Chiral Kinetic Theory and Quantum Transport of Chiral Fluids

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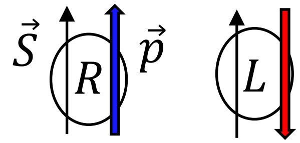
References :
Yoshimasa Hidaka, Shi Pu, DY, Phys. Rev. D95 (2017) no.9, 091901. arXiv:1612.04630
Yoshimasa Hidaka, Shi Pu, DY, Phys. Rev. D 97 (2018) no.1, 016004. arXiv:1710.00278
Yoshimasa Hidaka, DY, Phys. Rev. D98 (2018) no.1, 016012. arXiv:1801.08253
DY, Phys.Rev. D98 (2018) no.7, 076019. arXiv:1807.02395

Anomalous transport in chiral matter

- Weyl fermions :
  chirality=helicity



$$\mathbf{J}_V = \mathbf{J}_R + \mathbf{J}_L$$
$$\mathbf{J}_5 = \mathbf{J}_R - \mathbf{J}_L$$

(see Liao & Shovkovys' talks)

- Chiral anomaly :

$$\partial_\mu J^\mu_{R/L} = \pm \frac{\mathbf{E}\cdot\mathbf{B}}{4\pi^2} \Longrightarrow \partial_\mu J^\mu_5 = \frac{\mathbf{E}\cdot\mathbf{B}}{2\pi^2}$$

S. Adler, J. Bell, R. Jackiw, 69 K. Fujikawa, 79

- Anomalous transport (in chiral matter):

$\mathcal{P}$-odd $\mathcal{T}$-even

Chiral magnetic effect (CME) : $\mathbf{J}_V = \boxed{\frac{1}{2\pi^2}\mu_5}\mathbf{B}$

A. Vilenkin, 80
K. Fukushima, D. Kharzeev, H. Warringa, 08
D. Kharzeev, L. McLerran, H. Warringa, 08

Chiral separation effect (CSE) : $\mathbf{J}_5 = \frac{1}{2\pi^2}\mu_V \mathbf{B}$

Chiral vortical effect (CVE) : $\mathbf{J}_V = \frac{1}{\pi^2}\mu_5\mu_V \boldsymbol{\omega}$ $\quad \mathbf{J}_5 = \left(\frac{\mu_V^2 + \mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\boldsymbol{\omega}$

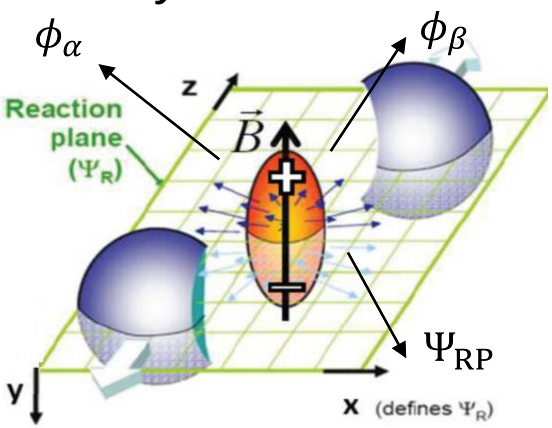
A. Vilenkin, 79 K. Landsteiner, E. Megias, F. Pena-Benitez, 11

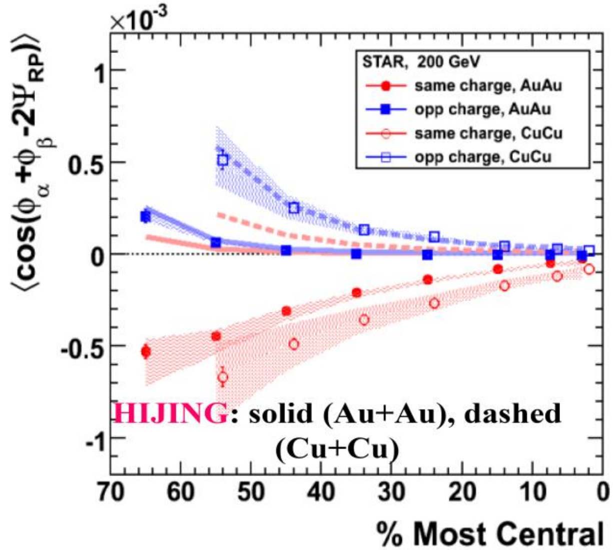
- The anomalous transport is derived from a variety of approaches : Kubo formulae, hydrodynamics, lattice QCD, AdS/CFT, etc.

