat{Q}_{R,a}, \hat{J}^{\mu}_{R,b}(x)\right] = if^{c}{}_{ab}\hat{J}^{\mu}_{R,c}(x), \quad \left[\hat{Q}_{R,a}, \hat{J}^{\mu}_{L,b}(x)\right] = 0$$

Low-energy theorem

- **Goldberger-Treiman relation**
- **Soft Pion theorem, ⋯**

Universal results for process with low-energy pion scattering!

If current algebra satisfies the above relations,
it does not matter whether UV theory is QCD, NJL model, or anything!

Current algebra and chiral anomaly

◆ **Current algebra in external EM fields for** $U(1)_V \times U(1)_A$

$$\left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = 0$$

$$\cancel{\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = 0}$$

Proof.

Definition of Noether current gives

$$\hat{J}^0(x) = -\frac{\partial \mathcal{L}}{\partial(\partial\phi)} i\hat{\phi} = -i\hat{\pi}(x)\hat{\phi}(x), \qquad \hat{J}^0_5(x) = -i\hat{\pi}(x)\gamma_5\hat{\phi}(x)$$

Using canonical commutation relation $\left[\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})\right] = i\delta(\boldsymbol{x}-\boldsymbol{y})$

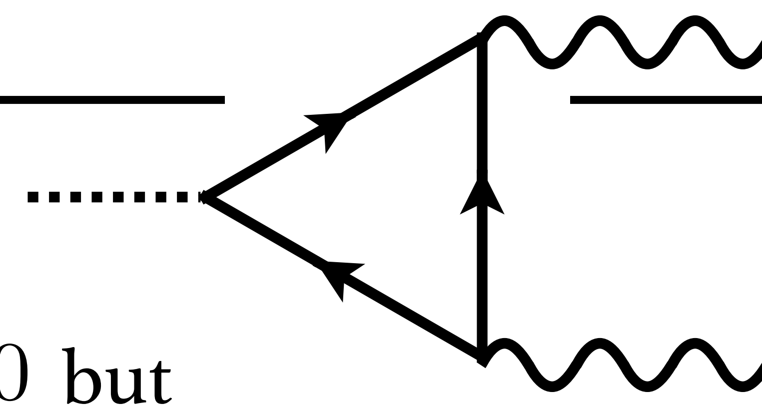
we can directly show the above current algebraic structure! $\square$

Current algebra and chiral anomaly

◆ **Current algebra in external EM fields for** $U(1)_V \times U(1)_A$

$$\left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = 0$$

$$\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y}) \partial_i^x \delta(\boldsymbol{x}-\boldsymbol{y})$$



Sketch of Proof.

Ward-Takahashi identity is not $\langle \partial_\mu J_5^\mu(x) \rangle_A = 0$ but

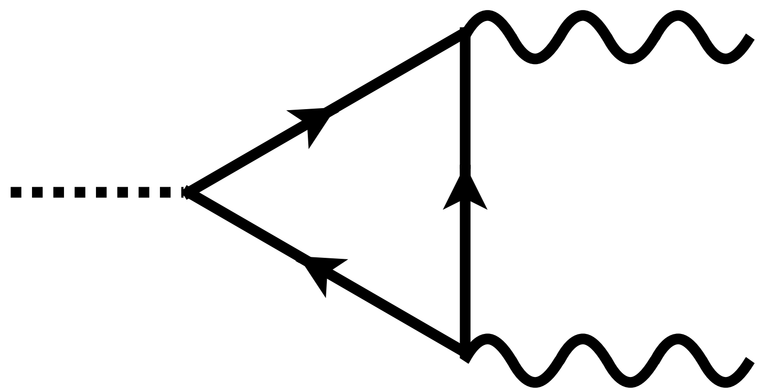
$$\langle \partial_\mu J_5^\mu(x) \rangle_A = C\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x) \sim C dA dA$$

