Chapter 1 Introduction: Measurement, Physical Quantities and Units

Galaxies are immense as atoms are small. Yet, the same laws of physics govern both, and all the rest of nature, an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature’s apparent complexity.

Physics: An introduction

The physical universe is enormously complex in its detail. Quarks, lightning, gravity, flowers, etc.

We have found that nature is remarkably cooperative – it exhibits the underlying order and simplicity that we so value.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text.

Science and the Realm of Physics

Science consists of the laws that are the general truths of nature and the body of knowledge they encompass.

Physics is the most basic of the sciences, concerning itself with the interactions of energy, matter, space, and time, and especially with questions of what underlies every phenomenon.

We discover physics by learning how to measure the quantities involved in physics. Among these quantities are length, time, mass, temperature, pressure and electrical current.

Applications of Physics

You need not be a scientist to use physics. On the contrary, a knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave oven works, etc.

Physics is the foundation of many important disciplines and contributes directly to other. Chemistry, for example – since it deals with the interactions of atoms and molecules – is rooted in atomic and molecular physics.

The disciplines of biology, chemistry, and physics are needed to understand membranes. [Human and Medical application]
Models, Theories, and Laws; The Role of Experimentation

A **model** is a mental image or analogy to objects or phenomena that we can experience directly.

A **theory** is usually a larger-scale and more broadly applicable generalization than a model and often seeks to describe nature with mathematical precision. Some theories include models to help visualize phenomena, whereas others do not.

The designation **law** is reserved for a concise and very general statement, such as the law that energy is conserved in any process, or Newton’s second law of motion.

Less broadly applicable statements are usually called **principles** (such as Pascal’s principle, which is applicable only in fluids, but the distinction between laws and principles often is not carefully made.

The models, theories, and laws that we devise sometimes imply the existence of objects or phenomena as yet unobserved. A theory or a law needs to be verified by an experiment. A law may be completely overturned, or may be modified, as a result of an experiment.

**The evolution of natural philosophy into modern physics**

Physics was not always a separate and distinct discipline and is not now isolated from other sciences. The word physics comes from Greek, meaning nature. They study of nature came to be called “naturally philosophy”. From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics and medicine. Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**.

**Modern physics** itself consists of two revolutionary theories, relativity and quantum mechanics. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope.

**Physical Quantities and Units**

We define a **physical quantity** either by specifying how it is measured or by stating how it is calculated from other measurements.

We measure each physical quantity in its own units, by comparison with a standard. The **unit** is a unique name we assign to measures of that quantity – for example, meter (m) for the quantity length. The **standard** corresponds to exactly 1.0 unit of the quantity.
SI Units: Fundamental and derived units

In 1971, the 14th General Conference on Weights and Measures picked seven quantities as base quantities, thereby forming the basis of the International System of Units, abbreviated SI from its French name and popularly known as the metric system. Table 1-1 shows the units for the three base quantities—length, mass, and time—that we use in the early chapters of this book. These units were defined to be on a “human scale.”

<table>
<thead>
<tr>
<th>Units for Three SI Base Quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Time</td>
</tr>
<tr>
<td>Mass</td>
</tr>
</tbody>
</table>

Many SI derived units are defined in terms of these base units. For example, the SI unit for power, called the watt (W), is defined in terms of the base units of mass, length and time.

\[ 1 \text{ watt} = 1 \text{ W} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3, \]

Units of Time, Length and Mass: The Second, Meter and Kilogram

Length

In 1792, the newborn Republic of France established a new system of weights and measures. Its cornerstone was the meter, defined to be one ten-millionth of the distance from the north pole to the equator. Later, for practical reasons, this Earth standard was abandoned and the meter came to be defined as the distance between two fine lines engraved near the ends of a platinum–iridium bar, the standard meter bar, which was kept at the International Bureau of Weights and Measures near Paris. Accurate copies of the bar were sent to standardizing laboratories throughout the world. These secondary standards were used to produce other, still more accessible standards, so that ultimately every measuring device derived its authority from the standard meter bar through a complicated chain of comparisons.

By 1983, however, the demand for higher precision had reached such a point that even the krypton-86 standard could not meet it, and in that year a bold step was taken. The meter was redefined as the distance traveled by light in a specified time interval. In the words of the 17th General Conference on Weights and Measures:

- The meter is the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second.

This time interval was chosen so that the speed of light \( c \) is exactly

\[ c = 299 792 458 \text{ m/s}. \]
Time

One second is the time taken by 9192631770 oscillations of the light (of a specified wavelength) emitted by a cesium-133 atom.

Mass

The Standard Kilogram

The SI standard of mass is a platinum–iridium cylinder (Fig. 1-4) kept at the International Bureau of Weights and Measures near Paris and assigned, by international agreement, a mass of 1 kilogram. Accurate copies have been sent to standardizing laboratories in other countries, and the masses of other bodies can be determined by balancing them against a copy. Table 1-5 shows some masses expressed in kilograms, ranging over about 83 orders of magnitude.

![Image of the kilogram cylinder](image)

Table 1-5: Some Approximate Masses

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass in Kilograms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known universe</td>
<td>$1 \times 10^{58}$</td>
</tr>
<tr>
<td>Our galaxy</td>
<td>$2 \times 10^{41}$</td>
</tr>
<tr>
<td>Sun</td>
<td>$2 \times 10^{30}$</td>
</tr>
<tr>
<td>Moon</td>
<td>$7 \times 10^{22}$</td>
</tr>
<tr>
<td>Asteroid Eros</td>
<td>$5 \times 10^{15}$</td>
</tr>
<tr>
<td>Small mountain</td>
<td>$1 \times 10^{12}$</td>
</tr>
<tr>
<td>Ocean liner</td>
<td>$7 \times 10^{7}$</td>
</tr>
<tr>
<td>Elephant</td>
<td>$5 \times 10^{3}$</td>
</tr>
<tr>
<td>Grape</td>
<td>$3 \times 10^{-3}$</td>
</tr>
<tr>
<td>Speck of dust</td>
<td>$7 \times 10^{-10}$</td>
</tr>
<tr>
<td>Penicillin molecule</td>
<td>$5 \times 10^{-17}$</td>
</tr>
<tr>
<td>Uranium atom</td>
<td>$4 \times 10^{-25}$</td>
</tr>
<tr>
<td>Proton</td>
<td>$2 \times 10^{-27}$</td>
</tr>
<tr>
<td>Electron</td>
<td>$9 \times 10^{-31}$</td>
</tr>
</tbody>
</table>

A Second Mass Standard

The masses of atoms can be compared with one another more precisely than they can be compared with the standard kilogram. For this reason, we have a second mass standard. It is the carbon-12 atom, which, by international agreement, has been assigned a mass of 12 atomic mass units (u). The relation between the two units is

$$1 \text{ u} = 1.66 054 02 \times 10^{-27} \text{ kg},$$

(1-8)

with an uncertainty of ±10 in the last two decimal places. Scientists can, with reasonable precision, experimentally determine the masses of other atoms relative to the mass of carbon-12. What we presently lack is a reliable means of extending that precision to more common units of mass, such as a kilogram.

