Magnetohydrodynamics with chiral anomaly: formulation and phases of collective excitations and instabilities

KH, Yuji Hirono (BNL→APCTP), Ho-Ung Yee (U. Illinois at Chicago), and Yi Yin (MIT),
arXiv:1711.08450 [hep-th]

Koichi Hattori
Fudan University → Yukawa Institute on Dec. 1

Workshop on Recent Developments in Chiral Matter and Topology
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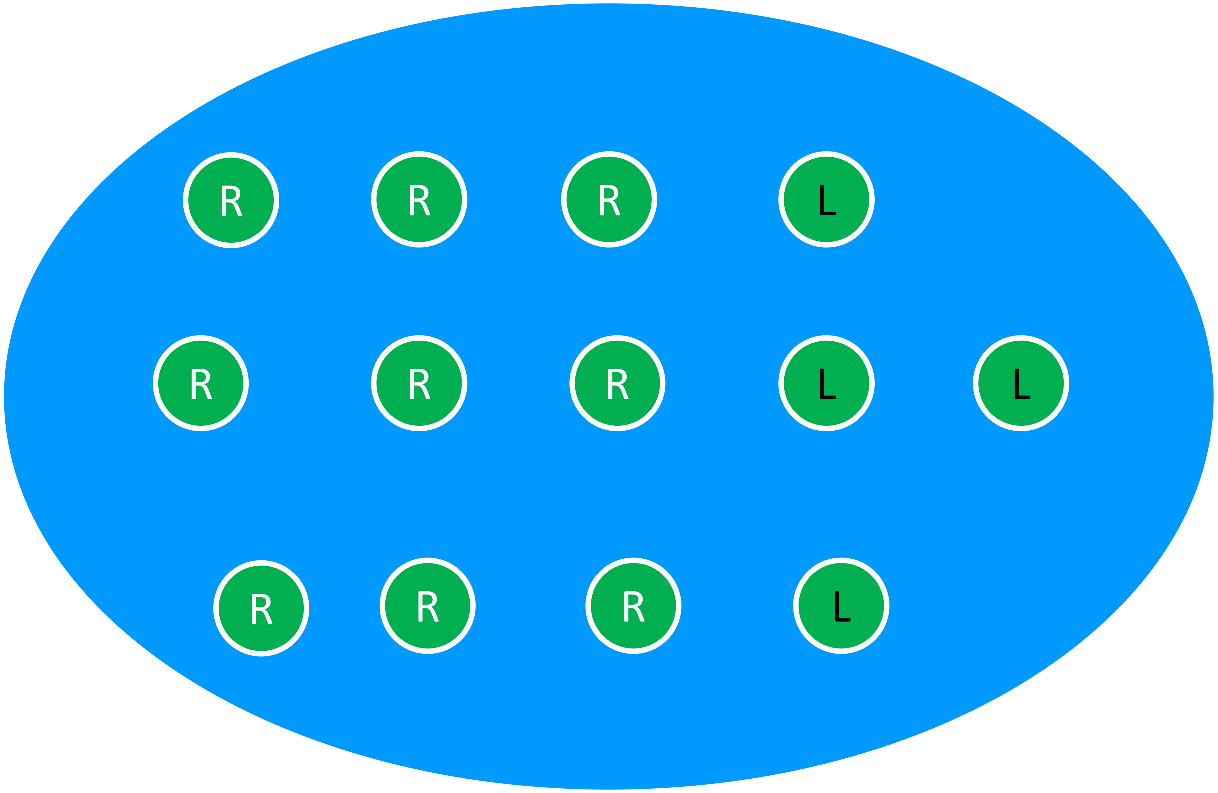
Chiral fluid

$n_R - n_L \neq 0,\ \mathrm{B} \neq 0$



$$\mu_A = (\mu_R - \mu_L)/2 \neq 0$$
$$\mu_V = (\mu_R + \mu_L)/2$$

Anomaly-induced transports in a magnetic OR vortex field

$$\mu_V = (\mu_R + \mu_L)/2$$
$$\mu_A = (\mu_R - \mu_L)/2$$

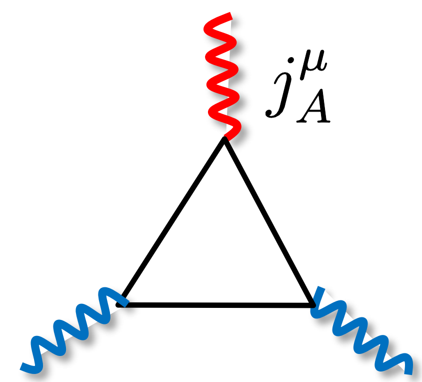
$$\begin{pmatrix} j_V^\mu \\ j_A^\mu \end{pmatrix} = C_A \begin{pmatrix} q_f \mu_A & \mu_V \mu_A \\ q_f \mu_V & (\mu_V^2 + \mu_A^2)/2 + C_A^{-1} T^2/12 \end{pmatrix} \begin{pmatrix} B^\mu \\ \omega^\mu \end{pmatrix}$$

$$B^\mu = \tilde{F}^{\mu\nu} u_\nu, \; \omega^\mu = \frac{1}{2} \epsilon^{\mu\alpha\beta\gamma} u_\alpha \partial_\beta u_\gamma$$

Non-dissipative transport phenomena with time-reversal even and nonrenormalizable coefficients.

Anomaly relation: $\partial_\mu j_A^\mu = q_f^2 C_A \boldsymbol{E} \cdot \boldsymbol{B}$

