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Chiral magnetic effect in two-band lattice model of Weyl semi-metal

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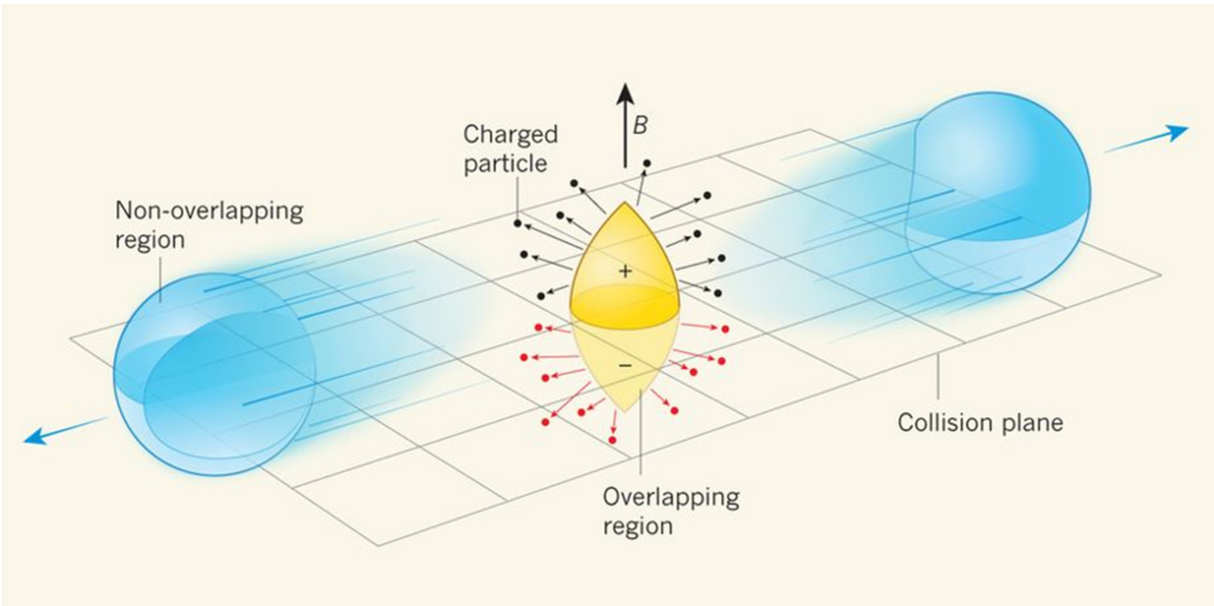
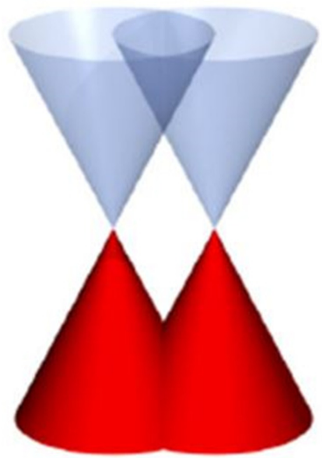
Chiral magnetic effect (Vilenkin, Phys Rev D **22**, 3080, 1980)

$$\boxed{\vec{J}_{CME} = -\alpha_B \vec{B}}$$

- Quark-gluon plasma in heavy ion collision
- Relativistic plasma in astrophysics
- Weyl semimetal
- …