Metric Prefixes
Measurements, accuracy, and uncertainty; significant figures

The uncertainty in a measurement is an estimate of the amount it can be off from the “true” value.

Using the method of significant figures, the rule is that the last digit written down is the first digit with some uncertainty. Special consideration is given to zeros when counting significant figures.

**Significant figures in calculations**

The result has the same number of significant figures as the quantity having the least significant figures entering into the calculation.

**Problems:**

1.2, 1.37

---

**TABLE 1-2**
Prefixes for SI Units

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{24}$</td>
<td>yotta-</td>
<td>Y</td>
</tr>
<tr>
<td>$10^{21}$</td>
<td>zetta-</td>
<td>Z</td>
</tr>
<tr>
<td>$10^{18}$</td>
<td>exa-</td>
<td>E</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>peta-</td>
<td>P</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera-</td>
<td>T</td>
</tr>
<tr>
<td>$10^9$</td>
<td>giga-</td>
<td>G</td>
</tr>
<tr>
<td>$10^6$</td>
<td>mega-</td>
<td>M</td>
</tr>
<tr>
<td>$10^3$</td>
<td>kilo-</td>
<td>k</td>
</tr>
<tr>
<td>$10^2$</td>
<td>hecto-</td>
<td>h</td>
</tr>
<tr>
<td>$10^1$</td>
<td>deka-</td>
<td>da</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>deci-</td>
<td>d</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>centi-</td>
<td>c</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli-</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro-</td>
<td>μ</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano-</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>pico-</td>
<td>p</td>
</tr>
<tr>
<td>$10^{-15}$</td>
<td>femto-</td>
<td>f</td>
</tr>
<tr>
<td>$10^{-18}$</td>
<td>atto-</td>
<td>a</td>
</tr>
<tr>
<td>$10^{-21}$</td>
<td>zepto-</td>
<td>z</td>
</tr>
<tr>
<td>$10^{-24}$</td>
<td>yocto-</td>
<td>y</td>
</tr>
</tbody>
</table>

*The most frequently used prefixes are shown in bold type.*
Chapter 2 Kinematics

Kinematics is defined to be the study of motion without regard to mass or force.

2.1 Displacement

We define **displacement** to be the change in position of an object

In 1D: \( \Delta x = x_2 - x_1 \)

Distance is defined to be the magnitude of size of displacement. Distance has no direction and, hence has no sign.

Vectors and Scalars

Any quantity with both magnitude and direction is defined to be a **vector**. For example, displacement is a vector.

Any quantity with a magnitude only, but no direction, is defined to be a **scalar**. For example, temperature and *pressure* are both scalars.

2.2 Time, Velocity and Speed

In physics the definition of time is simple – **time** is change. It is impossible to know that time has passed unless something changes.

We define **elapsed time** \( \Delta t \), to be the difference between the ending time and beginning time:

\[ \Delta t = t_2 - t_1 \]

**Average velocity** \( \bar{v} \) is precisely defined to be displacement (change in position) divided by the time of travel and to have the **same direction** as displacement.

\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \]

**Instantaneous velocity** \( v \) is the average velocity at a specific instant in time, i.e., \( \Delta t \to 0 \)
**Instantaneous speed** is simply defined to be the magnitude of instantaneous velocity.

### 2.3 Acceleration

**Acceleration** \( \ddot{a} \) is the rate at which the velocity changes. In symbols, **average acceleration** is defined to be

\[
\ddot{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}
\]

Acceleration is a vector in the *same direction* as the change in velocity.

**Instantaneous acceleration**, in symbol, is defined as

\[
\ddot{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v_2 - v_1}{t_2 - t_1} = \frac{dv}{dt}
\]

### 2.4 Motion equations for constant acceleration in one dimension

### 2.6 Falling objects

An object falling without air resistance or friction is defined to be in **Free-fall**.

\[
g = 9.8 \text{ m/s}^2
\]

*Taken from Fig. 2. 10*
Taken from Fig. 2.13

Problems: 2.16, 2.31
Chapter 3 Two-dimensional Kinematics

The graphical addition of vectors:

Adding vectors graphically (head to tail)

Vector can be added in any order. \( A + B = B + A \)
Subtracting vector graphically

\[ A - B = A + (-B) \]

Analytical methods of vector addition

\[ \vec{r} = \vec{a} + \vec{b}, \]

\[ r_x = a_x + b_x \]
\[ r_y = a_y + b_y \]
\[ r_z = a_z + b_z. \]

*Fig. 3-19*  
(a) Two vectors \( \vec{a} \) and \( \vec{b} \), with an angle \( \theta \) between them.  
(b) Each vector has a component along the direction of the other vector.

Taken from Fig. 3.19 (Halliday)

Unit vector
The scalar product of the vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-19a is written as $\vec{a} \cdot \vec{b}$ and defined to be
\[
\vec{a} \cdot \vec{b} = ab \cos \phi, \tag{3-20}
\]
\[
\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.
\]

Right-hand rule

\[\text{Fig. 3-20} \quad \text{Illustration of the right-hand rule for vector products. (a)} \quad \text{Sweep vector} \quad \vec{a} \quad \text{into vector} \quad \vec{b} \quad \text{with the fingers of your right hand. Your outstretched thumb shows the direction of vector} \quad \vec{c} = \vec{a} \times \vec{b}. \quad \text{(b)} \quad \text{Showing that} \quad \vec{b} \times \vec{a} \quad \text{is the reverse of} \quad \vec{a} \times \vec{b}.\]

The Vector Product

The vector product of $\vec{a}$ and $\vec{b}$, written $\vec{a} \times \vec{b}$, produces a third vector $\vec{c}$ whose magnitude is
\[
c = ab \sin \phi, \tag{3-27}
\]
where $\phi$ is the smaller of the two angles between $\vec{a}$ and $\vec{b}$. (You must use the smaller of the two angles between the vectors because $\sin \phi$ and $\sin(360^\circ - \phi)$ differ in algebraic sign.) Because of the notation, $\vec{a} \times \vec{b}$ is also known as the cross product, and in speech it is “a cross b.”

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.
**Projectile Motion**

**Horizontal**
\[ x - x_0 = v_{0x}t. \]
Because \( v_{0x} = v_0 \cos \theta_0 \), this becomes
\[ x - x_0 = (v_0 \cos \theta_0)t. \]

**Vertical**
\[ y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \]
\[ = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2; \]

**The Equation of the path**
\[ y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \] (trajectory).

**Horizontal range**
\[ R = (v_0 \cos \theta_0)t, \]
and
\[ 0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2. \]
Eliminating \( t \) between these two equations yields
\[ R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0. \]
Using the identity \( \sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0 \) (see Appendix E), we obtain
\[ R = \frac{v_0^2}{g} \sin 2\theta_0. \]
Relative velocity and classical relativity

![Diagram of relative velocity and classical relativity](image)

**Fig. 4-20** Alex (frame A) and Barbara (frame B) watch car P, as both B and P move at different velocities along the common x axis of the two frames. At the instant shown, \(x_{BA}\) is the coordinate of B in the A frame. Also, \(P\) is at coordinate \(x_{PB}\) in the B frame and coordinate \(x_{PA} = x_{PB} + x_{BA}\) in the A frame.

