Anti-de Sitter Space as Topological Insulator and Holography

S.H. Ho (NCTS)
Work with Prof. Feng-Li Lin (NTNU)

based on arXiv:1205.4185
Outline

- **Introduction**
  - Topological orders and symmetry protected topological orders
  - SPT phases and topological insulators/superconductors

- **Kitaev’s K-theory classification scheme**
  - General idea to classify SPT phases
  - Complex fermions
  - charge conjugation and time reversal symmetries
  - Real fermions

- **Bulk/edge correspondence and AdS/CFT**
  - Bulk/edge correspondence
  - Holographic fermions

- **AdS space as a topological insulator**

- **Discussion**
Introduction

• Phase transition:
  • symmetry breaking: order parameters, Nambu-Goldstone/Higgs modes
  • non-symmetry breaking: topological orders (gapped system), Fermi liquids...etc.
  • e.g. fractional quantum Hall effect: lots of different phases with the same symmetry
  • can be characterized by gapless boundary modes
  • robust under perturbations and smooth change of parameters

QFT of many-body systems, X.G. Wen
• Symmetry-protected topological (SPT) order:

• gapped quantum phase with certain symmetries

• cannot distinguish these phases if we remove the symmetries (all can be smoothly connected to the same trivial phase if no symmetries)

• e.g. non-interacting (gapped) free fermion systems such as topological insulator/superconductor

\[ H = \psi^\dagger h \psi = \psi^\dagger i (\alpha^i \partial_i + M) \psi \]

• topological insulator can be characterized by the gapless boundary modes protected by time-reversal symmetry
Kitaev’s K-theory classification scheme

- General idea for classifying topological orders
  - only know how to classify gapped system
  - gapless modes between different topological phases in the configuration space
  - topological phase is an equivalence class of perturbations which do not close the gap
Physically the gapless modes on the phase boundary in Fig. 1 cannot propagate in the bulk because the systems are gapped by definition. To host these modes we must provide some physical support to localize them.
How to classify? An intuitive example

- Start from $d=0$ (spatial dimension) gapped system with one orbital

  the number of different phases is two: occupied and empty (1 or 0)

  - Mathematically, consider $H^2 = 1$

    the number of positive and negative energy levels are $l$ and $m$, $n = l + m$

    $H = \begin{pmatrix} I_{l \times l} & 0 \\ 0 & -I_{m \times m} \end{pmatrix}$ up to an unitary transformation $H = U_{n \times n} \begin{pmatrix} I_{l \times l} & 0 \\ 0 & -I_{m \times m} \end{pmatrix} U_{n \times n}^\dagger$

    but $U_{n \times n}$ is not a one-to-one labeling

    the $H$ also invariant under $\begin{pmatrix} V_{l \times l} & 0 \\ 0 & W_{m \times m} \end{pmatrix}$

- The classification space $C_0$ is $\bigcup_m \frac{U(l+m)}{U(l) \times U(m)}$, large $n$ limit

  $C_0 \equiv \frac{U(l+m)}{U(l) \times U(m)} \times \mathbb{Z}$

  $\pi_0(C_0) = \mathbb{Z}$

  number of disconnected pieces of $C_0$

For \( d=1 \), we have flat band condition \( H^2 = 1 \) and \( H \) have to satisfy \( \gamma_1 H + H \gamma_1 = 0 \) with \( \gamma_1^2 = 1 \) and hermitian.

\[
\gamma_1 = \sigma^x \otimes I_{n \times n}
\]

\[
H = \sigma^z \otimes I_{n \times n}
\]

\[
H = e^{i \sigma^x \otimes A_{n \times n}} e^{i \sigma^0 \otimes B_{n \times n}} (\sigma^z \otimes I_{n \times n}) e^{-i \sigma^0 \otimes B_{n \times n}} e^{-i \sigma^x \otimes A_{n \times n}}
\]

because \( H \) invariant under \( e^{i \sigma^x \otimes A_{n \times n}} e^{i \sigma^0 \otimes B_{n \times n}} \in U(n) \times U(n) \) positive and negative eigenvalues always paired

\( C_1 \) is \( U(n) \times U(n)/U(n) = U(n) \)

