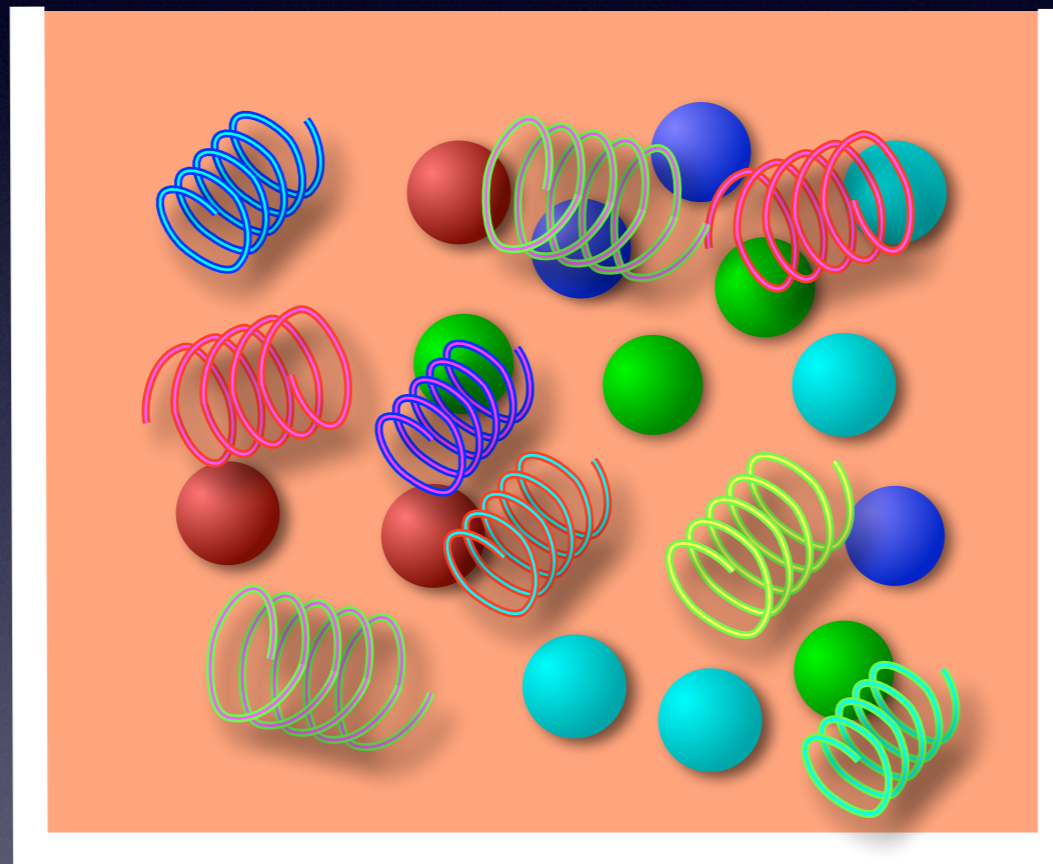


Particle productions from chiral matter

Yoshimasa Hidaka
(RIKEN)

Quark gluon plasma as chiral matter



 quark

 gluon

Temperature
200 MeV
 $\sim 2 \times 10^{13}$ K

Quarks are almost massless Dirac fermion

$$m_q/T \sim 0.03$$

Two topics

**Electric conductivity
in a magnetic field**

**Dilution production
in a magnetic field and vorticity**

Motivation

Strong magnetic field and rotation
in heavy ion collisions

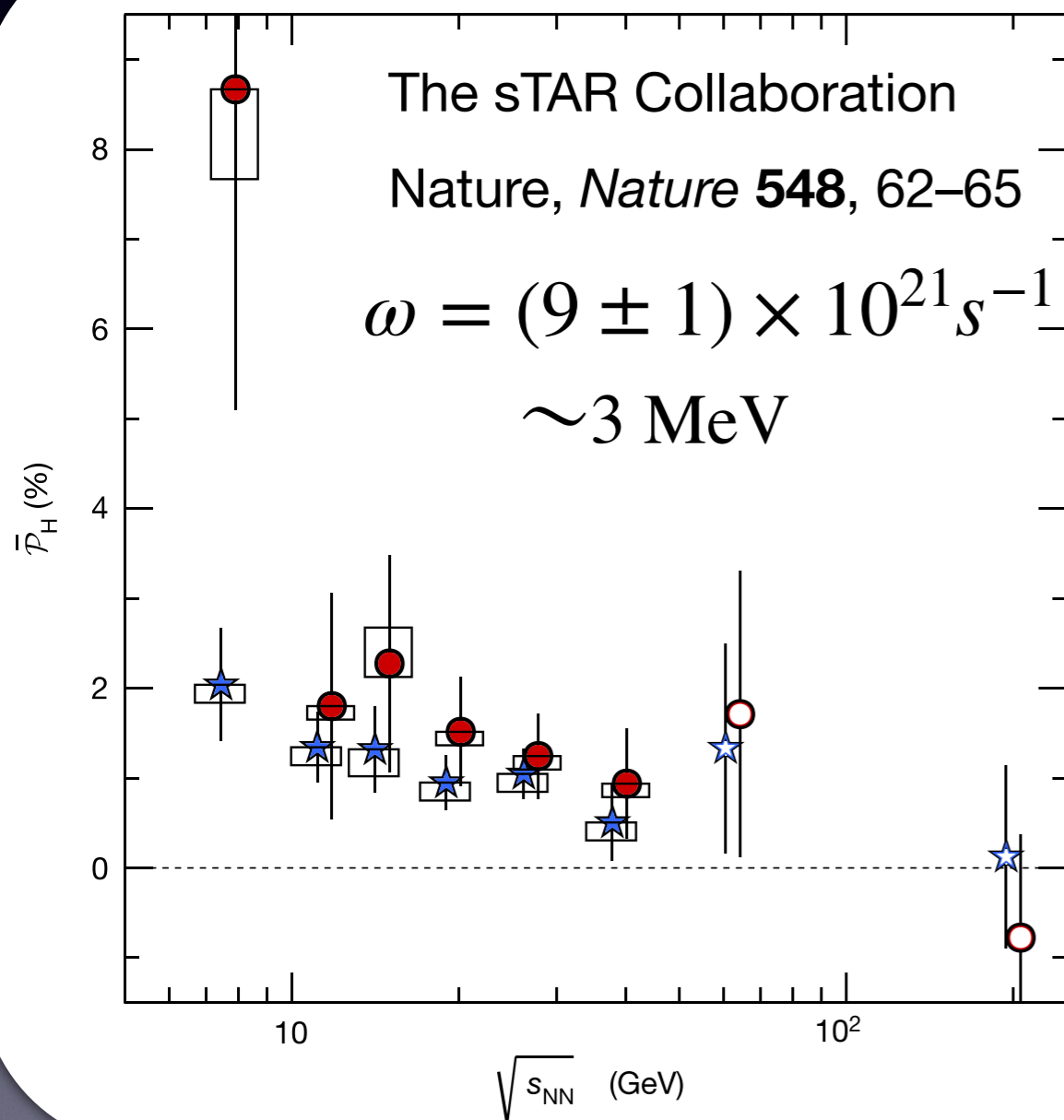


B 
 ω



Vorticity in HIC

Lambda polarization

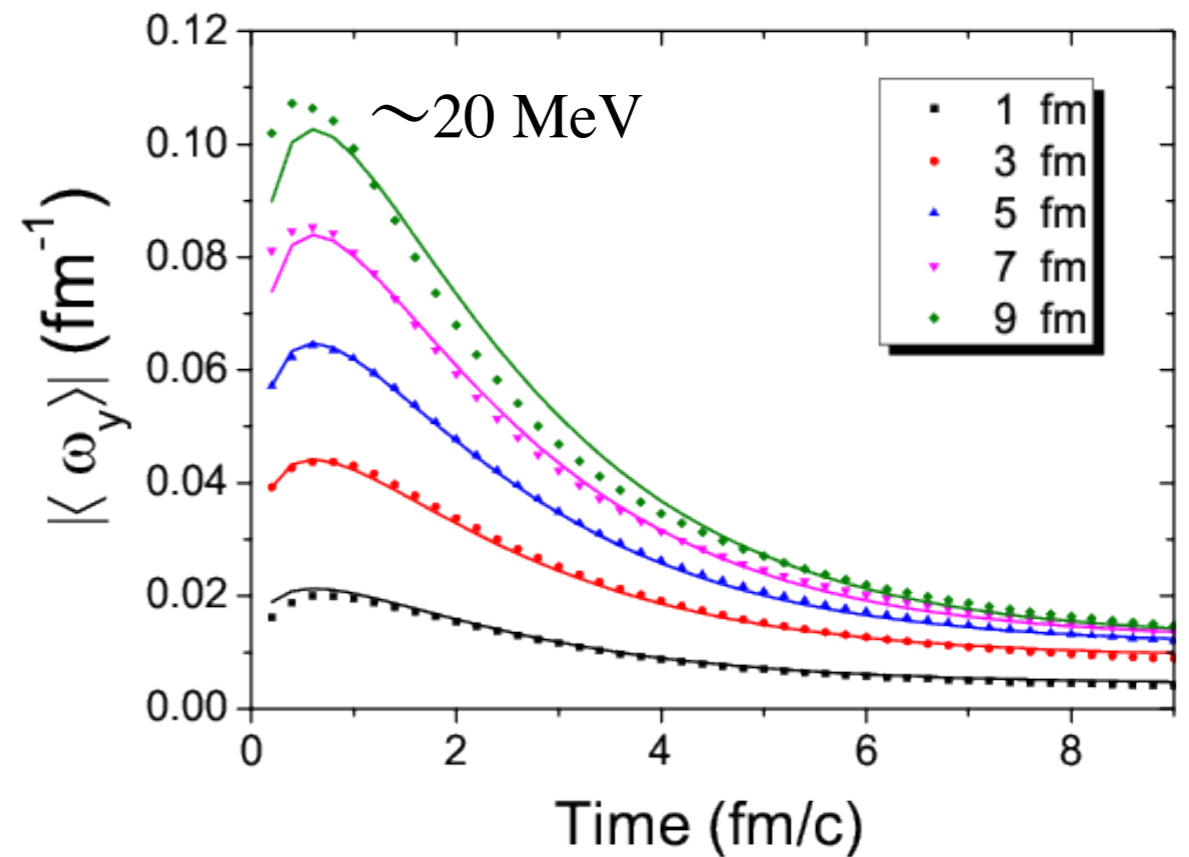


AMPT simulation

Jiang, Z. W. Lin and J. Liao

Phys. Rev. C 94, 044910 (2016)

Erratum: [Phys. Rev. C 95, no. 4, 049904 (2017)]



