Chapter 21

Electric Charge

Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.0360 N. Of the initial charges on the spheres, with a positive net charge, what was (a) the negative charge on one of them and (b) the positive charge on the other?

Ans:

\[ F_a = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = -k \frac{q_1 q_2}{r^2} \]

\[ q_1 q_2 = -\frac{r^2 F_a}{k} = -\frac{(0.500 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \times \frac{0.108 \text{ N}}{3.00 \times 10^{-12} \text{ C}^2} = -3.00 \times 10^{-12} \text{ C}^2 . \]

\[ F_b = \frac{1}{4\pi\epsilon_0} \frac{\left(\frac{q_1 + q_2}{2}\right)^2}{r^2} = k \frac{(q_1 + q_2)^2}{4r^2} . \]

\[ q_1 + q_2 = 2r \sqrt{\frac{F_b}{k}} = 2(0.500 \text{ m}) \sqrt{\frac{0.0360 \text{ N}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 2.00 \times 10^{-6} \text{ C} \]
\[ q_2 = \frac{-\left(3.00 \times 10^{-12} \text{ C}^2\right)}{q_1} \]

\[ q_1 - \frac{3.00 \times 10^{-12} \text{ C}^2}{q_1} = 2.00 \times 10^{-6} \text{ C}. \]

\[ q_1^2 - \left(2.00 \times 10^{-6} \text{ C}\right)q_1 - 3.00 \times 10^{-12} \text{ C}^2 = 0. \]

\[ q_1 = \frac{2.00 \times 10^{-6} \text{ C} \pm \sqrt{(-2.00 \times 10^{-6} \text{ C})^2 - 4(-3.00 \times 10^{-12} \text{ C}^2)}}{2}. \]

(a) Using \( q_2 = (-3.00 \times 10^{-12})/q_1 \) with \( q_1 = 3.00 \times 10^{-6} \text{ C} \),

we get \( q_2 = -1.00 \times 10^{-6} \text{ C} \).

(b) If we instead work with the \( q_1 = -1.00 \times 10^{-6} \text{ C} \) root,

then we find \( q_2 = 3.00 \times 10^{-6} \text{ C} \).
shell?

Problem 20

In Fig. 21-29, particles 1 and 2 of charge $q_1 = q_2 = +3.20 \times 10^{-19}$ C are on a y axis at distance $d = 17.0$ cm from the origin. Particle 3 of charge $q_3 = +6.40 \times 10^{-19}$ C is moved gradually along the x axis from $x = 0$ to $x = +5.0$ m. At what values of x will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?

Ans:

$$
\cos \theta = \frac{x}{\sqrt{x^2 + d^2}} .
$$

$$
F_{net} = 2F \cos \theta = \frac{2(2e)(4e)}{4\pi \varepsilon_0 (x^2 + d^2)} \frac{x}{\sqrt{x^2 + d^2}} = \frac{4e^2 x}{\pi \varepsilon_0 (x^2 + d^2)^{3/2}} .
$$

$$
F' = 0 = \frac{1}{(x^2 + d^2)^{3/2}} - \frac{3}{2} \frac{x \times 2x}{(x^2 + d^2)^{5/2}}
$$

$$
\Rightarrow 2(x^2 + d^2) - 3x^2 = 0 \Rightarrow x = \frac{d}{\sqrt{2}}
$$
In crystals of the salt cesium chloride, cesium ions $\text{Cs}^+$ form the eight corners of a cube and a chlorine ion $\text{Cl}^-$ is at the cube’s center (Fig. 21-32). The edge length of the cube is 0.40 nm. The $\text{Cs}^+$ ions are each deficient by one electron (and thus each has a charge of $+e$), and the $\text{Cl}^-$ ion has one excess electron (and thus has a charge of $-e$). (a) What is the magnitude of the net electrostatic force exerted on the $\text{Cl}^-$ ion by the eight $\text{Cs}^+$ ions at the corners of the cube? (b) If one of the $\text{Cs}^+$ ions is missing, the crystal is said to have a defect; what is the magnitude of the net electrostatic force exerted on the $\text{Cl}^-$ ion by the seven remaining $\text{Cs}^+$ ions? 

\[ F = 0 \]

\[ d' = \left(\sqrt{3}/2\right)a. \]

\[ F = k \frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = 1.9 \times 10^{-9} \text{ N}. \]
Three charged particles form a triangle: particle 1 with charge $Q_1 = 80.0 \, \text{nC}$ is at $xy$ coordinates $(0, 3.00 \, \text{mm})$, particle 2 with charge $Q_2$ is at $(0, -3.00 \, \text{mm})$, and particle 3 with charge $q = 18.0 \, \text{nC}$ is at $(4.00 \, \text{mm}, 0)$. In unit-vector notation, what is the electrostatic force on particle 3 due to the other two particles if $Q_2$ is equal to (a) $80.0 \, \text{nC}$ and (b) $-80.0 \, \text{nC}$?