For more general dimension \( d=p \), first pick \( p \) fixed hermitian matrix with \( \gamma_i \gamma_j + \gamma_j \gamma_i = 2 \delta_{i,j} \)

\( C_p \) is the space of hermitian matrix with condition \( H^2 = 1 \), \( \gamma_i H = -H \gamma_i \), \( i = 1, \ldots, p \)
• Kitaev’s K-theory analysis

- classify the configuration space of mass matrix $M$ of free Dirac Hamiltonian through the Clifford algebra formed by mass matrix $M$ and symmetry operators (if any).

$$
\hat{H} = \psi^\dagger h \psi \equiv \psi^\dagger (-i)(\alpha^k \partial_k + M) \psi
$$

fixed $\alpha^i$ find all possible solution for $M$

- Flat-band condition: only care about the number of positive and negative eigenvalue of Hamiltonian for topological consideration

$$
h^2 = (\vec{p} \cdot \vec{p} + 1) \otimes 1,
$$

- Clifford algebra

\[
\begin{align*}
\{\alpha^i \otimes 1, \alpha^j \otimes 1\} &= 2\delta^{ij} \otimes 1, \\
\{\alpha^i, M\} &= 0, \\
M^2 &= -1 \otimes 1.
\end{align*}
\]
• Complex fermions

- if no charge conjugation (C) and time reversal (T)
  \[
  \{\alpha^i \otimes 1, \alpha^j \otimes 1\} = 2\delta^{ij} \otimes 1,
  \{\alpha^i, M\} = 0,
  M^2 = -1 \otimes 1.
  \]
  whole story

- Clifford algebra \( Cl(d, 1) \) for complex matrices,
  the space of different ways to gap the system (the ways to write down \( M \))

- configuration space denoted \( \mathcal{M} = C_d \)

- the bulk phase \( \pi_0(C_{p=even}) = \mathbb{Z} \) labelled by some integer \( \mathbb{Z} \)

- the defect \( \pi_{d-d_b-1}(C_d) = \pi_0(C_{d_b+1}) \), \( d \) independent

Gauss law:
cover the defect by a closed surface

Bott periodicity \( \pi_n(C_p) = \pi_0(C_{p+n}) \)
• **C and T symmetries**

\[ H = \psi^\dagger h \psi \equiv \psi^\dagger (-i) (\alpha^k \partial_k + M) \psi \]

- **charge conjugation:**

\[ h(\bar{\theta}, \bar{x}) = -[U_c^\dagger h(-\bar{\theta}, \bar{x})U_c]^T, \quad \hat{C} \hat{\psi}(t, \bar{x}) \hat{C}^{-1} = U_c \hat{\psi}^*(t, \bar{x}), \]

\[ \alpha^i U_c = U_c (\alpha^i)^*, \]

\[ M U_c = U_c M^*, \]

- **time reversal:**

\[ h(\bar{\theta}, \bar{x}) = U_t^\dagger h^*(\bar{\theta}, \bar{x}) U_t. \quad \hat{T} \hat{\psi}(t, \bar{x}) \hat{T}^{-1} = U_t \hat{\psi}(-t, \bar{x}) \]

\[ (\alpha^i)^* U_t = -U_t \alpha^i, \]

\[ M^* U_t = -U_t M. \]
- \( C \) and \( T \) are \( \mathbb{Z}_2 \) symmetries, define

\[
U_c U_c^* = s_c,
\]

\[
U_t U_t^* = s_T,
\]

\[
U_t U_c = s_{TC} U_c^* U_t^*;
\]

\( s_C, s_T \) and \( s_{TC} \) are taking values of \( \pm 1 \).
• Real fermions

- C and T will not join $\alpha^i$'s and M to form an enlarged Clifford algebra in complex case

- first assume $\alpha^i$, M, $U_c$ and $U_t$ are all real

$$[U_c, M] = [U_c, \alpha^i] = 0, \quad \{U_t, \alpha^i\} = \{U_t, M\} = 0$$

$$U_c^2 = s_C, \quad U_t^2 = s_T \quad \text{and} \quad U_t U_c = s_{TC} U_c U_t.$$ 

1. $U_t$, $\alpha^i$ and M form a Clifford algebra,

2. $U_c$ and $U_t$ may combine to form a generator of Clifford algebra,

3. the configuration space $M_{s_{TC}, s_C}^{s_{TC}}$, determined by $s_C$, $s_T$ and $s_{TC}$.

