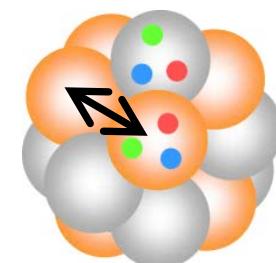
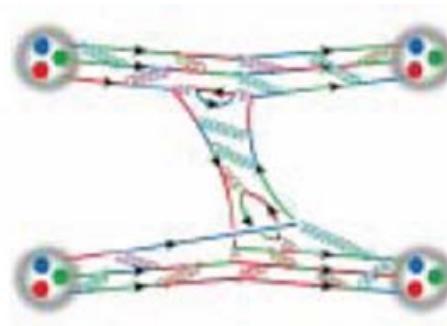
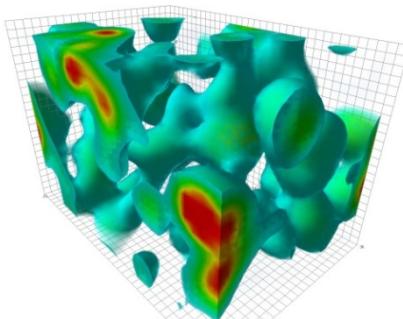
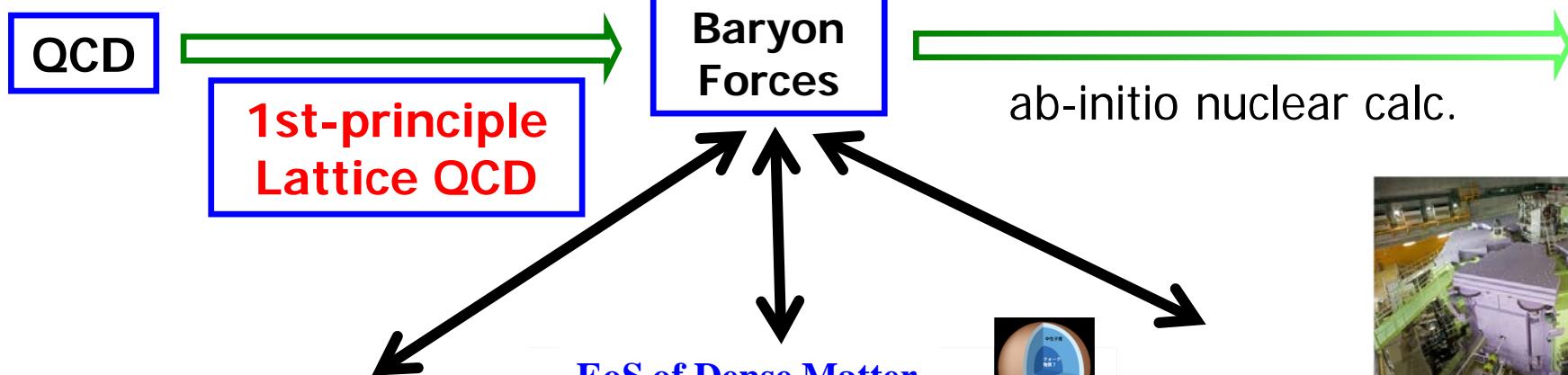
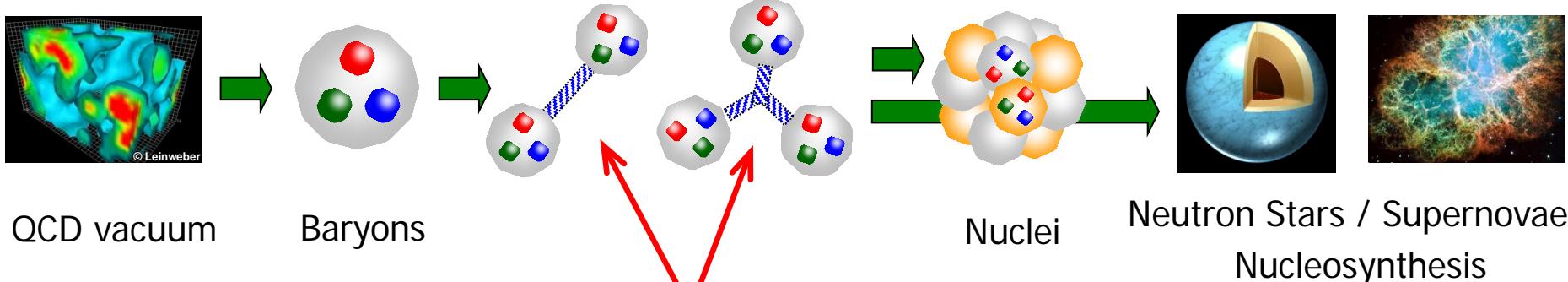


# Nuclear Physics from Lattice QCD

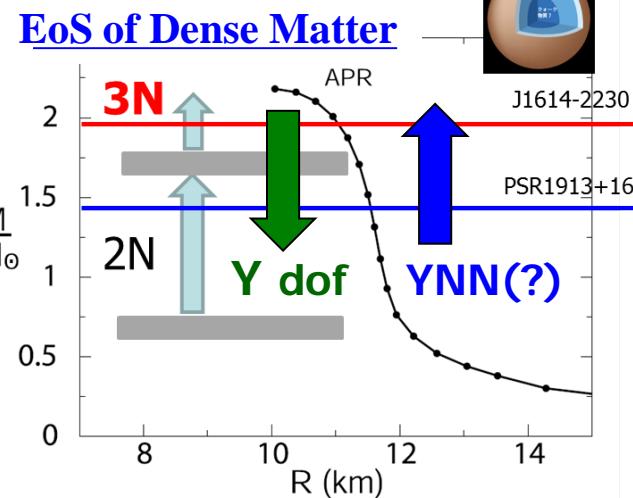
**Takumi Doi**  
(Nishina Center, RIKEN)



# The Odyssey from Quarks to Universe



## Nuclear Forces / Hyperon Forces

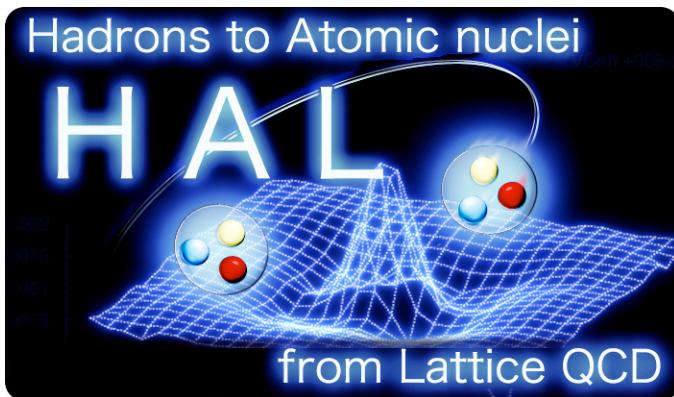


LIGO/Virgo  
KAGRA



- Outline

- Introduction
- Theoretical framework (HAL QCD method)
- (Results at heavy quark masses)
- Results at physical quark masses
- Summary / Prospects



**S. Aoki, T. Aoyama, D. Kawai,  
T. Miyamoto, K. Sasaki** (YITP)  
**T. Doi, T. M. Doi, S. Gongyo,  
T. Hatsuda, T. Iritani** (RIKEN)  
**F. Etminan** (Univ. of Birjand)  
**Y. Ikeda, N. Ishii, K. Murano,  
H. Nemura** (RCNP)  
**T. Inoue** (Nihon Univ.)

# [HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0 | N(\vec{r}) N(\vec{0}) | N(\vec{k}) N(-\vec{k}); \text{in} \rangle$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$

- phase shift at asymptotic region

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

Extended to multi-particle systems

M.Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467

N.Ishizuka, PoS LAT2009 (2009) 119

CP-PACS Coll., PRD71(2005)094504

S. Aoki et al., PRD88(2013)014036

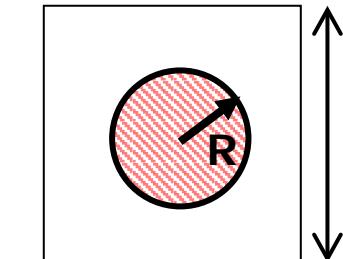
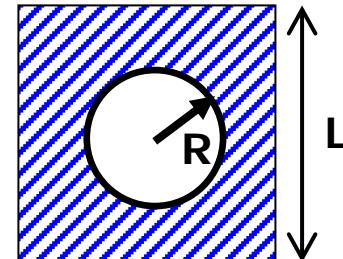
- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\vec{r}) = m \int d\vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}'), \quad r < R$$

- $U(\vec{r}, \vec{r}')$ : faithful to the phase shift by construction

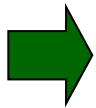
- $U(\vec{r}, \vec{r}')$ : E-independent, while non-local in general

- Non-locality → derivative expansion

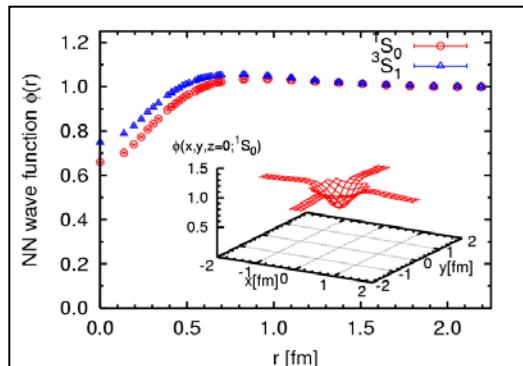


# HAL QCD method

Lattice QCD



NBS wave func.

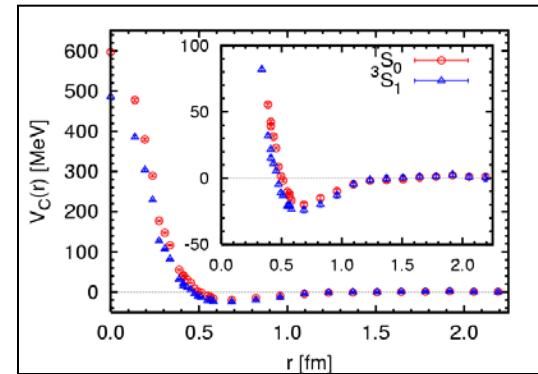


$$\begin{aligned}\psi_{NBS}(\vec{r}) &= \langle 0 | N(\vec{r}) N(0) | N(\vec{k}) N(-\vec{k}), in \rangle \\ &\simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)\end{aligned}$$

(at asymptotic region)

Analog to ...

