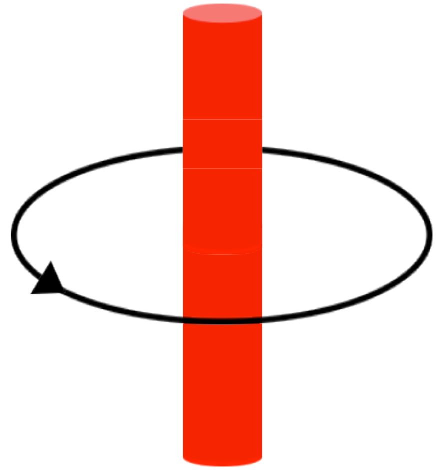


Topological order and color superconductivity

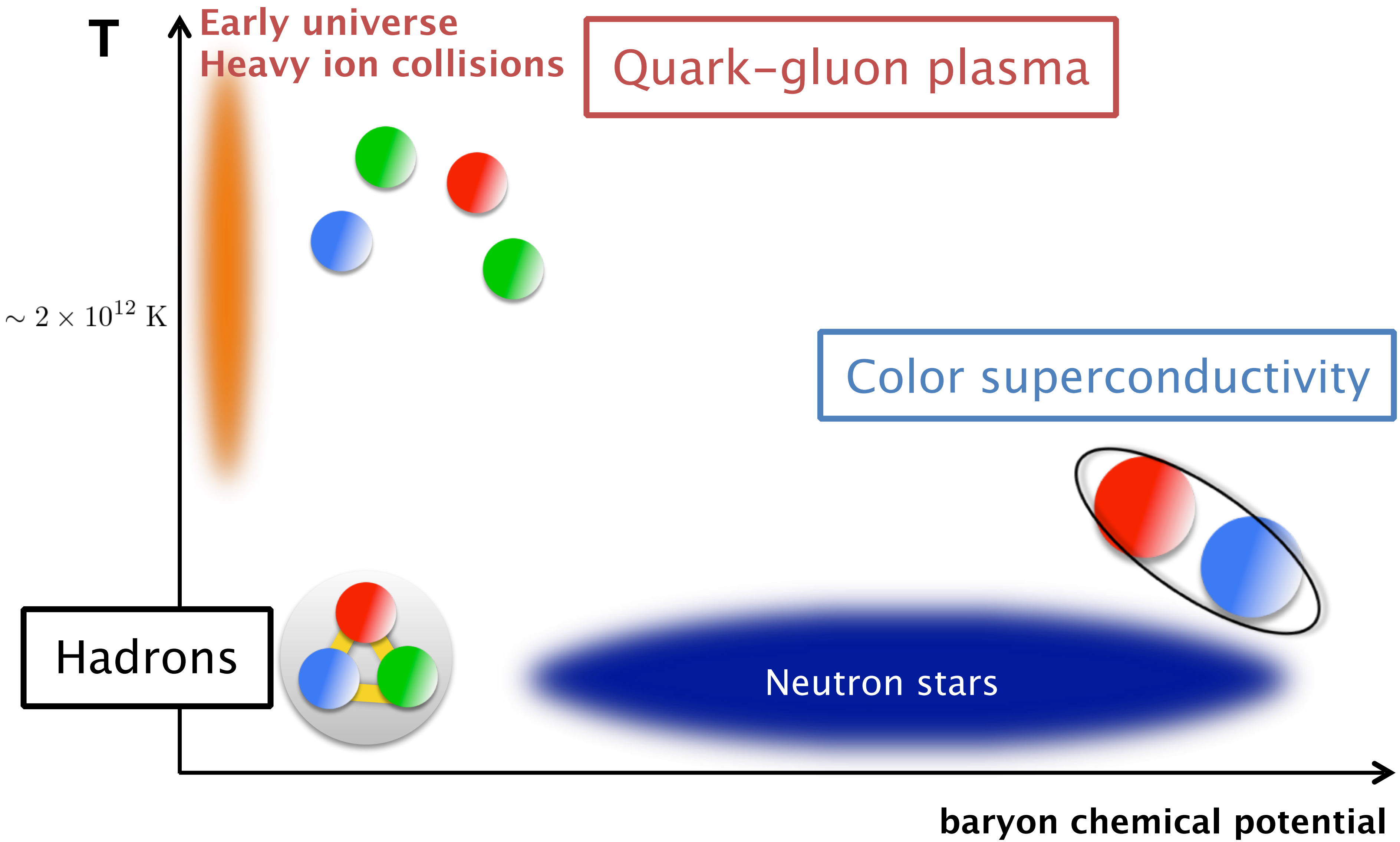
In collaboration with Yuya Tanizaki [1811.10608]