4. for the Clifford algebra $Cl(d, 1)$, the classification space denoted by $R^0_d$.
• Boundary excitation classified by: $\pi_{d-d_b-1}(\mathcal{M}^{STC}_{STSC})$.

Bott periodicity

$$R_p^q = R_{p-q \mod 8}^0,$$
$$\pi_n(R_p^0) = \pi_0(R_{p-n}^0).$$

**TABLE II:** The classifying spaces $\mathcal{M}^{STC}_{STSC}$.

<table>
<thead>
<tr>
<th>$\mathcal{M}^{STC}_{STSC}$</th>
<th>$\mathcal{M}^{+}_{++}$</th>
<th>$\mathcal{M}^{+}_{+-}$</th>
<th>$\mathcal{M}^{+}_{-+}$</th>
<th>$\mathcal{M}^{+}_{--}$</th>
<th>$\mathcal{M}^{-}_{++}$</th>
<th>$\mathcal{M}^{-}_{+-}$</th>
<th>$\mathcal{M}^{-}_{-+}$</th>
<th>$\mathcal{M}^{-}_{--}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Phase</td>
<td>$R_{d+1}^0$</td>
<td>$R_{d-1}^0$</td>
<td>$C_{d+1}$</td>
<td>$C_{d+1}$</td>
<td>$R_d^0$</td>
<td>$R_d^0$</td>
<td>$R_{d+2}^0$</td>
<td>$R_{d-2}^0$</td>
</tr>
<tr>
<td>Defects</td>
<td>$R_{d_b+2}^0$</td>
<td>$R_{d_b}^0$</td>
<td>$C_{d_b}$</td>
<td>$C_{d_b}$</td>
<td>$R_{d_b+1}^0$</td>
<td>$R_{d_b+1}^0$</td>
<td>$R_{d_b+3}^0$</td>
<td>$R_{d_b-1}^0$</td>
</tr>
</tbody>
</table>

only these two are relevant in our consideration

**TABLE III:** The space $R_p^0$ and its homotopy group $\pi_0(R_p^0)$.

<table>
<thead>
<tr>
<th>$p \mod 8$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_p^0$</td>
<td>$O^{(2n)}_{U(n)}$</td>
<td>$O(n)$</td>
<td>$O^{(l+m)}_{O(l) \times O(m)} \times \mathbb{Z}$</td>
<td>$U(n)$</td>
<td>$Sp(n)$</td>
<td>$Sp(n)$</td>
<td>$Sp^{(l+m)}_{Sp(l) \times Sp(m)} \times \mathbb{Z}$</td>
<td>$U^{(2n)}_{Sp(n)}$</td>
</tr>
<tr>
<td>$\pi_0(R_p^0)$</td>
<td>$\mathbb{Z}_2$</td>
<td>$\mathbb{Z}_2$</td>
<td>$\mathbb{Z}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\mathbb{Z}$</td>
</tr>
</tbody>
</table>
• Modified Kitaev’s K-theory in high energy physics

1. Lorentz invariance

\[ \{\alpha^i \otimes 1, \alpha^j \otimes 1\} = 2\delta^{ij} \otimes 1, \]
\[ \{\alpha^i, M\} = 0, \]
\[ M^2 = -1 \otimes 1. \]

automatically satisfied

\[ \alpha^i := \Gamma^0 \Gamma^i \quad M = \Gamma^0 \otimes (m + \Lambda \Phi) \]

2. Assume no non-trivial constituents for fermions

non-trivial fractionalization of the electrons are commonly postulated when considering the topological orders

\[ U_c \psi^* = \psi, \quad s_C = 1. \]

reality condition

\[ U_c U_c^* = s_C = 1 \]

unitary and \( U_c \) is symmetric.

exist a basis in which \( U_c = 1 \).
- Consider the Hamiltonian

\[ h = i\Gamma^0 (\Gamma^i \partial_i - m - \Lambda \Phi) := i(\alpha^i \partial_i - M) \quad i = 1, 2, \cdots, d \text{ and } \alpha^i := \Gamma^0 \Gamma^i. \]

\[ M = \Gamma^0 \otimes (m + \Lambda \Phi). \]

- In the $U_c = 1$ basis (Majorana representation),

\[ \alpha^i = \alpha^i, \quad M^* = M. \]

\[ \Gamma^\mu = \pm \Gamma^\mu, \quad (m + \Lambda \Phi)^* = \pm (m + \Lambda \Phi). \]

1. $U_c = 1$ trivial

2. only $s_T$ relevant if $U_t$ exists.