Chiral effect in QGP

$$J = \xi_B B + \xi_\omega \omega$$

Chiral magnetic effect

Kharzeev, McLerran, Warringa ('08)
Fukushima, Kharzeev, Warringa ('08)

$$\xi_B \sim \mu_5$$

Chiral vortical effect

Son, Surowka ('09),
Landsteiner, Megias, Pena-Benitez (11)

$$\xi_\omega \sim \mu_5 \mu$$

$$J_5 = \zeta_B B + \zeta_\omega \omega$$

Chiral separation effect

Son, Zhitnitsky ('04)
Metlitski, Zhitnitsky ('05)

related to chiral anomaly $\partial_\mu j_5^\mu = CE \cdot B$

Electric conductivity in B

Fukushima YH , Phys. Rev. Lett. 120 (2018) no.16, 162301

Current $J^i = \sigma^{ij} E^j + \dots$

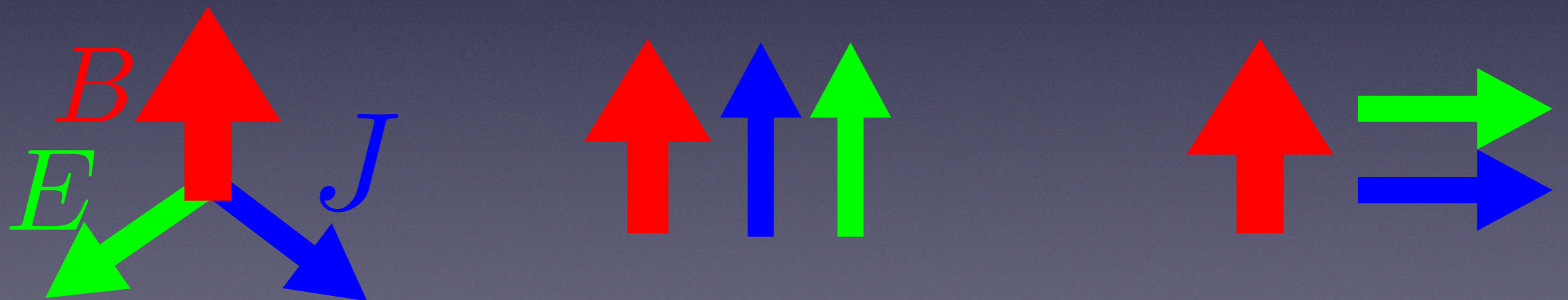
$$\sigma^{ij} = \sigma_H \epsilon^{ijk} \hat{B}^k + \sigma_{\parallel} \hat{B}^i \hat{B}^j + \sigma_{\perp} (\delta^{ij} - \hat{B}^i \hat{B}^j)$$

Hall

longitudinal

perpendicular

Related to CME



Strong B, chiral limit

Chiral anomaly

$$\partial_{\mu} j_{55}^{\mu} = CE \cdot B \longrightarrow n_5 \propto tE \cdot B$$

CME

$$j_{\text{CME}} \sim \mu_5 B \sim n_5 B \quad \text{implies } \sigma_{\parallel} \rightarrow \infty$$

in cond-mat

Interaction with
phonon or impurity

$$t \rightarrow \tau_R$$

$$\sigma_{\parallel} \propto \tau_R B^2 + \text{Ohmic term}$$

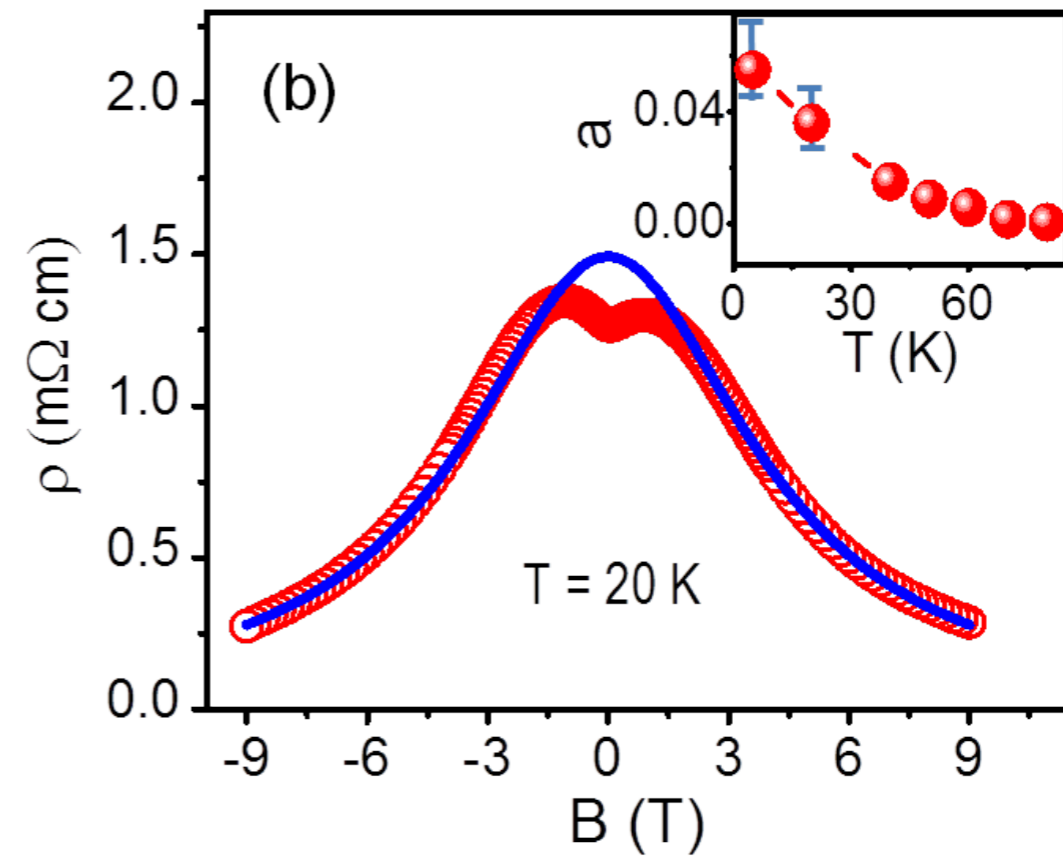
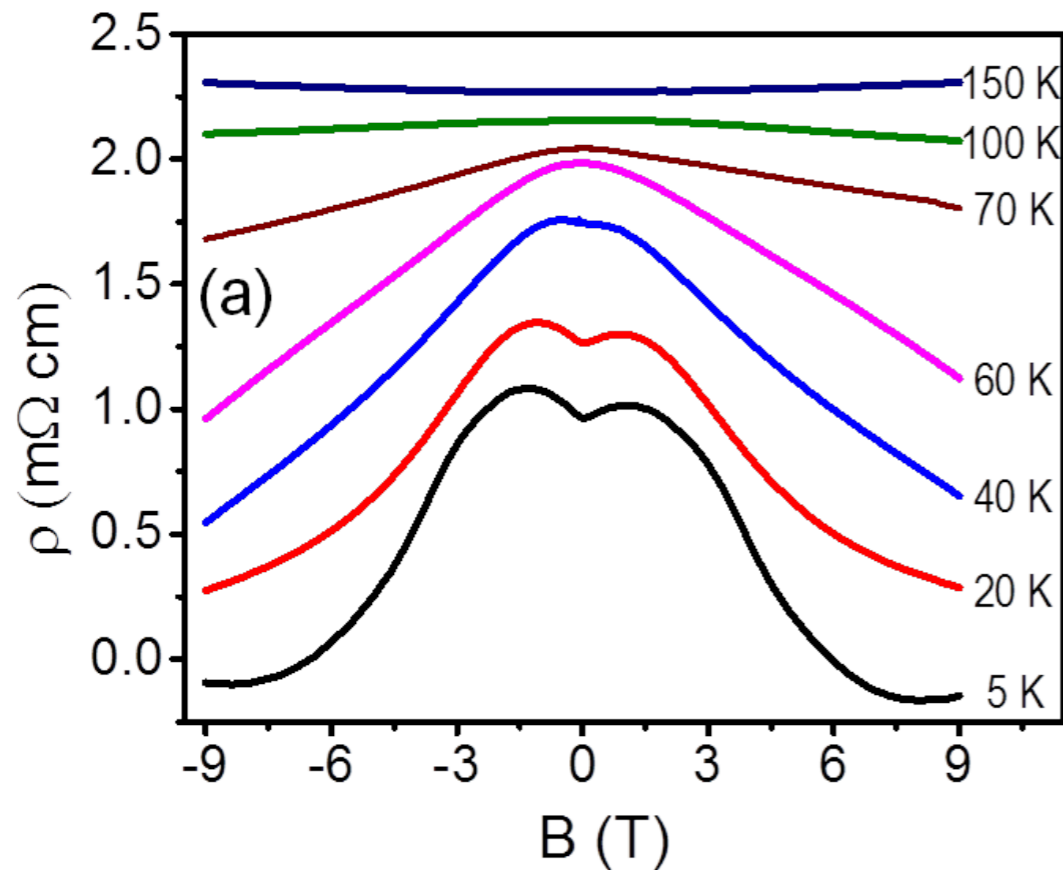
in QCD

explicit breaking

by m_q

chiral magnetic effect in cond-mat.

Q Li, et al, Nature Physics 12, 550-554 (2016)