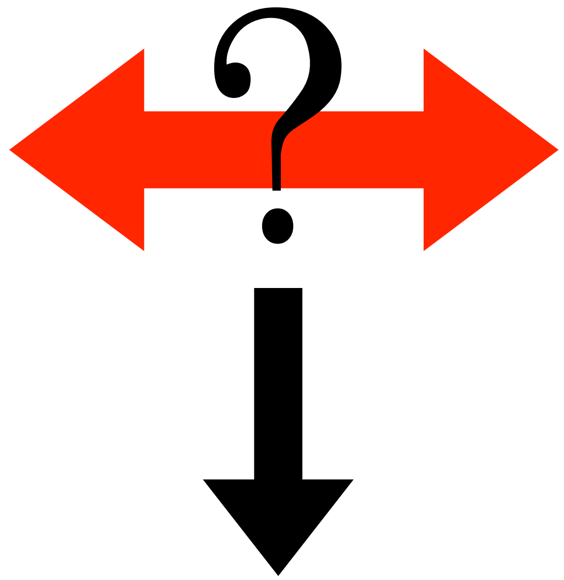
Variation w.r.t A_0 **gives** $\partial_\mu \langle J_5^\mu(x) J^0(y) \rangle_A \sim C ddA \sim C dB$

"Corr. function = T-product in operator formalism" gives the above □

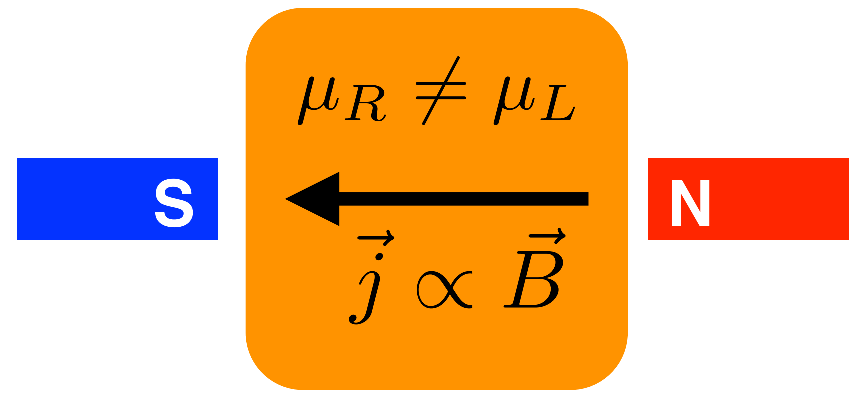
Anomaly and chiral transport

Quantum anomaly





CME



Can we understand this based on current algebra?

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$\rightarrow$ We have to generalize current algebra for $T \neq 0, \mu \neq 0$

