Chiral and helical Majorana fermions in non-centrosymmetric superconductors on honeycomb lattice

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Der-Hau Lee and CH C, Phys. Status Solidi B **2018**, 1800114

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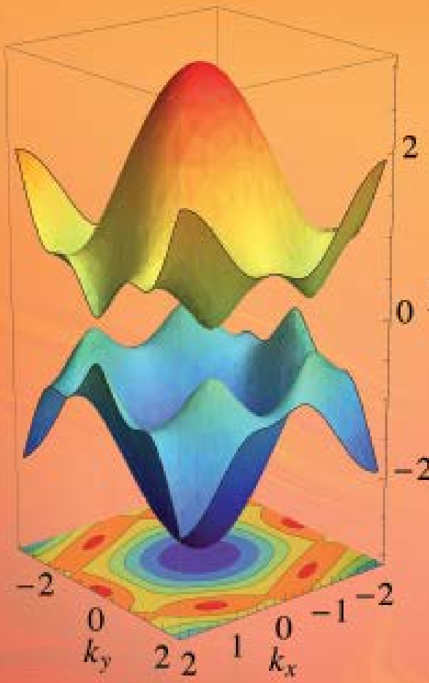
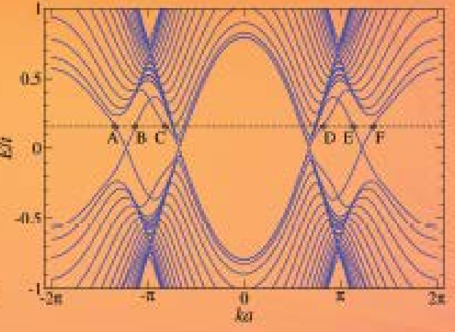
2018

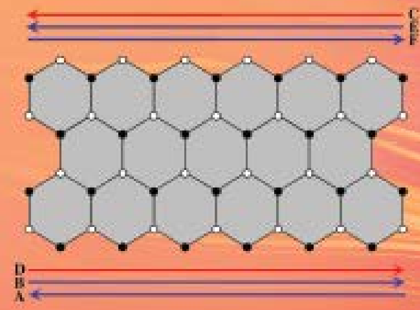
Editor's Choice

Non-Centrosymmetric Superconductors on Honeycomb Lattice

Der-Hau Lee and Chung-Hou Chung







WILEY-VCH

Outline

- Introduction to Non-centrosymmetric superconductors

- Topological superconductor on honeycomb lattice:

-- topological phase diagram of the superconducting Kane-Mele t-J model

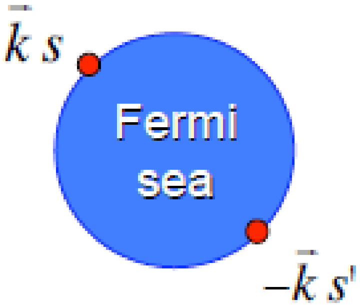
--singlet-triplet pairing mixture and coexisting helical-chiral Majorana zero mode

- Summary

M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Cooper pair symmetry

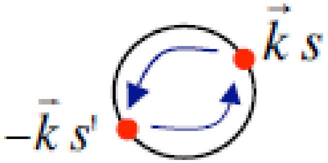
Symmetry of pairs of identical electrons:



$P_{tot}=0$

$$\Psi_{\vec{k}} = \langle c_{-\vec{k}s'} c_{\vec{k}s} \rangle = \underbrace{\phi(\vec{k})}_{\text{orbital}} \underbrace{\chi_{ss'}}_{\text{spin}}$$

Pauli principle:
wave function totally antisymmetric
under particle exchange



$\vec{k} \to -\vec{k}$ $\quad s \leftrightarrow s'$

	angular momentum	spin	
even parity:	$L = 0,2,4,\ldots,$ even	$S=0$ odd	spin singlet
odd parity:	$L = 1,3,5,\ldots,$ odd	$S=1$ even	spin triplet

Cooper pair symmetry

M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Symmetry of pairs of identical electrons:

$P_{tot}=0$

$$\Psi_{\vec k} = \langle c_{-\vec k s'} c_{\vec k s} \rangle = \phi(\vec k)\chi_{ss'}$$

Pauli p
wave fu
under p

Classification

L = 0, S = 0: most symmetric „conventional pairing"

L > 0 : lower symmetry „unconventional pairing"

	angular momentum	spin	
even parity:	*L* =0,2,4,... , even	*S=0* odd	spin singlet
odd parity:	*L* = 1,3,5,... , odd	*S=1* even	spin triplet

M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Cooper pair symmetry

Symmetry of pairs of identical electrons:

$P_{tot}=0$

$$\Psi_{\vec{k}} = \langle c_{-\vec{k}s'} c_{\vec{k}s} \rangle = \phi(\vec{k}) \chi_{ss'}$$

Pauli p

wave fu

under p

key symmetries for this classification

time reversal & inversion

	angular momentum	spin	
even parity:	$L=0,2,4,\ldots,$ even	$S=0$ odd	spin singlet
odd parity:	$L=1,3,5,\ldots,$ odd	$S=1$ even	spin triplet

Anderson's Theorems (1959,1984)

M. Sigrist's Talk “Key symmetries of
superconductivity
Inversion and time reversal symmetry”

Cooper pairs with total momentum P_{tot}=0
form from degenerate quasiparticle states.

$$|+\vec{k}s\rangle \qquad |-\vec{k}s'\rangle \quad \text{with} \quad \epsilon_{\vec{k}s} = \epsilon_{-\vec{k}s'}$$

How to guarantee existence of degenerate partners?

- Spin singlet pairing: time reversal symmetry

$$|\vec{k}\uparrow\rangle \rightarrow \hat{T}|\vec{k}\uparrow\rangle = |-\vec{k}\downarrow\rangle \Longleftrightarrow \epsilon_{\vec{k}\uparrow} = \epsilon_{-\vec{k}\downarrow}$$

harmful: magnetic impurities, ferromagnetism,
Zeemann fields (paramagnetic limiting)

- Spin triplet pairing: inversion symmetry

$$|\vec{k}\uparrow\rangle \rightarrow \hat{I}|\vec{k}\uparrow\rangle = |-\vec{k}\uparrow\rangle \Longleftrightarrow \epsilon_{\vec{k}\uparrow} = \epsilon_{-\vec{k}\uparrow}$$

harmful: crystal structure without inversion center

M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Key symmetries and band structure

Electron state: $|\vec{k}s\rangle$

$$\left\{ \begin{array}{ll} \text{time reversal:} & \hat{T}|\vec{k},s\rangle \rightarrow |-\vec{k},-s\rangle \\ \text{inversion:} & \hat{I}|\vec{k},s\rangle \rightarrow |-\vec{k},s\rangle \end{array} \right.$$

➡ orbital and spin part distinctly treated

Electron spectrum:

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{ c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'} \}$$

charge density

spin density

conserved

$$\left\{ \begin{array}{lll} \text{time reversal} & \epsilon_{\vec{k}} = \epsilon_{-\vec{k}} & \vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}} \\ \text{inversion} & \epsilon_{\vec{k}} = \epsilon_{-\vec{k}} & \vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}} \end{array} \right.$$

M. Sigrist's Talk “Key symmetries of superconductivity
Inversion and time reversal symmetry”

Superconducting phases

pair wave function $\hat{\Psi}_{\vec{k}} = \begin{pmatrix} \Psi_{\vec{k}\uparrow\uparrow} & \Psi_{\vec{k}\uparrow\downarrow} \\ \Psi_{\vec{k}\downarrow\uparrow} & \Psi_{\vec{k}\downarrow\downarrow} \end{pmatrix}$

spin singlet, even parity

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} 0 & \psi(\vec{k}) \\ -\psi(\vec{k}) & 0 \end{pmatrix}$$

$$= i\hat{\sigma}^y \psi(\vec{k})$$

1 configuration $\frac{\psi(\vec{k})}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$\psi(-\vec{k}) = \psi(\vec{k})$$

spin triplet, odd parity

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

$$= i\vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}}\hat{\sigma}^y$$

3 configurations $\begin{cases} (-d_x(\vec{k}) + id_y(\vec{k}))|\uparrow\uparrow\rangle \\ \frac{d_z(\vec{k})}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ (d_x(\vec{k}) + id_y(\vec{k}))|\downarrow\downarrow\rangle \end{cases}$

$$\vec{d}(-\vec{k}) = -\vec{d}(\vec{k})$$

Anderson theorem for perturbations

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^\dagger_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{c^\dagger_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'}\}$$

time reversal symmetry breaking

$$\alpha \vec{\lambda}_{\vec{k}} = -\mu_B \vec{H}$$

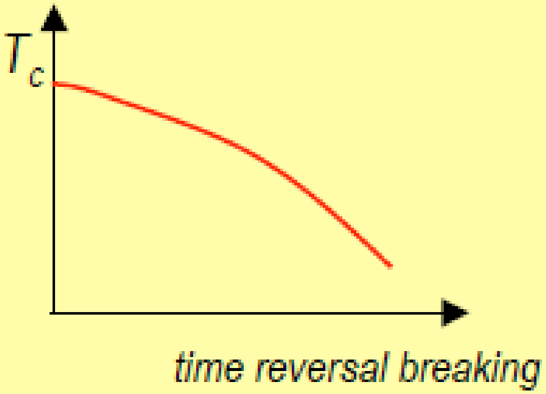
Zeeman coupling

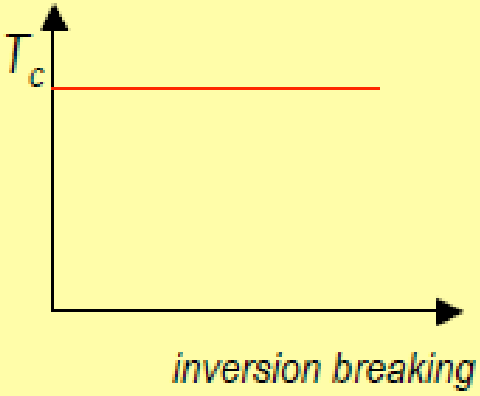
inversion symmetry breaking

$$\alpha \vec{\lambda}_{\vec{k}} = \alpha (\vec{k} \times \hat{z})$$

Rashba spin-orbit coupling

spin-singlet even-parity pairing





M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Anderson theorem for small perturbation

superconducting phase: bare $T_c = T_{c0}$ and $|\alpha \vec{\lambda}_{\vec{k}}| \sim k_B T_{c0}$

singlet pairing

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \left\langle |\psi(\vec{k})|^2 f(\rho_{\vec{k}}) \left\{1 + \hat{\lambda}_{\vec{k}} \cdot \hat{\lambda}_{-\vec{k}}\right\} \right\rangle_{\vec{k}}$$

$$T_c = T_{c0}$$

$$\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}}$$

no inversion

$$T_c \neq T_{c0}$$

$$\vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}}$$

no time reversal

$$f(\rho) = Re \sum_{n=1}^{\infty} \left(\frac{1}{2n-1+i\rho} - \frac{1}{2n-1} \right) \qquad \rho_{\vec{k}} = \frac{|\vec{\lambda}_{\vec{k}}|}{\pi k_B T_c} \qquad \hat{\lambda}_{\vec{k}} = \frac{\vec{\lambda}_{\vec{k}}}{|\vec{\lambda}_{\vec{k}}|}$$

M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Anderson theorem for perturbations

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{ c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'} \}$$

time reversal symmetry breaking

$$\alpha \vec{\lambda}_{\vec{k}} = -\mu_B \vec{H}$$

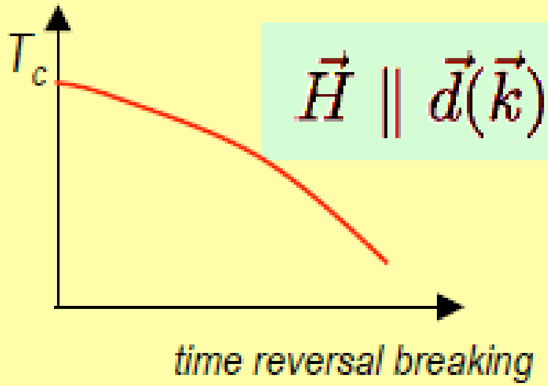
Zeeman coupling

inversion symmetry breaking

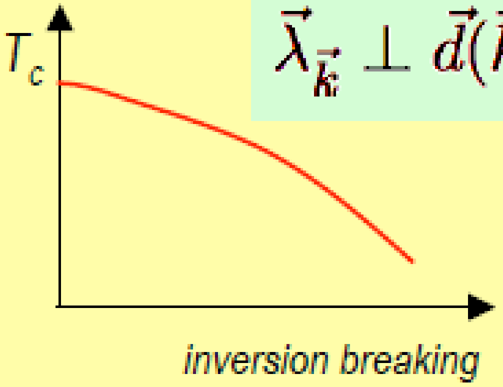
$$\alpha \vec{\lambda}_{\vec{k}} = \alpha (\vec{k} \times \hat{z})$$

Rashba spin-orbit coupling

spin-triplet odd-parity pairing



$$\vec{H} \parallel \vec{d}(\vec{k})$$



$$\vec{\lambda}_{\vec{k}} \perp \vec{d}(\vec{k})$$

M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Anderson theorem for perturbations

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{ c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'} \}$$

time reversal symmetry breaking

$$\alpha \vec{\lambda}_{\vec{k}} = -\mu_B \vec{H}$$

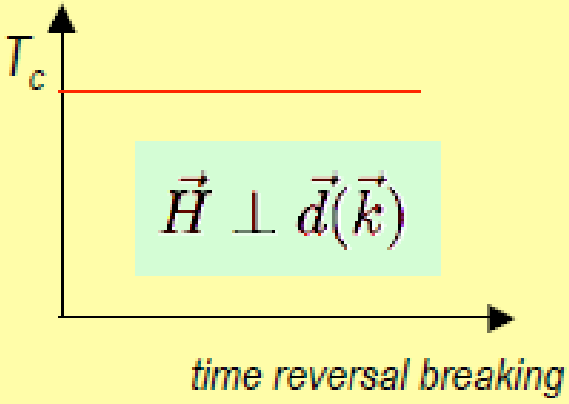
Zeeman coupling

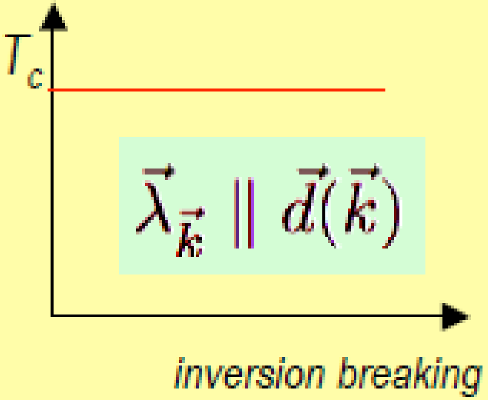
inversion symmetry breaking

$$\alpha \vec{\lambda}_{\vec{k}} = \alpha (\vec{k} \times \hat{z})$$

Rashba spin-orbit coupling

spin-triplet odd-parity pairing





M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

supplement

Anderson theorem for small perturbation

superconducting phase: bare $T_c = T_{c0}$ and $|\alpha \vec{\lambda}_{\vec{k}}| \sim k_B T_{c0}$

triplet pairing

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \left\langle 2f(\rho_{\vec{k}}) \left\{ (\hat{\lambda}_{\vec{k}} \cdot \vec{d}^*(\vec{k}))(\hat{\lambda}_{-\vec{k}} \cdot \vec{d}(\vec{k})) + |\vec{d}(\vec{k})|^2 \left[1 - \hat{\lambda}_{\vec{k}} \cdot \hat{\lambda}_{-\vec{k}} \right] \right\} \right\rangle_{\vec{k}}$$

$$\overbrace{\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}} \qquad \qquad \vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}}}^{T_c = T_{c0}} \qquad \qquad T_c \neq T_{c0}$$

no inversion — $\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$

no time reversal — $\vec{\lambda}_{\vec{k}} \perp \vec{d}(\vec{k})$

otherwise

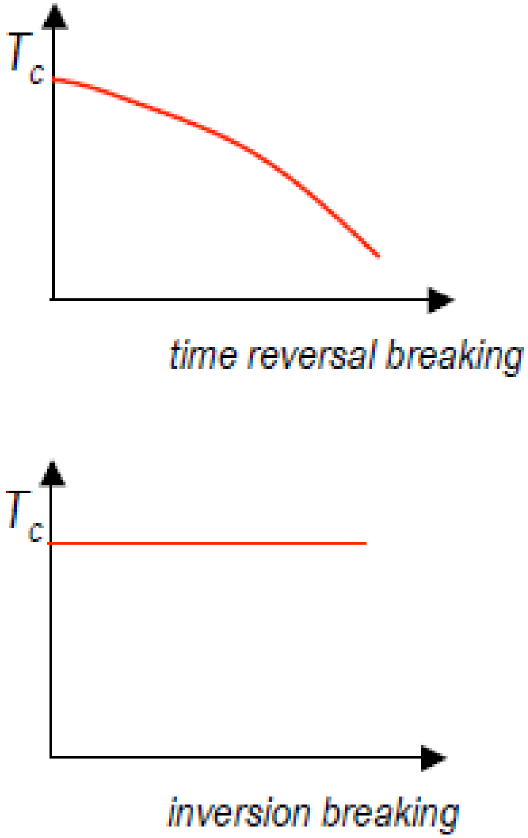
spin structure adapted to perturbation

$$f(\rho) = Re \sum_{n=1}^{\infty} \left(\frac{1}{2n-1+i\rho} - \frac{1}{2n-1} \right) \qquad \rho_{\vec{k}} = \frac{|\vec{\lambda}_{\vec{k}}|}{\pi k_B T_c} \qquad \hat{\lambda}_{\vec{k}} = \frac{\vec{\lambda}_{\vec{k}}}{|\vec{\lambda}_{\vec{k}}|}$$