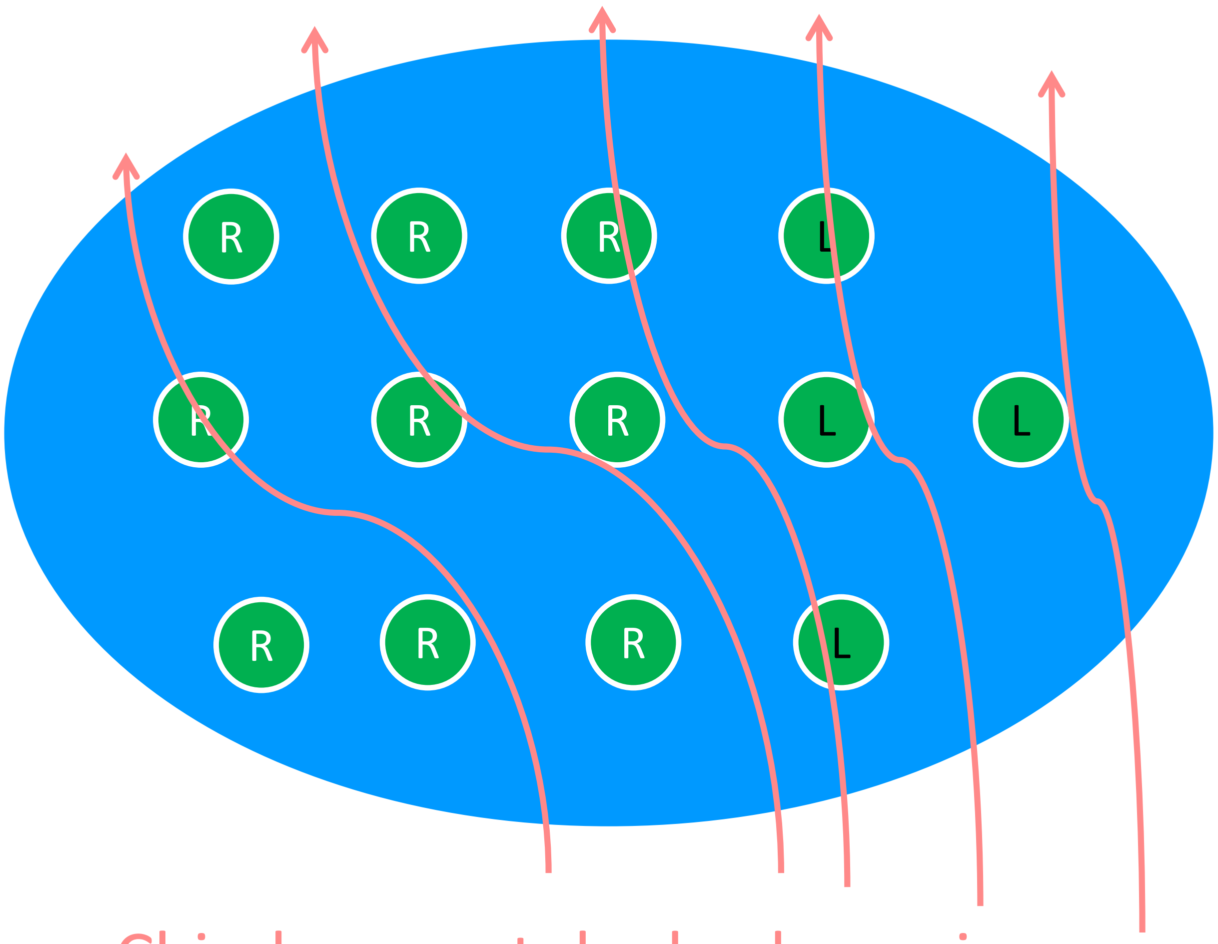
$$C_A = \frac{1}{2\pi^2}$$



Cf., An interplay between the B and ω leads to a new nonrenormalizable transport coefficient for the magneto-vorticity coupling.

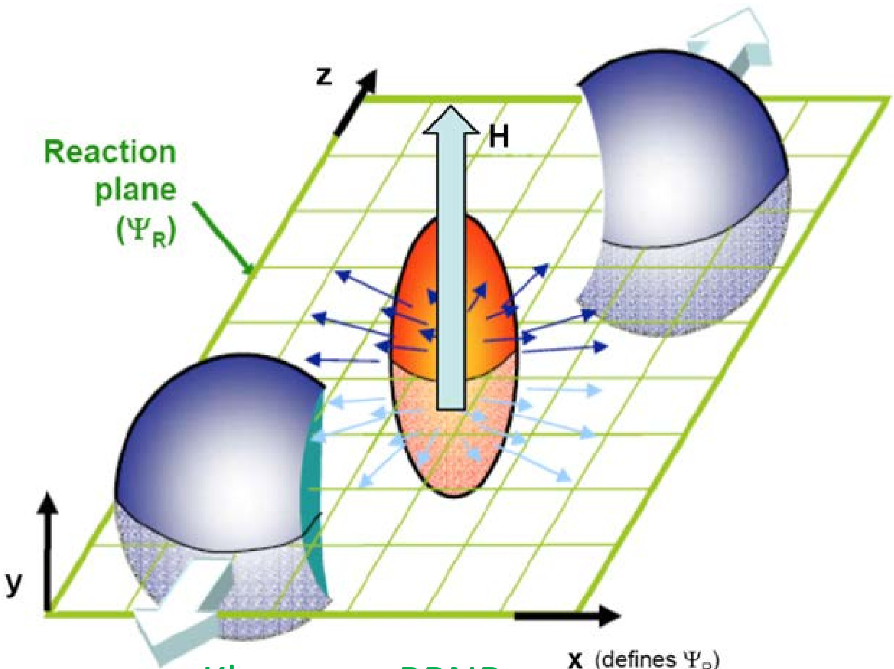
KH and Y.Yin, Phys.Rev.Lett. 117 (2016) 152002 [1607.01513 [hep-th]]

Low-energy effective theory of the chiral fluid in a dynamical magnetic field

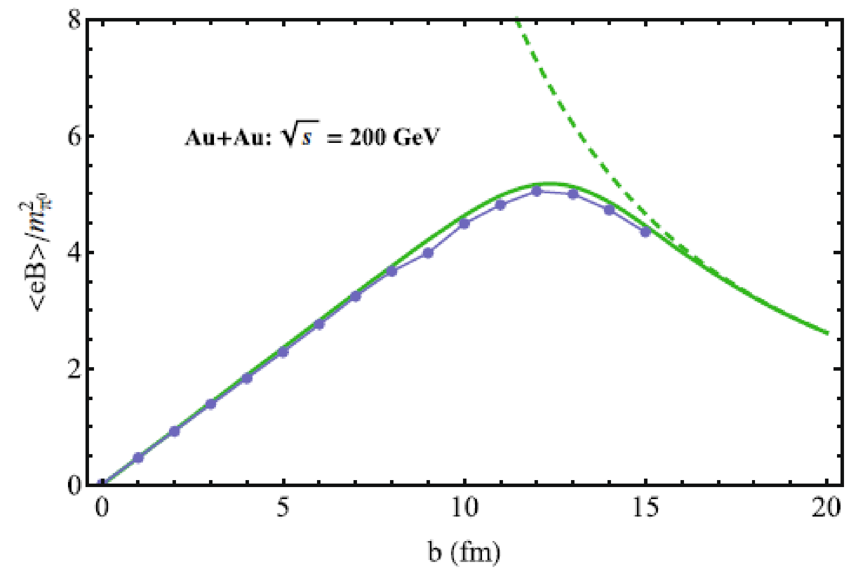


Chiral magnetohydrodynamics
(Chiral MHD, or anomalous MHD)

Strong magnetic fields
induced by relativistic heavy-ion collisions



Kharzeev, PPNP



W.-T. Deng & X.-G. Huang
KH and X.-G. Huang

Z ~ 80, v > 0.99999 c,
Length scale ~ $1/\Lambda_{QCD}$

$$eB \gtrsim m_\pi^2$$

One can study the interplay btw QCD and QED.

Besides,

- Weyl & Dirac semimetals
- Strong B field by lattice QCD simulations
- Neutron stars/magnetars
- High intensity laser fields
- Cosmology

Plan for the rest of talk

1. Formulation of the chiral magnetohydrodynamics (chiral MHD)

   --- Finite chirality imbalance ($n_R \neq n_L$)
   --- Dynamical magnetic field

2. Collective excitations with the linear analysis wrt δv and δB.
   (MHD has a fluctuation of dynamical magnetic field δB.)

