Pion distribution amplitude from Euclidean correlation functions

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arXiv: 1702.00008 (PRD, 17)
with J.-W. Chen, X. Ji, L. Jin and H.-W. Lin
and
arXiv: 1506.00248 (PRD, 15)
with X. Ji, A. Schäfer, X. Xiong

Introduction

- DIS experiments and other inclusive reactions can be used to access parton densities, but not sufficient for probing hadronic wave function

- Complementary to them are exclusive processes where the kinematical parameters of all initial/final state particles are specified

- Hadron distribution amplitudes are universal non-perturbative input in hard exclusive processes

  - For example, the pion distribution amplitude (DA)

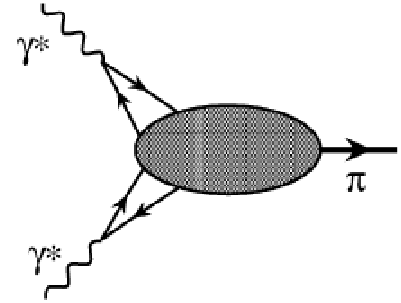
$$\phi_\pi(x) = \frac{i}{f_\pi}\int \frac{d\xi}{2\pi} e^{i(x-1)\xi\lambda\cdot P}\langle\pi(P)|\bar{\psi}(0)\lambda\cdot\gamma\gamma_5\Gamma(0,\xi\lambda)\psi(\xi\lambda)|0\rangle$$

  - $\lambda = (1,0,0,-1)/\sqrt{2}$

  - Represents probability amplitude of finding the valence $q\bar{q}$ Fock state in the pion with the quark carrying a fraction of x of total pion momentum

Introduction

- Experimental data are mostly on pion transition form factor
- It can be defined by the matrix element of the product of two electromagnetic currents



$$\int d^4y\, e^{iq_1\cdot y}\langle\pi^0(p)|T\{j_\mu(y)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F_{\gamma^*\gamma^*\to\pi^0}(q_1^2,q_2^2)$$

$$q_2 = q_1 + p, \quad q_1^2 = -Q^2, \quad q_2^2 = -q^2$$

- Collinear factorization

$$F_{\gamma^*\gamma^*\to\pi^0}(Q^2,q^2) = T_H(x,Q^2,q^2,\mu)\otimes\phi_\pi(x,\mu) + \mathcal{O}(1/Q^4)$$

- For $q^2 = 0$, the leading contribution can be written as

$$F_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 dx\, T_H(x,Q^2,\mu,\alpha_s(\mu))\phi_\pi(x,\mu)$$

with

$$T_H = \frac{1}{xQ^2}\left\{1 + C_F\frac{\alpha_S(\mu)}{2\pi}\left[\frac{1}{2}\ln^2 x - \frac{x\ln x}{2(1-x)} - \frac{9}{2} + \left(\frac{3}{2}+\ln x\right)\ln\frac{Q^2}{\mu^2}\right]\right\}$$

- At large scales, the pion DA is asymptotically

$$\phi_\pi^{\mathrm{as}}(x) = 6x(1-x), \qquad \int_0^1 dx\,\phi_\pi(x) = 1$$

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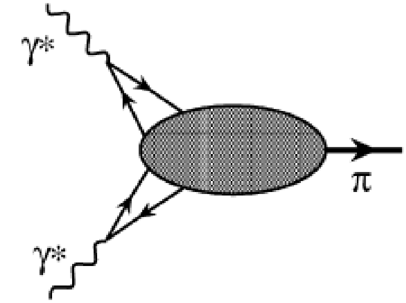

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- Measurement of pion transition form factor is ideal for testing QCD factorization approach

- One can also determine pion DA from it which can then be used to describe other exclusive processes

- Knowledge on pion DA from theory side is important and complementary

Pion DA from lattice QCD

- Traditional approach
  - Only the 2nd moment has been computed [Braun et al., PRD 15, Bali et al., PLB 17]
  - Higher moments are expected to be small
  - Lattice renormalization of local operators with many derivatives becomes too cumbersome

