Mediated interactions: From Yukawa to Efimov

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ULTRA-COLD ATOMS

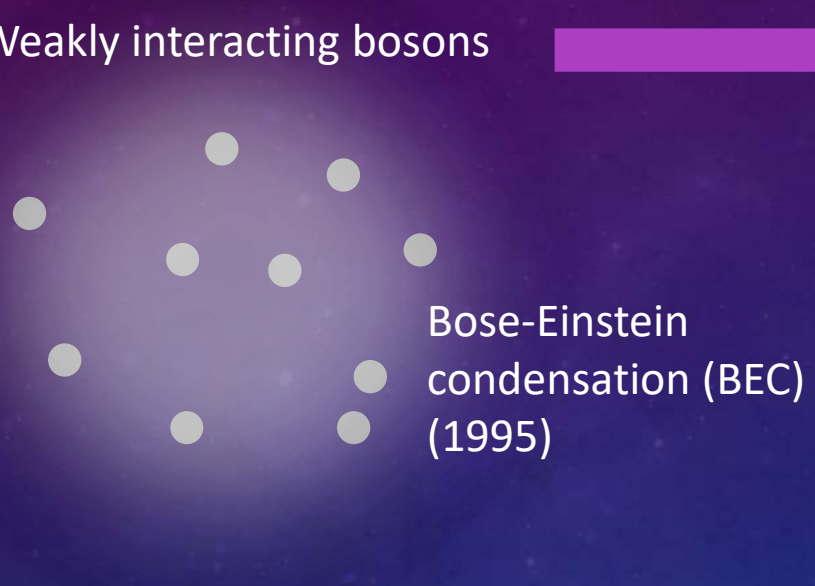
Fully quantum many-body systems → Quantum Field Theory

Interactions are controllable → Non-perturbative regime

ULTRA-COLD ATOMS

Examples:

Weakly interacting bosons



Bose-Einstein condensation (BEC) (1995)

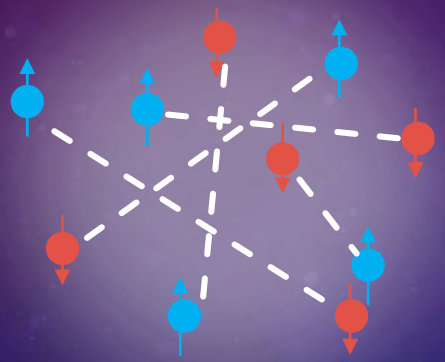
Strongly interacting bosons

Efimov trimers (2006)

ULTRA-COLD ATOMS

Examples:

Weakly interacting fermions → Strongly interacting fermions



Bardeen-Cooper-Schrieffer (BCS) pairing (2004)

BCS-BEC crossover

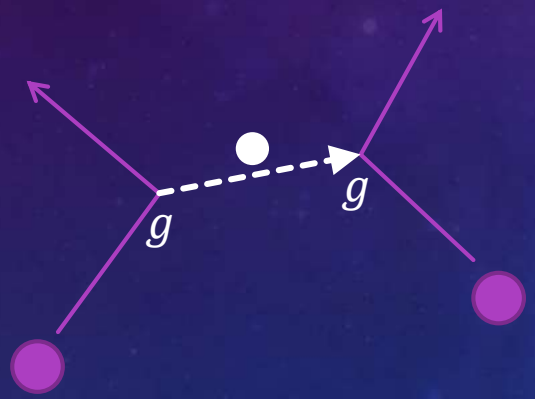
Unitary Fermi gas

Bose-Einstein condensate (BEC) of dimers

MEDIATED INTERACTIONS

Yukawa potential (1930)

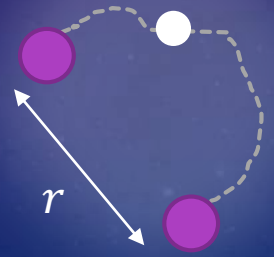
$$V(r) = -g^2 \frac{e^{-mr}}{r}$$



Many-body
Bosons are created/absorbed
“Exchange of virtual particles”

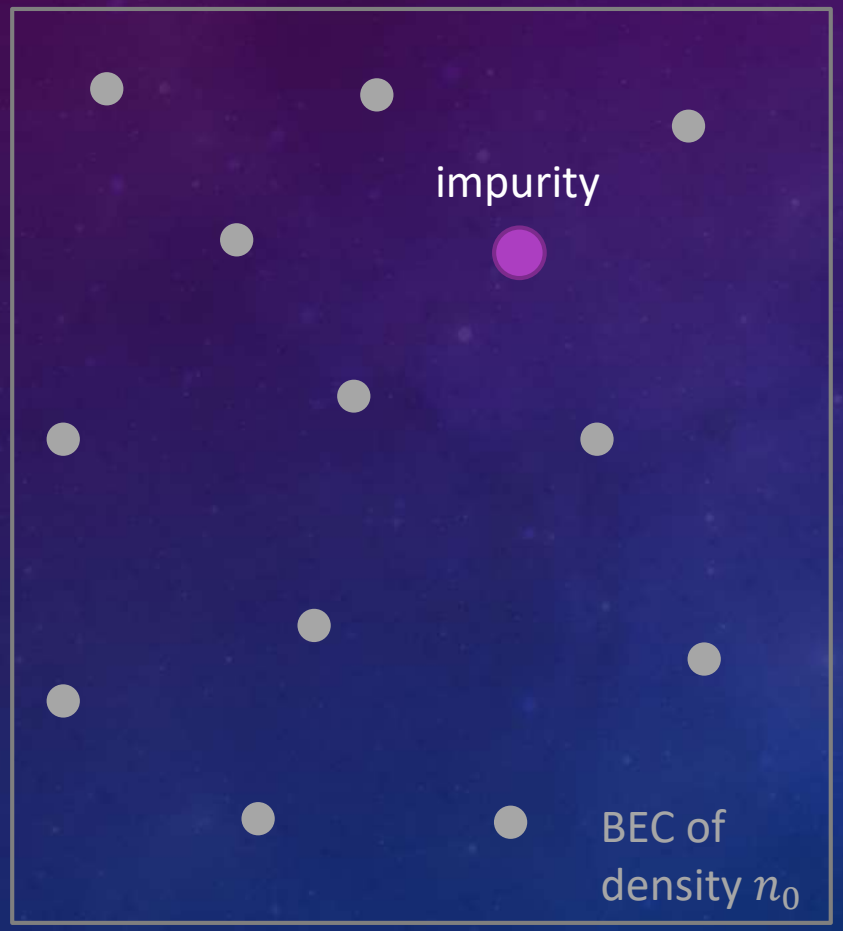
Efimov potential (1970)

$$V(r) = -\frac{\hbar^2}{2m}\frac{0.567^2}{r^2}$$

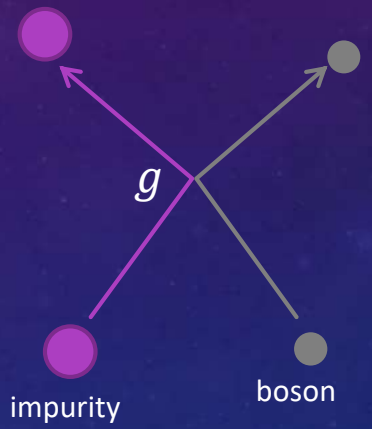


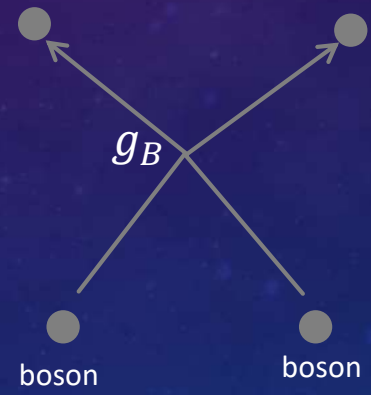
Three-body
Particle always there
“Exchange of a real particle”

