Moments of pion light-cone wavefunction using OPE on the lattice

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Outline

- Pion Light-Cone (LC) wave function and its moments
- Moments using lattice OPE with a valance heavy quark [1]
- Lattice correlators
- Exploratory numerical results
- Summary

[1] proposed by Detmold and Lin (Phys.Rev. D73 (2006)).

Pion LC wave function/distribution amplitude:

$$\langle \pi^+(p)|\bar{d}(\frac{z}{2})\ \gamma_5\gamma_\mu\ u(-\frac{z}{2})|0\rangle = -ip_\mu f_\pi \int_0^1 d\xi\ e^{i(\bar{\xi}p\frac{z}{2}-\xi p\frac{z}{2})}\phi_\pi(\xi)$$

$$\bar{\xi} = 1-\xi$$

$|\pi^+(p)\rangle \rightarrow$ Ground state of the pseudoscalar π^+ meson with on shell momentum $p^2 = m_\pi^2$.
fraction ξ of pion momentum is carried by u quark.

Moments:

$$a_n = \int_0^1 d\xi\ \xi^n\ \phi_\pi(\xi).$$

OPE:

$$\langle \pi^+(p)|O^{\mu_1..\mu_n}|0\rangle = f_\pi\ a_{n-1}\ [p^{\mu_1}\ldots p^{\mu_n} - \text{Traces}]$$

$$O^{\mu_1..\mu_n} = \bar{\psi}\gamma^{\{\mu_1}\gamma^5(iD^{\mu_2})\ldots(iD^{\mu_n\}})\psi - \text{Traces}$$

$$\langle \pi^+(p)|\overline{\psi}(0)\ \gamma_5\gamma_\mu\ \psi(0)|0\rangle = if_\pi p_\mu$$
$$\implies\ a_0 = 1$$

In the isospin limit $m_u = m_d$:

$$\phi_\pi(\xi) = \phi_\pi(\overline{\xi})$$

$\implies$ Odd moments vanish $\rightarrow$ lowest non-trivial moment is a_2.

Lattice calculations of the second moment are available $\rightarrow$ precision calculation of the higher moments are needed to get the correct shape of the $\phi_\pi(\xi)$.

Euclidean OPE with a valance heavy quark

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Heavy-light currents:

$$V^{\mu}_{\boldsymbol{\Psi},\psi} = \overline{\boldsymbol{\Psi}}\gamma^{\mu}\psi + \overline{\psi}\gamma^{\mu}\boldsymbol{\Psi}$$
$$A^{\mu}_{\boldsymbol{\Psi},\psi} = \overline{\boldsymbol{\Psi}}\gamma^{\mu}\gamma^{5}\psi + \overline{\psi}\gamma^{\mu}\gamma^{5}\boldsymbol{\Psi}$$

ψ: light quarks, $\boldsymbol{\Psi}$: fictitious, relativistic, valance quark which is heavy.

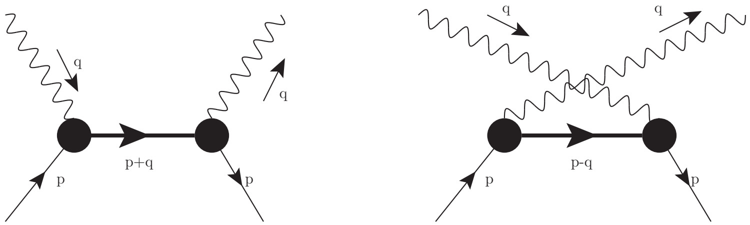
$\longrightarrow$Simplify the lattice calculation.

$\longrightarrow$Removes the higher twist contributions.

Scale hierarchy required:

$$\Lambda_{QCD} << m_{\boldsymbol{\Psi}} \sim \sqrt{q^2} << \frac{1}{a}$$

$\Longrightarrow$ Fine lattices are required.