Yuji Hirono

Asia Pacific Center for Theoretical Physics

apctp

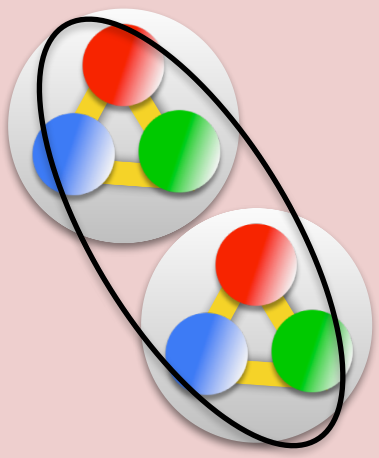
Phases of QCD matter



“Quark–hadron continuity”

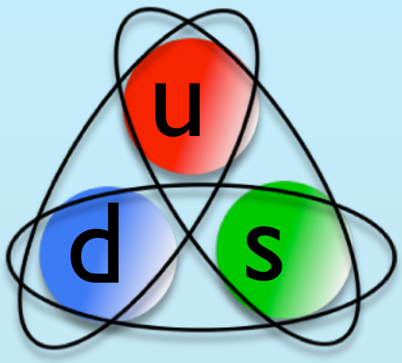
[Schafer, Wilczek '99]

“Topological order”



Nucleon superfluidity

?



Color superconductor

“CFL phase”

Outline

- Color SC & quark–hadron continuity
- Vortices in CFL phase
- Low–energy effective gauge theory
  - BF theory + massless phonons

Color superconductivity

- Three light quarks with degenerate mass
  - up, down, strange

- Order parameter: diquark condensate

$$\Phi_{\alpha i} = \epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\left\langle q^T_{\beta j} C q_{\gamma k}\right\rangle$$

color flavor

- Symmetry transformation

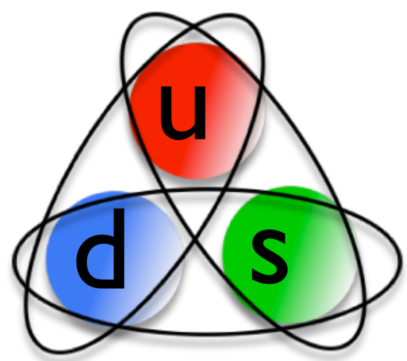
$$\Phi \to e^{\mathrm{i}\theta_{\mathrm{B}}} U_{\mathrm{c}} \Phi U_{\mathrm{f}}^T$$

$$e^{\mathrm{i}\theta_{\mathrm{B}}} \in U(1)_{\mathrm{B}} \qquad U_{\mathrm{c}} \in SU(3)_{\mathrm{c}} \qquad U_{\mathrm{f}} \in SU(3)_{\mathrm{f}}$$

Color–flavor locked phase

- Ground state

$$\Phi = \Delta \mathbf{1} \sim \mathrm{diag}\,[ud, ds, su]$$



- All the gluons are gapped: **color SC**
- SSB of global U(1): **superfluidity**

"Quark–hadron continuity"

[Schafer, Wilczek '99]

- Global symmetry

$$SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times U(1)_{\mathrm{B}} \to SU(3)_{\mathrm{L+R}}$$

The same in nucleon superfluid & CFL

- Light modes – Nambu–Goldstone bosons

U(1) phonons | **Pions**

"Quark–hadron continuity"

[Fukushima, Hatsuda '10]
[Yamamoto, Tachibana, Hatsuda, Baym '07]
......

