Chapter 29

Magnetic Fields Due to Currents

37. Figure 29-62 shows a cross section across a diameter of a long cylindrical conductor of radius $a = 2.00$ cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?

Ampere’s Law

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\text{enc}} \]

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \int B \cos \theta \, ds = B \int ds = B(2\pi r). \]

\[ B(2\pi r) = \mu_0 i \quad B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire)}. \]
\[ B(2 \pi r) = \mu_0 \frac{\pi r^2}{\pi a^2} i \]

\[ B = \mu_0 i r / 2\pi a^2 \quad ... r < a \]

\[ B = \mu_0 i / 2\pi r \quad ... r > a \]

(a) At \( r = 0 \), \( B = 0 \).

(b) At \( r = 0.0100 \text{m} \), \( B = \frac{\mu_0 i r}{2\pi a^2} = \frac{(4 \times 10^{-7} \text{T} \cdot \text{m/A})(170 \text{A})(0.0100 \text{m})}{2\pi (0.0200 \text{m})^2} = 8.50 \times 10^{-4} \text{T} \).

(c) At \( r = a = 0.0200 \text{m} \), \( B = \frac{\mu_0 i r}{2\pi a^2} = \frac{(4 \times 10^{-7} \text{T} \cdot \text{m/A})(170 \text{A})(0.0200 \text{m})}{2\pi (0.0200 \text{m})^2} = 1.70 \times 10^{-3} \text{T} \).

(d) At \( r = 0.0400 \text{m} \), \( B = \frac{\mu_0 i r}{2\pi r} = \frac{(4 \times 10^{-7} \text{T} \cdot \text{m/A})(170 \text{A})}{2\pi (0.0400 \text{m})} = 8.50 \times 10^{-4} \text{T} \).

**39** The current density \( \vec{I} \) inside a long, solid, cylindrical wire of radius \( a = 3.1 \text{ mm} \) is in the direction of the central axis, and its magnitude varies linearly with radial distance \( r \) from the axis according to \( J = J_0 r / a \), where \( J_0 = 310 \text{ A/m}^2 \). Find the magnitude of the magnetic field at (a) \( r = 0 \), (b) \( r = a / 2 \), and (c) \( r = a \).

For \( r \leq a \),

\[ B(r) = \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0}{2\pi} \int J(r) 2\pi r dr = \frac{\mu_0}{2\pi} \int_0^a J_0 \left( \frac{r}{a} \right) 2\pi r dr = \frac{\mu_0 J_0 r^2}{3a} \]

(a) At \( r = 0 \), \( B = 0 \).

(b) At \( r = a / 2 \), we have
\[ B(r) = \frac{\mu_0 J_0 r^2}{3a} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(310 \text{ A/m}^2)(3.1 \times 10^{-3} \text{ m}/2)^2}{3(3.1 \times 10^{-3} \text{ m})} = 1.0 \times 10^{-7} \text{ T}. \]

(c) At \( r = a \),

\[ B(r = a) = \frac{\mu_0 J_0 a}{3} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(310 \text{ A/m}^2)(3.1 \times 10^{-3} \text{ m})}{3} = 4.0 \times 10^{-7} \text{ T}. \]

**45** A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA. A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at 45.0° to the axial direction? (b) What is the magnitude of the magnetic field there?

\[ B = \mu_0 i_n \quad \text{(ideal solenoid)}. \]
(a)

\[ B_s = \mu_0 i_s n = B_w = \frac{\mu_0 i_w}{2\pi d} , \]

\[ d = \frac{i_w}{2\pi i_s n} = \frac{6.00 \text{ A}}{2\pi (20.0 \times 10^{-3} \text{ A}) (10 \text{ turns/cm})} = 4.77 \text{ cm} . \]

(b) The magnetic field strength is

\[ B = \sqrt{2} B_s = \sqrt{2} \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) (20.0 \times 10^{-3} \text{ A}) (10 \text{ turns} / 0.0100 \text{ m}) = 3.55 \times 10^{-5} \text{ T} . \]

67 A long, hollow, cylindrical conductor (inner radius 2.0 mm, outer radius 4.0 mm) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is co-axial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm, (b) 3.0 mm, and (c) 5.0 mm from the central axis of the wire and cylinder?