Note: need to break space-inversion symmetry

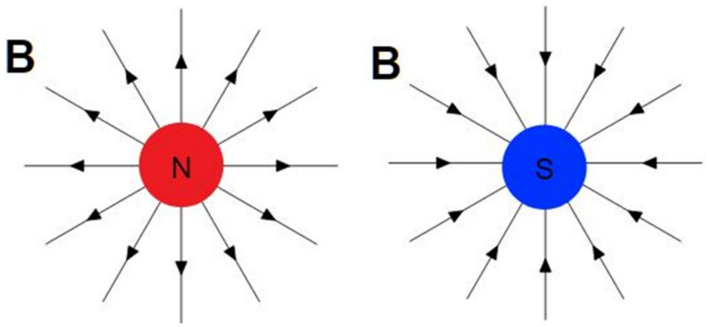




Figs from Dobrin Nature 2017, Chernodub arXiv 1002.1473, Vazifeh PRL 2013

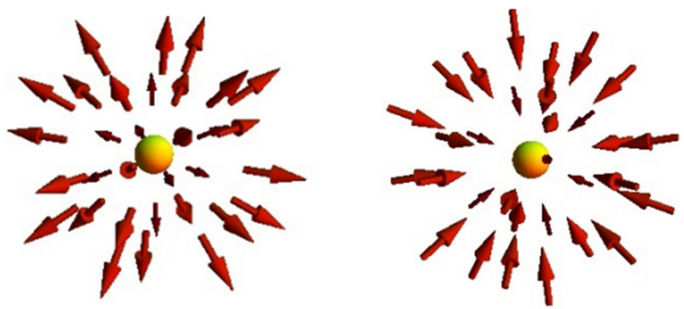
Monopole in Weyl semimetal

Dirac magnetic monopole

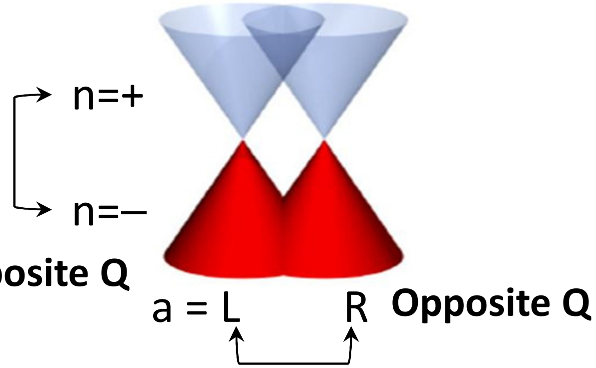


Weyl monopole

- A Weyl node is a monopole in momentum space



- The sign of a monopole charge Q_{na} depends on band-*n* and node-*a*



- Berry flux (or monopole charge) of a Weyl node is quantized

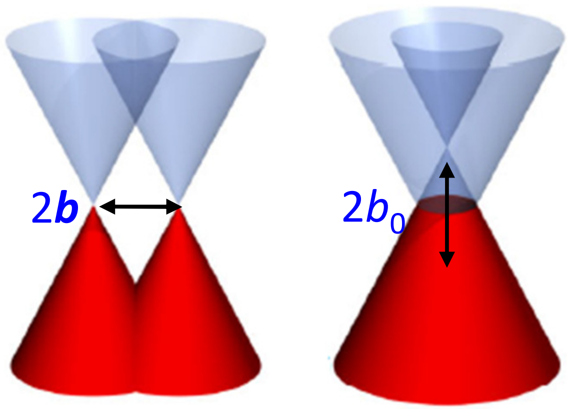
- Nielsen-Ninomiya theorem requires Weyl monopole (in a BZ) to appear in pairs with opposite chiralities

Chiral magnetic effect in Weyl semimetal

(Zyuzin and Burkov, Phys Rev B 2012)

- Low-energy effective theory for a pair of Weyl nodes

$$H = \tau_z \vec{\sigma}\cdot\vec{k} + \vec{\sigma}\cdot\vec{b} + \tau_z b_0$$



momentum separation

energy separation

- Effective electromagnetic action

$$S_\theta = \frac{1}{2\pi}\frac{e^2}{h}\int dtd^3x\,\theta(\vec{r},t)\vec{E}\cdot\vec{B}$$

with the axion field

$$\theta = 2(\vec{b}\cdot\vec{r} - b_0 t)$$

- Current density

$$\rightarrow \quad \vec{J} = \frac{1}{2\pi}\frac{e^2}{h}\left(\nabla\theta\times\vec{E} + \partial_t\theta\,\vec{B}\right)$$

$$= \frac{e^2}{2\pi h}\left(2\vec{b}\times\vec{E} - 2b_0\,\vec{B}\right)$$

Hall effect

Chiral magnetic effect

(Relativistic covariance requires AHE and CME to *both* exist)