IMPURITIES IN A BOSE-EINSTEIN CONDENSATE



Interactions





$g < 0$ can be large

(attraction)

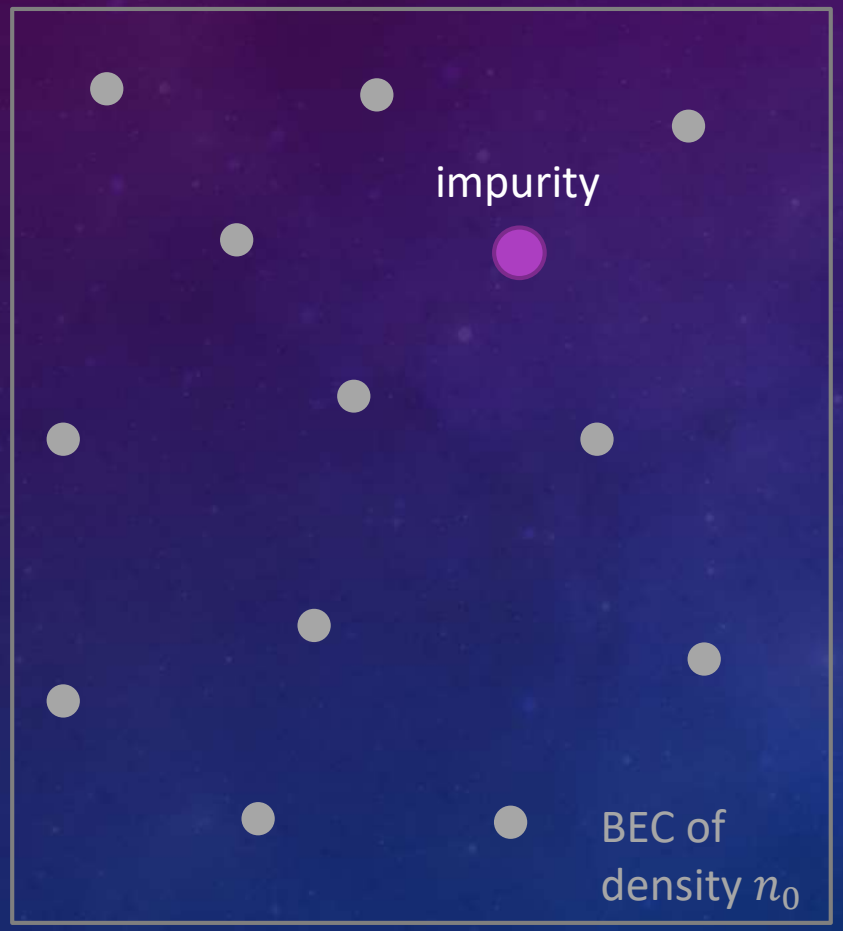
$g_B > 0$ is small

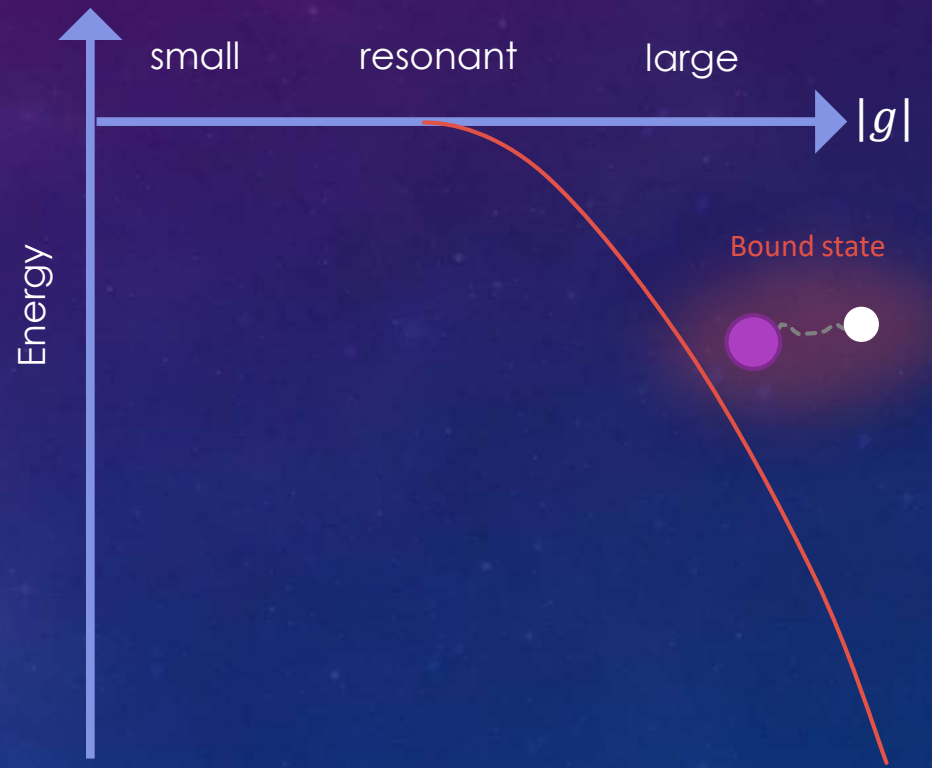
(weak repulsion)

$n_0 a_B^3 \ll 1$

Neglect direct interactions between impurities

IMPURITIES IN A BOSE-EINSTEIN CONDENSATE





$a \leq 0$ $a = \pm\infty$ $a > 0$

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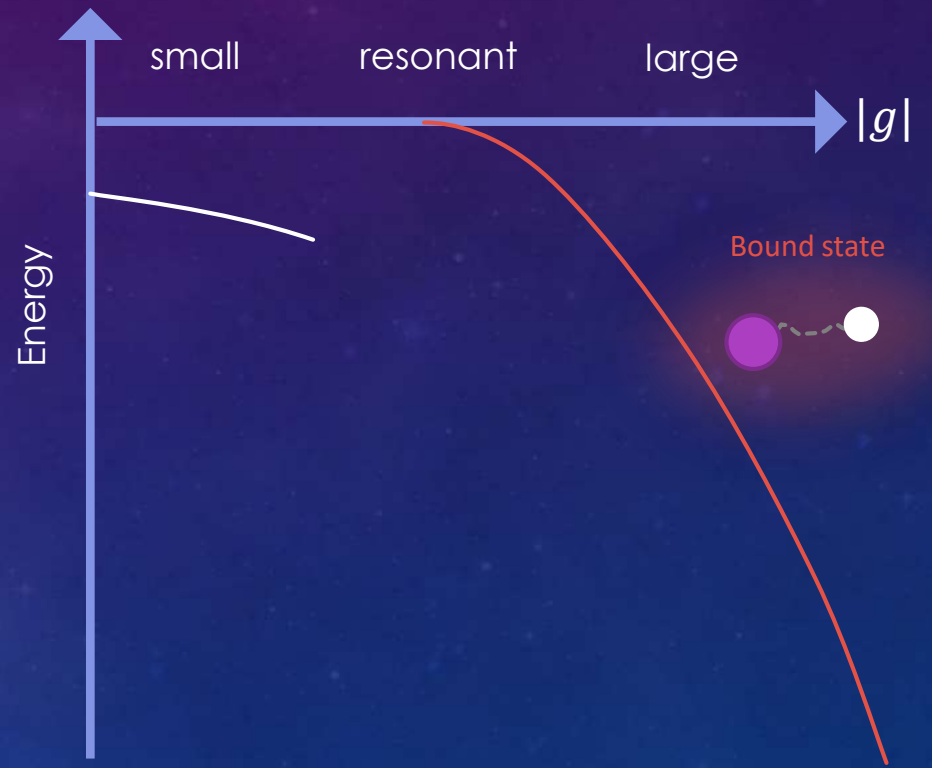