Ans:

$$|\vec{F}_{31}| = k \frac{q_3 |q_1|}{r_{31}^2} \quad \text{and} \quad |\vec{F}_{32}| = k \frac{q_3 q_2}{r_{32}^2}.$$

$r_{31} = r_{32} = 0.005 \, \text{m}.$

(a) $F = 0.518 \times (4/5) \times 2 \sim 0.829 \, \text{N}$ (i)

(b) $F = 0.518 \times (-3/5) \times 2 \sim 0.621 \, \text{N}$ (-j)
Chapter 22

Electric Fields

**Problem 19** Figure 22-38 shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole’s electric field at point $P$, located at distance $r \gg d$?

Ans:

(a)

\[
|\vec{E}_{\text{net}}| = 2E_1 \sin \theta = 2 \left[ \frac{1}{4\pi \varepsilon_0} \frac{q}{(d/2)^2 + r^2} \right] \frac{d/2}{\sqrt{(d/2)^2 + r^2}}
\]
\[
\frac{1}{4\pi \varepsilon_0} \frac{qd}{(d/2)^2 + r^2}^{3/2}
\]

\(r \gg d\), we write \([(d/2)^2 + r^2]^{3/2} \approx r^3\)

\(|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi \varepsilon_0} \frac{qd}{r^3}\).

(b) \(-J\)

**Sec. 22-7 The Electric Field Due to a Charged Disk**

*30* A disk of radius 2.5 cm has a surface charge density of 5.3 \(\mu\)C/m² on its upper face. What is the magnitude of the electric field produced by the disk at a point on its central axis at distance \(z = 12\) cm from the disk?

**Ans:**

\[
E = \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)
\]

\[
= \frac{5.3 \times 10^{-6} \text{ C/m}^2}{2 \left(8.85 \times 10^{-12} \text{ C}^2/N\cdot\text{m}^2\right)} \left[1 - \frac{12\text{ cm}}{\sqrt{(12\text{ cm})^2 + (2.5\text{ cm})^2}}\right]
\]

\[
= 6.3 \times 10^3 \text{ N/C}.
\]
**33** Suppose you design an apparatus in which a uniformly charged disk of radius \( R \) is to produce an electric field. The field magnitude is most important along the central perpendicular axis of the disk, at a point \( P \) at distance \( 2.00R \) from the disk (Fig. 22-47a). Cost analysis suggests that you switch to a ring of the same outer radius \( R \) but with inner radius \( R/2.00 \) (Fig. 22-47b). Assume that the ring will have the same surface charge density as the original disk. If you switch to the ring, by what percentage will you decrease the electric field magnitude at \( P \)?

\[
E_{(a)} = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{2R}{\sqrt{(2R)^2 + R^2}} \right).
\]

\[
E_{(b)} = E_{(a)} - \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{2R}{\sqrt{(2R)^2 + (R/2)^2}} \right)
\]

\[
\frac{E_{(a)} - E_{(b)}}{E_{(a)}} = \frac{1 - \frac{2}{\sqrt{4 + \frac{1}{4}}}}{1 - \frac{2}{\sqrt{4 + 1}}} = 0.283
\]

Approximately 28%
An electric dipole consisting of charges of magnitude 1.50 nC separated by 6.20 μm is in an electric field of strength 1100 N/C. What are (a) the magnitude of the electric dipole moment and (b) the difference between the potential energies for dipole orientations parallel and antiparallel to \( \vec{E} \)?