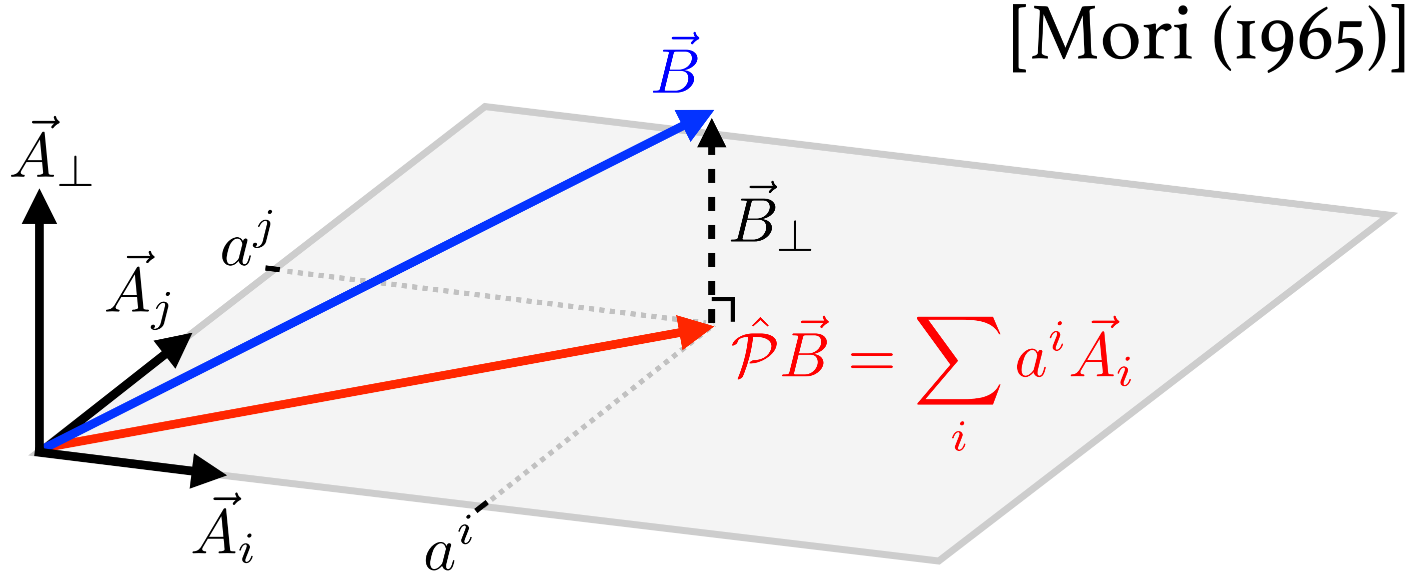
Mori's projection operator method

[Mori (1965)]

A method to write down

Equation of Motion (EoM)

only focusing on $\hat{A}_n(t)$



◆ EoM given by Mori's projection operator method

$$\partial_0 \hat{A}_n(t) = i\Omega_n{}^m \hat{A}_m(t) - \int_0^t ds \Phi_n{}^m(t-s)\hat{A}_m(s, \boldsymbol{y}) + \hat{R}_n(t)$$

Reversible　　Dissipative　　Noise

$$i\Omega_n{}^m = -\frac{i}{\beta}\langle[\hat{A}_n(0), \hat{A}^m(0)]\rangle + i\mu\big([\hat{N}, \hat{A}_n(0)], \hat{A}^m(0)\big)$$

Fluctuation Dissipation relation: $\Phi_n^m(t-s) = \big(\hat{R}_n(t-s), \hat{R}^m(0)\big)$

Mori's projection operator method

[Mori (1965)]

A method to write down

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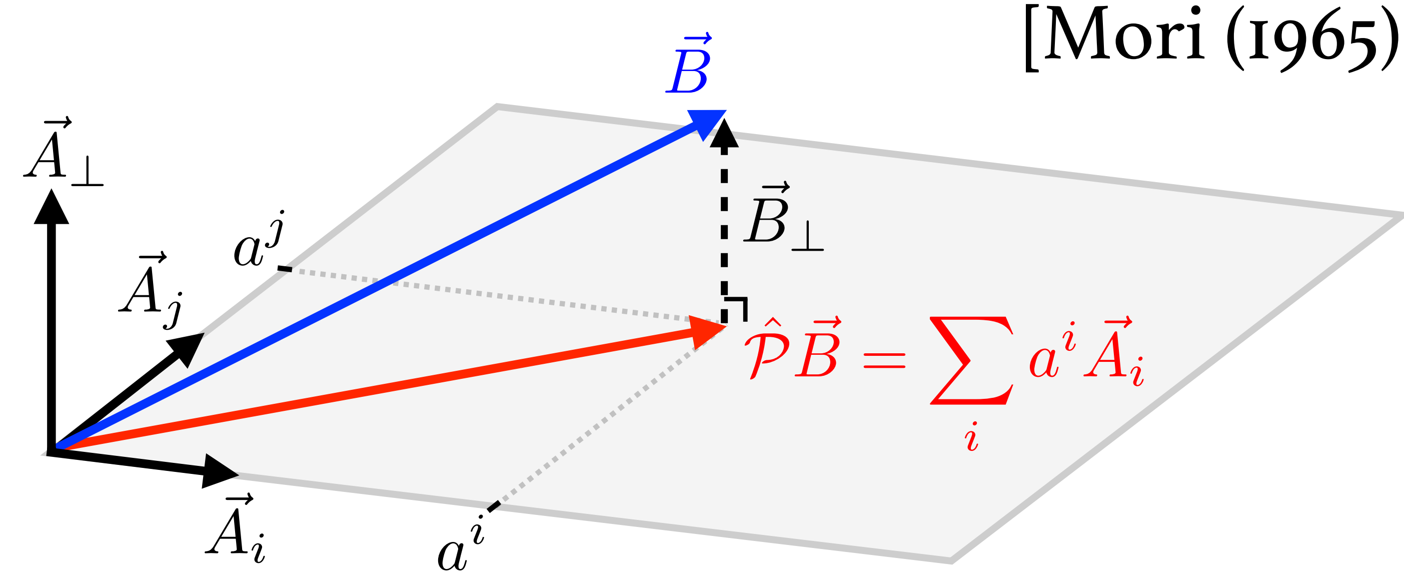