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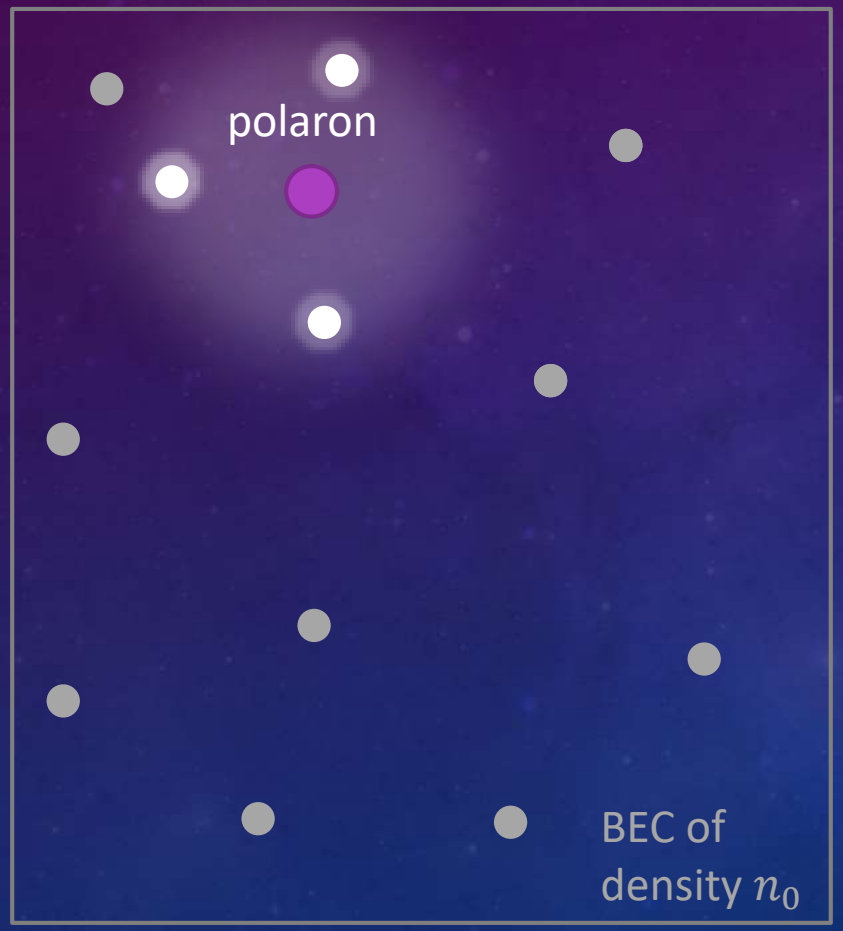