However, Results against (static) CME

$$\vec{J}_{CME} = -\alpha_B \vec{B}$$

- Semiclassical analysis (Zhou et al, Chinese Phys Lett 2013)

$$\alpha_B = \frac{e^2}{\hbar} \sum_n \int \frac{d^3k}{(2\pi)^3} \vec{v}_n \cdot \vec{\Omega}_n f_n$$

Berry curvature

$$= \frac{e^2}{h^2} \sum_n \int^{\mu} \frac{d\varepsilon}{2\pi} \Phi_n^{\Omega}(\varepsilon) f_n,$$

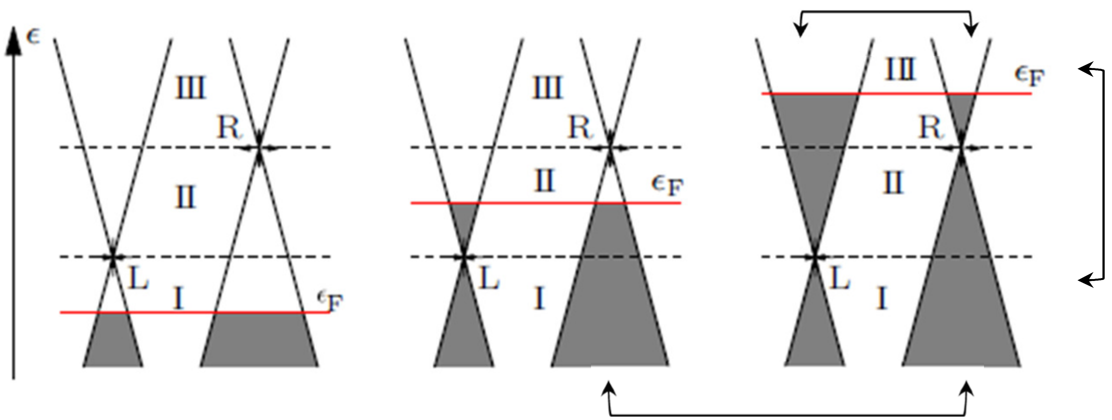
$$= 0$$

Energy-integrated Berry flux

$$\Phi_{na}^{\Omega}(\varepsilon) = 2\pi Q_{na}$$

Berry flux over an iso-energy surface (*a*-th node in *n*-th band)

Opposite monopole charges



Opposite Berry curvatures

Same Berry flux

- Numerical work on lattice (Vazifeh and Franz, PRL 2013)
- And more …

Argument against (static) CME

(Basar, Kharzeev, and Yee, PRB 2014)

$$\vec{J} = -\alpha_B \vec{B}$$

- Work done by field on charges

$$\frac{dP}{dt} = \vec{J} \cdot \vec{E} \sim \vec{E} \cdot \vec{B}$$ can be > 0 or < 0

Can extract energy out of equilibrium state!

Goswami and Tewari, 1311.1506
Chang and Yang, Phys Rev B 2015

To resolve this issue, we propose

- A minimum model with two bands
- Use linear response theory
- Consider both orders of taking the limits

static limit: $\lim_{q \to 0} \lim_{\omega \to 0} \alpha_B(\vec{q}, \omega)$

uniform limit: $\lim_{\omega \to 0} \lim_{q \to 0} \alpha_B(\vec{q}, \omega)$ ←

The usual **DC** conductivity is calculated using this limit (see, e.g., Mahan)

What we found (for a clean and infinite system)

- Static limit: $\alpha_B = 0$
- uniform limit: $\alpha_B \neq 0$

If there are impurities, the conclusion might change (later)

Quantities related to Berry curvature in Weyl semimetal

- $\int \frac{d^3k}{(2\pi)^3} \Omega_n f_n \quad = \left[\frac{\hbar}{e^2}\right] \sigma_H$ — Hall conductivity

- $\int d^3k\, \vec{\Omega}_n \cdot \vec{v}_n \left(-\frac{\partial f_n}{\partial \varepsilon_n}\right) = \left[\frac{1}{\hbar}\right] \Phi_n^{\Omega}(\varepsilon_F)$ — Berry flux through Fermi surface (→ chiral anomaly)