*Taken from Halliday*

**Inertial reference frames (constant velocity)**

\[
x_{PA} = x_{PB} + x_{BA}.
\]

\[
\frac{d}{dt} (x_{PA}) = \frac{d}{dt} (x_{PB}) + \frac{d}{dt} (x_{BA}),
\]

or (because \(v = dx/dt\))

\[
v_{PA} = v_{PB} + v_{BA}.
\]  \(4-41\)

\[
\frac{d}{dt} (v_{PA}) = \frac{d}{dt} (v_{PB}) + \frac{d}{dt} (v_{BA}).
\]

Because \(v_{BA}\) is constant, the last term is zero and we have

\[
a_{PA} = a_{PB}.
\]  \(4-42\)

In other words,

> Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

**Problems:** 3.42, 3.47
Chapter 4 Dynamics

Newton’s laws of Motion

4.1 Force

Restoring force

![Restoring force diagram]

Hooke’s law: \( F = -kx \)

Newton’s first law of motion

A body at rest remains at rest, or if in motion remains in motion at constant velocity, unless acted on by a net external force.

Mass

The property by which a body remains at rest or remain in motion with constant velocity is called inertia, and Newton’s first law is often called the law of inertia.
\[ a = \text{net } F \]
\[ a = \frac{\text{net } F}{m} \]

**Newton’s second law of motion**

\[ \text{net } F = ma \]

**Newton’s third law of motion**

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the one it exerts.

**Weight and the force of gravity**

\[ w = mg \]

*Free body diagrams*

*Taken from Figure 4.8*

**Weight on an incline, a two-dimensional problem**

**Friction**
The magnitude of static friction

\[ f_s \leq \mu_s N \text{, where } \mu_s \text{ is the coefficient of static friction} \]

The magnitude of kinetic friction

\[ f_k = \mu_k N \text{, where } \mu_k \text{ is the coefficient of kinetic friction} \]

Figure 4.21 shows a 75.0 kg man standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of 1.20 m/s\(^2\), and (b) if the elevator moves upward at a constant speed.
Sol:

a) net \( F = ma \)

\[ F_s - w = ma \]

\[ F_s = mg + ma \]

\[ F_s = (75.0 \text{ kg})(1.20 \text{ m/s}^2) + (75.0 \text{ kg})(9.80 \text{ m/s}^2) = 825 \text{ N} \]

b) \( F_s = (75.0 \text{ kg})(9.80 \text{ m/s}^2) = 735 \text{ N} \)

c) Free-fall?

Problems: 4.3, 4.30
Chapter 5 Statics, torque, and elasticity

5.1 The first condition for equilibrium

net \( F = 0 \)

5.2 The second condition for equilibrium

net \( \tau = 0 \)

Def. \( \tau = \mathbf{r} \times \mathbf{F} \)

Fig. 11-10 Defining torque. (a) A force \( \mathbf{F} \), lying in an \( xy \) plane, acts on a particle at point \( A \). (b) This force produces a torque \( \tau ( = \mathbf{r} \times \mathbf{F}) \) on the particle with respect to the origin \( O \). By the right-hand rule for vector (cross) products, the torque vector points in the positive direction of \( z \). Its magnitude is given by \( rFz \) in (b) and by \( rFz \) in (c).

Taken from Halliday

Center of Mass; center of gravity

\[
\mathbf{r}_{CM} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}
\]

\[
\mathbf{r}_{CG} = \frac{m_1 g_1 \mathbf{r}_1 + m_2 g_2 \mathbf{r}_2 + m_3 g_3 \mathbf{r}_3 + \ldots}{m_1 g_1 + m_2 g_2 + m_3 g_3 + \ldots}
\]
If \( \vec{g} \) is uniform, then \( r_{CM} = r_{CG} \).

**Stability**

A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement.

A system is in **unstable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.

A system is in **neutral equilibrium** if its equilibrium is independent of displacement from its original position.

**Forces and torques in muscles and joints**

![Diagram](image)

Taken from Figure 5.19
Figure 5.19 shows a forearm holding a book and a schematic diagram of an analogous lever system. The triceps is assumed to be relaxed.

Ex. Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in Fig. 5.19, and compare this force with the weight of the forearm plus its load.

Sol: net $\tau = 0$

Pivot: the joint

$$r_2w_a + r_3w_b = r_1F_b$$

$$\frac{r_2w_a + r_3w_b}{r_1} = F_b$$

$$F_b = 470 \text{ N}$$

$$\frac{F_b}{w_a + w_b} = \frac{470}{63.7} = 7.38!$$

Muscles exert bigger forces than you might think. Very large forces are also created in the joint. Because the way things are put together, joints and muscles often exert forces in opposite direction. Forces in muscles are largest when their load is a long distance from the joint. Racket sports, such as tennis, exaggerate this effect by extending the arm. Tennis elbow….

Ex. Do not lift with your back

Consider the person lifting a heavy box with his back, shown in Fig. 5.22. (a) Calculate the force in the back muscle $F_b$ needed to support the upper body plus the box and compare this with their weight. The mass of the upper body is 55.0 kg and that of the box is 30.0 kg (b) Calculate the magnitude and direction of the force $F_v$ exerted by the vertebrae on the spine at the indicated pivot point.

Sol:

(a) net $\tau = 0$ Using the perpendicular lever arms given in the figure.

$$=(35.0 \text{ cm})(55.0 \text{ kg})(9.80 \text{ m/s}^2) + (50.0 \text{ cm})(30.0 \text{ kg})(9.80 \text{ m/s}^2) = (8.00 \text{ cm})(F_b)$$
\( F_B = 4.20 \times 10^3 \text{ N} \)

\[
\frac{F_B}{w_{ub} + w_{box}} = \frac{4.20 \times 10^3}{833} = 5.04
\]

Taken from Figure 5.22

(b) More important in terms of its damage potential is the force on the vertebrae \( F_y \).

net \( F_y = \text{net} \ F_x = 0 \)

\[ F_{vy} - w_{ub} - w_{box} - F_B \sin 29.0^\circ = 0 \]

\[ F_{vy} = 2.87 \times 10^3 \text{ N} \]

Similarly \( F_{vx} - F_B \cos 29.0^\circ = 0 \)

\[ F_{vx} = 3.67 \times 10^3 \text{ N} \]

\[ F_y = \sqrt{F_{vx}^2 + F_{vy}^2} = 4.66 \times 10^3 \text{ N} \]
\[ \theta = \tan^{-1} \left( \frac{F_{vy}}{F_{vx}} \right) = 38.0^\circ \]

\[ \frac{F_y}{w_{ub} + w_{box}} = \frac{4.66 \times 10^3}{833} = 5.60 \]

This force is 5.60 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large – because similar forces are created in our hips, knees and ankles – but that our spines are relatively weak.

