# Chiral Matter and Topology in Astrophysics

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"Recent Developments in Chiral Matter and Topology"
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# Main topics

- Core-collapse supernova
- Chiral hydrodynamics
- Chiral turbulence in supernovae
- Photonic chiral vortical effect in pulsars

Units: $\hbar = c = k_{\mathrm{B}} = e = 1$

# Core-collapse supernovae explosions

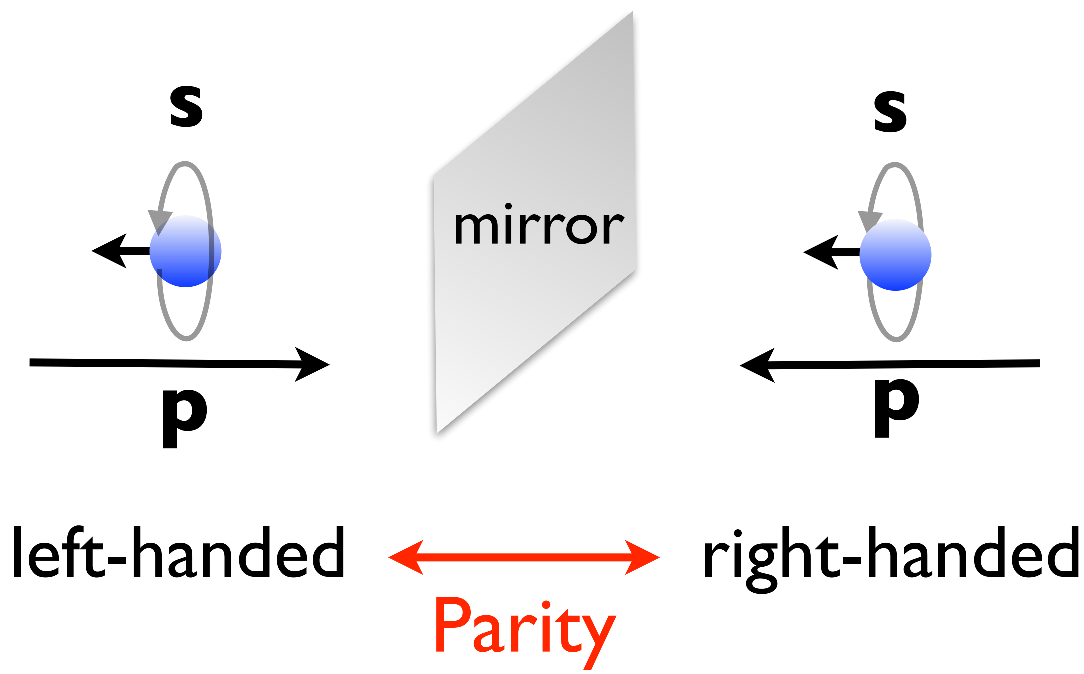
# Core-collapse supernova explosions

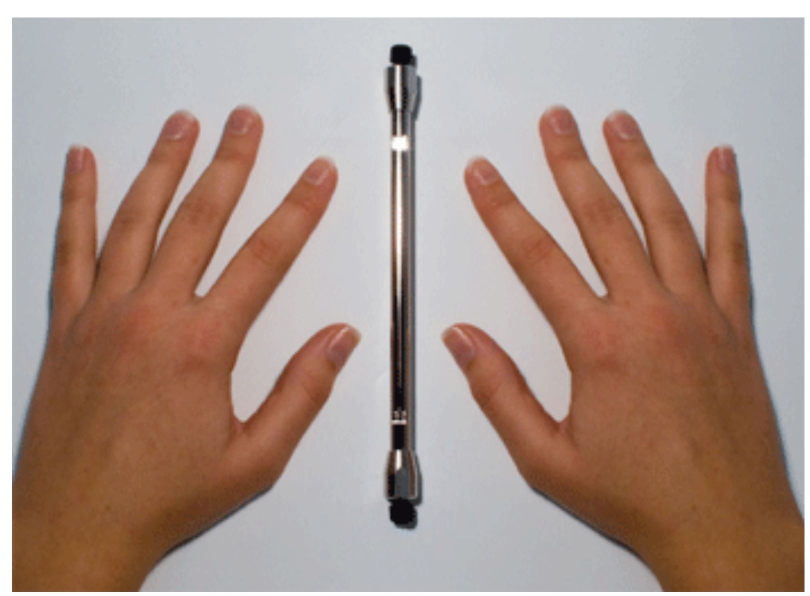
- One of the most energetic phenomena in the Universe
- Transition to neutron stars & origin of heavy elements
- But explosion is difficult in conventional 3D hydrodynamic theory

One of the puzzles in astrophysics

http://www.riken.jp/pr/press/2009/20091211/

# Chirality of fermions



# Why is "God" left-handed?

The laws of physics are left-right symmetric except for the weak interaction that acts only on left-handed particles.

W. Pauli

"God is just a weak left-hander."

# From micro to macro

Microscopic parity violation is reflected in macroscopic behavior:

