Mesonic and nucleon fluctuation effects in nuclear medium

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GF & A. Hosaka, Phys. Rev. D **94**, 036005 (2016)
GF & A. Hosaka, Phys. Rev. D **95**, 116011 (2017)

Outline

Motivation

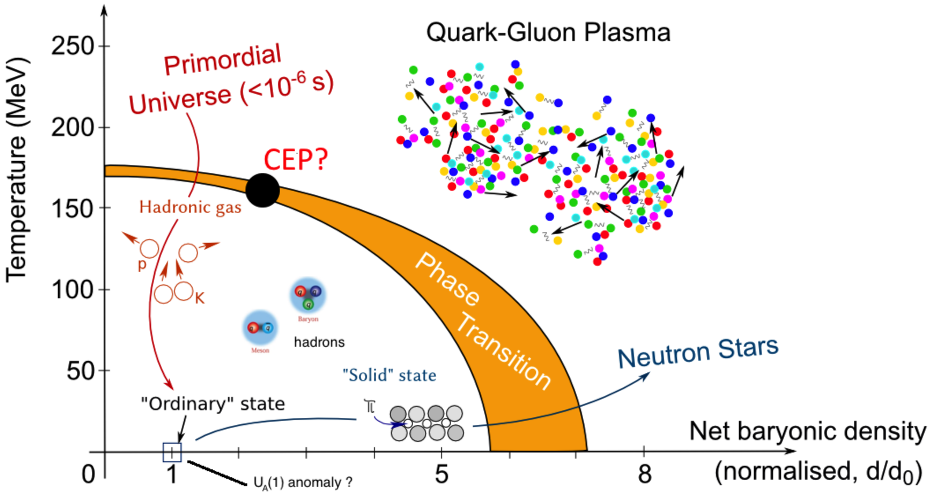
Functional Renormalization Group

Chiral effective nucleon-meson theory at finite μ_B

Numerical results

Summary

Motivation


Quark-Gluon Plasma
Primordial
Universe (<10^{-6} s)
CEP?
Hadronic gas
p
K
Baryon
Meson
hadrons
Phase
Transition
Neutron Stars
"Solid" state
"Ordinary" state
Temperature (MeV)
250
200
150
100
50
0
1
5
8
Net baryonic density
(normalised, d/d$_0$)
$U_A(1)$ anomaly ?

Motivation

AXIAL ANOMALY OF QCD:

- $U_A(1)$ anomaly: anomalous breaking of the $U_A(1)$ subgroup of chiral symmetry
  $\longrightarrow$ vacuum-to-vacuum topological fluctuations (instantons)

$$\partial_\mu j_A^{\mu a} = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[T^a F_{\mu\nu}F_{\rho\sigma}\right]$$

- $U_A(1)$ breaking interactions depend on instanton density
  $\longrightarrow$ suppressed at high T[1]
  $\longrightarrow$ calculations are trustworthy only at high temperature
  $\longrightarrow$ is the anomaly present at the phase transition?
- Very little is known at finite baryochemical potential (μ_B)[2]
  $\longrightarrow$ effective models have not been explored in this direction

[1] R. D. Pisarski, and L. G. Yaffe, Phys. Lett. B**97**, 110 (1980).
[2] T. Schaefer, Phys. Rev. D**57**, 3950 (1998).

η' - NUCLEON BOUND STATE:

- Effective models at finite T and/or density:
  $\longrightarrow$ effective models (NJL[3], linear sigma models[4]) predict a drop in $m_{\eta'}$ at finite μ_B
- Effective description of the mass drop:
  $\longrightarrow$ attractive potential in medium $\Rightarrow$ $\eta' N$ bound state
  $\longrightarrow$ Analogous to $\Lambda(1405) \sim \bar{K}N$ bound state

[3] P. Costa, M. C. Ruivo & Yu. L. Kalinovsky, Phys. Lett. B 560, 171 (2003).
[4] S. Sakai & D. Jido, Phys. Rev. C**88**, 064906 (2013).