VV type operator:

$$T^{\mu\nu}_{\Psi,\psi}(p,q) = \int d^4x \; e^{iqx} \; \langle \pi^+(p) | T[V^{\mu}_{\mathbf{\Psi},\psi}(x) V^{\nu}_{\mathbf{\Psi},\psi}(0)] | 0 \rangle$$

OPE:

$$\int d^4x \; e^{iqx} \;\; T[V^{\mu}_{\mathbf{\Psi},\psi}(x) V^{\nu}_{\mathbf{\Psi},\psi}(0)] = \overline{\psi}\gamma^{\mu} \frac{-i(i\not{D} + \not{q}) + m_{\Psi}}{(iD+q)^2 + m_{\Psi}^2} \gamma^{\nu} \psi + \overline{\psi}\gamma^{\nu} \frac{-i(i\not{D} - \not{q}) + m_{\Psi}}{(iD-q)^2 + m_{\Psi}^2} \gamma^{\mu} \psi$$

Taylor expansion:

$$\frac{-i(i\not{D} + \not{q}) + m_{\Psi}}{(iD-q)^2 + m_{\Psi}^2} = -\frac{-i(i\not{D} + \not{q}) + m_{\Psi}}{Q^2 + D^2 - m_{\Psi}^2} \sum_{n=0}^{\infty} \left(\frac{-2iq.D}{Q^2 + D^2 - m_{\Psi}^2} \right)^n$$

Higher twist terms $\rightarrow$ expansion parameter: $\left(\frac{-2iq.D+D^2}{Q^2-m_\Psi^2}\right)$ $\rightarrow$ Extra powers of p^2 $\rightarrow$ small.

Antisymmetric in μ and ν:

$$\int d^4x\; e^{iqx}\;\; T[V^\mu_{\Psi,\psi}(x)V^\nu_{\Psi,\psi}(0)] = \frac{i}{2}\sum_{n=0,even}^{\infty}\frac{1}{(\tilde{Q}^2)^{n+1}}\overline{\psi}\gamma_\lambda\gamma_5(i\not{D}_{\mu_1})\ldots(i\not{D}_{\mu_n})\psi\varepsilon_{\mu\nu\rho\lambda}$$

$\tilde{Q}^2 = Q^2 - M_\Psi^2 + \alpha$, $m_\Psi = M_\Psi - \frac{1}{2}\alpha$, $\quad M_\Psi \rightarrow$ mass of heavy-light pseudoscalar meson.

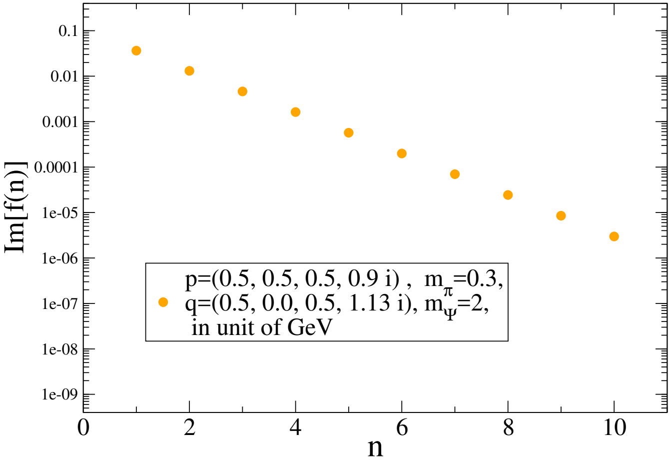
$$\int d^4x\; e^{iqx}\;\langle\pi^+(p)|\,T[V^\mu_{\Psi,\psi}(x)V^\nu_{\Psi,\psi}(0)]|0\rangle = \sum_{n=0,\;even}^{\infty} a_n\; f(n)\; f_\pi$$

$$f(n) = \frac{i}{2}\frac{\xi^{n+1}}{n+1}\left[\frac{2\eta\, C_n^2(\eta)(q^\rho p^\lambda)}{p.q}\right]\varepsilon_{\mu\nu\rho\lambda}$$