3. Summary

Formulating the chiral MHD

Anomalous hydrodynamics in STRONG & DYNAMICAL magnetic fields

-- Anomalous hydrodynamics $\mu_A \neq 0, \; B \sim \mathcal{O}(\partial A)$ and external

Son & Surowka

-- Anomalous magnetohydrodynamics (MHD) $\mu_A \neq 0, \; B \sim \mathcal{O}(1)$

This work.

and dynamical

Slow variables in chiral MHD:
$\{\epsilon, \; u^\mu, \; B^\mu, \text{ and } n_A\}$

n_A: # density of axial charge
Neutral plasma ($n_V = 0$)
No E-field in the global equilibrium

EoMs:

$$\partial_\mu T^{\mu\nu}_{\text{fluid+EM}} = 0,$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0,$$

$$\partial_\mu j^\mu_A = -C_A E^\mu B_\mu.$$

$$C_A = \frac{1}{2\pi^2}$$

Constitutive eqs. in the ideal order (zeroth order in derivative)

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - X\Delta^{\mu\nu} - Y B^\mu B^\nu$$

$$\tilde{F}^{\mu\nu}_{(0)} = B^\mu u^\nu - B^\nu u^\mu$$

$$j^\mu_{A(0)} = n_A u^\mu$$

$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \; (u_\mu \Delta^{\mu\nu} = 0)$

E-field is first order.

$B^{(\mu}u^{\nu)}$ is absent in $T^{\mu\nu}$ when $n_V = 0$.

From EoM + thermodynamic relation $ds = \frac{1}{T}(d\epsilon - \mu_A dn_A - H_\mu dB^\mu)$

$$\begin{aligned}\partial_\mu(su^\mu) &= u\cdot\partial s + s\partial\cdot u \\ &= (p - X)\partial\cdot u + (H^\mu - YB^\mu)B\cdot\partial u_\mu \\ &= 0 \quad \text{for the ideal part.}\end{aligned}$$

Therefore, $T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} - \mu^{-1}B^\mu B^\nu$

ϵ and p are the total (fluid+magnetic) energy and pressure.

Reproduces the "ideal MHD" when $\epsilon = \epsilon_{\text{fluid}} + \frac{1}{2}B^2$.

Constitutive eqs. and the entropy generation in the first order

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)}$$
$$\tilde{F}^{\mu\nu} = \tilde{F}^{\mu\nu}_{(0)} - \epsilon^{\mu\nu\alpha\beta} u_\alpha E_{(1)\beta}$$
$$j^\mu_A = j^\mu_{A(0)} + j^\mu_{A(1)}$$

Note that $\partial_\mu j^\mu_A = -C_A E^\mu_{(1)} B_\mu$.

The zeroth order term $T^{\mu\nu}_{(0)}$ reproduces the ideal MHD.

$$T^{\mu\nu}_{(1)},\ E^\mu_{(1)},\ j^\mu_{A(1)} \sim \mathcal{O}(\partial^1)$$

The second law of the thermodynamics $\partial_\mu (s u^\mu) \geq 0$ constrains possible first-order corrections.

Computing the entropy current,

$$\begin{aligned}\partial_\mu \left(s u^\mu + \mathcal{O}(\partial^1) \right) &= T^{\mu\nu}_{(1)} \partial_\mu (\beta u_\nu) - j^\mu_{A(1)} \partial_\mu (\beta \mu_A) \\ &\quad + E^\mu_{(1)} \{ \mu_A C_A B_\mu - \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha (\beta H^\beta) \} \\ &= E^\mu_{(1)} X_{\mu\nu} E^\nu_{(1)},\ \text{for example.}\end{aligned}$$

Insuring the semi-positivity with bilinear forms

Positivity is insured by a bilinear form: $E^{\mu}_{(1)} X_{\mu\nu} E^{\nu}_{(1)} \geq 0$

$$X_{\mu\nu} = \sigma_{\|} b_\mu b_\nu - \sigma_\perp (g_{\mu\nu} - u_\mu u_\nu + b_\mu b_\nu) - \sigma_{\mathrm{Hall}}\, \epsilon_{\mu\nu\alpha\beta} u^\alpha b^\beta$$

$b^\mu = -B^\mu / B^2$ breaks a spatial rotational symmetry.

$\sigma_{\|,\perp} \geq 0$, but $\sigma_{\mathrm{Hall}} \propto \mu_V$.

Therefore, we get a "constitutive eq." of the E-field:

$$E^{\mu}_{(1)} = X^{-1\mu\rho} \{ \mu_A C_A B_\rho - \epsilon_{\rho\nu\alpha\beta} u^\nu \partial^\alpha (\beta H^\beta) \}$$

KH, Hirono, Yee, Yin

Similarly,

$$T^{\mu\nu}_{(1)} \partial_\mu (\beta u_\nu) \geq 0$$

provides 5 dissipative and
2 non-dissipative (Hall) viscous coefficients

de Groot; Landau & Lifshitz; Huang, Sedrakian, & Rischke; Hernandez & Kovtun; …

$$-j^{\mu}_{A(1)} \partial_\mu (\beta \mu_A) \geq 0$$

3 diffusion coefficients

Conductivities: CME and dissipative terms

From the constitutive eq. of $E^{\mu}_{(1)}$ and the Maxwell eq.,

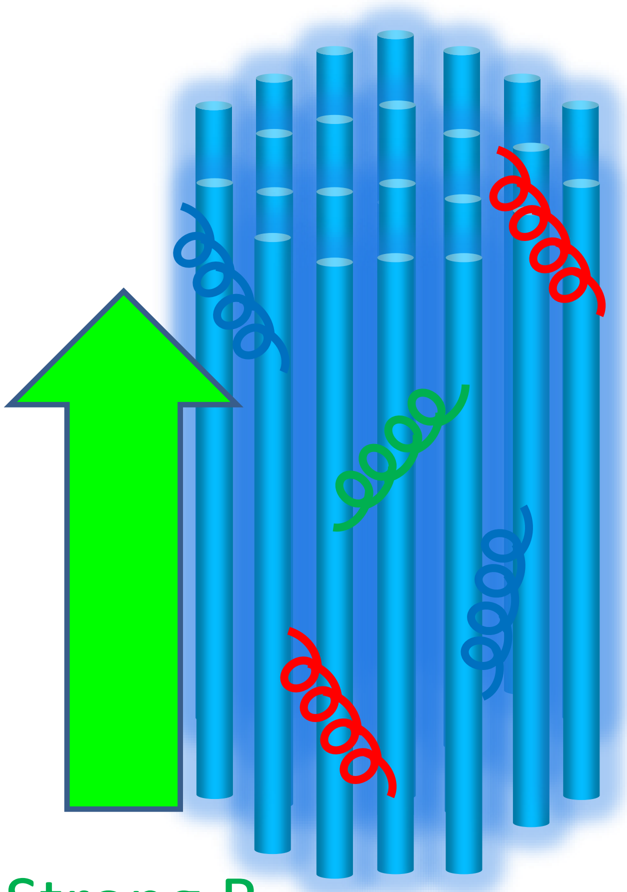
$$J^{\mu}_{V} = C_A \mu_A B^{\mu} + \left[\sigma_{\parallel} E^{\mu}_{\parallel} + \sigma_{\perp} E^{\mu}_{\perp} + \sigma_{\rm Hall} \epsilon^{\mu\nu\alpha\beta} u_{\nu} b_{\alpha} E_{\beta} \right] + \cdots$$

The CME current is completely fixed by C_A, and is necessary for insuring the semi-positive entropy production.

The CME has the universal form in the MHD regime as well.

There appear the longitudinal and transverse Ohmic conductivities due to the breaking of the rotational symmetry.

Conductivities and viscosities in strong B fields



In the LLL, charged fermions transport charges and momenta only along the B.
→ Effective dimensional reduction to (1+1) D in the fermion sector.

Computation by the perturbation theory at finite T and B

Longitudinal conductivity

KH, S.Li, D.Satow, H.-U. Yee, 1610.06839 [hep-ph];
KH, D.Satow, 1610.06818 [hep-ph].