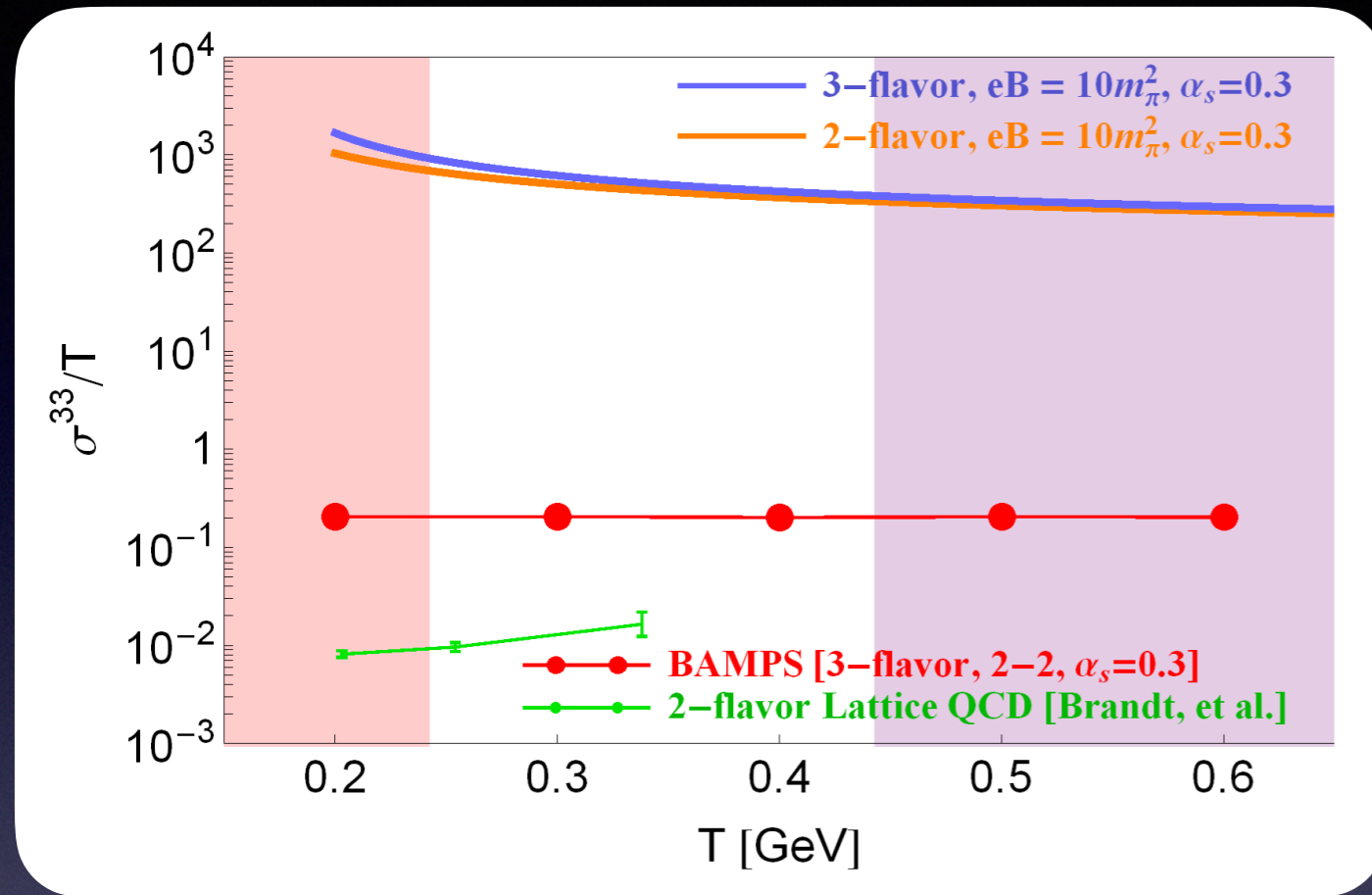


$$\text{resistance : } \rho = \frac{1}{\frac{\tau}{\chi}(CB)^2 + \sigma_{\text{Ohm}}}$$

Conductivity of QCD in strong B

Hattori, Satow, Phys. Rev. D94 (2016) 114032

Hattori, Li, Satow, Yee Phys.Rev. D95 (2017) no.7, 076008



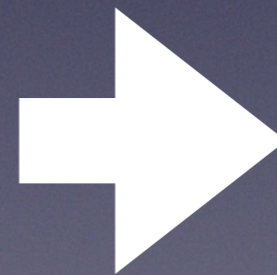
$$\sqrt{eB} \gg T$$

$$\mu = 0$$

$$\sigma_{\parallel}/T \sim \frac{eB}{m_q^2 g^2}$$

Strong B

Effective 1+1 dynamics
+chiral symmetry



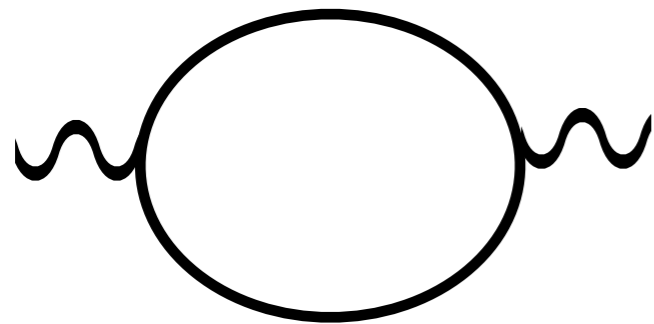
suppression of
interactions

Electric conductivity in B

current $J^i = \sigma^{ij} E^j + \dots$

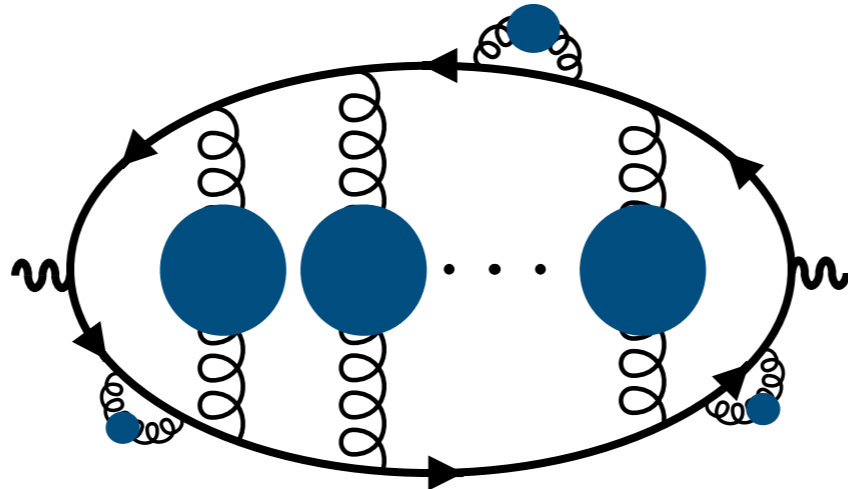
$$\sigma^{ij} = \underbrace{\sigma_H \epsilon^{ijk} \hat{B}^k}_{\text{Hall}} + \underbrace{\sigma_{\parallel} \hat{B}^i \hat{B}^j}_{\text{longitudinal}} + \underbrace{\sigma_{\perp} (\delta^{ij} - \hat{B}^i \hat{B}^j)}_{\text{perpendicular}}$$

one-loop



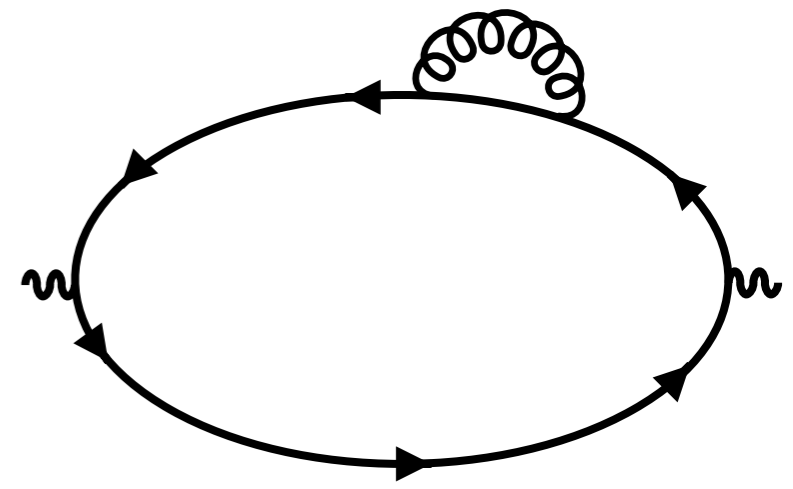
$$\sigma_H = \frac{n_e}{B}$$

resume



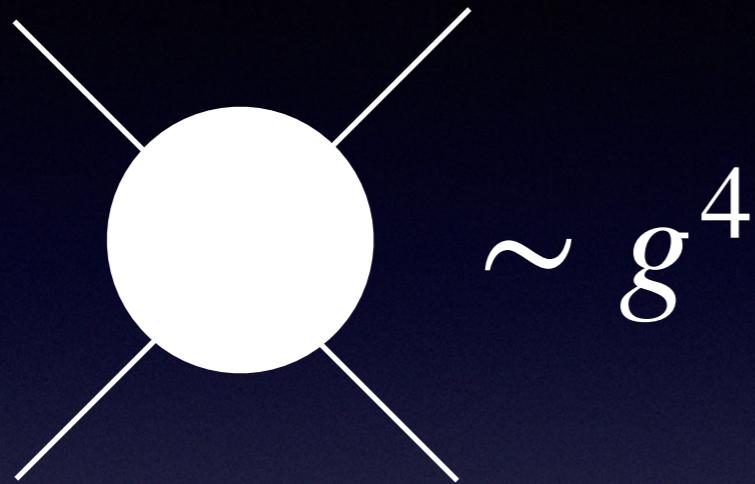
$$\frac{\sigma_{\parallel}}{T} \sim \frac{1}{g^n} F(T^2/eB)$$

two loop

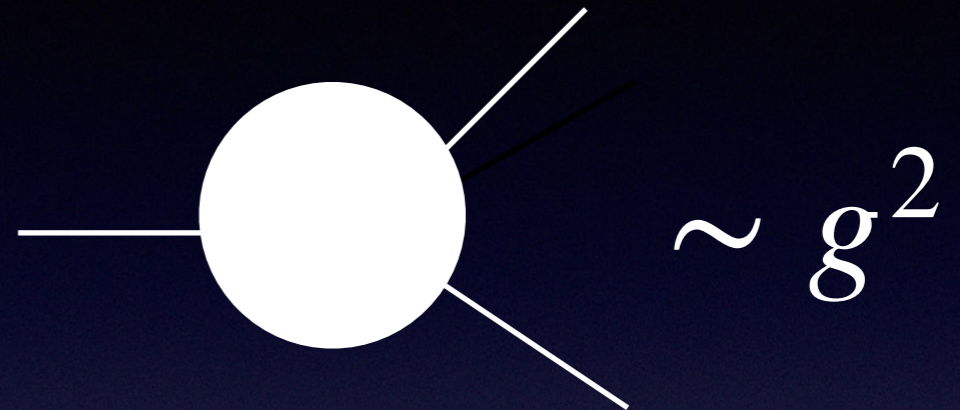


$$\frac{\sigma_{\perp}}{T} \sim \frac{g^2 T^2}{|eB|}$$

Scattering v.s. radiation



Leading contribution to conductivity



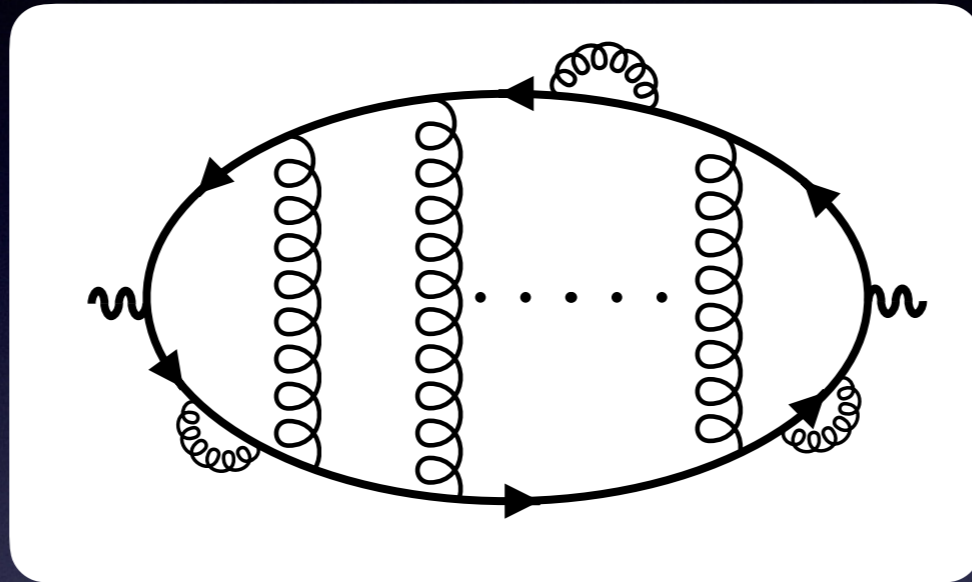
Usually suppress by kinematics

The situation is
different in B!