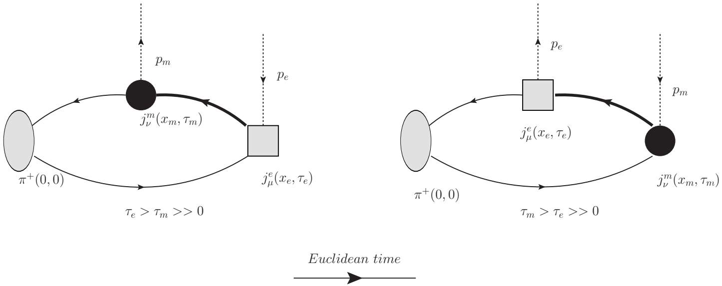
$$\xi = \frac{\sqrt{p^2q^2}}{\tilde{Q}^2},\;\; \eta = \frac{p.q}{\sqrt{p^2q^2}}$$

For simplicity, Wilson coefficients are set to one.

Identical result for the AA type correlator.


0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
Im[f(n)]
0
2
4
6
8
10
n
p=(0.5, 0.5, 0.5, 0.9 i) , m_π=0.3,
q=(0.5, 0.0, 0.5, 1.13 i), m_Ψ=2,
in unit of GeV

Lattice correlators



$$C_3^{\mu\nu}(\tau_m, \tau_e; \vec{p}_m, \vec{p}_e) = \int d^3x_m \int d^3x_e \; e^{i\vec{p}_m\cdot\vec{x}_m} \mathrm{e}^{-i\vec{p}_e\cdot\vec{x}_e} \langle 0|\mathrm{T}[j^\mu_m(\vec{x}_m, \tau_m) j^\nu_e(\vec{x}_e, \tau_e) \mathscr{O}^\dagger_\pi(\vec{0}, 0)]|0\rangle$$

$$C^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_m,\tau_e;\vec{p}_m,\vec{p}_e) \approx \frac{1}{2E_\pi}\delta^3(\vec{p}_\pi-(\vec{p}_m-\vec{p}_e))\times e^{-E_\pi\tau_e}\left\langle\pi(\vec{p}_\pi)\middle|\mathscr{O}^\dagger_\pi(\vec{0},0)\middle|0\right\rangle \int d^3x\, e^{i\vec{p}_m\cdot\vec{x}}\left\langle 0\middle|j^\mu_m(\vec{x},\tau_m-\tau_e)j^\nu_e(\vec{0},0)\middle|\pi(\vec{p}_\pi)\right\rangle .$$

The two point function:

$$C_\pi(\tau_\pi;\vec{p}_\pi) = \int d^3x\, e^{i\vec{p}_\pi\cdot\vec{x}}\left\langle 0\middle|\mathscr{O}_\pi(\vec{x},\tau)\mathscr{O}^\dagger_\pi(\vec{0},0)\middle|0\right\rangle \xrightarrow{\tau_\pi\to\infty} \frac{\left|\left\langle\pi(\vec{p}_\pi)\middle|\mathscr{O}^\dagger_\pi(\vec{0},0)\middle|0\right\rangle\right|^2}{2E_\pi}\times e^{-E_\pi\tau_\pi} .$$

We can take the ratio

$$R^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_m-\tau_e;\vec{p}_m,\vec{p}_\pi) = \frac{C^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_m,\tau_e;\vec{p}_m,\vec{p}_m-\vec{p}_\pi)}{C_\pi(\tau_e;\vec{p}_\pi)}\times\left\langle\pi(\vec{p}_\pi)\middle|\mathscr{O}^\dagger_\pi(\vec{0},0)\middle|0\right\rangle = \int d^3x\, e^{i\vec{p}_m\cdot\vec{x}}\left\langle 0\middle|j^\mu_m(\vec{x},\tau_m-\tau_e)j^\nu_e(\vec{0},0)\middle|\pi(\vec{p}_\pi)\right\rangle_{\tau_m>\tau_e} .$$

Perform the Fourier transform: $\int d\tau\, e^{iq_4\tau}$, $\tau=\tau_m-\tau_e$.