U(1) phonons ↔	U(1) phonons
Pions ↔	CFL pions
Baryons ↔	Quarks
Vector mesons ↔	Gluons

Topological order

[X. G. Wen '90]

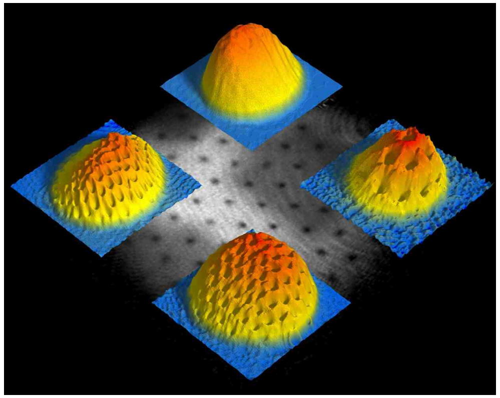
- Order that cannot be captured by local order parameters
  - Robust degeneracy of ground states
  - Ex) s-wave SC / FQHE
- Phase transition is needed btw. states with different topological order
- Order parameter: Wilson loop, etc
- "Higher-form symmetry" [Gaiotto, Kapustin, Seiberg, Willett '15] and its spontaneous breaking

Vortices in CFL

[Balachandran, Digal, Matsuura '06]
[Arata Yamamoto's talk tomorrow]

$$\Phi \sim \mathrm{diag}\left[e^{i\theta}, 1, 1\right]$$

- Quantized (1/3) superfluid circulation
- Color magnetic flux
- Rotating neutron star

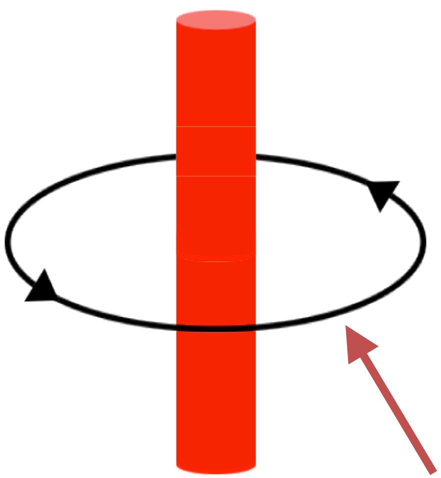


[http://cua.mit.edu/ketterle_group/Nice_pics.htm]

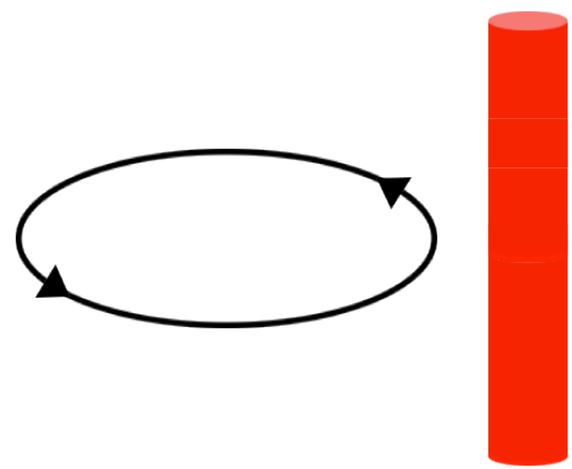
Fractional statistics of vortices & particles

[Cherman, Sen, Yaffe 1808.04827]

