Kinetic and magnetoresistance/ Weyl metal/ chiral anomaly

Sungkit Yip
Institute of Physics
Institute of Atomic and Molecular Sciences
Academia Sinica
National Center for Theoretical Sciences

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Attempt to formulate magnetoresistance in quasiclassical regime

Kinetic equation (no interference , localization / antilocalization)

Nielsen and Ninomiya (1983) -- quantum regime

Son and Spivak (2013) -- assumed local equilibrium at each Weyl node
each specified by chemical potential
chemical potential difference related to chiral anomaly
deduced magneto-resistance

Kim et al (Pohang, 2014) – Green's function, then kinetic equation

Other work: diffusive limit …

(Q Niu, M-C Chang +collaborators)

Starting point:

$$\dot{\mathbf{r}} = \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} - \dot{\mathbf{k}} \times \mathbf{\Omega}_{\mathbf{k}}$$

[left out m(k) · B]

$$\dot{\mathbf{k}} = -e\mathbf{E} - \frac{e}{c}\dot{\mathbf{r}} \times \mathbf{B}$$

$$\dot{\mathbf{r}} = \frac{1}{1 + \frac{e}{c}\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{B}} \left[\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + e\mathbf{E} \times \mathbf{\Omega}_{\mathbf{k}} + \frac{e}{c}(\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}})\mathbf{B} \right]$$

$$\dot{\mathbf{k}} = \frac{1}{1 + \frac{e}{c}\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{B}} \left[-e\mathbf{E} - \frac{e}{c}(\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} \times \mathbf{B}) - \frac{e^2}{c}(\mathbf{E} \cdot \mathbf{B})\mathbf{\Omega}_{\mathbf{k}} \right]$$

k-space volume $(1 + \frac{e}{c}\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{B})$ (conserved)

dimensionless

$$\sum_{\mathbf{k}} \to \int \frac{d^3k}{(2\pi)^3}(1 + \frac{e}{c}\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{B})$$

$$\frac{\partial}{\partial t}n(\mathbf{r}, \mathbf{k}, t) + \dot{\mathbf{r}}\frac{\partial}{\partial \mathbf{r}}n(\mathbf{r}, \mathbf{k}, t) + \dot{\mathbf{k}}\frac{\partial}{\partial \mathbf{k}}n(\mathbf{r}, \mathbf{k}, t) = I_{col}$$ (k, ..) *

Eqm: $n(\mathbf{r}, \mathbf{k}) = f(\epsilon_{\mathbf{k}} - \mu)$ *both sides vanish*

* one to one correspondence to Kubo formula in linear response

only then we have continuity equation

conservation of number of particles under collision

$$\Sigma_k \; I_{col}\; (r,k,t) = 0$$

$$\frac{\partial n(\mathbf{r},t)}{\partial t} + \nabla \cdot J(\mathbf{r},t) = 0$$

$$\mathbf{J}(\mathbf{r},t) = \sum_{\mathbf{k}} \frac{1}{1+\frac{e}{c}\mathbf{\Omega_k}\cdot\mathbf{B}} \left[\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + e\mathbf{E}\times\mathbf{\Omega_k} + \frac{e}{c}(\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}}\cdot\mathbf{\Omega_k})\mathbf{B}\right] n(\mathbf{r},\mathbf{k},t)$$

J = 0 in equilibrium (when E = 0)

[Appendix]

To first order in E:

$$\mathbf{J^e} = \mathbf{J^e_a} + \mathbf{J^e_{AH}}$$

$$\mathbf{J^e_{AH}} = (-e^2)\int \frac{d^3k}{(2\pi)^3}\mathbf{E}\times\Omega_{\mathbf{k}} f(\epsilon_{\mathbf{k}}-\mu)$$

anomalous Hall

$$\mathbf{J^e_a} = (-e)\int \frac{d^3k}{(2\pi)^3}\left[\frac{\partial \epsilon}{\partial \mathbf{k}} + \frac{e}{c}(\frac{\partial \epsilon}{\partial \mathbf{k}}\cdot\mathbf{\Omega_k})\mathbf{B}\right](n(\mathbf{k}) - f(\epsilon_{\mathbf{k}}-\mu))$$