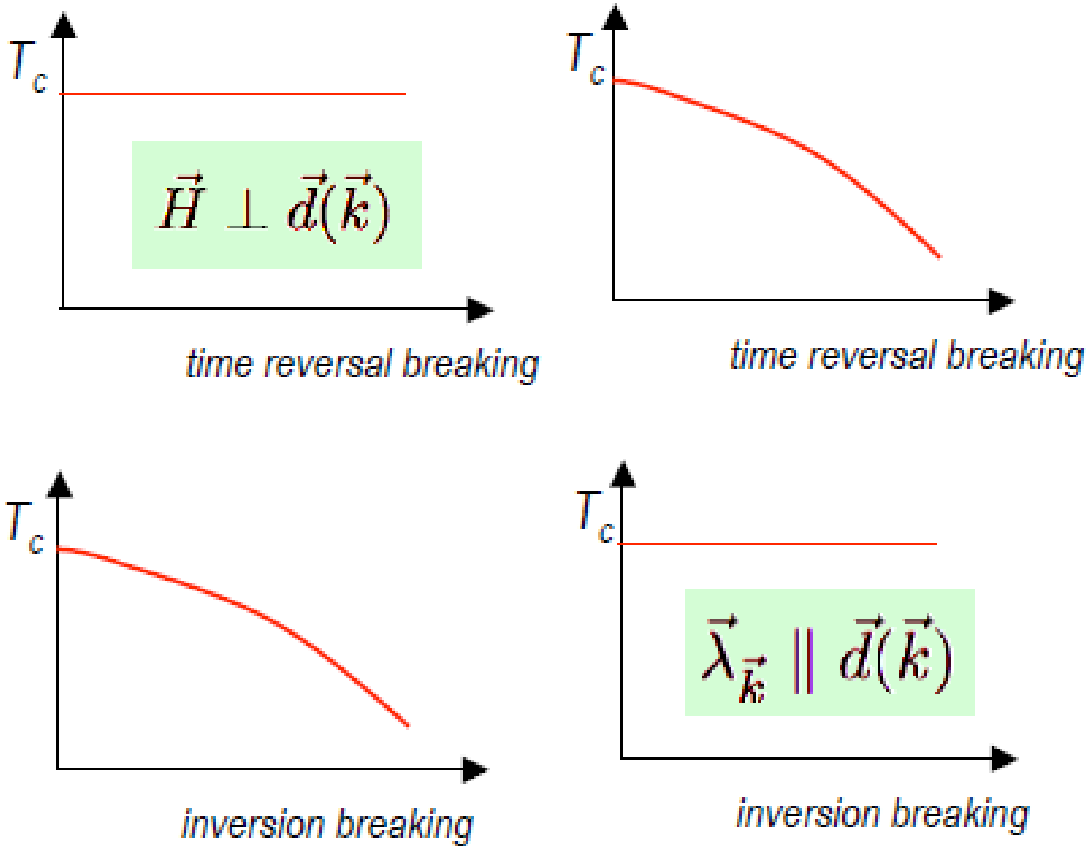
M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Anderson theorem for perturbations - summary

spin singlet even parity



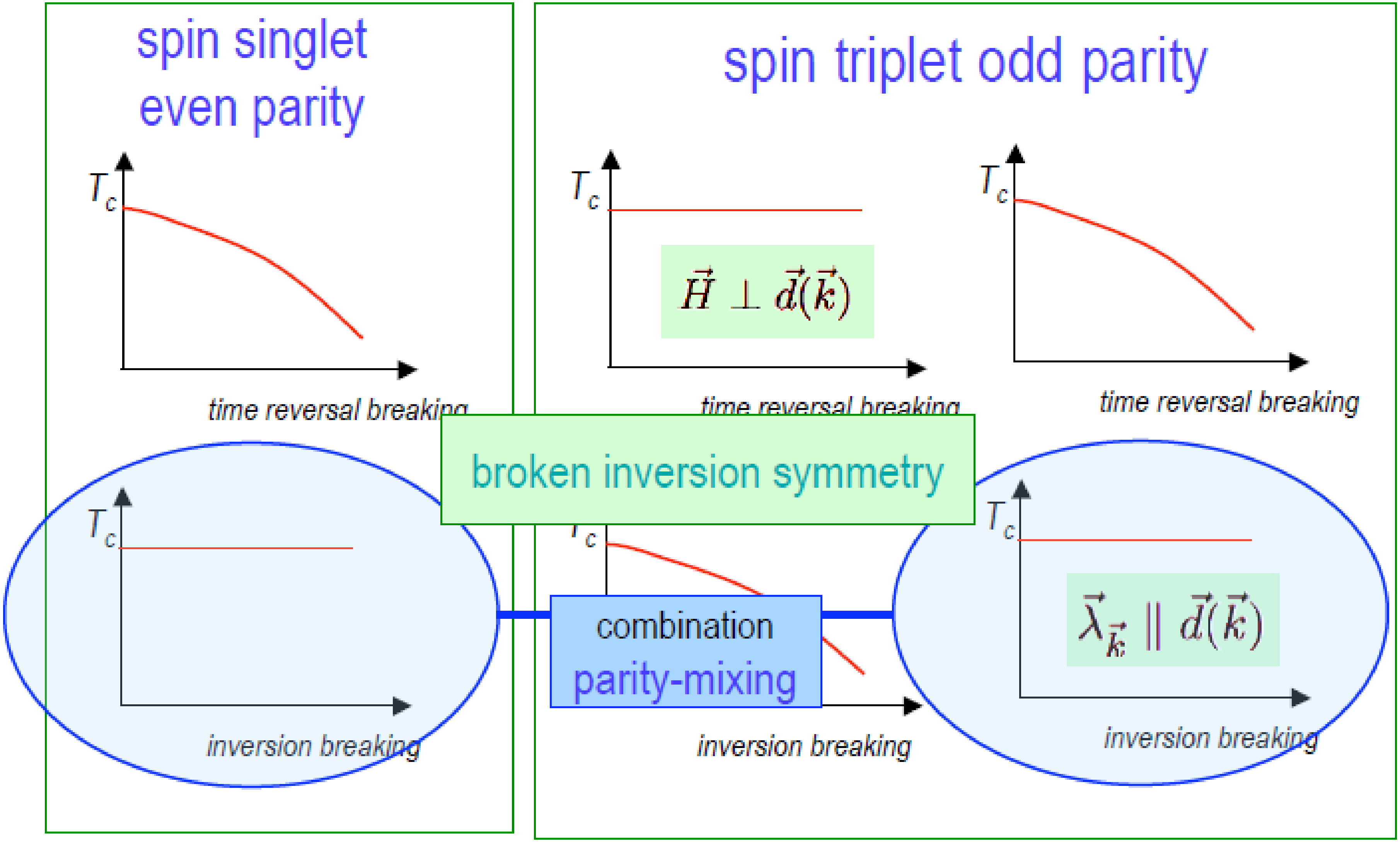
spin triplet odd parity



M. Sigrist's Talk “Key symmetries of superconductivity
Inversion and time reversal symmetry”

M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Anderson theorem for perturbations - summary



M. Sigrist's Talk "Key symmetries of superconductivity
Inversion and time reversal symmetry"

Mixed parity states are *non-unitary*

unitary superconducting states: $\hat{\Psi}_{\vec{k}}\hat{\Psi}^{\dagger}_{\vec{k}} = |\Psi_{\vec{k}}|^2 \hat{\sigma}_0 \propto$ *2x2 unit matrix*

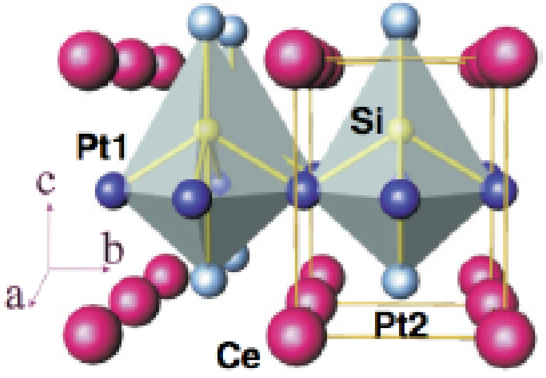
$$\hat{\Psi}_{\vec{k}} = \left\{ \psi(\vec{k}) + \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}} \right\} i\hat{\sigma}^y$$

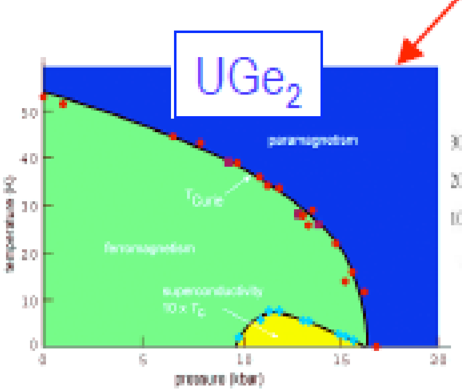
$$\hat{\Psi}_{\vec{k}}\hat{\Psi}^{\dagger}_{\vec{k}} = (|\psi|^2 + |\vec{d}|^2)\hat{\sigma}_0 + \underbrace{\{\psi^*\vec{d} + \psi\vec{d}^*\}}_{\propto \vec{\lambda}_{\vec{k}}} \cdot \hat{\vec{\sigma}} + i\underbrace{\{\vec{d} \times \vec{d}^*\}}_{\propto \vec{H}} \cdot \hat{\vec{\sigma}}$$

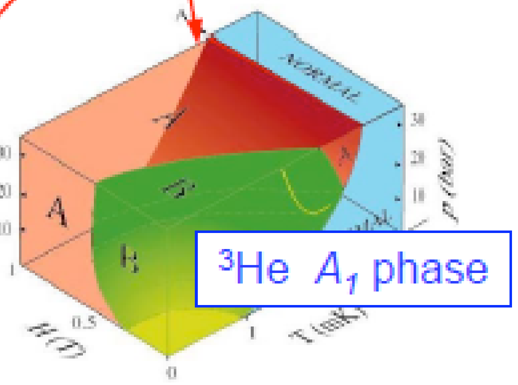
inversion symmetry violated

time reversal symmetry violated

CePt$_3$Si




UGe$_2$


^{3}He A_1 phase

Non-centrosymmetric superconductor on honeycomb lattice

Topological superconductor on honeycomb lattice
with mixed chiral and helical Majorana zero modes

3D Topological insulators

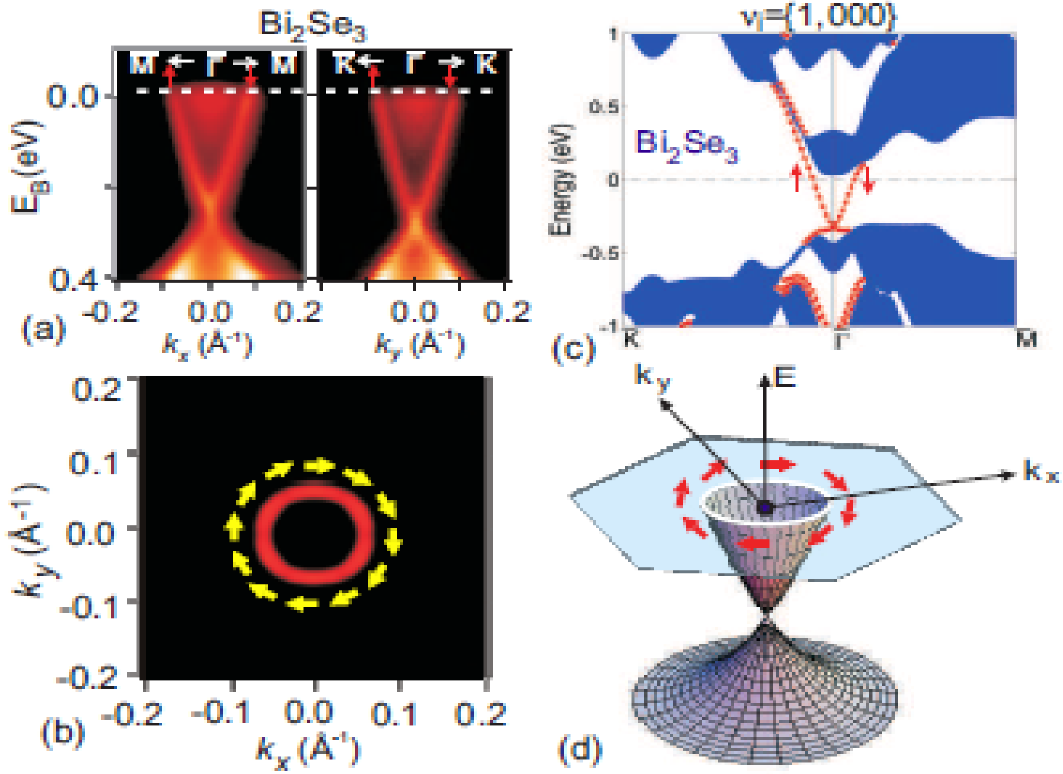
Gapless surface states due to Spin-orbit coupling

Chern invariant odd integer

$$n_m = \frac{1}{2\pi}\int d^2\mathbf{k}\mathcal{F}_m.$$

$$\mathcal{F}_m = \nabla \times \mathcal{A}_m$$

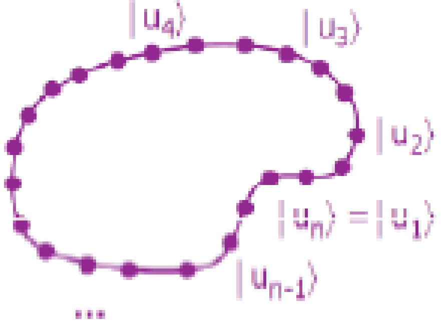
$$\mathcal{A}_m = i\langle u_m|\nabla_k|u_m\rangle$$

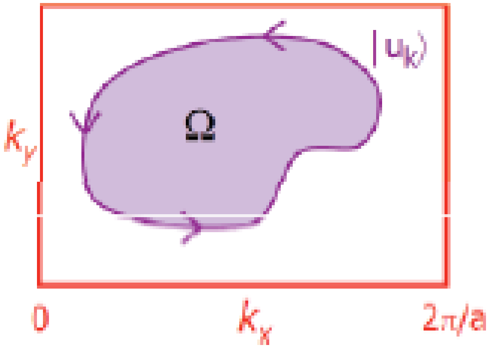


Hassn, Kane, 2010

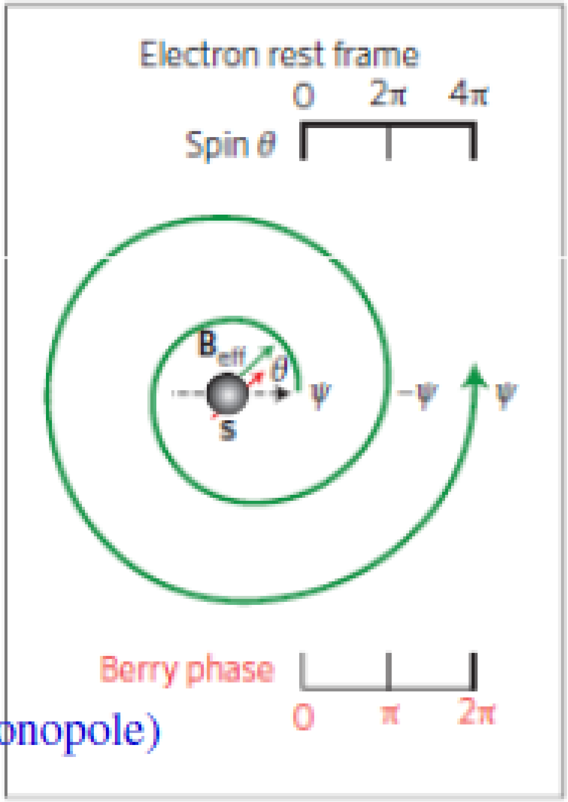
Berry phase, Hall conductivity and Chern number

Geometrical properties of the Hamiltonian
→ a phase difference acquired over the course of a cycle





Berry phase of π electrons in Graphene



Berry curvature $\Omega = -2\text{Im}\left\langle \frac{\partial u}{\partial k_x} \middle| \frac{\partial u}{\partial k_y} \right\rangle$

(→ magnetic field)

Berry phase $\phi = \int_{BZ} \Omega d\text{k}$

(→ magnetic flux)

Hall conductivity (non-trivial topology → magnetic monopole)

$$\sigma_{xy} = -\frac{e^2}{4\pi^2\hbar}\int_{BZ} \Omega d\text{k} = -\frac{e^2}{4\pi^2\hbar}\phi = -\frac{e^2}{4\pi^2\hbar}(2\pi C)$$

σ_{xy} is protected by C.

C: TKNN or charge Chern number (number of edge modes)

2D symmetry protected Topological insulator—
Quantum Spin Hall (Helical) Edge States in Kane-Mele model

QSHE: the Kane-Mele model

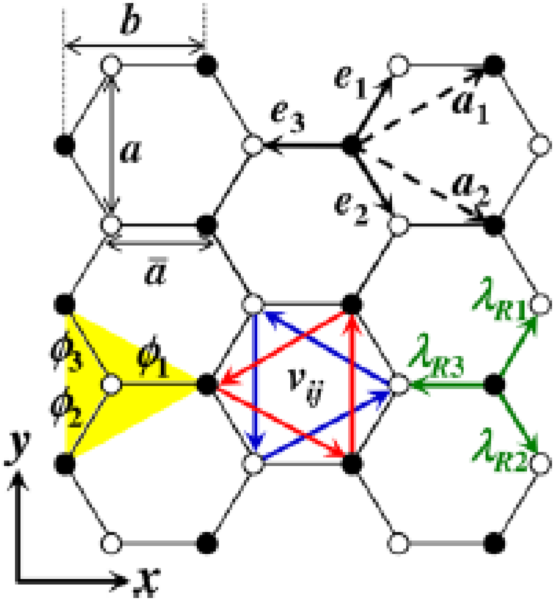
PRL 95, 226801 (2005) PHYSICAL REVIEW LETTERS week ending 25 NOVEMBER 2005

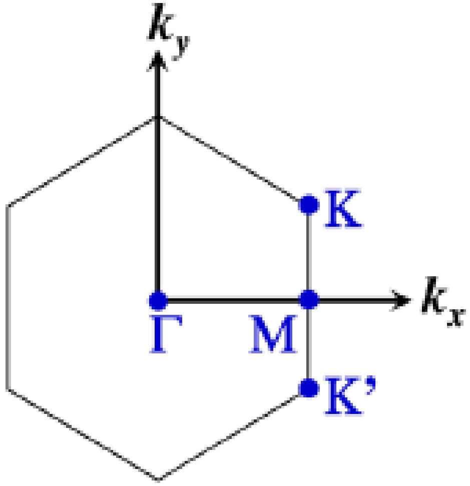
Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele
Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

(Received 29 November 2004; published 23 November 2005)

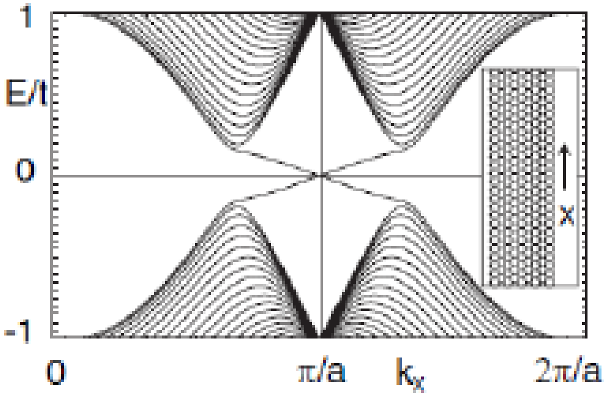
$$\boxed{H_{KM}=-t\sum_{\langle ij\rangle}c_i^{+}c_j+i\lambda\sum_{\langle\langle ij\rangle\rangle}\sum_{\sigma\sigma'}v_{ij}\sigma^{z}_{\sigma\sigma'}c^{+}_{i\sigma}c_{j\sigma'}}$$

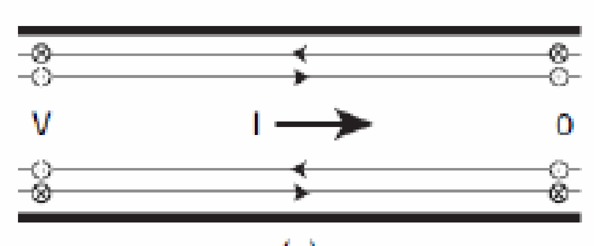




2D symmetry protected Topological insulator
--Quantum Spin Hall (Helical) Edge States in KM model

Kane, Mele, PRL 2005



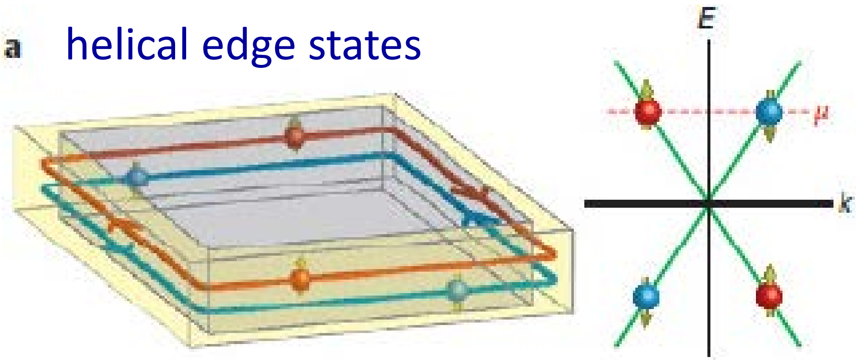


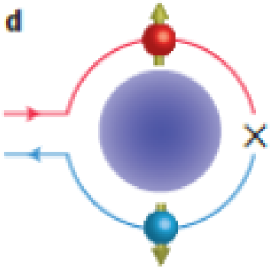
(a)

Time-reversal invariant →

Pair of counter-propagating edge states

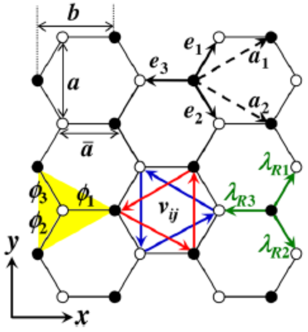
helical edge states

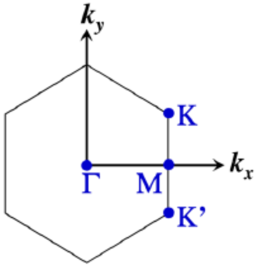




Backscattering process on a TI surface is prohibited→ dissipationless metallic edge.