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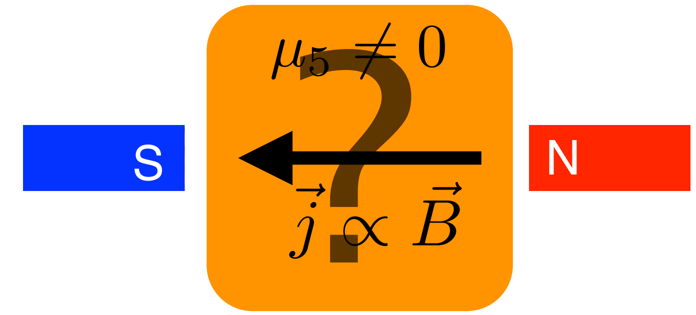
Reversible　　Dissipative　　Noise

$$\begin{cases} i\Omega_n{}^m = -\dfrac{i}{\beta}\langle[\hat{A}_n(0), \hat{A}^m(0)]\rangle + i\mu([\hat{N}, \hat{A}_n(0)], \hat{A}^m(0)) \\ \text{Fluctuation Dissipation relation: } \Phi_n^m(t-s) = (\hat{R}_n(t-s), \hat{R}^m(0)) \end{cases}$$

Outline

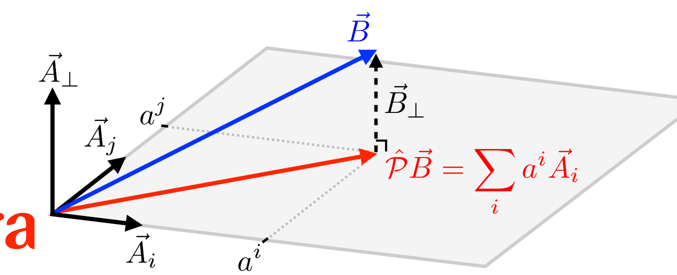
MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra



Anomalous commutation: $\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

Chiral Magnetic Effect in operator formalism:

Sound and Chiral Magnetic Wave as a family of Type-B NG mode

Mori's method and current algebra

◆ **Leading Order term in EOM**

$$\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$$ (χ^{lm} : inv. suscep.)

◆ **Current algebra related to relativistic hydrodynamics**

Choose $\hat{A}_n(t)$ **as conserve charges:** $\hat{A}_n(t) = \{\hat{T}^0{}_0(t, \boldsymbol{x}), \hat{T}^0{}_i(t, \boldsymbol{x})\}$

➡ **EoM(LO) is controlled by energy-momentum density algebra!**

Current algebra

$$\begin{cases} \left[\hat{T}^0{}_0(t, \boldsymbol{x}), \hat{T}^0{}_0(t, \boldsymbol{y})\right] = -i\left(\hat{T}^k{}_0(t, \boldsymbol{x}) + \hat{T}^k{}_0(t, \boldsymbol{y})\right)\partial_k \delta(\boldsymbol{x} - \boldsymbol{y}) \\ \left[\hat{T}^0{}_0(t, \boldsymbol{x}), \hat{T}^0{}_i(t, \boldsymbol{y})\right] = -i\left(\hat{T}^j{}_i(t, \boldsymbol{x})\partial_j - \hat{T}^0{}_0(t, \boldsymbol{y})\partial_i\right)\delta(\boldsymbol{x} - \boldsymbol{y}) \\ \left[\hat{T}^0{}_i(t, \boldsymbol{x}), \hat{T}^0{}_j(t, \boldsymbol{y})\right] = -i\left(\hat{T}^0{}_j(t, \boldsymbol{x})\partial_i + \hat{T}^0{}_i(t, \boldsymbol{y})\partial_j\right)\delta(\boldsymbol{x} - \boldsymbol{y}) \end{cases}$$