steady state: $$-\frac{1}{(1+\frac{e}{c}\mathbf{\Omega_k}\cdot\mathbf{B})}\left[e\mathbf{E} + \frac{e}{c}(\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}}\times\mathbf{B}) + \frac{e^2}{c}(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega_k}\right]\cdot\frac{\partial n}{\partial \mathbf{k}} = I_{col}\,(\mathrm{k})$$

$$-\frac{1}{(1+\frac{e}{c}\mathbf{\Omega}_{\mathbf{k}}\cdot\mathbf{B})}\left[e\mathbf{E}+\frac{e}{c}(\frac{\partial\epsilon_{\mathbf{k}}}{\partial\mathbf{k}}\times\mathbf{B})+\frac{e^2}{c}(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}_{\mathbf{k}}\right]\cdot\frac{\partial n}{\partial\mathbf{k}}=I_{col}(\mathrm{k})$$

ordinary Hall effect, ignored from here on ~ $(\omega_c \tau)^2$

1. No Weyl / no chiral anomaly / single Fermi surface
2. Weyl / with chiral anomaly / multiple Fermi surfaces

1. Single Fermi surface

$$-\frac{1}{(1+\frac{e}{c}\mathbf{\Omega_k}\cdot\mathbf{B})}\left[e\mathbf{E}+\frac{e^2}{c}(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega_k}\right]\cdot\frac{\partial n}{\partial \mathbf{k}}=I_{col}(\mathrm{k})$$

relaxation time approx:

$$I_{col}=-\frac{[\,n(\mathbf{k})-f(\epsilon_\mathbf{k}-\mu)\,]}{\tau_0}$$

$$=-\frac{\delta n(\mathbf{k})}{\tau_0}$$

Generally $\Sigma_{k'}\, n(k') / \tau_{k'k}$

$$\delta n(\mathbf{k})\equiv n(\mathbf{k})-f(\epsilon_\mathbf{k}-\mu)$$

relax to equilibrium

I_{col} = 0 at equilibrium

$$\sum_\mathbf{k}\delta n(\mathbf{k})=0$$

[can be checked posteriori →
satisfied if μ is the equilibrium chemical potential
density not modified by E]

advantage:

$\delta n(\mathbf{k})$ available explicitly, since on LHS, n(k) in $\frac{\partial n}{\partial \mathbf{k}}$ can be replaced by f(ε) in linear response

$$\mathbf{J}_{\mathrm{a}}^{\mathrm{e}} = e^2 \int \frac{d^3k}{(2\pi)^3} \left(-\frac{\partial f}{\partial \epsilon}\right) \tau_0 \left[\frac{\partial \epsilon}{\partial \mathbf{k}} + \frac{e}{c}(\frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}})\mathbf{B}\right] \left[\frac{\partial \epsilon}{\partial \mathbf{k}} + \frac{e}{c}(\frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}})\mathbf{B}\right] \cdot \mathbf{E} \, \frac{1}{(1 + \frac{e}{c}\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{B})}$$

from which conductance (tensor) can be read off

possible off-diagonal components

Term linear in B vanishes if time-reversal obeyed $\mathbf{\Omega}_{\mathbf{k}} = -\mathbf{\Omega}_{-\mathbf{k}}$ ε(k) even

positive magneto-conductance

$$\sigma_{a,ij}^{(0)} = e^2 \int \frac{d\omega_{\hat{k}}}{4\pi} \tau_0 \rho_{\hat{k}} \frac{\partial \epsilon}{\partial k_i} \frac{\partial \epsilon}{\partial k_j}$$