Lat Baryon Force



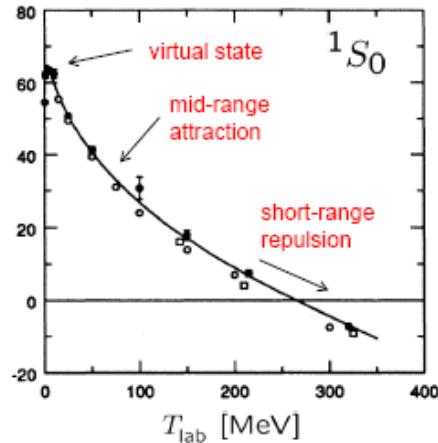
$$(k^2/m_N - H_0) \psi(\vec{r}) = \int d\vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}')$$

**E-indep (& non-local) Potential:**  
Faithful to phase shifts

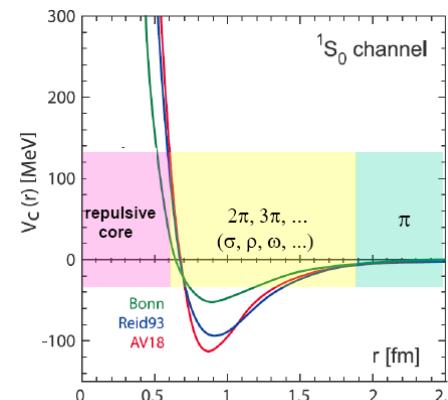
Scattering Exp.



Phase shifts



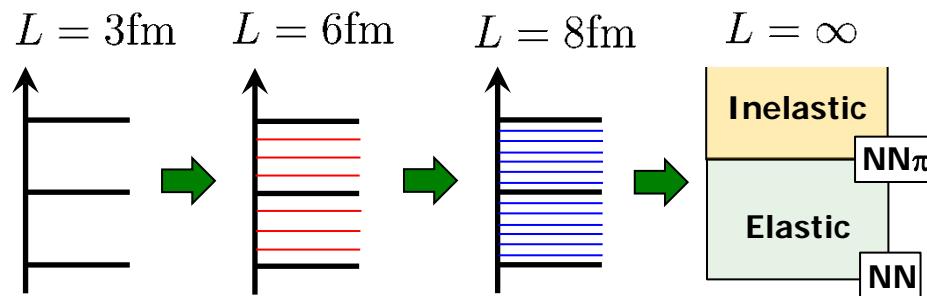
Phen. Potential



# The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

- (almost) No Excitation Energy
- LQCD method based on G.S. saturation impossible



Signal/Noise issue

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m}_N - \mathbf{3/2} \mathbf{m}_\pi) \times t]$$

Parisi('84), Lepage('89)

$$L=8\text{fm} @ \text{physical point} \quad (E_1 - E_0) \simeq 25\text{MeV} \implies t > 10\text{fm}$$

$$S/N \sim 10^{-32}$$

Naïve plateau fitting at  $t \sim 1\text{fm}$  is unreliable ("mirage" of true signal)

# Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

**E-indep of potential  $U(r,r')$**   $\rightarrow$  (excited) scatt states share the same  $U(r,r')$   
*They are not contaminations, but signals*

## Original (t-indep) HAL method

$$G_{NN}(\vec{r}, t) = \langle 0 | N(\vec{r}, t) N(\vec{0}, t) \overline{\mathcal{J}_{\text{src}}(t_0)} | 0 \rangle$$

$$R(\mathbf{r}, t) \equiv G_{NN}(\mathbf{r}, t)/G_N(t)^2 = \sum A_{W_i} \psi_{W_i}(\mathbf{r}) e^{-(W_i - 2m)t}$$

$$\int d\mathbf{r}' \mathbf{U}(\mathbf{r}, \mathbf{r}') \underline{\psi_{W_0}(\mathbf{r}')} = (\underline{E_{W_0}} - H_0) \underline{\psi_{W_0}(\mathbf{r})}$$

$$\int d\mathbf{r}' \mathbf{U}(\mathbf{r}, \mathbf{r}') \underline{\psi_{W_1}(\mathbf{r}')} = (\underline{E_{W_1}} - H_0) \underline{\psi_{W_1}(\mathbf{r})}$$

...

← Many states contribute

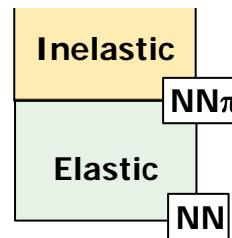
## New t-dep HAL method

All equations can be combined as

$$\int d\mathbf{r}' \mathbf{U}(\mathbf{r}, \mathbf{r}') \underline{R(\mathbf{r}', t)} = \left( -\frac{\partial}{\partial t} + \frac{1}{4m} \frac{\partial^2}{\partial t^2} - H_0 \right) \underline{R(\mathbf{r}, t)}$$

~~G.S. saturation~~  $\rightarrow$  "Elastic state" saturation

[Exponential Improvement]

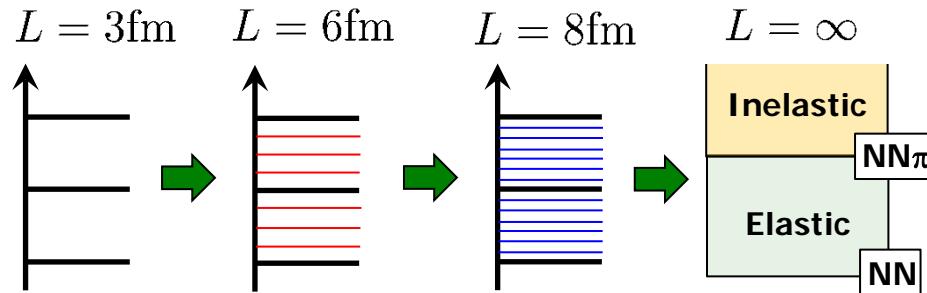


potential

# The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

- (almost) No Excitation Energy
- LQCD method based on G.S. saturation impossible



OCD

HAL QCD method

Baryon Forces

“Time-dependent method”

N.Ishii et al. PLB712(2012)437

G.S. saturation NOT required w/ E-indep pot



Direct method

G.S. saturation required

Savage et al. (NPL Coll.)  
Yamazaki et al.

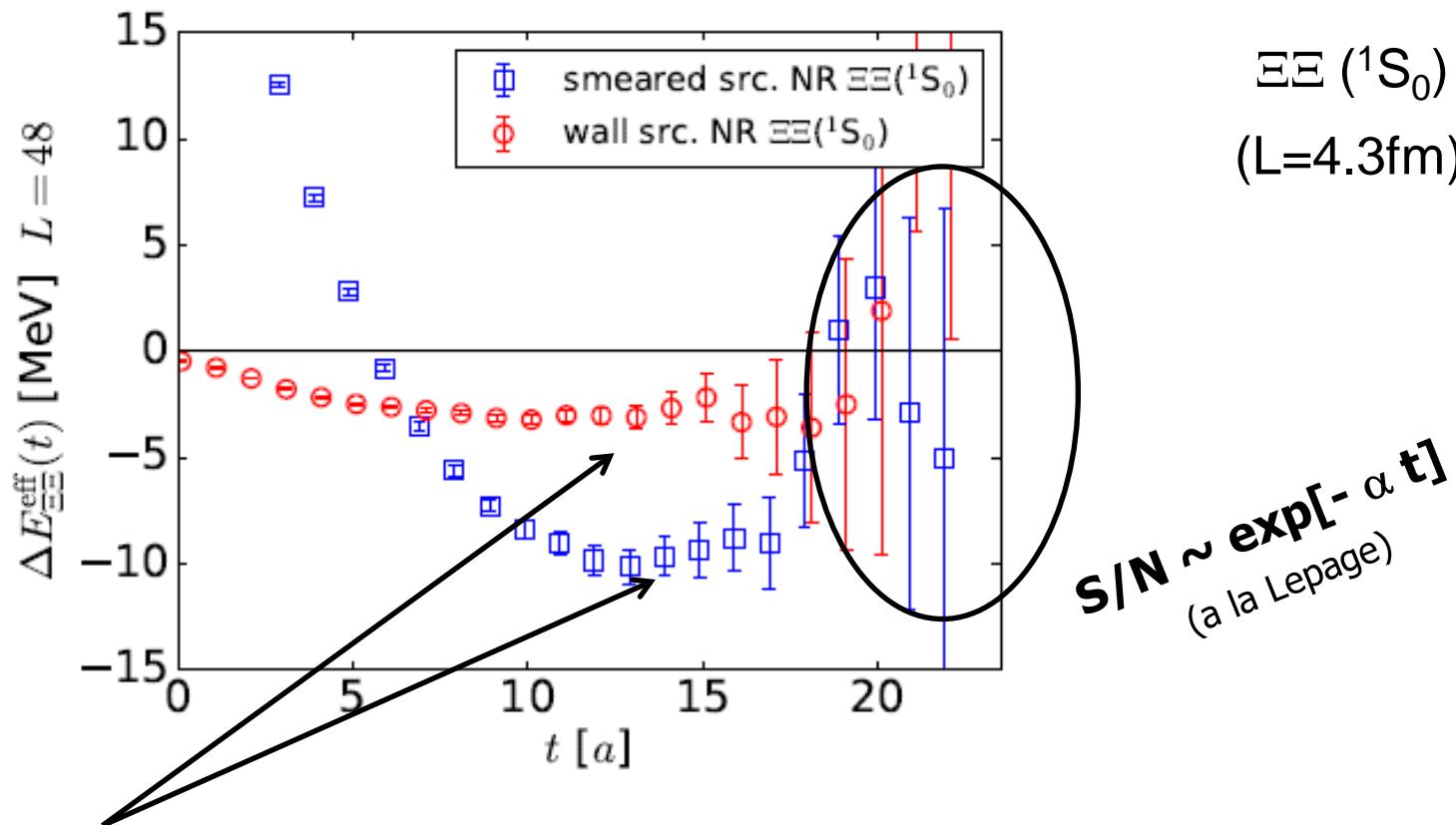
Experiments

# Example of failure of the direct method

$$R(t) = \sum_{\vec{r}} \sum_{\vec{x}} \langle 0 | B(\vec{r} + \vec{x}, t) B(\vec{x}, t) \overline{\mathcal{J}_{\text{src}}(0)} | 0 \rangle / \{G_B(t)\}^2$$