**Micro** Chirality of fermions (e, ν) in Standard Model

↓

Chiral kinetic theory

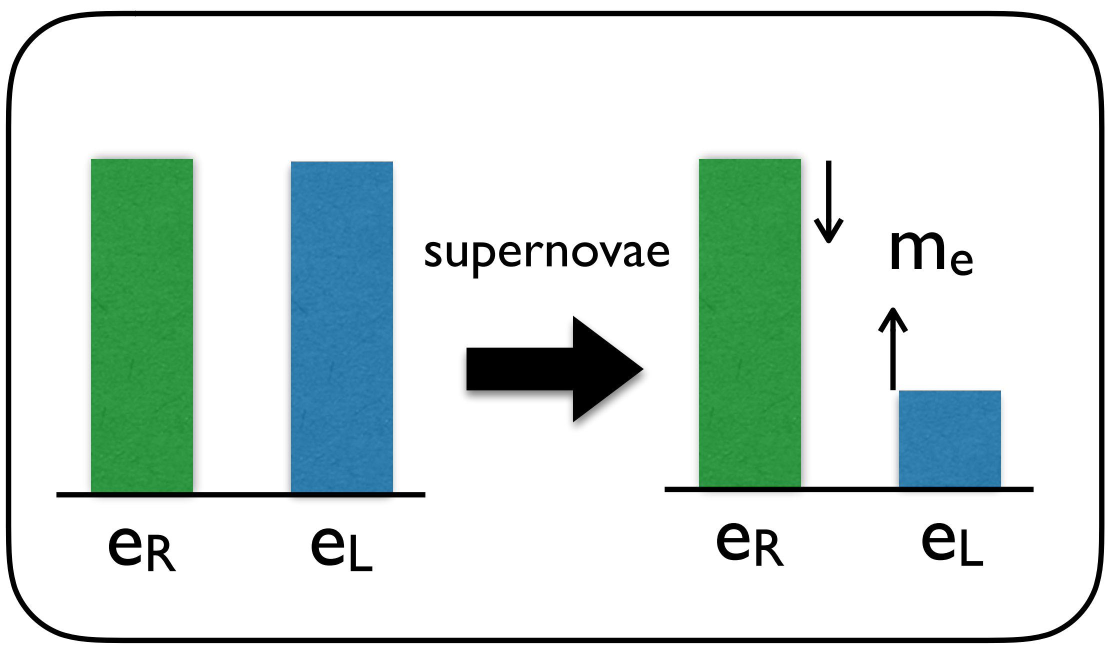
↓ Son, Yamamoto (2012); Stephanov, Yin (2012); J. W. Chen, S. Pu, Q. Wang, X. N. Wang (2013), …

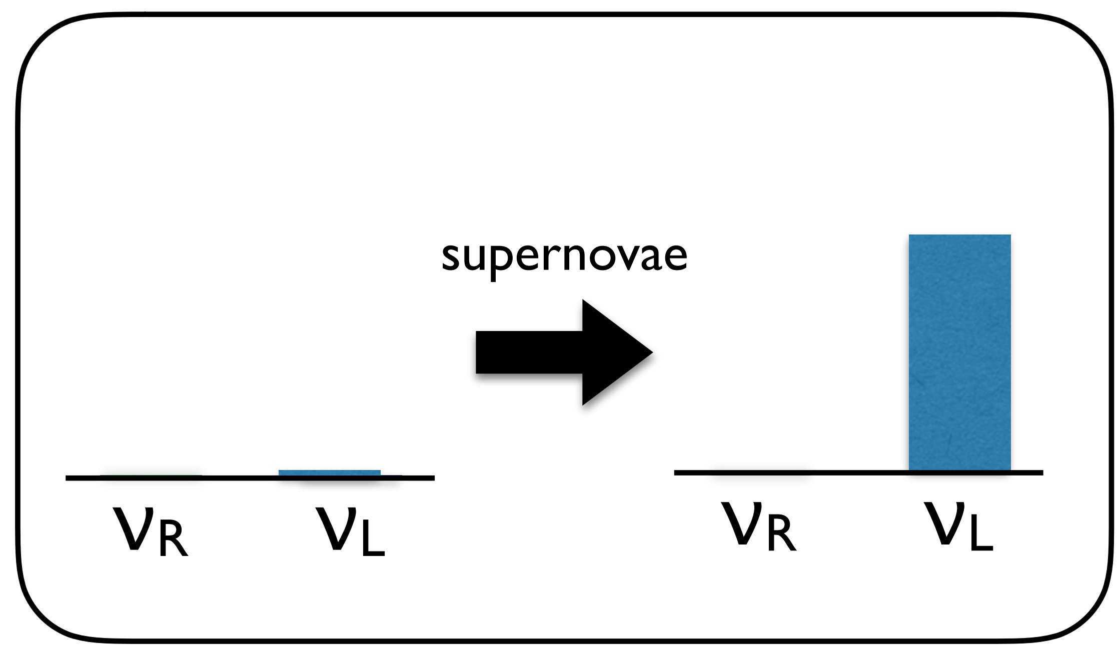
**Macro** Evolution of core-collapse supernovae (giant P violation)

Yamamoto (2016)