B // x:

$$\sigma_{a,xx}^{(2)} = e^2 \int \frac{d\omega_{\hat{k}}}{4\pi} \tau_0 \rho_{\hat{k}} \left(\frac{e}{c}\right)^2 (\frac{\partial \epsilon}{\partial k_y} \Omega_y + \frac{\partial \epsilon}{\partial k_z} \Omega_z)^2 B_x^2$$

$$\sigma_{a,yy}^{(2)} = e^2 \int \frac{d\omega_{\hat{k}}}{4\pi} \tau_0 \rho_{\hat{k}} \left(\frac{e}{c}\right)^2 (\frac{\partial \epsilon}{\partial k_y} \Omega_x)^2 B_x^2$$

$$\frac{\sigma^{(2)}_{a,jj}}{\sigma^{(0)}_{a,jj}} \sim \left(\frac{e}{\hbar c}\tilde{\Omega}B\right)^2 \qquad \frac{\Delta\sigma}{\sigma(0)} \approx 1.79 \times 10^{-11} \left(\frac{\tilde{\Omega}}{\text{Bohr}^2}\right)^2 \left(\frac{B}{\text{Testla}}\right)^2$$

$3 \times 10^4/\text{Bohr}^2$	1 T	1.6%
10^3 /Bohr ^{2}	3 T	0.016%

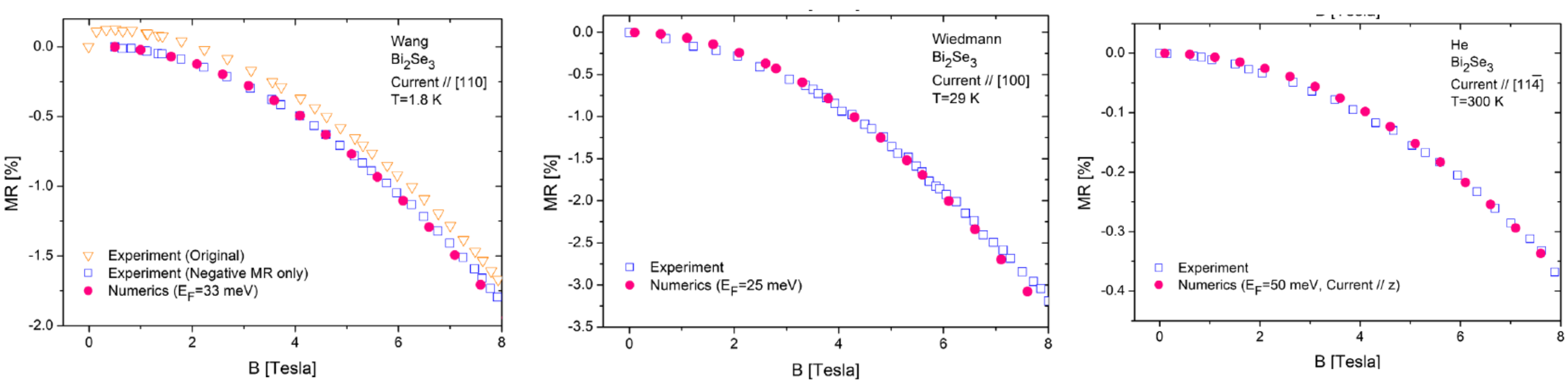
recent claim:

with modifications, including m(k) · B

PRL **119,** 166601 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

Negative Magnetoresistance without Chiral Anomaly in Topological Insulators

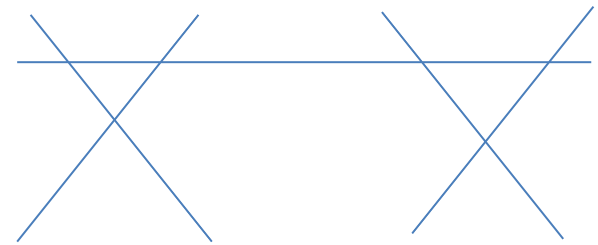
Xin Dai,[1] Z. Z. Du,[2,3,4] and Hai-Zhou Lu[2,4,*]

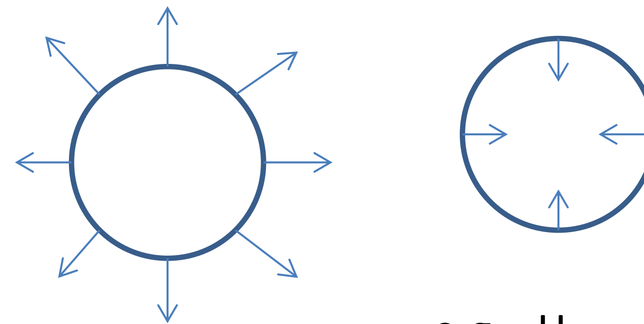


~0.1- 0.5 % in 3 T

2. *Weyl, with multiple Fermi surfaces:*

[not nec separated in k]