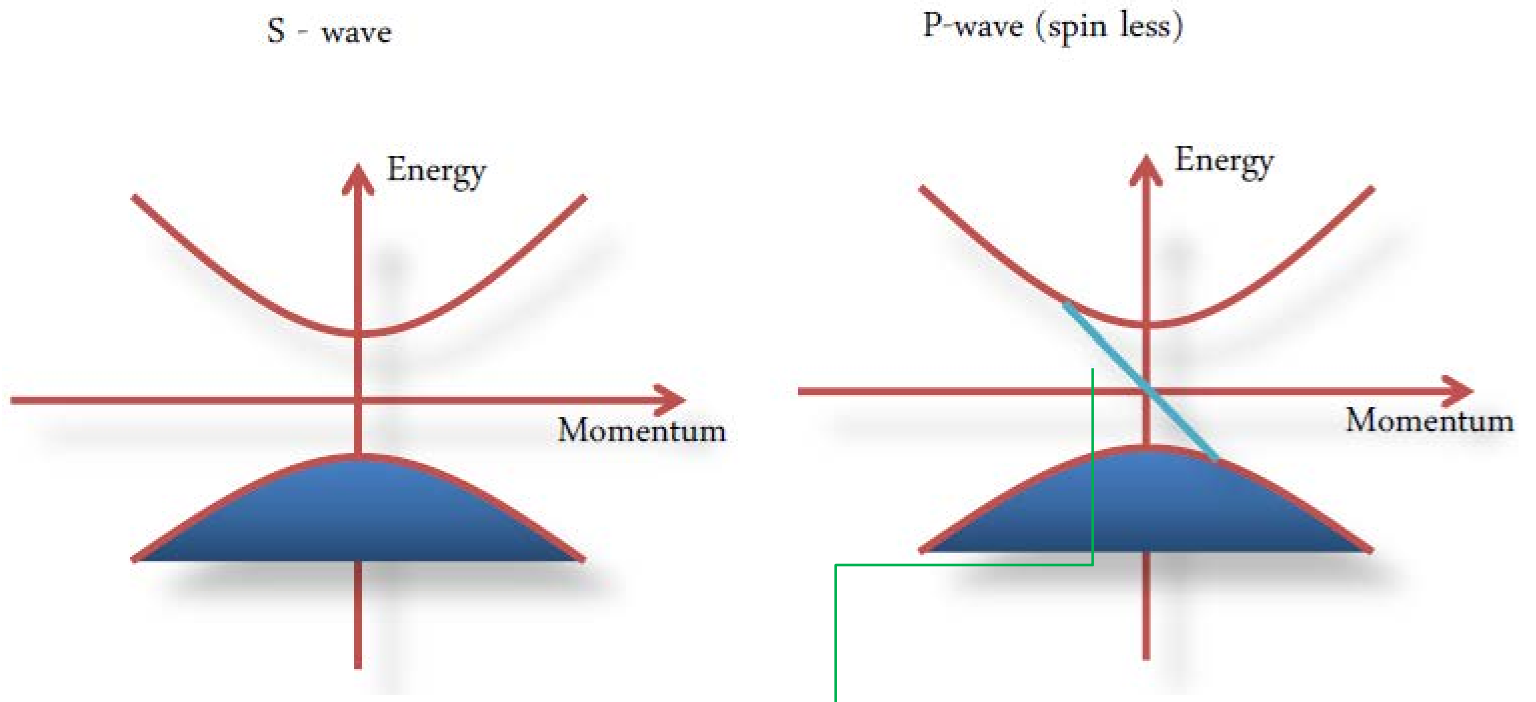
Desheng Kong and Yi Cui, Nature Chemistry 3, 845 (2011)





Topological superconductors : Superconductors with large SO coupling

M.Z. Hasan, C.Kane, RMP 2010,
X.L. Qi, S.C. Zhang, RMP 2011



Majorana fermions:

Gapless quasi-particle edge states with Dirac spectrum
Particle-hole symmetry

What is Majorana fermion

Majorana Fermion $=$ Real Dirac fermion

1. Dirac Hamiltonian

$$\mathcal{H}(\boldsymbol{k}) = \boldsymbol{\sigma}\cdot\boldsymbol{k}, \text{ or } \quad \mathcal{H}(k_x) = ck_x$$

2. Reality condition

$$\Psi = C\Psi^*$$

particle = antiparticle

- It was introduced as elementary particles.
- But now, it can be realized in topological superconductors.

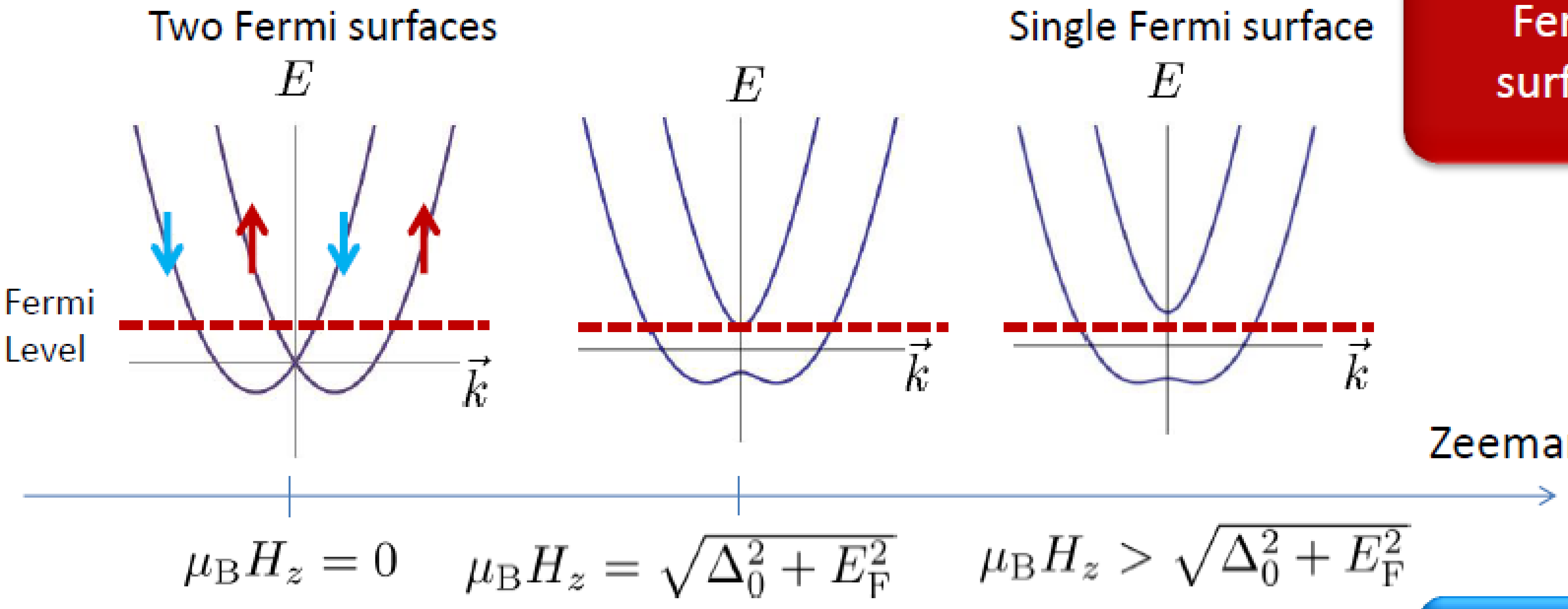
Read-Green(00), Stone-Roy(04),Qi et al (09), Schnyder et al (08), MS(09), MS-Fujimoto(09,10), Tanaka et al (09), Law-Ng-Lee(09), Akhmerov et al(09), J. Sau et al(10), Y. Oreg et al(10)

Superconducting state with SO interaction

Topological Edge state

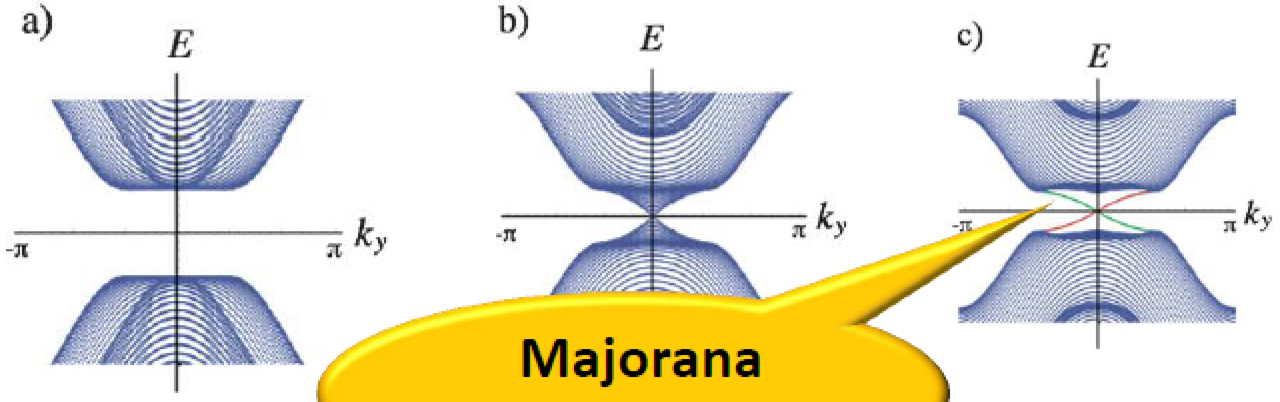
Sato-Takahashi-Fujimoto (09, 10)

Fermi surface

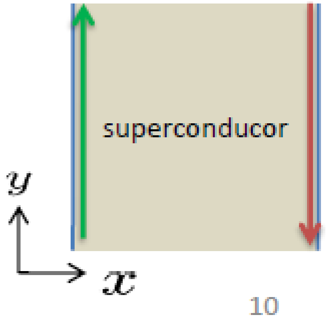


$\mu_{\rm B}H_z = 0$ $\quad$ $\mu_{\rm B}H_z = \sqrt{\Delta_0^2 + E_{\rm F}^2}$ $\quad$ $\mu_{\rm B}H_z > \sqrt{\Delta_0^2 + E_{\rm F}^2}$

Edge state



Majorana fermion



Prof. M. Sato’s talk, Dec. 13, 2013, NCTU, Taiwan

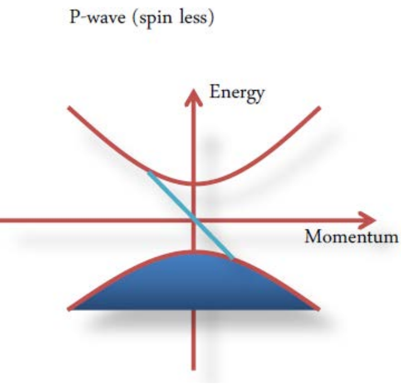
s-wave superconductors + 1D wire with strong SO coupling

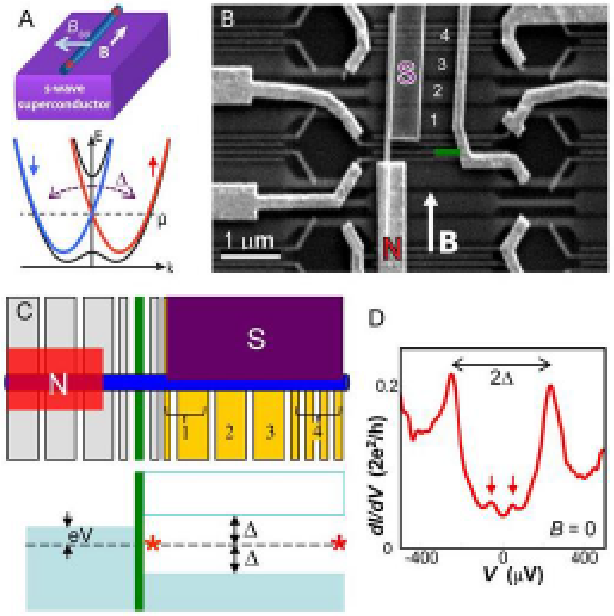
Sciencexpress **In Sb**

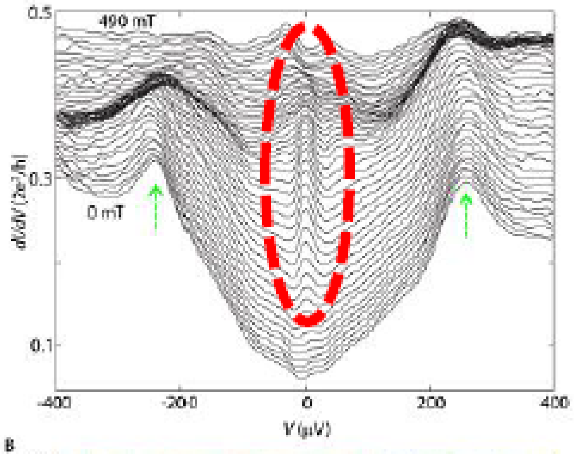
Sciencexpress / http://www.sciencemag.org/content/early/recent / 12 April 2012 / Page 3 / 10.1126/science.1222360

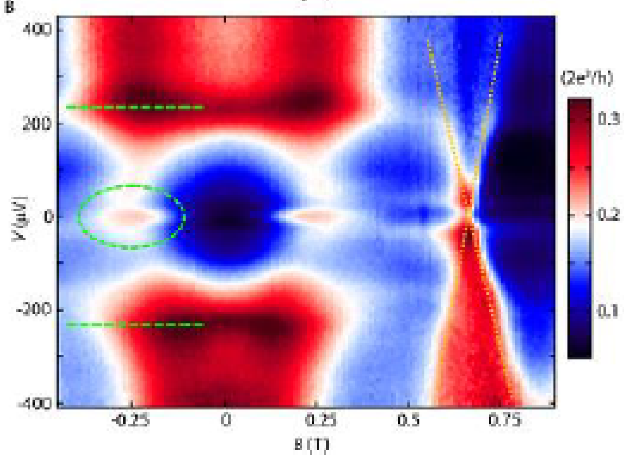
Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers Kouwenhoven[1†]





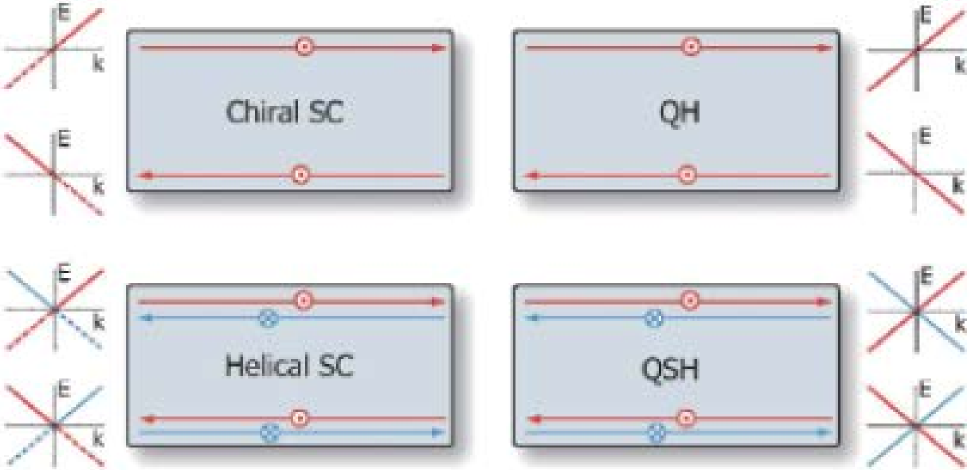




Prof. Yuval Oreg’s talk, Dec. 13, 2013, NCTU, Taiwan

Topological Numbers in Two Dimensions

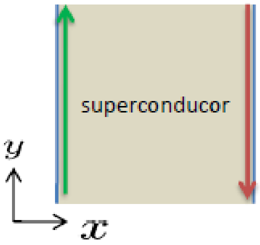
	Time-reversal-symmetric insulators/ superconductors	Time-reversal-breaking insulators/ superconductors
topological number	$\mathbb{Z}_2$ number (Kane-Mele)	1st Chern number (TKNN)
1D fermionic edge modes	Helical Dirac fermions/ Helical Majorana fermions	Chiral Dirac fermions/ Chiral Majorana fermions



Qi, X.-L., T. L. Hughes, S. Raghu, and S.-C. Zhang, 2009

Proposals for Majorana fermions

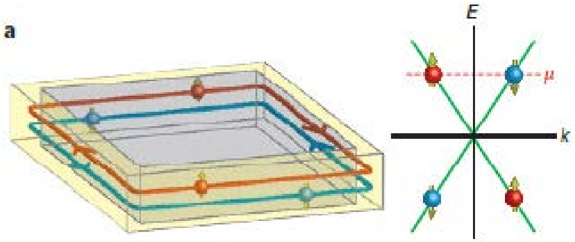
Chiral Majorana fermions



Spin triplet, p-wave superconductor in a magnetic field

Time-reversal broken topological superconductors

Can helical Majorana fermions exist?