# Supernova = Giant Parity Breaker

$$p + e^{\mathrm{L}} \to n + \nu_e^{\mathrm{L}}$$



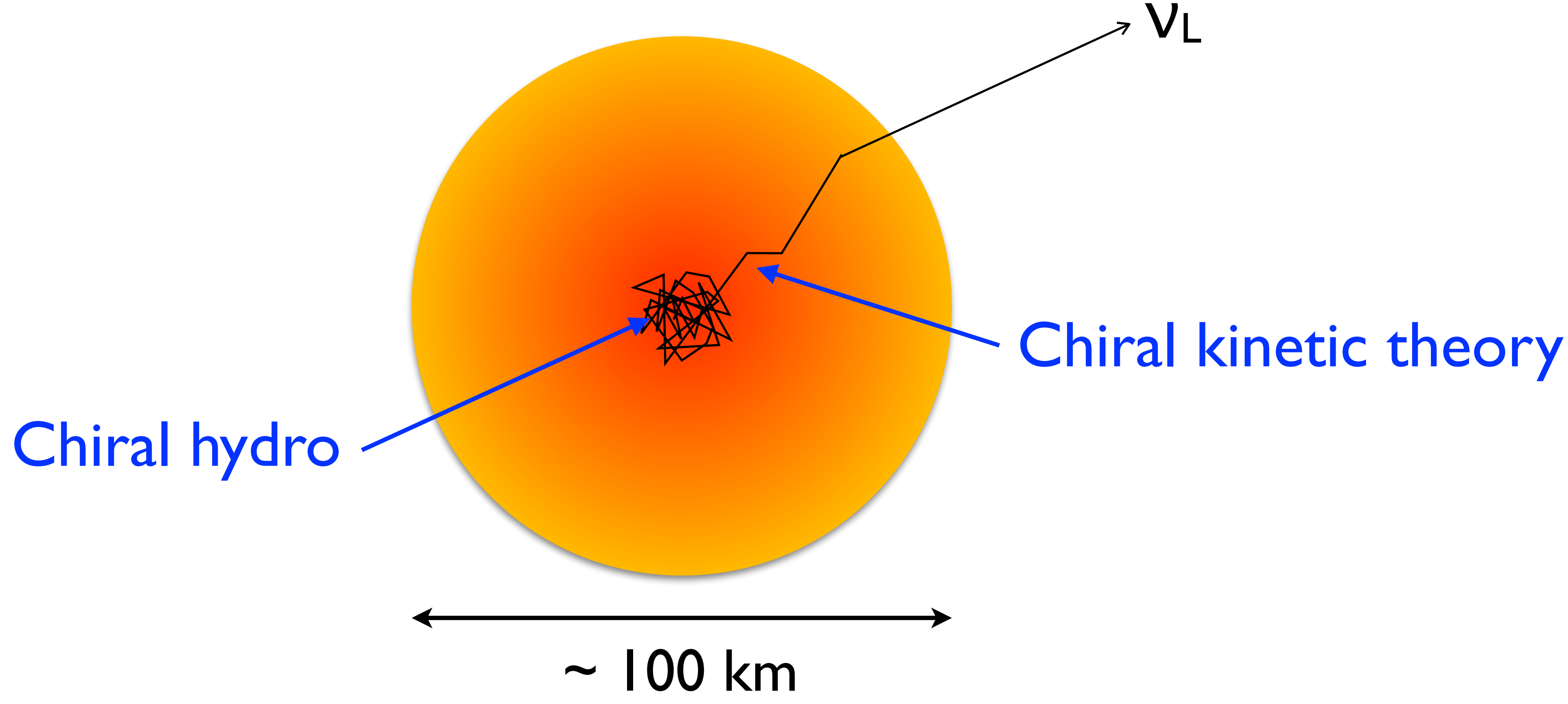




Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015); Sigl, Leite (2016), …

# Neutrino matter in supernovae

- Neutrino mean free path ~ 1cm at core ($\rho_N \sim 10^{15}$ g/cm$^3$).
- Neutrino matter = Chiral liquid ($\mu_\nu \sim 200$ MeV $\gg$ T~10 MeV)

  = 3D topological matter



# Chiral hydrodynamics

# Chiral magnetic effect

$$\boldsymbol{j} = \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B} \equiv \frac{\mu_5}{2\pi^2}\boldsymbol{B}$$

$$\boldsymbol{j}_5 = \frac{\mu_{\mathrm{R}} + \mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B} \equiv \frac{\mu}{2\pi^2}\boldsymbol{B}$$

Vilenkin (1980); Nielsen, Ninomiya (1983); Fukushima, Kharzeev, Warringa (2008), …

# Chiral vortical effect

$$\boldsymbol{j} = \frac{\mu \mu_5}{2\pi^2} \boldsymbol{\omega}$$

$$\boldsymbol{j}_5 = \left( \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12} \right) \boldsymbol{\omega}$$

vorticity $\boldsymbol{\omega} \equiv \boldsymbol{\nabla} \times \boldsymbol{v}$

Vilenkin (1979); Erdmenger et al. (2009); Banerjee et al. (2011); Son, Surowka (2009); Landsteiner et al. (2011)

# Lorentz covariant chiral hydro

Energy-momentum conservation: $\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$

Anomaly relation: $\partial_\mu j_5^\mu = C E^\mu B_\mu$

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu} + \text{(dissipation)}$$

$$j^\mu = n u^\mu + \xi_B B^\mu + \xi \omega^\mu + \text{(dissipation)}$$

$$j_5^\mu = n_5 u^\mu + \xi_{B5} B^\mu + \xi_5 \omega^\mu + \text{(dissipation)}$$

$$B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}, \quad \omega^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$$

Son, Surowka (2009); Sadofyev, Isachenkov (2011); Neiman, Oz (2011)

# Helicity conservation

Yamamoto (2016); see also Avdoshkin et al., (2016)

$$\partial_\mu j_5^\mu = C \boldsymbol{E} \cdot \boldsymbol{B}$$

# Helicity conservation

Yamamoto (2016); see also Avdoshkin et al., (2016)

$$\frac{d}{dt}\int d^3\boldsymbol{x}\left(j_5^0 + \frac{C}{2}\boldsymbol{A}\cdot\boldsymbol{B}\right) = 0$$

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CVE CME

$$j_5^0 = n_5 + \xi_5 \boldsymbol{v}\cdot\boldsymbol{\omega} + \xi_{B5}\boldsymbol{v}\cdot\boldsymbol{B}$$

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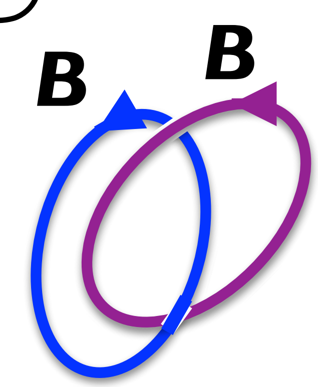
$$\frac{d}{dt}Q_{\rm tot} = 0, \quad Q_{\rm tot} \equiv Q_{\rm chi} + Q_{\rm mag} + Q_{\rm flu} + Q_{\rm mix}$$