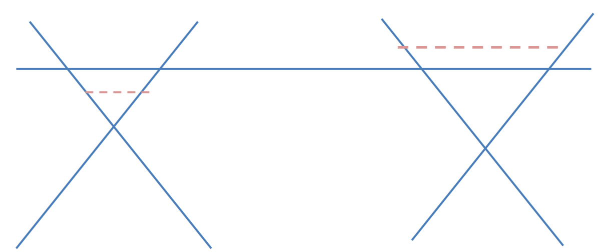


e.g. H= $v^* \mathbf{k} \cdot \tau$

$$\dot{\mathbf{k}} = \frac{1}{1 + \frac{e}{c}\Omega_{\mathbf{k}} \cdot \mathbf{B}} \left[-e\mathbf{E} - \frac{e}{c}\left(\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} \times \mathbf{B}\right) - \frac{e^2}{c}(\mathbf{E} \cdot \mathbf{B})\Omega_{\mathbf{k}} \right]$$

sink and source if (E · B) ≠ 0;

expect chemical potential differences between pockets



Ω_k above and below opposite

Particles disappear / appear from the lower band, total conserved

PHYSICAL REVIEW B **88**, 104412 (2013)

Chiral anomaly and classical negative magnetoresistance of Weyl metals

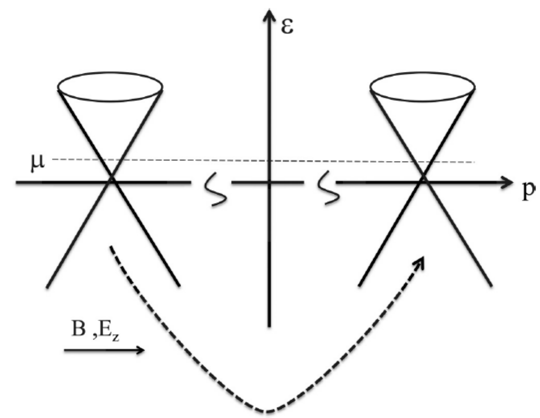
D. T. Son[1] and B. Z. Spivak[2]

$$\frac{\partial n^{(i)}(\epsilon)}{\partial t} + \frac{k^{(i)}}{\rho^{(i)}(\epsilon)} \frac{e^2}{4\pi^2\hbar^2 c}(\mathbf{E}\cdot\mathbf{B})\frac{\partial n^{(i)}(\epsilon)}{\partial \epsilon} = I^{(i)}_{\rm coll}\{n^{(i)}(\epsilon)\}$$

$$\frac{\partial N^{(i)}}{\partial t} + \nabla\cdot\mathbf{j}^{(i)} = k^{(i)}\frac{e^2}{4\pi^2\hbar^2 c}(\mathbf{E}\cdot\mathbf{B}) - \frac{\delta N^{(i)}}{\tau}$$

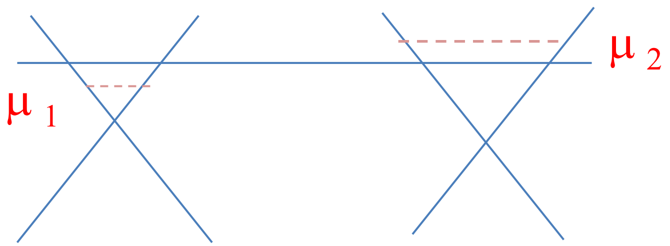
$$\delta n^{(i)}(\epsilon) = -\frac{k^{(i)}}{\rho^{(i)}(\epsilon)}\frac{e^2\tau}{4\pi^2\hbar^2 c}(\mathbf{E}\cdot\mathbf{B})\frac{\partial n_0(\epsilon)}{\partial\epsilon}$$

$$\sigma_{zz} = \frac{e^2}{4\pi^2\hbar c}\frac{v}{c}\frac{(eB)^2 v^2}{\mu^2}\tau.$$