T. Iritani et al. (HAL)  
JHEP1610(2016)101

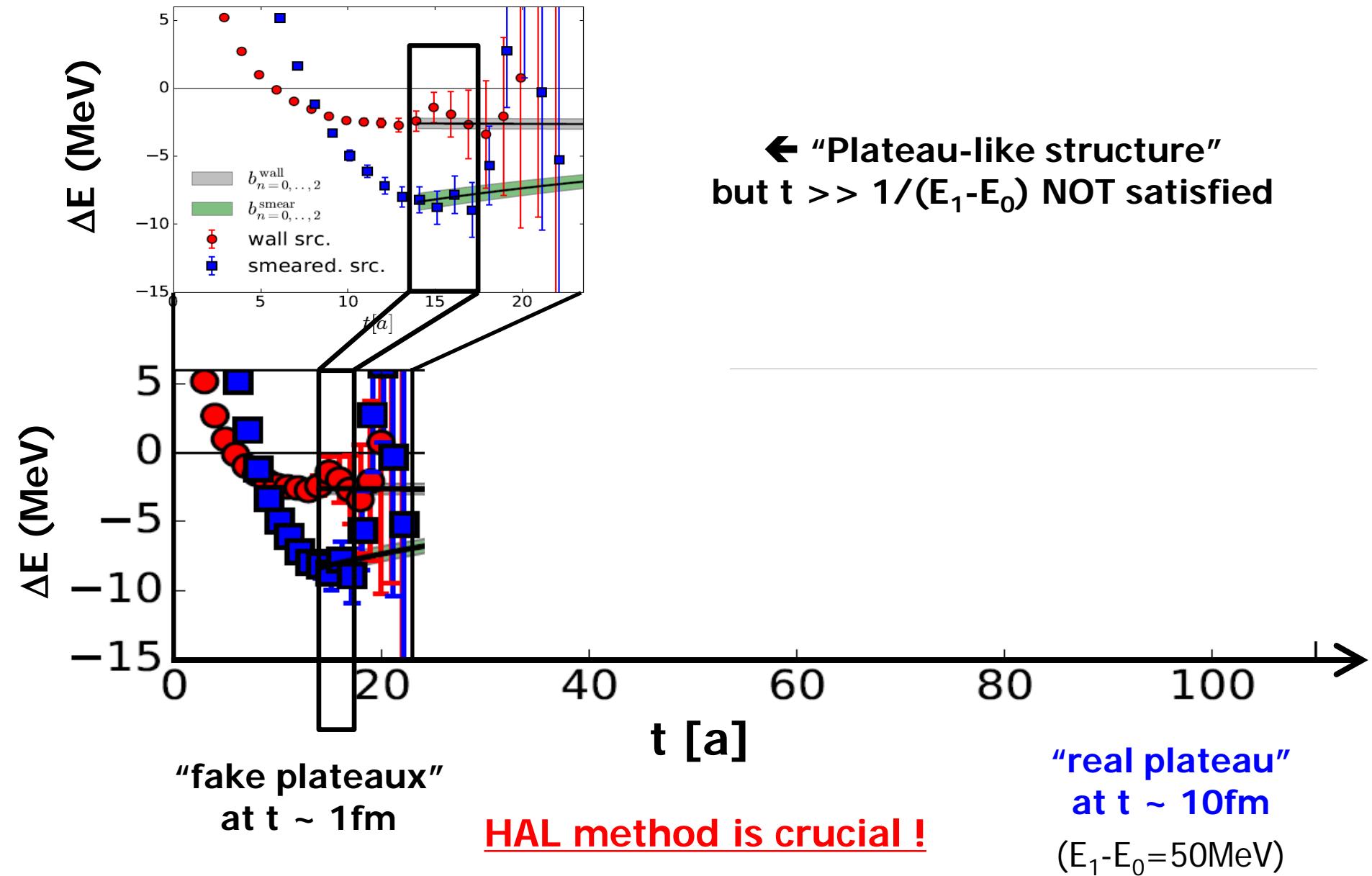
Physics should NOT depend on source op.



Wall and Smeared are Inconsistent:  
one cannot judge which (or neither) is reliable

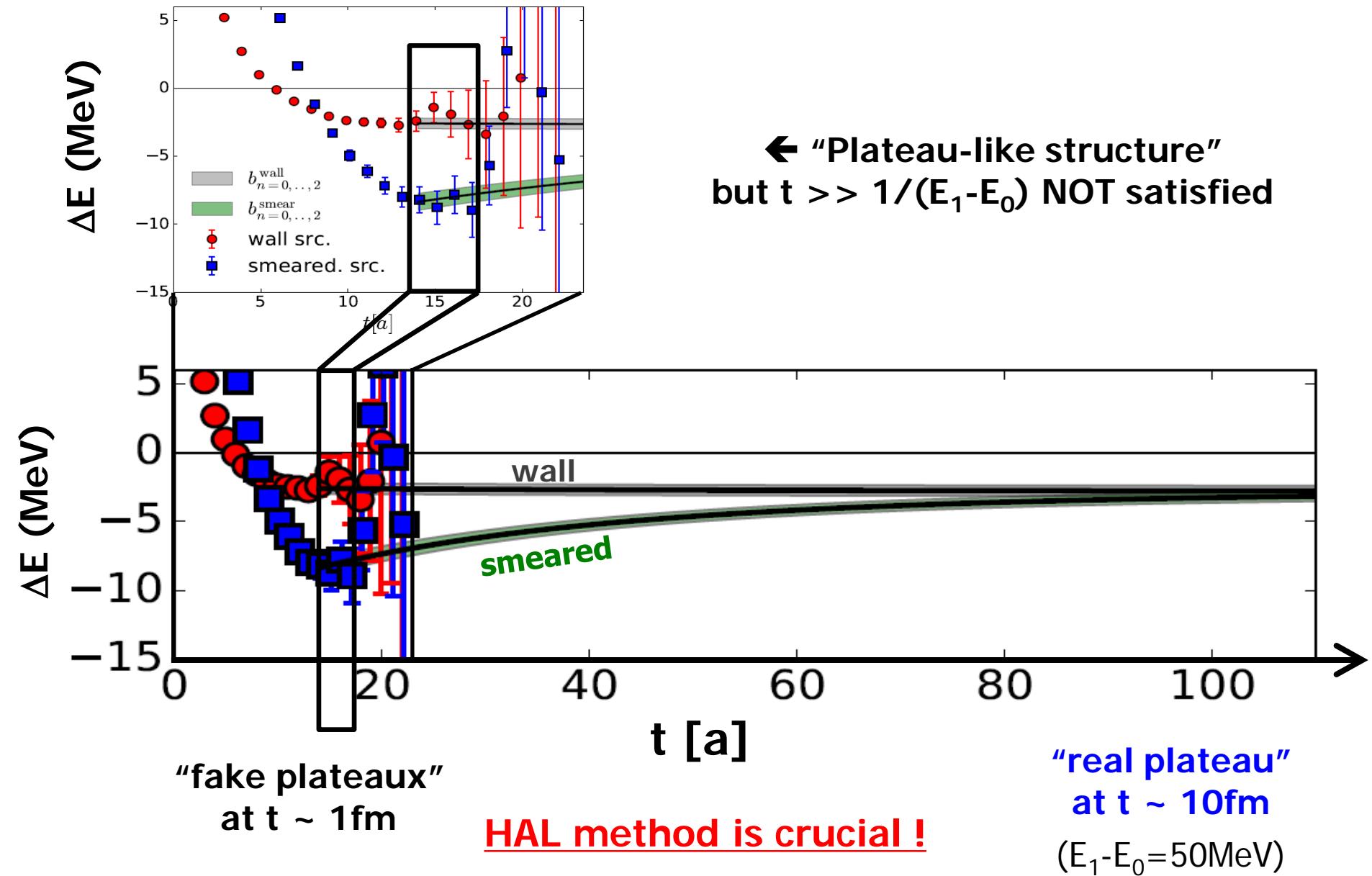
# “Anatomy” of symptom in direct method

T. Iritani (HAL Coll.), arXiv:1710.06147



# “Anatomy” of symptom in direct method

T. Iritani (HAL Coll.), arXiv:1710.06147



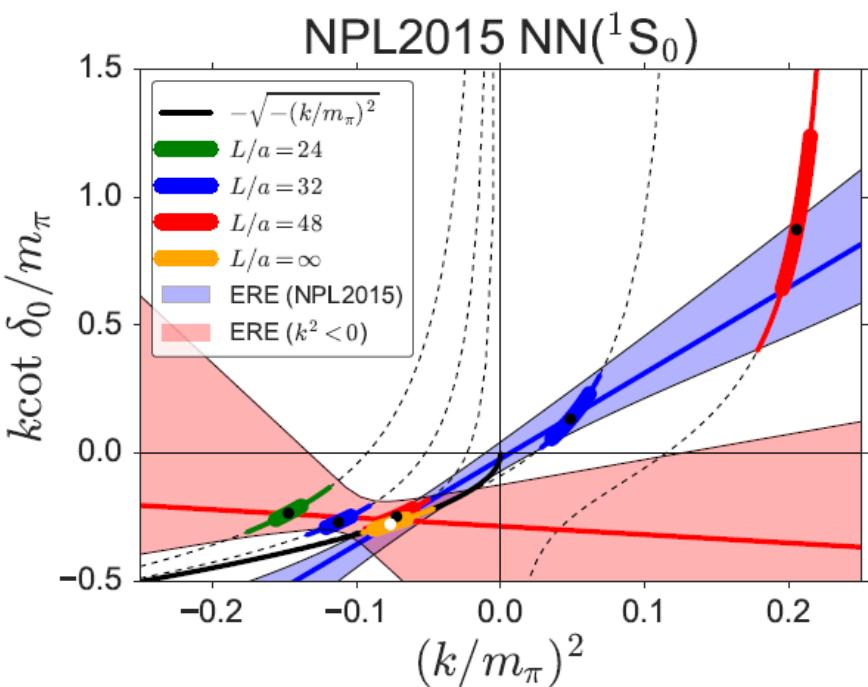
# “Sanity Check” for results from direct method