**Elastic**

*Fig. 12-11 (a) A cylinder subject to tensile stress stretches by an amount \( \Delta L \). (b) A cylinder subject to shearing stress deforms by an amount \( \Delta x \), somewhat like a pack of playing cards would. (c) A solid sphere subject to uniform hydraulic stress from a fluid shrinks in volume by an amount \( \Delta V \). All the deformations shown are greatly exaggerated.*

\[ \frac{F}{A} = E \frac{\Delta L}{L} \]

\[ \frac{F}{A} = G \frac{\Delta x}{L} \]

\[ p = B \frac{\Delta V}{V} \]

Taken from Halliday.
Stress = \gamma \times \text{Strain}

Problems: 5.19, 5.35
Chapter 6 Work, energy and power

Work and kinetic energy

\[ F_x = ma_x, \]  
(7-3)

where \( m \) is the bead’s mass. As the bead moves through a displacement \( \vec{d} \), the force changes the bead’s velocity from an initial value \( \vec{v}_0 \) to some other value \( \vec{v} \). Because the force is constant, we know that the acceleration is also constant. Thus, we can use Eq. 2-16 to write, for components along the \( x \) axis,

\[ v^2 = v_0^2 + 2a_x d. \]  
(7-4)

Solving this equation for \( a_x \), substituting into Eq. 7-3, and rearranging then give us

\[ \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d. \]  
(7-5)

The first term on the left side of the equation is the kinetic energy \( K_f \) of the bead at the end of the displacement \( d \), and the second term is the kinetic energy \( K_i \) of the bead at the start of the displacement. Thus, the left side of Eq. 7-5 tells us the kinetic energy has been changed by the force, and the right side tells us the change is equal to \( F_x d \). Therefore, the work \( W \) done on the bead by the force (the energy transfer due to the force) is

\[ W = F_x d. \]  
(7-6)

\[ W = \vec{F} \cdot \vec{d} \quad \text{(work done by a constant force)}, \]  
(7-8)

Work–Kinetic Energy Theorem

Equation 7-5 relates the change in kinetic energy of the bead (from an initial \( K_i = \frac{1}{2}mv_0^2 \) to a later \( K_f = \frac{1}{2}mv^2 \)) to the work \( W \) \((= F_x d \) done on the bead. For such particle-like objects, we can generalize that equation. Let \( \Delta K \) be the change in the kinetic energy of the object, and let \( W \) be the net work done on it. Then we can write

\[ \Delta K = K_f - K_i = W, \]  
(7-10)

which says that

\[ \left( \text{change in the kinetic energy of a particle} \right) = \left( \text{net work done on the particle} \right). \]

We can also write

\[ K_f = K_i + W, \]  
(7-11)

which says that

\[ \left( \text{kinetic energy after the net work is done} \right) = \left( \text{kinetic energy before the net work} \right) + \left( \text{the net work done} \right). \]

These statements are known traditionally as the work–kinetic energy theorem

Taken from Halliday
\[ W_g = mgd \cos \phi \quad \text{(work done by gravitational force).} \quad (7-12) \]

For a rising object, force \( \vec{F}_g \) is directed opposite the displacement \( \vec{d} \), as indicated in Fig. 7-6. Thus, \( \phi = 180^\circ \) and

\[ W_g = mgd \cos 180^\circ = mgd(-1) = -mgd. \quad (7-13) \]

The minus sign tells us that during the object’s rise, the gravitational force acting on the object transfers energy in the amount \( mgd \) from the kinetic energy of the object. This is consistent with the slowing of the object as it rises.

After the object has reached its maximum height and is falling back down, the angle \( \phi \) between force \( \vec{F}_g \) and displacement \( \vec{d} \) is zero. Thus,

\[ W_g = mgd \cos 0^\circ = mgd(+1) = +mgd. \quad (7-14) \]

\[ \text{Fig. 7-6} \quad \text{Because the gravitational force} \ \vec{F}_g \ \text{acts on it, a particle-like tomato of mass} \ m \ \text{thrown upward slows from velocity} \ v_0 \ \text{to velocity} \ v' \ \text{during displacement} \ \vec{d}. \ \text{A kinetic energy gauge indicates the resulting change in the kinetic energy of the tomato, from} \ K_i = \frac{1}{2}mv_0^2 \ \text{to} \ K_f = \frac{1}{2}mv^2. \]

**Taken from Halliday**

**Conservative forces and potential energy**

A **conservative force** is one, like gravity, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

\[ W = \int_{x_i}^{x_f} F(x) \, dx \quad \text{(work: variable force).} \quad (7-32) \]

Spring force \( \vec{F} = -k\vec{x} \)
From Eq. 7-21, the force magnitude $F_x$ is $kx$. Thus, substitution leads to

\[ W_s = \int_{x_i}^{x_f} -kx \, dx = -k \int_{x_i}^{x_f} x \, dx \]
\[ = (-\frac{1}{2} k)[x^2]_{x_i}^{x_f} = (\frac{1}{2} k)(x_f^2 - x_i^2). \]  \hspace{1cm} (7-24)

Multiplied out, this yields

\[ W_s = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2 \hspace{1cm} \text{(work by a spring force)}. \]  \hspace{1cm} (7-25)

Nonconservative forces: open systems

A crate plus a friction floor

Power

\[ P_{\text{avg}} = \frac{W}{\Delta t} \hspace{1cm} \text{(average power)}. \]  \hspace{1cm} (7-42)

The instantaneous power $P$ is the instantaneous time rate of doing work, which we can write as

\[ P = \frac{dW}{dt} \hspace{1cm} \text{(instantaneous power)}. \]  \hspace{1cm} (7-43)

Conservation of energy

Kinetic energy

Potential energy

Mechanical energy

Problems: 6.15, 6.33
Chapter 7 Linear Momentum

Linear momentum and force

\[ p = mv \]

\[ \Delta p = \Delta (mv) = m \Delta v \quad \text{if } m \text{ is constant} \]

\[ \text{net } F = \frac{\Delta p}{\Delta t} = \frac{m \Delta v}{\Delta t} = ma \]

9-7 Conservation of Linear Momentum

Suppose that the net external force \( \vec{F}_{\text{net}} \) (and thus the net impulse \( \vec{J} \)) acting on a system of particles is zero (the system is isolated) and that no particles leave or enter the system (the system is closed). Putting \( F_{\text{net}} = 0 \) in Eq. 9-27 then yields \( dP/dt = 0 \), or

\[ \vec{P} = \text{constant} \quad \text{(closed, isolated system)}. \quad (9-42) \]

In words,

If no net external force acts on a system of particles, the total linear momentum \( \vec{P} \) of the system cannot change.