Ans:

(a)

\[ p = qd = (1.50 \times 10^{-9} \text{ C})(6.20 \times 10^{-6} \text{ m}) = 9.30 \times 10^{-15} \text{ C} \cdot \text{m}. \]

(b)

\[ U(180°) - U(0) = 2pE = 2(9.30 \times 10^{-15})(1100) = 2.05 \times 10^{-11} \text{ J}. \]
Chapter 23

Gauss’ Law

**27** Figure 23-37 is a section of a conducting rod of radius \( R_1 = 1.30 \text{ mm} \) and length \( L = 11.00 \text{ m} \) inside a thin-walled coaxial conducting cylindrical shell of radius \( R_2 = 10.0 R_1 \) and the (same) length \( L \). The net charge on the rod is \( Q_1 = +3.40 \times 10^{-12} \text{ C} \); that on the shell is \( Q_2 = -2.00 Q_1 \). What are the (a) magnitude \( E \) and (b) direction (radially inward or outward) of the electric field at radial distance \( r = 2.00 R_2 \)? What are (c) \( E \) and (d) the direction at \( r = 5.00 R_1 \)? What is the charge on the (e) interior and (f) exterior surface of the shell?

**Ans:** (a)

The flux through the surface is \( \Phi = 2\pi r L E \).

\[
q_{\text{enc}} = Q_1 + Q_2 = -Q_1 = -3.40 \times 10^{-12} \text{ C}.
\]

\[
E = \frac{q_{\text{enc}}}{2\pi \varepsilon_0 L r}
\]

\[
= \frac{-3.40 \times 10^{-12} \text{ C}}{2\pi (8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)(11.0 \text{ m})(20.0 \times 1.30 \times 10^{-3} \text{ m})} = -0.214 \text{ N/C},
\]
(b) The negative sign in $E$ indicates that the field points inward.

(c) Next, for $r = 5.00 \text{ R}_1$, the charge enclosed by the Gaussian surface is $q_{enc} = Q_1 = 3.40 \times 10^{-12} \text{ C}$. Consequently, Gauss' law yields $2\pi r^2 \varepsilon_0 LE = q_{enc}$, or

$$E = \frac{q_{enc}}{2\pi \varepsilon_0 Lr} = \frac{3.40 \times 10^{-12} \text{ C}}{2\pi (8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2) (11.0 \text{ m})(5.00 \times 1.30 \times 10^{-2} \text{ m})} = 0.855 \text{ N/C}.$$  

(d) The positive sign indicates that the field points outward.

(e) We consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss’ law). Since the central rod has charge $Q_1$, the inner surface of the shell must have charge $Q_m = -Q_1 = -3.40 \times 10^{-12} \text{ C}$.

(f) Since the shell is known to have total charge $Q_2 = -2.00 Q_1$, it must have charge $Q_{out} = Q_2 = Q_m = Q_1 = 3.40 \times 10^{-12} \text{ C}$ on its outer surface.
**35** In Fig. 23-41, two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge densities of opposite signs and magnitude $7.00 \times 10^{-22} \text{ C/m}^2$. In unit-vector notation, what is the electric field at points (a) to the left of the plates, (b) to the right of them, and (c) between them?

**Fig. 23-41** Problem 35.

**Ans:**

(a) To the left of the plates:

$$\vec{E} = \left( \frac{\sigma}{2\varepsilon_0} \right) \hat{i} \text{ (from the right plate)} + \left( \frac{\sigma}{2\varepsilon_0} \right) \hat{i} \text{ (from the left one)} - 0.$$

(b) To the right of the plates:

$$\vec{E} = \left( \frac{\sigma}{2\varepsilon_0} \right) \hat{i} \text{ (from the right plate)} + \left( \frac{\sigma}{2\varepsilon_0} \right) (-\hat{i}) \text{ (from the left one)} = 0.$$

(c) Between the plates:

$$\vec{E} = \left( \frac{\sigma}{2\varepsilon_0} \right) (-\hat{i}) + \left( \frac{\sigma}{2\varepsilon_0} \right) (-\hat{i}) = \left( \frac{\sigma}{\varepsilon_0} \right) (-\hat{i}) = -\left( \frac{7.00 \times 10^{-22} \text{ C/m}^2}{8.85 \times 10^{-12} \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}} \right) \hat{i} = (-7.91 \times 10^{-11} \text{ N/C}) \hat{i}.$$
Figure 23-52 shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius $R$. Point $P$ lies on a line connecting the centers of the spheres, at radial distance $R/2.00$ from the center of sphere 1. If the net electric field at point $P$ is zero, what is the ratio $q_2/q_1$ of the total charge $q_2$ in sphere 2 to the total charge $q_1$ in sphere 1?