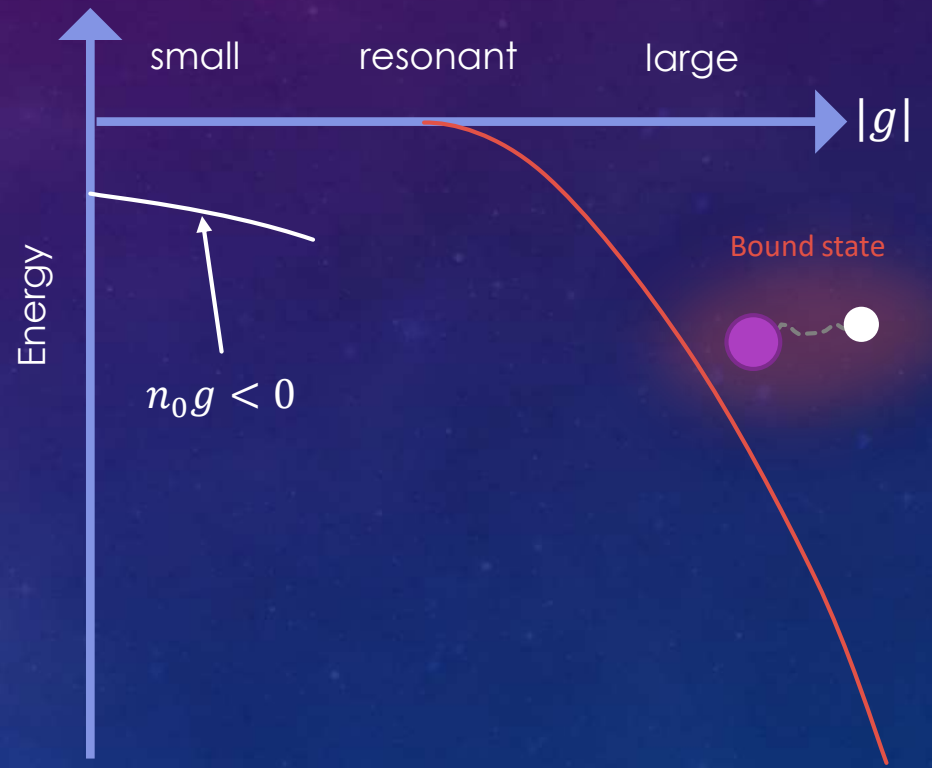

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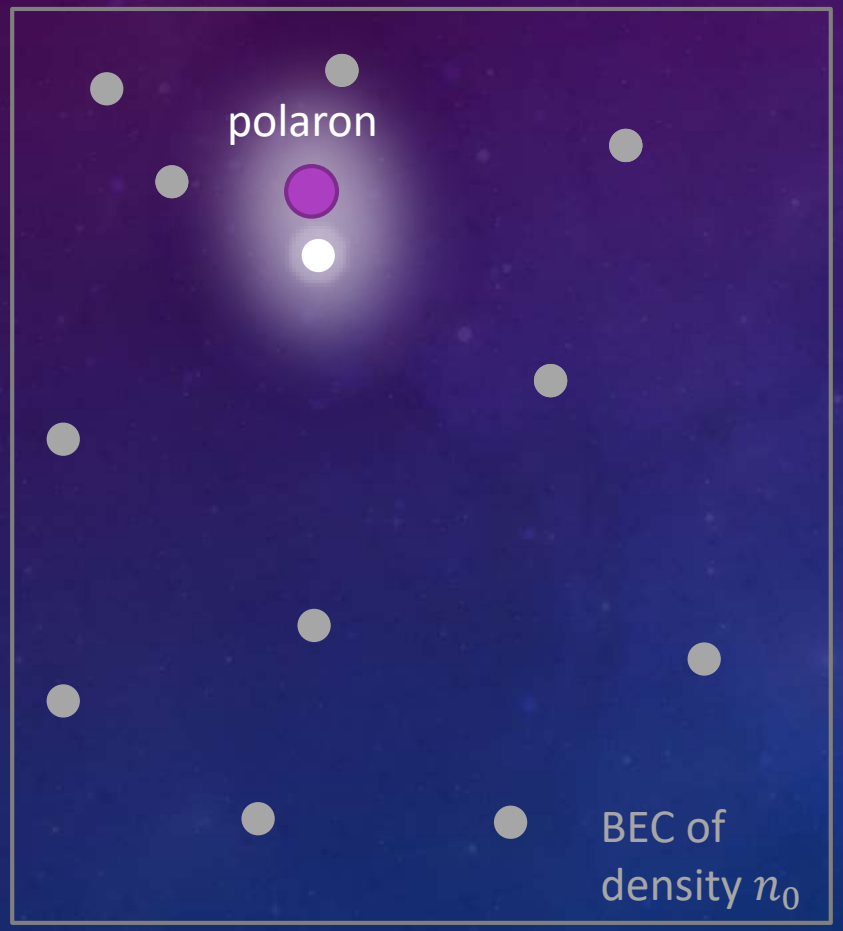
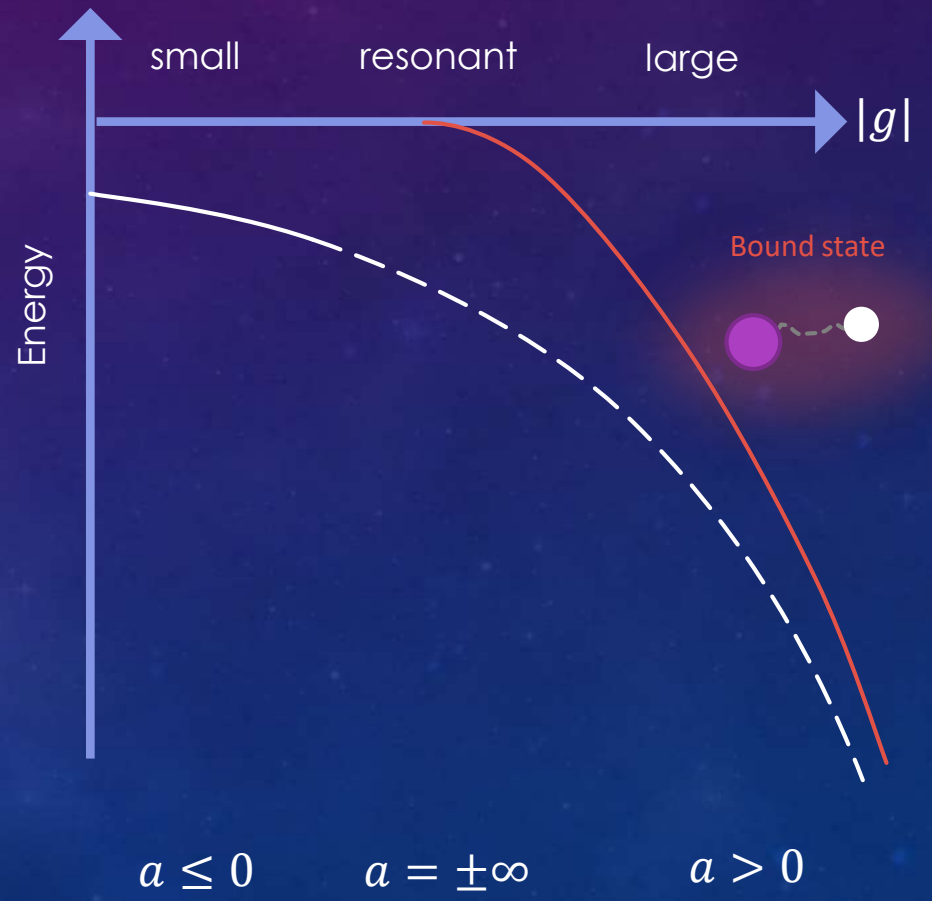
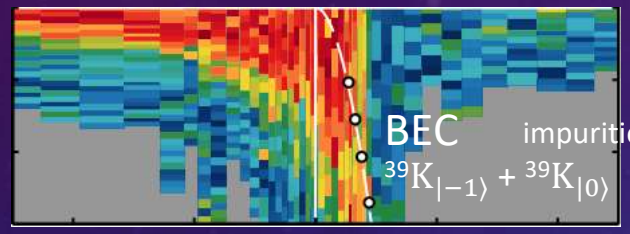


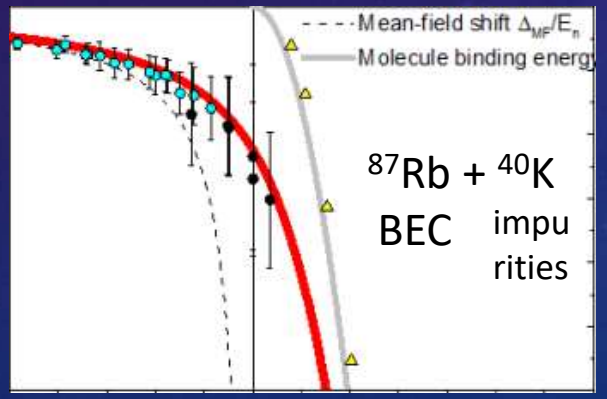


IMPURITIES IN A BOSE-EINSTEIN CONDENSATE

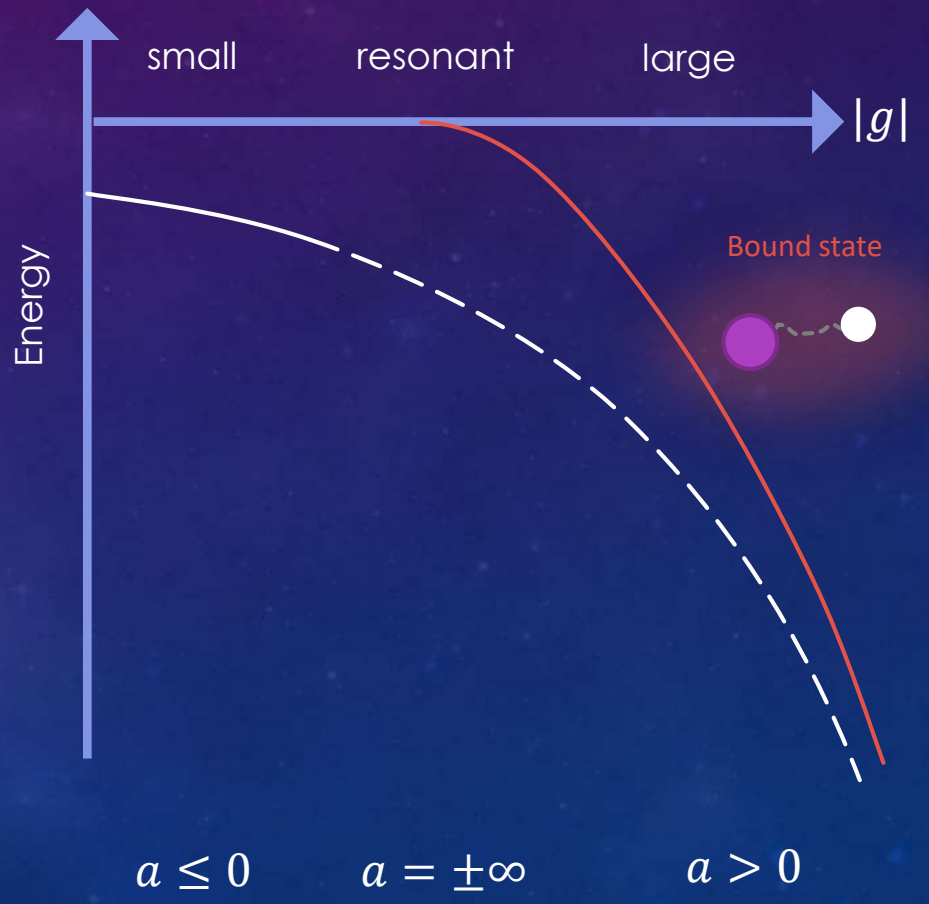
Bose polaron recently observed



Jørgensen et al, PRL 117, 055302 (2016)