This result is called the law of conservation of linear momentum. It can also be written as

\[ \vec{P}_i = \vec{P}_f \quad \text{(closed, isolated system)}. \quad (9-43) \]

Elastic collision: the total kinetic energy is conserved.

the total linear momentum is conserved as well.

\[ \left( \text{total kinetic energy} \right)_{\text{before the collision}} = \left( \text{total kinetic energy} \right)_{\text{after the collision}} \]

This does not mean that the kinetic energy of each colliding body cannot change. Rather, it means this:

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

Inelastic collision: the total kinetic energy is not conserved.

the total linear momentum is conserved.
Elastic collision: Stationary target

\[ m_1v_{1i} = m_1v_{1f} + m_2v_{2f} \]  \hspace{2cm} (linear momentum). \hspace{2cm} (9-63)

If the collision is also elastic, then the total kinetic energy is conserved and we can write that conservation as

\[ \frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \]  \hspace{2cm} (kinetic energy). \hspace{2cm} (9-64)

and Eq. 9-64 as

\[ m_1(v_{1i} - v_{1f}) = m_2v_{2f} \] \hspace{2cm} (9-65)

After dividing Eq. 9-66 by Eq. 9-65 and doing some more algebra, we obtain

\[ v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \] \hspace{2cm} (9-67)

and

\[ v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \] \hspace{2cm} (9-68)

Special cases:

1. *Equal masses* If \( m_1 = m_2 \), Eqs. 9-67 and 9-68 reduce to

\[ v_{1f} = 0 \text{ and } v_{2f} = v_{1i}. \]

2. *A massive target* In Fig. 9-19, a massive target means that \( m_2 \gg m_1 \). For example, we might fire a golf ball at a stationary cannonball. Equations 9-67 and 9-68 then reduce to

\[ v_{1f} \approx -v_{1i} \text{ and } v_{2f} \approx \left( \frac{2m_1}{m_2} \right) v_{1i}. \] \hspace{2cm} (9-69)

3. *A massive projectile* This is the opposite case; that is, \( m_1 \gg m_2 \). This time, we fire a cannonball at a stationary golf ball. Equations 9-67 and 9-68 reduce to

\[ v_{1f} \approx v_{1i} \text{ and } v_{2f} \approx 2v_{1i}. \] \hspace{2cm} (9-70)

Moving targets

![Diagram](image)

*Fig. 9-20* Two bodies headed for a one-dimensional elastic collision.
For the situation of Fig. 9-20, the conservation of linear momentum is written as

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}, \quad (9.71)$$

and the conservation of kinetic energy is written as

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2. \quad (9.72)$$

To solve these simultaneous equations for $v_{1f}$ and $v_{2f}$, we first rewrite Eq. 9.71 as

$$m_1(v_{1i} - v_{1f}) = -m_2(v_{2i} - v_{2f}), \quad (9.73)$$

and Eq. 9.72 as

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f}). \quad (9.74)$$

After dividing Eq. 9.74 by Eq. 9.73 and doing some more algebra, we obtain

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \quad (9.75)$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}. \quad (9.76)$$

Elastic collisions in 2D

![Diagram of elastic collision](image)

**Fig. 9-22** An elastic collision between two bodies in which the collision is not head-on. The body with mass $m_2$ (the target) is initially at rest.

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}. \quad (9.77)$$

If the collision is also elastic (a special case), then the total kinetic energy is also conserved:

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}. \quad (9.78)$$

Taken from Halliday
Rocket

A system with variable mass

Rocket scientist?

Fig. 9-23 (a) An accelerating rocket of mass $M$ at time $t$, as seen from an inertial reference frame. (b) The same but at time $t + dt$. The exhaust products released during interval $dt$ are shown.

**Taken from Halliday**

Conservation of the linear momentum

$$P_f = P_i,$$  \hspace{1cm} (9-82)

where the subscripts $i$ and $f$ indicate the values at the beginning and end of time interval $dt$. We can rewrite Eq. 9-82 as

$$Mv = -dM \ U + (M + dM)(v + dv),$$  \hspace{1cm} (9-83)
\[
\left( \text{velocity of rocket} \right)_{\text{relative to frame}} = \left( \text{velocity of rocket} \right)_{\text{relative to products}} + \left( \text{velocity of products} \right)_{\text{relative to frame}}.
\]

In symbols, this means
\[
(v + dv) = v_{\text{rel}} + U,
\]
or
\[
U = v + dv - v_{\text{rel}}. \tag{9-84}
\]

Substituting this result for \(U\) into Eq. 9-83 yields, with a little algebra,
\[
-dM v_{\text{rel}} = M \, dv.
\]

Dividing each side by \(dt\) gives us
\[
-\frac{dM}{dt} \, v_{\text{rel}} = M \frac{dv}{dt}. \tag{9-86}
\]

We replace \(dM/dt\) (the rate at which the rocket loses mass) by \(-R\), where \(R\) is the (positive) mass rate of fuel consumption, and we recognize that \(dv/dt\) is the acceleration of the rocket. With these changes, Eq. 9-86 becomes
\[
Rv_{\text{rel}} = Ma \quad \text{(first rocket equation).} \tag{9-87}
\]

Equation 9-87 holds at any instant, with the mass \(M\), the fuel consumption rate \(R\), and the acceleration \(a\) evaluated at that instant.

**Finding the Velocity**

How will the velocity of a rocket change as it consumes its fuel? From Eq. 9-85 we have
\[
dv = -v_{\text{rel}} \frac{dM}{M}.
\]

Integrating leads to
\[
\int_{v_i}^{v_f} dv = -v_{\text{rel}} \int_{M_i}^{M_f} \frac{dM}{M},
\]
in which \(M_i\) is the initial mass of the rocket and \(M_f\) its final mass. Evaluating the integrals then gives
\[
v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \quad \text{(second rocket equation)} \tag{9-88}
\]

**Problems:** 7.11, 7.35
Chapter 26 Special Relativity

Albert Einstein

Inertial reference frames: constant velocity

Space and time are actually entangled: the time between two events depends on how far apart they occur, and vice versa.

1. The Relativity Postulate: The laws of physics are the same for observers in all inertial reference frames. No one frame is preferred over any other.

2. The Speed of Light Postulate: The speed of light in vacuum has the same value \( c \) in all directions and in all inertial reference frames.

The Ultimate speed

The existence of a limit to the speed of accelerated electrons was shown in a 1964 experiment by W. Bertozzi, who accelerated electrons to various measured speeds and—by an independent method—measured their kinetic energies. He found that as the force on a very fast electron is increased, the electron’s measured kinetic energy increases toward very large values but its speed does not increase appreciably (Fig. 37-2). Electrons have been accelerated in laboratories to at least 0.999 999 999 95 times the speed of light but—close though it may be—that speed is still less than the ultimate speed \( c \).

This ultimate speed has been defined to be exactly

\[
    c = 299 792 458 \text{ m/s.}
\]  

(37-1)

![Fig. 37-2](image)

The dots show measured values of the kinetic energy of an electron plotted against its measured speed. No matter how much energy is given to an electron (or to any other particle having mass), its speed can never equal or exceed the ultimate limiting speed \( c \). (The plotted curve through the dots shows the predictions of Einstein’s special theory of relativity.)

Taken from Halliday
Testing the speed of light postulate

Neutral pion

\[ \pi^0 \rightarrow \gamma + \gamma. \quad (37-2) \]

In 1964, physicists at CERN, the European particle-physics laboratory near Geneva, generated a beam of pions moving at a speed of 0.999 75c with respect to the laboratory. The experimenters then measured the speed of the gamma rays emitted from these very rapidly moving sources. They found that the speed of the light emitted by the pions was the same as it would be if the pions were at rest in the laboratory, namely c.