(a)

\[ i_w = 24 \text{ A} \text{ and } r = 0.0010 \text{ m} . \]

\[ |\vec{B}| = \frac{\mu_0 i_w}{2\pi r} = 4.8 \times 10^{-3} \text{ T} \]
(b)

\[ r = 0.0030 \text{ m}, \quad R_i = 0.0020 \text{ m}, \quad R_o = 0.0040 \text{ m} \text{ and } i_c = 24 \text{ A.} \]

\[
\left| \bar{B} \right| = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_{\text{c}}}{2\pi r} \left( \frac{\pi r^2 - \pi R_i^2}{\pi R_o^2 - \pi R_i^2} \right)
\]

\[
\left| \bar{B} \right| = 9.3 \times 10^{-4} \text{ T.}
\]

(c)

since \( i_c = i_w \rightarrow B = 0 \)
Chapter 30

Induction and Inductance

Problem 5. In Fig. 30-37, a 120-turn coil of radius 1.8 cm and resistance 5.3 Ω is coaxial with a solenoid of 220 turns/cm and diameter 3.2 cm. The solenoid current drops from 1.5 A to zero in time interval Δt = 25 ms. What current is induced in the coil during Δt?

\[ ε = -N \frac{dΦ_B}{dt} = -NA \left( \frac{dB}{dt} \right) = -NA \frac{d}{dt} (μ_0ni) \]

\[ = -Nμ_0nA \frac{di}{dt} = -Nμ_0n(πr^2) \frac{di}{dt} = 0.16 \text{ V.} \]

\[ i = |ε| / R = 0.016 \text{ V} / 5.3Ω = 0.030 \text{ A}. \]
A solenoid that is 85.0 cm long has a cross-sectional area of 17.0 cm². There are 950 turns of wire carrying a current of 6.60 A. (a) Calculate the energy density of the magnetic field inside the solenoid. (b) Find the total energy stored in the magnetic field there (neglect end effects).

(a)

\[ u_B = \frac{B^2}{2\mu_0}, \quad B = \mu_0ni, \]

\[ n = \frac{950 \text{ turns}}{0.850 \text{ m}} = 1.118 \times 10^3 \text{ m}^{-1}. \]

\[ u_n = \frac{1}{2} \mu_0 n^2 i^2 = \frac{1}{2} (4\pi \times 10^{-7} \text{T} \cdot \text{m/A}) (1.118 \times 10^3 \text{ m}^{-1})^2 (6.60 \text{ A})^2 = 34.2 \text{ J/m}^3. \]

(b)

\[ U_B = (34.2 \text{ J/m}^3)(17.0 \times 10^{-4} \text{ m}^2)(0.850 \text{ m}) = 4.94 \times 10^{-2} \text{ J}. \]

At time \( t = 0 \), a 12.0 V potential difference is suddenly applied to the leads of a coil of inductance 23.0 mH and a certain resistance \( R \). At time \( t = 0.150 \text{ ms} \), the current through the inductor is changing at the rate of 280 A/s. Evaluate \( R \).

\[ L = \frac{N\Phi_B}{i}, \quad \varepsilon_L = -\frac{d(N\Phi_B)}{dt}, \quad \varepsilon_L = -L \frac{di}{dt} \]

\[ \frac{di}{dt} + Ri = \varepsilon \]
\[\tau_L = L/R \text{ (Eq. 30-42), } L = 0.023 \text{ H}\]

\[\epsilon = 12 \text{ V}, \quad t = 0.00015 \text{ s}, \quad \frac{di}{dt} = 280 \text{ A/s,}\]

\[i = \frac{\epsilon}{R} \left(1 - e^{-t/\tau_L}\right) \quad \text{(rise of current).}\]

\[\frac{di}{dt} = \left(\frac{\epsilon}{R\tau_L}\right) e^{-t/\tau_L} = \left(\frac{\epsilon}{L}\right) e^{-t/\tau_L}.\]

\[\Rightarrow R = 95.4 \Omega.\]
Switch S in Fig. 30-80 is closed for \( t < 0 \) and is opened at \( t = 0 \). When current \( i_1 \) through \( L_1 \) and current \( i_2 \) through \( L_2 \) are first equal to each other, what is their common value? (The resistors have the same resistance \( R \)).

\[
\begin{align*}
\text{t<0} & \quad \Rightarrow \quad i_2 = 0 \quad \text{and} \quad i_1 = \frac{\varepsilon}{R}. \\
\text{t=0+} & \quad \Rightarrow \quad \varepsilon - i_1 R - L_1 \frac{di_1}{dt} - i_2 R - L_2 \frac{di_2}{dt} = 0, \\
\text{when} \ i_1 & \sim i_2 \quad \Rightarrow \\
L_1 \frac{di_1}{dt} & \approx -L_2 \frac{di_2}{dt}. \\
\frac{di_1}{dt} & \approx \frac{\Delta i_1}{\Delta t} = \frac{i - \varepsilon/R}{\Delta t}; \quad \frac{di_2}{dt} \approx \frac{\Delta i_2}{\Delta t} = \frac{i - 0}{\Delta t}. \\
L_1 \left( i - \frac{\varepsilon}{R} \right) & = -L_2 (i - 0) \Rightarrow i = \frac{\varepsilon L_1}{L_2 R_1 + L_1 R_2} = \frac{L_1}{L_1 + L_2} \frac{\varepsilon}{R}. 
\end{align*}
\]