3. $s_{TC}$ irrelevant since $U_t^* = s_{TC} U_t$.

\[ s_{TC} = 1 \quad U_t \text{ real} \]

\[ s_{TC} = -1 \quad U_t \text{ imaginary} \quad \text{redefine } U_t \]
1. Q: when $U_c = 1$ (real representation exists)?

\[ \Phi \text{ real rep.} \quad \Rightarrow \quad \text{real rep. for Dirac gamma matrix} \]
\[ \Phi \text{ pseudo-real rep.} \quad \Rightarrow \quad \text{pseudo-real rep. for Dirac gamma matrix} \]

combined to form a real rep.

<table>
<thead>
<tr>
<th>real Dirac gamma matrix</th>
<th>d=0,1,2,3,7 mod 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>pseudo-real Dirac gamma matrix</td>
<td>d=3,4,5,6,7 mod 8</td>
</tr>
</tbody>
</table>

2. Q: when $U_t$ exists?

\[ U_t = U_c^* \Gamma^0 \Gamma \quad \Gamma = (-1)^{(d-1)/4} \Gamma^0 \Gamma^1 \cdots \Gamma^d \quad \{\Gamma, \Gamma^\mu\} = 0 \]

<table>
<thead>
<tr>
<th>real Dirac gamma matrix</th>
<th>$U_t$ exists when d=1,3,7 mod 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>pseudo-real Dirac gamma matrix</td>
<td>$U_t$ exists when d=3,5,7 mod 8</td>
</tr>
<tr>
<td>$d \mod 8$</td>
<td>$d = 0$</td>
</tr>
<tr>
<td>----------------</td>
<td>--------</td>
</tr>
<tr>
<td>R or PR</td>
<td>R</td>
</tr>
<tr>
<td>$\Gamma^\mu$</td>
<td>Im</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>$R_0^0$</td>
</tr>
<tr>
<td>$\pi_0(\mathcal{M})$</td>
<td>$\mathbb{Z}_2$</td>
</tr>
<tr>
<td>$\mathcal{M}_{db}$</td>
<td>$R_{db+1}^0$</td>
</tr>
</tbody>
</table>

**TABLE IV:** Classification of symmetry-protected topological order of real free fermions in $(d + 1)$-dimensional Minkowski space. Here $\mathcal{M}_{db}$ denotes the configuration space for the topological phases on the $(d_b + 1)$-dimensional defect, and the edge modes are then classified by $\pi_0(\mathcal{M}_{db})$. The second row indicates the scalar condensate $\Phi$ is “real” (R) or “pseudo-real” (PR). The third row indicates the Dirac gamma matrices are real or purely imaginary. The table is mod 8 implied by Bott’s periodicity. However, $d = 0$ is exceptional with possible real configurations of R(Im), R(Re) (omitted) and PR(Im) (omitted), but all classified by $\Pi_0(R_0^0) = \mathbb{Z}_2$. See the discussions in the main text for details.
Bulk/edge correspondence and AdS/CFT

- **Bulk/edge correspondence**
  - an idea that relates the topological properties of the bulk to the number of gapless modes on the boundary
  - examples: quantum Hall effect

![Diagram of quantum Hall effect](image)


the difference between right moving and left moving mode is fixed by the bulk topology information, here is the change of the Chern number across the boundary.
• Holographic fermions

\[ S_f = \int d^{d+1}x \sqrt{-g} \ i(\bar{\psi} \slashed{D} \psi - \Lambda \bar{\psi} \Phi \psi - m \bar{\psi} \psi) \]

To have a well-defined variation principle, the boundary term is added

\[ \pm \ i \int d^d x \sqrt{-\gamma} \ \bar{\psi} \psi = \pm \ i \int d^d x \sqrt{-\gamma} (\bar{\psi}_+ \psi_- + \bar{\psi}_- \psi_+) . \]

\[ \psi := \psi_+ + \psi_- \quad \Gamma^r \psi_\pm = \pm \psi_\pm \]

corresponds to impose Dirichlet boundary condition on (standard and alternative quantization scheme)

\[ \psi_+ \text{ for } m > 1/2 \]
\[ \psi_- \text{ for } m < -1/2 \]