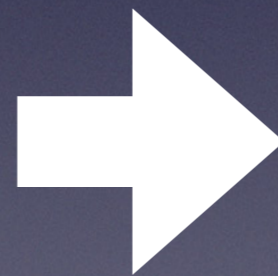
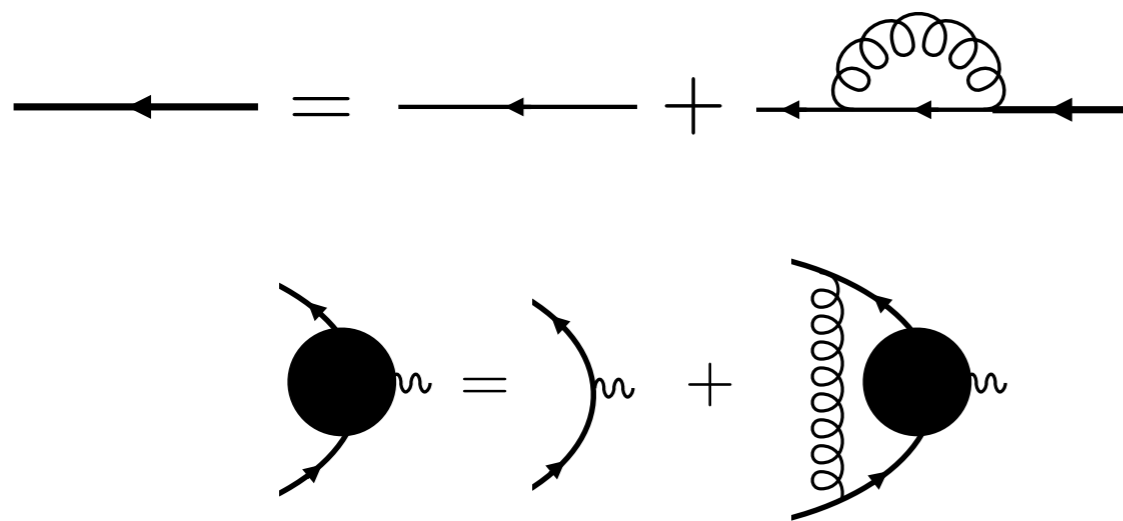
ex) Synchrotron radiation

Longitudinal conductivity $\sigma_{||}$

Infinitely diagrams contribute to the conductivity



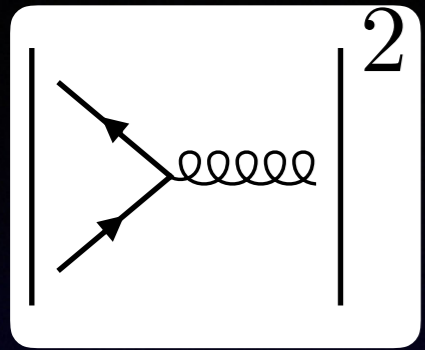
which is generated by



Solving linearized
Boltzmann Eq.

cf. Jeon, Phys Rev. D 52 (1995) 3591
Hidaka, Kunihiro, Phys. Rev. D83 (2011) 076004
Fukushima, YH (2018)

Just complicated



$$\sim \sum_{l,s,c} \int \frac{d^2 p_{\perp}}{(2\pi)^2} |\mathcal{M}_{p+p' \rightarrow k}|^2 = X(n, n', \xi)$$

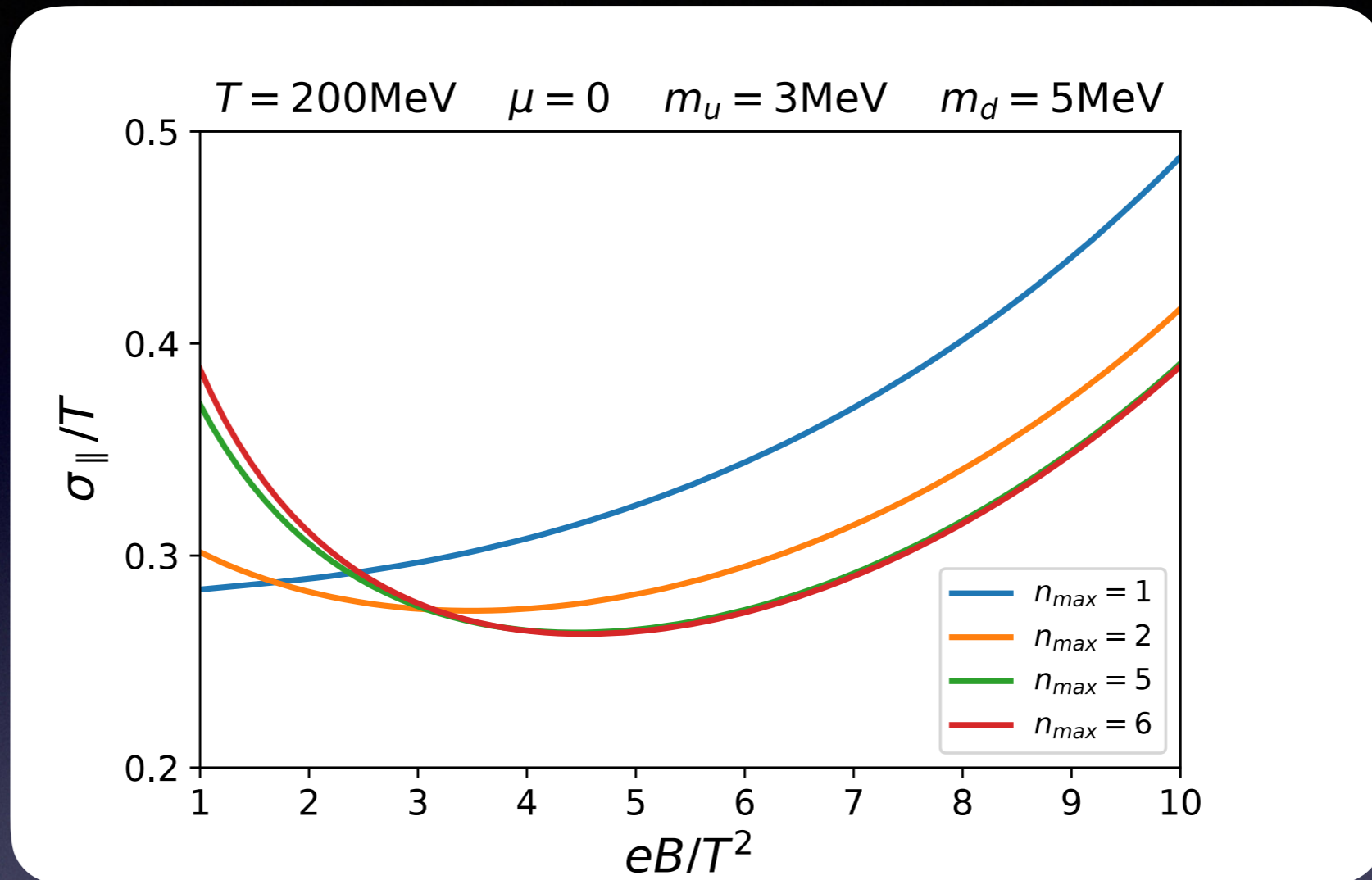
$$X(n, n', \xi) = g^2 N_c C_F \frac{|q_f B|}{2\pi} e^{-\xi} \frac{n!}{n'} \xi^{n'-n} \times \left\{ \left[4m_f^2 - 4|q_f B| (n + n' - \xi) \frac{1}{\xi} (n + n') \right] F(n, n', \xi) + 16|q_f B| n'(n + n') \frac{1}{\xi} L_n^{(n'-n)}(\xi) L_{n-1}^{(n'-n)}(\xi) \right\}$$

$$F(n, n', \xi) = \left[L_n^{(n'-n)}(\xi) \right]^2 + \frac{n'}{n} \left[L_{n-1}^{(n'-n)}(\xi) \right]^2$$

Laguerre Polynomials

$$\xi = \frac{(\varepsilon_{fn} + \varepsilon_{fn'})^2 - (p_z + p'_z)^2}{2|q_f B|}$$