  $\Phi_{na}^{\Omega}(\varepsilon) = 2\pi Q_{na} = \pm 2\pi$

- $\int d^3k\, \vec{\Omega}_n \cdot \vec{v}_n f_n \quad = \left[\frac{1}{\hbar}\right] \int^{\mu} \frac{d\varepsilon}{2\pi} \Phi_n^{\Omega}(\varepsilon) f_n$ — Energy-integrated Berry flux $\boxed{\Phi_n^{\Omega}}$ (→ static CME)

- $\int d^3k\, \vec{m}_n \cdot \vec{v}_n \left(-\frac{\partial f_n}{\partial \varepsilon_n}\right) = \left[\frac{1}{\hbar}\right] \Phi_n^{m}(\varepsilon_F)$ — m-flux (→ dynamic CME) “Not related to topology”

  ↑ Magnetic moment of a Bloch electron

$\vec{m}_{\pm} = \pm d_{\vec{k}} \vec{\Omega}_{\pm} \frac{e}{\hbar}$ in 2-band model

CME coefficient: linear response theory

2-band model $H = \varepsilon_0(\vec{k}) + \vec{d}(\vec{k}) \cdot \vec{\sigma}$

- *static limit*

$$\lim_{q\to 0}\lim_{\omega\to 0} \bar{\alpha}(\vec{q},\omega) = \frac{e^2}{\hbar}\sum_{n=\pm}\int [d\vec{k}]\left(\vec{v}_0 \cdot \vec{\Omega}_n f_n - n d_{\vec{k}} \vec{v}_n \cdot \vec{\Omega}_n \frac{\partial f_n}{\partial \varepsilon_n}\right)$$

$$= \frac{e^2}{\hbar}\sum_{n=\pm}\int [d\vec{k}] \vec{v}_n \cdot \vec{\Omega}_n f_n \quad \leftarrow \boxed{\Phi^{\Omega}}$$ (energy-integrated)

$$= \begin{cases} 0 & \text{Equilibrium} \\ \frac{e^2}{h^2}(\mu_L - \mu_R) & \text{Non-equilibrium} \end{cases}$$

- *Uniform limit*

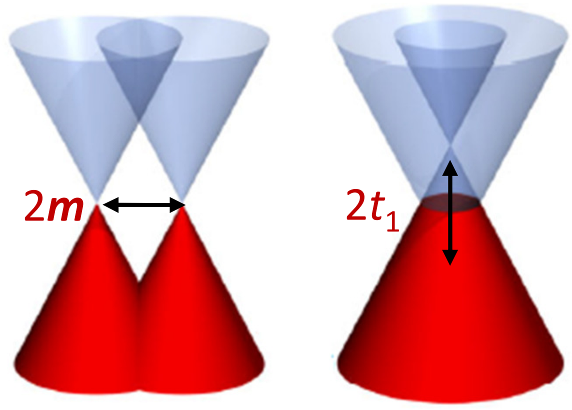
$$\lim_{\omega\to 0}\lim_{q\to 0} \bar{\alpha}(\vec{q},\omega) = \frac{e^2}{\hbar}\sum_{n=\pm}\int [d\vec{k}]\left(\vec{v}_0 \cdot \vec{\Omega}_n f_n - \frac{1}{3} n d_{\vec{k}} \vec{v}_n \cdot \vec{\Omega}_n \frac{\partial f_n}{\partial \varepsilon_n}\right)$$