Cf., Landau-level resummation, Fukushima, Hidaka.

Longitudinal bulk viscosity

KH, X.-G.Huang, D.Satow, D.Rischke, 1708.00515 [hep-ph].

Strong B

"Mismatched dimensions"

Quarks live in (1+1) D

Gluons live in (3+1) D

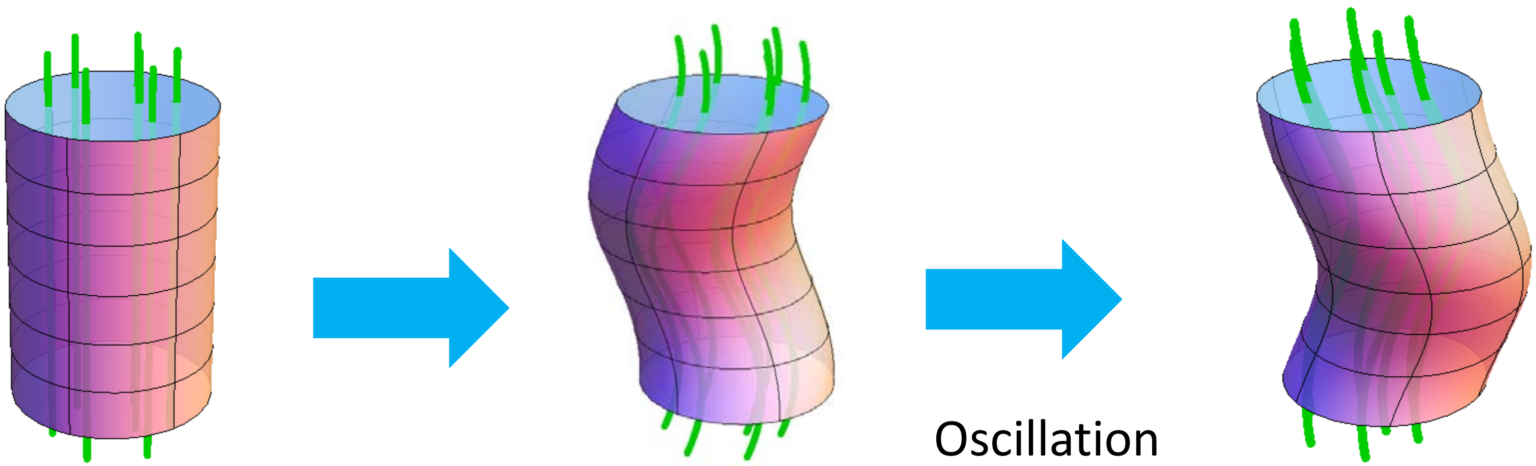
Phases of the collective excitations and instabilities from a linear analysis

Collective excitations in MHD without anomaly

2 transverse waves (Alfven waves)

4 longitudinal waves (fast and slow magneto-sonic waves)

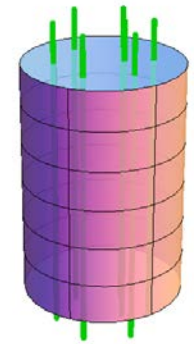
* Magnetic lines move together with the fluid volume.



Tension of B-field → Restoring force
Fluid energy (mass) density → Inertia

Transverse Alfven wave

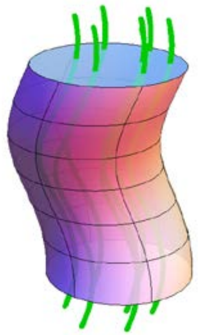
Alfven wave from a linear analysis



$$\boldsymbol{B}_0 \neq 0\,,\ T > 0\,,\ \mu_V = 0$$

0. Stationary solutions

$$u^\mu = (1, \mathbf{0}),\ \ B^\mu = (0, \boldsymbol{B}_0),\ \ j^\mu = (0, \mathbf{0})$$

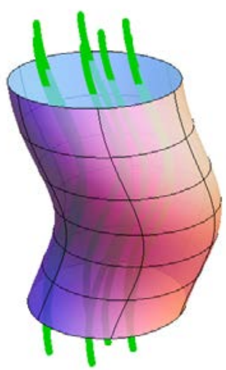


1. Transverse perturbations

$$\boldsymbol{v} \to \boldsymbol{v} + \delta \boldsymbol{v}$$

$$\boldsymbol{B}_0 \to \boldsymbol{B}_0 + \delta \boldsymbol{B}$$

Linearlize the set of hydrodynamic eqs. with respect to the perturbation.



2. Wave equation

$$\partial_t^2 \delta \boldsymbol{B}(t,z) = \frac{B_0^2}{\epsilon + p} \partial_z^2 \delta \boldsymbol{B}(t,z)$$

Alfven wave velocity

Transverse wave propagating along background B_0

$$\boldsymbol{B}_0 \parallel \boldsymbol{k}$$

Same wave equation for δv

→ Fluctuations of B and v propagate together.

How does the CME change the hydrodynamic waves in chiral fluid?

--- Drastic changes by only one term in the current

$$j^{\mu} = \sigma_{\rm CME} B^{\mu}$$

Eigenmodes of chiral MHD

$$\psi^T = \underbrace{(c_s\,\delta\tilde{\epsilon}_f, \delta v_1, \delta v_2, \delta v_3, \delta b_1, \delta b_2)}_{\text{6 degrees of freedom}}$$

ϵ (1 d.o.f.)
$\boldsymbol{v}$ (3 d.o.f.)
$\boldsymbol{B}$ $(\nabla \cdot \boldsymbol{B} = 0)$ (2 d.o.f.)

$$M\psi = V\psi \quad \text{where } \omega = Vk$$

6 × 6 matrix from the linearlized EoMs

$$M = M_0 + \epsilon_A\, M_A$$

$$\epsilon_A = \sigma_{\rm CME}/\sigma$$

M_A: Modification by a finite μA

When $\mu_A = 0$, we have $M = M_0$.
The solutions reproduce the Alfven and magneto-sonic waves in MHD.