B dependence



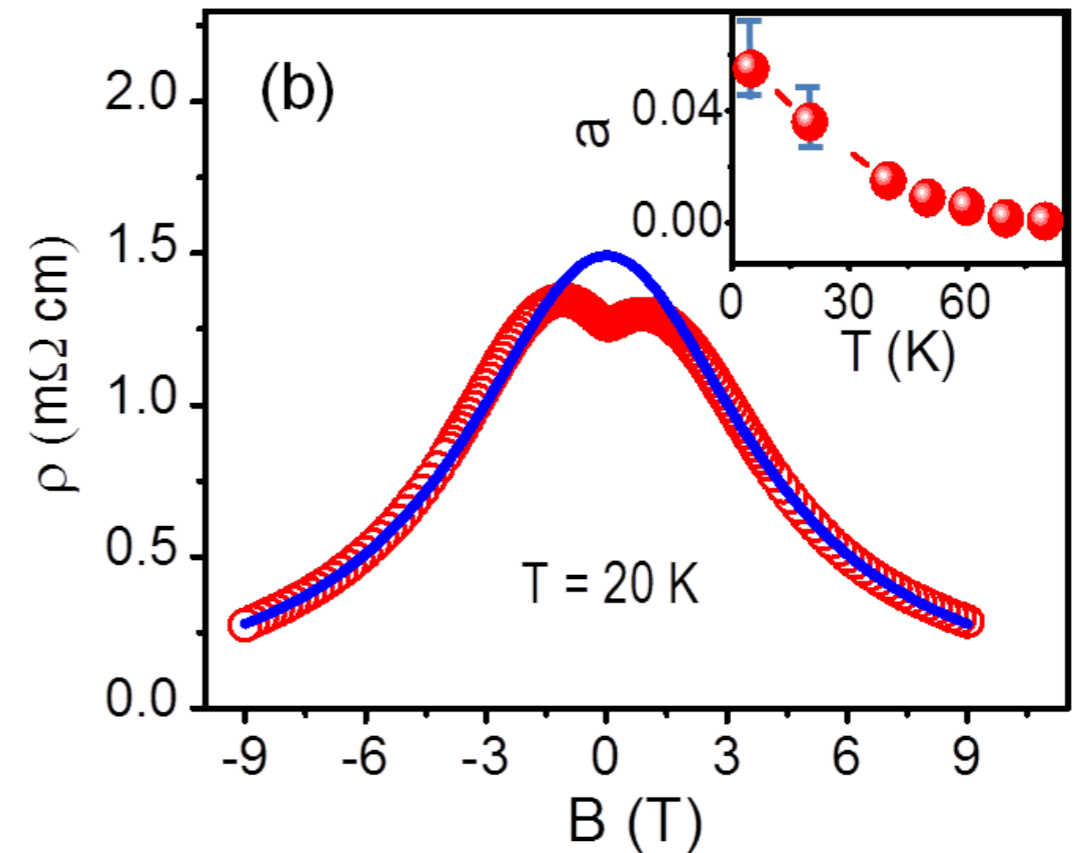
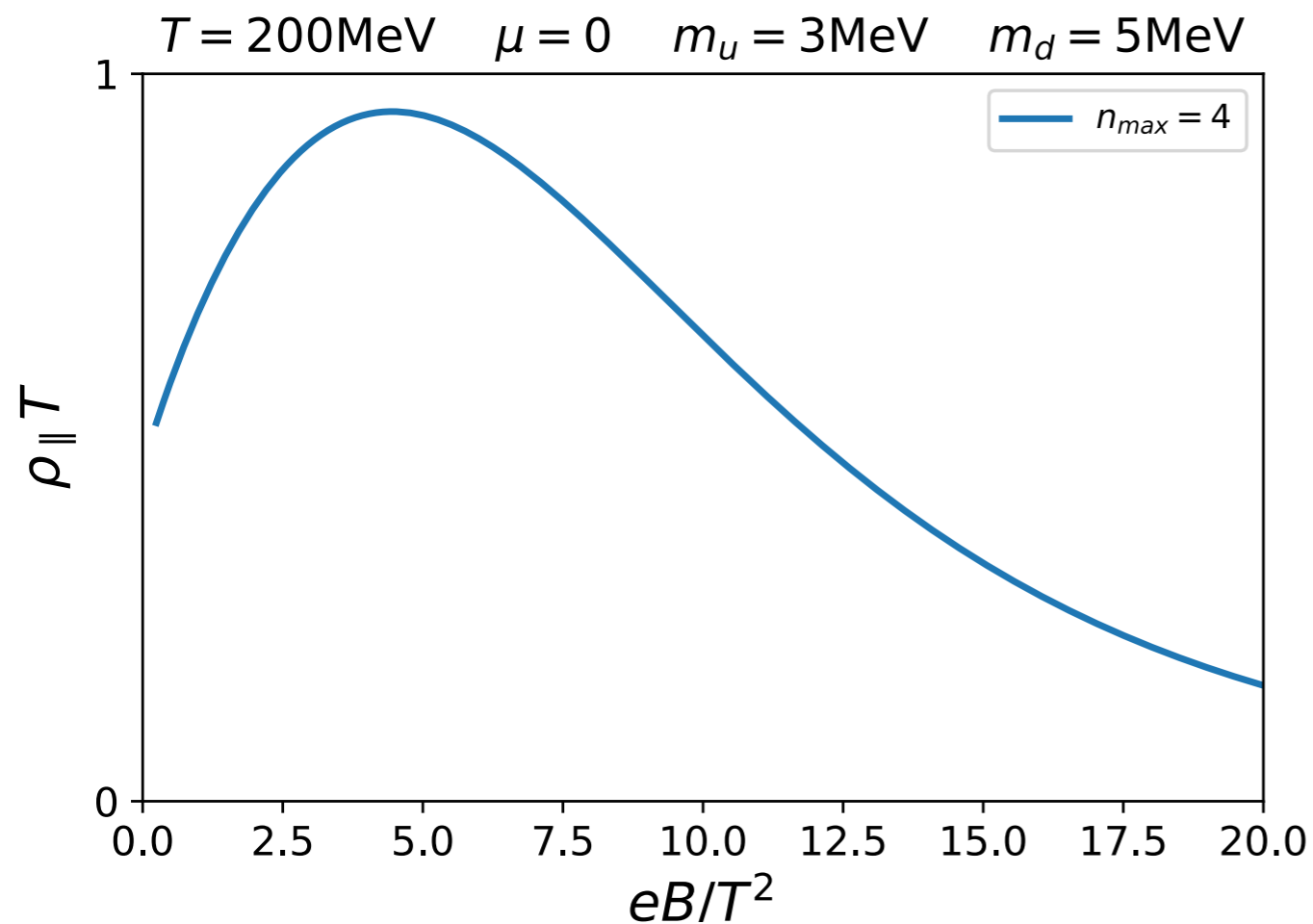
non-monotonic behavior

Degrees of freedom $\sim \frac{eB}{2\pi}$
v.s.

Higher Landau level is suppressed by Boltzmann factor: $\exp(-\sqrt{eBn}/T)$

B dependence

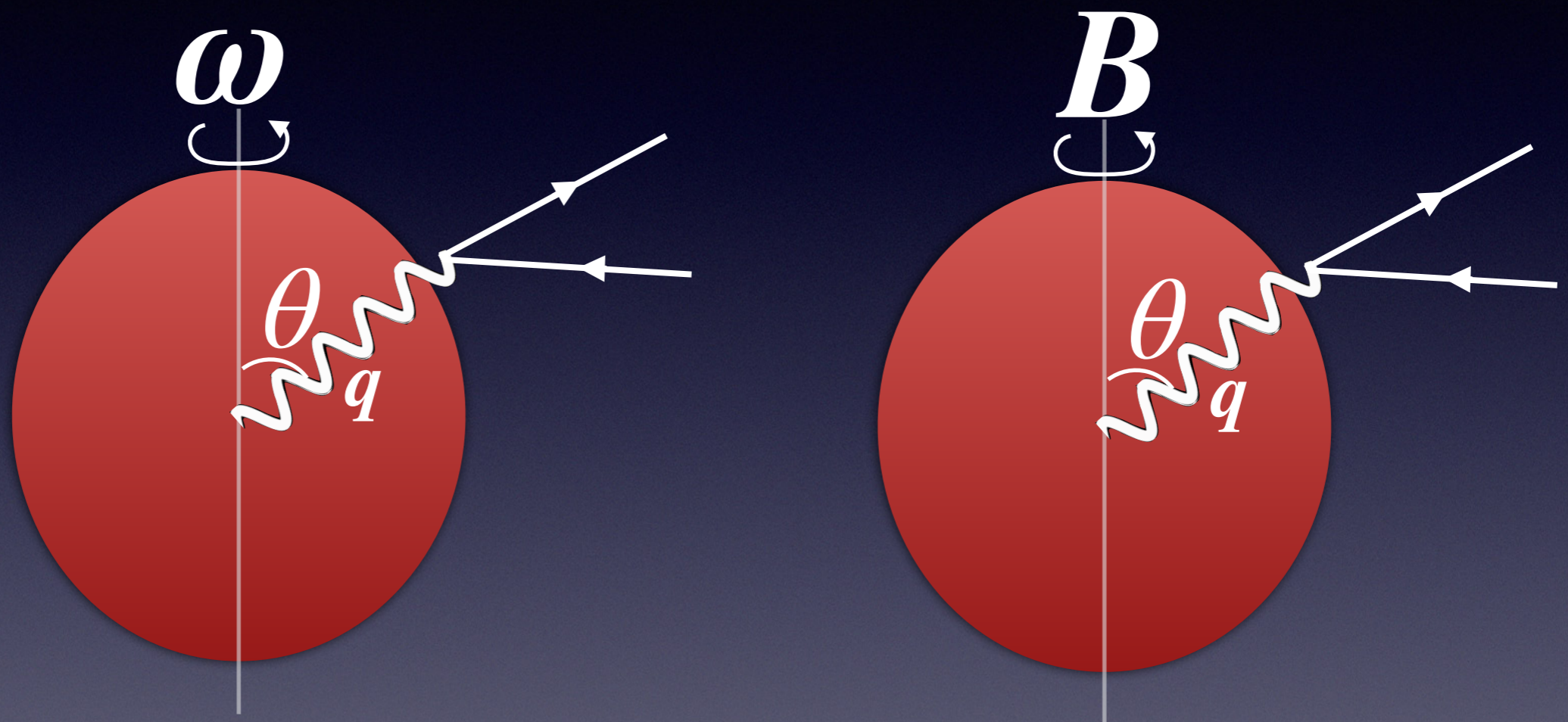
Li Li, Kharzeev, Zhang, Huang, Pletikosic,
Fedorov, Zhong, Schneeloch,
Gu, Valla, Nature Phys. 12, 550 (2016)



Similar behavior

although physical processes are different

Dilepton production



Chiral Kinetic theory

Son, Yamamoto ('12) Stephanov, Yin ('12)

Chiral kinetic equation (CKE)

$$(\partial_t + \dot{\mathbf{x}} \cdot \nabla_x + \dot{\mathbf{p}} \cdot \nabla_p) f = C[f]$$

Equations of motion $\dot{\mathbf{x}} = \hat{\mathbf{p}} + \dot{\mathbf{p}} \times \boldsymbol{\Omega}$

$$\dot{\mathbf{p}} = \dot{\mathbf{x}} \times \mathbf{B} + \mathbf{E}$$

Berry curvature $\boldsymbol{\Omega} = \nabla_p \times \mathbf{a} = \frac{\hat{\mathbf{p}}}{2p^2}$

Anomaly

$$\partial_\mu j^\mu = \frac{1}{4\pi^2} \mathbf{E} \cdot \mathbf{B}$$

Covariant version of Chiral kinetic equation (CKE)

YH, Shi Pu, Yang ('16) ('17)

$$\Delta_{\mu} S^{<\mu} = \Sigma_{\mu}^{<} S^{>\mu} - \Sigma_{\mu}^{>} S^{<\mu} \quad \Delta_{\mu} = \partial_{\mu} + F_{\nu\mu} \partial_{p_{\nu}}$$

$$S^{<\mu} = 2\pi\epsilon(p \cdot n) \left[\delta(p^2) (p^{\mu} + S_n^{\mu\nu} \mathcal{D}_{\nu}) + p_{\nu} \tilde{F}^{\mu\nu} \delta'(p^2) \right] f$$

spin: $S_n^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \frac{p_{\alpha} n_{\beta}}{p \cdot n}$ $\mathcal{D}_{\mu} f = \Delta_{\mu} f + \Sigma_{\mu}^{<} f - \Sigma_{\mu}^{>} \bar{f}$

(local) Equilibrium

$$\text{Current: } J^\mu = 2 \int \frac{d^4 p}{(2\pi)^4} S^{<\mu}(p, X)$$

$$\Rightarrow J = nu + \underbrace{\sigma_B B}_{\text{CME}} + \underbrace{\sigma_\omega \omega}_{\text{CVE}}$$

Dissipative current

CKE with relaxation time approximation

Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy ('16)

Chen, Ishii, Pu, Yamamoto ('16)

YH, Pu, Yang ('17)