**Ans:**

\[
E_1 = \frac{|q_1|}{4\pi \varepsilon_0 R^3} r_1 = \frac{|q_1|}{4\pi \varepsilon_0 R^3} \left(\frac{R}{2}\right) = \frac{|q_1|}{8\pi \varepsilon_0 R^2} .
\]

\[
E_2 = \frac{|q_2|}{4\pi \varepsilon_0 r^2} = \frac{|q_2|}{4\pi \varepsilon_0 (1.5 R)^2} .
\]

\[
\frac{q_2}{q_1} = \frac{9}{8} = 1.125.
\]
A charge distribution that is spherically symmetric but not uniform radially produces an electric field of magnitude \( E = Kr^2 \), directed radially outward from the center of the sphere. Here \( r \) is the radial distance from that center, and \( K \) is a constant. What is the volume density \( \rho \) of the charge distribution?

**Ans:**

\[
E(r) = \frac{q_{\text{enc}}}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0 r^2} \int_0^r \rho(r') 4\pi r'^2 dr' 
\]

\[
\rho(r) = \frac{\varepsilon_0}{r^2} \frac{d}{dr} \left[ r^2 E(r) \right] = \frac{\varepsilon_0}{r^2} \frac{d}{dr} \left( Kr^6 \right) = 6K\varepsilon_0 r^3. 
\]
Chapter 24

Electric Potential

24 In Fig. 24-40, what is the net electric potential at the origin due to the circular arc of charge $Q_1 = +7.21 \text{ pC}$ and the two particles of charges $Q_2 = 4.00Q_1$ and $Q_3 = -2.00Q_1$? The arc’s center of curvature is at the origin and its radius is $R = 2.00 \text{ m}$; the angle indicated is $\theta = 20.0^\circ$.

Ans:

$$V = \frac{1}{4\pi\varepsilon_0} \frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0} \frac{+4Q_1}{2R} + \frac{1}{4\pi\varepsilon_0} \frac{-2Q_1}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{R}$$

$$= \frac{(8.99 \times 10^9)(7.21 \times 10^{-12})}{2.00} = 3.24 \times 10^{-2} \text{ V.}$$
41. In the rectangle of Fig. 24-49, the sides have lengths 5.0 cm and 15 cm, \( q_1 = -5.0 \ \mu\text{C} \), and \( q_2 = +2.0 \ \mu\text{C} \). With \( V = 0 \) at infinity, what is the electric potential at (a) corner A and (b) corner B? (c) How much work is required to move a charge \( q_3 = +3.0 \ \mu\text{C} \) from B to A along a diagonal of the rectangle? (d) Does this work increase or decrease the electric potential energy of the three-charge system? Is more, less, or the same work required if \( q_3 \) is moved along a path that is (e) inside the rectangle but not on a diagonal and (f) outside the rectangle?

**Fig. 24-49** Problem 41.

**Ans:**

(a)

\[
V_A = \frac{1}{4\pi\varepsilon_0} \left[ \frac{q_1}{\ell} + \frac{q_2}{w} \right] = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \left[ \frac{-5.0 \times 10^{-6} \text{ C}}{0.15 \text{ m}} + \frac{2.0 \times 10^{-6} \text{ C}}{0.050 \text{ m}} \right] = 6.0 \times 10^4 \text{ V.}
\]

(b)

\[
V_B = \frac{1}{4\pi\varepsilon_0} \left[ \frac{q_1}{\ell} + \frac{q_2}{w} \right] = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \left[ \frac{-5.0 \times 10^{-6} \text{ C}}{0.050 \text{ m}} + \frac{2.0 \times 10^{-6} \text{ C}}{0.15 \text{ m}} \right] = -7.8 \times 10^4 \text{ V.}
\]

(c)

\[
W = U_A - U_B = q_3(V_A - V_B) = (3.0 \times 10^{-6} \text{ C})(6.0 \times 10^4 \text{ V} + 7.8 \times 10^4 \text{ V}) = 2.5 \text{ J.}
\]

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.
*53* What is the excess charge on a conducting sphere of radius \( r = 0.15 \) m if the potential of the sphere is 1500 V and \( V = 0 \) at infinity?  