Eigenvalues V: Dispersion relations
Eigenvectors ψ: Polarizations

“Phase diagram” of the eigenmodes

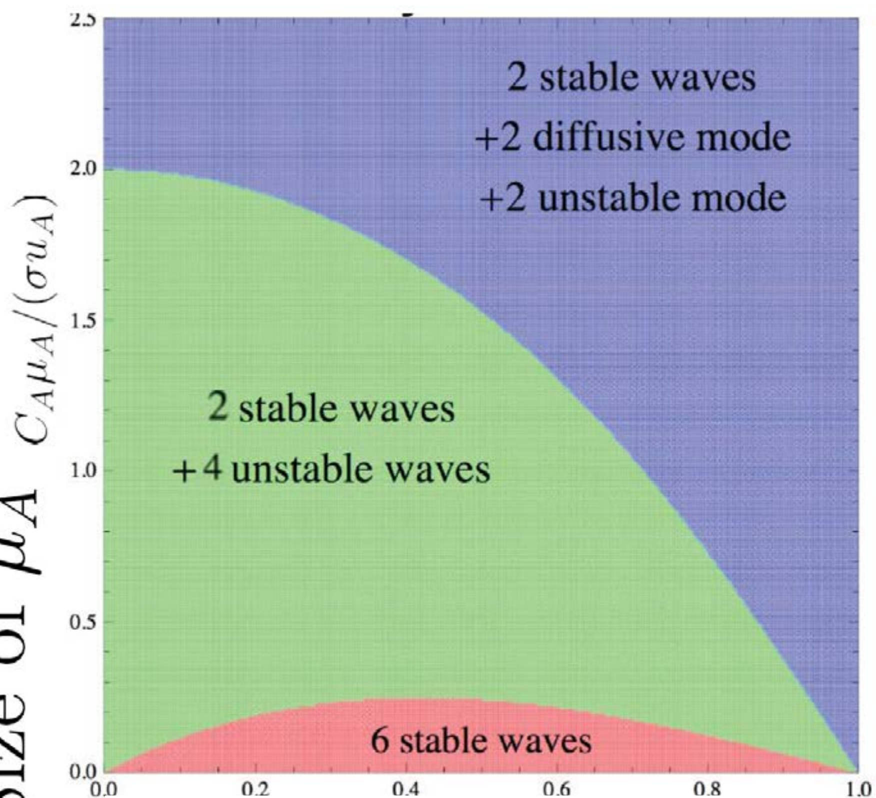
Secular eq. is a cubic eq. of ω^2

--- 3 modes propagating in the opposite directions (6 solutions in total)

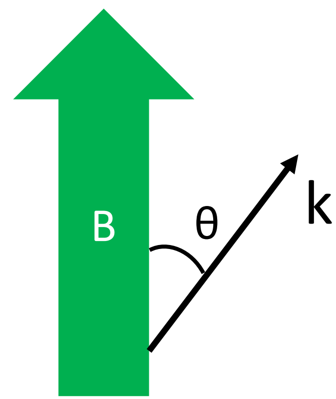
$$(\omega^2 - x_1)(\omega^4 + b\omega^2 + c) = 0 \qquad x_1\text{: Real solution}$$

Stability of the waves from classification of solutions

Size of μ_A



Direction of wave wrt B (θ)



1 real and 2 pure imag. sols.

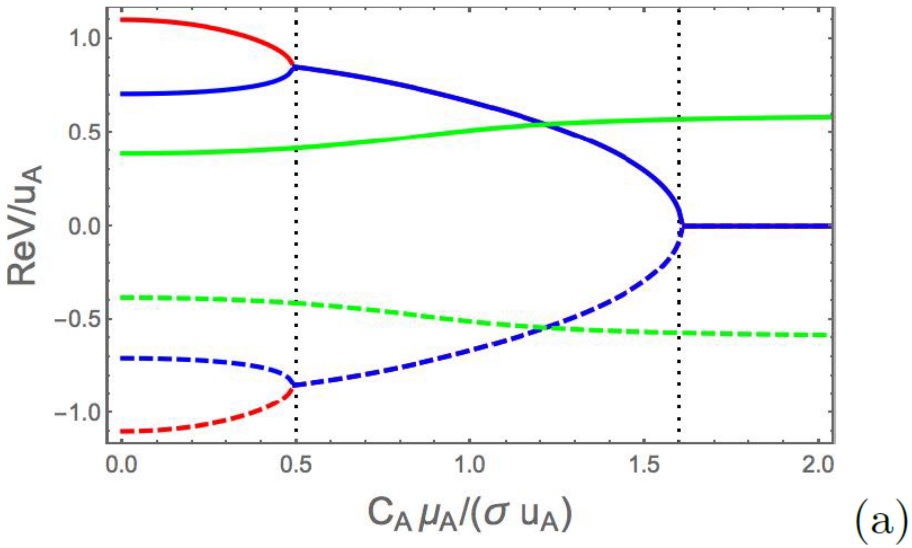
1 real and 2 complex sols.

3 real solutions

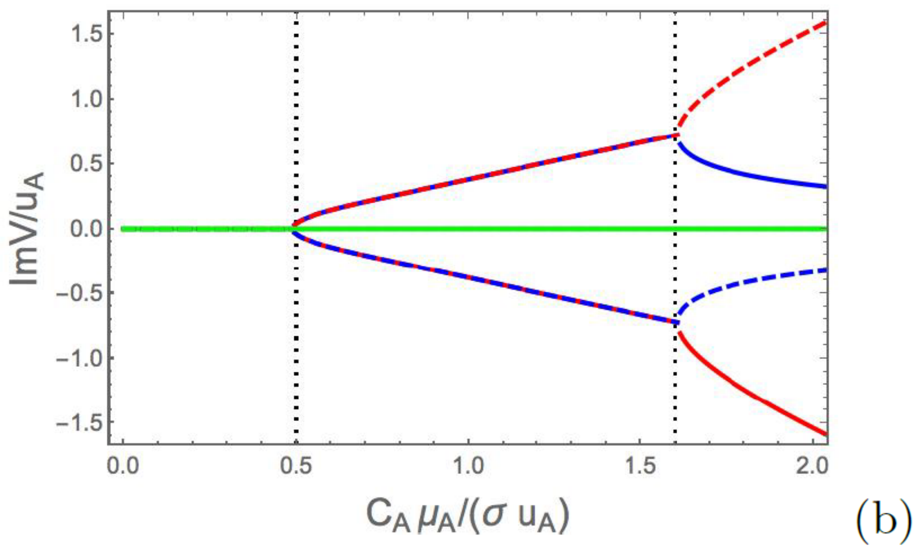
Alfven and magneto-sonic waves

Dispersion relations of the waves

Real part of V



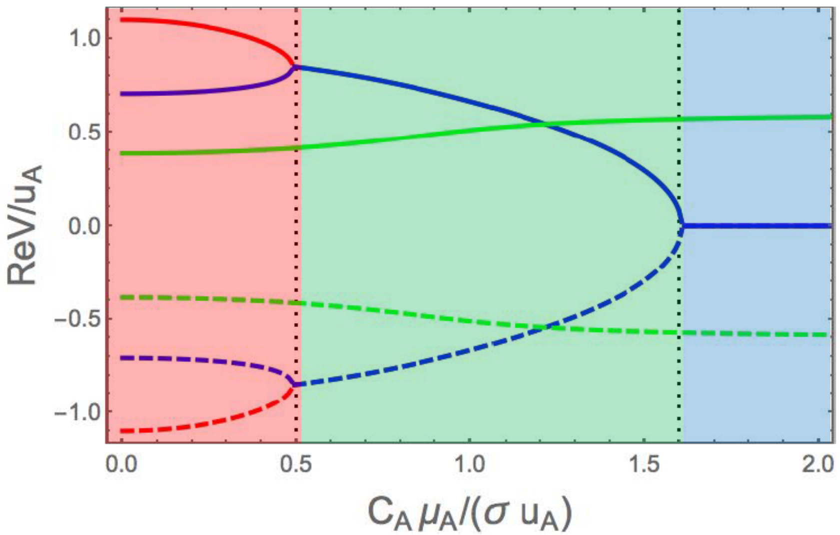
Imaginary part of V



There is a pair of modes (green) which are stable in any phase.
[Will not be focused hereafter.]