➡ **EoM for perfect fluid (Sound wave) is derived!!**

Perfect fluid from Mori's method

◆ **Relativistic hydrodynamic from Mori's method**

[Minami-Hidaka (2013)]

$$\partial_0 \delta \hat{T}^0{}_0 = -ik^i \delta \hat{T}^0{}_i$$

$$\partial_0 \delta \hat{T}^0{}_i = -ik_i h_{\mathrm{eq}} \chi^{ee} \delta \hat{T}^0{}_0 - \left[k_i k^k \left(\frac{\zeta}{h_{\mathrm{eq}}} + \frac{d-3}{d-1} \frac{\eta}{h_{\mathrm{eq}}} \right) + \boldsymbol{k}^2 \delta_i^k \frac{\eta}{h_{\mathrm{eq}}} \right] \delta \hat{T}^0{}_k + \hat{R}_{\pi_i}$$

◆ **Green-Kubo formula for transport coefficients (viscosity)**

$$\zeta = \beta_{\mathrm{eq}} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{0}))$$

$$\eta = \frac{\beta_{\mathrm{eq}}}{(d+1)(d-2)} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{\pi}_{ik}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{\pi}_{jl}(0, \boldsymbol{0})) \Delta^{ij} \Delta^{kl}$$

➡ **Reversible → Sound wave / Dissipative → Diffusion mode**

CME from anomalous commutation

For EoM: $\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$

Choose $\hat{A}_n(t) = \{\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{x}), J^0(t,\boldsymbol{x}), J^0_5(t,\boldsymbol{x})\}$

Current algebra with anomalous commutation rel.

$$\begin{cases} \left[\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -i\hat{J}^0(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ \left[\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = -i\hat{J}^0_5(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\dfrac{i}{2\pi^2}B^i(t,\boldsymbol{y})\partial_i^x\delta(\boldsymbol{x}-\boldsymbol{y}) \\ \left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = 0 \end{cases}$$

◆ **EoM for** $\hat{J}^0(t,\boldsymbol{x})$

$$\partial_0 \hat{J}^0(x) + \partial_i^x\left[\frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2}B^i(x)\right] + \cdots = 0$$

CME from anomalous commutation

$$\partial_0 \hat{J}^0(x) + \partial_i^x \underbrace{\left[\frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2} B^i(x)\right]}_{=\hat{J}^i(x)} + \cdots = 0$$

◆ Summary of result

- Conservation law: $\partial_\mu \hat{J}^\mu(x) = 0$

- Const. relation: $\hat{J}^i(x) = \dfrac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2} B^i(x)$ ⬅ Chiral Magnetic Effect (CME)

Chiral Magnetic Wave (CMW)