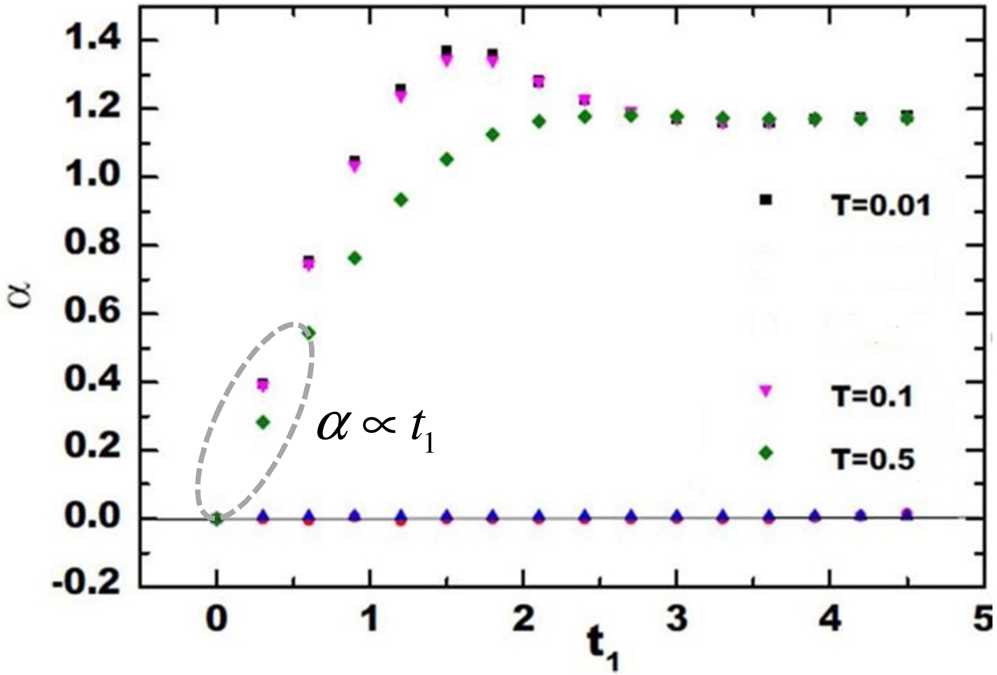
$$= \bar{\alpha}_{static} + \frac{2}{3} e \sum_{n=\pm}\int [d\vec{k}] n d_{\vec{k}} \vec{v}_n \cdot \vec{\Omega}_n \frac{\partial f_n}{\partial \varepsilon_n}$$

- Not zero (for a clean and infinite system)
- Later, semiclassical analysis shows that this should be interpreted as dynamic CME, instead of static CME

A two-band model

$$H = t_1 \cos k_z + H_{so} + H'$$
$$H_{so} = t_{so}(\sin k_x \sigma_x + \sin k_y \sigma_y + \sin k_z \sigma_z)$$
$$H' = \left(m + 2 - \cos k_x - \cos k_y\right)\sigma_z$$





← Saturation since at large t_1, $v_z \approx t_1$

$\Omega_z \approx t_1^{-1}$

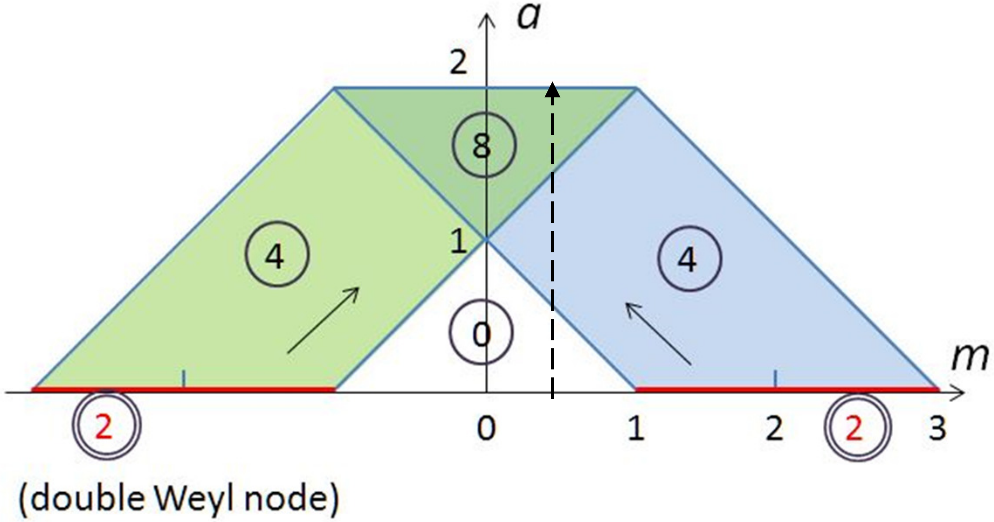
Data from static limit

- No energy separation (between nodes), no CME
- Filled bands (insulator) don't have CME
- In *this* model, if no Weyl nodes, then no CME