Below: formulation on the same basis as I gave before:

Remark: can apply kinetic equation if μ not too close to Weyl points



k-space volume $(1 + \frac{e}{c}\boldsymbol{\Omega}_{\mathbf{k}} \cdot \mathbf{B})$

diverges at Weyl point but not a concern since total particles conserved

Collision integrals:

intra pocket: $I_{col}^{(11)} = -\frac{\delta n(\mathbf{k})}{\tau_0}$ $\delta n(\mathbf{k}) = n(\mathbf{k}) - f(\epsilon - \mu_1)$

unknown at this stage

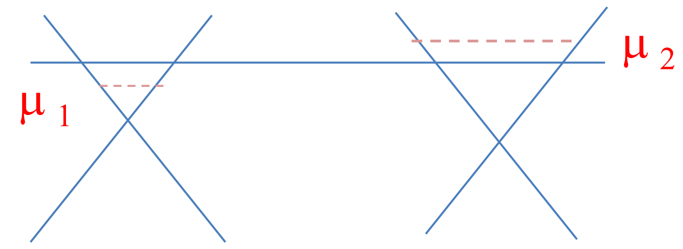
conservation of particles $\sum_{\mathbf{k}\in 1} \delta n(\mathbf{k}) = 0$

interpockets:

1 to 2:

$$I_{col}^{(12)}(\mathrm{k}) = (n(\mathbf{k}) - f(\epsilon - \mu_2))/\tau_k^{12}$$

↓ from pocket 2 to 1



and similarly for 2 to 1

conservation:

$$\sum_{\mathbf{k}\in 1} \frac{n(\mathbf{k}) - f(\epsilon - \mu_2)}{\tau_k^{12}} + \sum_{\mathbf{k}\in 2} \frac{n(\mathbf{k}) - f(\epsilon - \mu_1)}{\tau_k^{21}} = 0$$

determines μ's after n(k) solved

if k independent:

$$\sum_{\mathbf{k}\in 1} \frac{f(\epsilon - \mu_1) - f(\epsilon - \mu_2)}{\tau^{12}} + \sum_{\mathbf{k}\in 2} \frac{f(\epsilon - \mu_2) - f(\epsilon - \mu_1)}{\tau^{21}} = 0$$

used $\sum_{\mathbf{k}\in 1} \delta n(\mathbf{k}) = 0$ and similarly for 2

$$\sum_{\mathbf{k}\in 1} \frac{f(\epsilon-\mu_1) - f(\epsilon-\mu_2)}{\tau^{12}} + \sum_{\mathbf{k}\in 2} \frac{f(\epsilon-\mu_2) - f(\epsilon-\mu_1)}{\tau^{21}} = 0$$

Define: $\tilde{D}_1(B) \equiv \int_{k\in 1} \frac{d\omega_{\hat{k}}}{4\pi} \rho_{\hat{k}} (1 + \frac{e}{c} \mathbf{\Omega_k} \cdot \mathbf{B})$ (density of states)

similarly for 2; not necessarily equal

Restriction: $\frac{\tilde{D}_1(B)}{\tau^{12}} = \frac{\tilde{D}_2(B)}{\tau^{21}}$!

τ^{12} τ^{21} field dependent if field dependent density of states not equal

while if define: $\frac{1}{\tau^{12}} = \frac{1}{\tau^{X}} \frac{\sqrt{D_1 D_2}}{\tilde{D}_1(B)}$ ← (B = 0 values)

↓ *can* be field independent

τ^{12} τ^{21} in general not equal

$$I_{col}^{(11)} = -\frac{\delta n(\mathbf{k})}{\tau_0}$$

$$I_{col}^{(12)}(\mathrm{k}) = (n(\mathbf{k}) - f(\epsilon - \mu_2))/\tau_k^{12}$$

$$\delta n(\mathbf{k}) = n(\mathbf{k}) - f(\epsilon - \mu_1)$$

$$I_{col}^1 = -\frac{\delta n(\mathbf{k})}{\tilde{\tau}_1} + \frac{\partial f}{\partial \epsilon}\frac{1}{\tau^X}\frac{\sqrt{D_1 D_2}}{\tilde{D}_1}(\mu_1 - \mu_2)$$

$$\frac{1}{\tilde{\tau}_1} = \frac{1}{\tau_0} + \frac{1}{\tau^X}\frac{\sqrt{D_1 D_2}}{\tilde{D}_1(B)}$$