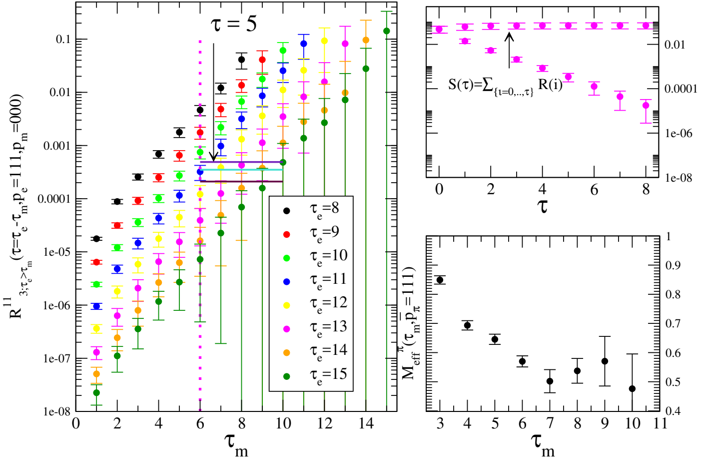
exploratory numerical result

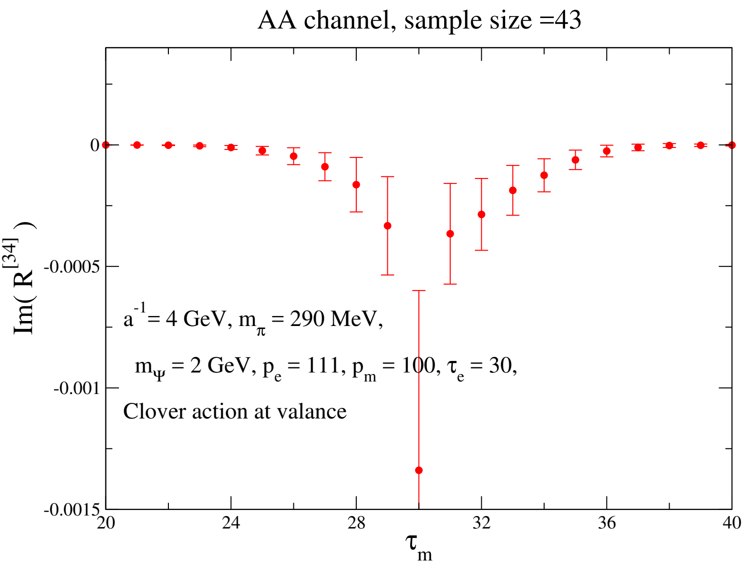
Quenched calculation

VA channel:

$a^{-1} \sim 2$ GeV, $L^3 \times T = 24^3 \times 48$, naive Wilson action at valance, $m_\pi \sim 370$ MeV, $m_\Psi \sim 1.1$ GeV,

sample size= 24




AA channel, sample size =43
Im(R[34])
τm
0
-0.0005
-0.001
-0.0015
20
24
28
32
36
40
a-1= 4 GeV, mπ = 290 MeV,
mΨ = 2 GeV, pe = 111, pm = 100, τe = 30,
Clover action at valance

Summary

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- I have described a new method to calculate the moments of pion distribution amplitude by studying the Euclidean OPE on lattice.
- A valance heavy quark is used to make lattice calculation simpler and to give more flexibility.
- I have shown our preliminary numerical data and demonstrate our strategy to analyze those.
- The method described here can also be used to study parton distribution function of the pion and nucleon.

Thanks for your attention!