T. Iritani et al. (HAL Coll.) PRD96(2017)034521

$k \cot \delta(k)$  vs  $k^2$  plot

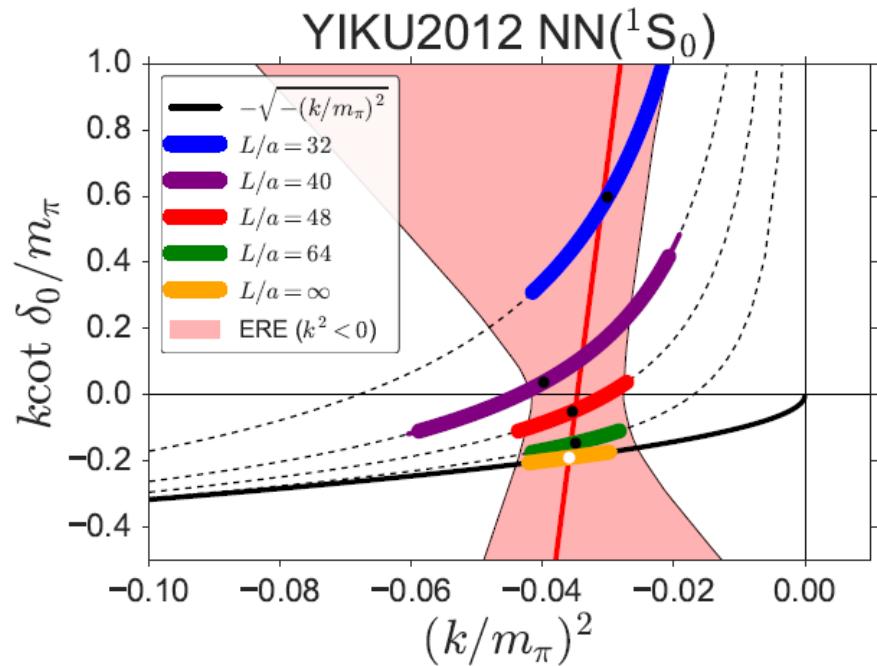
ERE:  $k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2} \mathbf{r} k^2 + \dots$

Data from NPL Coll. ('15)



Inconsistent ERE  
Unphysical pole residue

Data from Yamazaki et al ('12)



FAILED

Singular behaviors

$r \simeq \pm\infty$

12

# The fate of the direct method (check on NN)

T. Iritani et al. (HAL Coll.) PRD96(2017)034521

Data	$NN(^1S_0)$					$NN(^3S_1)$				
	Source independence	Sanity check			Source independence	Sanity check			(i)	(ii)
		(i)	(ii)	(iii)		(i)	(ii)	(iii)		
YKU2011 [24]	†	No	No	*	†	No	No	*		
YIKU2012 [25]	No	†	No	*	No	†	No	*		
YIKU2015 [26]	†	†	No	*	†	†	No	No		
NPL2012 [27]	†	†	No	*	†	†	*	*		
NPL2013 [28, 29]	No	*	*	No	No	*	*	?		
NPL2015 [30]	†	No	*	No	†	No	*	No		
CalLat2017 [31]	No	?	*	No	No	?	*	No		

All data for NN by the direct method fail these “minimum” tests so far

➔ Studies w/ the variational method are mandatory

- Outline

- Introduction
- Theoretical framework (HAL QCD method)
- (Results at heavy quark masses)
- Results at physical quark masses
  - Nuclear forces and Hyperon forces
  - Impact on dense matter
- Summary / Prospects

- Baryon Forces from LQCD Ishii-Aoki-Hatsuda (2007)
- Exponentially better S/N Ishii et al. (2012)
- Coupled channel systems Aoki et al. (2011,13)

**[Theory] = HAL QCD method**

## Baryon Interactions at Physical Point

### [Hardware]

= K-computer [10PFlops]

- + FX100 [1PFlops] @ RIKEN
- + HA-PACS [1PFlops] @ Tsukuba

- HPCI Field 5 “Origin of Matter and Universe”



### [Software]

= Unified Contraction Algorithm

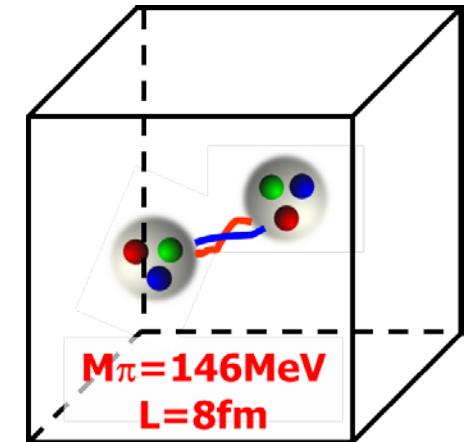
- Exponential speedup Doi-Endres (2013)

$^3\text{H}/^3\text{He}$	:	$\times 192$
$^4\text{He}$	:	$\times 20736$
$^8\text{Be}$	:	$\times 10^{11}$

# Lattice QCD Setup

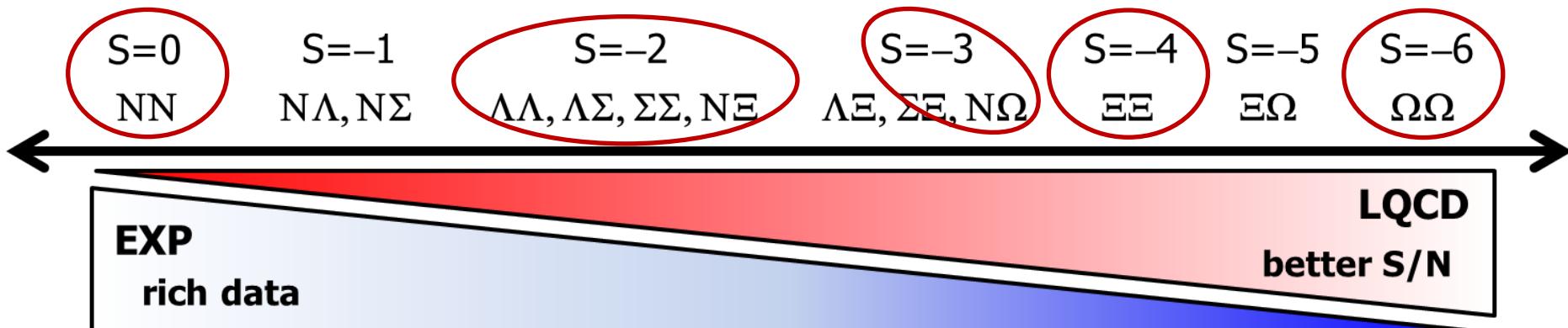
- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge w/ stout smearing
  - $V=(8.1\text{fm})^4$ ,  $a=0.085\text{fm}$  ( $1/a = 2.3 \text{ GeV}$ )
  - $m(\pi) \approx 146 \text{ MeV}$ ,  $m(K) \approx 525 \text{ MeV}$
  - #traj  $\approx 2000$  generated

PACS Coll., PoS LAT2015, 075



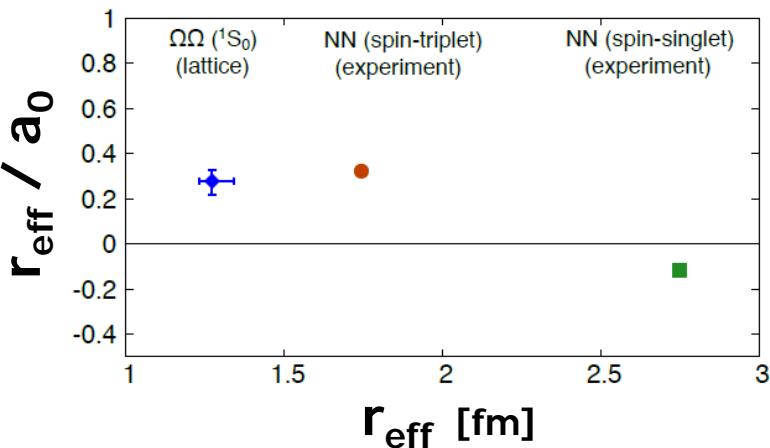
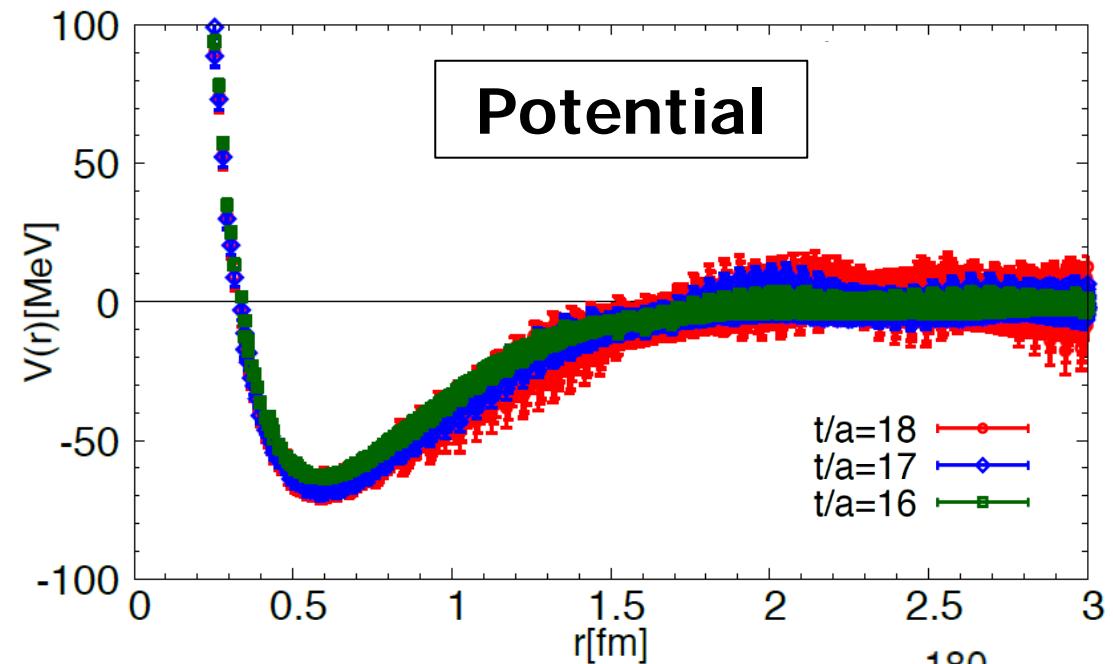
- **Measurement**
  - All of NN/YN/YY for central/tensor forces in P=(+) (S, D-waves)