$\nabla\mu, \nabla T$ correction

$$\delta J = C_1 \mathbf{E} \times \nabla\mu + C_2 \mathbf{E} \times \nabla T + C_3 \nabla\mu \times \nabla T$$

$$C_i \sim \tau_R$$

low-T



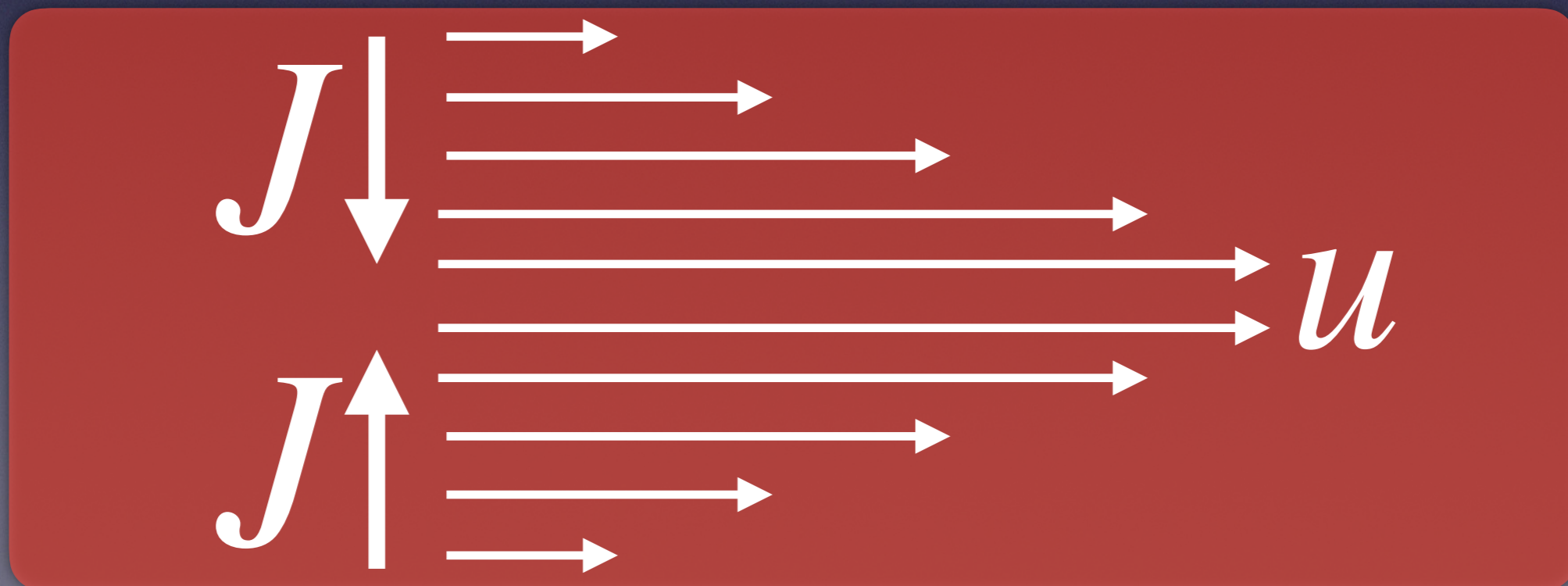
high-T

Dissipative current

YH, Yang ('18)

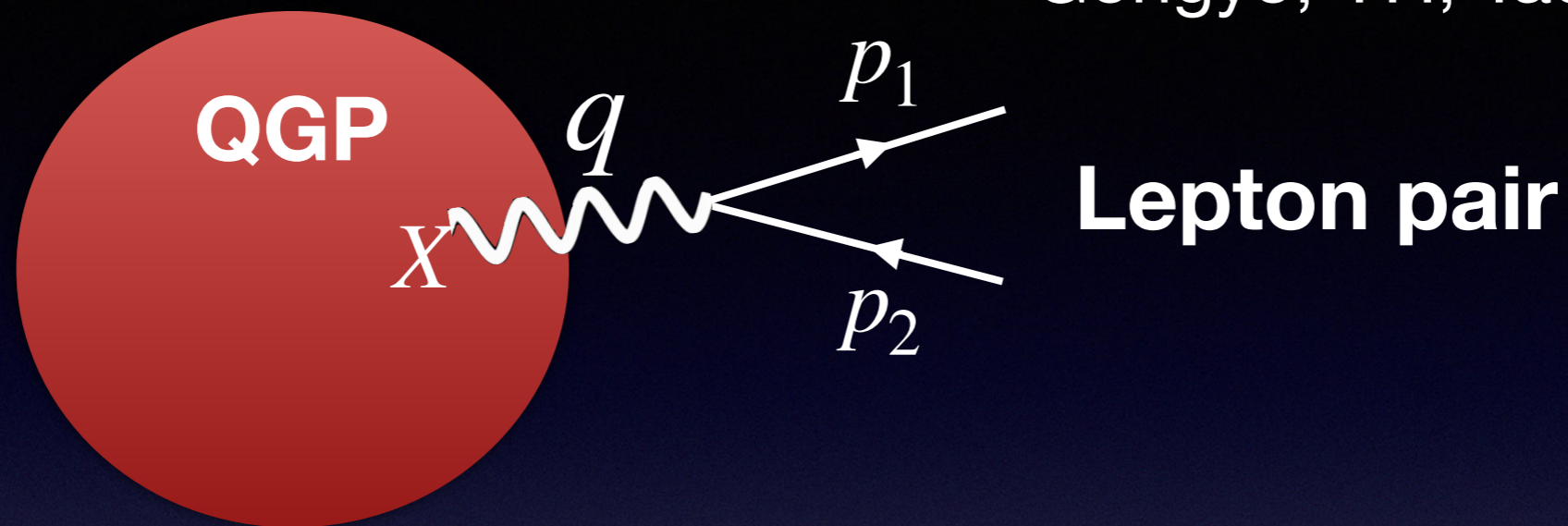
Shear and bulk correction

$$\delta J^i = C_4 \pi^{ij} B_j + C_5 \pi^{ij} \omega_j + C_6 (\nabla \cdot \mathbf{u}) B^i + C_7 (\nabla \cdot \mathbf{u}) \omega^i$$



Dilepton production

Gongyo, YH, Tachibana ('18)



Photon polarization function

$$\Pi^{<\mu\nu}(X, q) = \int d^4s e^{iq \cdot s} \langle j^\nu(X - s/2) j^\mu(X + s/2) \rangle$$

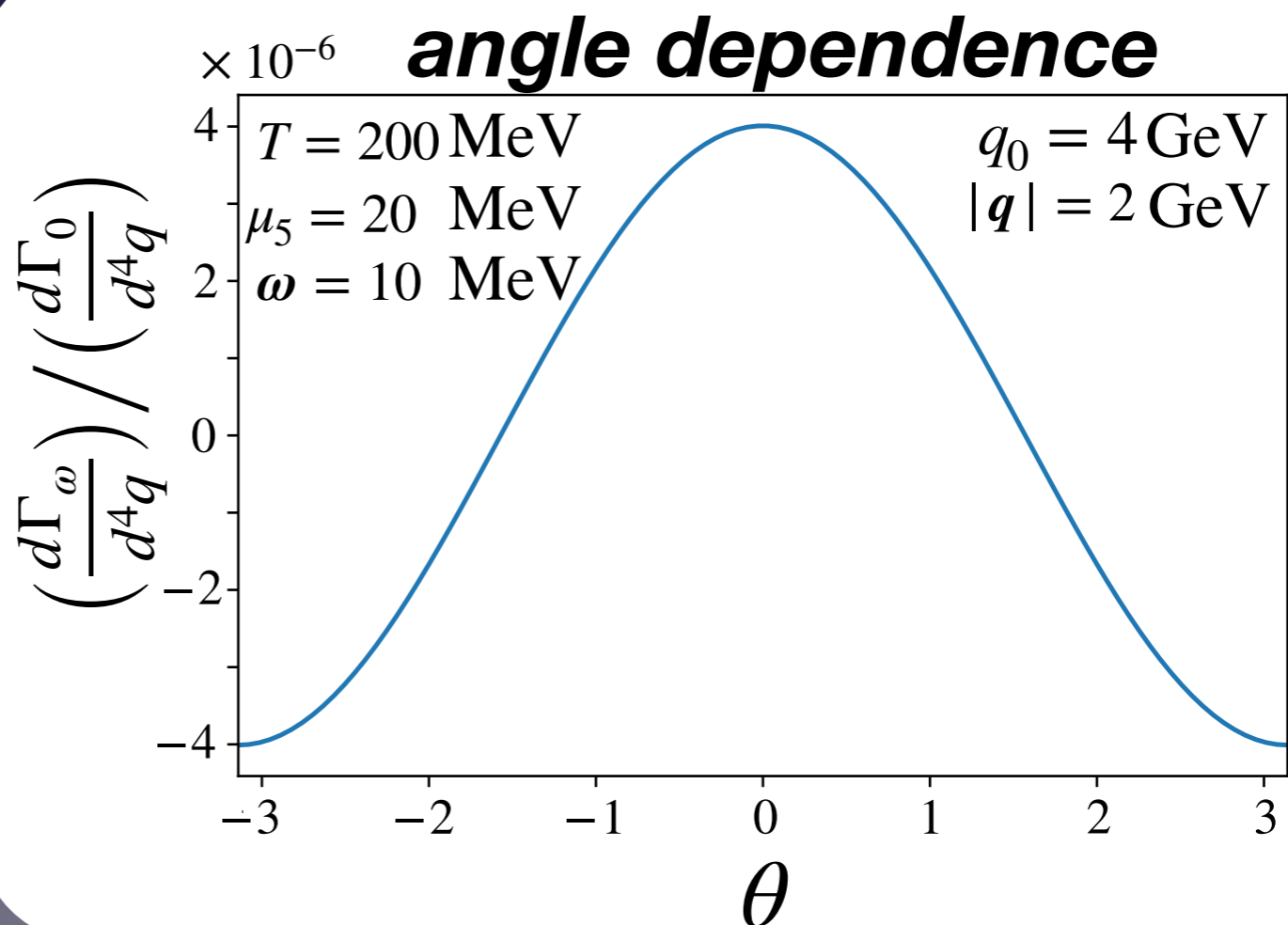
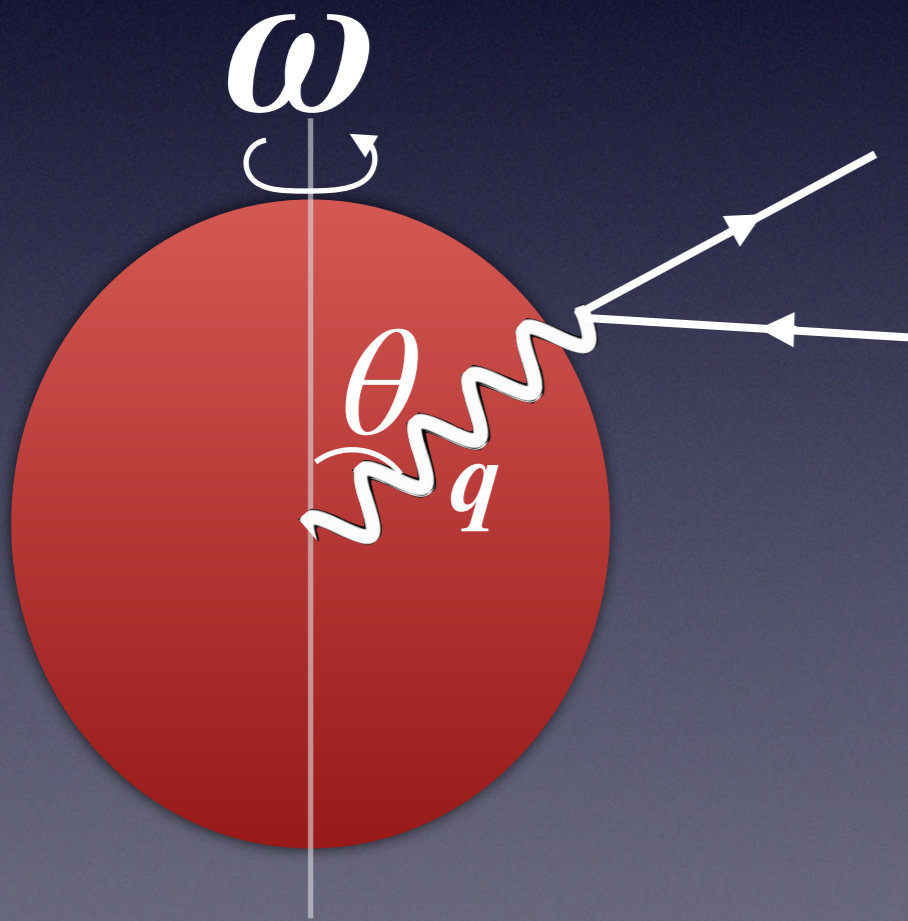
Dilepton production rate