Measuring an event

An event is something that happens, to which an observer can assign three space coordinates and one time coordinate.

The speed of light!! We need to construct an imaginary array of measuring rods and clocks throughout the observer's frame.

1. The Space Coordinates. We imagine the observer's coordinate system fitted with a close-packed, three-dimensional array of measuring rods, one set of rods parallel to each of the three coordinate axes. These rods provide a way to determine coordinates along the axes. Thus, if the event is, say, the turning on of a small light bulb, the observer, in order to locate the position of the event, need only read the three space coordinates at the bulb's location.

2. The Time Coordinate. For the time coordinate, we imagine that every point of intersection in the array of measuring rods includes a tiny clock, which the observer can read because the clock is illuminated by the light generated by the event. Figure 37.2 suggests one plane in the "jungle gym" of clocks and measuring rods we have described.

The array of clocks must be synchronized properly. It is not enough to assemble a set of identical clocks, set them all to the same time, and then move them to their assigned positions. We do not know, for example, whether moving the clocks will change their rates. (Actually, it will.) We must put the clocks in place and then synchronize them.
The relativity of simultaneity

If two observers are in relative motion, they will not, in general, agree as to whether two events are simultaneous. If one observer finds them to be simultaneous, the other generally will not, and conversely.

Fig. 37-4 The spaceships of Sally and Sam and the occurrences of events from Sam’s view. Sally’s ship moves rightward with velocity \( v \). (a) Event Red occurs at positions \( RR' \) and event Blue occurs at positions \( BB' \); each event sends out a wave of light. (b) Sam simultaneously detects the waves from event Red and event Blue. (c) Sally detects the wave from event Red. (d) Sally detects the wave from event Blue.

The relativity of time

The time interval between two events depends on how far apart they occur in both space and time; that is, their spatial and temporal separations are entangled.

An interesting experiment

Sally on the train with constant velocity. She has a light source, a mirror, and a clock.

Sam in the station

They want to measure the time interval between two events: event 1 and event 2.
Fig. 37-5  (a) Sally, on the train, measures the time interval $\Delta t_0$ between events 1 and 2 using a single clock $C$ on the train. That clock is shown twice: first for event 1 and then for event 2. (b) Sam, watching from the station as the events occur, requires two synchronized clocks, $C_1$ at event 1 and $C_2$ at event 2, to measure the time interval between the two events; his measured time interval is $\Delta t$.

Taken from Halliday

$$\Delta t_0 = \frac{2D}{c} \quad \text{(Sally)}.$$  

The time interval measured by Sam between the two events is

$$\Delta t = \frac{2L}{c} \quad \text{(Sam)},$$  

in which

$$L = \sqrt{(\frac{1}{2}v \Delta t)^2 + D^2}. \quad \text{(37-5)}$$

From Eq. 37-3, we can write this as

$$L = \sqrt{(\frac{1}{2}v \Delta t)^2 + (\frac{1}{2}c \Delta t_0)^2}. \quad \text{(37-6)}$$

If we eliminate $L$ between Eqs. 37-4 and 37-6 and solve for $\Delta t$, we find

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - (v/c)^2}}. \quad \text{(37-7)}$$
When two events occur at the same location in an inertial reference frame, the time interval between them, measured in that frame, is called the proper time interval. Measurements of the same time interval from any other inertial reference frame are always greater.

\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - (v/c)^2}}.
\]  

(37-8)

With these replacements, we can rewrite Eq. 37-7 as

\[
\Delta t = \gamma \Delta t_0 \quad \text{(time dilation)}.
\]  

(37-9)

Tests:

Microscopic clocks

1. Microscopic Clocks. Subatomic particles called muons are unstable; that is, when a muon is produced, it lasts for only a short time before it decays (transforms into particles of other types). The lifetime of a muon is the time interval between its production (event 1) and its decay (event 2). When muons are stationary and their lifetimes are measured with stationary clocks (say, in a laboratory), their average lifetime is 2.200 μs. This is a proper time interval because, for each muon, events 1 and 2 occur at the same location in the reference frame of the muon—namely, at the muon itself. We can represent this proper time interval with \( \Delta t_1 \); moreover, we can call the reference frame in which it is measured the rest frame of the muon.

If, instead, the muons are moving, say, through a laboratory, then measurements of their lifetimes made with the laboratory clocks should yield a greater average lifetime (a dilated average lifetime). To check this conclusion, measurements were made of the average lifetime of muons moving with a speed of 0.9994 c relative to laboratory clocks. From Eq. 37-8, with \( \beta = 0.9994 \), the Lorentz factor for this speed is

\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - (0.9994)^2}} = 28.87.
\]

Equation 37-9 then yields, for the average dilated lifetime,

\[
\Delta t = \gamma \Delta t_0 = (28.87)(2.200 \mu s) = 63.51 \mu s.
\]

Macroscopic clocks

Example: Your starship passes Earth with a relative speed of 0.9990 c. After traveling 10.0 year (your time), you stop at lookout post LP13, turn, and then travel back to Earth with the same relative speed. The trip back takes another 10.0 year (your time). How long does the round trip take according to measurements made on Earth?
Sol:
Your time is the proper time.

\[
\Delta t = \frac{\Delta \tilde{t}}{\sqrt{1 - (\nu/c)^2}} = \frac{10.0 \text{ y}}{\sqrt{1 - (0.9950c/c)^2}} = (22.37)(10.0 \text{ y}) = 224 \text{ y}.
\]

On the return trip, we have the same situation and the same data. Thus, the round trip requires 20 y of your time but

\[
\Delta t_{\text{total}} = (2)(224 \text{ y}) = 448 \text{ y} \quad \text{(Answer)}
\]

Back to the future!!
Taken from Urone

The Relativity of length

\[
L = L_0 \sqrt{1 - \beta^2} = \frac{L_0}{\gamma} \quad \text{(length contraction).} \quad (37-13)
\]

The length \(L_0\) of an object measured in the rest frame of the object is its proper length or rest length. Measurements of the length from any reference frame that is in relative motion parallel to that length are always less than the proper length.
Proof of Eq. 37-13

Length contraction is a direct consequence of time dilation. Consider once more our two observers. This time, both Sally, seated on a train moving through a station, and Sam, again on the station platform, want to measure the length of the platform. Sam, using a tape measure, finds the length to be \( L_0 \), a proper length because the platform is at rest with respect to him. Sam also notes that Sally, on the train, moves through this length in a time \( \Delta t = L_0/v \), where \( v \) is the speed of the train; that is,

\[
L_0 = v \Delta t \quad \text{(Sam).} \tag{37-14}
\]

This time interval \( \Delta t \) is not a proper time interval because the two events that define it (Sally passes the back of the platform and Sally passes the front of the platform) occur at two different places, and therefore Sam must use two synchronized clocks to measure the time interval \( \Delta t \).