- Large momentum effective theory [Ji, PRL 13 & Sci. China 14, Ji, JHZ and Zhao, NPB, 17]
  - full Bjorken-x dependence of PDFs from lattice QCD
    - Lin, Chen, Cohen and Ji, PRD 14, Chen, Cohen, Ji, Lin and JHZ, NPB 16 (frontier article), Chen, JHZ et al., 17, Lin, Chen, Ishikawa and JHZ, 17;
    - Alexandrou et al. 14, PRD 15, PRD 16, NPB 17;
    - Ji and JHZ, PRD 15, Jia and Xiong, PRD 16, H.-N. Li, 16, Ishikawa et al., 16, 17, Chen, Ji and JHZ, 16, Manohan and Orginos, JHEP 17, Carlson and Freid, PRD 17, Briceno, Hansen and Monahan, PRD 17, Radyushkin, PLB 17, Xiong et al., 17, Constantinou et al., PRD 17, Rossi and Testa, PRD 17, Green et al., 17, Wang et al., 17, Stewart and Zhao, 17, Monahan, 17…
  - See talks by J.-W. Chen and C. Monahan

  - First computation of full pion DA, not its moments, from lattice QCD [JHZ et al., PRD 17]

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- Lattice cross sections [Ma and Qiu, 14 & 17]

- Pseudo-PDFs [Radyushkin, PRD 17]
  - Orginos et al., 17, Radyushkin, 17

- Current-current correlations
  - Detmold and Lin, PRD 06, K.-F. Liu, 16, Chambers et al., PRL 17, Braun and Müller, EPJC 08

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What is large momentum effective theory?

- An effective theory framework that allows to compute light-cone or parton observables from Euclidean quantities [Ji, PRL 13 & Sci. China 14]

- For a light-cone observable, e.g. the PDFs or the hadron DAs, construct a (frame-dependent) Euclidean quasi observable

$$\tilde{O}(P,\mu) \stackrel{P\to\infty}{\longrightarrow} O(\mu)$$

  - The choice of $\tilde{O}(P,\mu)$ is not necessarily unique

  - Instead of computing the light-cone observable directly, one can compute the quasi observable at a finite hadron momentum P

  - The difference between quasi and light-cone observables is in finite (but large) or infinite momentum, hence they shall have the same IR physics

  - Their difference can be perturbatively computed and captured in a matching factor Z

$$\tilde{O}(P,\mu) = Z(\frac{P}{\mu})O(\mu) + \mathcal{O}(\frac{M^2}{P^2})$$

with M a typical hadronic mass scale

What is large momentum effective theory?

- An effective theory framework that allows to compute light-cone or parton observables from Euclidean quantities [Ji, PRL 13 & Sci. China 14]

- Analogous to HQET. The role of heavy quark mass is now played by the large hadron momentum

- Parton model is an effective theory for the nucleon moving at large momentum

- Summary:
  - Suitably constructed Euclidean quasi observable is practically computable on the lattice
  - Perturbative matching allows to extract light-cone observable from the quasi observable
  - A good approximation of the light-cone observable can be achieved at a moderately large momentum

Application: pion DA

- Light-cone pion DA

$$\phi_\pi(x) = \frac{i}{f_\pi}\int \frac{d\xi}{2\pi} e^{i(x-1)\xi\lambda\cdot P}\langle \pi(P)|\bar{\psi}(0)\lambda\cdot\gamma\gamma_5\Gamma(0,\xi\lambda)\psi(\xi\lambda)|0\rangle$$

  - $\lambda = (1,0,0,-1)/\sqrt{2}$

- The pion quasi-DA can be constructed as

$$\tilde{\phi}(x,P_z) = \frac{i}{f_\pi}\int \frac{dz}{2\pi} e^{-i(x-1)P_z z}\langle \pi(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z)|0\rangle$$

  - z is a spatial direction
  - It approaches $\phi_\pi(x)$ in the limit $P_z \to \infty$
  - An alternative choice is to replace $\gamma^z \to \gamma^0$