## Predictions for Hyperon forces



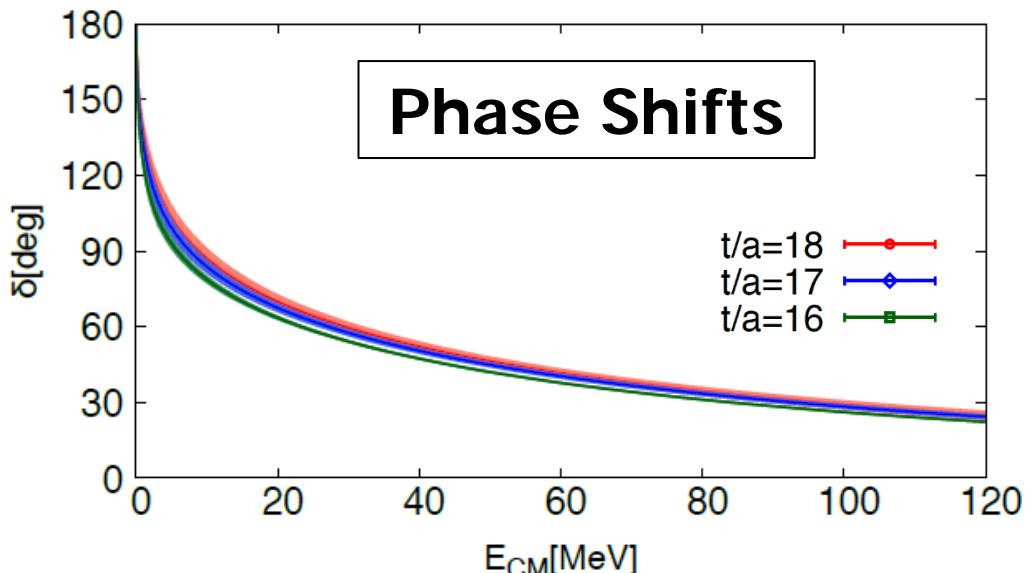
# $\Omega\Omega$ system ( $^1S_0$ )

The “most strange”  
dibaryon system

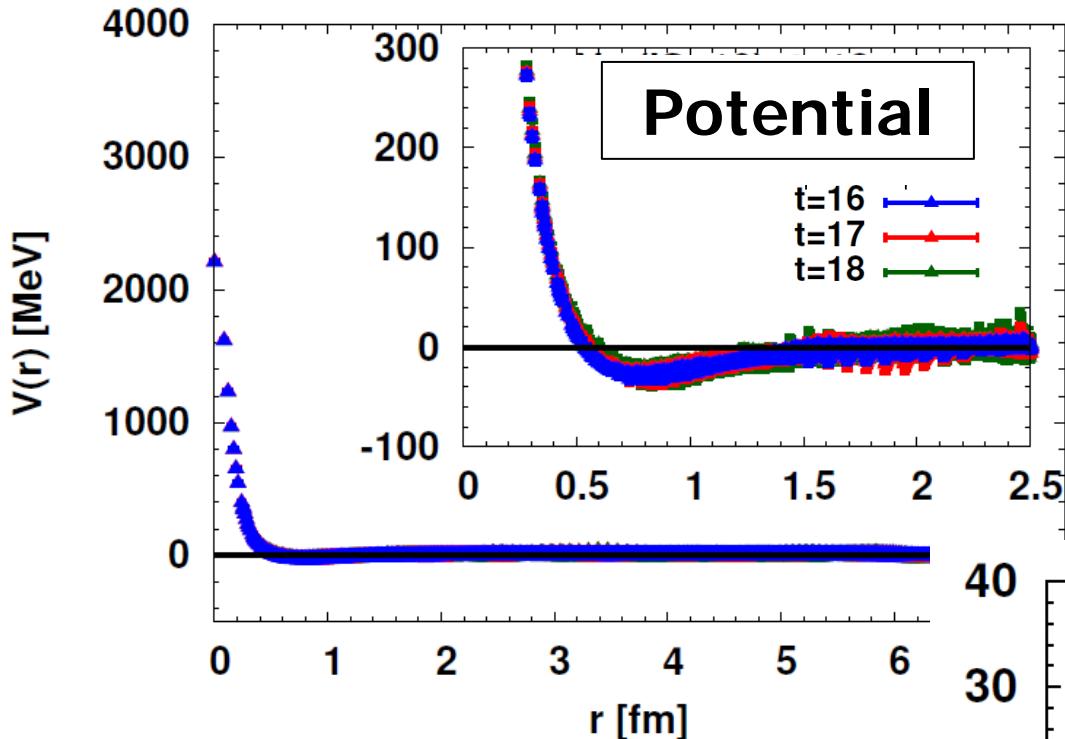


Strong Attraction

→ Vicinity of bound/unbound  
[~ Unitary limit]

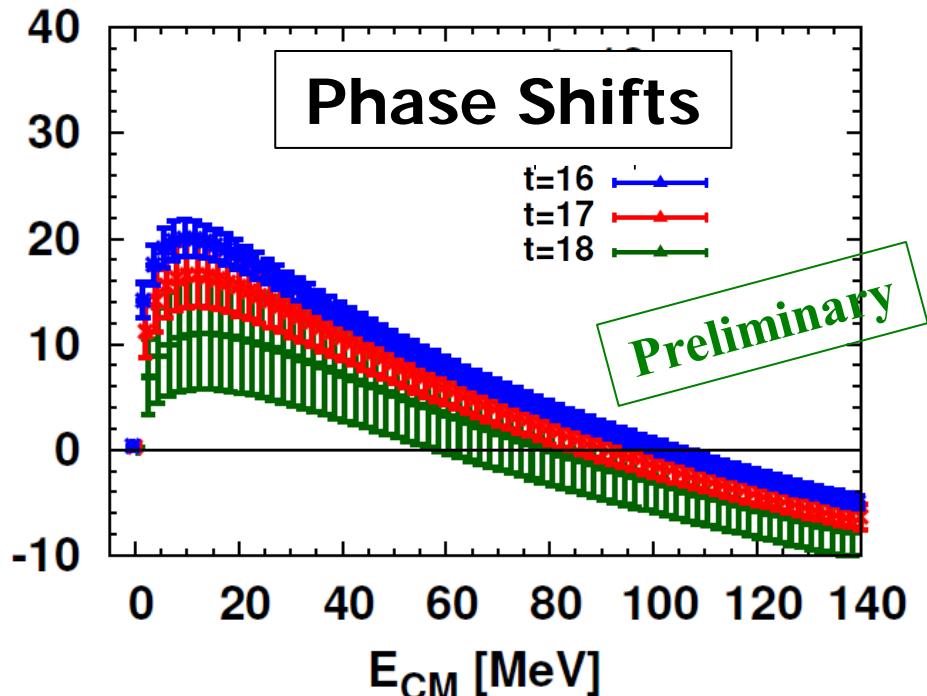


# $\Xi\Xi$ system ( $^1S_0$ )



Flavor SU(3)-partner of dineutron

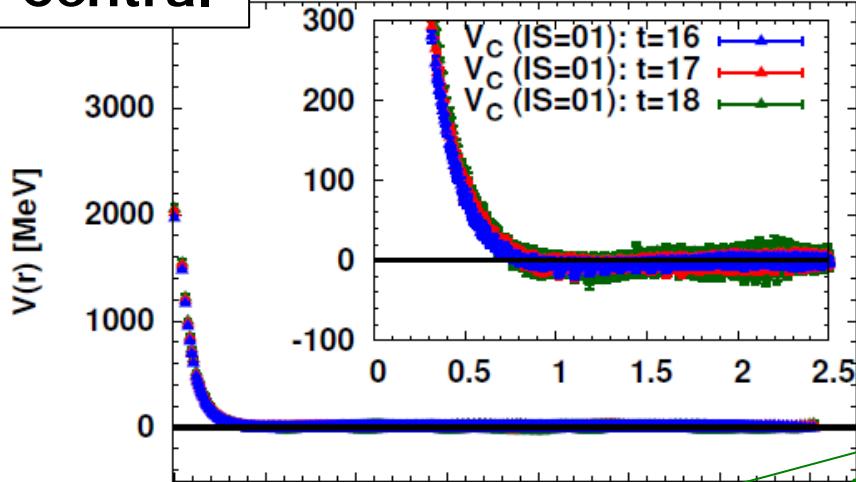
- ⇒ • “Doorway” to NN-forces
- Bound by SU(3) breaking ?



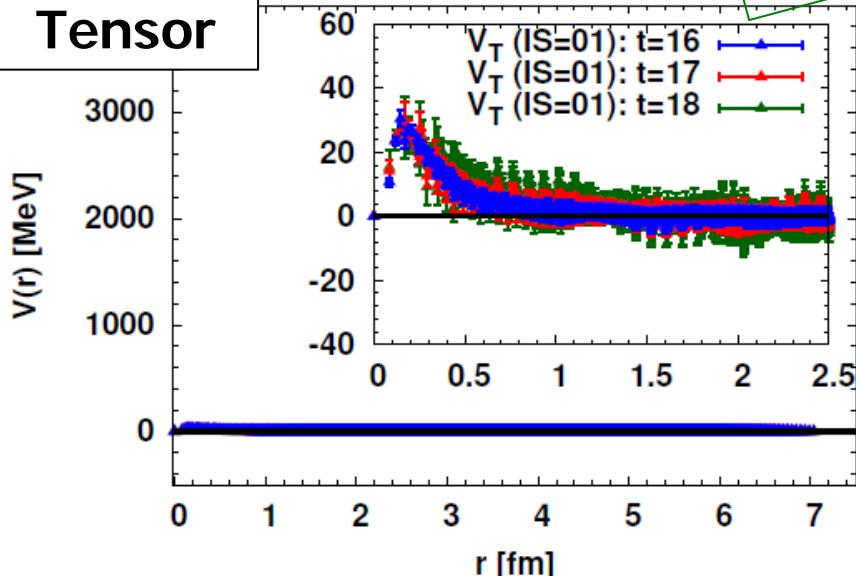
# $\Sigma\Sigma$ system ( $^3S_1$ - $^3D_1$ )

## Potentials

Central



Tensor



10plet  $\Leftrightarrow$  unique w/ hyperon DoF

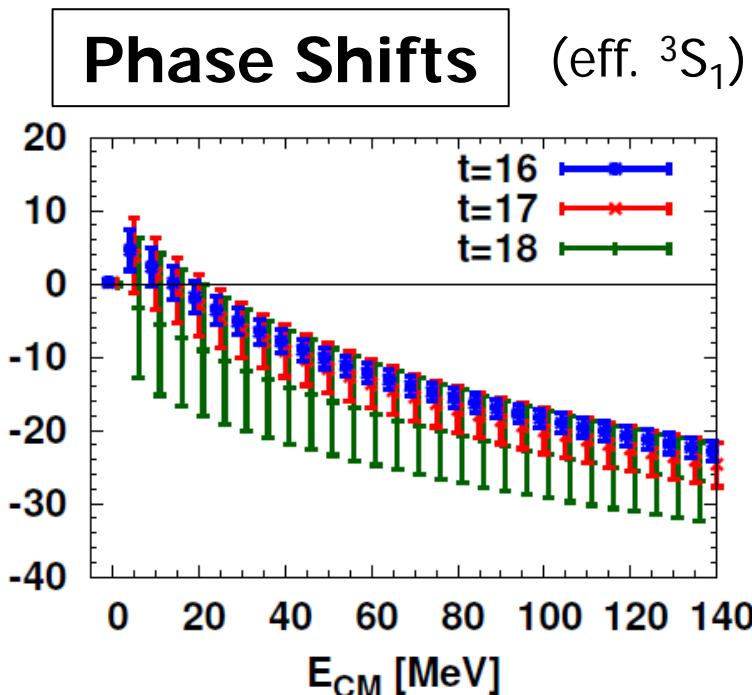
Flavor SU(3)-partner of  $\Sigma^-$  n

$\Rightarrow \bullet \Sigma^-$  in neutron star ?