Solve kinetic equation for $\delta n(\mathbf{k})$

$$\sum_{\mathbf{k}\in 1} \delta n(\mathbf{k}) = 0 \quad \longrightarrow$$

$$\delta\mu_1 - \delta\mu_2 = -\tau^X \; \frac{\int_{\mathbf{k}\in 1} \frac{d\omega_{\hat{k}}}{4\pi}\rho_{\hat{k}}\left[e\mathbf{E}\cdot\frac{\partial\epsilon}{\partial\mathbf{k}} + \frac{e^2}{c}(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}_{\mathbf{k}}\cdot\frac{\partial\epsilon}{\partial\mathbf{k}}\right]}{(D_1 D_2)^{1/2}}$$

contribution from 1st valley, similar expression for 2nd , total = sum:

$$\mathbf{J}^{\mathbf{e}}_{\mathbf{a1}} = e^2\tilde{\tau}_1 \int_{k\in 1} \frac{d\omega_{\hat{k}}}{4\pi}\rho_{\hat{k}} \frac{\tilde{\mathbf{v}}[\tilde{\mathbf{v}}\cdot\mathbf{E}]}{(1+\frac{e}{c}\mathbf{\Omega_k}\cdot\mathbf{B})} + \frac{e^2\tau^X}{2D}\left(1-2\frac{\tilde{\tau}_1}{\tau^X}\frac{D}{\tilde{D}}\right)\left[\int_{k\in 1}\frac{d\omega_{\hat{k}}}{4\pi}\rho_{\hat{k}}\tilde{\mathbf{v}}\right]\left[\int_{k\in 1}\frac{d\omega_{\hat{k}}}{4\pi}\rho_{\hat{k}}\tilde{\mathbf{v}}\cdot\mathbf{E}\right]$$

$$\tilde{\mathbf{v}} \equiv \left[\frac{\partial\epsilon}{\partial\mathbf{k}} + \frac{e}{c}(\frac{\partial\epsilon}{\partial\mathbf{k}}\cdot\mathbf{\Omega}_{\mathrm{k}})\mathbf{B}\right]$$

1st term: part 1

2nd term:

$$\left[\int_{k\in 1}\frac{d\omega_{\hat{k}}}{4\pi}\rho_{\hat{k}}\tilde{\mathbf{v}}\right] \longrightarrow \int_{k\in 1}\frac{d\omega_{\hat{k}}}{4\pi}\rho_{\hat{k}}\frac{\partial\epsilon}{\partial\mathbf{k}}\cdot\Omega(\mathbf{k}) \times \mathbf{B}$$

monopole charge

$$\longrightarrow \sim \tau^X \frac{e^4\tilde{M}^2}{(4\pi^2)^2c^2D}\mathbf{B}(\mathbf{B}\cdot\mathbf{E})$$

close to Weyl point: $D = k_F^2/(2\pi^2 v_F) = \mu^2/2\pi^2 v_F^3$

~ Son and Spivak

2nd term, continued: $$\tau^X \frac{e^4 \tilde{M}^2}{(4\pi^2)^2 c^2 D} \mathbf{B}(\mathbf{B}\cdot\mathbf{E})$$