Z3 braiding phases



$$= e^{\frac{2\pi i}{3}}$$

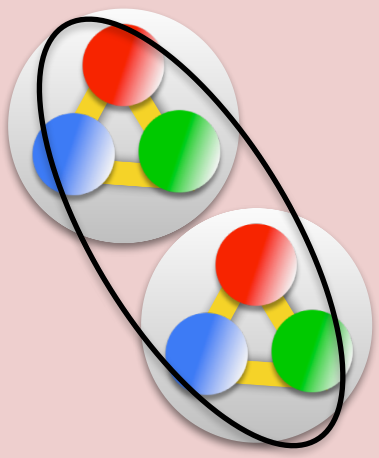


color Wilson loop

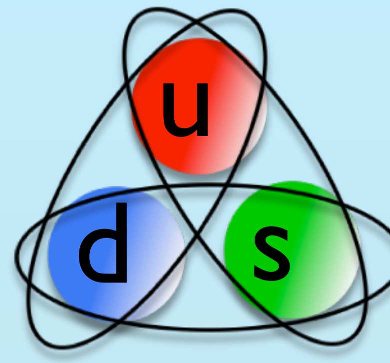
Fractional statistics of vortices & particles

[Cherman, Sen, Yaffe 1808.04827]

Z3 braiding phases



Nucleon superfluidity



Color superconductor
“CFL phase”

Low-effective theory for CFL

- We consider degenerate masses for u, d, s
- Massless degrees of freedom:

U(1) phonons

- Fractional statistics

- Correlation of **U(1) circulation**
  & **color holonomy**

Dual effective gauge theory – SC

BF theory for superconductivity

- Abelian Higgs model

$$S = \frac{1}{2g^2}|\mathrm{d}\phi + ka|^2 - \frac{1}{2}|\mathrm{d}a|^2$$

$$\Downarrow$$

$$\frac{g^2}{8\pi^2}\int h \wedge *h - \frac{\mathrm{i}}{2\pi}\int h \wedge (\mathrm{d}\phi + ka)$$

- EOM for ϕ: $\mathrm{d}h = 0 \Rightarrow h = \mathrm{d}b$

$$S = \frac{g^2}{8\pi^2}|\mathrm{d}b|^2 + \frac{\mathrm{i}k}{2\pi}\int b \wedge \mathrm{d}a$$

BF theory for superconductivity

$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi} \int b \wedge \mathrm{d}a$$

- Physical observables

Wilson loop operator

$$W(C) = \exp \mathrm{i} \int_C a$$

C : world line of a quasiparticle

Vortex operator

$$V(S) = \exp \mathrm{i} \int_S b$$

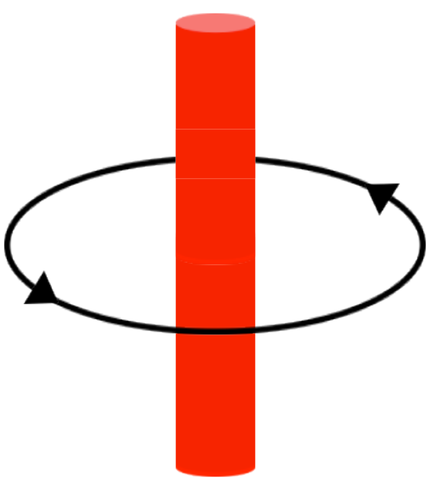
S : world-sheet of a vortex

BF theory for superconductivity

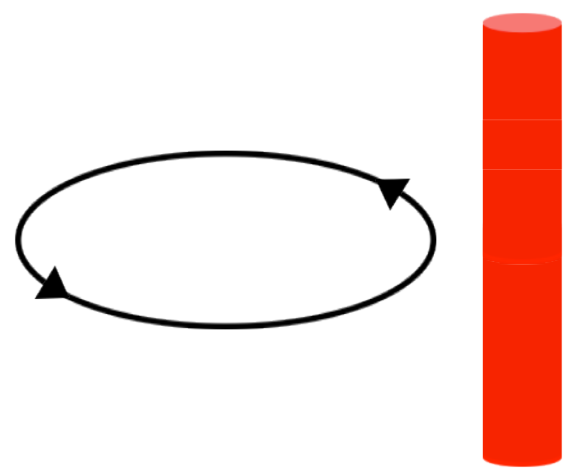
$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi}\int b \wedge \mathrm{d}a$$