Central: Strong Repulsion

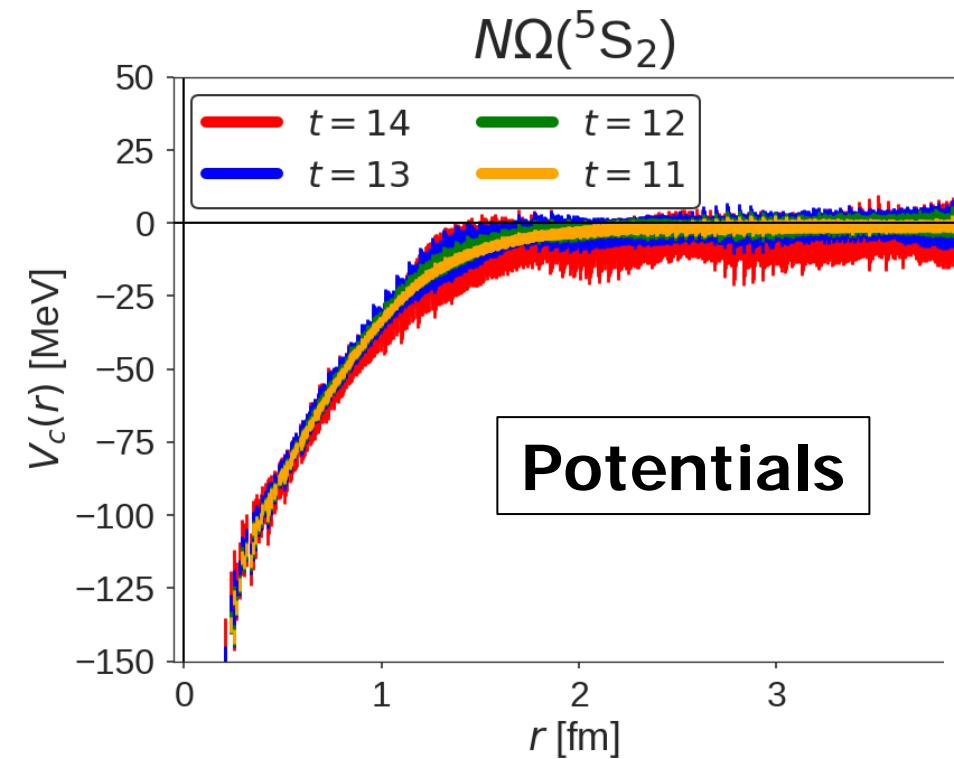
Tensor: Weak

Phase Shift [°]



(2-gauss + 2-OBEP)

# $N\Omega$ system ( ${}^5S_2$ )

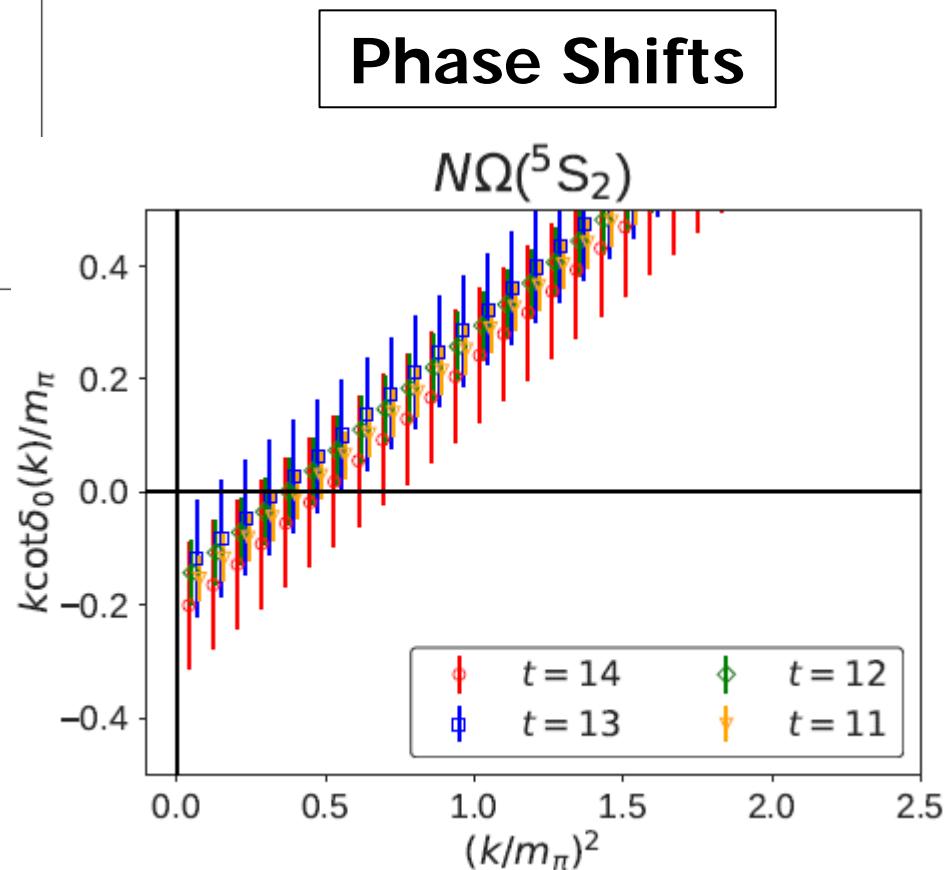


Strong Attraction  
possibly "Bound"

$\iff$   $N\Omega$  correlation in HIC

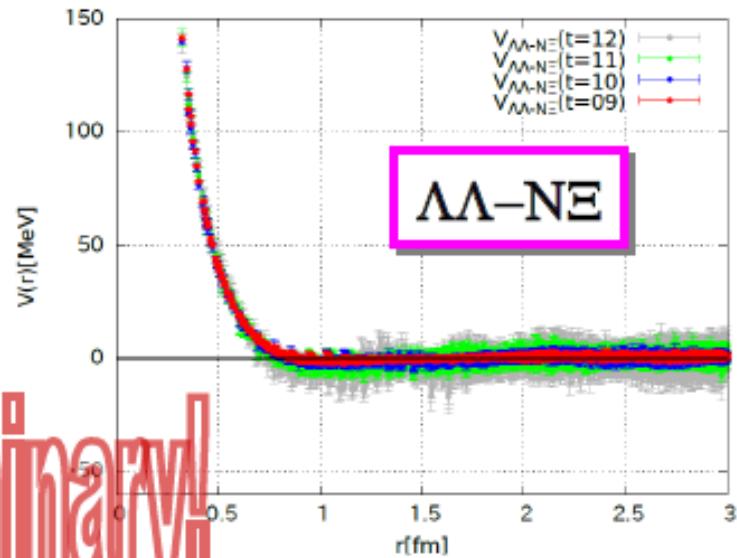
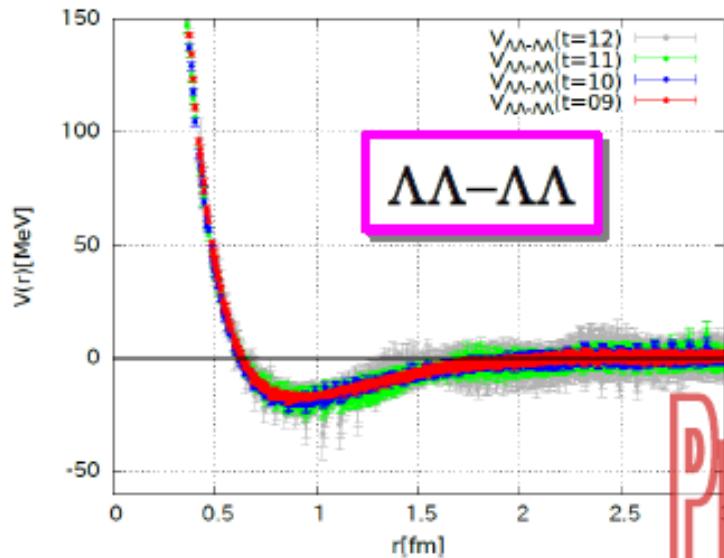
(200conf x 4rot x 48src)

preliminary



[T. Iritani]

# $\Lambda\Lambda$ , $N\Xi$ , ( $\Sigma\Sigma$ ) coupled channel $\rightarrow$ H-dibaryon channel



Preliminary!