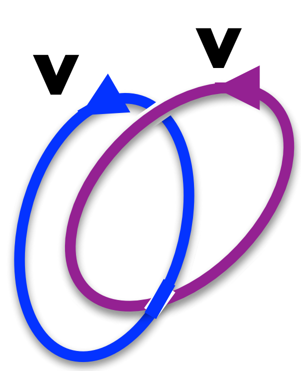
chiral charge

$$Q_{\rm chi} = \int d^3\boldsymbol{x}\; n_5$$

magnetic helicity

$$Q_{\rm mag} = \int d^3\boldsymbol{x}\frac{C}{2}\boldsymbol{A}\cdot\boldsymbol{B}$$



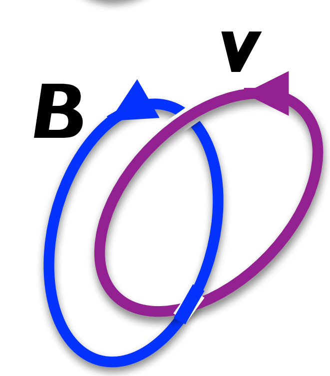




fluid helicity

$$Q_{\rm flu} = \int d^3\boldsymbol{x}\;\xi_5\boldsymbol{v}\cdot\boldsymbol{\omega}$$

mixed helicity

$$Q_{\rm mix} = \int d^3\boldsymbol{x}\;\xi_{B5}\boldsymbol{v}\cdot\boldsymbol{B}$$



# Neutrino chiral hydro

- Chiral hydrodynamic equations for pure neutrino matter:

$$(\epsilon + P)(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla} P + \nu \boldsymbol{\nabla}^2 \boldsymbol{v}$$

$$\partial_t(n + \underline{\xi \boldsymbol{v} \cdot \boldsymbol{\omega}}) + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0, \quad \boldsymbol{j} = n\boldsymbol{v} + \underline{\xi \boldsymbol{\omega}}$$

CVE CVE

- Neutrino number + fluid helicity is conserved.
- Generation of fluid helicity is numerically observed.

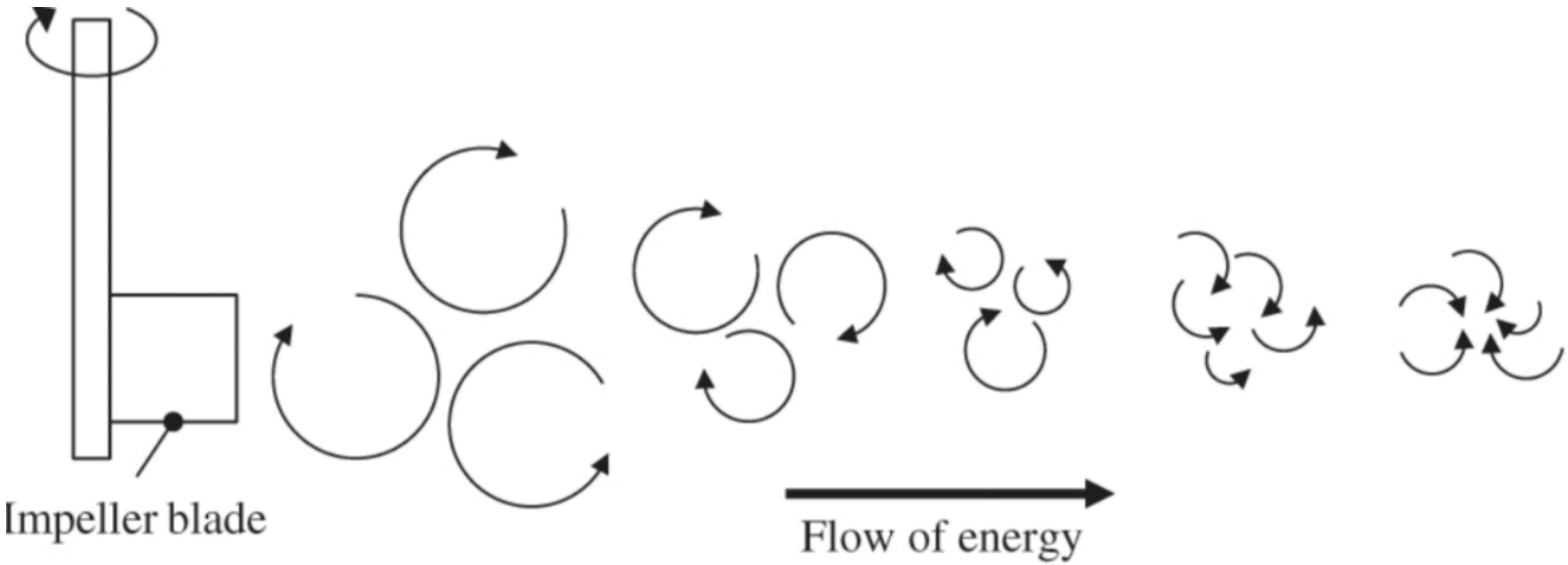
Kobayashi, Okuno, Yamamoto, in preparation