(see e.g. A. Yamamoto, 11)

Anomalous transport in the real world

- Heavy ion collisions :





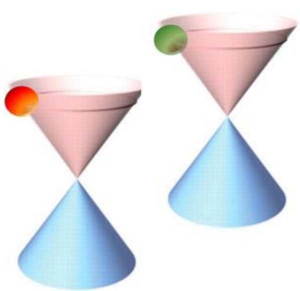
(see Liao & Shovkovys' talks)

CME signal :
azimutal charged-partiale correlations S.Voloshin '04

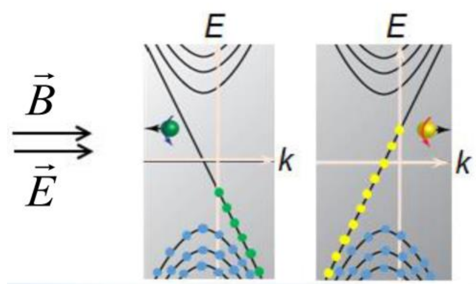
caution:
strong background!

- Weyl semimetals :

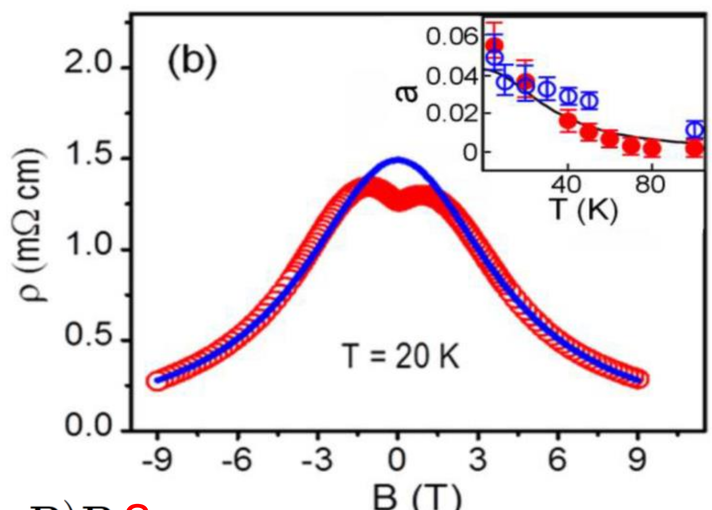
(see Dai & Shens' talks)