Ans:

\[
V = \frac{q}{4\pi \varepsilon_0 r},
\]

\[
q = 4\pi \varepsilon_0 r V = \frac{(0.15 \text{ m})(1500 \text{ V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 2.5 \times 10^{-8} \text{ C}.
\]
Chapter 25
Capacitance

17. In Fig. 25-33, the capacitances are \( C_1 = 1.0 \ \mu F \) and \( C_2 = 3.0 \ \mu F \), and both capacitors are charged to a potential difference of \( V = 100 \ \text{V} \) but with opposite polarity as shown. Switches \( S_1 \) and \( S_2 \) are now closed. (a) What is now the potential difference between points \( a \) and \( b \)? What now is the charge on capacitor (b) 1 and (c) 2? [SSM WWW]

Ans:

(a)

\[
q_1 = C_1 V = \left( 1.0 \times 10^{-6} \ \text{F} \right) \left( 100 \ \text{V} \right) = 1.0 \times 10^{-4} \ \text{C} \\
q_2 = C_2 V = \left( 3.0 \times 10^{-6} \ \text{F} \right) \left( 100 \ \text{V} \right) = 3.0 \times 10^{-4} \ \text{C},
\]

the net charge on the combination is \( 3.0 \times 10^{-4} \ \text{C} - 1.0 \times 10^{-4} \ \text{C} = 2.0 \times 10^{-4} \ \text{C} \).

\[
C_{\text{eq}} = C_1 + C_2 = 4.0 \times 10^{-6} \ \text{F}.
\]

\[
V_{ab} = \frac{2.0 \times 10^{-4} \ \text{C}}{4.0 \times 10^{-6} \ \text{F}} = 50 \ \text{V}.
\]
(b) The charge on capacitor 1 is now \( q_1 = C_1 V_{ab} = (1.0 \times 10^{-6} \text{ F})(50 \text{ V}) = 5.0 \times 10^{-5} \text{ C}. \)

(c) The charge on capacitor 2 is now \( q_2 = C_2 V_{ab} = (3.0 \times 10^{-6} \text{ F})(50 \text{ V}) = 1.5 \times 10^{-4} \text{ C}. \)

29 Assume that a stationary electron is a point of charge. What is the energy density \( u \) of its electric field at radial distances (a) \( r = 1.00 \text{ mm} \), (b) \( r = 1.00 \mu \text{m} \), (c) \( r = 1.00 \text{ nm} \), and (d) \( r = 1.00 \text{ pm} \)? (e) What is \( u \) in the limit as \( r \to 0 \)?

**Ans:**

The energy per unit volume is

\[
  u = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \left( \frac{e}{4 \pi \varepsilon_0 r^2} \right)^2 = \frac{e^2}{32 \pi^2 \varepsilon_0 r^4}.
\]

(a) At \( r = 1.00 \times 10^{-3} \text{ m} \), with \( e = 1.60 \times 10^{-19} \text{ C} \) and \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 \), we have \( u = 9.16 \times 10^{-18} \text{ J/m}^3 \).

(b) Similarly, at \( r = 1.00 \times 10^{-6} \text{ m} \), \( u = 9.16 \times 10^{-6} \text{ J/m}^3 \).

(c) at \( r = 1.00 \times 10^{-9} \text{ m} \), \( u = 9.16 \times 10^6 \text{ J/m}^3 \), and

(d) at \( r = 1.00 \times 10^{-12} \text{ m} \), \( u = 9.16 \times 10^{10} \text{ J/m}^3 \).

(e) From the expression above \( u \propto r^{-4} \). Thus, for \( r \to 0 \), the energy density \( u \to \infty \).
A certain parallel-plate capacitor is filled with a dielectric for which $\kappa = 5.5$. The area of each plate is 0.034 m$^2$, and the plates are separated by 2.0 mm. The capacitor will fail (short out and burn up) if the electric field between the plates exceeds 200 kN/C. What is the maximum energy that can be stored in the capacitor?

Ans:

$\sigma = q/A$, we have

$$|\vec{E}| = \frac{q}{\kappa \varepsilon_0 A} = 200 \times 10^3 \text{ N/C}$$

$q = 3.3 \times 10^{-7} \text{ C}$.

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa \varepsilon_0 A} = 6.6 \times 10^{-5} \text{ J}.$$
A common flashlight bulb is rated at 0.30 A and 2.9 \text{ V} \ (the \ values \ of \ the \ current \ and \ voltage \ under \ operating \ conditions). \ If \ the \ resistance \ of \ the \ tungsten \ bulb \ filament \ at \ room \ temperature \ \(20{\degree}\text{C}\) \ is \ 1.1 \ \Omega, \ what \ is \ the \ temperature \ of \ the \ filament \ when \ the \ bulb \ is \ on? \ [ILW]

Ans:

\[ R = \frac{V}{i} = \frac{2.9 \text{ V}}{0.30 \text{ A}} = 9.67 \ \Omega. \]

\[ R - R_0 = R_0 \alpha (T - T_0), \]

\[ T = T_0 + \frac{1}{\alpha} \left( \frac{R}{R_0} - 1 \right) = 20{\degree}\text{C} + \left( \frac{1}{4.5 \times 10^{-3} / \text{K}} \right) \left( \frac{9.67 \Omega}{1.1 \Omega} - 1 \right) = 1.9 \times 10^3 \ \text{C}. \]
An electrical cable consists of 125 strands of fine wire, each having 2.65 \( \mu \Omega \) resistance. The same potential difference is applied between the ends of all the strands and results in a total current of 0.750 A. (a) What is the current in each strand? (b) What is the applied potential difference? (c) What is the resistance of the cable?