However, in other 2-band models, we found

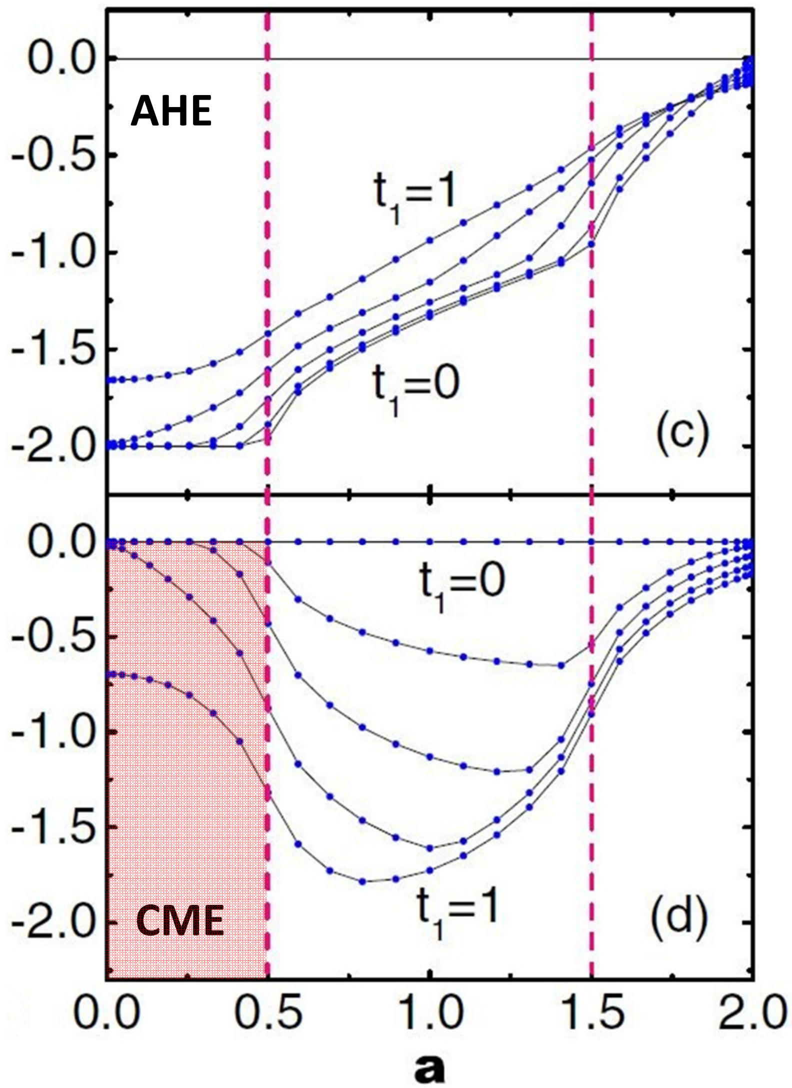
CME in the absence of Weyl node

Phase diagram and number of Weyl nodes



Note: chiral anomaly still requires Weyl nodes.

m=0.5



Chang and Yang, Phys Rev B 2015

Semiclassical analysis (Xiao et al, Rev Mod Phys 2010)

- E and B can oscillate in space/time $(\hbar\omega << \varepsilon_{gap})$
- Easier to consider *finite* ***q***, ω, and include *relaxation* τ

Equations of motion:

$$\begin{cases} \dot{\mathbf{x}} = \frac{1}{\hbar}\frac{\partial \tilde{\varepsilon}_n}{\partial \mathbf{k}} - \dot{\mathbf{k}} \times \Omega_n(\mathbf{k}), \\ \dot{\mathbf{k}} = -\frac{e}{\hbar}\tilde{\mathbf{E}} - \frac{e}{\hbar}\dot{\mathbf{x}} \times \mathbf{B}. \end{cases}$$

(quantities with ~ are modified by a m.B term)