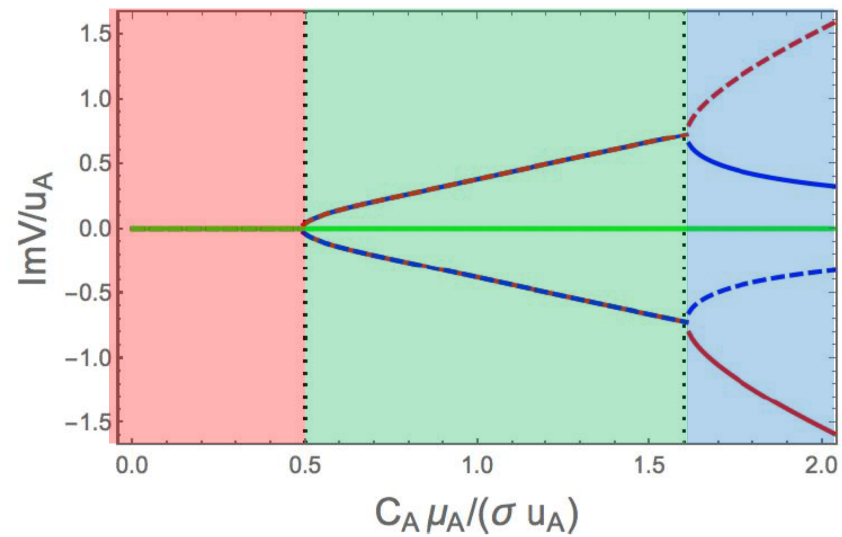
Dispersion relations of the waves

Real part of V



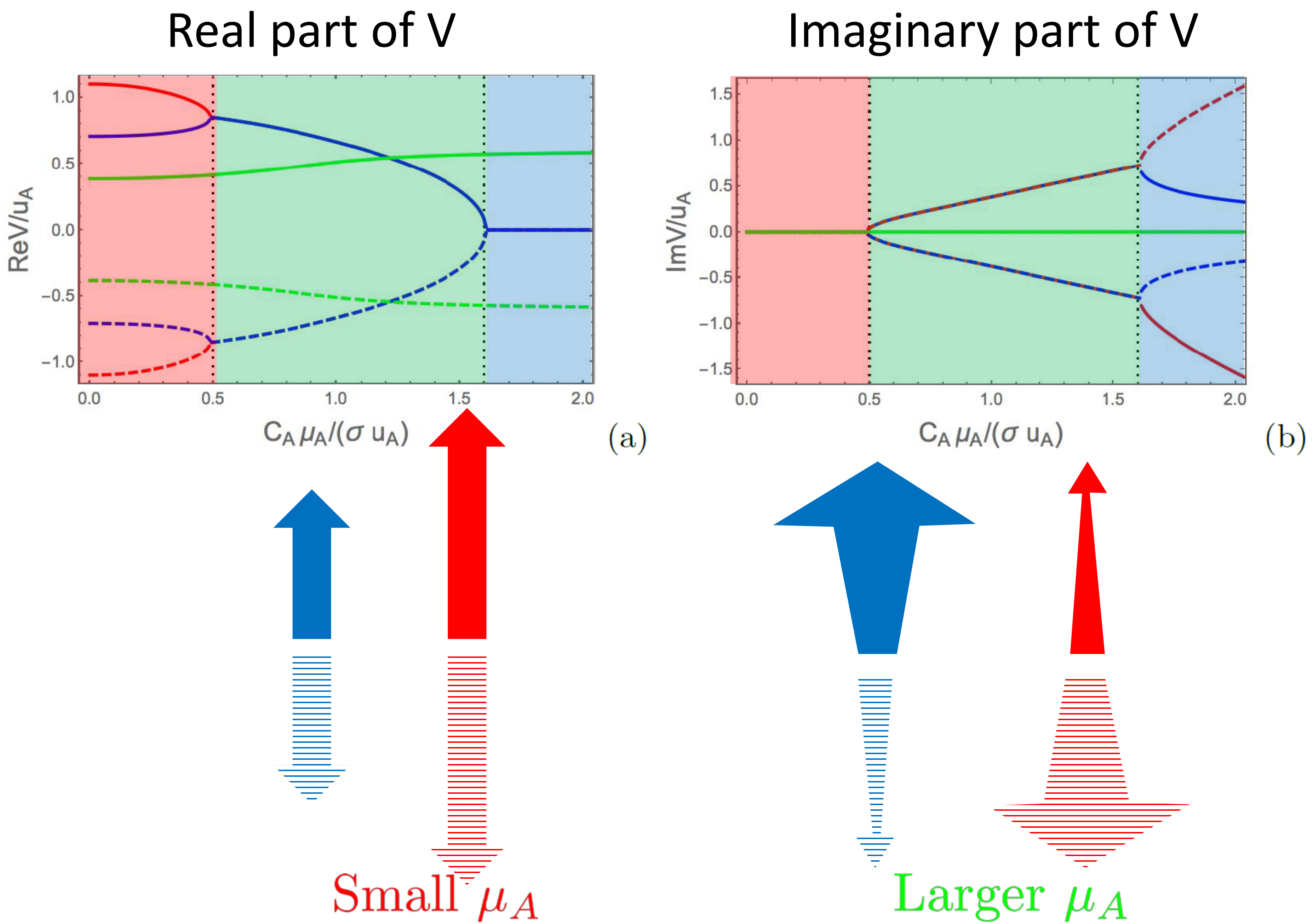
(a)

Imaginary part of V



(b)