- When coupled to charged sector, fluid helicity ~ $\mu_5$ for electrons

$$\boldsymbol{j} \sim (\boldsymbol{v} \cdot \boldsymbol{\omega})\boldsymbol{B}$$

# Chiral MHD turbulence in supernovae

# Turbulence and cascade



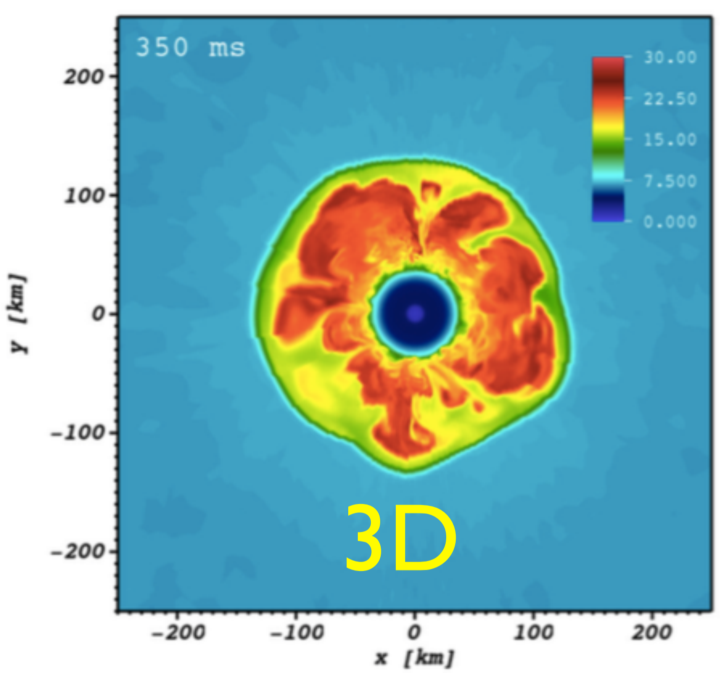

https://doi.org/10.1515/htmp-2016-0043

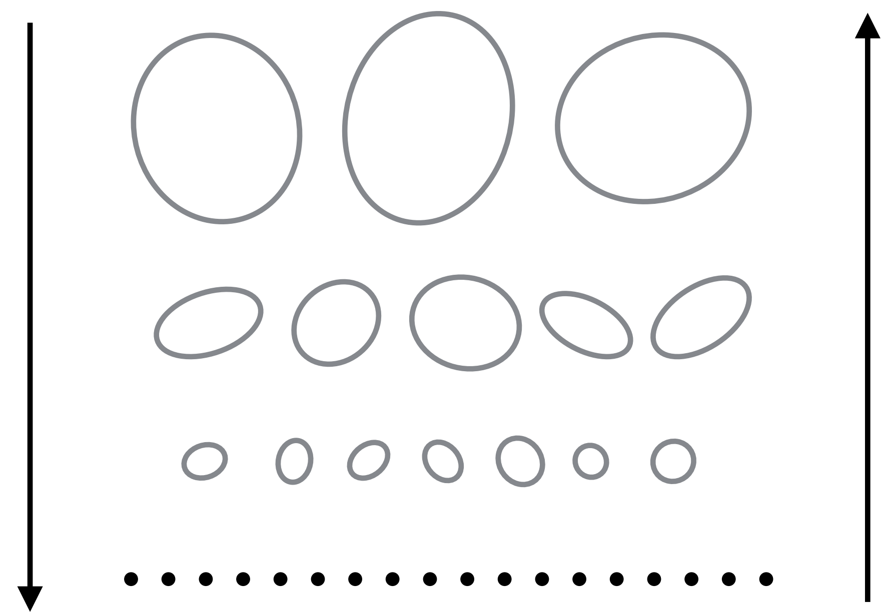
- The structure becomes smaller, and eventually dissipates (direct cascade)
- Similar in magneto-hydrodynamics (MHD)

# Cascade and explosion

Direct cascade
(3D usual matter)

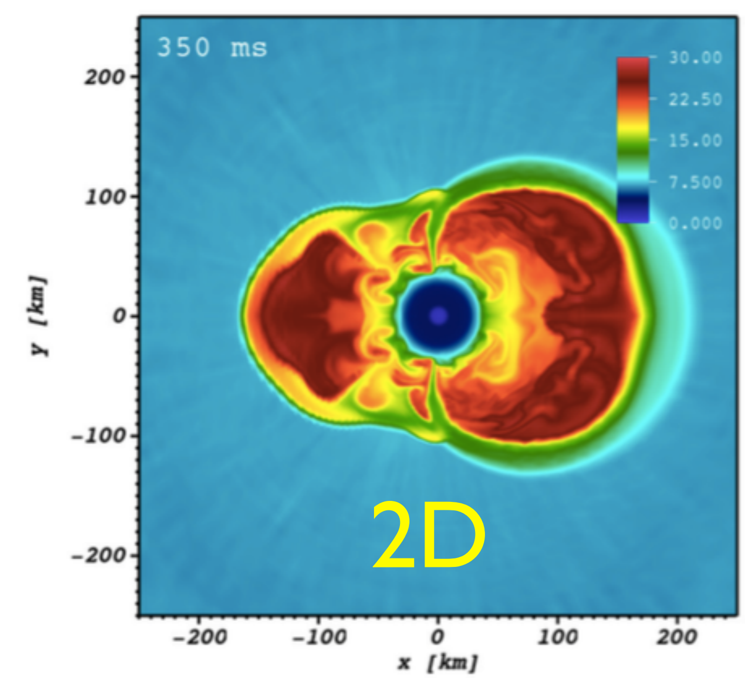
explosion difficult



Inverse cascade
(2D usual matter)