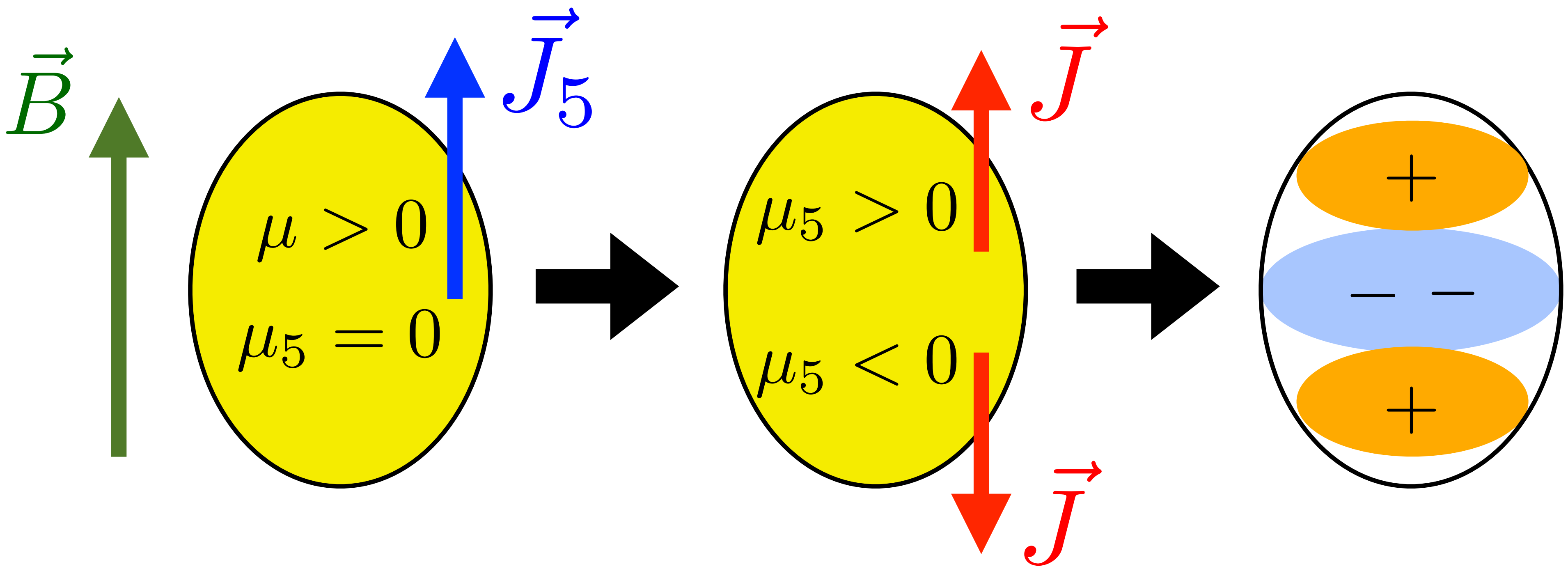
Chiral Magnetic Effect

$$\hat{j}^i(x) = \frac{\chi^{nn_5}\hat{j}_5^0(x)}{2\pi^2}B^i(x)$$

Chiral Separation Effect

$$\hat{j}_5^i(x) = \frac{\chi^{n_5 n}\hat{j}^0(x)}{2\pi^2}B^i(x)$$

Ex.



Charge propagation along B

||

Collective excitation

→ **Chiral Magnetic Wave**

[Kharzeev, Yee, (2011)]

Interpretation as Type-B NG mode

◆ **Spontaneous symmetry breaking & Nambu-Goldstone mode**

For some conserved charge $\hat{Q}_a$

$$\exists\, \hat{\Phi}(x) \text{ such that } \langle [i\hat{Q}_a, \hat{\Phi}_i(x)] \rangle \equiv \mathrm{Tr}\big(\hat{\rho}[i\hat{Q}_a, \hat{\Phi}_i(x)]\big) \neq 0$$

Spontaneous Symmetry Breaking (SSB)

➡ **Massless mode = Nambu-Goldstone (NG) mode appears!**

◆ **Classification of NG mode** [Hidaka (2012), Watanabe-Murayama(2012)]

- Type-A NG mode : $\forall\, \hat{Q}_b$ satisfy $\langle [i\hat{Q}_a, \hat{Q}_b] \rangle = 0$
- Type-B NG mode : $\exists\, \hat{Q}_b$ such that $\langle [i\hat{Q}_a, \hat{Q}_b] \rangle \neq 0$

➡ **Generalization of Nambu-Goldstone's theorem for type-B NG mode**

CMW ≒ Type-B NG mode?