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- Problem with effective model calculations: they treat model parameters as environment independent constants
  $\longrightarrow$ „$a \cdot v$" type of terms decrease (a-constant, v-decreases)
  $\longrightarrow$ evolution of „a" at finite T and μ_B?
- What is the role of fluctuations?

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Motivation

NEUTRON STARS:

- Two solar mass neutron star observations:
  J1614-2230: mass $= (1.928 \pm 0.017)M_s$
  J0348+0432: mass $= (2.01 \pm 0.04)M_s$
- Theoretical challenge: describe stiffness of the equation of state of cold dense matter

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- Theoretical challenge: describe stiffness of the equation of state of cold dense matter

- Effective model calculations stop at mean field level
  $\longrightarrow$ quantum and density fluctuations?
- Hyperon puzzle
  $\longrightarrow$ hyperons cause an undesired softening of the EoS
  $\longrightarrow$ few times normal nucl. density $\Rightarrow$ hyperons should appear

Motivation

FLUCTUATION EFFECTS IN FIELD THEORY:

- 2nd order transitions in statistical field theory
  $\longrightarrow$ diverging correlation length invalidates pert. theory
  $\longrightarrow$ solution: Wilson's momentum space RG
  $\longrightarrow$ explanation of universality, critical exponents, etc.

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  $\longrightarrow$ it can fail even for small couplings $\Rightarrow$ resummation!
  $\longrightarrow$ solution: Gell-Mann - Low RG

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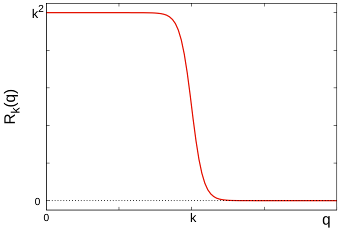
- Problem in QCD: running coupling grows as energy decreases
  $\longrightarrow$ RG cannot provide a generic solution
  $\longrightarrow$ effective models can help, but are also strongly coupled
- Non-perturbative methods are necessary
  $\longrightarrow$ Functional Renormalization Group (FRG)

Functional Renormalization Group

Mathematical implementation:

- Scale dependent partition function:

$$Z_k[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi] + \int J\phi)} \times e^{-\frac{1}{2}\int \phi R_k \phi}$$



- Scale dependent effective action:

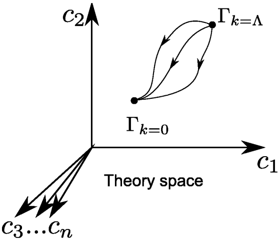
$$\Gamma_k[\bar{\phi}] = -\log Z_k[J] - \int J\bar{\phi} - \frac{1}{2}\int \bar{\phi} R_k \bar{\phi}$$

$\longrightarrow$ $k \approx \Lambda$: no fluctuations included

$\Rightarrow \Gamma_k[\bar{\phi}]|_{k=\Lambda} = \mathcal{S}[\bar{\phi}]$

$\longrightarrow$ $k = 0$: all fluctuations included

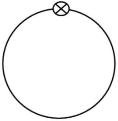
$\Rightarrow \Gamma_k[\bar{\phi}]|_{k=0} = \Gamma[\bar{\phi}]$



Motivation

- Flow equation of the effective action:

$$\partial_k \Gamma_k = \frac{1}{2} \int_{q,p}^{(T)} \partial_k R_k(q,p)(\Gamma_k^{(2)} + R_k)^{-1}(p,q) = \frac{1}{2}$$



- One-loop structure with dressed and regularized propagators

  $\longrightarrow$ RG change in the *n*-point vertices are described by one-loop diagrams

  $\longrightarrow$ exact relation, approximations are necessary

[5] D. Litim, Phys. Rev. D**64**, 105007 (2001).