- TRITOPs: Time-Reversal-Invariant-Topological-Superconductors
  1. Spin singlet, d-wave, s-wave + SO coupling via proximity effect

     Oreg 2014, Flensberg 2014, Law 2012

  2. spin-triplet $p_x \pm ip_y$ superconductors   S.C. Zhang 2009

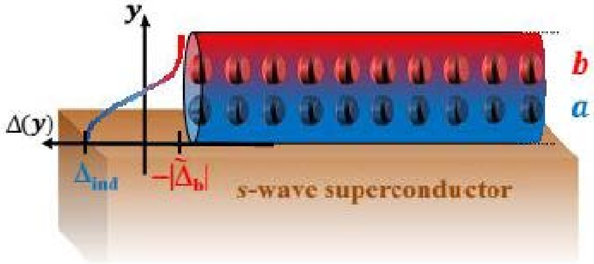
- New route: Helical Majorana fermions without TRS

  Via doping the Mott insulator with SO coupling

Helical Majorana fermions exist in TRS breaking chiral d-wave superconductors in doped Kane-Mele t-J model

Helical MFs in Time-Reversal-Topological-Superconductors (TRITOPs)

K-T Law et al PRB 2013

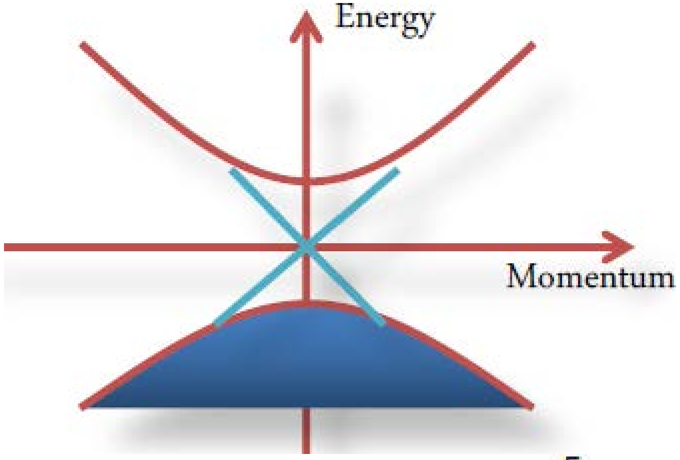


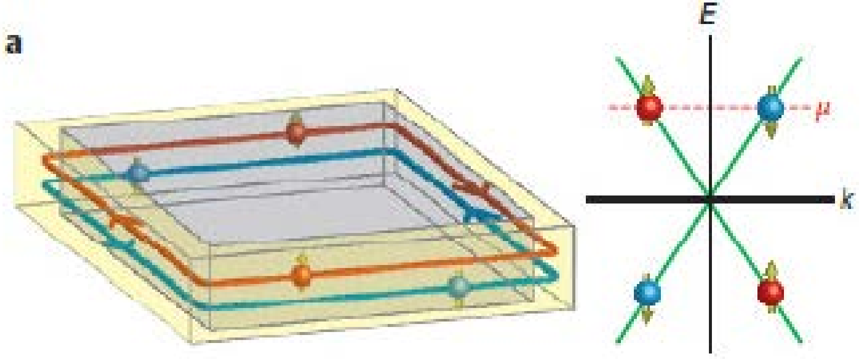
Y. Oreg et. al PRB 2014

Y.Oreg's talk, Dec. 13, 2013, NCTU, Taiwan

X-L Qi et al PRL 2009

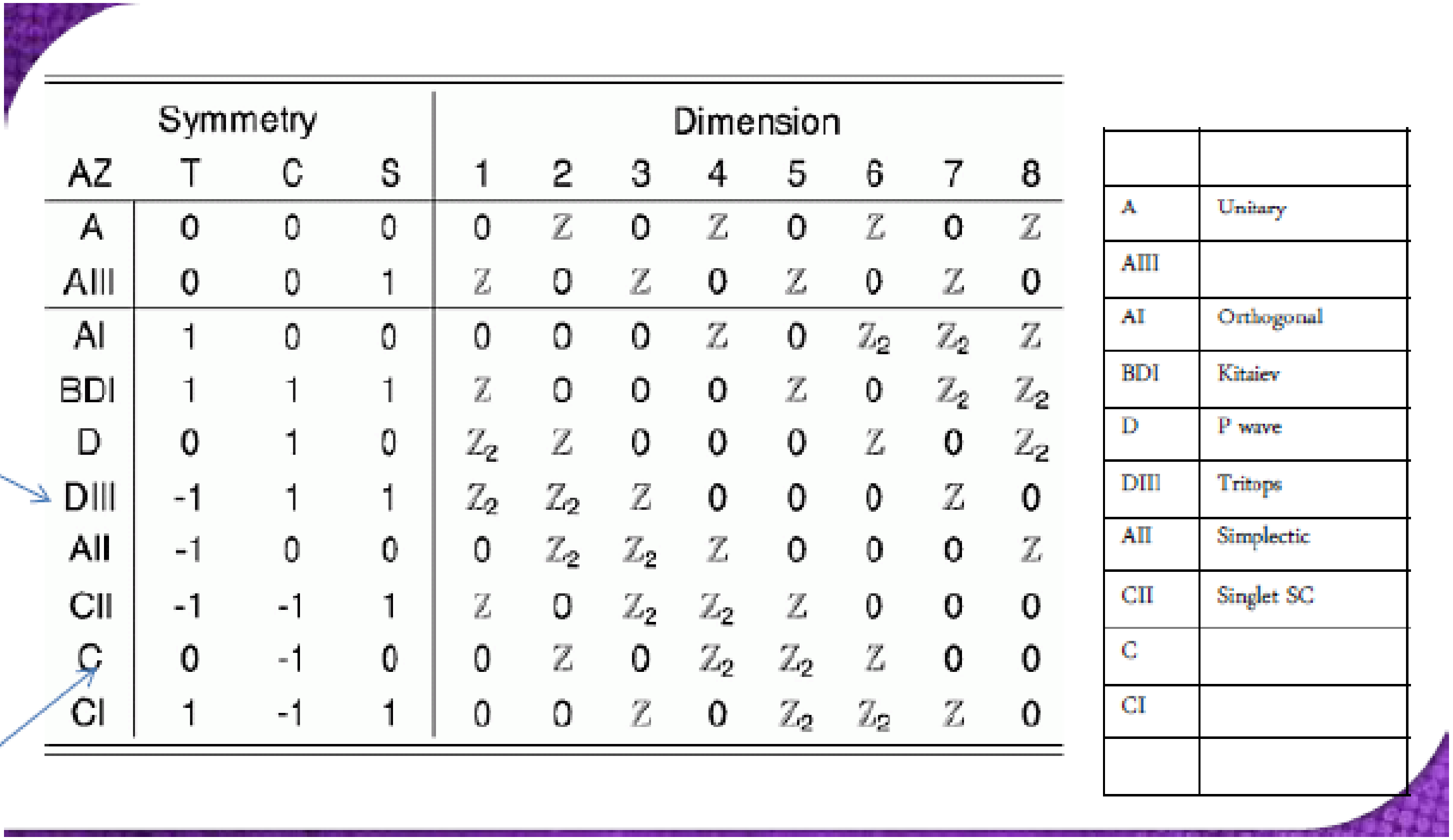
2D spin-triplet $p_x \pm ip_y$ superconductors





TRITOPS

Classification of topological superconductors



Symmetry				Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

A	Unitary
AIII	
AI	Orthogonal
BDI	Kitaiev
D	P wave
DIII	Tritops
AII	Simplectic
CII	Singlet SC
C	
CI	

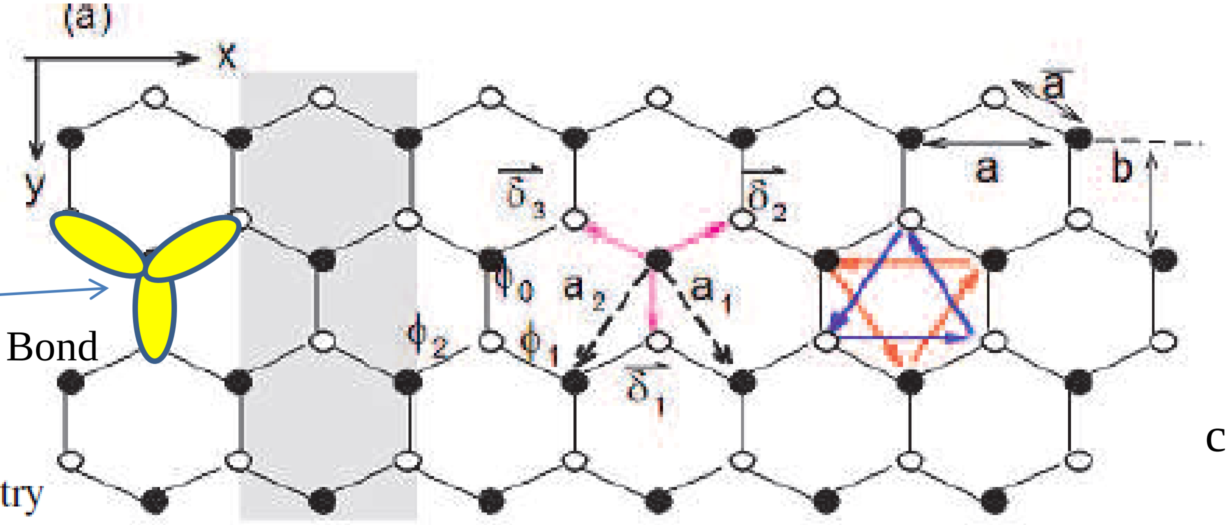
TRITOPs

Chiral d-wave sperconductor in graphene and doped KM t-J model

Schnyder et al (08)
Teo-Kane (10)

Pairing symmetry of superconductivity on correlated honeycomb lattice

nearest-neighbour pairing $\Delta_\eta(i) = \sum_l f_\eta(\boldsymbol{\delta}_l)\left(\hat{c}_{i\uparrow}\hat{c}_{i+\delta_l\downarrow} \pm \hat{c}_{i\downarrow}\hat{c}_{i+\delta_l\uparrow}\right)$



Resonating Valence Bond

lattice D_{6h} symmetry

t $c_i^{\dagger\alpha} c_j^\alpha$

J $\vec{S}_i \cdot \vec{S}_j$

graphene

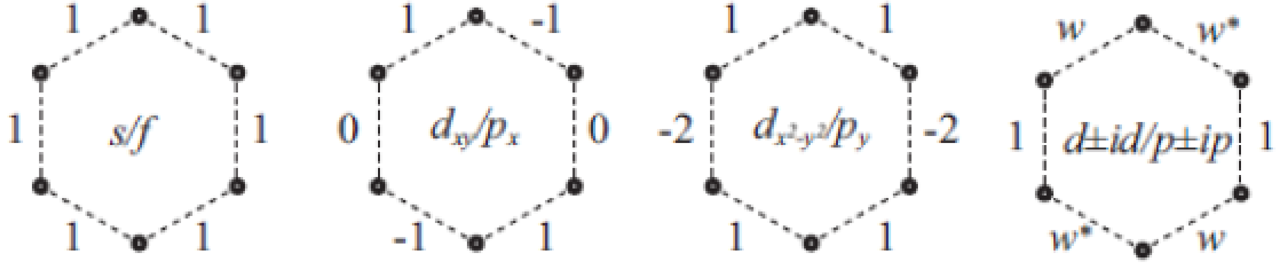
t~J

correlated regime

$w = \exp(\pm i\, 2\pi/3)$

$U = 3.3t$

Coulomb U



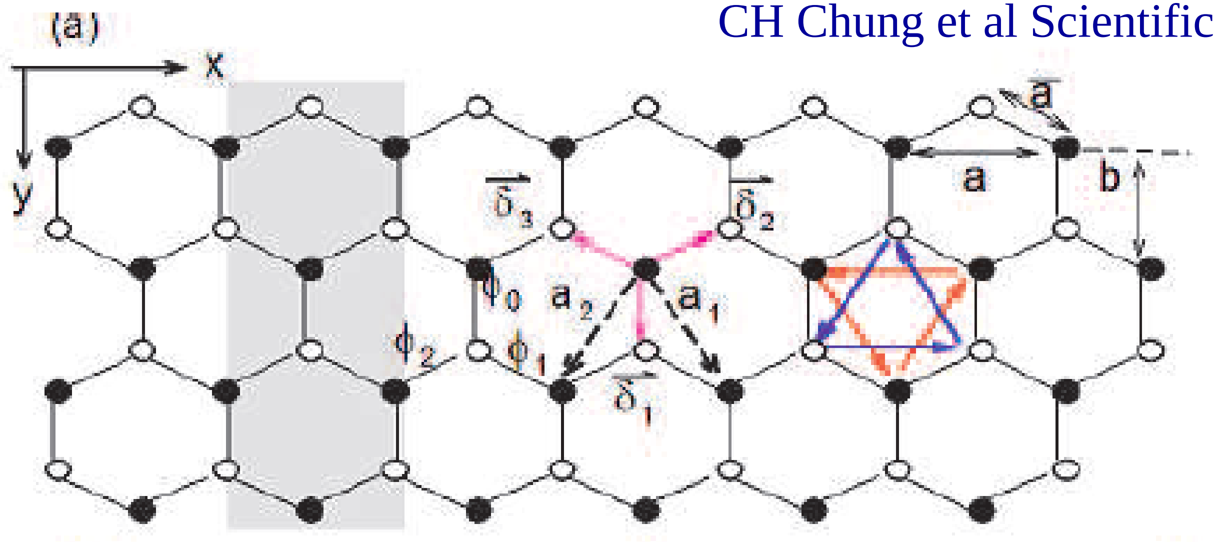
Stefan Wessel et al., PRB 94, 115105 (2016)

s, d_{xy}, $d_{x^2-y^2}$ and $d \pm id$ singlet pairing

p_x, p_y, $p \pm ip$ and f triplet

Helical Majorana fermions in chiral-d-wave superconductors in doped KM t-J model

CH Chung et al Scientific Reports 2016

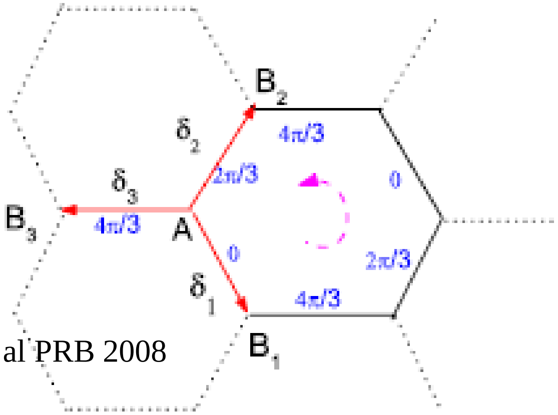


N=8

$$\phi_0 = 0, \phi_{1(2)} = -(+)2\pi/3$$

Superconducting pairing order parameter

$d_{x^2-y^2}+id'_{xy}$ wave pairing



J.P. Hu et al PRB 2008

$$\Delta(\vec{k}) = \sum_a \Delta_{\vec{\delta}_a} e^{i\vec{k}\cdot\vec{\delta}_a}$$

s-wave pairing $\Delta_{\vec{\delta}_a} = \Delta, \; a = 1,2,3$

$d_{x^2-y^2}+id'_{xy}$ wave pairing $\Delta_{\vec{\delta}_i} = \Delta e^{2ia\pi/3}, \; a = 1,2,3$

Renormalized Mean-Field Theory (RMFT)

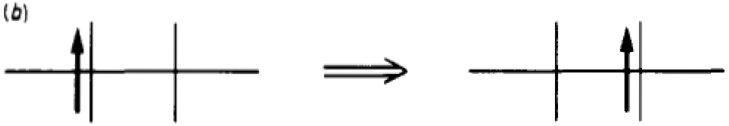
F C Zhang, C Gros, T M Rice and H Shiba

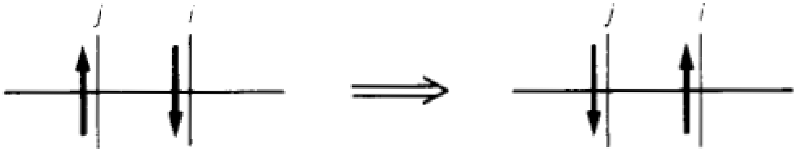
Supercond. Sci. Technol. 1 (1988) 36–46.

$$|\varphi\rangle = P_d|\varphi_0\rangle$$

$$|\varphi_0\rangle = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$$

Gutzwiller projected BCS wave function





$$\begin{aligned} H &= H_{KM} + H_J, \\ H_{KM} &= tg_t \sum_{\langle ij\rangle,\sigma} c^\dagger_{i\sigma} c_{j\sigma} - \mu \sum_i c^\dagger_i c_i, \\ &+ ig_t\lambda_{SO} \sum_{\langle\langle ij\rangle\rangle,\sigma} \nu_{ij} c^\dagger_{i\sigma} s^z c_{j\sigma} + h.c. \\ H_J &= Jg_s \sum_{<i,j>} S_i S_j \end{aligned}$$

$$\begin{aligned} H_J &= H_\chi + H_\Delta + H_{const} \\ H_\chi &= \sum_{i,\alpha,a} \chi_{\vec{\delta}_a} c^{\dagger\alpha}_{A,i} c^\alpha_{B,i+\vec{\delta}_a} + h.c. \\ H_\Delta &= \sum_{i,a} \Delta_{\vec{\delta}_a} [c^{\dagger\uparrow}_{A,i} c^{\dagger\downarrow}_{B,i+\vec{\delta}_a} - c^{\dagger\downarrow}_{A,i} c^{\dagger\uparrow}_{B,i+\vec{\delta}_a}] + h.c. \\ H_{const} &= N_s \sum_{a=1,2,3} [\frac{|\chi_{\vec{\delta}_a}|^2}{\frac{3}{4}g_s J} + \frac{|\Delta_{\vec{\delta}_a}|^2}{\frac{3}{4}g_s J}] - 2N_s\mu\delta \end{aligned}$$

RMFT: $\begin{cases} t \rightarrow tg_t \\ J \rightarrow g_s J \end{cases}$

$$g_t = 2\delta/(1+\delta)$$

$$g_s = 4/(1+\delta)^2$$

$$\boxed{H_k = \psi_k^\dagger \mathcal{M}_k \psi_k}$$

$$\psi_k = (c^{\uparrow}_{A,k}, c^{\uparrow}_{B,k}, c^{\dagger\downarrow}_{A,-k}, c^{\dagger\downarrow}_{B,-k})$$

$$E_k = \pm\sqrt{(\sqrt{|\gamma_k|^2 + |\epsilon_k|^2} \pm \mu)^2 + |\Delta_k|^2}$$

$$\mathcal{M}_k = \begin{pmatrix} \hat{h}_k & \hat{\Delta}_k \\ \hat{\Delta}_k^\dagger & -\hat{h}^*_{-k} \end{pmatrix}$$

$$\Delta(\vec{k}) = \sum\nolimits_{a=1,2,3} \Delta_{\vec{\delta}_a} e^{i\vec{k}\cdot\vec{\delta}_a}$$

$$, \hat{h}_k = \begin{pmatrix} h_k^+ & \\ & \hat{h}_k^- \end{pmatrix}, \hat{\Delta}_k = \begin{pmatrix} 0 & \bar{\Delta}_k \\ -\bar{\Delta}_k & 0 \end{pmatrix}$$

$$\gamma_k = 2g_t\lambda_{SO}[-\sin(\sqrt{3}k_y) + 2\cos(3k_x/2)\sin(\sqrt{3}k_y/2)]$$

$$\epsilon_k = -(tg_t + \chi)\sum\nolimits_{a=1,2,3} e^{ik\cdot\vec{\delta}_a}$$

$$h_k^\pm = \begin{pmatrix} \pm\gamma_k - \mu & \epsilon_k \\ \epsilon_k^* & \mp\gamma_k - \mu \end{pmatrix}, \bar{\Delta}_k = \begin{pmatrix} 0 & \Delta_k \\ \Delta_{-k} & 0 \end{pmatrix}$$

Symmetries of H_k

- Particle-Hole Symmetry C:

C: $C_k \rightarrow C_k^\dagger$

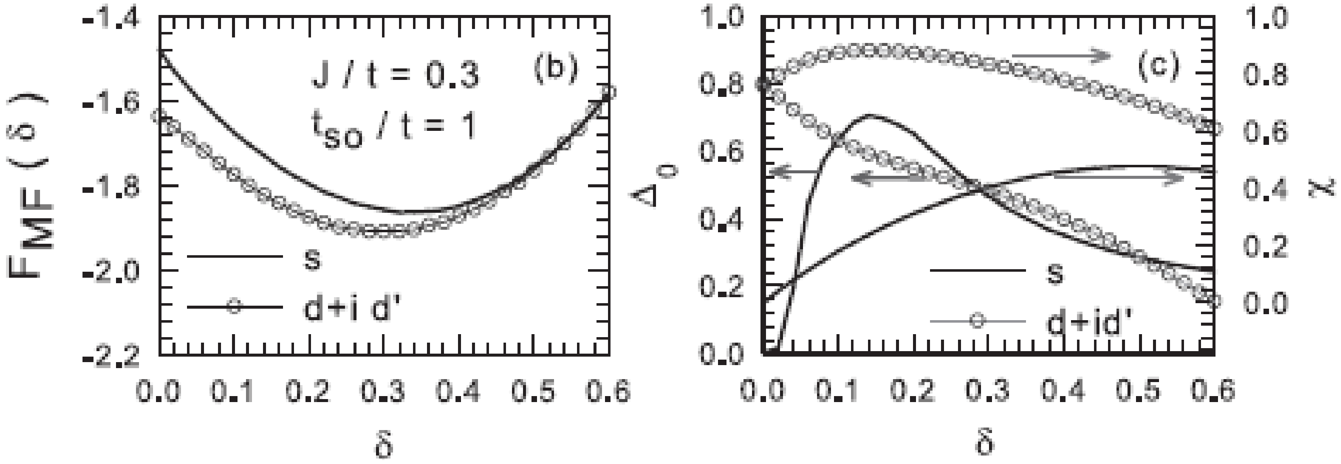
$$\boxed{C^{-1} H_k C = - H_{-k}}$$

- Sub-lattice symmetry S:

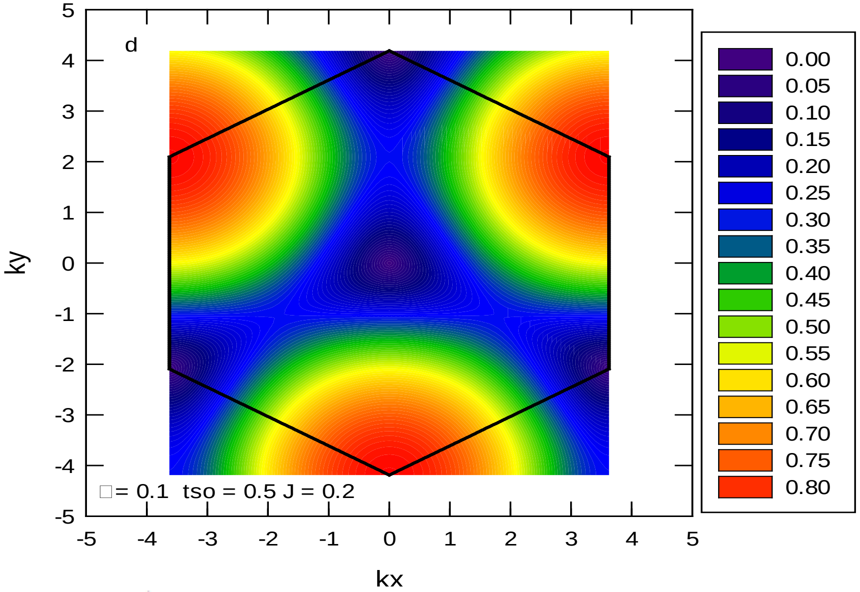
S: $C_A(k) \rightarrow C_B(k)$

$$\boxed{S^{-1} H_k S = H_{-k}}$$

d+id'-wave superconductivity

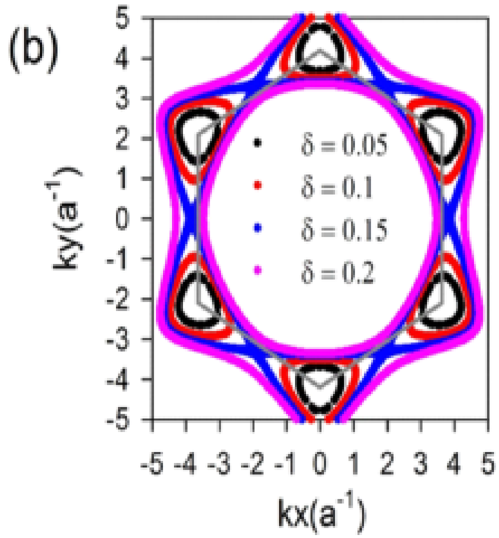


TRS breaking $\Delta_k^{d\pm id'}$



CH Chung et al Scientific Reports 2016

Normal state Fermi surface



$$\Delta_k^{d+id'} = \cos(\pi/3)\Delta_{dx^2-y^2}(k) + i\sin(\pi/3)\Delta_{d_{xy}}(k)$$

$$\Delta_{dx^2-y^2}(k) = \Delta_0(2e^{ik_x/2}e^{-i\sqrt{3}k_y/2} - e^{ik_x/2}e^{i\sqrt{3}k_y/2} - e^{-ik_x})$$

$$\Delta_{dxy}(k) = \Delta_0(e^{ik_x/2}e^{i\sqrt{3}k_y/2} - e^{-ik_x})$$

Time-reversal symmetric (TRS) v.s. Time-reversal broken (TRB) superconductors

TRS: $T^{-1}\ H_k\ T\ =\ H_{-k}$ $T = i s^y K$

$$\Delta^*(-k) = \Delta(k)$$

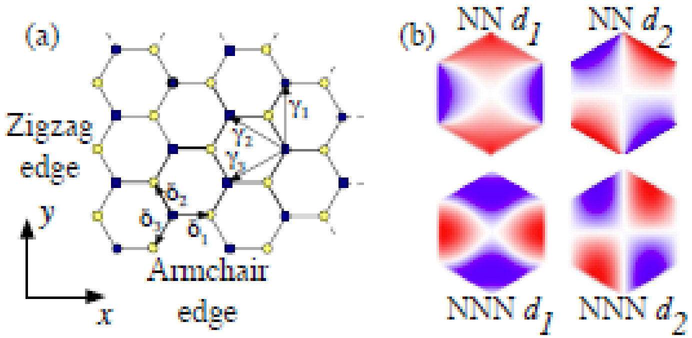
$$\begin{cases} \Delta_{dx^2-y^2}(k) = \Delta_0(e^{-i\frac{k_y}{\sqrt{3}}} - e^{i\frac{k_y}{2\sqrt{3}}}\cos(\frac{k_x}{2})) \\ \Delta_{dxy}(k) = i\Delta_0 e^{i\frac{k_y}{2\sqrt{3}}}\sin(\frac{k_x}{2}). \end{cases}$$

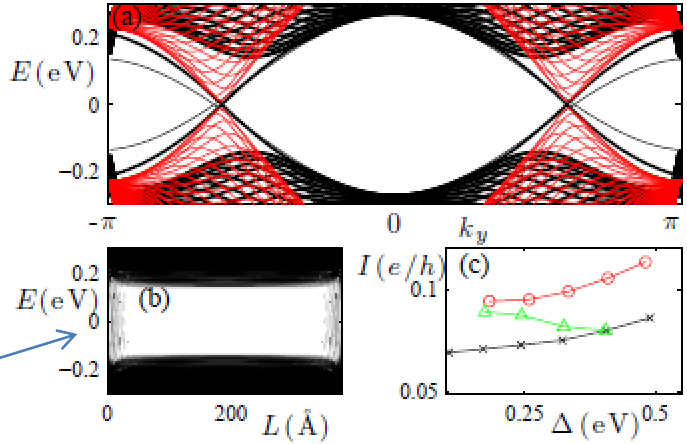
TRB: $$\Delta_k^{d\pm id'} = \cos(\pi/3)\Delta_{dx^2-y^2}(k) \pm i\sin(\pi/3)\Delta_{xy}(k)$$

$t_{so} \ll \Delta_0$ chiral edge state

$d_{x^2-y^2}+id'_{xy}$ wave pairing in graphene superconductivity

Black-Schaffer PRL 2013





chiral edge state

TKNN number in spin-singlet chiral-d-wave superconductors in graphene (t_{SO}→0)

$$\Delta_k^{d\pm id'} = \cos(\pi/3)\Delta_{dx^2-y^2}(k) \pm i\sin(\pi/3)\Delta_{xy}(k)$$

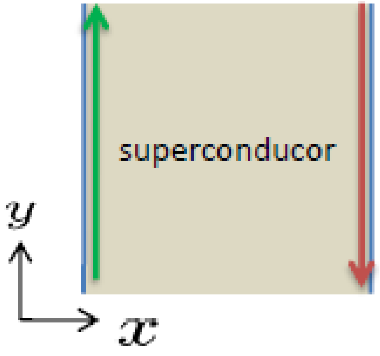
d +i d-wave pairing breaks time-reversal symmetry (TRS) and parity (P) symmetry

TKNN number: $\mathcal{N} = \frac{1}{4\pi}\int_{\mathrm{BZ}} d^2k\, \hat{\mathrm{m}} \cdot \left(\frac{\partial \hat{\mathrm{m}}}{\partial k_x} \times \frac{\partial \hat{\mathrm{m}}}{\partial k_y}\right)$

$$\hat{\mathrm{m}} = \frac{1}{\sqrt{\varepsilon(\mathbf{k})^2 + |\Delta(\mathbf{k})|^2}} \begin{pmatrix} \mathrm{Re}\,\Delta(\mathbf{k}) \\ \mathrm{Im}\,\Delta(\mathbf{k}) \\ \varepsilon(\mathbf{k}) \end{pmatrix}$$

spin-singlet $d_{x^2-y^2} + id_{xy}$ state with $\Delta(\mathbf{k}) \propto \cos 2\phi \pm i \sin 2\phi$

$$\phi = \arctan(k_y/k_x)$$



$$\mathcal{N} = \pm 2$$

chiral superconductors

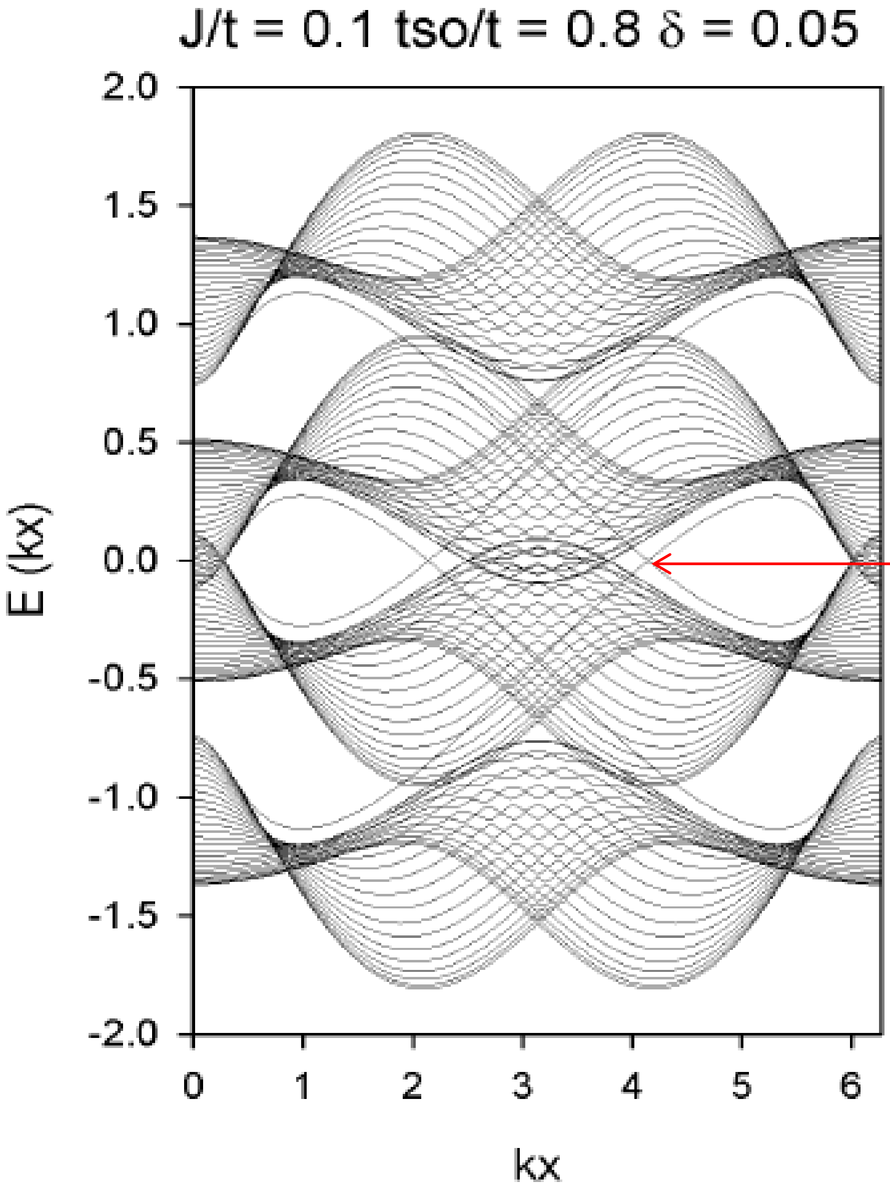
2 co-propagating chiral edge modes

Black-Schaffer et al. J. Phys. C 2014

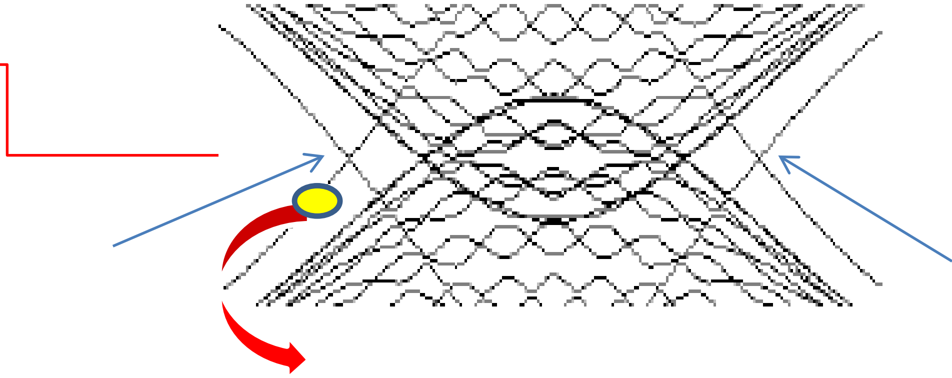
Bogoliubov excitation spectrum at large tso

$\Delta_0 < t_{so}$

CH Chung et al Scientific Reports 2016

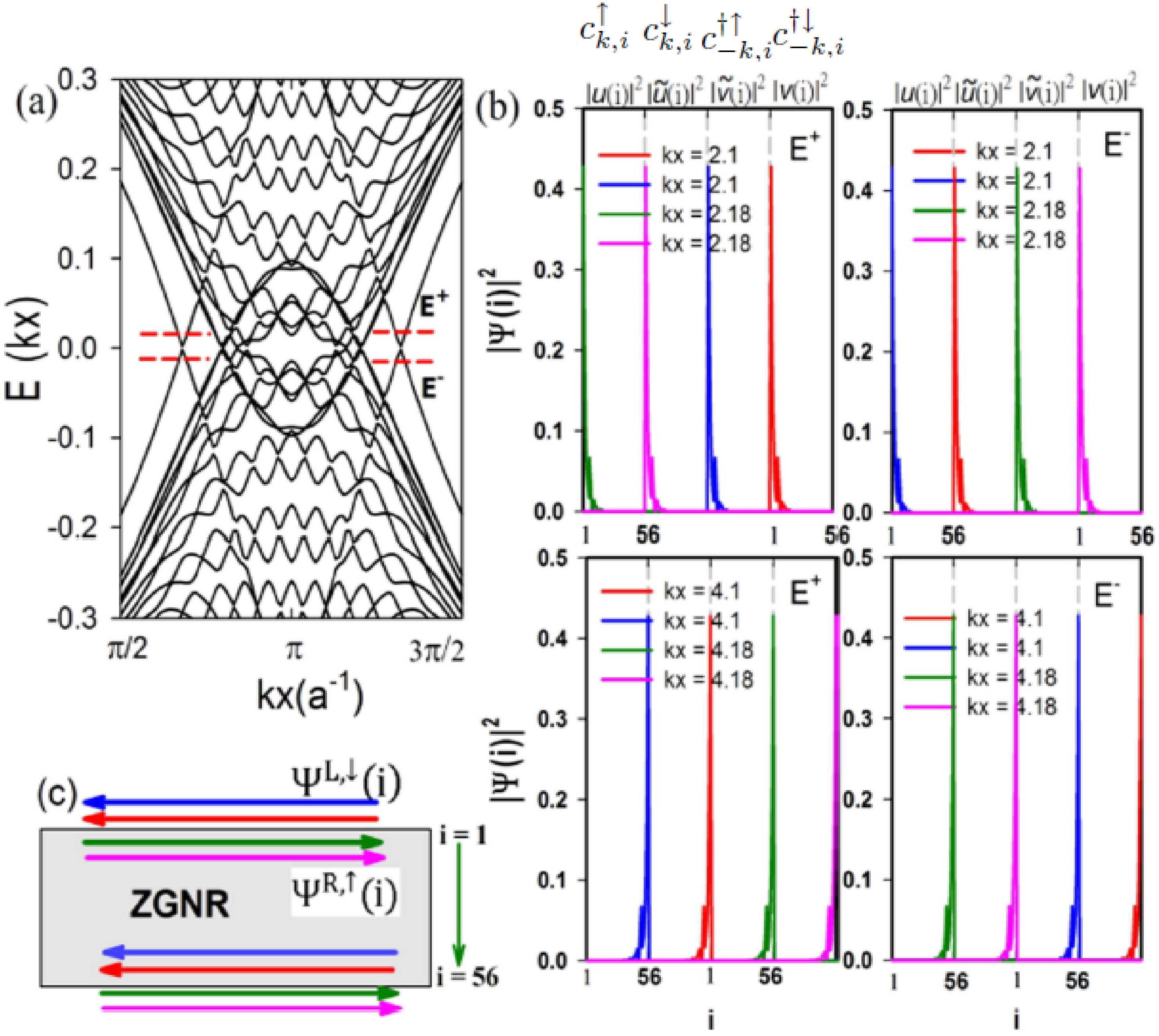


2 Dirac-dispersed lines



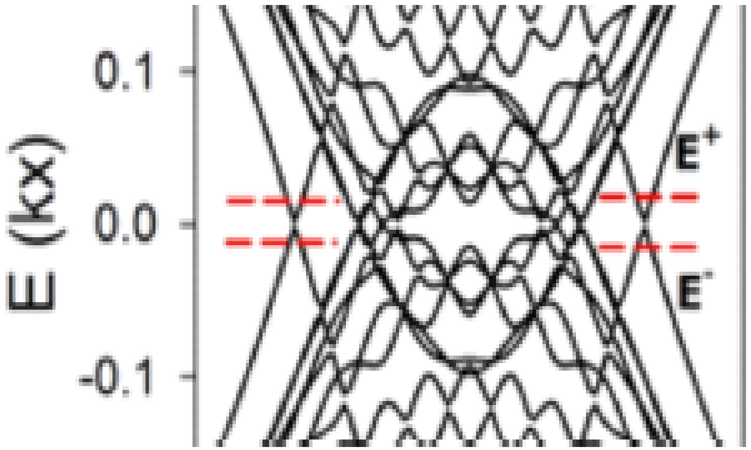
2-fold degeneracy at each kx

Helical Majorana zero modes in d+id'-wave superconductivity in doped KM t-J ribbon