- Type-B NG mode： $\exists \hat{Q}_b$ such that $\langle [i\hat{Q}_a, \hat{Q}_b] \rangle \neq 0$

Hydrodynamics of chiral plasma contains

massless collective excitation known as
- Sound wave
- Chiral Magnetic Wave (CMW)

◆ Origin of Sound wave and CMW

$$\langle \big[\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{y}) \big] \rangle = -i \big(\langle \hat{T}^j{}_i(t,\boldsymbol{x}) \rangle \partial_j - \langle \hat{T}^0{}_0(t,\boldsymbol{y}) \rangle \big) \partial_k \delta(\boldsymbol{x}-\boldsymbol{y}) \neq 0$$

$$\langle \big[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y}) \big] \rangle = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y}) \partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y}) \neq 0$$

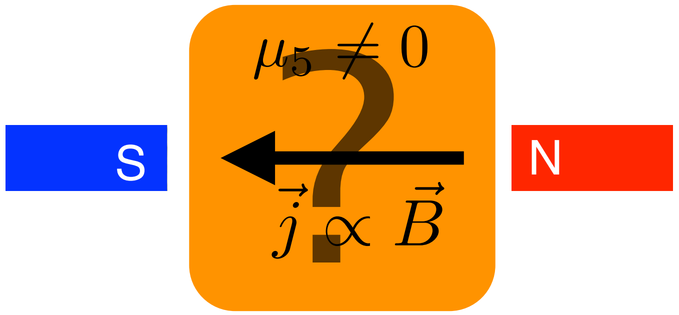
The above definition states they are a friend of Type-B NG mode!

$$\partial_0 \hat{A}_n(t) = -i\chi^{lm} \langle [\hat{A}_n(0), \hat{A}_m(0)] \rangle \hat{A}_l(t) + \cdots$$

Summary

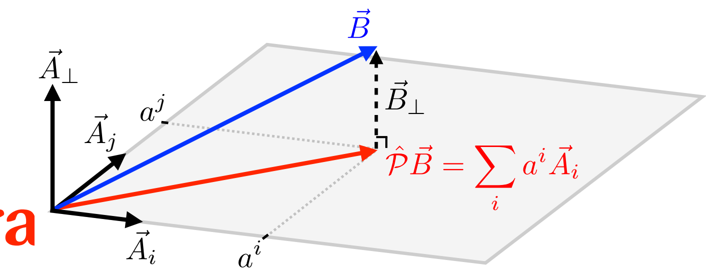
MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra



Anomalous commutation: $[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

Chiral Magnetic Effect in operator formalism: $\hat{J}^i(x) = \frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2} B^i(x)$

Sound and Chiral Magnetic Wave as a friend of Type-B NG mode

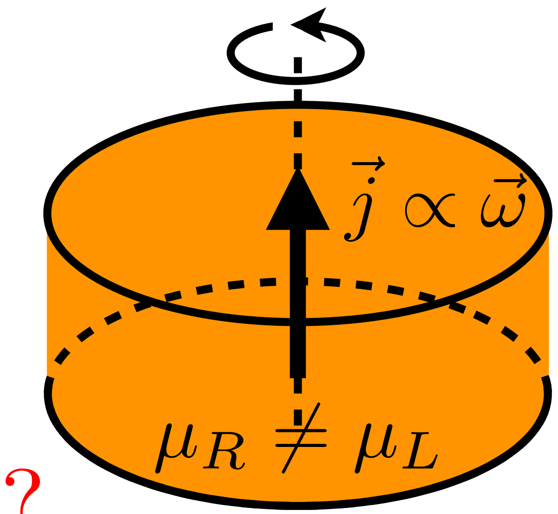
Outlook 1

◆ Chiral vortical effect

$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B} + \left(\frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\omega}$$

Originated from chiral anomaly

Origin of this term? $\tilde{R}R$??