$$\langle W(C)V(S)\rangle = \exp\frac{2\pi\mathrm{i}}{k}\mathrm{link}(C,S)$$

Fractional statistics of quasiparticles & vortices



$$= e^{\frac{2\pi\mathrm{i}}{k}}$$



BF theory for superconductivity

$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi} \int b \wedge \mathrm{d}a$$

- Emergent Z_k 1-form & 2-form symmetry

$$a \mapsto a + \frac{1}{k} a' \qquad \mathrm{d}a' = 0 \qquad \int a' \in 2\pi\mathbb{Z}$$

$$b \mapsto b + \frac{1}{k} b' \qquad \mathrm{d}b' = 0 \qquad \int b' \in 2\pi\mathbb{Z}$$

BF theory for superconductivity

$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi}\int b \wedge \mathrm{d}a$$

- Emergent Z_k 1-form & 2-form symmetry

$$a \mapsto a + \frac{1}{k}a' \qquad e^{\frac{2\pi \mathrm{i}}{k}} W(C)$$

$$b \mapsto b + \frac{1}{k}b' \qquad e^{\frac{2\pi \mathrm{i}}{k}} V(S)$$

Charged objects

BF theory for superconductivity

$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi} \int b \wedge \mathrm{d}a$$

- 1-form & 2-form symmetries are **spontaneously broken**

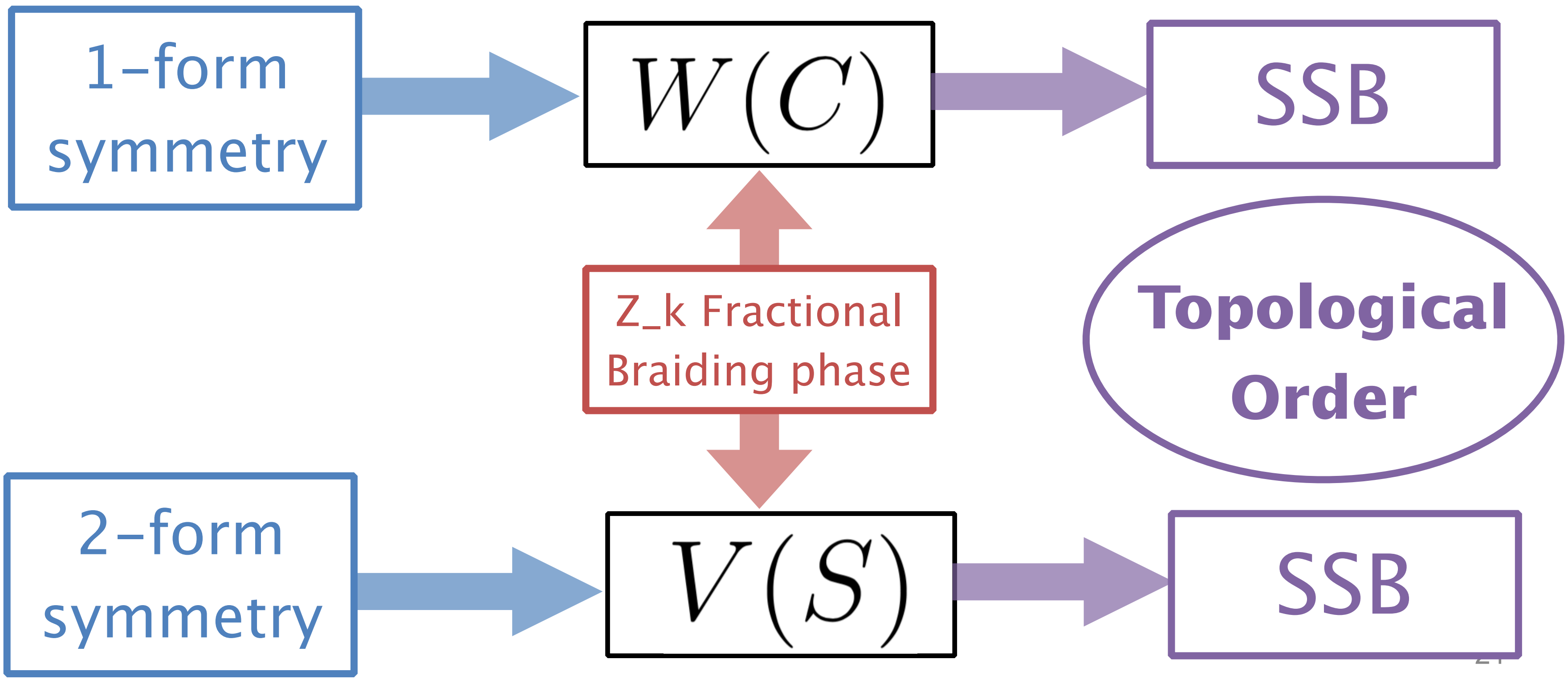
$$\langle W(C) \rangle \sim e^{-\kappa\, \mathrm{perimeter}(C)}$$