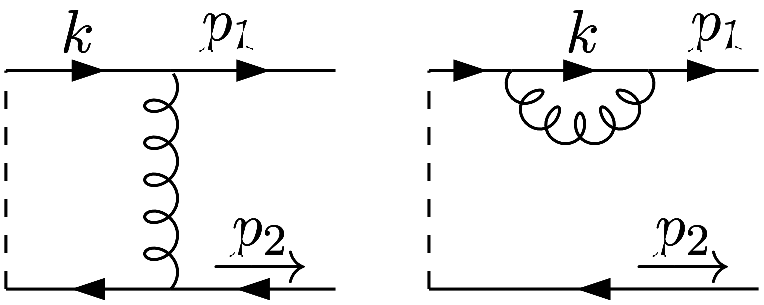
Application: pion DA

- Matching between pion quasi-DA and DA

$$\tilde{\phi}(x,\Lambda,P_z) = \int_0^1 dy\, Z_\phi(x,y,\Lambda,\mu,P_z)\phi(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_z^2}, \frac{m_\pi^2}{P_z^2}\right)$$

  - Matching kernel can be perturbatively computed analogously to evolution kernel

$$Z_\phi(x,y) = \delta(x-y) + \frac{\alpha_s}{2\pi}\overline{Z}_\phi(x,y) + \mathcal{O}\left(\alpha_s^2\right)$$



  - Extraction of $\phi(x)$

$$\phi(x) \simeq \tilde{\phi}(x) - \frac{\alpha_s}{2\pi}\int_{-\infty}^{\infty} dy\left[Z_\phi^{(1)}(x,y)\,\tilde{\phi}(y) - Z_\phi^{(1)}(y,x)\,\tilde{\phi}(x)\right]$$

Application: pion DA

$$Z_\phi^{(1)}(x,y,\mu,p^z)/C_F = G_1(x,y,\mu,p^z)\theta(x<0) + G_2(x,y,\mu,p^z)\theta(0<x<y)$$
$$+ G_3(x,y,\mu,p^z)\theta(y<x<1) + G_4(x,y,\mu,p^z)\theta(x>1)$$

$$G_1(x,y,\mu,p^z) = \left(\frac{x}{2(1-y)} + \frac{1-x}{2y}\right)\ln\frac{x}{x-1} + \left(\frac{x}{2(1-y)} - \frac{1-x}{2y} + \frac{1}{y-x}\right)\ln\frac{(x-y)^2}{x(x-1)}$$
$$+ \frac{\mu}{p^z(x-y)^2},$$
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$$+ \left(\frac{x}{2(y-1)} + \frac{x-1}{2y}\right)\ln[4x(1-x)] + \left(\frac{x}{2(y-1)} + \frac{1-x}{2y} - \frac{1}{y-x}\right)\ln\frac{1-x}{x}$$
$$+ \frac{1}{y} - \frac{1}{y-x} + \frac{\mu}{p^z(x-y)^2},$$
$$G_3(x,y,\mu,p^z) = G_2(1-x,1-y,\mu,p^z),$$
$$G_4(x,y,\mu,p^z) = G_1(1-x,1-y,\mu,p^z),$$

- Does not vanish for $x > 1$ and $x < 0$
- Collinear divergence only in 0<x<1 and cancels between quasi-DA and DA
- Logarithmic term coincides with ERBL evolution kernel
- Linear divergence

Application: pion DA

$$Z_\phi^{(1)}(x,y,\mu,p^z)/C_F = G_1(x,y,\mu,p^z)\theta(x<0) + G_2(x,y,\mu,p^z)\theta(0<x<y) + G_3(x,y,\mu,p^z)\theta(y<x<1) + G_4(x,y,\mu,p^z)\theta(x>1)$$