AHE — static CME

$$\begin{cases} \dot{\mathbf{x}} = \frac{1}{D_n(\mathbf{k})}\left[\tilde{\mathbf{v}}_n + \frac{e}{\hbar}\tilde{\mathbf{E}} \times \Omega_n(\mathbf{k}) + \frac{e}{\hbar}(\Omega_n(\mathbf{k}) \cdot \tilde{\mathbf{v}}_n)\mathbf{B}\right] \\ \dot{\mathbf{k}} = \frac{1}{D_n(\mathbf{k})}\left[-\frac{e}{\hbar}\tilde{\mathbf{E}} - \frac{e}{\hbar}\tilde{\mathbf{v}}_n \times \mathbf{B} - \frac{e^2}{\hbar^2}(\tilde{\mathbf{E}} \cdot \mathbf{B})\Omega_n(\mathbf{k})\right] \end{cases}$$

Chiral anomaly

Density of (***x***,***k***) phase space

Non-Abelian generalization:
J.W. Chen et al, Phys Rev D 2014

Boltzmann equation (with relaxation)

$$\frac{\partial n_{\mathbf{k}}}{\partial t} + \dot{\mathbf{x}} \cdot \frac{\partial n_{\mathbf{k}}}{\partial \mathbf{x}} + \dot{\mathbf{k}} \cdot \frac{\partial n_{\mathbf{k}}}{\partial \mathbf{k}} = -\frac{\delta n_{\mathbf{k}}}{\tau} \qquad n_{\mathbf{k}} = \tilde{f}_n(\mathbf{k}) + \delta n_{\mathbf{k}}$$

Intraband (dominated by intravalley)

- Consider dynamic electromagnetic field

$$\mathbf{E}, \mathbf{B} \propto \exp\{i(\mathbf{q} \cdot \mathbf{x} - \omega t)\}$$

➡ Chiral magnetic effect (E=0, finite τ):

$$\alpha_{ij}(\omega) = \alpha_{static}\delta_{ij} + \frac{\omega\tau}{\omega\tau + i} e \sum_n \int [d\vec{k}] v_{ni} m_{nj} \frac{\partial f_n}{\partial \varepsilon_n}$$

$$\lim_{q\to 0}\lim_{\omega\to 0} \alpha(\vec{q},\omega) = \lim_{\omega\to 0}\lim_{q\to 0} \alpha(\vec{q},\omega)$$

$$= \frac{e^2}{\hbar} \sum_n \int [d\vec{k}] \vec{v}_n \cdot \vec{\Omega}_n f_n \qquad \boxed{\omega\tau << 1} \qquad \leftarrow \boxed{\Phi^{\Omega}}$$

(energy integrated)

- Finite τ removes the non-analyticity of α(0,0)
- No static CME under both limits (in equilibrium)

$\boxed{\omega\tau >> 1}$ (high frequency, or clean)

Dynamic CME, or Gyrotrpic Magnetic Effect

$$\bar{\alpha}(\omega) = \alpha_{static} + \frac{1}{3} e \sum_n \int [d\vec{k}] \vec{v}_n \cdot \vec{m}_n \frac{\partial f_n}{\partial \varepsilon_n} \qquad \leftarrow \Phi^m_{FS}$$

2/3 in LRT

Magnetic moment of a Bloch electron

- Dynamic *B* field induces an *E* field

(Need to put ***E*** back, and redo the semiclassical calculation)

$$\vec{J}^{\mathbf{B}} \left(= -i\omega \vec{P}^{\mathbf{B}} \right) = -\alpha \vec{B}$$

Semiclassical analysis gives

$$\vec{M}^{\mathbf{E}} = \beta \vec{E}, \quad \beta = -\frac{1}{i\omega}\alpha$$

magnetoelectric effect

$$\rightarrow \quad \vec{J}^{\mathbf{E}} = i\vec{q} \times \vec{M}^{\mathbf{E}}$$