$$\langle V(S) \rangle \sim e^{-\kappa'\, \mathrm{perimeter}(S)}$$

➡ **"Topological order"**

BF theory for superconductivity

$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi}\int b \wedge \mathrm{d}a$$



Dual effective gauge theory – CFL

[Hirono, Tanizaki 1811.10608]

GL model for CFL

- GL Lagrangian

$$S = \frac{1}{2g_{\mathrm{YM}}^2}|G|^2 + \frac{1}{2}|(\mathrm{d} + \mathrm{i}a_{SU(3)})\Phi|^2 + V_{\mathrm{eff}}(\Phi^\dagger\Phi, \det(\Phi))$$

- Drop amplitude fluctuations kinetic term of the gauge field
- Fix the gauge so that $\Phi = \Delta_0 \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi_1} & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\phi_2} & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\phi_3} \end{pmatrix}$

$$S = \frac{1}{2g_0^2}\left(|\mathrm{d}\phi_1 + a_3 + a_8|^2 + |\mathrm{d}\phi_2 - a_3 + a_8|^2 + |\mathrm{d}\phi_3 - 2a_8|^2\right)$$

- Go to dual: $\phi_i \to b_i$

Dual theory for CFL

$$S = \frac{1}{2} G_{ij} \mathrm{d}(b_0)_i \wedge * \mathrm{d}(b_0)_j + \frac{\mathrm{i}}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

Phonons BF term

$i = 1, 2, 3 \quad A = 1, 2$

- Topological BF theory coupled with massless superfluid phonons
- K matrix
  - not square
  - dim coker K = (# of massless phonons)

Dual theory for CFL

$$S = \frac{1}{2} G_{ij} \mathrm{d}(b_0)_i \wedge *\mathrm{d}(b_0)_j + \frac{\mathrm{i}}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

- Physical observables

$$W_{\boldsymbol{q}}(C) = \exp \mathrm{i}\, q_A \int_C a_A$$

$$V_{\boldsymbol{p}}(S) = \exp \mathrm{i}\, p_i \int_S b_i$$

Phonons

Dual theory for CFL

$$S = \frac{1}{2} G_{ij} \mathrm{d}(b_0)_i \wedge *\mathrm{d}(b_0)_j + \frac{\mathrm{i}}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

$$\frac{\langle W_{\boldsymbol{q}}(C) V_{\boldsymbol{p}}(S) \rangle}{\langle W_{\boldsymbol{q}}(C) \rangle \langle V_{\boldsymbol{p}}(S) \rangle} = \exp\left[2\pi\mathrm{i}\; q_A K^{+}_{Ai}\, p_i \;\mathrm{link}(C, S)\right]$$