Bogoliubov quasi-particle operators

CH Chung et al Scientific Reports 2016



$$H_{edge} = \sum_{\bar{k},\tau=\uparrow,\downarrow} \bar{k} \, (\gamma_{\bar{k}}^{\dagger R\tau} \gamma_{\bar{k}}^{R\tau} - \gamma_{\bar{k}}^{\dagger L\tau} \gamma_{\bar{k}}^{L\tau}),$$

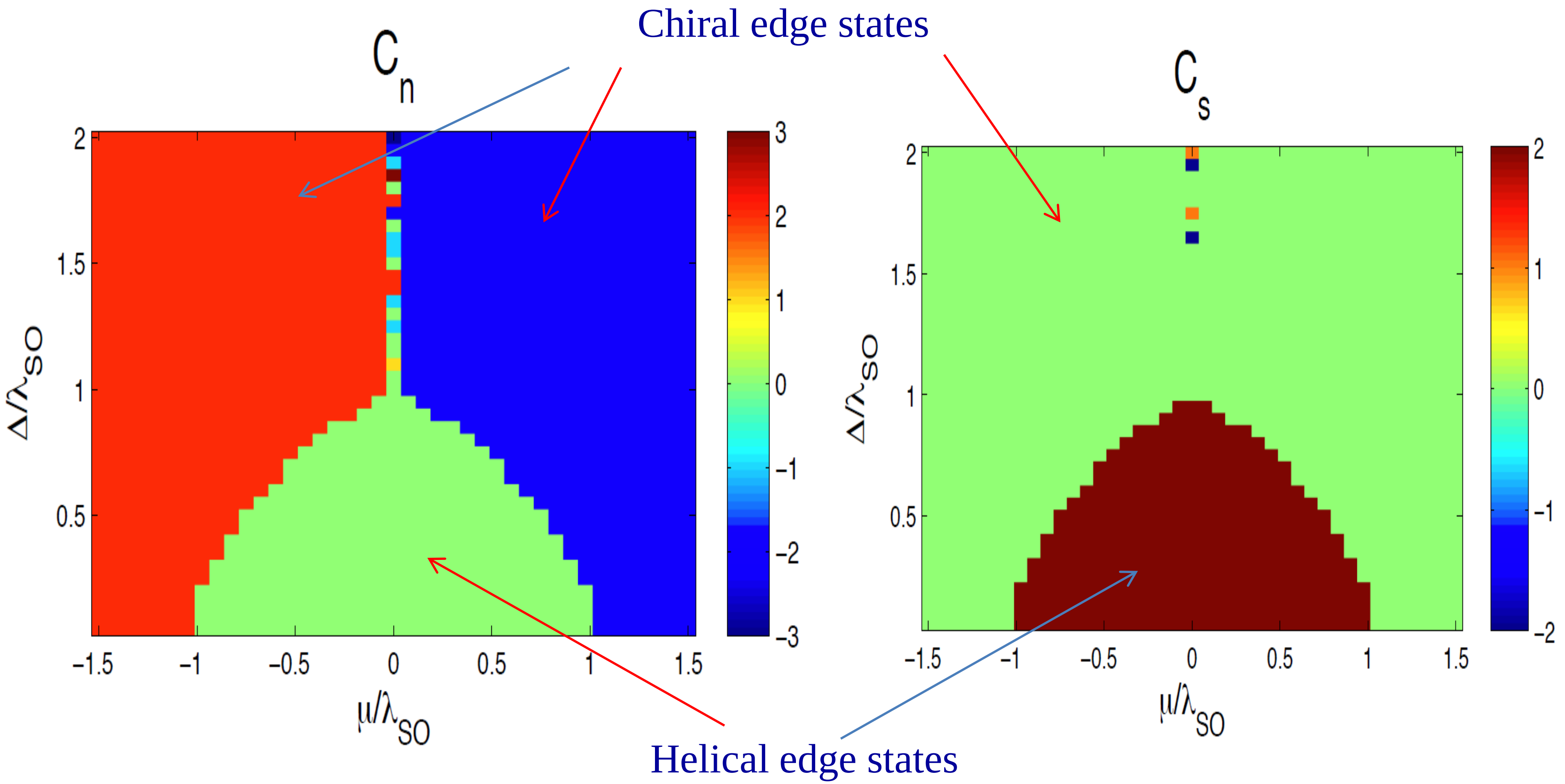
$$\begin{aligned} \gamma_k^{R\tau} &= u_{k,i}^{\tau} c_{k,i}^{\uparrow} + \tilde{u}_{k,i}^{\tau} c_{k,i}^{\downarrow} + \tilde{v}_{k,i}^{\tau} c_{-k,i}^{\dagger\uparrow} + v_{k,i}^{\tau} c_{-k,i}^{\dagger\downarrow}, \\ \gamma_k^{L\tau} &= -v_{k,i}^{\tau} c_{-k,i}^{\uparrow} + \tilde{v}_{k,i}^{\tau} c_{-k,i}^{\downarrow} - \tilde{u}_{k,i}^{\tau} c_{k,i}^{\dagger\uparrow} + u_{k,i}^{\tau} c_{k,i}^{\dagger\downarrow}, \end{aligned}$$

Helical Majorana fermions protected by the additional symmetry

Topological quantum phase diagram

Charge Chern number

Pseudo-Spin Chern number

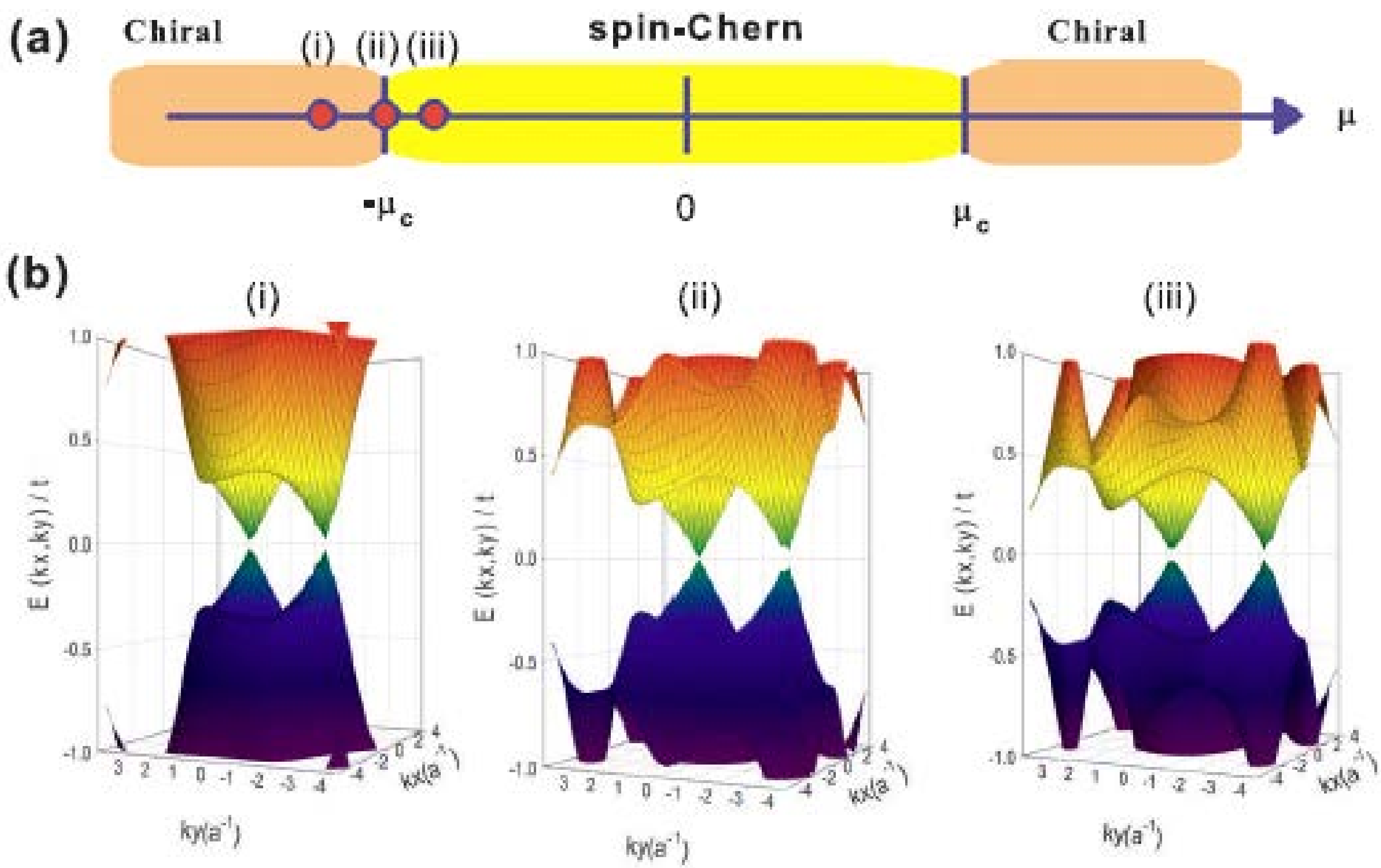

Chiral edge states

Helical edge states

S M Huang, CY Mou, Wei-Feng-Tsai, CH Chung, PRB, 2016

Topological phase transition between choral and spin-Chern phases

N_{TKNN} =2 $N_{spin\text{-}Chern}$ =2 N_{TKNN} =2



Bulk band gap closes at the phase transition

CH Chung et al Scientific Reports 2016

Pseudo-spin Symmetry protects helical Majorana

$$c^{\uparrow}_{SC,k} = (c^{\uparrow}_{A,k}, c^{\uparrow}_{B,k}, -c^{\dagger\downarrow}_{A,-k}, c^{\dagger\downarrow}_{B,-k}) \qquad c^{\downarrow}_{SC,k} = (c^{\downarrow}_{A,k}, c^{\downarrow}_{B,k}, -c^{\dagger\uparrow}_{A,-k}, c^{\dagger\uparrow}_{B,-k});$$

$$H^{\uparrow}_{SC} = \begin{pmatrix} \gamma_k - \mu & \epsilon_k & 0 & \Delta_k \\ \epsilon^*_k & -\gamma_k - \mu & -\Delta_{-k} & 0 \\ 0 & -\Delta^*_{-k} & -\gamma_k + \mu & \epsilon_k \\ \Delta^*_k & 0 & \epsilon^*_k & \gamma_k + \mu \end{pmatrix};$$

CH Chung et al
Scientific Reports 2016

Pseudo-spin Symmetry $\boxed{\tau_z^{-1} H^{\uparrow(\downarrow)}_{SC} \tau_z = H^{\uparrow(\downarrow)}_{SC}}$

$$\underset{8\times 8}{\boxed{H_k = \psi_k^{\dagger} \mathcal{M}_k \psi_k}} \Rightarrow \begin{bmatrix} \overset{4\times 4}{H^{\uparrow}_{SC}} & 0 \\ 0 & \underset{4\times 4}{H^{\downarrow}_{SC}} \end{bmatrix}$$

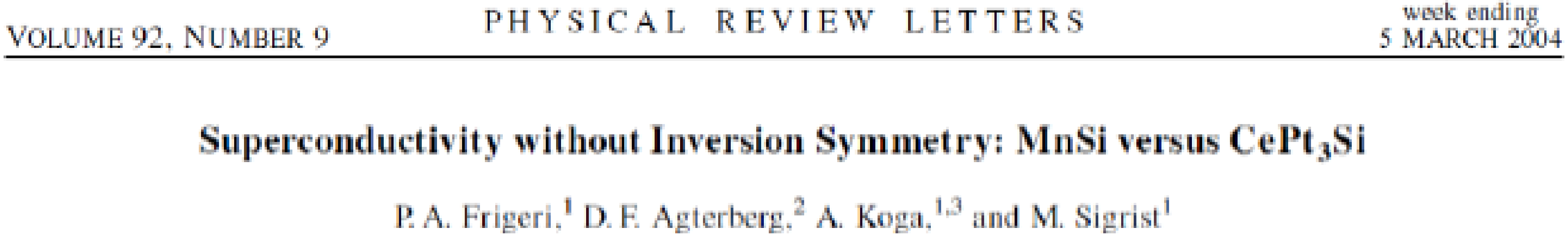
pseudo-spin Chern number $C_w = (C_n - C_{\bar{n}})/2$ $\boxed{C_w = 1}$

n and $\bar{n}$

class D topological superconductors

$$(c^{\uparrow}_{A,k}, c^{\uparrow}_{B,k}) \rightarrow (-c^{\dagger\downarrow}_{A,-k}, c^{\dagger\downarrow}_{B,-k})$$

p-wave pairing state & Rashba SOC



VOLUME 92, NUMBER 9 PHYSICAL REVIEW LETTERS week ending 5 MARCH 2004

Superconductivity without Inversion Symmetry: MnSi versus $CePt_3Si$

P. A. Frigeri,[1] D. F. Agterberg,[2] A. Koga,[1,3] and M. Sigrist[1]

Calculate T_C using gap equation:

The triplet (p-wave) component is aligned with the Rashba coupling.

p wave pairing

$$H_p = \alpha \sum_{\boldsymbol{k},s,s'} \boldsymbol{g}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}_{ss'} c^{\dagger}_{\boldsymbol{k}s} c^{\dagger}_{\boldsymbol{k}s'},$$

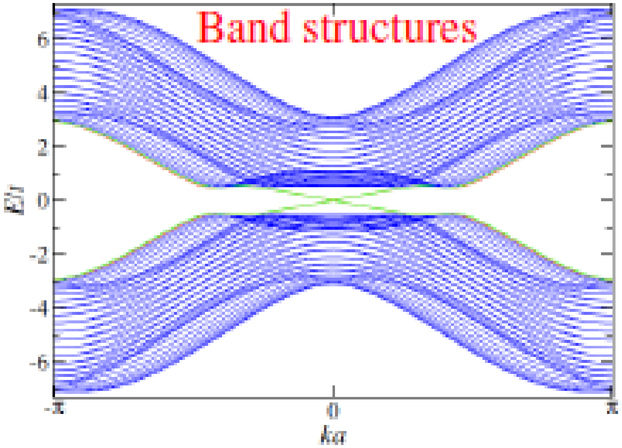
aligned

Rashba coupling

$$H_R = \lambda_R \sum_{k\sigma\sigma'} \tilde{R}_k \cdot \tilde{\sigma} a^{\dagger}_{k\sigma} b_{k\sigma'} + h.c.$$

Square lattice in stripe geometry

Rashba SOC + Zeeman coupling



The gapless edge states form a Kramers pair.

M. Sato and S. Fujimoto, PRB 79, 094504 (2009)

p+ip-wave triplet pairing and Rashba SOC on *honeycomb* lattice

p+ip wave pairing

$$H_p = \alpha \sum_{k,s,s'} g_k \cdot \sigma_{ss'} c^{\dagger}_{ks} c^{\dagger}_{ks'},$$

Rashba coupling

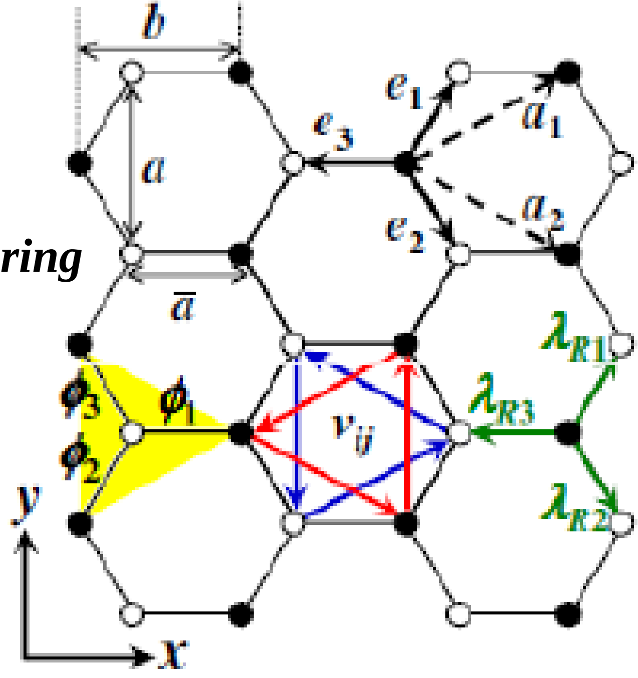
$$H_R = \lambda_R \sum_{k\sigma\sigma'} \tilde{R}_k \cdot \tilde{\sigma} a^{\dagger}_{k\sigma} b_{k\sigma'} + h.c.$$

The triplet component is aligned with the Rashba coupling.