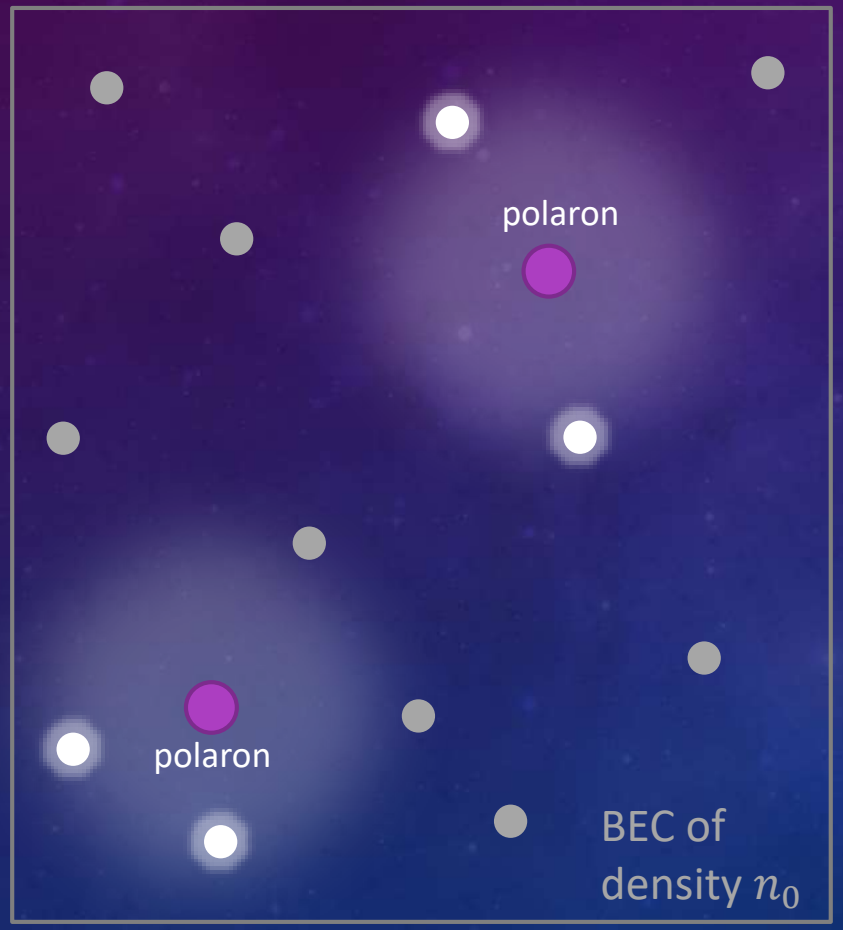


Ming-Guang Hu et al, PRL 117, 055301 (2016)



POLARONIC INTERACTION

for weak coupling g



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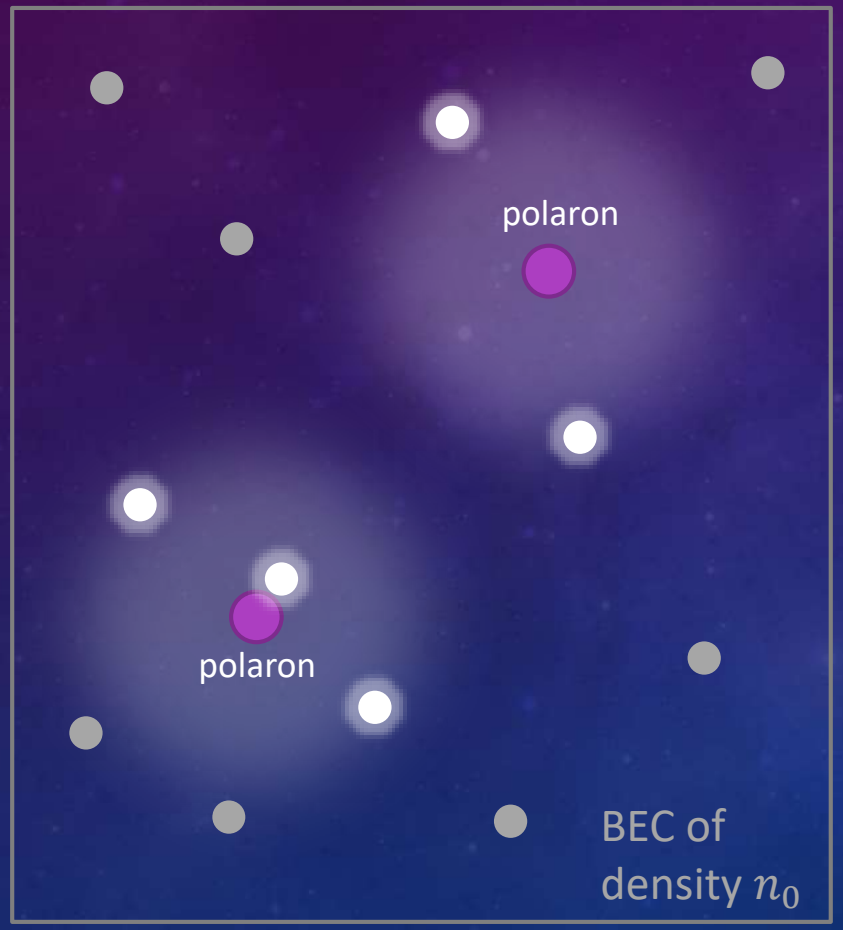