K^{+}_{Ai} is the Moore–Penrose inverse of K_{iA}

Massless phonons

$$(b_0)_i = P^b_{ij}\, b_j \qquad P^b_{ij} = \delta_{ij} - [K K^{+}]_{ij}$$

Dual theory for CFL

$$S = \frac{1}{2} G_{ij} \mathrm{d}(b_0)_i \wedge * \mathrm{d}(b_0)_j + \frac{\mathrm{i}}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

$$K = \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 0 & -2 \end{pmatrix} \Rightarrow K^+ = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \end{pmatrix}$$

Physical charge vectors

$$\boldsymbol{q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad \boldsymbol{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow q_A K^+_{Ai} p_i$$

Z3 fractional statistics

Dual theory for CFL

$$S = \frac{1}{2} G_{ij} \mathrm{d}(b_0)_i \wedge *\mathrm{d}(b_0)_j + \frac{\mathrm{i}}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

Discrete (Z3) 2-form symmetry

$$b_i \mapsto b_i + q_A K^+_{Ai} \lambda$$

$$\mathrm{d}\lambda = 0 \qquad \int \lambda \in 2\pi\mathbb{Z}$$

No discrete 1-form symmetry

2-form symmetry is unbroken

- Vortices are log-confined because of massless phonons

$$\langle V(S) \rangle \sim \exp[-cTL\ln L]$$

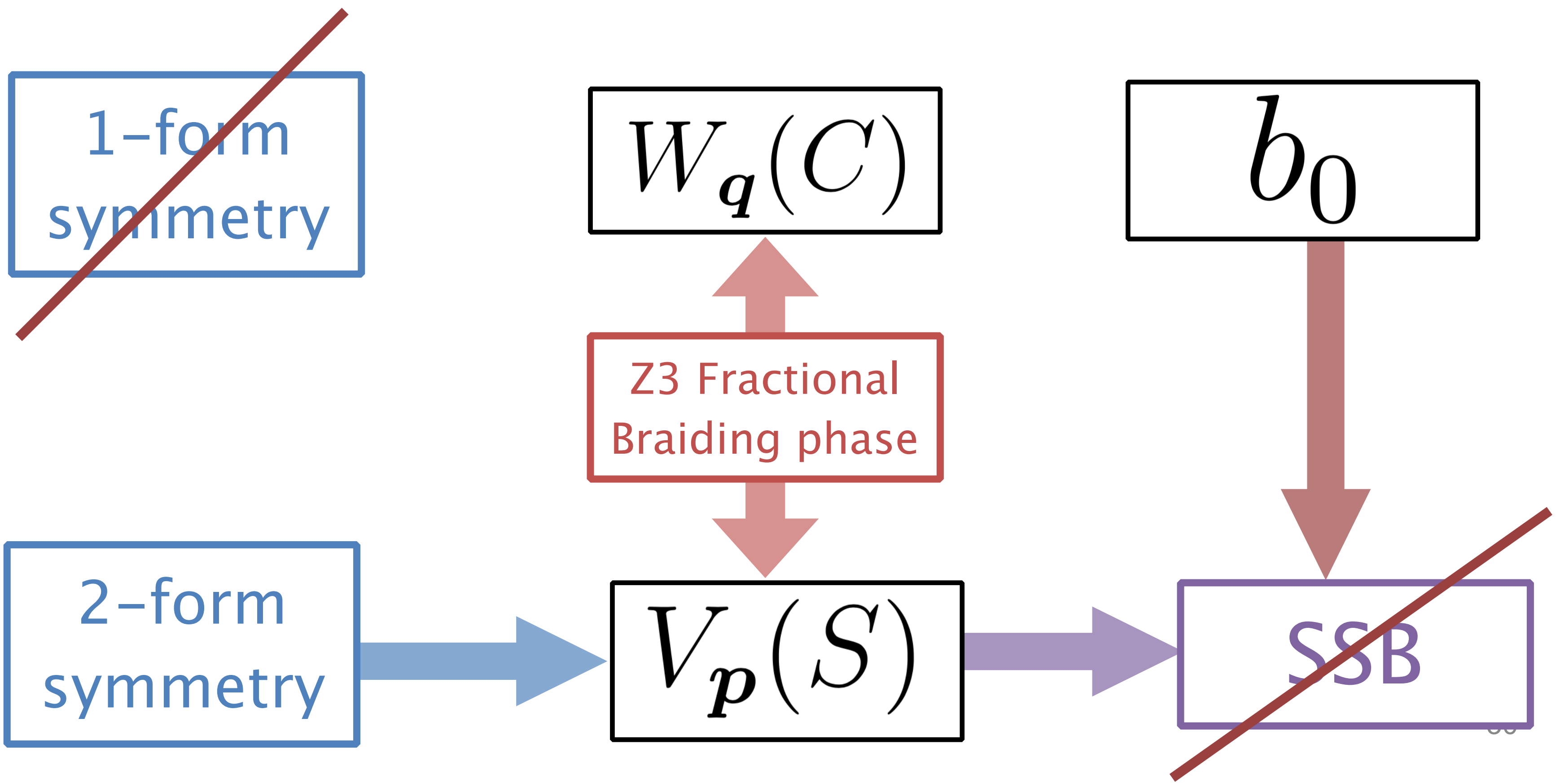
No topological degeneracy of the ground states