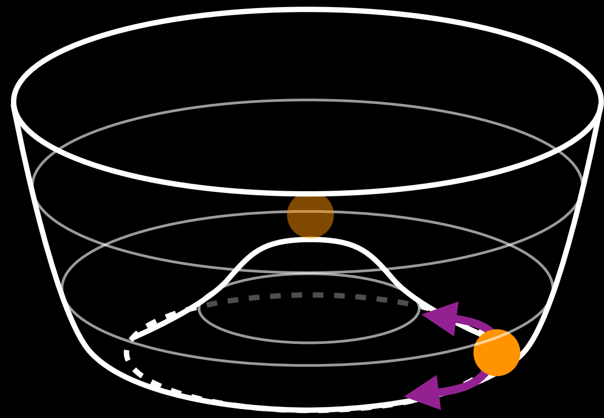
◆ Path-integral treatment?

	Operator formalism	Path-integral formalism
QCD	CA w/ Anomalous CR	Chiral pert. w/ Wess-Zumino term
Hydro	Mori's projection w/ Anomalous CR	MSRJD effective lagrangian w/ ? ? [Crossley et al. (2015)]

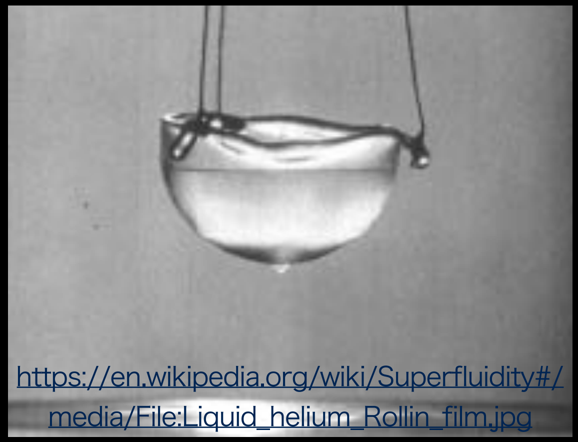
Outlook 2: Chiral superfluid

◆ Spontaneous symmetry breaking

Micro：Selecting vacuum

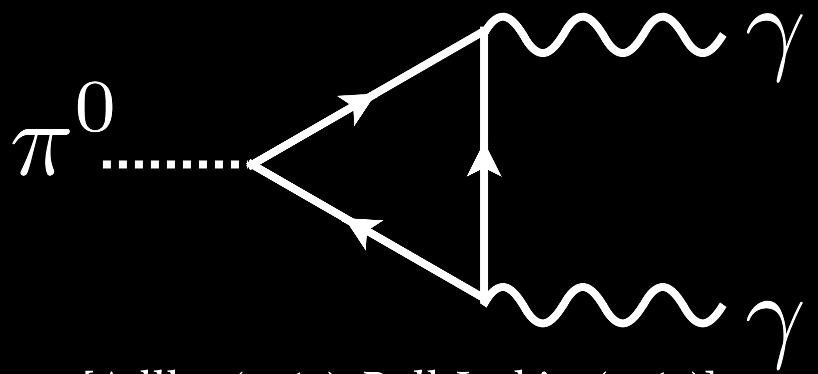


Macro：Superfluid



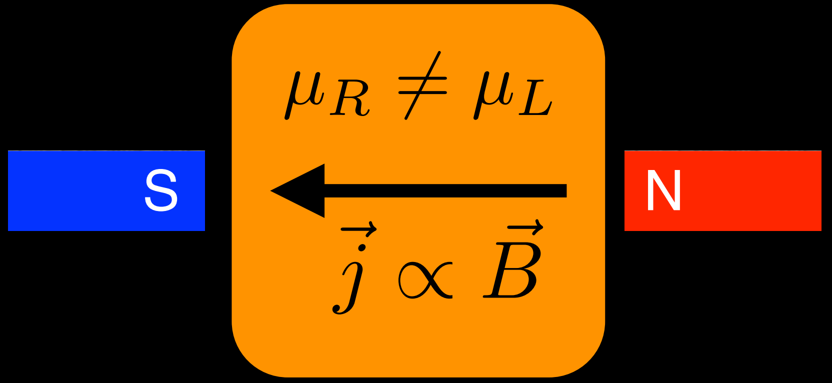
◆ Symmetry breaking by quantum anomaly

Micro：π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Macro：Anomalous transport



[Erdmenger et al. (2008), Son-Surowka (2009)]

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