Via KM SO, ferromagnetic spin coupling on NNN sites

-J’ $\vec{S}_i \cdot \vec{S}_j$

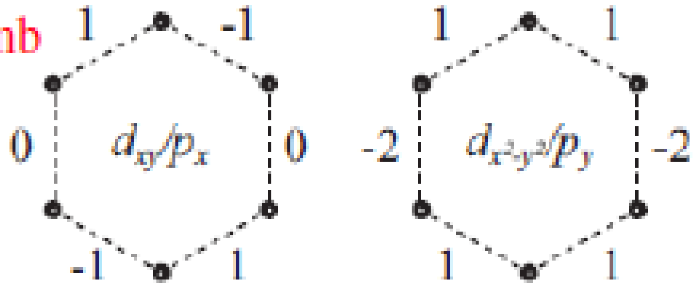
triplet pairing



Rashba terms: $(\bar{\sigma} \times e_{ij})_z$

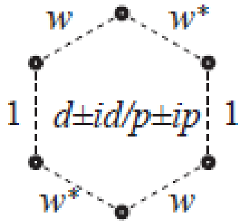
$$\boxed{\begin{aligned} \lambda_{R1} &= (\sigma_y - \sqrt{3}\sigma_x)/2, \\ \lambda_{R2} &= (\sigma_y + \sqrt{3}\sigma_x)/2, \\ \lambda_{R3} &= -\sigma_y, \end{aligned}}$$

Honeycomb lattice



pairing amplitude for NN pairing channels

***p+ip-wave triplet* pairing**



A. M. Black-Schaffer, S. Doniach, *Phys. Rev. B* **2007**, *75*, 134512.

NCS on *honeycomb* lattice

CH Chung et al pssb 2018

$$\boxed{H = H_t + H_{KM} + H_R + H_\mu + H_B + H_\Delta,}$$

$$H_t = t\sum_{k\sigma} g_k a^\dagger_{k\sigma} b_{k\sigma} + \text{h.c.},$$

$$H_{KM} = \lambda_{SO}\sum_{k\sigma}\gamma_k\sigma_z\left(a^\dagger_{k\sigma}a_{k\sigma} - b^\dagger_{k\sigma}b_{k\sigma}\right),$$

$$H_R = \lambda_R\sum_{k\sigma\sigma'}\tilde{R}_k\cdot\tilde{\sigma}a^\dagger_{k\sigma}b_{k\sigma'} + \text{h.c.},$$

$$H_\mu = \mu\sum_{k\sigma}\left(a^\dagger_{k\sigma}a_{k\sigma} + b^\dagger_{k\sigma}b_{k\sigma}\right),$$

$$H_B = \mu_B B_z\sum_{k\sigma}\sigma_z\left(a^\dagger_{k\sigma}a_{k\sigma'} + b^\dagger_{k\sigma}b_{k\sigma'}\right),$$

$$H_\Delta = \frac{1}{2}\sum_{k\sigma\sigma'}\left[\Delta(k)a^\dagger_{k\sigma}b^\dagger_{-k\sigma'} + \Delta^*(k)\,a_{-k\sigma}b_{k\sigma'}\right].$$

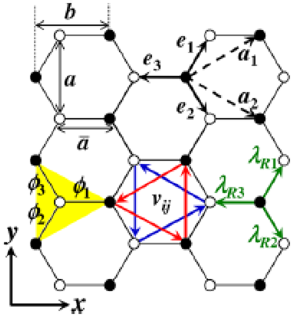
$$\tilde{R}_k = \sum_{l=1}^{3}\left(-e_{l,y}, e_{l,x}\right)e^{i(k\cdot e_l+\pi/2)},$$

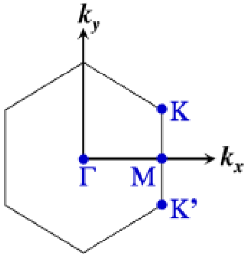
Singlet-triplet mixing superconducting pairing

$$\boxed{\Delta(k) = i\Delta_d L_k\sigma_y + i\Delta_t\tilde{R}_k\cdot\tilde{\sigma}\sigma_y}$$

Δ_d **: d + *i* d-wave singlet pairing**

Δ_t **: p + *i* p-wave triplet pairing**



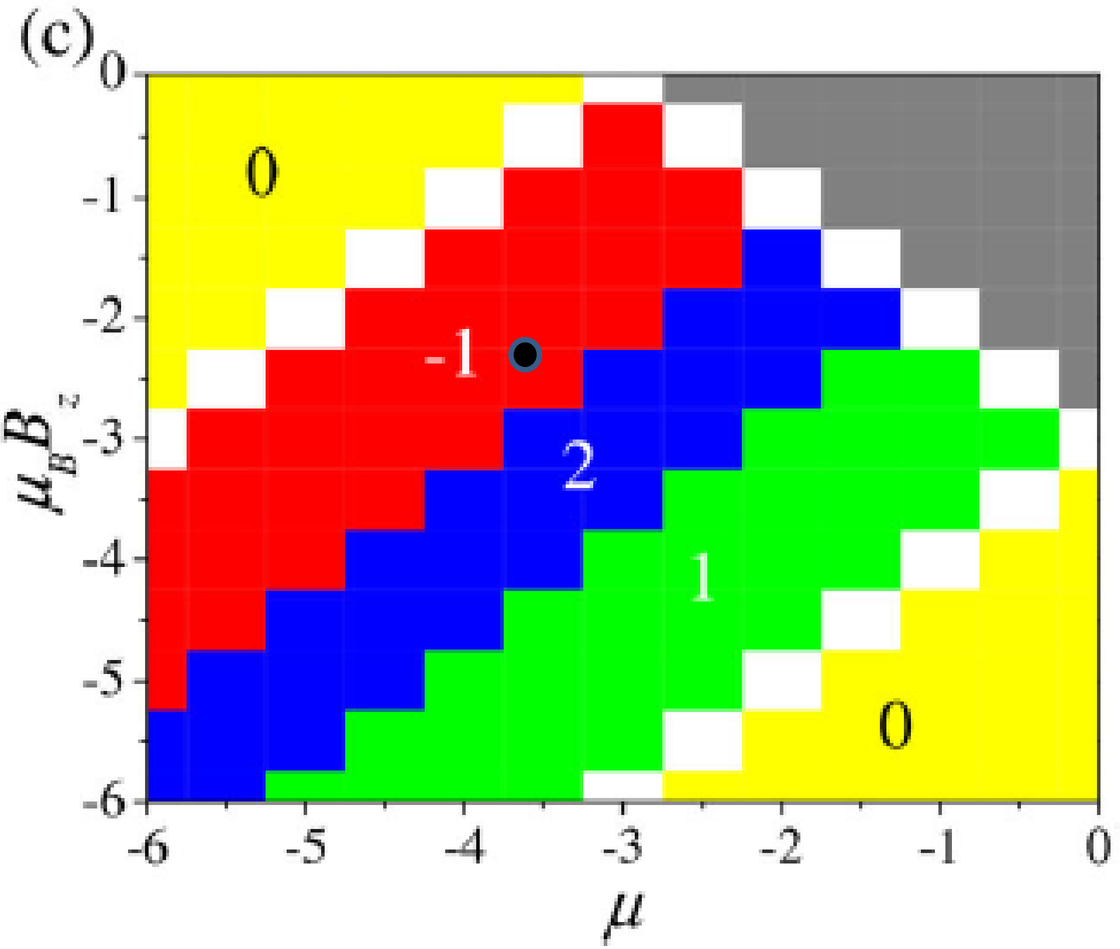


Topological phase diagram

CH Chung et al pssb 2018

charge Chern number

$$C = -\frac{1}{\pi}\int\int dk_x dk_y \sum_{E_\alpha<0<E_\beta} \frac{\mathrm{Im}\,\langle\alpha|\,\partial_{k_y}\tilde{h}\,(k)\,|\beta\rangle\,\langle\beta|\,\partial_{k_y}\tilde{h}\,(k)\,|\alpha\rangle}{\left(E_\alpha - E_\beta\right)^2}$$



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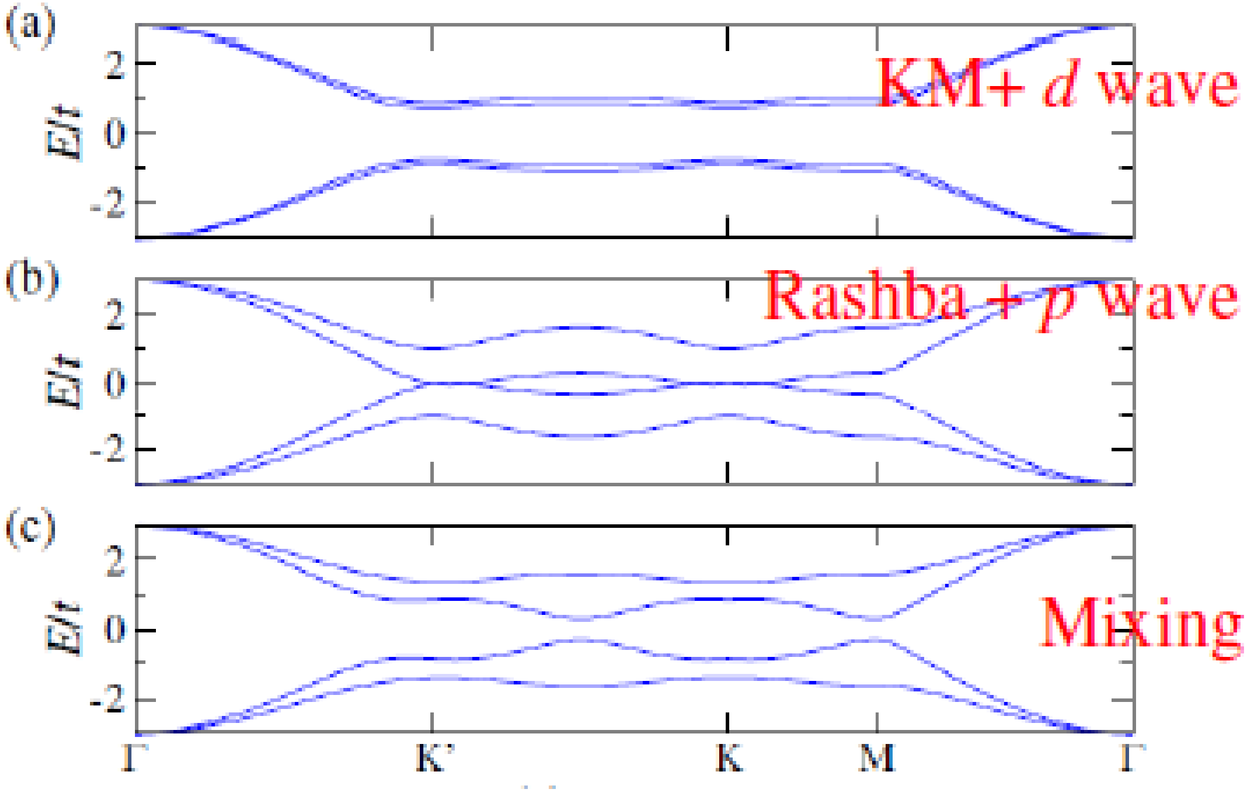
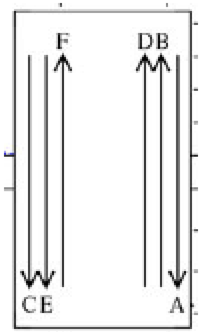
Dispersion relation along high symmetry points $(\mu = \mu_B B_z = -3)$

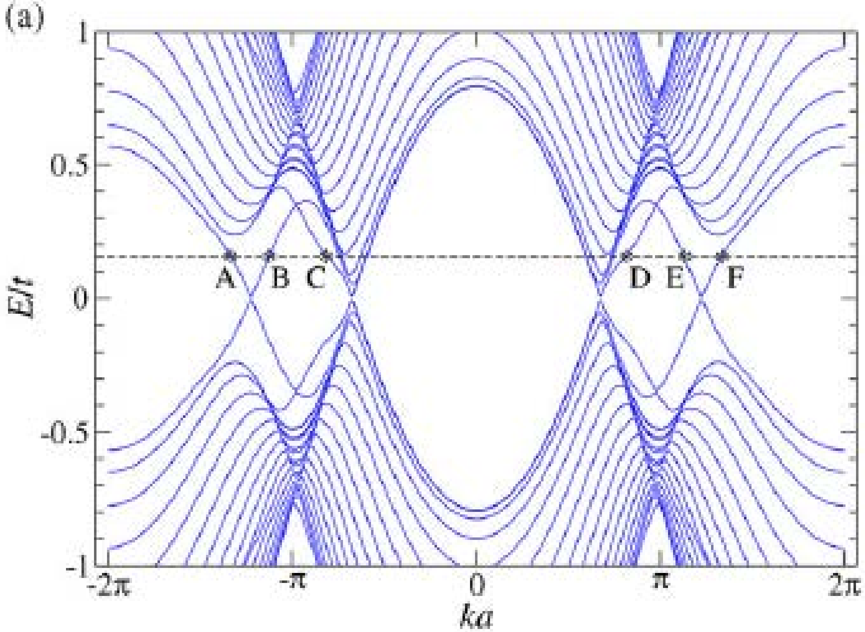


Edge states and mixed helical/chiral Majorana modes on a ribbon

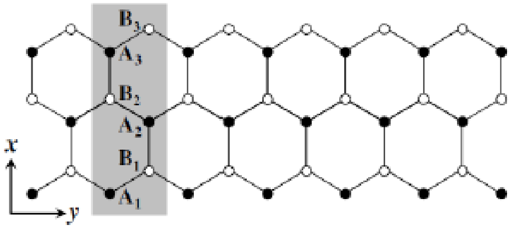
(A,B), (E,F):
helical Majorana

(C,D):
chiral Majorana

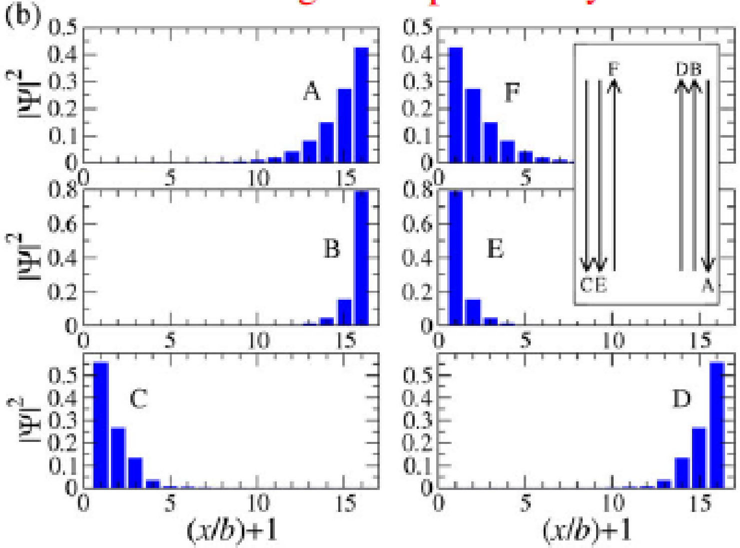




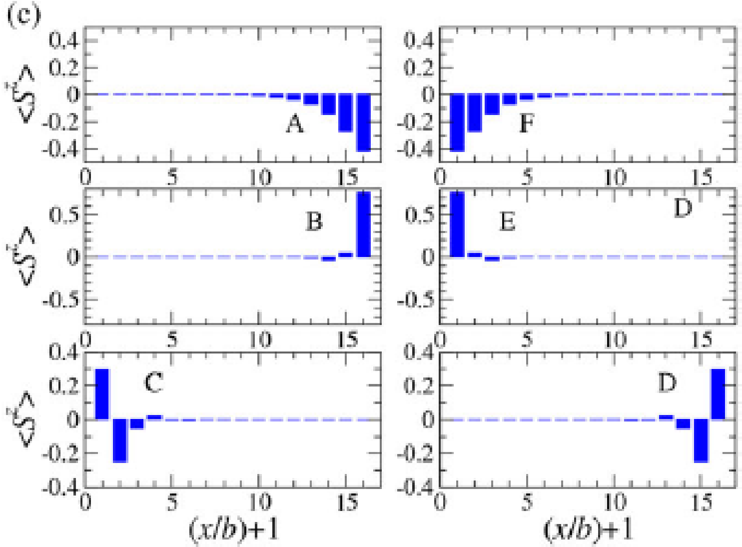
CH Chung et al pssb 2018



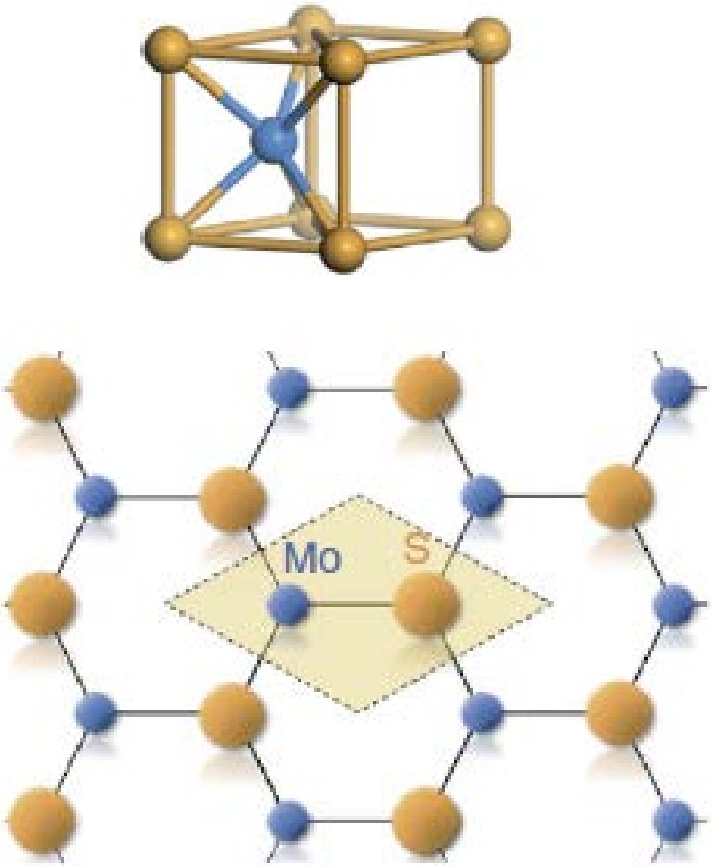
Edge state probability

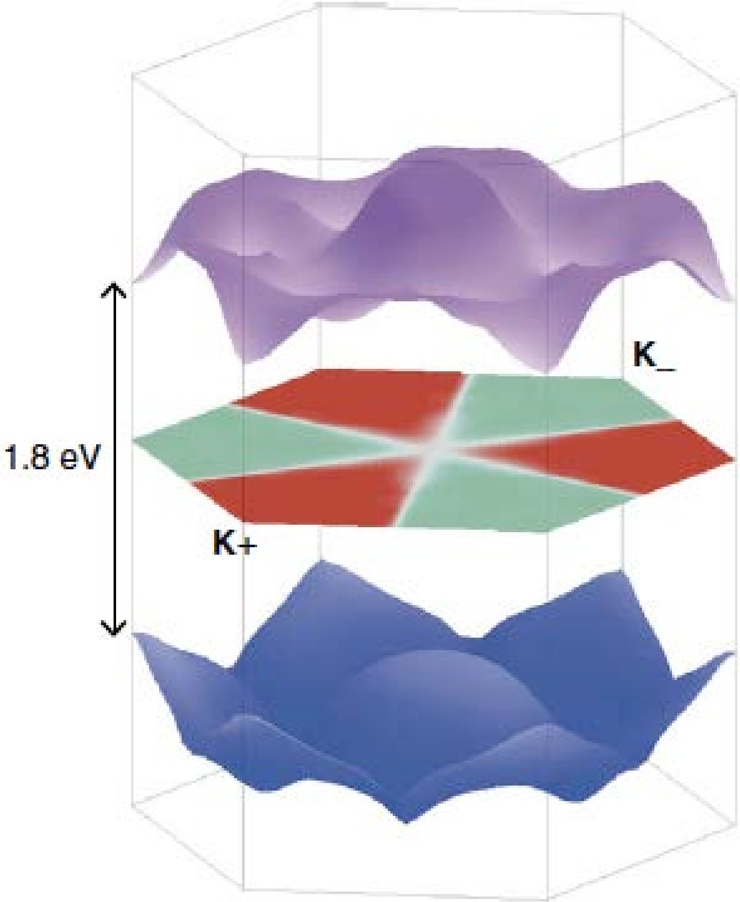


Spin polarization



Monolayer MoS_2



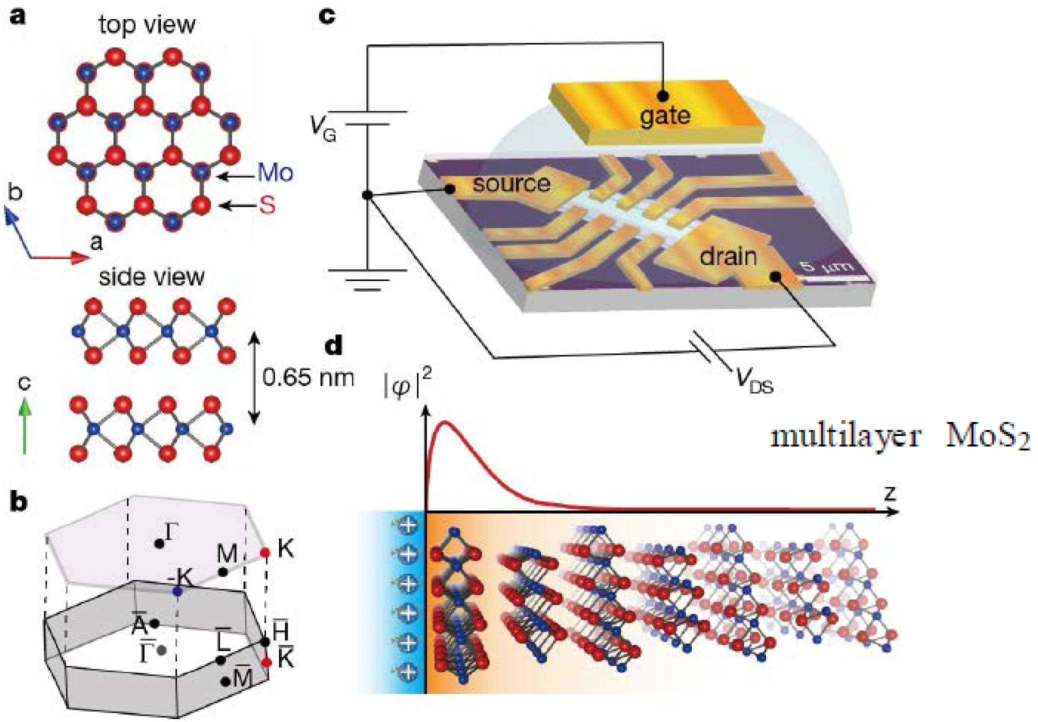


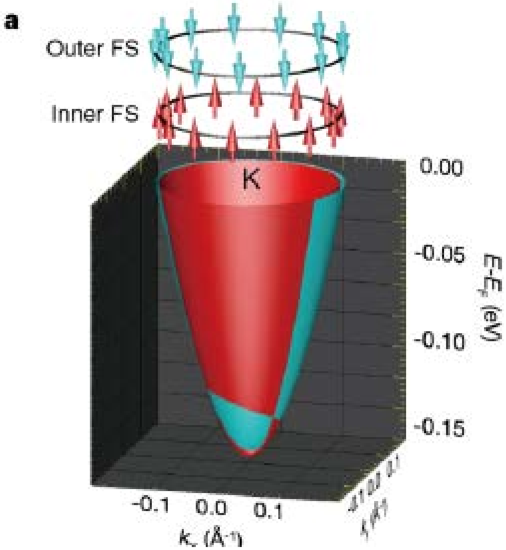
Direct band gap 1.8eV
at Dirac points

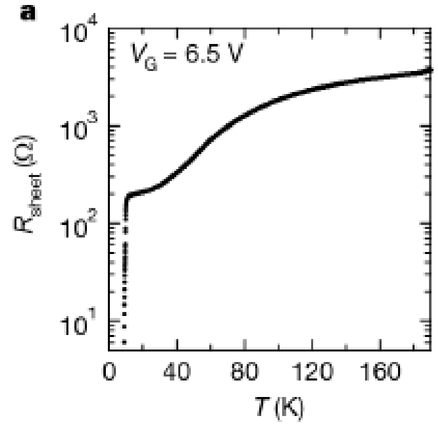
K. F. Mak et al., Phys. Rev. Lett. 105, 136805 (2010)
T. Cao et al. Nature Communication, 3, 887 (2012)