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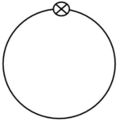

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- Derivative expansion (local potential approximation):

$$\Gamma_k = \int_x \left[Z_k \partial_i \Phi \partial_i \Phi + V_k(\Phi; x) \right]$$

$\longrightarrow$ „optimized" regulator[5]: $R_k(q) = Z_k(k^2 - q^2)\Theta(k^2 - q^2)$

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Chiral effective nucleon-meson model

- Chiral symmetry of QCD: $U(3) \times U(3)$ $(\Psi_{L/R} \to U_{L/R}\Psi_{L/R})$
- Effective model of mesons (M) and the nucleon (N):
  [M: π, K, η, η' and a_0, κ, f_0, σ, N: n, p, ω for short range $N - N$ int.]

$$
\begin{aligned}
\mathcal{L} &= \mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] + \mu^2\,\mathrm{Tr}\,(M^\dagger M) + \frac{g_1}{9}[\,\mathrm{Tr}\,(M^\dagger M)]^2 \\
&+ \frac{g_2}{3}\,\mathrm{Tr}\,(M^\dagger M)^2 + \text{higher order terms in } M \\
&+ a(\det M^\dagger + \det M) - \mathrm{Tr}\,[H(M^\dagger + M)] \\
&+ \bar{N}(-\partial_i\gamma_i - \mu_B\gamma_0)N - g\bar{N}\tilde{M}_5 N \\
&+ \frac{1}{4}(\partial_i\omega_j + i \leftrightarrow j)^2 + \frac{1}{2}m_\omega^2\omega_i\omega_i - ig_\omega\bar{N}\omega_i\gamma_i N
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- Model parameters:
  $\longrightarrow$ meson mass parameter (μ^2), quartic couplings (g_1, g_2),
  $\longrightarrow$ explicit breaking ($H = h_0 T^0 + h_8 T^8$), $U_A(1)$ anomaly (a),
  $\longrightarrow$ Yukawa couplings (g, g_ω), vector meson mass (m_ω^2)

Chiral effective nucleon-meson model

- Local potential approximation:

$$\Gamma_k = \int_x \Big[V_k[M] + \mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] - \mathrm{Tr}\,[H(M + M^\dagger)] + \bar{N}(-\partial_i\gamma_i + \mu_B\gamma_0)N - g\bar{N}\tilde{M}_5 N + \frac{1}{4}(\partial_i\omega_j + i \leftrightarrow j)^2 + \frac{1}{2}m_\omega^2\omega_i\omega_i - ig_\omega\bar{N}\omega_i\gamma_i N \Big]$$

$$V_k[M] = V_k^{\mathrm{mes}}[M] + V_k^{\mathrm{ferm}}[\tilde{M}]$$

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$$V_k[M] = V_k^{\mathrm{mes}}[M] + V_k^{\mathrm{ferm}}[\tilde{M}]$$

- Chiral invariant expansion: $\rho_{\mathrm{det}} = \det M^\dagger + \det M, \quad \rho_2 = \mathrm{Tr}\,[M^\dagger M],$
$\rho_4 = \mathrm{Tr}\,[M^\dagger M - \mathrm{Tr}\,[M^\dagger M]/3]^2$

$$V_k^{\mathrm{mes}} = U_k(\rho_2) + C_k(\rho_2)\cdot\rho_4 + A_k(\rho_2)\cdot\rho_{\mathrm{det}}, \quad V_k^{\mathrm{ferm}} = \tilde{U}_k(\tilde{\rho}_2)$$

- Projecting the flow equation onto various operators, one derives individual flow equations for U_k, C_k, A_k, $\tilde{U}_k$

Chiral effective nucleon-meson model

- Baryon Silver Blaze property:
  $\longrightarrow$ no change in thermodynamics for $\mu_B < m_N - B \equiv \mu_{B,c}$

[6] M. Drews and W. Weise, Prog. Part. Nucl. Phys. 93, 69 (2017).

