The flavor structure of nucleon sea from lattice QCD

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Feynman's Parton Model



The momentum distributions of partons (quarks, antiquarks and gluons) become one dimensional distributions in the infinite momentum frame.

Measuring Parton Distributions Using DIS experiments



Parton Distribution Function (PDF) in QCD



Parton Distribution Function (PDF) in QCD



The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^-P^+} \left\langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^-\lambda) \right| P \right\rangle$$

Current Status of Proton PDFs

How do momentum and spin distribute among partons?

- Exp: 1d mom. dist. largely mapped out (up to parameterizations of the functional forms); largest sys. uncertainty in Higgs production.
 improve 1d(spin)+3d: BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...
- Theory: Only first few moments could be computed directly from QCD!!!

PDFs from QCD---why is it so hard?

- Quark PDF in a proton: $(\lambda^2 = 0)$ $q(x, \mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P | \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) | P \rangle$
- Non-perturbative, infinite dof, need lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators $t^2 r^2 = 0$
- Moments of PDF given by local twist-2 $-t_E^2 \mathbf{r}^2 = 0$ operators (twist = dim - spin); limited to first few moments but carried out successfully

$$a_n = \int_{-1}^1 dx \, x^{n-1} q(x)$$
 and $q(-x) = -\bar{q}(x)$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x -dependence directly. (variation: pseudo-PDF, Radyushkin)

Ji's idea

• Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{a\xi}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \right| P \right\rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

• Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

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- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching----LaMET (Large Momentum Effective Theory)

Review: Ji's LPDF (LaMET)

$$\begin{split} \widetilde{q}(x,\mu^2,P^z) &= \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(z\lambda) \right| P \right\rangle \\ &\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z)P^z \end{split}$$

$$\lambda^{\mu} = (0, 0, 0, 1)$$

• Taylor expansion yields $\overline{\psi}\lambda \cdot \gamma\Gamma \left(\lambda \cdot D\right)^n \psi = \lambda_{\mu_1}\lambda_{\mu_2}\cdots \lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$ op. symmetric but not traceless $\left(\lambda_{\mu_1}\lambda_{\mu_2} - g_{\mu_1\mu_2}\lambda^2/4\right)$

Review: Ji's LPDF (LaMET)

$$\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

- LHS: trace, twist-4 $O(\Lambda_{QCD}^2/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$ilde{q}(x,\Lambda,P_z) = \int rac{dy}{|y|} Z\left(rac{x}{y},rac{\mu}{P_z},rac{\Lambda}{P_z}
ight) q(y,\mu) + \mathcal{O}\left(rac{\Lambda^2_{ ext{QCD}}}{P_z^2},rac{M^2}{P_z^2}
ight) + \dots$$

What do we expect to see on the lattice?

• Suppose LPDF were the CTEQ PDF at $P^z \to \infty$





First (isovector) LPDF Computation

• Lattice: $24^3 \times 64$

 $a \approx 0.12 \text{ fm}$ $L \approx 3 \text{ fm}$

• Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

 $N_f = 2 + 1 + 1$ $M_\pi \approx 310 \text{ MeV}$

• Gauge fields/links: hypercubic (HYP) smearing, 461 config.

$$P^{z} = \frac{2\pi}{L}n = n \times 0.43 \ GeV$$
 $n = 1, 2, 3...$

(high momentum smearing: Bali, Lang, Musch, Schafer)



Quasi-PDF (unpolarized)



$$M_{\pi} \approx 310 \text{ MeV}$$
$$P^{z} = \frac{2\pi}{L}n = n \times 0.43 \text{ GeV} \quad n = 1, 2, 3.$$

$$\mathcal{O}(M^2/(P^z)^2)$$
 ·Corrections

$$P^z = \frac{2\pi}{L}n = n \times 0.43 \ GeV$$

• Computed to all orders in $\mathcal{O}(M^2/(P^z)^2)$ ·

$$q(x) = \sqrt{1+4c} \sum_{n=0}^{\infty} \frac{f_{-}^{n}}{f_{+}^{n+1}} \Big[(1+(-1)^{n}) \tilde{q} \Big(\frac{f_{+}^{n+1}x}{2f_{-}^{n}} \Big) + (1-(-1)^{n}) \tilde{q} \Big(\frac{-f_{+}^{n+1}x}{2f_{-}^{n}} \Big) \Big]$$

$$f_{\pm} = \sqrt{1 + 4c} \pm 1$$
 $c = M^2/4P_z^2$

 $\mathcal{O}(\Lambda^2_{QCD}/(P^z)^2)$ Corrections

• Twist-4:

$$q_{tr}(x,\mu^2,P^z) = \frac{\lambda^2}{8\pi} \int_{-\infty}^{\infty} dz \int_{0}^{1} \frac{dt}{t} e^{i\frac{zk^2}{t}} \left\langle P \left| \widetilde{\mathcal{O}}_{tr}(z) \right| P \right\rangle$$

$$\widetilde{\mathcal{O}}_{tr}(z) = \int_{0}^{z} dz_{1} \overline{\psi}(0) \left[\gamma^{\nu} \Gamma(0, z_{1}) D_{\nu} \Gamma(z_{1}, z) + \int_{0}^{z_{1}} dz_{2} \lambda \cdot \gamma \Gamma(0, z_{2}) D^{\nu} \Gamma(z_{2}, z_{1}) D_{\nu} \Gamma(z_{1}, z) \right] \psi(z\lambda)$$
Parameterized ($\alpha(x) + \beta(x)/P_{z}^{2}$)