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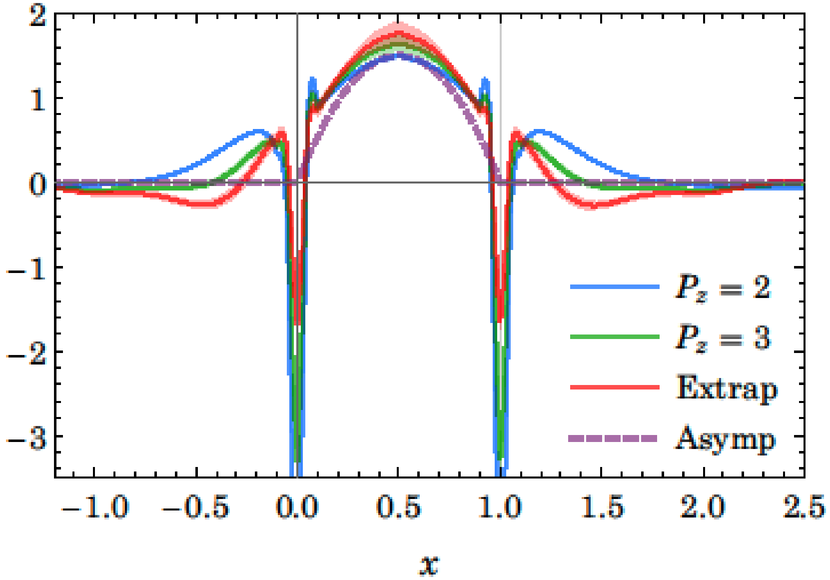
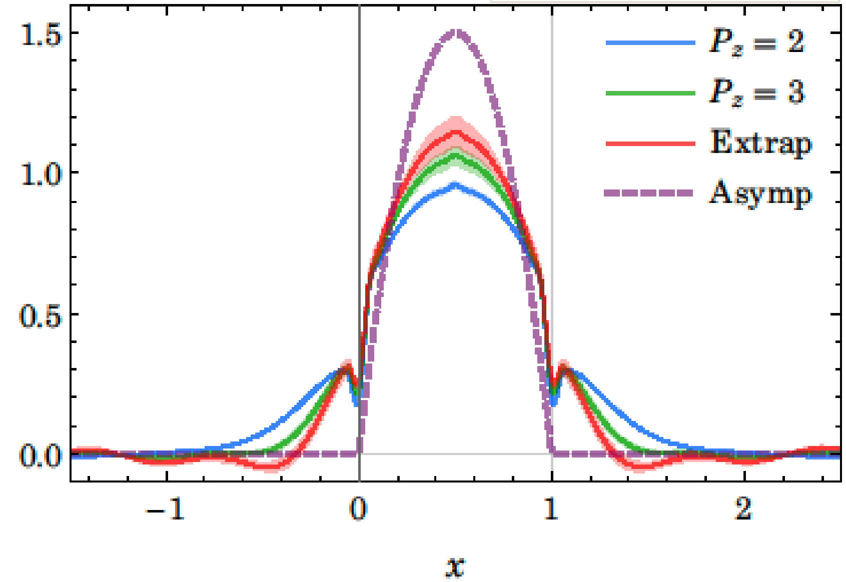
- Linear divergence can be removed by Wilson line mass renormalization