Chiral effective nucleon-meson model

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  $\longrightarrow$ no change in thermodynamics for $\mu_B < m_N - B \equiv \mu_{B,c}$
- At $\mu_B = \mu_{B,c}$:[6]
  $\longrightarrow$ 1st order phase transition from nuclear gas to liquid
  $\longrightarrow$ nuclear density jumps from zero to $n_0 \approx 0.17\,\mathrm{fm}^{-3}$
  $\longrightarrow$ non-strange chiral condensate jumps from f_π to $v_{ns,nucl}$
  (Landau mass $M_L \approx 0.8 m_N \Rightarrow v_{ns,nucl} \approx 69.5\,\mathrm{MeV}$)

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  (Landau mass $M_L \approx 0.8 m_N \Rightarrow v_{ns,nucl} \approx 69.5\,\mathrm{MeV}$)
- The first order transition is related to the condensation of the timelike component of the ω vector particle
- ω couples to v_{ns} that couples to v_s
  $\longrightarrow$ jump in all order parameters

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Chiral effective nucleon-meson model

- Step I.: solve equations in the vacuum $\Rightarrow$ determine model parameters

  $\longrightarrow$ physical masses of π, K, η, η' and N are used

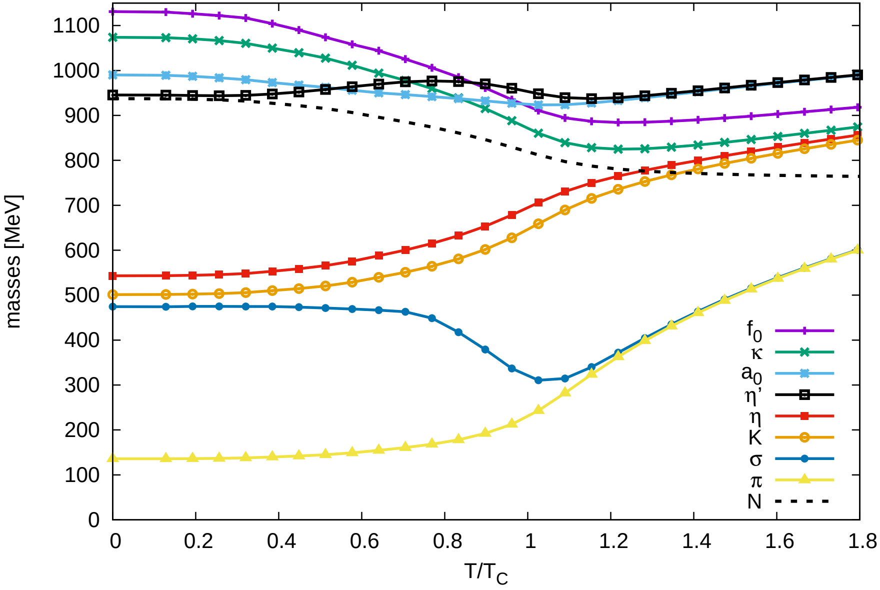
  $\longrightarrow$ PCAC relations leads to $H \Leftrightarrow (h_0, h_8)$

$$m_\alpha^2 f_\alpha \pi_\alpha \;=\; \partial_\mu J_\alpha^{5\mu} = -\frac{\partial}{\partial \theta_A^\alpha} \mathrm{Tr}\left(H(M + M^\dagger)\right)$$

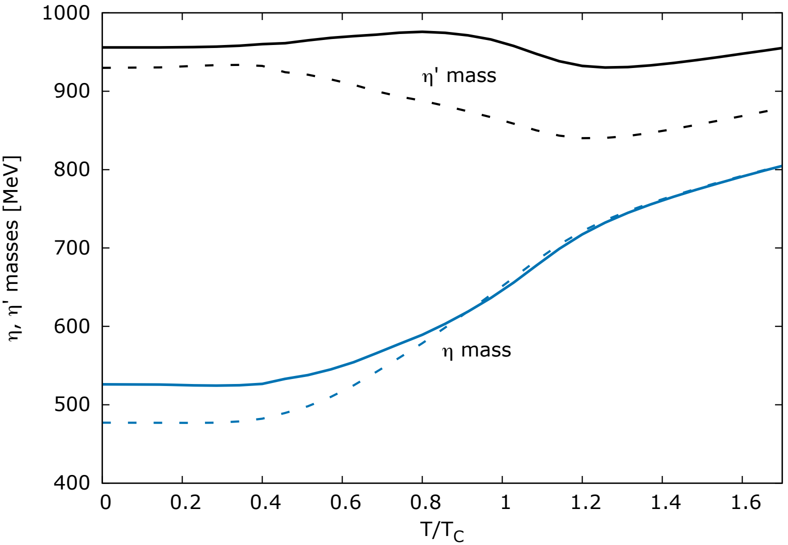
  $\longrightarrow$ inputs from the nuclear liquid gas transition:
  sat. density, Landau mass, binding energy, surface tension