$$\frac{d\Gamma}{d^4q} = - \frac{\alpha}{24\pi^4} \Pi^{<\mu}_{\mu}(q, X)$$

Di-lepton production in ω

Gongyo, YH, Tachibana ('18)

$$\frac{d\Gamma}{d^4q} = \frac{d\Gamma_0}{d^4q} + \frac{d\Gamma_\omega}{d^4q} \quad \text{with} \quad \frac{d\Gamma_\omega}{d^4q} = (\Omega_\gamma \cdot \omega) C_\omega(q)$$
$$C_\omega \sim \mu_5 \quad \Omega_\gamma = \frac{\hat{q}}{|q|^2}$$

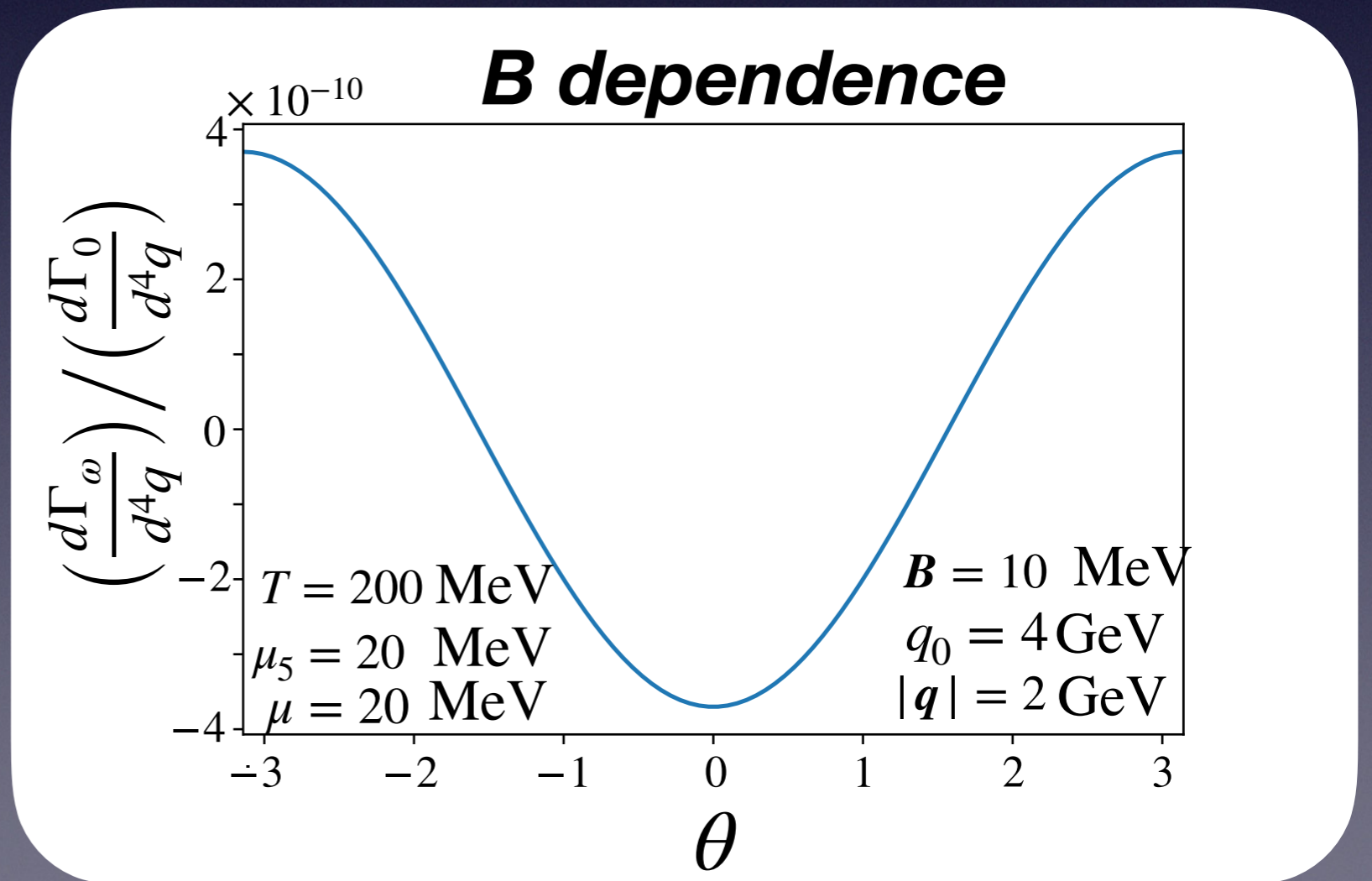
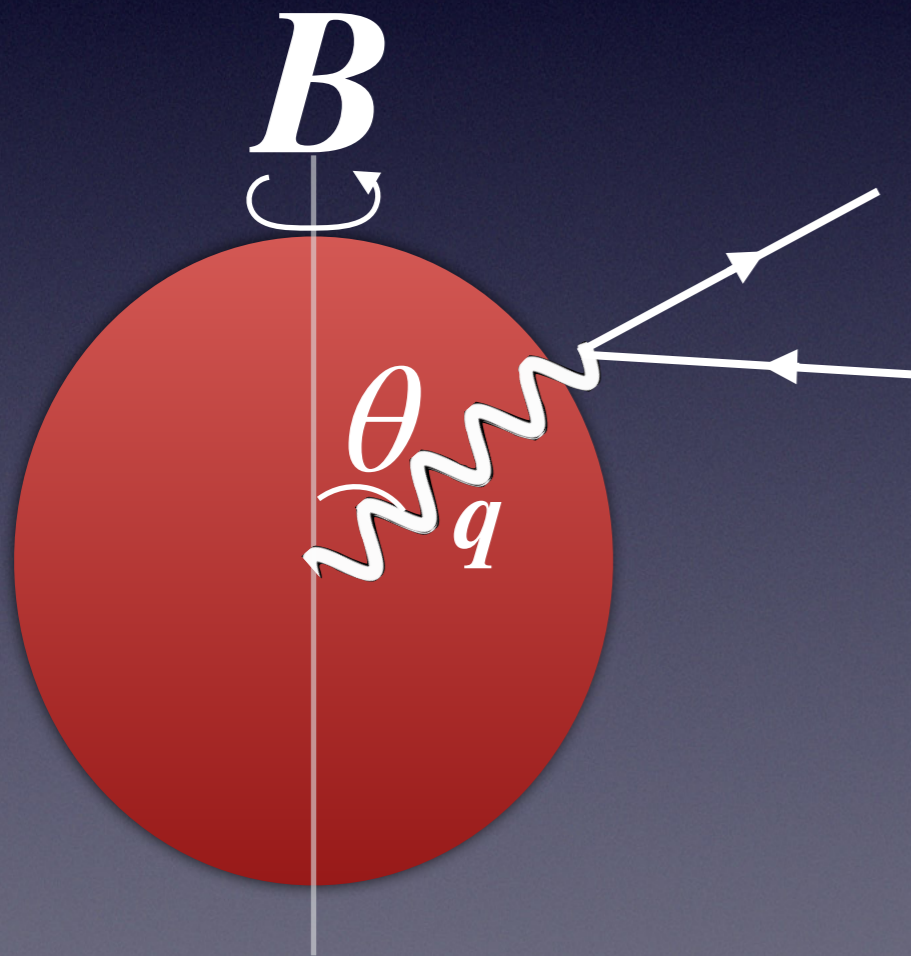


Di-lepton production in B

Gongyo, YH, Tachibana ('18)