For Sally, however, the platform is moving past her. She finds that the two events measured by Sam occur at the same place in her reference frame. She can time them with a single stationary clock, and so the interval \( \Delta t_0 \) that she measures is a proper time interval. To her, the length \( L \) of the platform is given by

\[
L = v \Delta t_0 \quad \text{(Sally).} \tag{37-15}
\]

If we divide Eq. 37-15 by Eq. 37-14 and apply Eq. 37-9, the time dilation equation, we have

\[
\frac{L}{L_0} = \frac{v \Delta t_0}{v \Delta t} = \frac{1}{\gamma},
\]

or

\[
L = \frac{L_0}{\gamma}, \tag{37-16}
\]

which is Eq. 37-13, the length contraction equation.

Galilean and Lorentz transformations

![Diagram showing Galilean and Lorentz transformations](image)

*Fig. 37-9* Two inertial reference frames: frame \( S' \) has velocity \( v \) relative to frame \( S \).

\[
x' = x - vt \quad \text{(Galilean transformation equations; approximately valid at low speeds)}.
\]

\[
t' = t
\]
\[ x' = \gamma(x - vt), \]
\[ y' = y, \]
\[ z' = z, \]
\[ t' = \gamma(t - vx/c^2) \]

(Lorentz transformation equations; valid at all physically possible speeds). \hspace{1cm} (37-21)

If \( \gamma \to \infty \) then \( \gamma \to 1 \)

Galilean transformation is equivalent to Lorentz transformation.

**Consequences of the Lorentz transformation**

**TABLE 37-2**

<table>
<thead>
<tr>
<th>The Lorentz Transformation Equations for Pairs of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \Delta x = \gamma(\Delta x' + v\Delta t') )</td>
</tr>
<tr>
<td>2. ( \Delta t = \gamma(\Delta t' + v\Delta x'/c^2) )</td>
</tr>
</tbody>
</table>

\[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} = \frac{1}{\sqrt{1 - \beta^2}} \]

Frame \( S' \) moves at velocity \( v \) relative to frame \( S \).

as measured by an observer in \( S \), and

\[ \Delta x' = x'_2 - x'_1 \quad \text{and} \quad \Delta t' = t'_2 - t'_1, \]

as measured by an observer in \( S' \).

**Simultaneity**

Consider Eq. 2 of Table 37-2,

\[ \Delta t = \gamma \left( \Delta t' + \frac{v \Delta x'}{c^2} \right). \]  \hspace{1cm} (37-23)

If two events occur at different places in reference frame \( S' \) of Fig. 37-9, then \( \Delta x' \) in this equation is not zero. It follows that even if the events are simultaneous in \( S' \) (thus \( \Delta t' = 0 \)), they will not be simultaneous in frame \( S \). (This is in accord with our conclusion in Section 37-4.) The time interval between the events in \( S \) will be

\[ \Delta t = \gamma \frac{v \Delta x'}{c^2} \quad \text{(simultaneous events in \( S' \)).} \]

**Time Dilation**

Suppose now that two events occur at the same place in \( S' \) (thus \( \Delta x' = 0 \)) but at different times (thus \( \Delta t' \neq 0 \)). Equation 37-23 then reduces to

\[ \Delta t = \gamma \Delta t' \quad \text{(events in same place in \( S' \)).} \]  \hspace{1cm} (37-24)
\[
\Delta t = \gamma \Delta t_0 \quad \text{(time dilation)}.
\]

which is exactly Eq. 37-9, the time dilation equation.

**Length Contraction**

Consider Eq. 1’ of Table 37-2,

\[
\Delta x' = \gamma (\Delta x - v \Delta t).
\]  \hspace{1cm} (37-25)

If a rod lies parallel to the \( x \) and \( x' \) axes of Fig. 37-9 and is at rest in reference frame \( S' \), an observer in \( S' \) can measure its length at leisure. One way to do so is by subtracting the coordinates of the end points of the rod. The value of \( \Delta x' \) that is obtained will be the proper length \( L_0 \) of the rod because the measurements are made in a frame where the rod is at rest.

Suppose the rod is moving in frame \( S \). This means that \( \Delta x \) can be identified as the length \( L \) of the rod in frame \( S \) only if the coordinates of the rod’s end points are measured *simultaneously*—that is, if \( \Delta t = 0 \). If we put \( \Delta x' = L_0 \), \( \Delta x = L_0 \), and \( \Delta t = 0 \) in Eq. 37-25, we find

\[
L = \frac{L_0}{\gamma} \quad \text{(length contraction).} \]  \hspace{1cm} (37-26)

which is exactly Eq. 37-13, the length contraction equation.

The relativity of velocities

Fig. 37-11 Reference frame \( S' \) moves with velocity \( v \) relative to frame \( S \). A particle has velocity \( \vec{u}' \) relative to reference frame \( S' \) and velocity \( \vec{u} \) relative to reference frame \( S \).

Taken from Halliday

Lorentz transformation
\[ \Delta x = \gamma (\Delta x' + v \Delta t') \]

and

\[ \Delta t = \gamma \left( \Delta t' + \frac{v \Delta x'}{c^2} \right). \]

If we divide the first of these equations by the second, we find

\[ \frac{\Delta x}{\Delta t} = \frac{\Delta x' + v \Delta t'}{\Delta t' + v \Delta x'/c^2}. \]

Dividing the numerator and denominator of the right side by \( \Delta t' \), we find

\[ \frac{\Delta x}{\Delta t} = \frac{\Delta x'/\Delta t' + v}{1 + v(\Delta x'/\Delta t')/c^2}. \]

However, in the differential limit, \( \Delta x/\Delta t \) is \( u \), the velocity of the particle as measured in \( S \), and \( \Delta x'/\Delta t' \) is \( u' \), the velocity of the particle as measured in \( S' \). Then we have, finally,

\[ u = \frac{u' + v}{1 + u'v/c^2} \quad \text{(relativistic velocity transformation)} \quad (37-29) \]

as the relativistic velocity transformation equation. This equation reduces to the classical, or Galilean, velocity transformation equation,

\[ u = u' + v \quad \text{(classical velocity transformation).} \quad (37-30) \]

when we apply the formal test of letting \( c \to \infty \). In other words, Eq. 37-29 is correct for all physically possible speeds but Eq. 37-30 is approximately correct for speeds much less than \( c \).

**Doppler effect for light**

Let \( f_0 \) represent the proper frequency of the source—that is, the frequency that is measured by an observer in the rest frame of the source. Let \( f \) represent the frequency detected by an observer moving with velocity \( \vec{v} \) relative to that rest frame. Then, when the direction of \( \vec{v} \) is directly away from the source,

\[ f = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}} \quad \text{(source and detector separating),} \quad (37-31) \]

where \( \beta = v/c \). When the direction of \( \vec{v} \) is directly toward the source, we must change the signs in front of both \( \beta \) symbols in Eq. 37-31.
**Low-Speed Doppler Effect**

For low speeds ($\beta \ll 1$), Eq. 37-31 can be expanded in a power series in $\beta$ and approximated as

$$f = f_0 (1 - \beta + \frac{1}{2} \beta^2) \quad \text{(source and detector separating, } \beta \ll 1). \quad (37-32)$$

The corresponding low-speed equation for the Doppler effect with sound waves (or any waves except light waves) has the same first two terms but a different coefficient in the third term. Thus, the relativistic effect for low-speed light sources and detectors shows up only with the $\beta^2$ term.