- Step II.: solve the same equations at finite T and μ_B
  $\longrightarrow$ mass spectrum in medium
  $\longrightarrow$ details of symmetry restoration
  $\longrightarrow$ $U_A(1)$ anomaly

Numerical results: mass spectrum at finite T

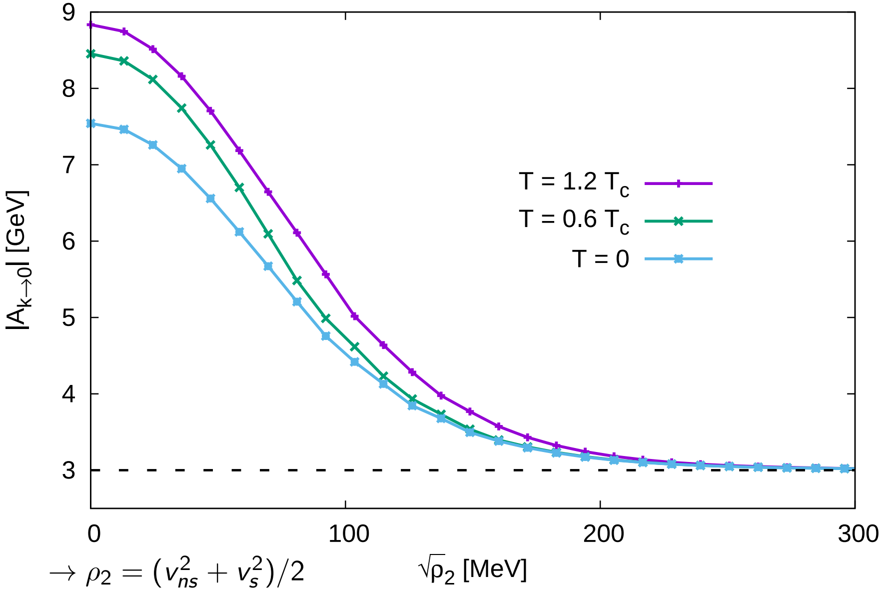


Numerical results: $\eta - \eta'$ system at finite T



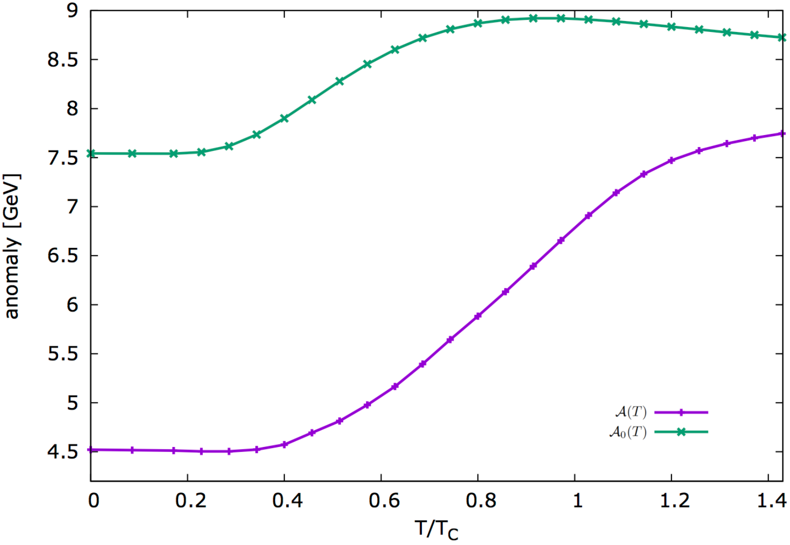
⟶ solid: full solution, dashed: field- and T independent anomaly