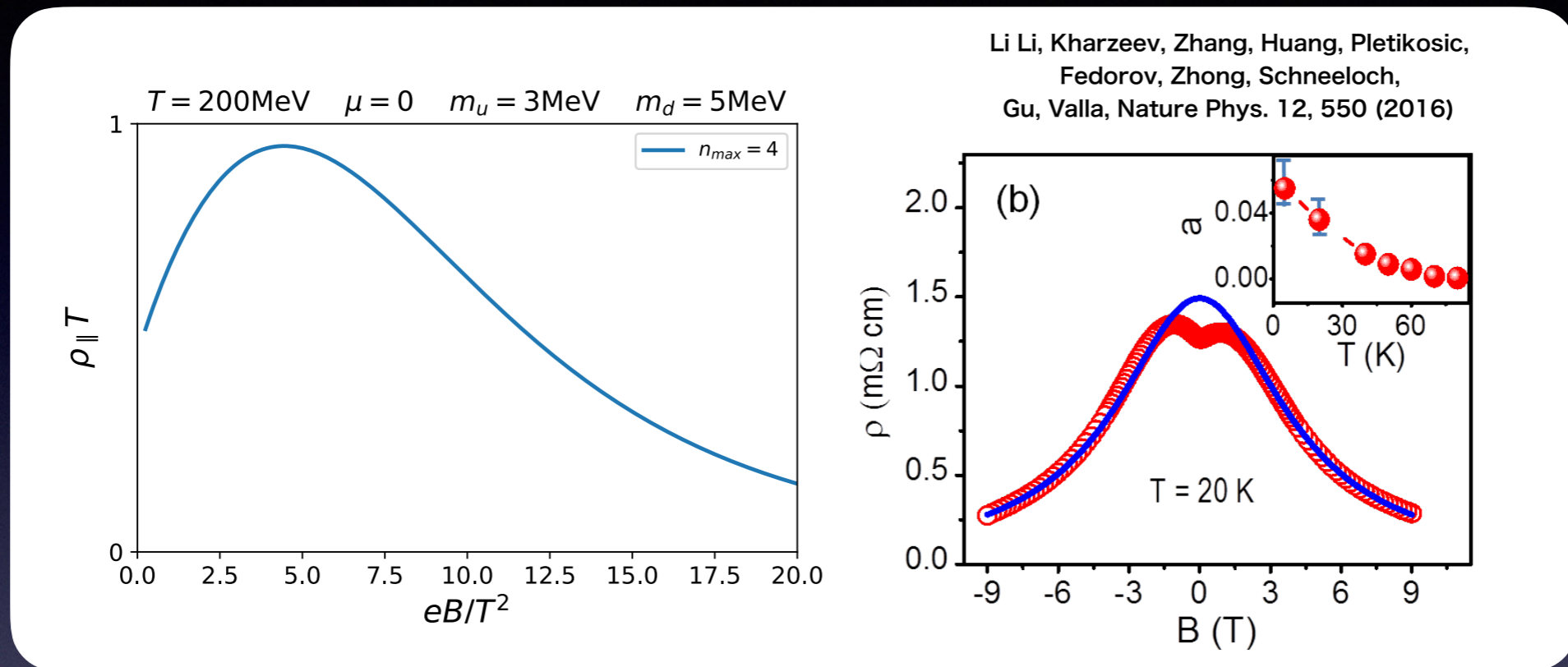
$$\frac{d\Gamma}{d^4q} = \frac{d\Gamma_0}{d^4q} + \frac{d\Gamma_B}{d^4q} \quad \text{with} \quad \frac{d\Gamma_B}{d^4q} = (\boldsymbol{\Omega}_\gamma \cdot \mathbf{B}) C_B(q)$$

$$C_\omega \sim \mu_5 \mu \quad \boldsymbol{\Omega}_\gamma = \frac{\hat{q}}{|q|^2}$$



Summary

Electric conductivity



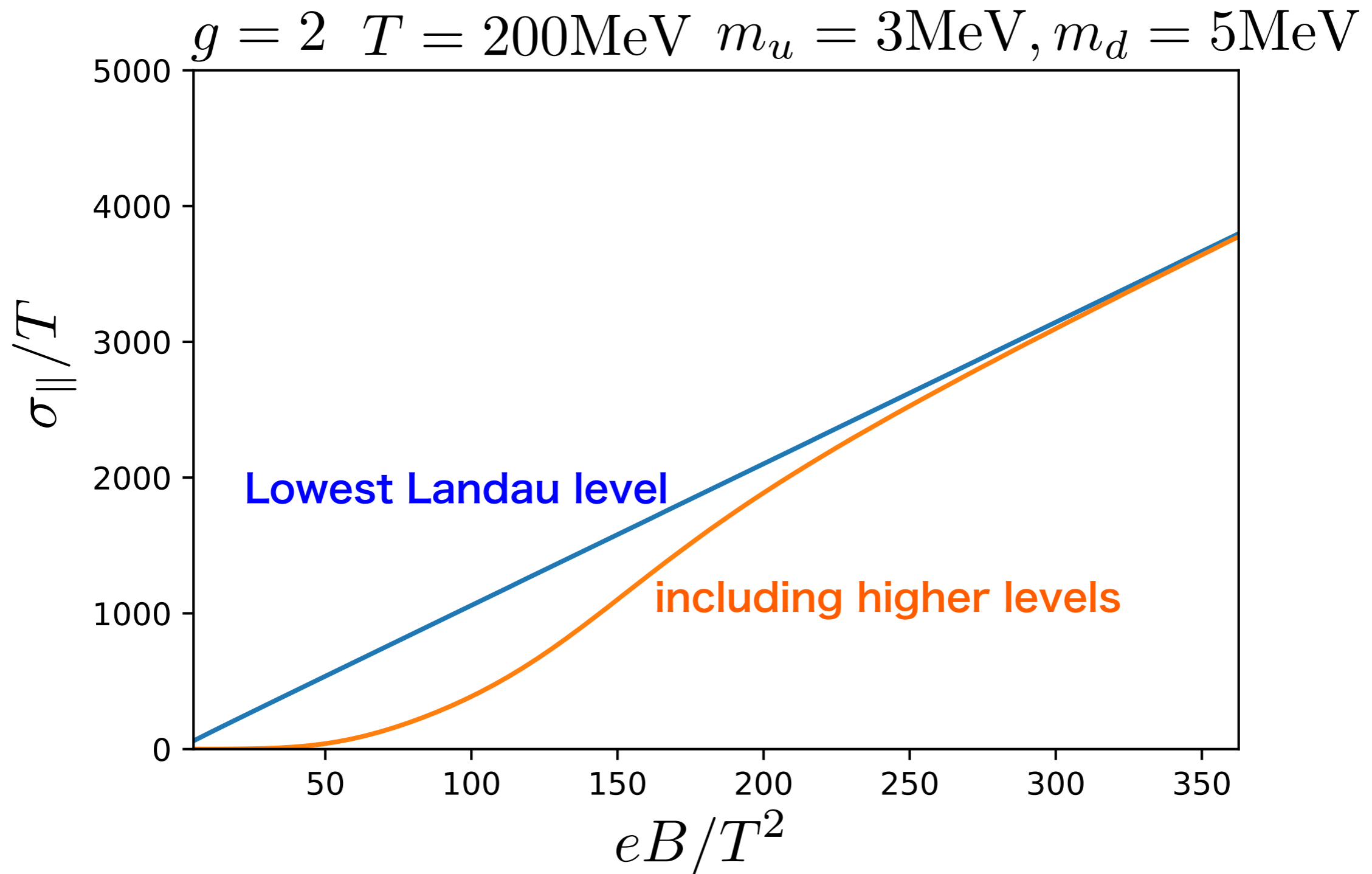
Particle production: Novel chiral effects:

$$\frac{d\Gamma_{\omega}}{d^4q} = (\boldsymbol{\omega} \cdot \boldsymbol{\Omega}_{\gamma}) C_{\omega} \quad \frac{d\Gamma_B}{d^4q} = (\mathbf{B} \cdot \boldsymbol{\Omega}_{\gamma}) C_B$$

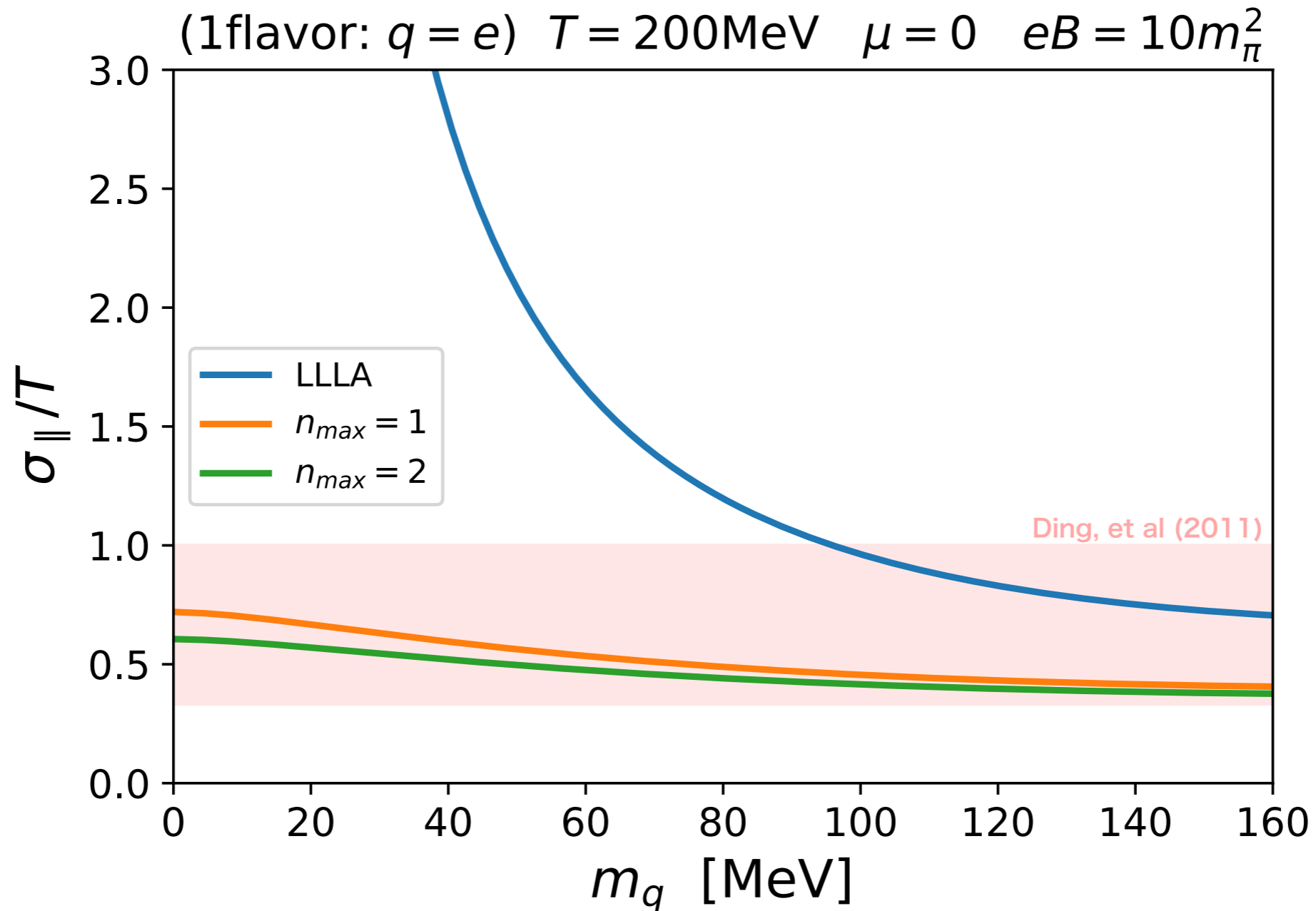
$$\boldsymbol{\Omega}_{\gamma} = \frac{\hat{\mathbf{q}}}{|\mathbf{q}|} \quad C_{\omega} \sim \mu_5 \quad C_B \sim \mu_5 \mu$$

Backup

Large B behavior



Quark mass dependence



For small current mass, higher Landau levels are important

$$\frac{d\Gamma_{\omega}}{d^4q} = (\mathbf{\Omega}_{\gamma} \cdot \boldsymbol{\omega}) C(q)$$