explosion easier

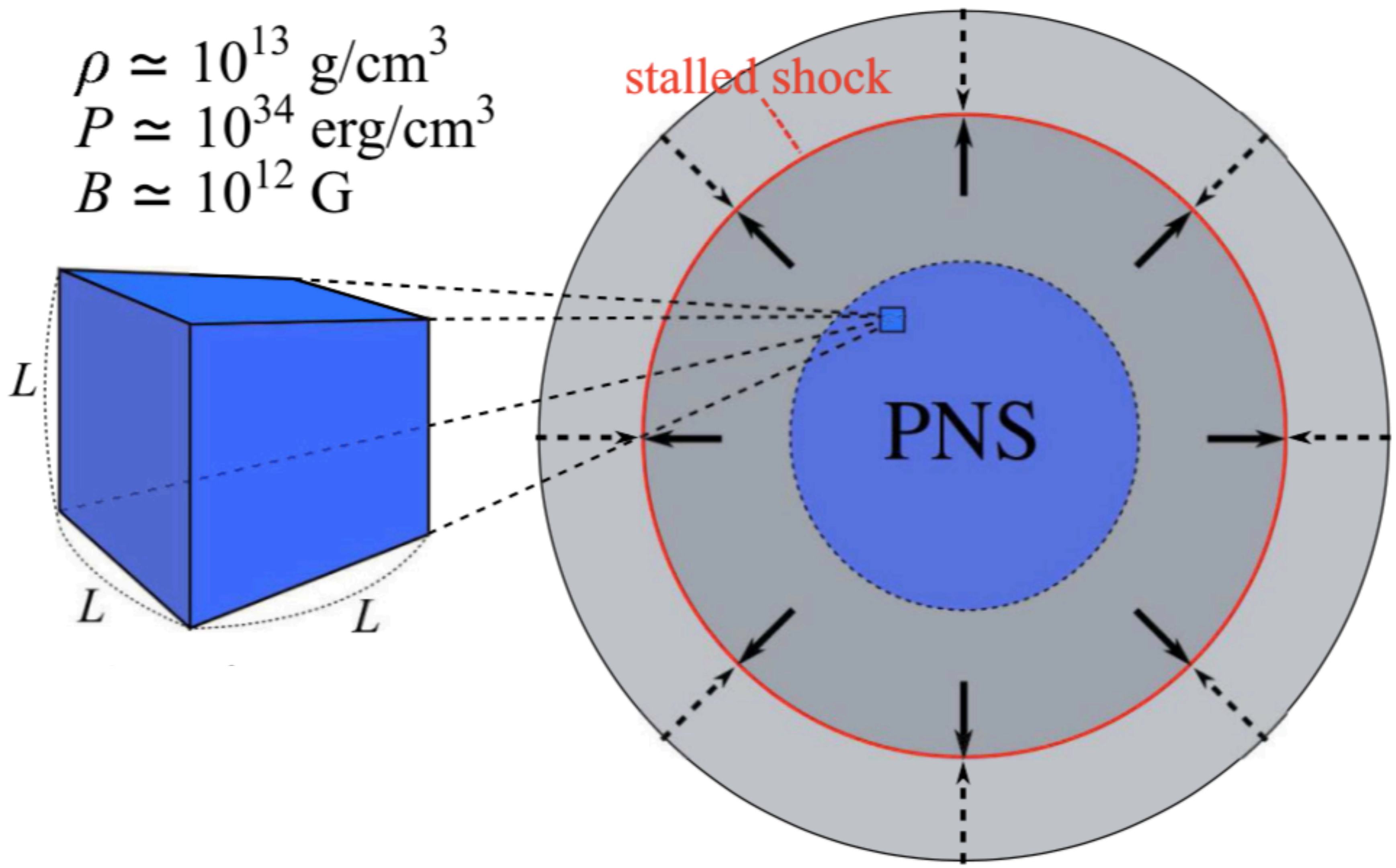



F. Hanke (2014)

What about 3D chiral matter?

# Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419




Proto-neutron star (PNS)

# Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419

- Chiral MHD w/o vorticity at the core (proton, $e_R$, $e_L$):

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$

$$\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \boldsymbol{v}) = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \text{(dissipation)}$$

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \boldsymbol{\nabla} \times \underline{(\xi_B \boldsymbol{B})}$$

"CME"

$$\partial_t n_5 = \underline{\frac{\eta}{2\pi^2} (\boldsymbol{\nabla} \times \boldsymbol{B} - \xi_B \boldsymbol{B}) \cdot \boldsymbol{B}}$$

chiral anomaly

- Setup for proto-neutron stars (100 MeV = 1) :

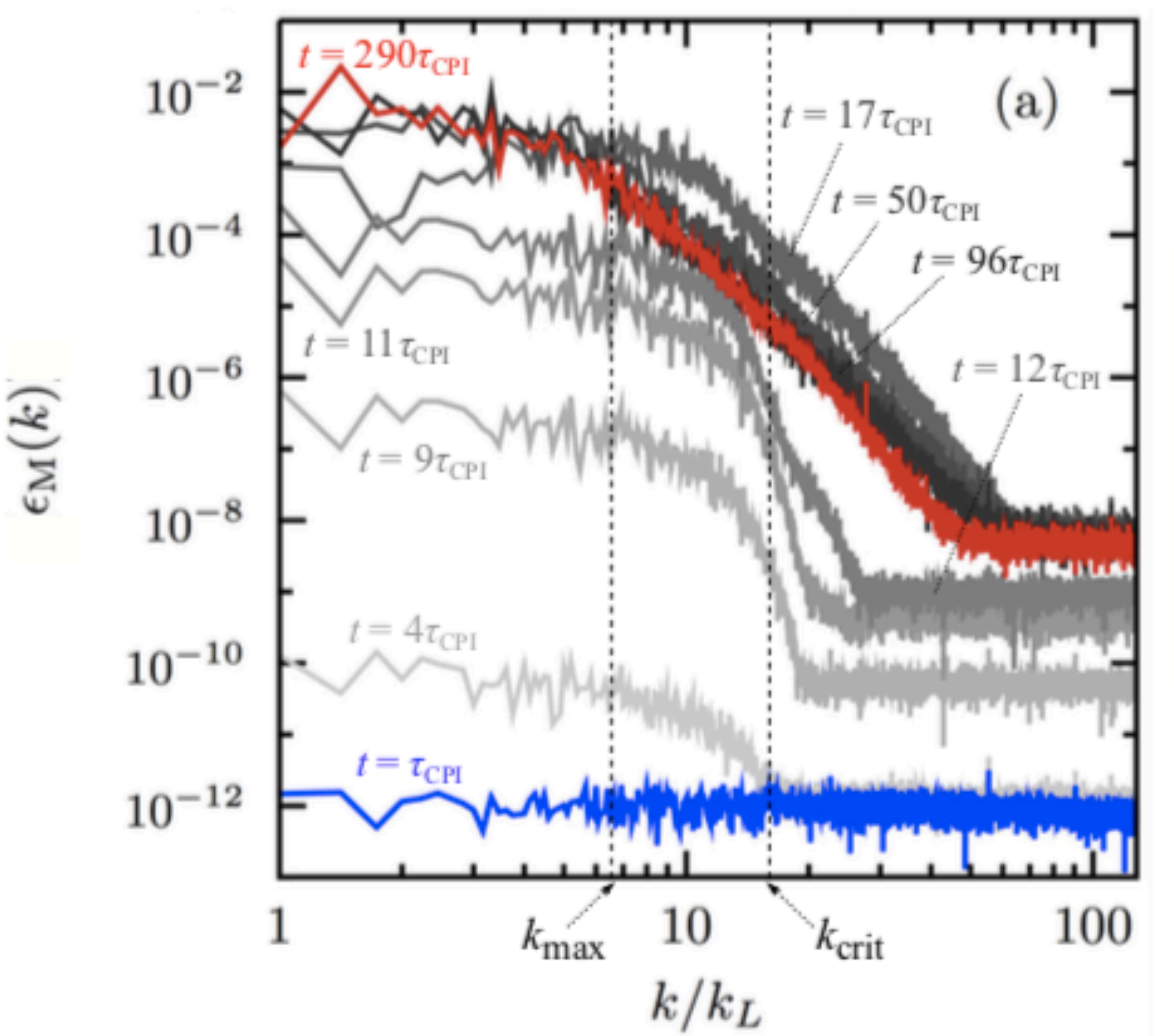
$$\rho_0 = 5.0, \ P_0 = 1.0, \ \xi_{B0} = 4.2 \times 10^{-3}, \ \eta = 100.0$$