Numerical results: anomaly at finite T

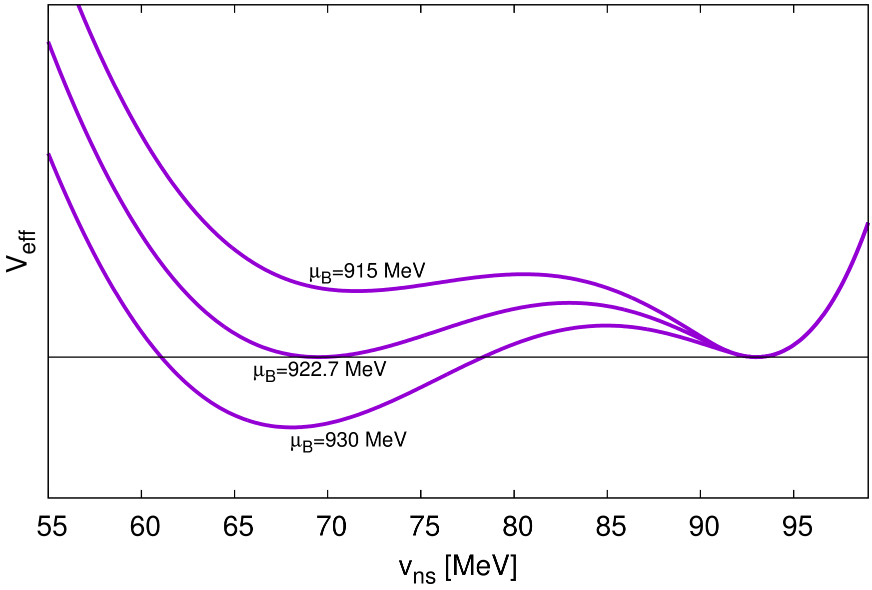


$\rightarrow \rho_2 = (v_{ns}^2 + v_s^2)/2$

Numerical results: anomaly at finite T



Numerical results: symmetry restoration at finite μ_B



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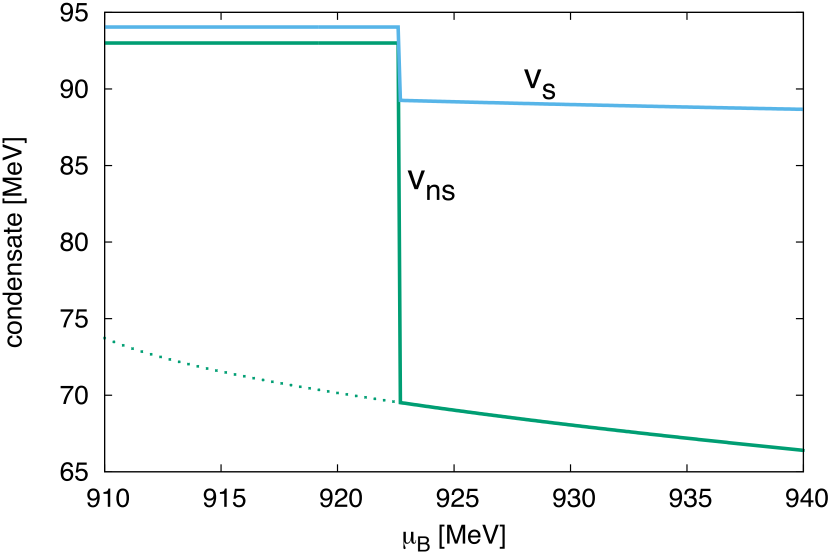
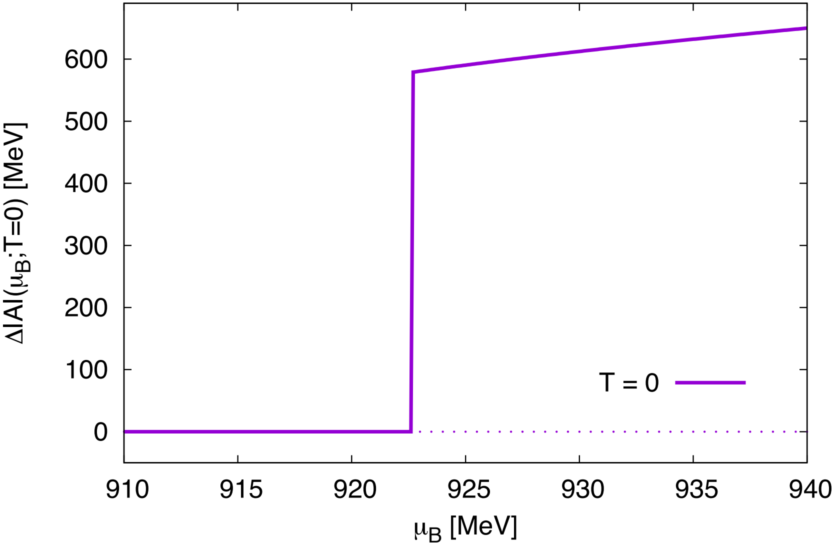