TaAs
NbAs
NbP
TaP



charge pumping via parallel
E & B : generate $\mu_5 \sim n_5 \sim E \cdot B$



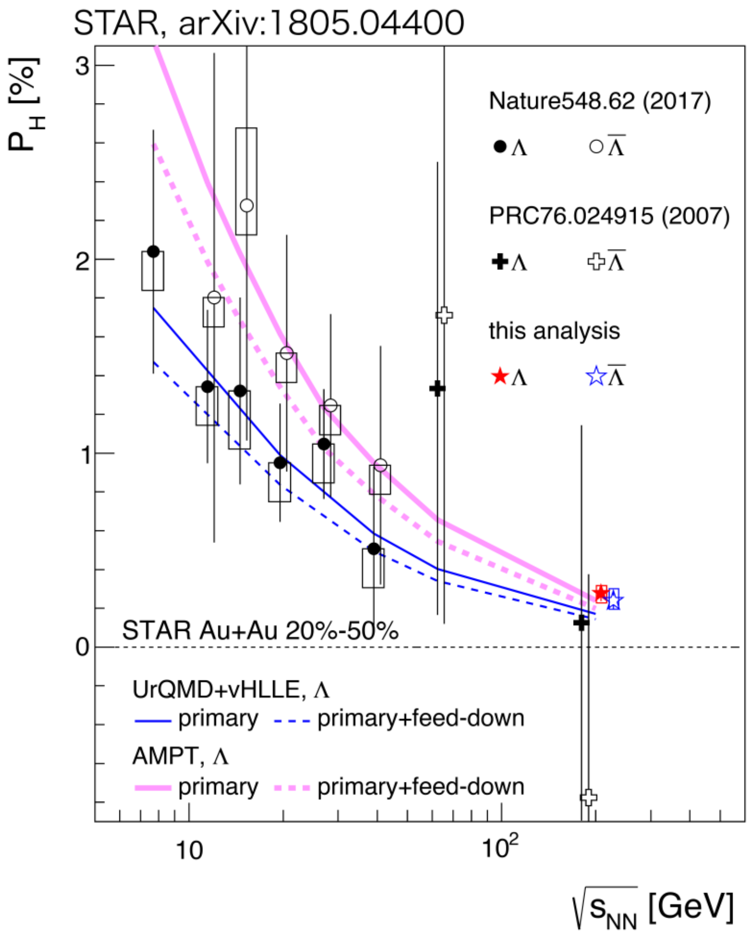
CME signal : negative magnetoresistance

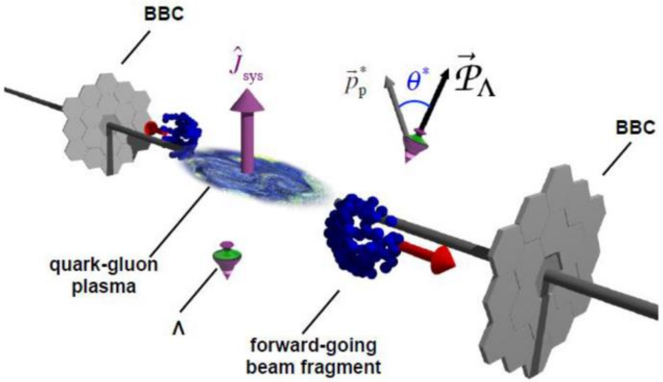
Qiang Li, et.al., Nature Phys. 12 (2016) 550-554 $\mathbf{J}_V \propto (\mathbf{E}\cdot\mathbf{B})\mathbf{B}$?

Rotating fluids with spins

- Global polarization of Λ hyperons : (see Liao's talk)

STAR, Nature 548 (2017) 62-65





STAR, 07

Theoretical studies :

- ❖ Spin-orbit coupling model
  Z.-T. Liang & X.-N. Wang, 05
- ❖ Wigner-function method
  Q. Wang, et.al. 16
- ❖ Statistical-hydro model
  F. Becattini, et.al. 13

See e.g. QM 17 review by Q. Wang
Or QM 18 by F. Becattini

- ➢ Related to axial currents led by CVE/CSE?

(albeit Λ or s quarks are massive)

Astrophysical applications

- Evolution of cosmic magnetic fields

  H. Tashiro, T. Vachaspati, A. Vilenkin, Phys.Rev. D86 (2012) 105033

  A. Boyarsky, J. Frohlich, and O. Ruchayskiy, Phys. Rev. Lett. 108, 031301 (2012).

- Neutrino/lepton transport in core-collapse supernovae

  (see N. Yamamoto's talk tomorrow)

  N. Yamamoto, Phys. Rev. D93, 065017 (2016)

  Y. Masada, K. Kotake, T. Takiwaki, and N. Yamamoto, Phys. Rev. D 98, 083018 (2018)

Outline

- Recent development in chiral kinetic theory (CKT) : (QED with BFs)
  - Covariant CKT with background fields & collisions from QFT (Wigner-function approach) (weakly coupled sys. & weak fields)
    Hidaka, Pu, DY, 16, 17
- Applications : non(near)-equilibrium transport for chiral fluids
  - Quantum transport in equilibrium
  - Non-linear (2^{nd}-order) responses for anomalous transport
    Hidaka, Pu, DY, 17
    Hidaka, DY, 18
    DY, 18
    (2^{nd}-order anomalous hydro)
- Mass corrections? (preliminary)
  Hidaka & DY

(I will mostly focus on R-handed fermions.)

Chiral kinetic theory

- Standard kinetic theory : $q^\mu\left(\partial_\mu + F_{\nu\mu}\frac{\partial}{\partial q_\nu}\right)f = \mathcal{C}[f] \Rightarrow \partial_\mu J^\mu = 0$
- CKT : D. T. Son and N. Yamamoto,12 M. Stephanov and Y. Yin, 12 $\Longrightarrow \partial_\mu J^\mu = \frac{\hbar}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$ (for R-handed fermions)

❖ The semi-classical approach : classical action + Berry phase

$$\left[(1+\hbar\mathbf{B}\cdot\mathbf{\Omega_p})\,\partial_t + (\tilde{\mathbf{v}} + \hbar\mathbf{E}\times\mathbf{\Omega_p} + \hbar(\tilde{\mathbf{v}}\cdot\mathbf{\Omega_p})\mathbf{B}) + \left(\tilde{\mathbf{E}} + \tilde{\mathbf{v}}\times\mathbf{B} + \hbar(\tilde{\mathbf{E}}\cdot\mathbf{B})\mathbf{\Omega_p}\right)\cdot\frac{\partial}{\partial\mathbf{p}}\right]f = 0$$

Berry curvature : $\mathbf{\Omega_p} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$ $\quad \tilde{\mathbf{v}} = \partial\epsilon_{\mathbf{p}}/\partial\mathbf{p}$ $\quad \tilde{\mathbf{E}} = \mathbf{E} - \partial\epsilon_{\mathbf{p}}/\partial\mathbf{x}$

(L-handed fermions : flip the signs for $\mathcal{O}(\hbar)$ corrections)

❖ Lorentz covariance : modified L.T. and side jumps (spin-orbit int.)