Dispersion relations of the waves

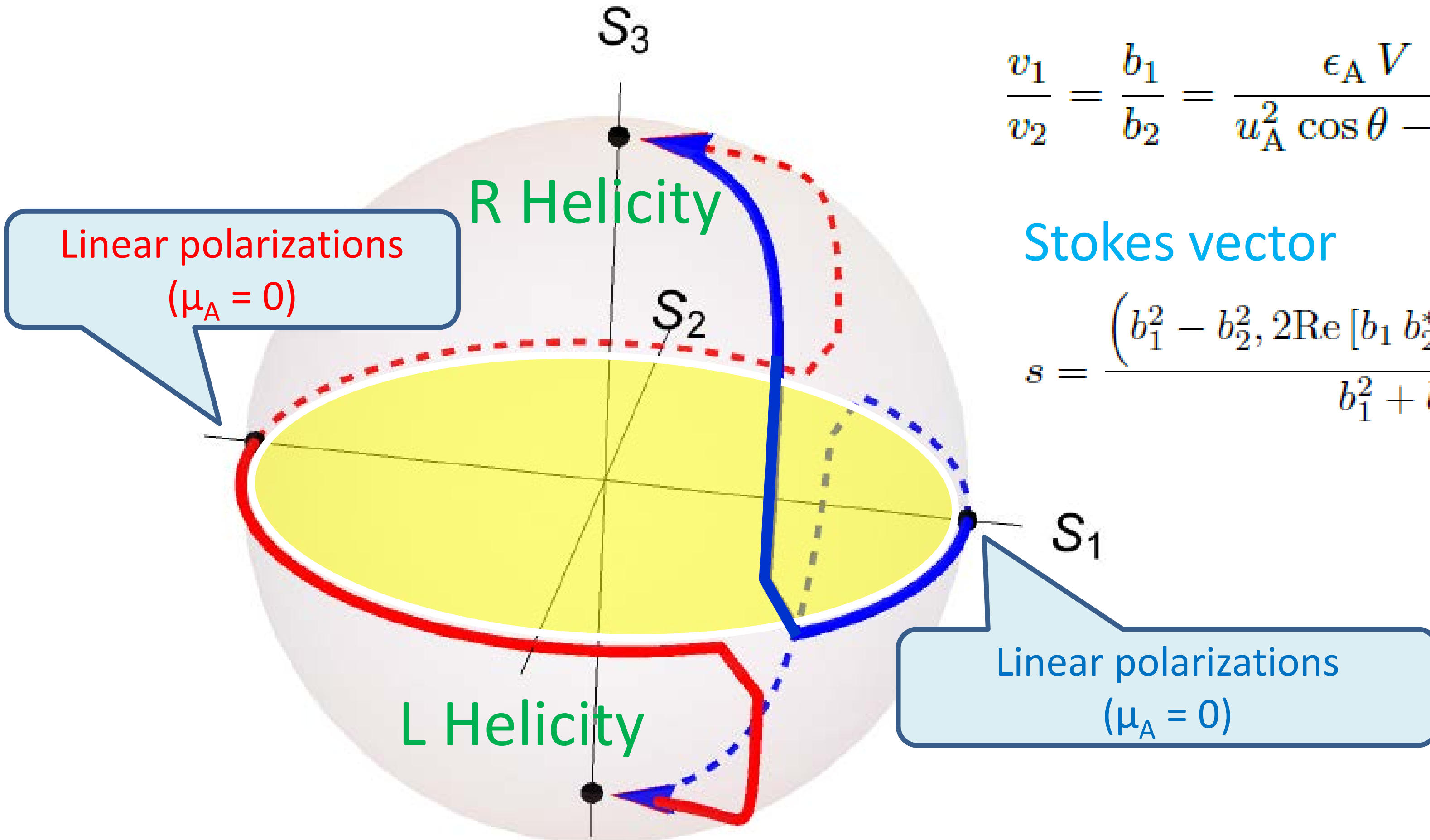


Polarizations on the Poincare sphere with a varying μ_A

$$\frac{v_1}{v_2} = \frac{b_1}{b_2} = \frac{\epsilon_{\mathrm{A}}\, V}{u_{\mathrm{A}}^2 \cos\theta - V^2}$$

Stokes vector

$$s = \frac{\left(b_1^2 - b_2^2, 2\mathrm{Re}\left[b_1\, b_2^*\right], 2\,\mathrm{Im}\left[b_1\, b_2^*\right]\right)}{b_1^2 + b_2^2}$$



Equator: Linear polarizations

Upper and lower hemispheres: R and L polarizations

(Poles: R and L circular polarizations)

The unstable modes have helical nature.

New hydrodynamic instability in a chiral fluid

Signs of the imaginary parts
(Damping/growing modes in the hydrodynamic time evolution)

Positive
(Damping)

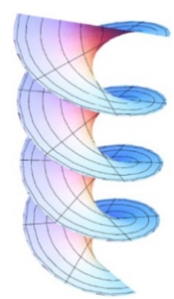
Negative
(Growing)

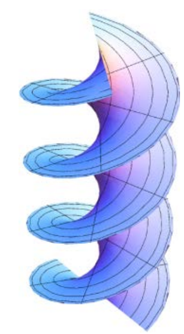
When $\mu_A > 0$

When $\mu_A < 0$

LH mode

RH mode





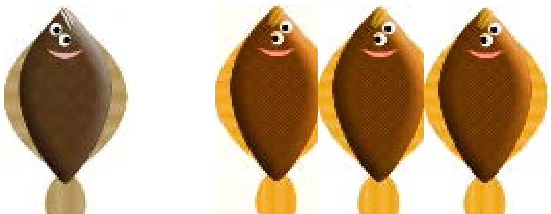
Helicity decomposition
(Circular R/L polarizations)

$$\nabla \times \boldsymbol{e}_{R/L} = \pm \boldsymbol{e}_{R/L}$$

A helicity selection, depending on the sign of μ_A .

Helicity conversions as the topological origin of the instability

Difference btw the # of R and L fermions: "Chiral imbalance" $n_R - n_L$



Chiral Plasma Instability (CPI)

Akamatsu&Yamamoto

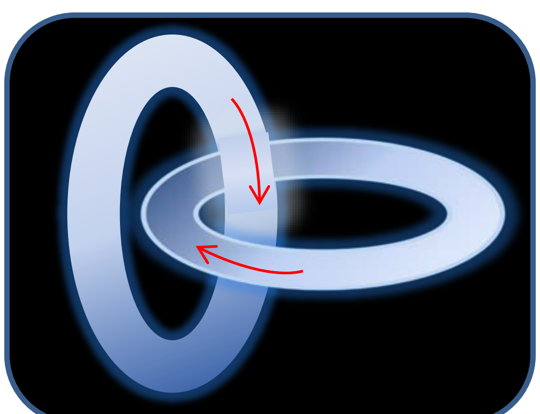
Magnetic helicity

$$\int_V d^3x \, \boldsymbol{B} \cdot \boldsymbol{A}$$

Real-time & beyond-linear analysis demanded.

Hirono

$$\int_V d^3x \, \boldsymbol{\omega} \cdot \boldsymbol{v}_{\mathrm{fluid}}$$



Fluid helicity (structures of vortex strings)

Summary

Formulation

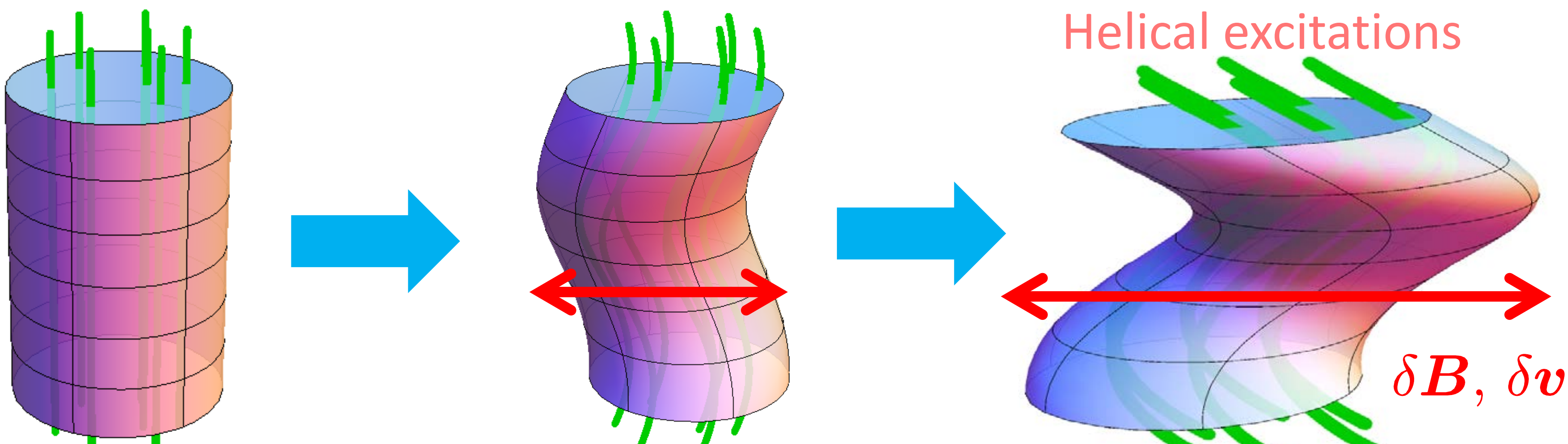
Second law of thermodynamics determines the form of the CME current, reproducing the universal form.