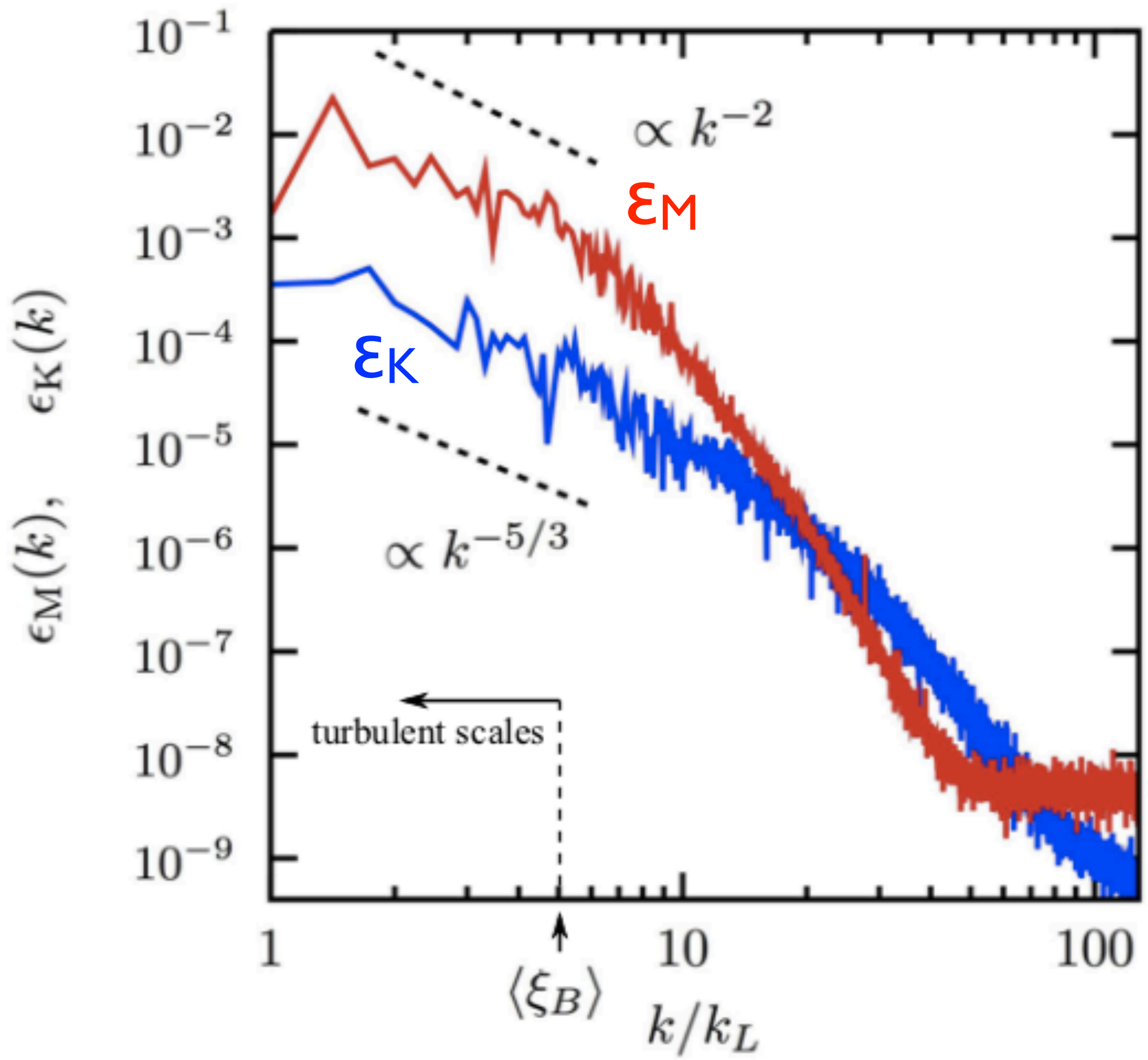
Movies of 3D simulations are available at:

http://www.kusastro.kyoto-u.ac.jp/~masada/movie.mp4

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419

# Energy spectra




- As time passes, energy in small-*k* and large-*k* regions grows
- Eventually, $\varepsilon_M \sim k^{-2}$, $\varepsilon_K \sim k^{-5/3}$