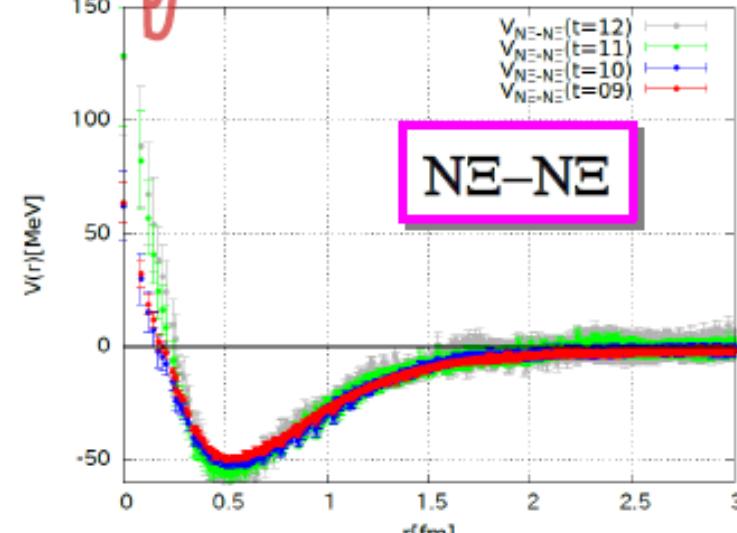
$m_{\Sigma\Sigma} = 2380$  MeV

2x2 Potentials

$m_{N\Xi} = 2260$  MeV

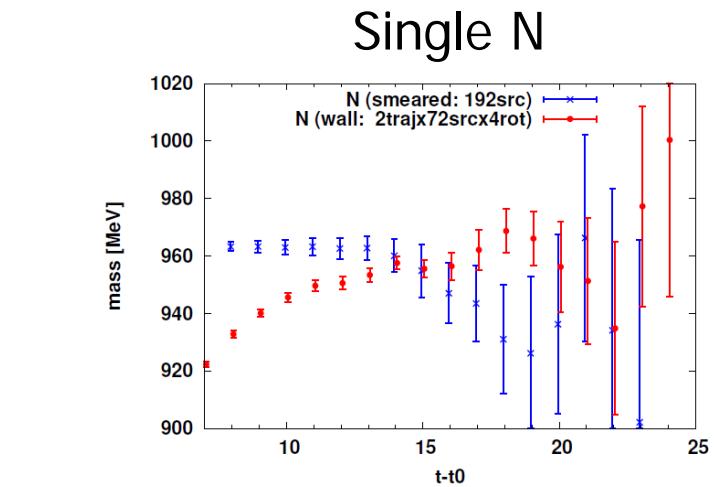
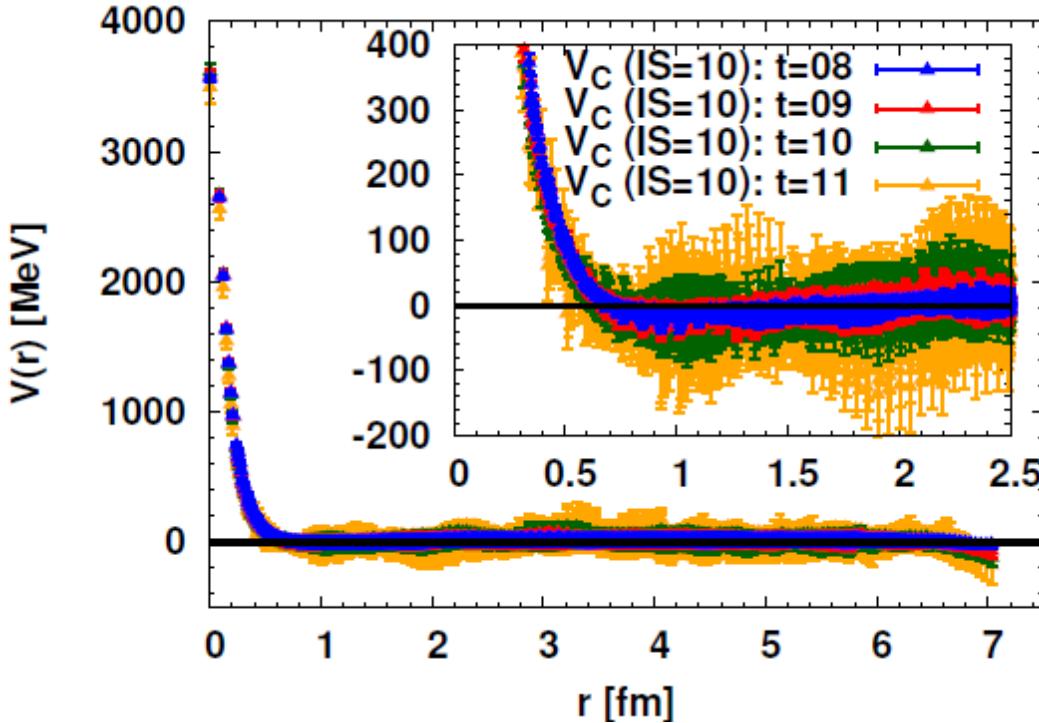
H-resonance (?)

$m_{\Lambda\Lambda} = 2230$  MeV



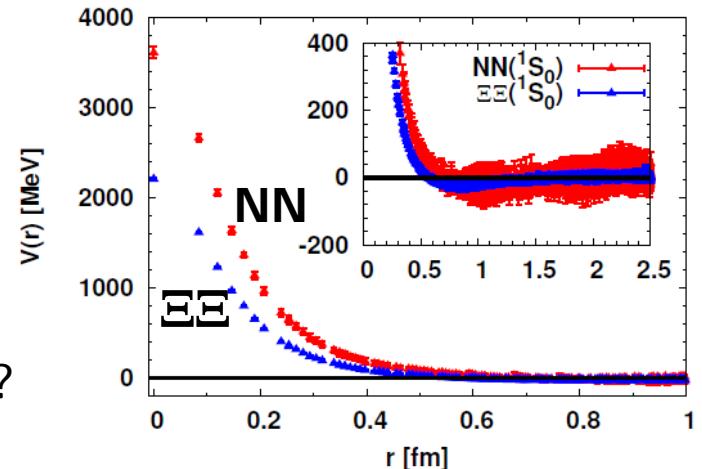
[K. Sasaki]

# Central Potential NN ( ${}^1S_0$ )

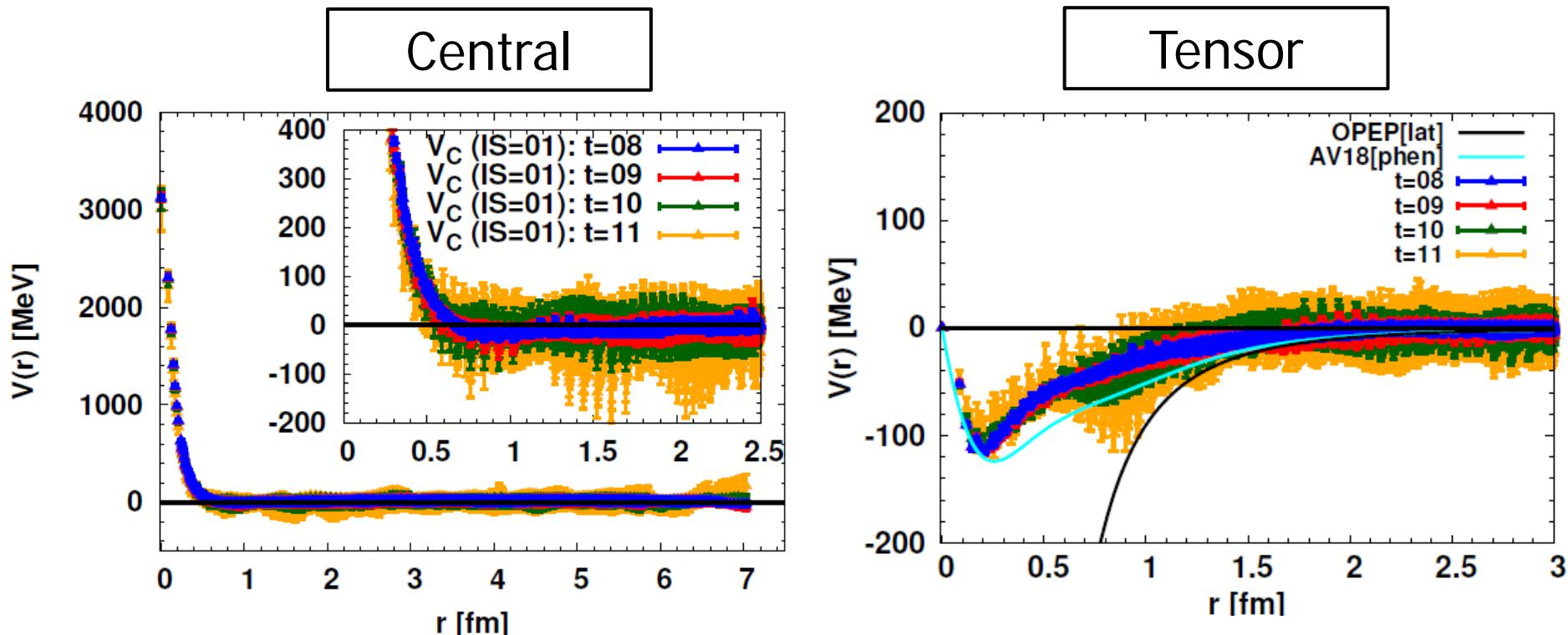


## The effect of SU(3)f breaking

NN( ${}^1S_0$ ) and  $\Xi\Xi({}^1S_0)$  belong to 27-plet



# Central/Tensor Potentials NN ( $^3S_1$ - $^3D_1$ )



Repulsive core  
observed

Attraction at  
mid-long range

Strong Tensor Force is  
clearly visible !

preliminary

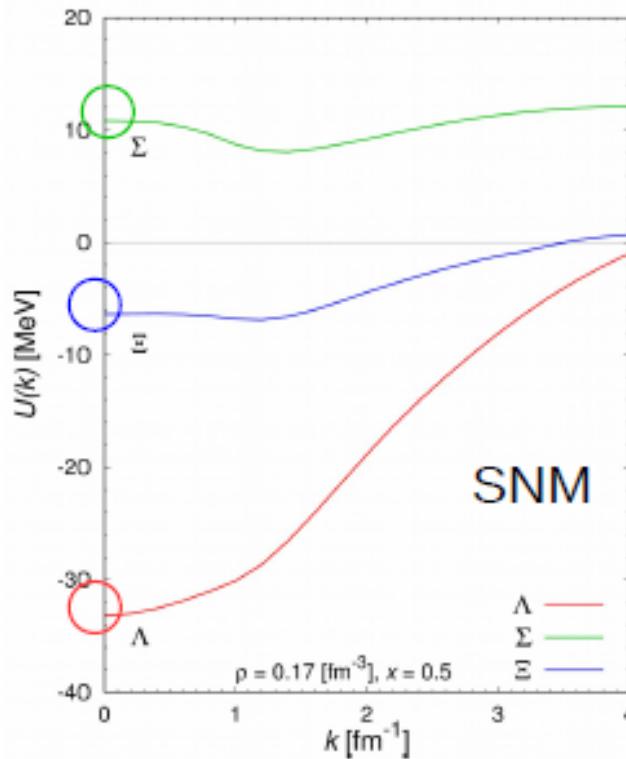
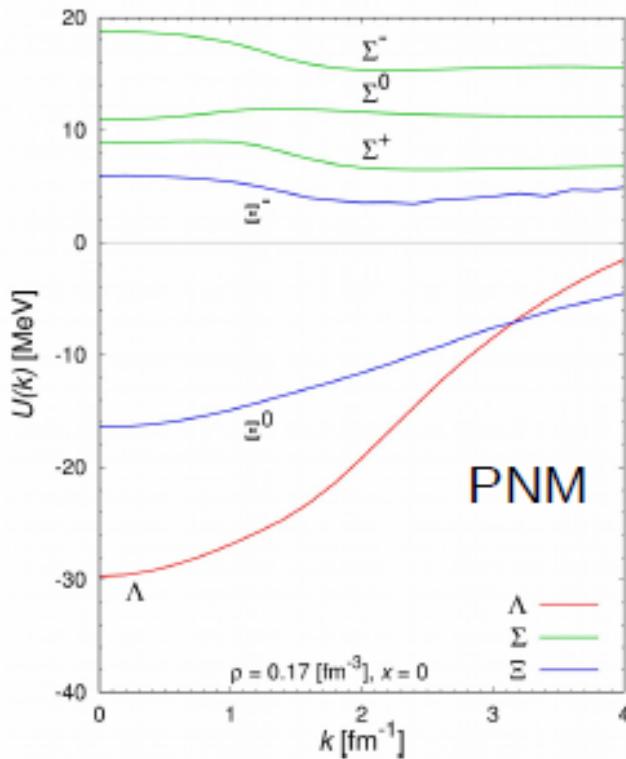
# Impact on dense matter