Stay tuned for a microscopic derivation of MHD. Hongo & KH

Phases of the collective excitations and instabilities

The CME drastically changes the time evolution of the chiral fluid in a B-field.

- Chiral fluid is not stable against a small perturbation on v and B.
- One of the helicities is strongly favored against the other due to a finite μA.



Backup slides

Hydrodynamic variables when μV = 0

$$\partial_t n_V = -\nabla \cdot \boldsymbol{j}_V = -\sigma \nabla \cdot \boldsymbol{E} = -\sigma n_V$$

$\partial_\mu j_V^\mu = 0$ Ohm's law Gauss's law

$$n_V = n(t=0)\exp(-\sigma t)$$

Therefore, when $t \gtrsim 1/\sigma$, $n_V \sim 0$.

$$\boldsymbol{E} = \frac{1}{\sigma}\boldsymbol{J} \to 0$$

E^μ in the rest frame is damped out quickly in a highly conducting plasma.

We work in the world after the E-field is damped.

$E^\mu \sim \mathcal{O}(\partial^1)$, and is given by a function of the hydrodynamic variables, a "constitutive equation."

Estimate of the relaxation time of n_A

Steady state: $J_{Ohm} = J_{CME}$ $$E^\mu = \frac{C_A \mu_A}{\sigma} B^\mu$$

$$\partial_t n_A = -\frac{C_A^2 \left(-B^2\right)}{\sigma} \mu_A$$

⇩

$$\tau_A = (\sigma\chi) \, / \, \left[C_A^2 \left(-B^2\right)\right]$$

$$\chi = (\partial n_A / \partial \mu_A)$$

(Relaxation time of E ~ 1/σ) << (Our time scale) << (Relaxation time of nA ~ σ)

The window is wider for a larger σ .

Collective excitations in chiral MHD

$$M\psi = V\psi \quad \text{where } \omega = Vk$$

$$\psi^T = (c_s\,\delta\tilde{\epsilon}_f, \delta v_L, \delta v_2, \delta b_2, \delta v_1, \delta b_1)$$

$$\delta\tilde{\epsilon}_f = \delta\epsilon_f/(\epsilon_{f0} + p_{f0} + B_0^2)$$

$$\epsilon_A = C_A\mu_A/\sigma$$

$$u_A^2 = B_0^2/(\epsilon_f + p_f + B_0^2)$$

$$M = M_0 + \epsilon_A\, M_A$$

$$M_0 = \begin{pmatrix} 0 & c_s & 0 & 0 & 0 & 0 \\ c_s & 0 & 0 & -u_A\sin\theta & 0 & 0 \\ 0 & 0 & 0 & -u_A\cos\theta & 0 & 0 \\ 0 & -u_A\sin\theta & -u_A\cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -u_A\cos\theta \\ 0 & 0 & 0 & 0 & -u_A\cos\theta & 0 \end{pmatrix}, \quad M_A = \frac{1}{u_A}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

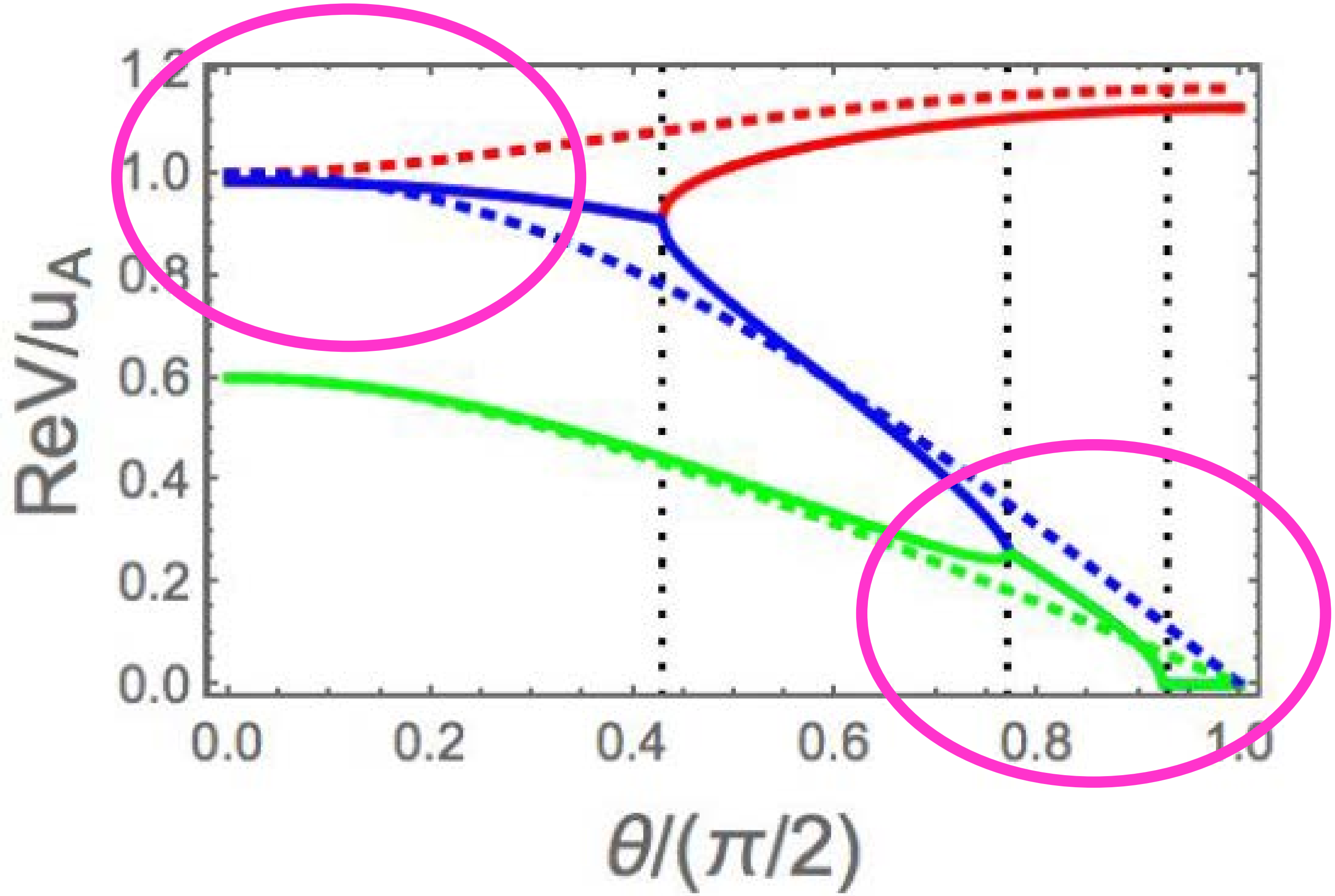
When $u_A \ll 1$,

$$(w - \cos^2\theta)\left[w^2 - \{1 + (c_s/u_A)^2\}w + (c_s/u_A)^2\cos^2\theta\right] + (\epsilon_A/u_A)^2\, w\{w - (c_s/u_A)^2\} = 0$$

$w \equiv V^2/u_A^2$

Effects of anomaly

Alfven wave, fast and slow magneto-sonic waves, when $\epsilon_A = 0$.



Dotted: Without anomaly effects
[Alfven (red), fast sonic (blue), slow sonic (green)]

Solid: With anomaly effects which mix the waves