Masada et al., arXiv:1805.10419; see also Brandenburg et al., arXiv:1707.03385

# Neutrino chiral radiation hydro

Yamamoto, work in progress

$$\nabla_\alpha (T^{\alpha\beta}_{\rm hyd} + T^{\alpha\beta}_\nu) = 0$$

Stress tensor for N & e

(Hydro)

Stress tensor for ν

(Chiral kinetic theory)

$$T^{ij}_\nu = \int_{\boldsymbol{p}} |\boldsymbol{p}| \left( \hat{p}^i \hat{p}^j n_\nu - \frac{1}{2} p^i \epsilon^{jk\ell} \Omega^k_{\boldsymbol{p}} \partial_\ell n_\nu - \frac{1}{2} p^j \epsilon^{ik\ell} \Omega^k_{\boldsymbol{p}} \partial_\ell n_\nu \right)$$

Berry curvature of ν : $\boldsymbol{\Omega}_{\boldsymbol{p}} = -\frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}$

# Photonic chiral vortical effect

Avkhadiev-Sadofiev (2017); Yamamoto (2017); V. A. Zyuzin (2017);
Chernodub, Cortijo, Landsteiner (2018), …

# Helicity and Berry curvature

- Spin-momentum locking ⇄ helicity λ
  - chiral fermions (λ=±1/2)
  - photons (λ=±1) e.g., Onoda, Murakami, Nagaosa (2004)
  - gravitons (λ=±2) Yamamoto (2018)
- Berry curvature (adiabatic approximation): $\boldsymbol{\Omega}_{\boldsymbol{p}} = \lambda \frac{\hat{\boldsymbol{p}}}{|\boldsymbol{p}|^2}$

# Photon gas under rotation

Yamamoto, arXiv:1702.08886

- Semi-classical equations of motion in a rotating frame:

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}_{\boldsymbol{p}}$$

$$\dot{\boldsymbol{p}} = 2|\boldsymbol{p}|\dot{\boldsymbol{x}} \times \boldsymbol{\omega} + O(\omega^2)$$

Coriolis force

$$\longrightarrow$$

$$\sqrt{G}\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + 2\boldsymbol{\omega}|\boldsymbol{p}|(\hat{\boldsymbol{p}} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})$$

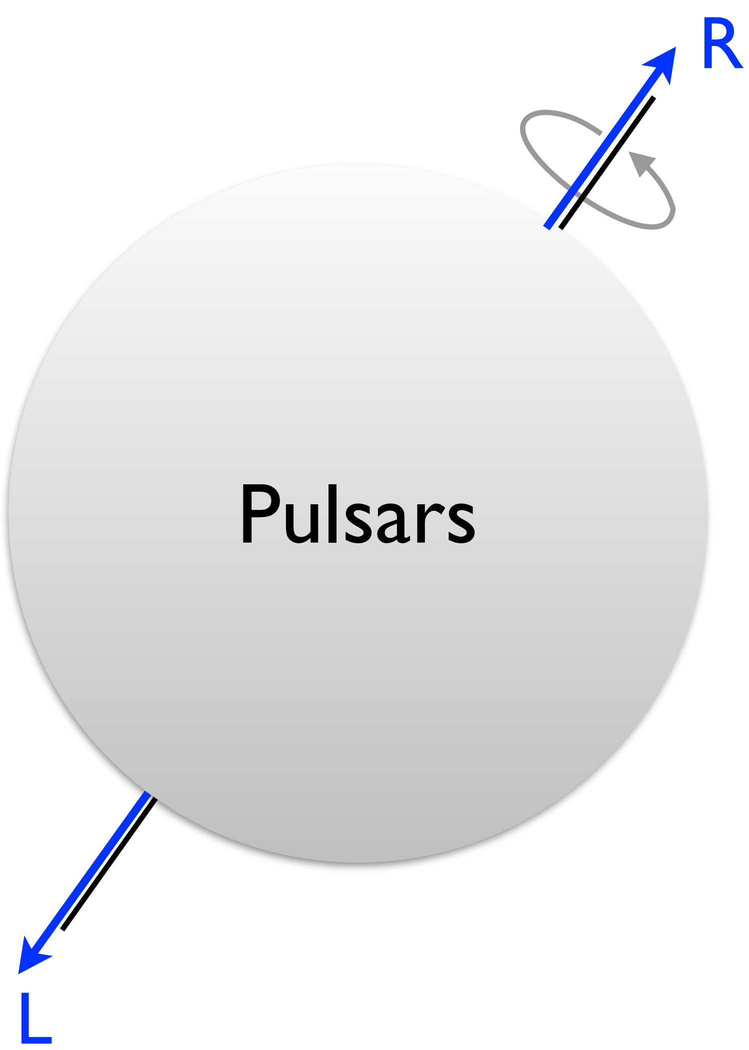
$$G = (1 + 2|\boldsymbol{p}|\boldsymbol{\omega} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})^2$$

- Photonic chiral current along a rotation:

$$\boldsymbol{j}^{\pm}_{\mathrm{CVE}} = 2\boldsymbol{\omega} \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} |\boldsymbol{p}|(\hat{\boldsymbol{p}} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}}) n^{\pm}_{\boldsymbol{p}} = \pm \frac{T^2}{6} \boldsymbol{\omega}$$

non-equilibrium equilibrium

# X-ray pulsars




$$T \sim 10\ \mathrm{keV},\ \omega \sim 10^3\ \mathrm{Hz}$$

→ Polarized photon flux: $f^{\pm} \sim 10^{21}/\mathrm{s}\cdot\mathrm{cm}^2$

cf) photon flux from sun: $f_{\odot} \sim 10^{17}/\mathrm{s}\cdot\mathrm{cm}^2$

# Conclusion

- Chiral effects of e & ν may help the supernova explosion
- Photonic chiral vortical effect in pulsars
- Quantum correction to gravit. lensing of gravit. waves
  ~ Berry curvature

Yamamoto, arXiv:1708.03113