Application: pion DA

- Unimproved vs. improved

Chen, JHZ et al., 17, $m_\pi = 310\ MeV, L = 3fm, a = 0.12fm$

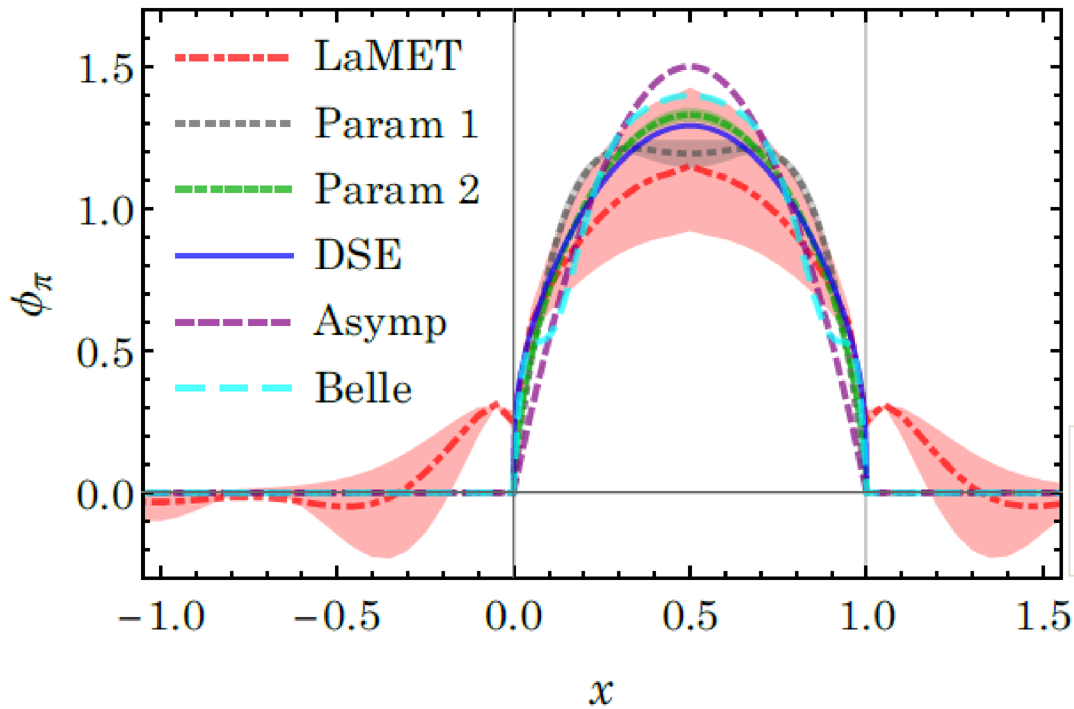




- One-loop correction, and also mass correction included, $P_z = \{2, 3\} \times 2\pi/L$
- Extrapolation to infinite momentum using a simple form $\alpha(x) + \beta(x)/P_z^2$, but has residual contribution outside [0,1]
- Left: using the unrenormalized quasi correlation, linear divergence contained in the matching factor, unphysical oscillatory behavior near $x = 0, 1$
- Right: improved, there are still small kinks in the unphysical region, expected to vanish when higher-order matching is included/higher momentum is reached

Application: pion DA

- Improved pion DA result



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- Param 1: $\phi_\pi(x) = 6x(1-x)[1 + a_2 C_2^{3/2}(2x-1)]$
- Param 2: $\phi_\pi(x) = A[x(1-x)]^B$
- Consistent with previous studies, favors a single-hump shape
- Error dominated by uncertainty in Wilson line mass renormalization, can be improved by computing at different lattice spacings

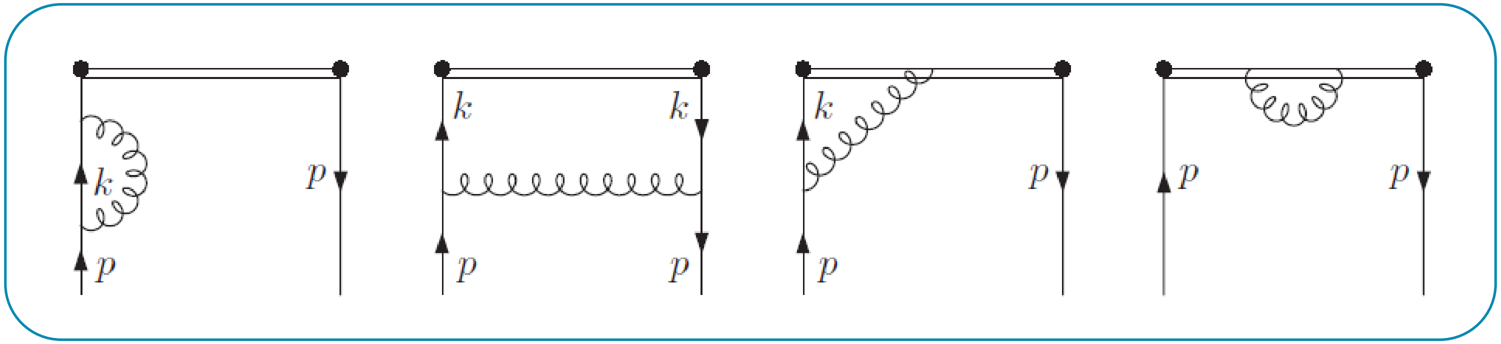
Summary

- PDFs or hadron DAs are difficult to compute due to their non-perturbative and intrinsically Minkowskian nature
- Large momentum effective theory offers a practical possibility to directly compute them from Euclidean lattice
  - Appropriately constructed Euclidean quasi-quantity + perturbative matching
- Rapid progress in the past few years, more effort needed for lattice results to reach an accuracy comparable with phenomenological studies
  - Finer lattice spacing
  - Larger hadron momentum