Additional complications? E.g. Radyushkin RG of Wilson Coefficient $\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{P_z^2},\frac{M_N^2}{P_z^2}\right) + \dots$

Xiong, Ji, Zhang, Zhao (GPD: Ji, Schafer, Xiong, Zhang; Xiong, Zhang) Factorization (Ma, Qiu; Li), Linear divergence & LPT (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang; Xiong, Luu, Meissner; Rossi, Testa; Constantinou et al.), RI (Monahan & Orginos; Yong & Stewart; Constantinou et al.), NPR(Constantinou et al.; LP3; Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida; Green, Jansen, Steffens), E vs. M spaces (Carlson et al.; Briceno et al.)

Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)



 $P^{z} = \frac{2\pi}{L}n = n \times 0.43 \; GeV$ n = 2 (upper) & 3

Unpolarized Isovector Proton PDF



 $\alpha(x) + \beta(x)/P_z^2$





Quark mass effect!

Follow-up works (Alexandrou et. al.:1504.07455+1610.03689)



Isovector Proton Helicity and Transversity



Isovector Proton Helicity



Isovector Proton Transversity



More on the power divergence in the matching kernel

Improved Quasi-PDF's (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang)



Improved Quasi-PDF's (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang)



All orders (Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida)



 $\tilde{q}_{\rm imp}(x,\Lambda,p^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z - \delta m|z|} \langle p|\overline{\psi}(0,0_{\perp},z)\gamma^z L(z,0)\psi(0)|p\rangle$



NPR(RI/MOM)+1100p matching



NPR w/o Pz corrections



Green, Jansen, Steffens, 1707.07152





1708.05301(LP3) attempts to address the long tail issue, physical pion mass (also 1710.6408 EMTC)

Outlook

- Further tests (non-singlet): long tail (L Pz large by taking Pz large? small x: large Nz); wee partons (smaller quark mass); factorization proof.
 - Know whether it works within 5 years (~20%)?
- Singlet PDF's: s, c, b and gluons Additional 3-5 yrs?
- If it works, complimentary to exp.: PDF (isov. sea, small and large x's, non-valence partons), DA, GPD, TMD ...

Backup slides

Pion Light Cone DA-Zhang, JWC, Ji, Jin, Lin



 $\phi_{\pi}(x, \mu) + 3\phi_{\eta}(x, \mu) = 2[\phi_{K^{+}}(x, \mu) + \phi_{K^{-}}(x, \mu)]$ = 2[\phi_{K^{0}}(x, \mu) + \phi_{\overline{K}^{0}}(x, \mu)], No leading chiral log JWC, Iain W. Stewart, Phys.Rev.Lett. 92 (2004) 202001



Improved Quasi-PDF's Ishikawa, Ma, Qiu, Yoshida: x-space JWC, Ji, Zhang: p-space

$$ilde{q}_{ ext{imp}}(x, a_L, p^z) = \int_{-1}^1 rac{dy}{|y|} Z\left(rac{x}{y}, p^z a_L, rac{\mu}{p^z}
ight) q(y, \mu) + \mathcal{O}\left(\Lambda_{ ext{QCD}}^2/(p^z)^2, M^2/(p^z)^2
ight)$$

$$Z(\xi) = \delta(\xi - 1) + \frac{\alpha_s}{2\pi} \left[Z^{(1)}(\xi) - \int dy Z^{(1)}(y) \,\delta(\xi - 1) \right] + \dots$$

$$Z^{(1)}/C_F = \begin{cases} \left(\frac{1+\xi^2}{1-\xi}\right) \ln \frac{\xi}{\xi-1} + 1 , & \xi > 1 ,\\ \left(\frac{1+\xi^2}{1-\xi}\right) \ln \frac{(p^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right) \ln \left[4\xi(1-\xi)\right] - \frac{2\xi}{1-\xi} + 1 , \ 0 < \xi < 1 ,\\ \left(\frac{1+\xi^2}{1-\xi}\right) \ln \frac{\xi-1}{\xi} - 1 , & \xi < 0 , \end{cases}$$

Stewart & Zhang: NP RI/MOM renorm. + one-loop RI/MOM MS-bar matching

Helicity and Transversity (isovector)



Quasi-PDF (Helicity and Transversity)



$$P^{z} = \frac{2\pi}{L}n = n \times 0.43 \ GeV$$
 n = 1, 2, 3.

Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)



 $P^{z} = \frac{2\pi}{L}n = n \times 0.43 \ GeV$ n = 2 (upper) & 3