Here we only consider the AdS metric \( ds^2 = r^2 (-dt^2 + \delta_{ab} dx^a dx^b) + \frac{dr^2}{r^2} \).
The equation of motion in Hamiltonian form:

\[ h\psi = \omega \psi \]

\[ h = i \Gamma^0 \left[ r^2 \Gamma_r \partial_r - r (\Gamma^a \partial_a + m + \Lambda \Phi) \right] + i \frac{d}{2} r \Gamma^0 \Gamma^r \]

1. interpreted as the energy of the boundary fermion theory by the holographic dictionary

2. solution used to construct the Green’s function of boundary fermion operator

Q: CFT is a gapless system by definition, its dual (AdS space) should also be a gapless system. How do we use K-theory to classify a gapless (bulk) system?
• AdS space as a topological insulator

1. Topological orders can be characterized by gapless boundary modes

2. Cosmological constant as an external field which breaks the translational invariance along radial direction

   create a co-dimensional one boundary defect

3. The Dirichlet condition imposed in AdS/CFT picks up the non-normalizable zero modes \((\omega = 0)\).

   localized mode on AdS boundary

4. These non-normalizable modes corresponds to dual CFT operators, which should be topological sector of CFT.

5. Bulk/edge correspondence:

   edge gapless mode \(\longleftrightarrow\) topological properties of the bulk

   here the bulk/edge correspondence is manifest

   the defect is classified by \(\pi_{d-1-d_b}(\mathcal{M})\), \(d_b\) is the spatial dimension of the defect

   here \(d_b = d - 1\)

   \(\rightarrow\) the defect is classified by \(\pi_0(\mathcal{M})\). \(\rightarrow\) also the classification of bulk topological phases
FIG. 2: A sketch of the idea of viewing the Minkowski and AdS spaces as topological insulators related by turning on the “external field”, here the cosmological constant. The symbols in the figure are some topological excitations. When there is no cosmological constant present, the non-trivial topological ordered phases (hence those topological excitations) can be created by adding fermions in the Minkowski bulk, which is a gapped system and can be classified by the K-theory. When we turn on the cosmological constant, the Minkowski space turns into the AdS space. Imposing the Dirichlet boundary condition on the non-normalizable modes as requested by AdS/CFT correspondence, these topological excitations are naturally localized on the boundary. Hence we can treat the AdS space as a topological insulator with the UV boundary as a defect on which the dual CFTs live. Due to the bulk/edge correspondence, classifying these boundary excitations is equivalent to classifying the bulk topological phases in Minkowski space by K-theory.

information about the bulk topological phases hidden in the mass matrix
• Holographic fermions and eigenvalue of mass matrix

zero mode equation

\[
\left( r^2 \Gamma^r \partial_r - r(\Gamma^a \partial_a + m + \Lambda \Phi) + \frac{d}{2} r \Gamma^r \right) \psi_0 = 0 \quad (\omega = 0)
\]

\[
\psi = \psi_+ + \psi_- \quad \text{with} \quad \Gamma^r \psi_\pm = \pm \psi_\pm
\]

the asymptotic behavior of the solution:

\[
\psi_+(r, \vec{k}) = A(\vec{k}) r^{-\frac{d}{2} - m} + B(\vec{k}) r^{-\frac{d}{2} - m - 1}, \\
\psi_-(r, \vec{k}) = C(\vec{k}) r^{-\frac{d}{2} + m - 1} + D(\vec{k}) r^{-\frac{d}{2} - m},
\]

conformal dimension of dual operator

1. we consider many fermion flavors \( m \) becomes a matrix

2. for topological consideration, only the sign of the eigenvalue of \( m \) relevant

\[
m^2 = 1
\]

3. \( m = 1 \), \( \psi_+ \) non-normalizable, \( \psi_- \) normalizable

4. \( m = -1 \), \( \psi_- \) non-normalizable, \( \psi_+ \) normalizable

which to choose?
• Standard description of AdS/CFT tells us which pattern is localized on the boundary

How?

by fixing the boundary action, “standard” or the “alternative” quantization schemes (Dirichlet boundary condition imposed on the (non-normalizable) mode)

e.g. for \( m=1 \), we chose standard quantization scheme

\[ \text{Dirichlet BC on } \psi_+ \longrightarrow \text{identify as a localized mode} \]

• Once the boundary action is chosen, we know which (zero) modes localized on the boundary because we know the eigenvalues of \( m \).