(responsible for the magnetization current & part of CVE)

J.-Y. Chen, et.al. 14 J.-Y. Chen, D. T. Son, and M. Stephanov, 15

❖ QFT derivation (WF) : limited conditions (steady state or high density)

J.-W. Chen, S. Pu, Q. Wang, X.-N. Wang, 12, D. T. Son & N. Yamamoto, 12

❖ A more general version : (non-)equilibrium + collisions Hidaka, Pu, DY, 16

Wigner functions (WF)

- less (greater) propagators :

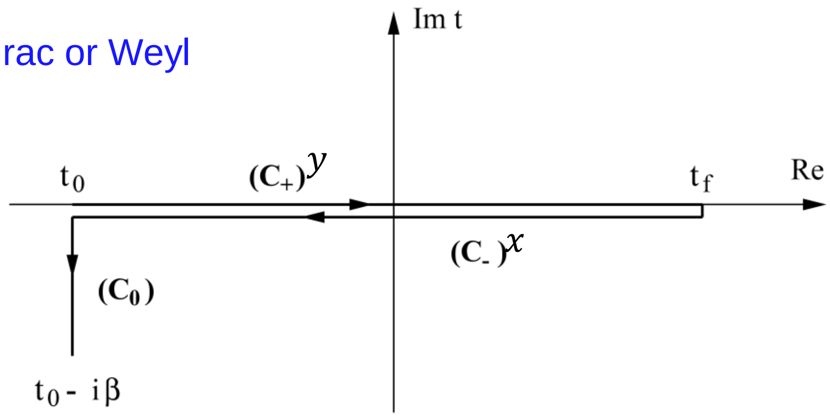
Dirac or Weyl

$$S^{>}(x,y) = \langle \psi(x) \mathcal{P}\mathcal{U}^{\dagger}(A_\mu, x, y) \psi^{\dagger}(y) \rangle$$

$$S^{<}(x,y) = \langle \psi^{\dagger}(y) \boxed{\mathcal{P}\mathcal{U}(A_\mu, x, y)} \psi(x) \rangle$$

gauge link

(see Wang's talk)



$X = \frac{x+y}{2}$, $Y = x - y$

review : J. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528

Wigner functions : $\grave{S}^{<(>)}(q, X) = \int d^4Y e^{\frac{iq\cdot Y}{\hbar}} S^{<(>)}\left(X + \frac{Y}{2}, X - \frac{Y}{2}\right)$

- Wigner functions are always covariant : $J^\mu = \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\left(\sigma^\mu \grave{S}^{<}\right)$

- Kadanoff-Baym (KB) equations up to $\mathcal{O}(\hbar)$: ($q \gg \partial$: weak fields)

$$\sigma^\mu \left(q_\mu + \frac{i\hbar}{2}\Delta_\mu\right) \grave{S}^{<} = \frac{i\hbar}{2}\left(\Sigma^{<}\grave{S}^{>} - \Sigma^{>}\grave{S}^{<}\right), \qquad \Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$$

systematically include collisions

$$\left(q_\mu - \frac{i\hbar}{2}\Delta_\mu\right) \grave{S}^{<}\sigma^\mu = -\frac{i\hbar}{2}\left(\grave{S}^{>}\Sigma^{<} - \grave{S}^{<}\Sigma^{>}\right).$$

(R-handed fermions)

Covariant CKT with collisions from QFT

- Wigner functions : $\grave{S}^{<}(q,X) = \bar{\sigma}^{\mu}\grave{S}^{<}_{\mu}(q,X)$
- Perturbative sol :

CVE or non-equilibrium effects CME in equilibrium

$$\grave{S}^{<\mu}(q,X) = 2\pi\bar{\epsilon}(q\cdot n)\left(q^{\mu}\delta(q^2)f^{(n)}_q + \boxed{\hbar\delta(q^2)S^{\mu\nu}_{(n)}\mathcal{D}_{\nu}f^{(n)}_q} + \hbar\epsilon^{\mu\nu\alpha\beta}q_{\nu}F_{\alpha\beta}\frac{\partial\delta(q^2)}{2\partial q^2}f^{(n)}_q\right),$$

$\mathcal{D}_{\beta}f^{(n)}_q = \Delta_{\beta}f^{(n)}_q - \mathcal{C}_{\beta}, \quad C_{\beta}[f] = \Sigma^{<}_{\beta}\bar{f} - \Sigma^{>}_{\beta}f$

side-jump term : magnetization current

Hidaka, Pu, DY, 16

spin tensor : $S^{\mu\nu}_{(n)} = \dfrac{\epsilon^{\mu\nu\alpha\beta}}{2(q\cdot n)}q_{\alpha}n_{\beta}$ $\quad (n_{\mu}\sigma^{\mu} = I)$: choice of the spin basis

(Son & Yamamoto, 12 : $n^{\mu} = (1,\mathbf{0})$ & onshell)