POLARONIC INTERACTION

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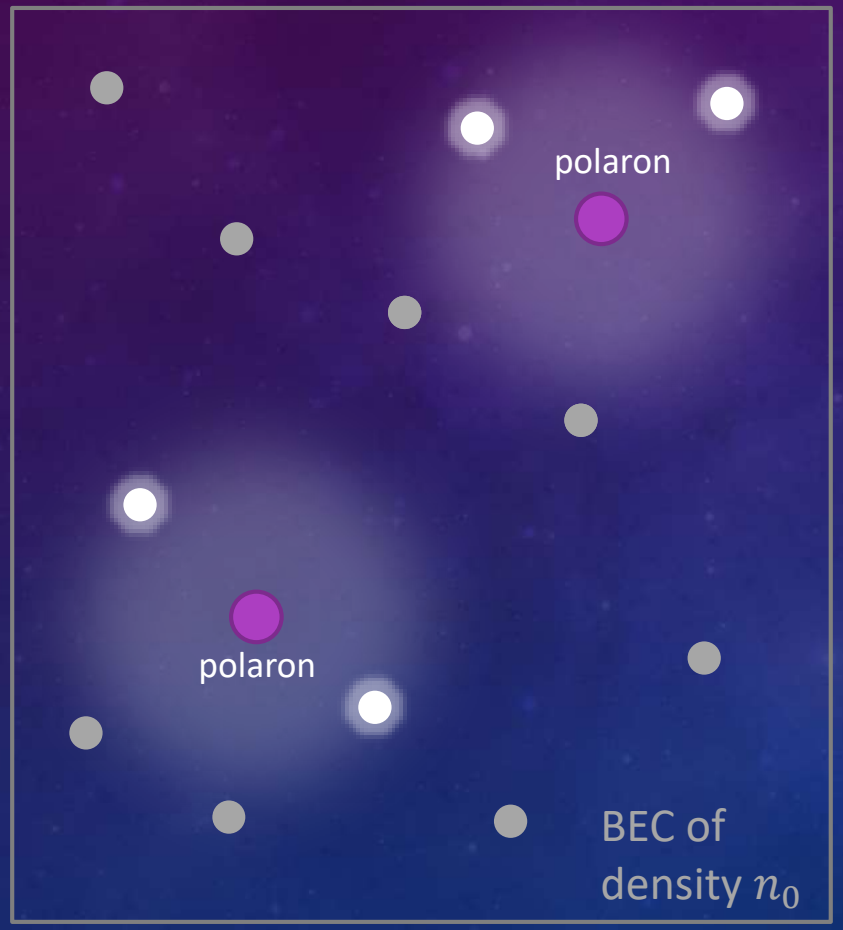
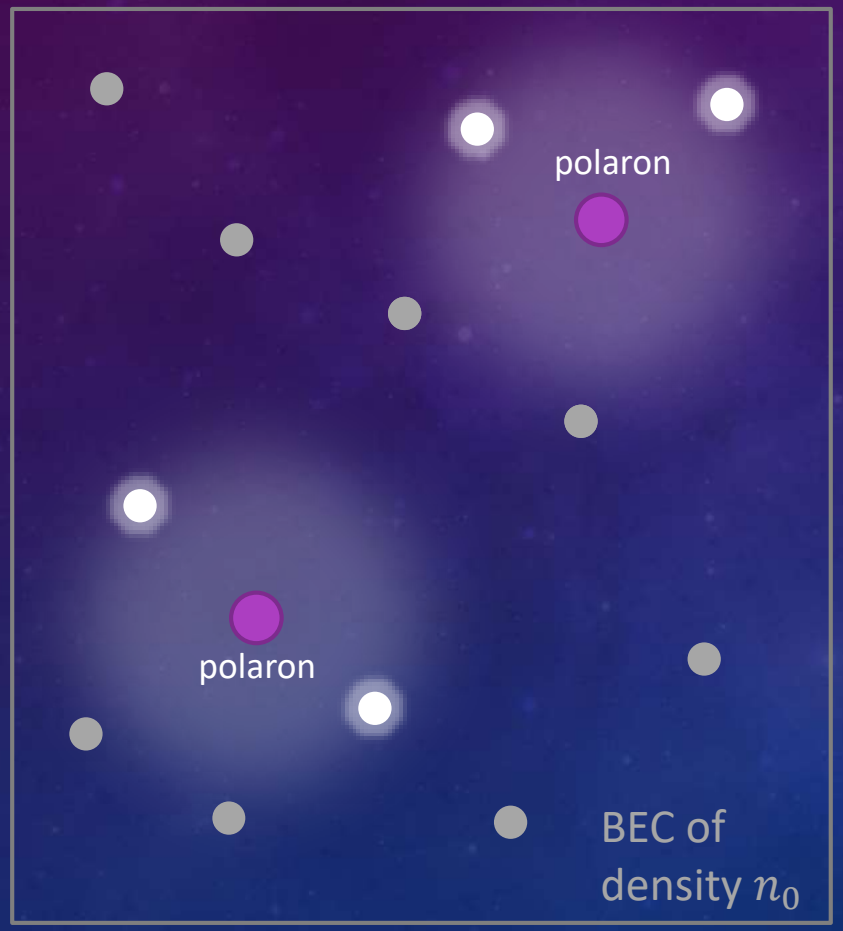
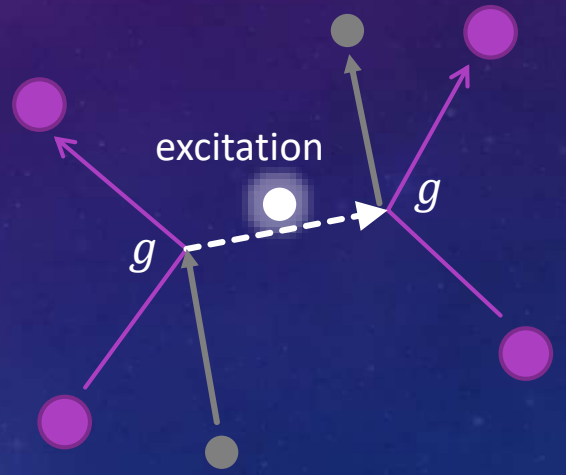


POLARONIC INTERACTION

for weak coupling g



The Bogoliubov excitations of the BEC can mediate a Yukawa potential



To second-order in perturbation theory:

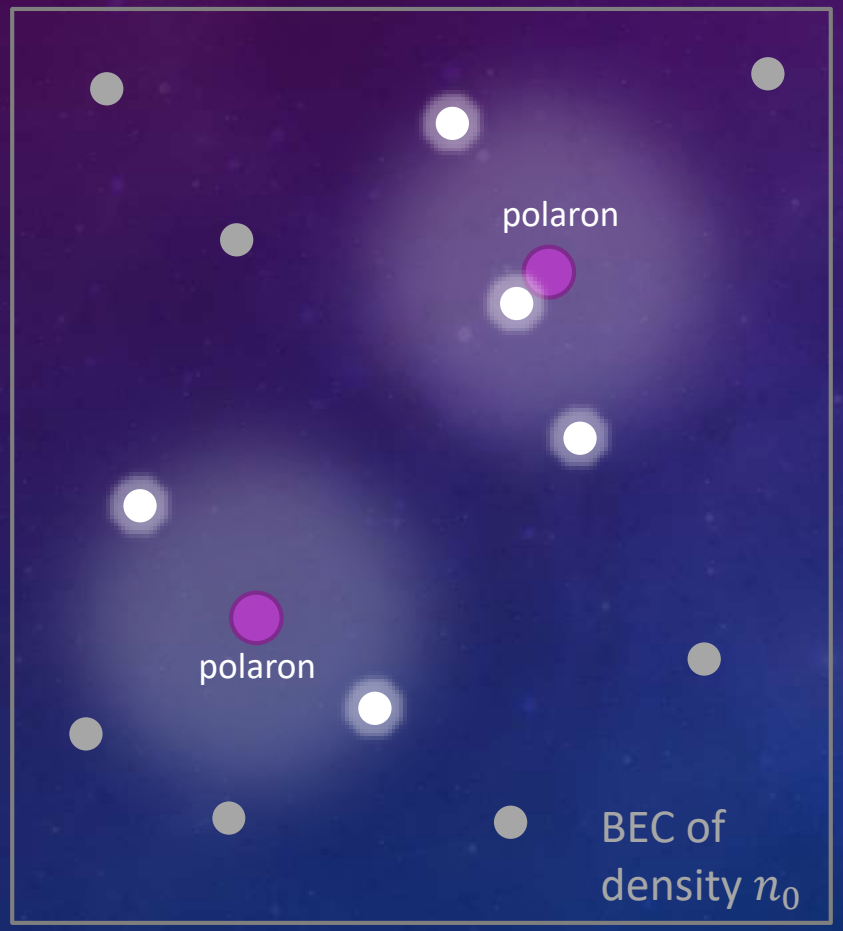
$$V(r) \propto -g^2 n_0 \frac{e^{-r\sqrt{2}/\xi}}{r}$$