A police radar unit employs the Doppler effect with microwaves to measure the speed $v$ of a car. A source in the radar unit emits a microwave beam at a certain (proper) frequency $f_0$ along the road. A car that is moving toward the unit intercepts that beam but at a frequency that is shifted upward by the Doppler effect due to the car's motion toward the radar unit. The car reflects the beam back toward the radar unit. Because the car is moving toward the radar unit, the detector in the unit intercepts a reflected beam that is further shifted up in frequency. The unit compares that detected frequency with $f_0$ and computes the speed $v$ of the car.

**Transverse Doppler effect**

So far, we have discussed the Doppler effect, here and in Chapter 17, only for situations in which the source and the detector move either directly toward or directly away from each other. Figure 37-12 shows a different arrangement, in which a source $S$ moves past a detector $D$. When $S$ reaches point $P$, the velocity of $S$ is perpendicular to the line joining $P$ and $D$, and at that instant $S$ is moving neither toward nor away from $D$. If the source is emitting sound waves of frequency $f_0$, $D$ detects that frequency (with no Doppler effect) when it intercepts the waves that were emitted at point $P$. However, if the source is emitting light waves, there is still a Doppler effect, called the **transverse Doppler effect**. In this situation, the detected frequency of the light emitted when the source is at point $P$ is

$$f = f_0 \sqrt{1 - \beta^2} \quad \text{(transverse Doppler effect).} \quad (37-37)$$

For low speeds ($\beta \ll 1$), Eq. 37-37 can be expanded in a power series in $\beta$ and approximated as

$$f = f_0 (1 - \frac{1}{2} \beta^2) \quad \text{(low speeds).} \quad (37-38)$$

*Fig. 37-12* A light source $S$ travels with velocity $v$ past a detector at $D$. The special theory of relativity predicts a transverse Doppler effect as the source passes through point $P$, where the direction of travel is perpendicular to the line extending through $D$. Classical theory predicts no such effect.
The transverse Doppler effect is really another test of time dilation. If we rewrite Eq. 37.37 in terms of the period $T$ of oscillation of the emitted light wave instead of the frequency, we have, because $T = 1/f$,

$$T = \frac{T_0}{\sqrt{1 - \beta^2}} = \gamma T_0,$$  \hspace{1cm} (37.39)

in which $T_0 (= 1/f_0)$ is the proper period of the source. As comparison with Eq. 37.9 shows, Eq. 37.39 is simply the time dilation formula because a period is a time interval.

Taken from Halliday

Relativistic momentum

Consider a particle moving with constant speed $v$ in the positive direction of an $x$ axis. Classically, its momentum has magnitude

$$p = mv = m \frac{\Delta x}{\Delta t} \quad \text{(classical momentum),}$$  \hspace{1cm} (37.40)

in which $\Delta x$ is the distance it travels in time $\Delta t$. To find a relativistic expression for momentum, we start with the new definition

$$p = m \frac{\Delta x}{\Delta t_0}.$$

Here, as before, $\Delta x$ is the distance traveled by a moving particle as viewed by an observer watching that particle. However, $\Delta t_0$ is the time required to travel that distance, measured not by the observer watching the moving particle but by an observer moving with the particle. The particle is at rest with respect to this second observer, thus that measured time is a proper time.

Using the time dilation formula, $\Delta t = \gamma \Delta t_0$ (Eq. 37.9), we can then write

$$p = m \frac{\Delta x}{\Delta t_0} = m \frac{\Delta x}{\Delta t} \cdot \frac{\Delta t}{\Delta t_0} = m \frac{\Delta x}{\Delta t} \gamma.$$

However, since $\Delta x/\Delta t$ is just the particle velocity $v$, we have

$$p = \gamma mv \quad \text{(momentum),}$$  \hspace{1cm} (37.41)

Relativistic energy

An object's mass $m$ and the equivalent energy $E_0$ are related by

$$E_0 = mc^2,$$  \hspace{1cm} (37.43)

which, without the subscript 0, is the best-known science equation of all time. This energy that is associated with the mass of an object is called mass energy or rest energy. The second name suggests that $E_0$ is an energy that the object has even when it is at rest, simply because it has mass. (If you continue your study of physics beyond this book, you will see more refined discussions of the relation between mass and energy. You might even encounter disagreements about just what that relation is and means.)
\[ E = E_0 + K = mc^2 + K. \]  
\[ (37-47) \]

Although we shall not prove it, the total energy \( E \) can also be written as

\[ E = \gamma mc^2, \]  
\[ (37-48) \]

where \( \gamma \) is the Lorentz factor for the object’s motion.

**Relativistic kinetic energy**

\[ K = E - mc^2 = \gamma mc^2 - mc^2 \]
\[ = mc^2(\gamma - 1) \quad \text{(kinetic energy)}, \]  
\[ (37-52) \]

where \( \gamma = \frac{1}{\sqrt{1 - (\nu/c)^2}} \) is the Lorentz factor for the object’s motion.

---

**Momentum and Kinetic Energy**

In classical mechanics, the momentum \( p \) of a particle is \( mv \) and its kinetic energy \( K \) is \( \frac{1}{2}mv^2 \). If we eliminate \( \nu \) between these two expressions, we find a direct relation between momentum and kinetic energy:

\[ p^2 = 2Km \quad \text{(classical).} \]
\[ (37-53) \]

We can find a similar connection in relativity by eliminating \( \nu \) between the relativistic definition of momentum (Eq. 37-41) and the relativistic definition of kinetic energy (Eq. 37-52). Doing so leads, after some algebra, to

\[ K + mc^2 = \gamma mc^2 \]

\[ (K + mc^2)^2 = \gamma^2 m^2 c^4 \]
\[ K^2 + 2Kmc^2 + m^2c^4 = \gamma^2 m^2c^4 \]

\[ K^2 + 2Kmc^2 = (\gamma^2 - 1)m^2c^4 = \left(\frac{c^2}{c^2 - v^2} - \frac{c^2 - v^2}{c^2 - v^2}\right)(m^2c^4) = \left(\frac{v^2}{c^2 - v^2}\right)(m^2c^4) \]

\[ p = \gamma mv \]

\[ p^2 = \gamma^2 m^2v^2 = \frac{m^2v^2}{c^2 - v^2} = \frac{m^2c^2v^2}{c^2 - v^2} \]

\[ p^2c^2 = \frac{m^2c^4v^2}{c^2 - v^2} \]

\[ K^2 + 2Kmc^2 = p^2c^2 \]

\[ E = mc^2 + K \]

\[ E^2 = m^2c^4 + 2mc^2K + K^2 = (pc)^2 + (mc^2)^2 \]

**Fig. 37-15** A useful memory diagram for the relativistic relations among the total energy \( E \), the rest energy or mass energy \( mc^2 \), the kinetic energy \( K \), and the momentum magnitude \( p \).

Taken from Halliday

Problems: 26.4, 26.19, 26.23