- A general form of CKT (for $n^{\mu} = n^{\mu}(X)$) : $\Delta_{\mu}\grave{S}^{<\mu} = \Sigma^{<}_{\mu}\grave{S}^{>\mu} - \Sigma^{>}_{\mu}\grave{S}^{<\mu}$

$$\Rightarrow \quad \delta\left(q^2 - \hbar\frac{B\cdot q}{q\cdot n}\right)\left[q\cdot\tilde{\mathcal{D}} + \frac{\hbar S^{\mu\nu}_{(n)}E_{\mu}}{q\cdot n}\mathcal{D}_{\nu} + \hbar S^{\mu\nu}_{(n)}(\partial_{\mu}F_{\rho\nu})\partial^{\rho}_q + \hbar(\partial_{\mu}S^{\mu\nu}_{(n)})\mathcal{D}_{\nu}\right]f^{(n)}_q = 0$$

$\tilde{\mathcal{D}}_{\mu}f^{(n)}_q = \Delta_{\mu}f^{(n)}_q - \tilde{\mathcal{C}}_{\mu} \qquad \tilde{\mathcal{C}}^{\mu} = \mathcal{C}^{\mu} + \hbar\dfrac{\epsilon^{\mu\nu\alpha\beta}n_{\nu}}{2q\cdot n}\left(\bar{f}^{(n)}_q\Delta^{>}_{\alpha}\Sigma^{<}_{\beta} - f^{(n)}_q\Delta^{<}_{\alpha}\Sigma^{>}_{\beta}\right)$

Hidaka, Pu, DY, 17

(see also J. Liao et.al. 18)

- Energy-momentum tensor and current :

$$T^{\mu\nu} = \int\frac{d^4q}{(2\pi)^4}\left[q^{\mu}\grave{S}^{<\nu} + q^{\nu}\grave{S}^{<\mu}\right], \quad J^{\mu} = 2\int\frac{d^4q}{(2\pi)^4}\grave{S}^{<\mu}.$$

Applications : Chiral fluids

- Anomalous hydrodynamics (R-handed) : $\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho, \quad \partial_\mu J^\mu = \frac{\hbar}{4\pi^2}(\mathbf{E}\cdot\mathbf{B})$
- Constitutive equations :

$$T^{\mu\nu} = \underbrace{u^\mu u^\nu \epsilon - p\Theta^{\mu\nu}}_{\mathcal{O}(1)} + \underbrace{\Pi^{\mu\nu}_{\rm non}}_{\mathcal{O}(\hbar)} + \underbrace{\Pi^{\mu\nu}_{\rm dis}}_{\mathcal{O}(1)+\mathcal{O}(\hbar)}, \quad J^\mu = \underbrace{N_0 u^\mu}_{\mathcal{O}(1)} + \underbrace{\nu^\mu_{\rm non}}_{\mathcal{O}(\hbar)} + \underbrace{\nu^\mu_{\rm dis}}_{\mathcal{O}(1)+\boxed{\mathcal{O}(\hbar)}}, \quad \Theta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu$$

Son & Surowka, 09

equilibrium : CME/CVE

our focus

- In local equilibrium : (in the co-moving frame $n^\mu = u^\mu$)

$$f_q^{\mathrm{eq}(u)} = \left(\exp\left[\beta(q\cdot u - \mu) + \boxed{\frac{\hbar q\cdot\omega}{2q\cdot u}}\right] + 1\right)^{-1}, \quad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu (\partial_\alpha u_\beta).$$

spin-vorticity coupling

J.-Y. Chen, et.al. 15

Hidaka, Pu, DY, 17

- Equilibrium anomalous transport :

$$\nu^\mu_{\rm non} = \hbar\sigma_B B^\mu + \hbar\sigma_\omega \omega^\mu \qquad \Pi^{\mu\nu}_{\rm non} = \hbar\xi_\omega(\omega^\mu u^\nu + \omega^\nu u^\mu) + \hbar\xi_B(B^\mu u^\nu + B^\nu u^\mu)$$

$$\sigma_\omega = \frac{T^2}{12}\left(1 + \frac{3\bar{\mu}^2}{\pi^2}\right), \quad \sigma_B = \frac{\mu}{4\pi^2}, \quad \xi_\omega = \frac{T^3}{6}\left(\bar{\mu} + \frac{\bar{\mu}^3}{\pi^2}\right), \quad \xi_B = \frac{T^2}{24}\left(1 + \frac{3\bar{\mu}^2}{\pi^2}\right).$$

CVE CME Hidaka, Pu, DY, 17

(agree with different approaches, e.g. Son & Surowka, 09. K. Landsteiner, et.al. Lect. Notes, 13.)