Enhancement in $\sigma \sim \tau^X (eB/c)^2 / D$

$$D = k_F^2/(2\pi^2 v_F) = \mu^2/2\pi^2 v_F^3$$

c.f. $\sigma(B=0) \sim \tau_0 D v^2$

Ratio: $\tau^X/\tau_0 \times (eB/c)^2 / (vD)^2$

$\sim \tau^X/\tau_0 \times (eB/c)^2 / k_F^4$

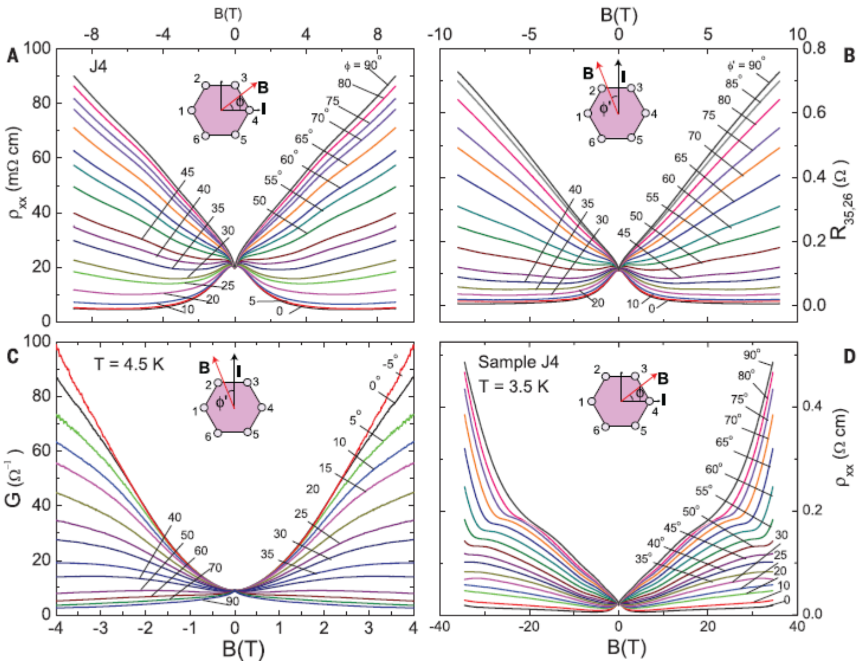
$\sim \tau^X/\tau_0 \times (eB/c)^2 \ \Omega^2$

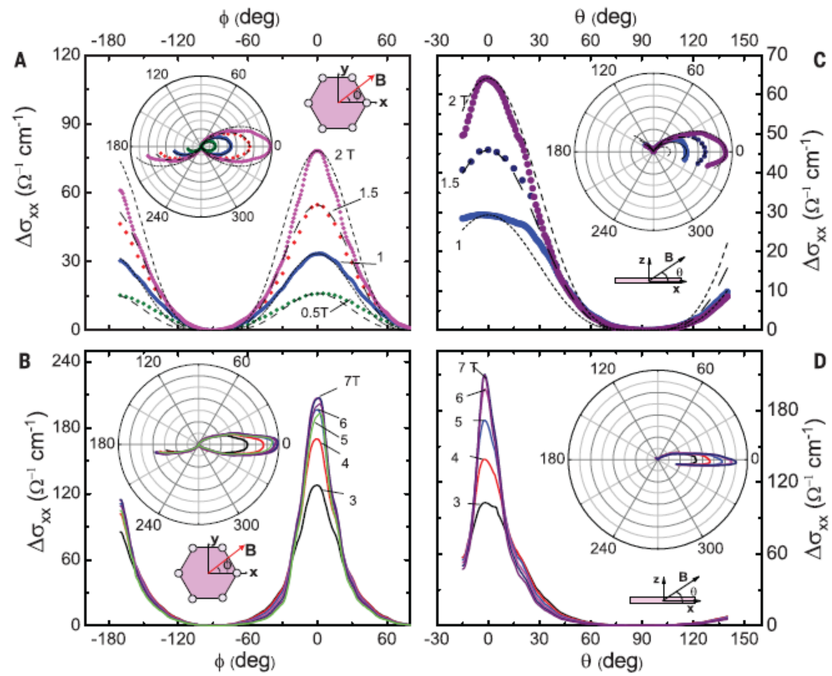
seen before

Evidence for the chiral anomaly in the Dirac semimetal Na_3Bi

Jun Xiong, Satya K. Kushwaha, Tian Liang, Jason W. Krizan, Max Hirschberger, Wudi Wang, R. J. Cava and N. P. Ong

Science **350** (6259), 413-416. (2015)





! ?

Quasiclassical kinetic equation

Particle number conservation