BEC coherence length

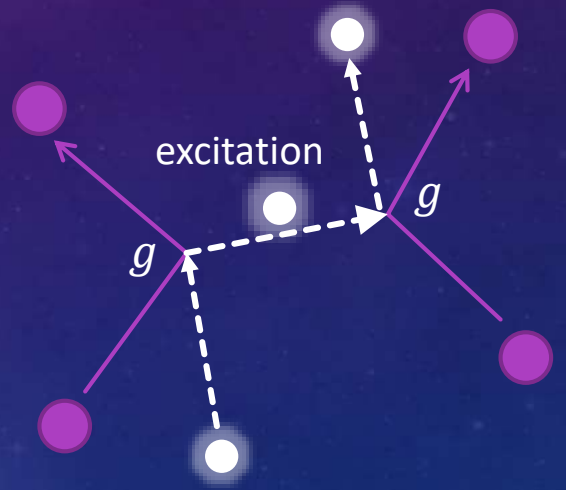
$$\xi = \frac{1}{\sqrt{8\pi n_0 a_B}}$$

POLARONIC INTERACTION

for resonant coupling g



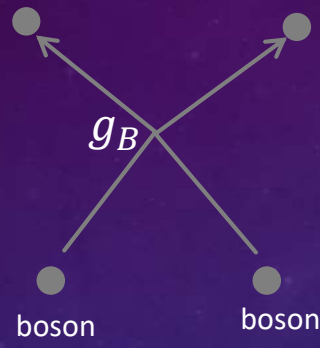
The Bogoliubov excitations of the BEC can also mediate an Efimov potential

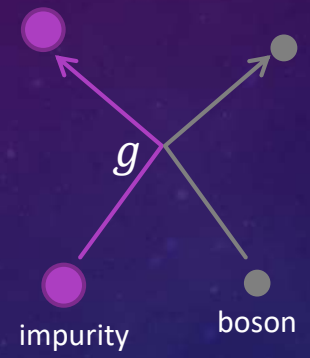


Non-perturbative!

$$V(r) \propto -\frac{\hbar^2}{2m}\frac{1}{r^2}$$

HAMILTONIAN





$$H = \underbrace{\sum_k \epsilon_k b_k^\dagger b_k + \frac{g_B}{2V}\sum_{k,k',p} b_{k'-p}^\dagger b_{k+p}^\dagger b_k b_{k'}}_{\text{Bosons}} + \underbrace{\sum_k \varepsilon_k c_k^\dagger c_k}_{\text{Impurities}} + \underbrace{\frac{g}{V}\sum_{k,k',p} b_{k'-p}^\dagger c_{k+p}^\dagger c_k b_{k'}}_{\text{Impurity-boson}}$$

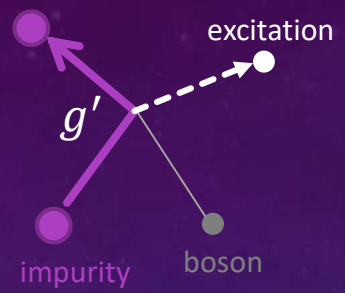
$$\epsilon_k = \frac{\hbar^2 k^2}{2m} \qquad \varepsilon_k = \frac{\hbar^2 k^2}{2M}$$

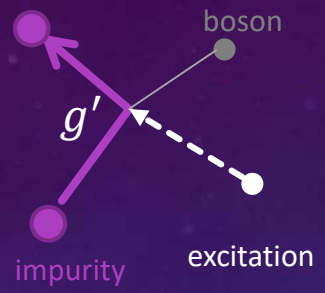
Bogoliubov approach

$b_0 = \sqrt{N_0}$ condensate

$b_k = u_k \beta_k - v_k \beta_k^\dagger$ Bogoliubov excitation

HAMILTONIAN





Free excitations

Dressed impurities

$$H = E_0 + \sum_k E_k \beta_k^\dagger \beta_k + \sum_k (\varepsilon_k + g n_0) c_k^\dagger c_k + \sqrt{N_0}\frac{g}{V}\sum_{k,p}\left(u_p \beta_{-p}^\dagger - v_p \beta_p\right) c_{k+p}^\dagger c_k + h.c.$$

Yukawa (Fröhlich)

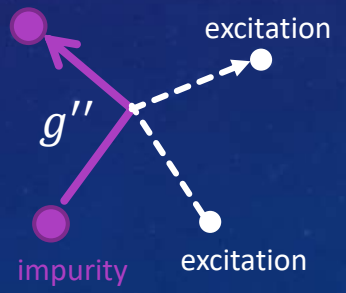
Bogoliubov excitation energy

$$E_k = \sqrt{\epsilon_k(\epsilon_k + 2 g_B n_0)}$$

$$+\frac{g}{V}\sum_{k,k',p}(u_{k'-p}u_{k'}\beta_{k'-p}^\dagger\beta_{k'} + v_{k'-p}v_{k'}\beta_{p-k'}\beta_{-k'}^\dagger)c_{k+p}^\dagger c_k$$

Efimov (Scattering)

$$+\frac{g}{V}\sum_{k,k',p}(u_{k'-p}v_{k'}\beta_{k'-p}^\dagger\beta_{-k'}^\dagger + v_{k'-p}u_{k'}\beta_{p-k'}\beta_{k'})c_{k+p}^\dagger c_k$$

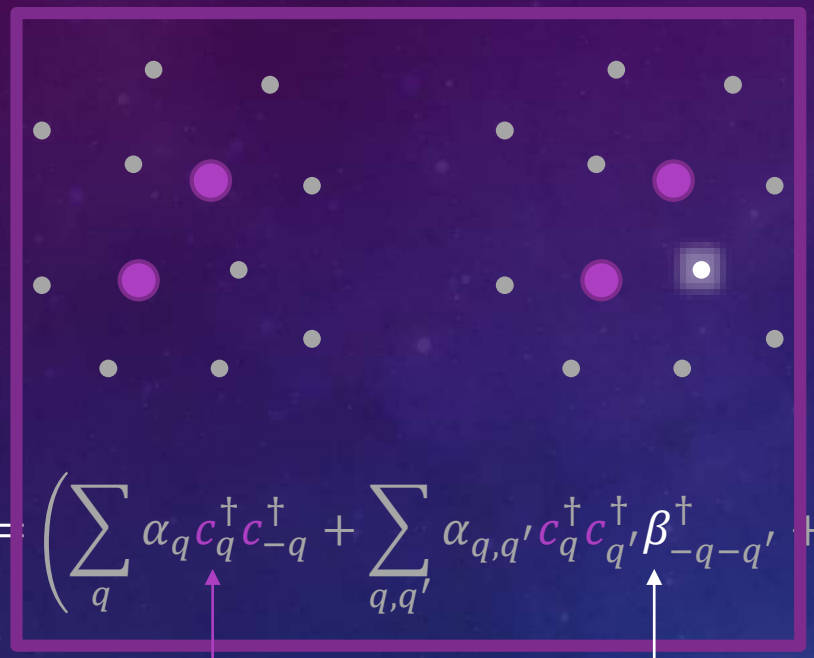


Bogoliubov approach

$b_0 = \sqrt{N_0}$ condensate

$b_k = u_k \beta_k - v_k \beta_k^\dagger$ Bogoliubov excitation

NON-PERTURBATIVE METHOD: TRUNCATED BASIS



$$|\Psi\rangle = \left(\sum_{q} \alpha_q c_q^\dagger c_{-q}^\dagger + \sum_{q,q'} \alpha_{q,q'} c_q^\dagger c_{q'}^\dagger \beta_{-q-q'}^\dagger + \sum_{q,q',q''} \alpha_{q,q',q''} c_q^\dagger c_{q'}^\dagger \beta_{q''}^\dagger \beta_{-q-q'-q''}^\dagger + \cdots \right) |\Phi\rangle$$