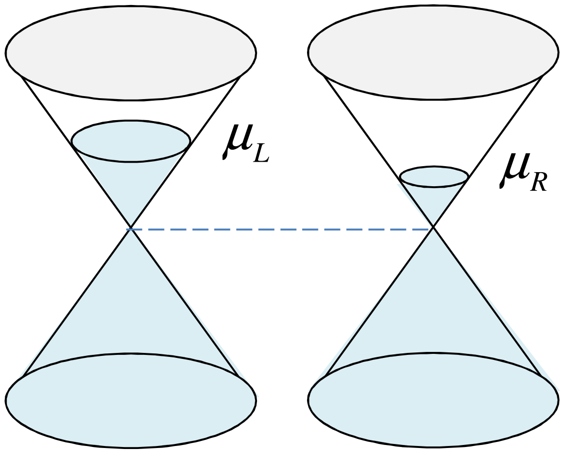
$$= i\vec{q} \times \beta \vec{E}, \quad \vec{q} \times \vec{E} = \omega \vec{B}$$

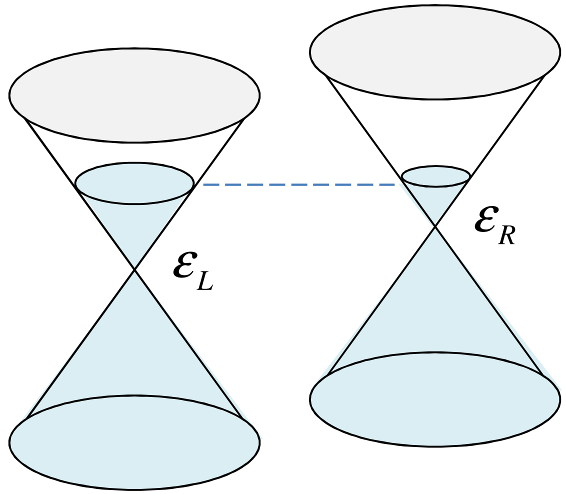
$$= -\alpha \vec{B}$$

Thus, current is doubled.

Also, see Kharzeev et al, Phys Rev D 2017

Summary: Different versions of CME





- Different chemical potentials

$$\vec{J} = \frac{e^2}{h^2} \Delta\mu \vec{B}$$

Magnitude: $J \sim 0.01$ (A/mm^2)
if $\Delta\mu$=0.01 meV, B=0.1 T

→ Negative magneto-resistance

(Zhang et al, Nature Comm 2015)

- Same chemical potential
  - Static B field: no current
  - Dynamic B field (non-equilibrium):

    can have CME current

    (related to natural gyrotropic effect)

Basar et al, Phys Rev B 2014
Zhong et al, Phys Rev Lett 2016

Dynamic CME and natural optical gyrotropy

(Ma and Pesin, Phys Rev B 2015; Zhong et al, PRL 2016)

- **Dielectric function** $\varepsilon_{ij}(\omega,\vec{q}) = \delta_{ij} + \frac{1}{\varepsilon_0\omega^2}\Pi_{ij}(\omega,\vec{q})$ Current response due to A

- **Antisymmetric part** $\Pi^A_{ij}(\omega,\vec{q}) = \Pi^A_{ij}(\omega,0) + \Pi^A_{ij\ell}(\omega)q_\ell$

  TR odd (Faraday rotation) TR even (natural optical rotation)

  ⬍

- **Cubic symmetry, or higher**

  $\alpha_{ij} = \bar{\alpha}\delta_{ij}$ (dynamical CME)

  $\Pi^A_{ij\ell}(\omega) = i\bar{\alpha}\varepsilon_{ij\ell}$ (totally antisymmetric)

- **Rotary power**

$$\gamma \equiv \frac{\pi}{\lambda}\mathrm{Re}(n_- - n_+)$$

$$= \frac{1}{\varepsilon_0 c^2}\mathrm{Re}\,\bar{\alpha} \approx 0.4\left(\frac{\mathrm{rad}}{\mathrm{mm}}\right)$$

for 2 Weyl nodes with $\Delta\varepsilon$=0.1 eV (e.g. $SrSi_2$)

Tsirkin, Puente, and Souza, Phys Rev B 2018

Landau and Lifshitz, *Electrodynamics of continuous media*

- No CME at static B field $(\omega\tau << 1)$
- CME from dynamic B field $(\omega\tau >> 1)$
- Dynamic CME: No Weyl node required, but need Fermi surface

  (also, need to break *space-inversion symmetry*)
- Connection between dynamic CME and optical gyrotropy

Thank you~