• These edge modes are topological since \( m \) encodes the topological information of the bulk phases.

• To consider the real fermions, we just make sure \( \psi_\pm \) is also real.

\[ \text{Majorana (real) representation } \Gamma^\mu \text{ & real } \Gamma^r \]
<table>
<thead>
<tr>
<th>\text{AdS}_{d+1}/\text{CFT}_d</th>
<th>\text{CFT}_1</th>
<th>\text{CFT}_2</th>
<th>\text{CFT}_3</th>
<th>\text{CFT}_4</th>
<th>\text{CFT}_5</th>
<th>\text{CFT}_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(s_C, s_T, s_{TC})$</td>
<td>$(+, +, +)$</td>
<td>$(+, \times, \times)$</td>
<td>$(+, -, +)$</td>
<td>$(\times, -, \times)$</td>
<td>$(+, +, +)$</td>
<td>$(+, \times, \times)$</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R/PR</td>
<td>PR</td>
<td>PR</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>$R_2^0$</td>
<td>$R_2^0$</td>
<td>$R_2^0$</td>
<td>$C_5$</td>
<td>$R_6^0$</td>
<td>$R_6^0$</td>
</tr>
<tr>
<td>$\pi_0(\mathcal{M})$</td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
<td>0</td>
<td>Z</td>
<td>Z</td>
</tr>
</tbody>
</table>

\textbf{TABLE V}: Classification of symmetry-protected topological order of real fermions in holographic CFTs. The letters R and PR denote as “real” $\Phi$ and “pseudo-real” $\Phi$, respectively. We restrict ourselves to $d \leq 6$ since AdS space with dimension higher than this may not be stable due to the large back reaction from the perturbations.
• Summary

1. start with Minkowski bulk free fermion (gapped system)

2. turn on cosmological constant: Minkowski $\rightarrow$ AdS

3. solution of Dirac equation in AdS space

$$\psi_+(r, \vec{k}) = A(\vec{k})r^{-\frac{d}{2}+m} + B(\vec{k})r^{-\frac{d}{2}-m-1}, \quad \psi_-(r, \vec{k}) = C(\vec{k})r^{-\frac{d}{2}+m-1} + D(\vec{k})r^{-\frac{d}{2}-m}.$$ 

note $m$ are eigenvalues of mass matrix $(m + \Lambda \Phi)$ with $(m + \Lambda \Phi)^2 = 1$

4. the non-normalizable part of solutions can be viewed as localized modes on the AdS time-like boundary $\rightarrow$ localized edge mode

5. Bulk/edge correspondence tells us the number of localized gapless modes should be encoded in the pattern of mass matrix (topological info)

6. $m \leftrightarrow -m, \quad \psi_+ \leftrightarrow \psi_-$ how do we know which should be localized mode?

   the quantization scheme in AdS/CFT tells us how many localized boundary mode are 
   e.g. standard quantization is chosen $\psi_+$ is picked up (for $m=1$)

7. number of positive/negative eigenvalues of mass matrix = number of localized edge modes

   manifest bulk/edge correspondence

8. in AdS/CFT description, these edge modes act as sources coupled to the dual CFT fermionic operators. These fermionic operators should correspond to the topological sector of dual CFT
• Discussions

1. The massive free fermions of many flavors in AdS space as a topological insulator with a co-dimensional one defect.

2. The non-normalizable (zero) modes picked up by the AdS/CFT description localized on the AdS boundary, which corresponds to dual CFT operators and should be topological sector of CFT.

3. Generally, we only know how to classify gapped systems. With the help of AdS/CFT and the bulk/edge correspondence, this scenario can be used to classify SPT of holographic CFTs.

Gapped bulk phase in Minkowski space

↑

edge modes on the AdS boundary

↓

gapless system (holographic CFTs)