- Z3 2-form symmetry $\subset U(1)$ 2-form symmetry
- Continuous 2-form symmetry cannot be broken in 4D (Coleman-Mermin-Wagner theorem)
  - p-form symmetry cannot be broken if d – p ≦ 2

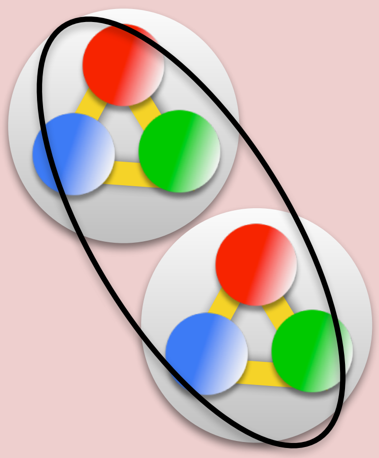
Summary

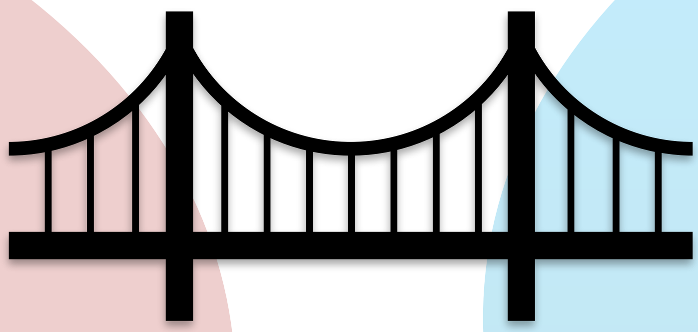
$$S = \frac{1}{2} G_{ij} \mathrm{d}(b_0)_i \wedge * \mathrm{d}(b_0)_j + \frac{\mathrm{i}}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

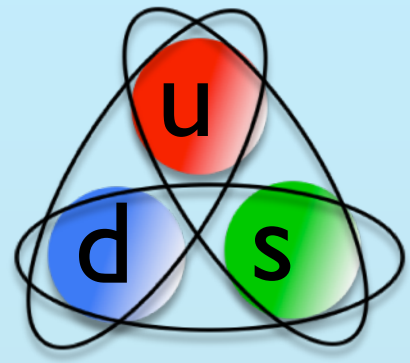
massless phonons



Summary







Nucleon superfluidity

Color superconductor
“CFL phase”