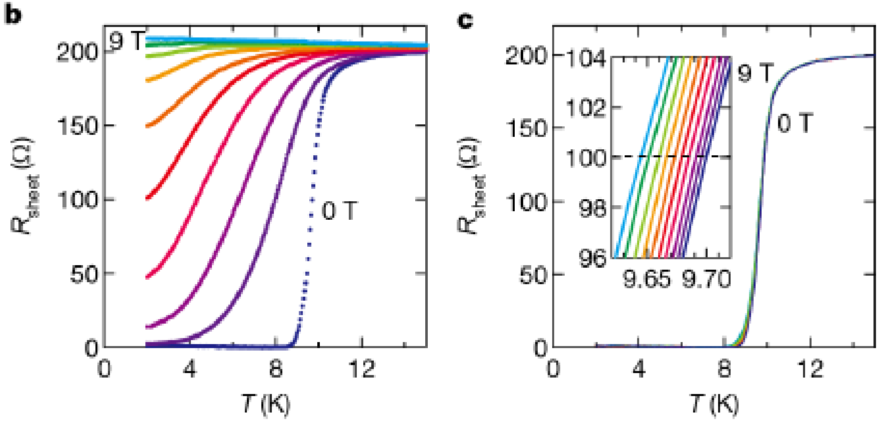
Gate-tuned superconductivity in MoS2

Y. Saito et al. Nature Physics **volume12**, pages144-149 (2016)





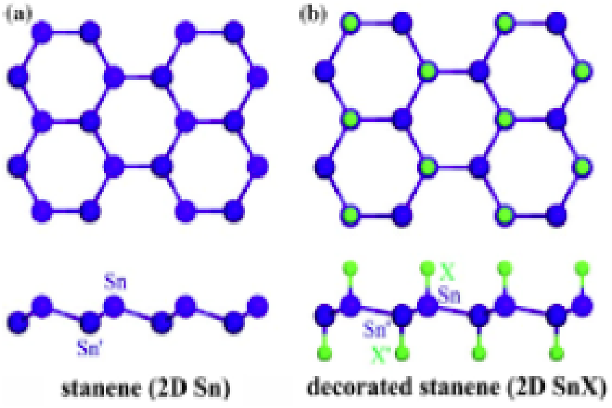


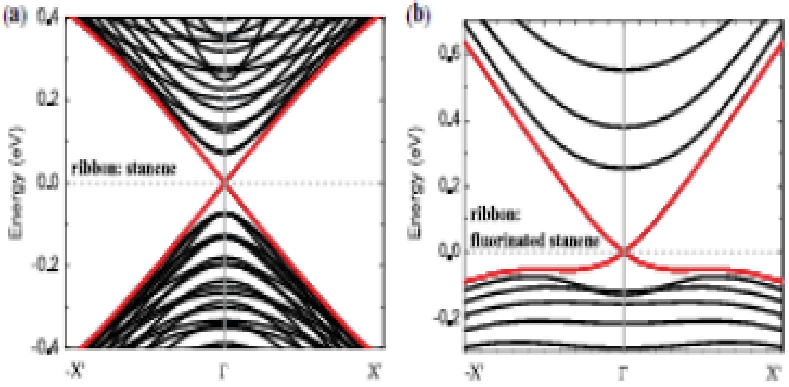


Possible realizations of QSHIs

Sn (Tin) film: Stanene

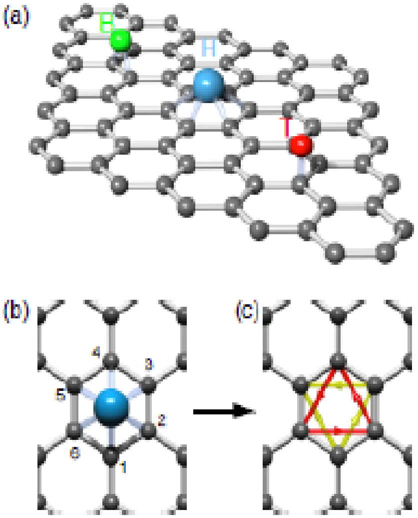
SO Gap~ 0.3eV



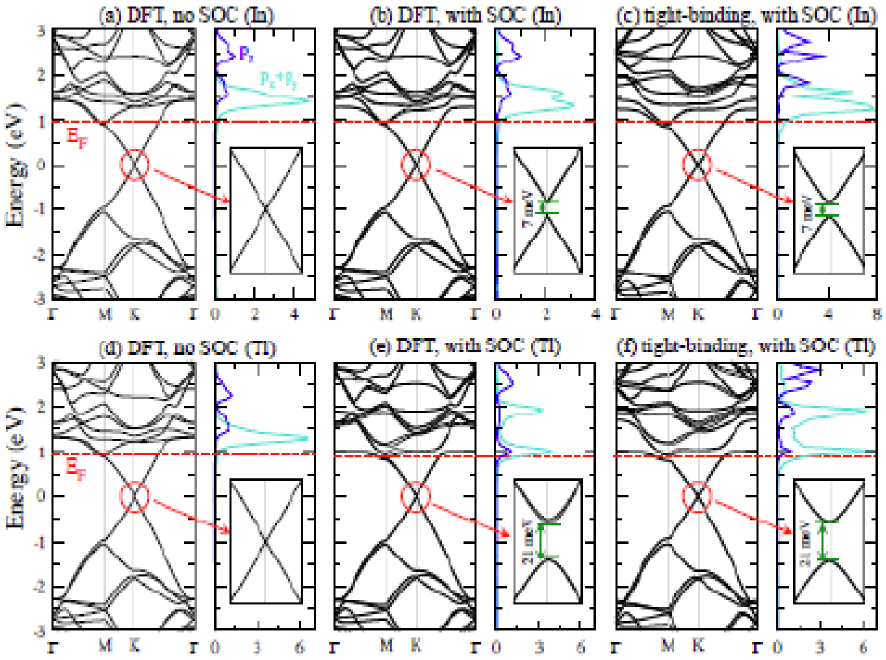


SC Zhang et al, PRL 2013

Adatom (Indium, Thallium doped) in graphene



SO Gap~ 7~21meV



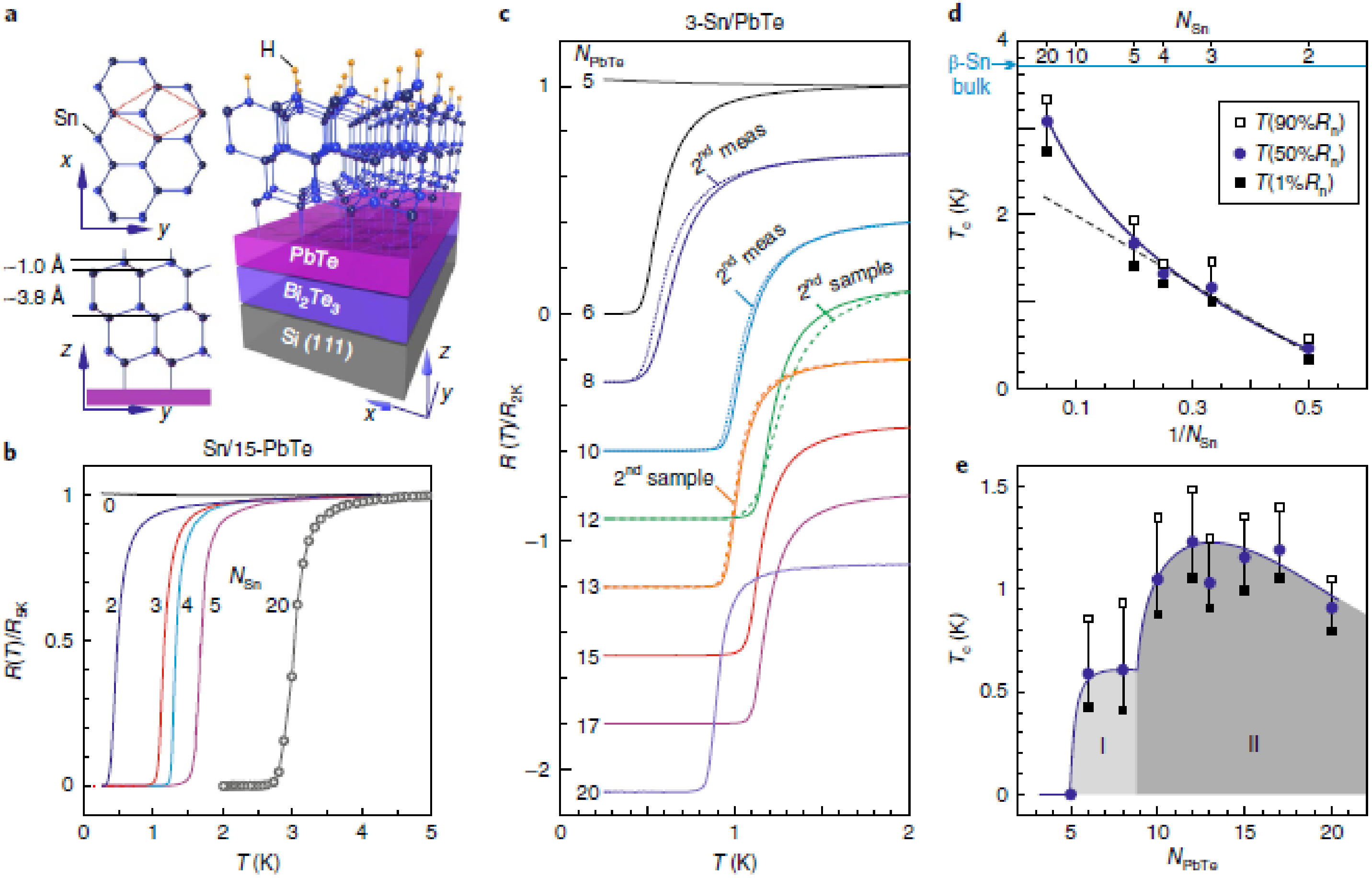
J. Alicea et al, Phys. Rev. X (2011)

LETTERS
https://doi.org/10.1038/s41567-017-0031-6
nature physics

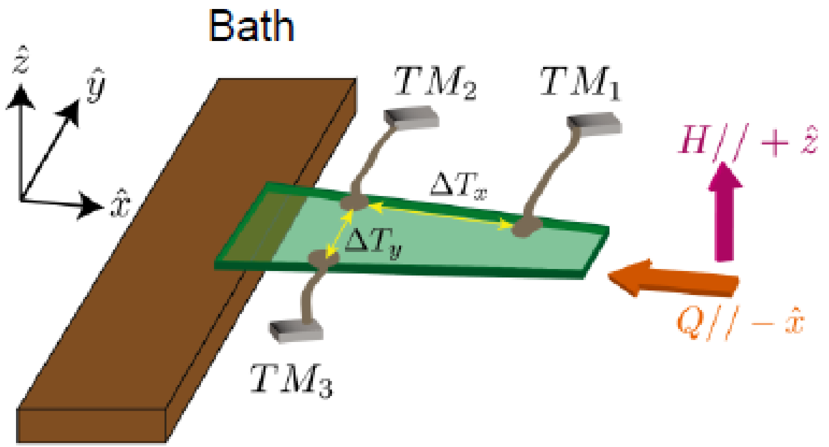
Nat. Phys. 2018

Superconductivity in few-layer stanene

Menghan Liao[1], Yunyi Zang[1], Zhaoyong Guan[1], Haiwei Li[1], Yan Gong[1], Kejing Zhu[1], Xiao-Peng Hu[1,2], Ding Zhang[1,2]*, Yong Xu[1,2,3]*, Ya-Yu Wang[1,2], Ke He[1,2], Xu-Cun Ma[1,2], Shou-Cheng Zhang[4] and Qi-Kun Xue[1,2]*



Thermal and thermal Hall conductivities



$$\mathbf{j}_\mathrm{T} = \kappa(-\nabla T)$$

$$\kappa = \begin{pmatrix} \kappa_{xx} & \kappa_{xy} \\ -\kappa_{xy} & \kappa_{xx} \end{pmatrix}$$

Thermal conductivity κ_{xx}

$$\kappa_{xx} = C v_s \ell$$

Thermal Hall conductivity κ_{xy}

Thermal analog of electronic Hall effect

Thermal conductivity can sensitively probe ***low energy "itinerant" excitations*** at low temperature.

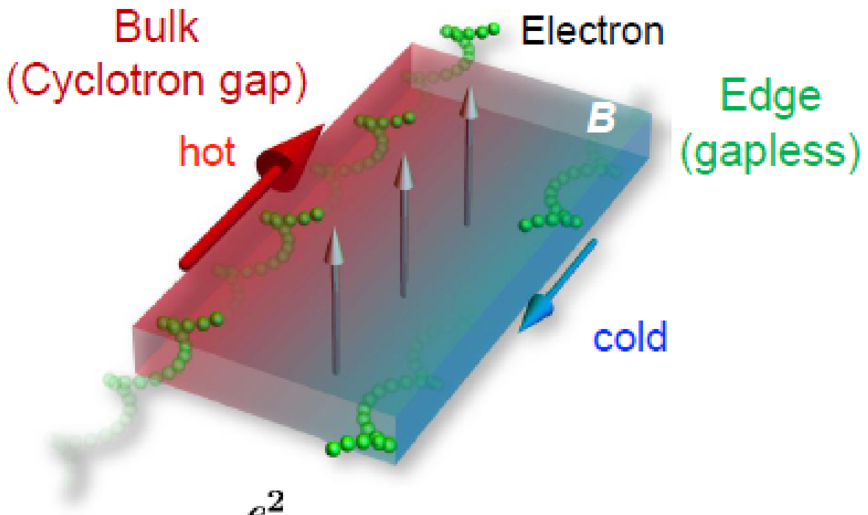
Not contaminated by localized impurities

Thermal Hall conductivity may be a unique probe to study ***nontrivial excitations*** in spin liquid states

Y. Matsuda's talk, IoP, Academia Sinica, Taipei, Nov. 29, 2018

2D electron gas and Kitaev spin liquid

2D electron gas
Chern insulator



$$\sigma_{xy}^{\mathrm{2D}} = \nu \frac{e^2}{\underline{h}} \qquad \nu = 1, 2, 3 \cdots$$

Chiral edge currents of ***electrons***

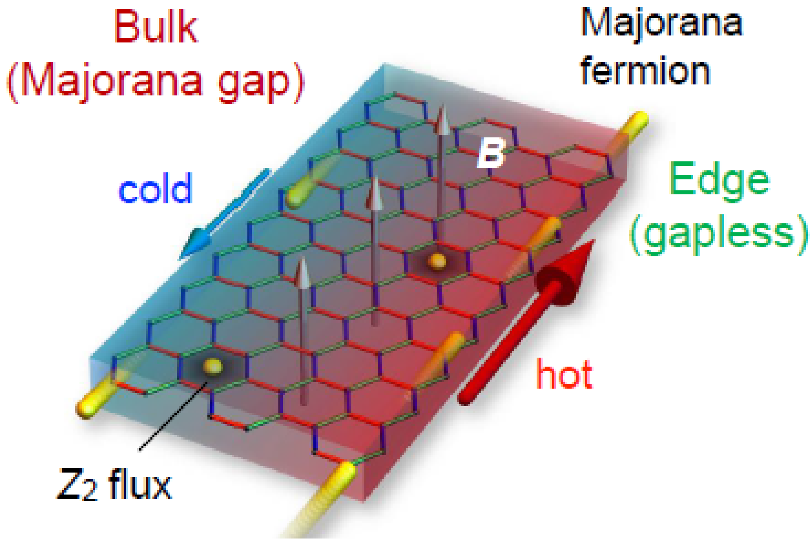
Integer QHE

$$\frac{\kappa_{xy}^{2D}}{T} = \nu \left(\frac{\pi}{6} \frac{k_B^2}{\underline{\hbar}} \right)$$

Integer thermal QHE

M. Banerjee *et al.* Nature **545**, 75 (2017).

Kitaev quantum spin liquid
Majorana Chern insulator



Chiral edge currents of **neutral Majorana fermions**

$$\frac{\kappa_{xy}^{2D}}{T} = \frac{1}{2} \left(\frac{\pi}{6} \frac{k_B^2}{\underline{\hbar}} \right)$$

Half integer thermal QHE

One Majorana = “*half*” of a fermion
Non-Abelian anyon

Y. Matsuda’s talk, IoP, Academia Sinica, Taipei, Nov. 29, 2018

Summary

- Doped Kane-Mele t-J model

--TRB d+d'-wave singlet superconductivity in doped Kane-Mele t-J model, protected by pseudospin symmetry due to A-B sublattice

--Topological phase transition chiral→ helical Majorana fermions with increasing KM SO

- Rashba SO on correlated honeycomb lattice favors p+ip-wave triplet superconductivity

- Doped Kane-Mele + Rashba t-J model on honeycomb lattice

--coexistence between d+i d singlet and p+ip-wave triplet superconductivity
--coexistence between helical and choral Majorana zero modes