BACKUP SLIDES

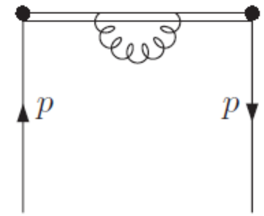
Renormalization of power divergence

- Linear divergence present in matching kernel needs to be removed
  - Not suppressed since $\Lambda > P_z$ always holds
  - Can spoil validity of perturbative expansion
- Origin: Wilson line self energy, this is clear in covariant Feynman gauge
- Such diagrams are common in quasi-PDF and pion DA, use the quasi-PDF to illustrate the renormalization of power divergence



Renormalization of power divergence

- Power divergence comes from Wilson line self energy [Ishikawa et al. 16′, Chen et al. 16′]
  - At one-loop, a linear div. is associated with



  - It is well-known that linear divergence associated with Wilson line can be removed by a mass renormalization (e.g. in auxiliary z-field formalism)
  - In a sense, the auxiliary field can be understood as a Wilson line extending between $[z, \infty]$

$$Z(z) = L(z, \infty), \qquad [\partial_z - igA_z(z)]\, Z(z) = 0$$

  - Analogous to a heavy quark field
  - Non-local Wilson line can be interpreted as a two-point function of z-field

$$L(z, 0) = Z(z) Z^{\dagger}(0)$$

  - Renormalizes analogously to a heavy quark two-point function [Dotsenko and Vergeles 80′, Dorn 86′]

$$L^{\rm ren}(z, 0) = \mathcal{Z}_Z^{-1} e^{-\delta m |z|} L(z, 0)$$

Renormalization of power divergence

- One-loop illustration
  - The Wilson line self energy diagram gives ($\bar{x}$=1-x)

$$\lim_{\epsilon\to 0}\int dk_z \frac{\alpha_s C_F \Lambda}{2\pi}\frac{[\delta(k_z-\bar{x}p_z)-\delta(\bar{x}p_z)]p_z}{k_z^2+\epsilon^2}$$

  - Mass counterterm contributes

$$-\int \frac{dz}{2\pi}p_z\, e^{i(x-1)p_z z}|z|\,\delta m = -\lim_{\epsilon\to 0}\int \frac{dz}{2\pi}p_z\, e^{-i\bar{x}p_z z}\frac{1-e^{-\epsilon|z|}}{\epsilon}\delta m$$
$$= -\lim_{\epsilon\to 0}\int \frac{dk_z}{\pi}p_z \frac{\delta(\bar{x}p_z)-\delta(k_z-\bar{x}p_z)}{k_z^2+\epsilon^2}\delta m.$$

  - Therefore

$$\delta m = -\frac{\alpha_s C_F}{2\pi}(\pi\Lambda)$$

  - It is gauge-independent

- Can be extended to higher-loop orders

Improved pion quasi-DA

- We can define an improved pion quasi-DA without power divergence

$$\tilde{\phi}_{\text{imp}}(x, P_z) = \frac{i}{f_\pi} \int \frac{dz}{2\pi} e^{-i(x-1)P_z z - \delta m|z|} \langle \pi(P)|\bar{\psi}(0)\gamma^z \gamma_5 \Gamma(0,z)\psi(z)|0\rangle$$

- Multiplicative renormalization [Musch et al. 11′]

$$O^{\text{ren}}_{\Gamma,q}[\mathcal{C}_l] = \underbrace{Z_\psi^{-1} Z^2_{(\psi z)} Z_z^{-1}}_{Z^{-1}_{\psi,z}} e^{-\delta m \ell[\mathcal{C}_l]} O_{\Gamma,q}[\mathcal{C}_l].$$

- Renormalization factors include quark field renormalization, quark-gauge link vertex, endpoint renormalization, which are all local and an exponential mass renormalization to remove power divergence