More on anomalous transport in equilibrium

- Entropy-density current in equilibrium : DY, 18

Boltzmann H function : $\mathcal{H}(f) = -f\ln f - (1-f)\ln(1-f)$

from WF :

$$s^{\mu}_{\rm Rleq} = 2\int \frac{d^4q}{(2\pi)^3}\bar{\epsilon}(q\cdot u)\left(\delta(q^2)\left(q^{\mu}+\hbar S^{\mu\nu}_{(u)}\Delta_{\nu}\right)+\hbar\epsilon^{\mu\nu\alpha\beta}q_{\nu}F_{\alpha\beta}\frac{\partial\delta(q^2)}{2\partial q^2}\right)\mathcal{H}(f^{{\rm leq}(u)}_q)$$

$$= \frac{1}{T}\left(u^{\mu}p_R + T^{\mu\nu}_{\rm Rleq}u_{\nu} - \mu_R J^{\mu}_{\rm Rleq} + \hbar D_{BR}B^{\mu} + \hbar D_{\omega R}\omega^{\mu}\right),$$

$$\boxed{D_{BR} = \frac{1}{8\pi^2}\left(\mu_R^2+\frac{\pi^2T^2}{3}\right) = \frac{\xi_{BR}}{T},\quad D_{\omega R} = \frac{1}{12}\left(T^2\mu_R+\frac{\mu_R^3}{\pi^2}\right) = \frac{\xi_{\omega R}}{2T}.} \Rightarrow \partial_{\mu}s^{\mu}_{\rm Rleq} = 0$$

no entropy production

(see also e.g. Son & Surowka, 09)

- Side-jump induced spin-orbit int. : $\boxed{T^{\mu\nu}_A = \frac{\hbar}{2}N_A\left(\omega^{\mu}u^{\nu}-\omega^{\nu}u^{\mu}\right)}$

(Antisymmetric component of canonical EM tensor in global equilibrium)

$$\partial_{\lambda}M^{\lambda\mu\nu}_C = 0. \Rightarrow \underbrace{\boxed{-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_{\lambda}J_{5\rho}}}_{\text{spin}} + \underbrace{\boxed{2T^{\mu\nu}_A}}_{\text{orbit}} = 0$$

(AM conservation)

Non-equilibrium charged currents

- 2^{nd}-order corrections on currents :

$J^{\mu}_{Q\perp}$	$\mathcal{O}(\partial)$	$\mathcal{O}(\partial^2)$ (RT approx.)
$\mathcal{O}(\hbar)$	$\sigma_B B^{\mu} + \sigma_{\omega}\omega^{\mu}$	$\tau_R\Big[\epsilon^{\mu\nu\alpha\beta}u_{\nu}\big(\hat{\gamma}_E\partial_{\alpha}E_{\beta} + \hat{\gamma}_{\mu}E_{\alpha}\partial_{\beta}\mu$ $+\hat{\gamma}_T E_{\alpha}\partial_{\beta}T + \hat{\gamma}_{T\mu}(\partial_{\alpha}T)(\partial_{\beta}\mu)\big)$ $+\delta\hat{\sigma}_{BL}\theta B^{\mu} + \delta\hat{\sigma}_{BH}\pi^{\mu\nu}B_{\nu}$ $+\delta\hat{\sigma}_{\omega L}\theta\omega^{\mu} + \delta\hat{\sigma}_{\omega H}\pi^{\mu\nu}\omega_{\nu}\Big]$

$\theta \equiv \partial\cdot u$

$\pi^{\mu\nu} \equiv P^{\mu}_{\rho}P^{\nu}_{\sigma}(\partial^{\rho}u^{\sigma}+\partial^{\sigma}u^{\rho}-2\eta^{\rho\sigma}\theta/3)/2$

Hidaka, Pu, DY, 17
Hidaka, DY, 18

Viscous corrections for CME/CVE

see also,
Gorbar, Shovkovy, et.al.,16
Chen, Ishii, Pu, Yamamoto,16

- 2^{nd}-order quantum transport coefficients : $\mathcal{P}, \mathcal{T} - odd$

(dissipative)