LQCD YN/YY-forces + Phen NN-forces (AV18)  
used in Brueckner-Hartree-Fock (BHF)

→ Single-particle energy of Hyperon in nuclear matter

(Only diagonal YN/YY forces in SU(3) irrep used)

# Hyperon single-particle potentials



Preliminary

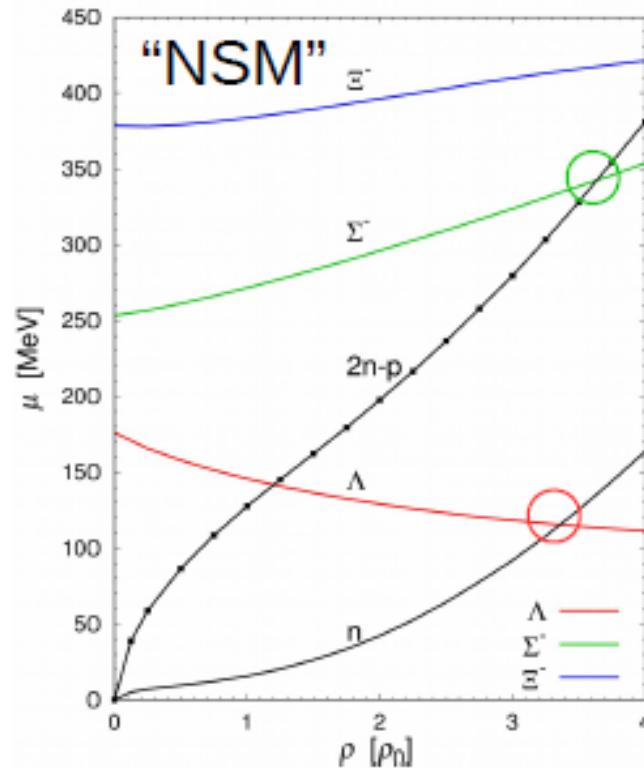
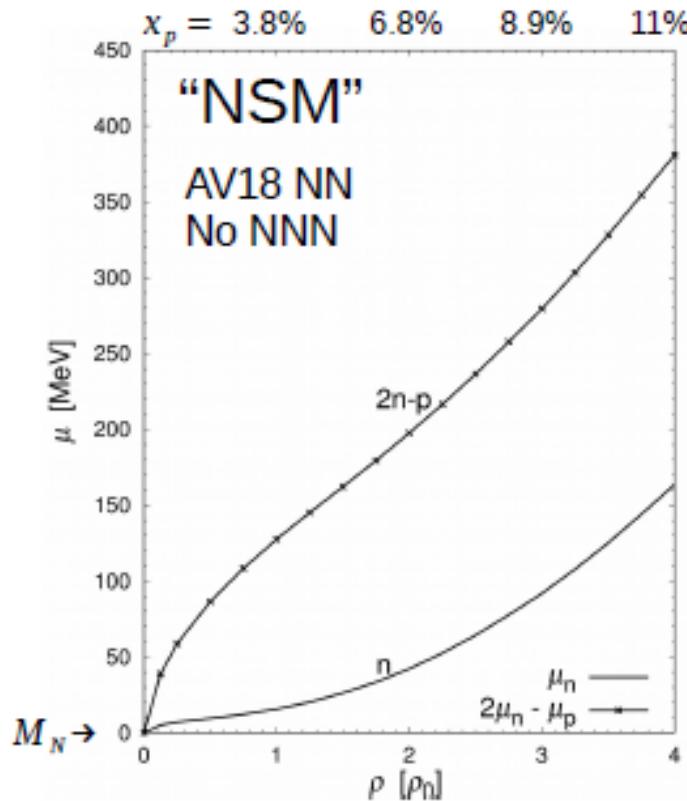
- obtained by using YN,YY forces from QCD.
- Results are compatible with experimental suggestion.

$$U_{\Lambda}^{\text{Exp}}(0) \simeq -30, \quad U_{\Xi}(0)^{\text{Exp}} \simeq -10, \quad U_{\Sigma}^{\text{Exp}}(0) \geq +20 \quad [\text{MeV}]$$

attraction                      attraction small                      repulsion

# Hyperon onset

(just for a demonstration)



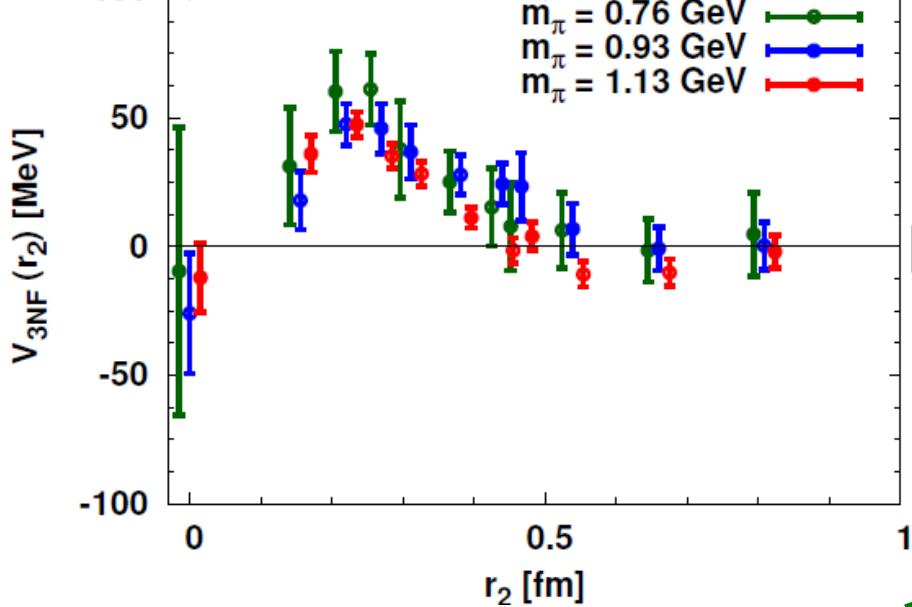
- "NSM" is matter w/ n, p, e,  $\mu$  under  $\beta$ -eq and  $Q=0$ .

[ T. Inoue ]

[Missing]  
P-wave/LS forces  
3-baryon forces

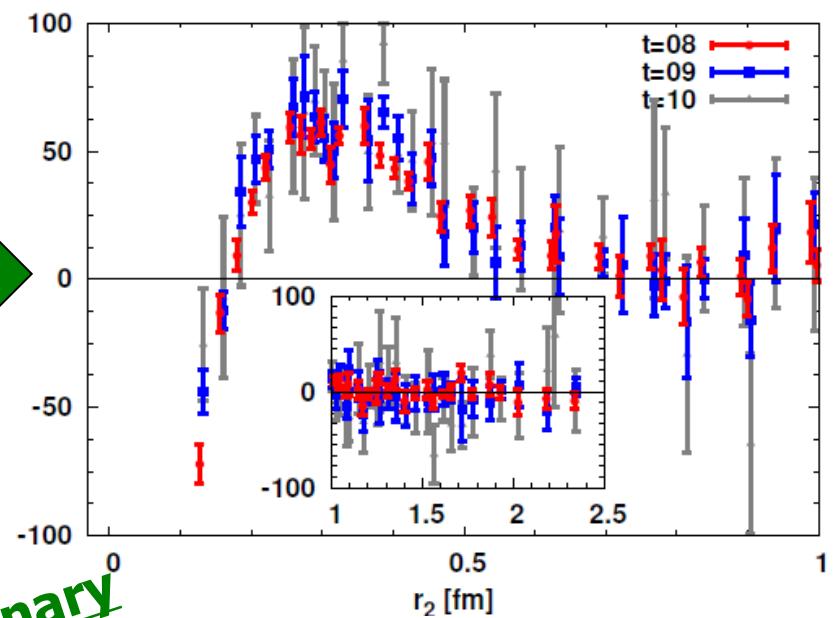
# 3N-forces (3NF)

Nf=2,  $m\pi=0.76-1.1$  GeV



Triton channel

Nf=2+1,  $m\pi=0.51$  GeV



Preliminary



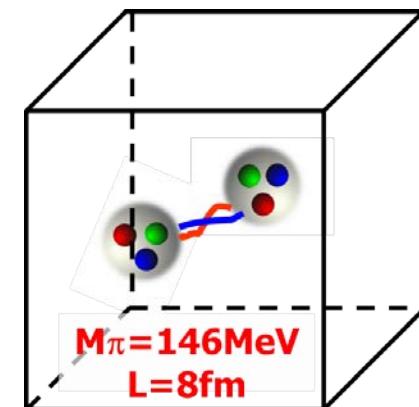
Magnitude of 3NF is similar for all masses  
Range of 3NF tend to get longer (?) for  $m(\pi)=0.5$ GeV

Kernel: ~50% efficiency achieved !

# Summary

- Baryon forces: Bridge between particle/nuclear/astro-physics
- HAL QCD method crucial for a reliable calculation
  - Direct method suffers from excited state contaminations
- The 1st LQCD for Baryon Interactions at  $\sim$  phys. point
  - $m(\pi) \sim= 146$  MeV,  $L \sim= 8$  fm,  $1/a \sim= 2.3$  GeV
  - Central/Tensor forces for NN/YN/YY in  $P=(+)$  channel

Nuclear Physics from LQCD  
New Era is dawning !



- Prospects
  - Exascale computing Era  $\sim$  2020s
  - LS-forces,  $P=(-)$  channel, 3-baryon forces, etc., & EoS