- Eventually the pion DA determined from the improved pion quasi-DA will be normalized to 1, this roughly means an implementation of the renormalization

Improved pion quasi-DA

- We can define an improved pion quasi-DA without power divergence

$$\tilde{\phi}_{\rm imp}(x, P_z) = \frac{i}{f_\pi}\int \frac{dz}{2\pi} e^{-i(x-1)P_z z - \delta m|z|}\langle \pi(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z)|0\rangle$$

- Multiplicative renormalization [Musch et al. 11′]

$$\mathcal{O}^{\rm ren}_{\Gamma,q}[\mathcal{C}_l] = \underbrace{Z_\psi^{-1} Z^2_{(\psi z)} Z_z^{-1}}_{Z^{-1}_{\psi,z}} e^{-\delta m \ell[\mathcal{C}_l]}\mathcal{O}_{\Gamma,q}[\mathcal{C}_l].$$

- The matching factor for the improved quasi-DA is obtained from the previous result by removing linear divergence

- Intuitively, the exponential renormalization factor $e^{-\delta m|z|}$ will increase the weight of matrix elements with relatively large z, and therefore will increase the contribution at relatively small momentum when Fourier transforming to momentum space

Improved pion quasi-DA

- Determination of δm [Musch et al. 11′]
  - Static heavy quark-antiquark potential can be obtained from asymptotic behavior of a rectangular Wilson loop

$$W(R,T) = c(R)e^{-V(R)T} + \text{higher excitations},$$

  - Choose a Wilson loop long in t-direction such that higher excitations are sufficiently suppressed

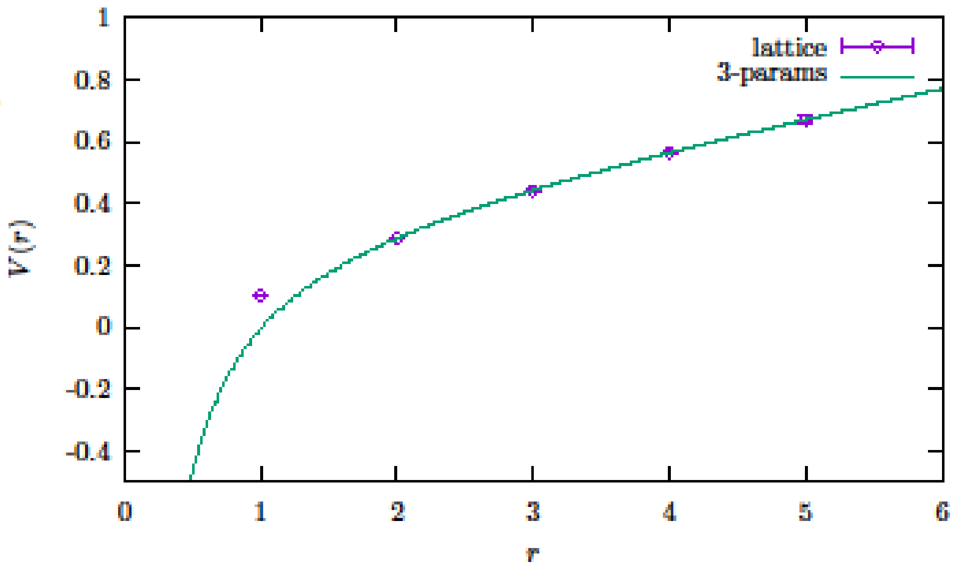
  - Fit the quark potential

$$V(r) = -\frac{1}{a}\lim_{t\to\infty} \ln \frac{\langle \mathrm{Tr}[W(t,r)]\rangle}{\langle \mathrm{Tr}[W(t-a,r)]\rangle}$$

  - to

$$V(r) = \frac{c_1}{r} + c_2 + c_3 r$$

  - c_1 is the Coulomb potential
  - c_3 is the confinement linear potential
  - c_2 twice the rest mass of heavy quark, expected to be $c_2 = \tilde{c}/a + O(\Lambda_{QCD})$,



$\delta m = -c_2/2 \approx -260 \pm 200$ MeV