CME Hall current

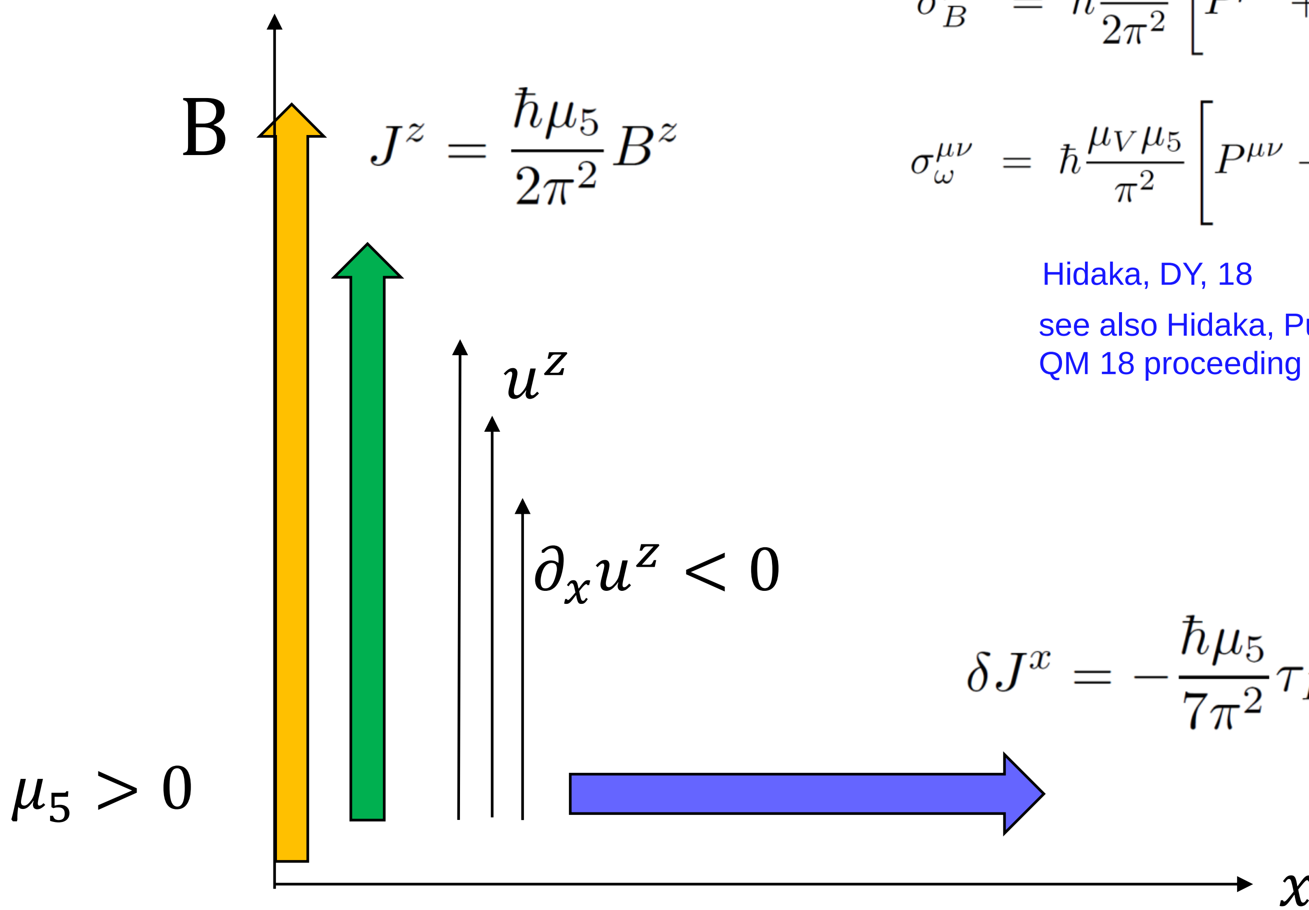
- Viscous corrections on CME/CVE ($\bar{\mu} \ll 1$) : $J^{\mu}_{Q\perp} = \sigma^{\mu\nu}_{B} B_{\nu} + \sigma^{\mu\nu}_{\omega}\omega_{\nu}$,

$$\sigma^{\mu\nu}_{B} = \hbar\frac{\mu_5}{2\pi^2}\left[P^{\mu\nu} + \tau_R\left(\frac{10P^{\mu\nu}}{9}\theta - \frac{2\pi^{\mu\nu}}{7}\right)\right],$$

$$\sigma^{\mu\nu}_{\omega} = \hbar\frac{\mu_V\mu_5}{\pi^2}\left[P^{\mu\nu} - \tau_R\left(\frac{37P^{\mu\nu}}{264}\theta + \frac{10\pi^{\mu\nu}}{63}\right)\right].$$