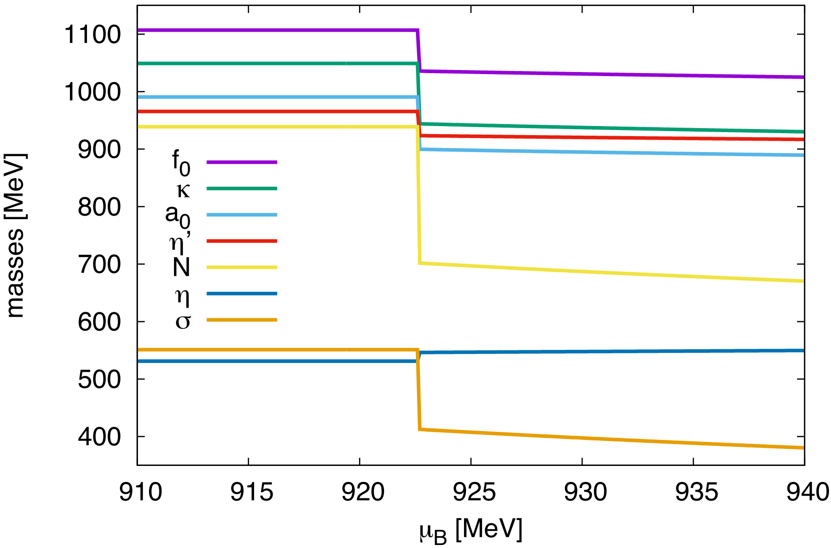
$\longrightarrow$ no phase transition beyond $\mu_{B,crit} \approx 922.7\,\mathrm{MeV}$

Numerical results: anomaly at finite μ_B



$\longrightarrow$ vacuum value $A_{k=0} \approx 4.7\,\mathrm{GeV}$ $\Rightarrow$ increases more than 10%

Numerical results: mass spectrum at finite μ_B



$\longrightarrow \eta'$ mass drop is rather small $\Rightarrow \eta' - N$ bound state in doubt

Summary

- Mesonic and nucleon fluctuations effects on chiral symmetry, axial anomaly and mesonic spectrum in nuclear medium
- Method: Functional Renormalization Group (FRG) approach
- Findings:

  $\longrightarrow$ mesonic fluctuations make the anomaly coefficient condensate dependent

- Nucl. liquid gas transition: $\sim 20\%$ drop in (n.s.) chiral cond.

  $\longrightarrow$ (partial) restoration of chiral symmetry seem to increase the anomaly ($\Delta|A| \gtrsim 10\%$ relative difference)

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- Important:

  $\longrightarrow$ no instanton effects have been included!

  $\longrightarrow$ environment dependence of the bare anomaly coefficient could be relevant!