Impurity creation operator

Excitation creation operator

BEC ground state

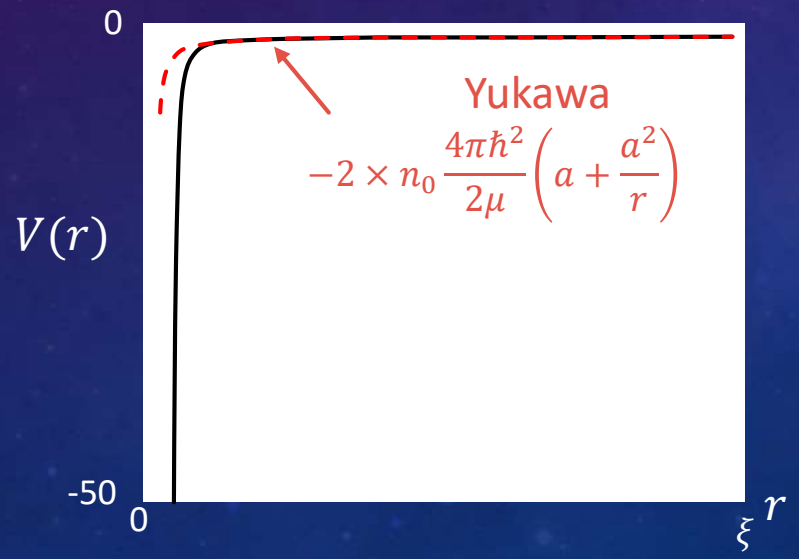
→ Coupled equations for α_q and to $\alpha_{q,q'}$ → Effective 3-body equation

RESULT: POLARONIC POTENTIAL

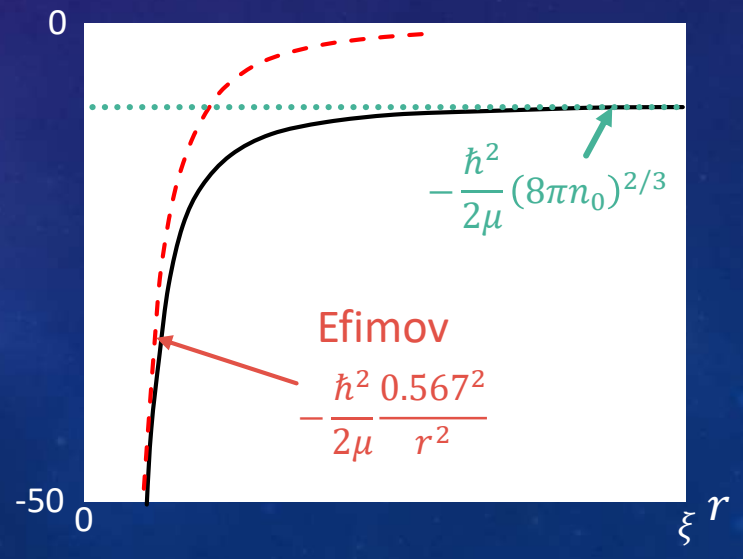
Effective potential (Born-Oppenheimer) between polarons:

$$V(r) = \frac{\hbar^2\kappa^2}{2\mu} \qquad \frac{1}{a} - \kappa + \frac{1}{r}e^{-\kappa r} + \frac{8\pi n_0}{\kappa^2} = 0 \qquad (a_B \to 0)$$

For small $a \lesssim 0$



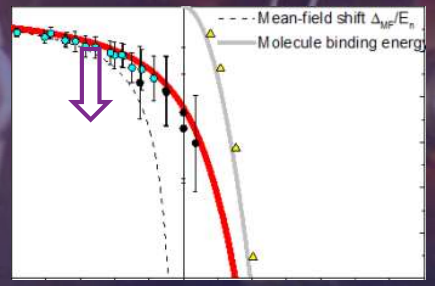
At resonance $a = \pm\infty$



POSSIBLE EXPERIMENTAL OBSERVATIONS

Heavy impurities in a condensate of light bosons (e.g. ^{133}Cs + ^{7}Li, Yb + ^{7}Li)

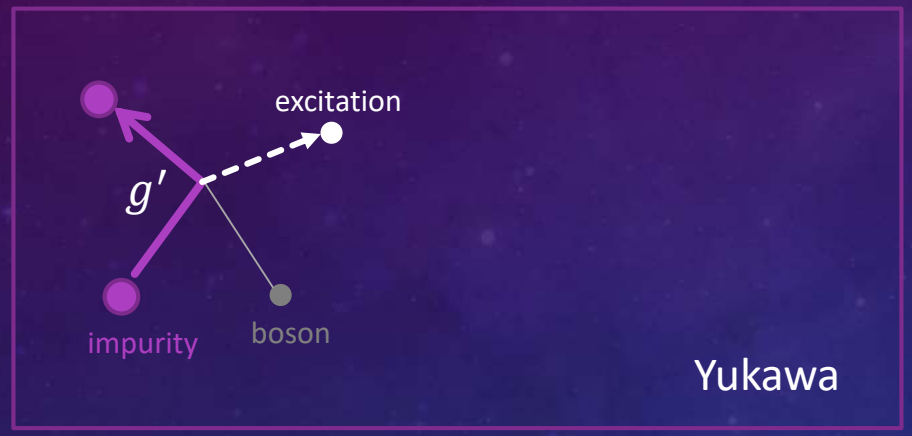
- Polaron RF spectroscopy: mean-field shift with the impurity density

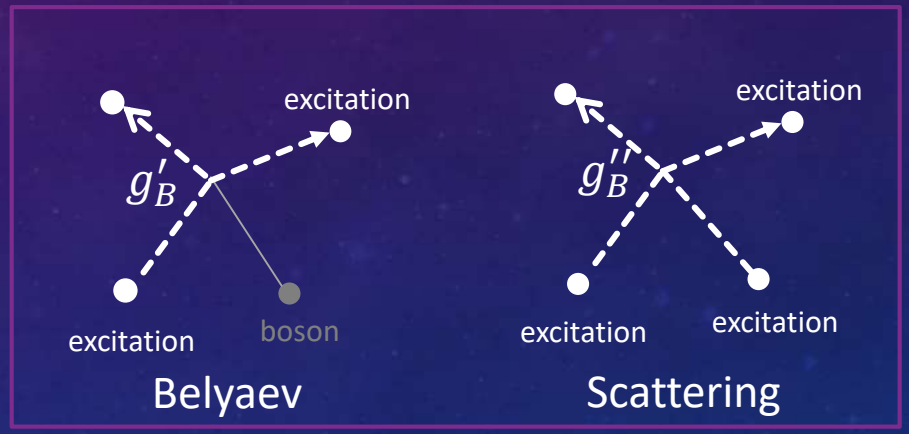


- Loss by recombination: shift of the loss peak with the condensate density

OUTLOOK

Strong interaction between bosons:





CONCLUSION

arXiv:1607.04507

- A Bose-Einstein condensate of atoms can mediate interactions that go from weak Yukawa-type to strong Efimov-type.

- Fermionic impurities in a BEC : atomic analogues of nucleons and mesons

- Analogues of quarks and gluons?