Ans:

(a) The current in each strand is \( i = 0.750 \text{ A}/125 = 6.00 \times 10^{-3} \text{ A} \).

(b) The potential difference is \( V = iR = (6.00 \times 10^{-3} \text{ A}) (2.65 \times 10^{-6} \Omega) = 1.59 \times 10^{-8} \text{ V} \).

(c) The resistance is \( R_{\text{total}} = 2.65 \times 10^{-6} \Omega/125 = 2.12 \times 10^{-8} \Omega \).

A heating element is made by maintaining a potential difference of 75.0 V across the length of a Nichrome wire that has a \( 2.60 \times 10^{-6} \text{ m}^2 \) cross section. Nichrome has a resistivity of \( 5.00 \times 10^{-7} \Omega \cdot \text{m} \). (a) If the element dissipates 5000 W, what is its length? (b) If 100 V is used to obtain the same dissipation rate, what should the length be?

Ans:

(a) From \( P = V^2/R = AV^2/\rho L \), we solve for the length:

\[
L = \frac{AV^2}{\rho P} = \frac{(2.60 \times 10^{-6} \text{ m}^2)(75.0 \text{ V})^2}{(5.00 \times 10^{-7} \Omega \cdot \text{m})(500 \text{ W})} = 5.85 \text{ m}.
\]
(b) Since \( L \propto V^2 \) the new length should be

\[
L' = L \left( \frac{V'}{V} \right)^2 = (5.85 \text{ m}) \left( \frac{100 \text{ V}}{75.0 \text{ V}} \right)^2 = 10.4 \text{ m}.
\]
In Fig. 27-43, two batteries of emf $\mathcal{E} = 12.0 \, \text{V}$ and internal resistance $r = 0.300 \, \Omega$ are connected in parallel across a resistance $R$. (a) For what value of $R$ is the dissipation rate in the resistor a maximum? (b) What is that maximum?

**Fig. 27-43** Problems 31 and 32.

Ans:

(a)

\[
\epsilon - ir - 2ir = 0 \Rightarrow i = \frac{\epsilon}{r + 2R}.
\]

\[
P = (2i)^2 R = \frac{4\epsilon^2 R}{(r + 2R)^2}.
\]

\[
\frac{dP}{dR} = \frac{4\epsilon^2}{(r + 2R)^3} - \frac{16\epsilon^2 R}{(r + 2R)^3} = \frac{4\epsilon^2 (r - 2R)}{(r + 2R)^3}.
\]

The derivative vanishes (and $P$ is a maximum) if $R = r/2$. With $r = 0.300 \, \Omega$, we have $R = 0.150 \, \Omega$. 
(b) We substitute $R = r/2$ into $P = 4e^2R/(r + 2R)^2$ to obtain

$$P_{\text{max}} = \frac{4e^2(r/2)}{[r + 2(r/2)]^2} = \frac{e^2}{2r} = \frac{(12.0 \text{ V})^2}{2(0.300 \ \Omega)} = 240 \text{ W.}$$

**43** In Fig. 27-51, $R_s$ is to be adjusted in value by moving the sliding contact across it until points $a$ and $b$ are brought to the same potential. (One tests for this condition by momentarily connecting a sensitive ammeter between $a$ and $b$: if these points are at the same potential, the ammeter will not deflect.) Show that when this adjustment is made, the following relation holds: $R_x = R_1R_2/R_1$. An unknown resistance ($R_x$) can be measured in terms of a standard ($R_s$) using this device, which is called a Wheatstone bridge.

Ans:

The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$.

$\rightarrow i_1R_1 = i_2R_s, \ i_2 = i_1R_1/R_s$

$$(R_1 + R_2)i_1 = (R_x + R_s)\frac{R_1}{R_s}i_1 \Rightarrow R_x = \frac{R_2R_s}{R_1}.$$