Hidaka, DY, 18

see also Hidaka, Pu ,DY
QM 18 proceeding



$$\delta J^x = -\frac{\hbar\mu_5}{7\pi^2}\tau_R B^z \partial_x u^z$$

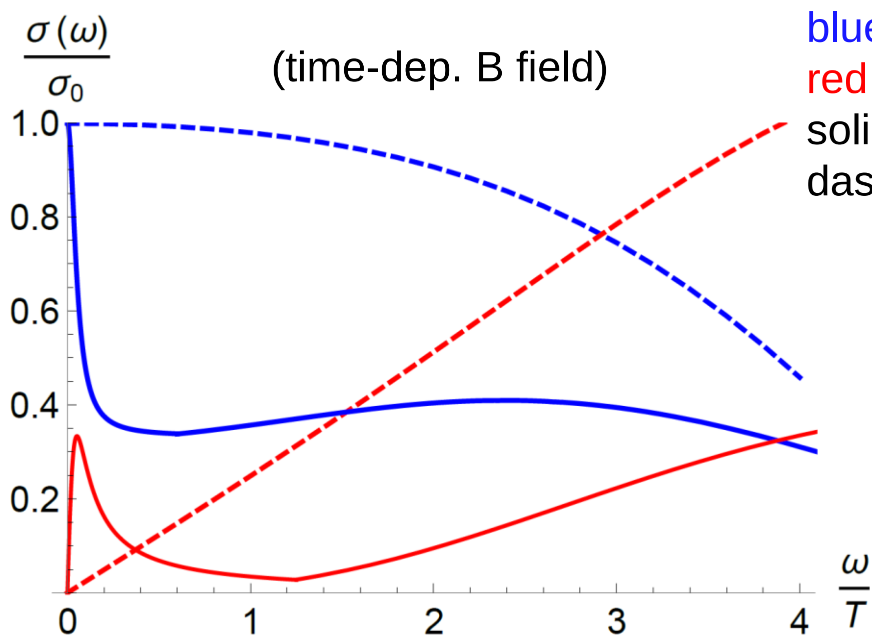
Out-of-equilibrium transport

- The origin of viscous corrections : time-dep. B

$$\partial_\nu \tilde{F}^{\mu\nu} = 0$$

$$\Rightarrow \quad \boxed{u\cdot\partial B^\rho + B^\rho \partial\cdot u - B\cdot\partial u^\rho + u^\rho B^\mu u\cdot\partial u_\mu} + \epsilon^{\rho\mu\alpha\beta}\left(u_\beta\partial_\mu E_\alpha + u_\mu E_\alpha u\cdot\partial u_\beta\right) = 0$$

- AC conductivity of CME :



blue : real part
red : imaginary part
solid : kinetic theory
dashed : AdS/CFT

D. Kharzeev & H. Warringa, 09
D. Satow & H. U. Yee, 14.
D. Kharzeev, et.al., 17,

$$\sigma(\omega) = \sigma_0\left(1 - \frac{2}{3}\frac{\omega}{\omega + i\tau_R^{-1}}\right)$$

int. dep. : dissipative

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Mass corrections

- No massless fermions in our universe
- Mass corrections on chiral anomaly : $\partial_\mu J_5^\mu = \frac{\hbar \mathbf{E}\cdot\mathbf{B}}{2\pi^2} + \boxed{2im\bar{\psi}\gamma_5\psi}$
- Relevant studies for the mass corrections on CSE/CVE (axial-charge currents) :

E. Gobar, V. Miransky, I. Shovkovy, X. Wang, 13 : Radiative Corrections to CSE

R.-h. Fang, L.-g. Pang, Q. Wang, and X.-n. Wang, 16 : Polarization of massive fermions in a vortical fluid

A. Flachi, K. Fukushima, 17 : CVE with finite rotation, temperature, and curvature

S. Lin, L. Yang, 18 : On Mass correction to CVE and CSE

- No chiral imbalance is involved
- Mass corrections to CME? (Hidaka, D.-L. preliminary)

Introduce a mass

- Mass corrections to WFs? (Hidaka, D.-L. preliminary)
- Dirac eqs : $i\sigma^\mu D_\mu \psi_R - m\psi_L = 0, \quad i\bar{\sigma}^\mu D_\mu \psi_L - m\psi_R = 0.$

$$\Rightarrow \quad \grave{S}^< = \langle \psi\bar{\psi} \rangle = \begin{pmatrix} \boxed{\grave{S}^<_{LR}} & \grave{S}^<_{LL} \\ \grave{S}^<_{RR} & \boxed{\grave{S}^<_{RL}} \end{pmatrix}$$

Chirality mixing

- Collisionless KB eqs :

$$\sigma^\mu \left(i\frac{\hbar}{2}\Delta_\mu + q_\mu \right) \grave{S}^<_{RR}(q,X) = m\grave{S}^<_{LR}(q,X), \quad \bar{\sigma}^\mu \left(i\frac{\hbar}{2}\Delta_\mu + q_\mu \right) \grave{S}^<_{LL}(q,X) = m\grave{S}^<_{RL}(q,X),$$

$$\sigma^\mu \left(i\frac{\hbar}{2}\Delta_\mu + q_\mu \right) \grave{S}^<_{RL}(q,X) = m\grave{S}^<_{LL}(q,X), \quad \bar{\sigma}^\mu \left(i\frac{\hbar}{2}\Delta_\mu + q_\mu \right) \grave{S}^<_{LR}(q,X) = m\grave{S}^<_{RR}(q,X).$$

- Decomposition : $\grave{S}^< = \langle \psi\bar{\psi} \rangle = \mathcal{S} + \mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^\mu\gamma^5 + \mathcal{S}_{\mu\nu}\Sigma^{\mu\nu}$

$$\grave{S}^<_{RR} = \bar{\sigma}\cdot(\mathcal{V} - \mathcal{A}), \quad \grave{S}^<_{LL} = \sigma\cdot(\mathcal{V} + \mathcal{A}),$$

$$\grave{S}^<_{RL} = \mathcal{S} + \mathcal{P} + \frac{i}{2}\bar{\sigma}^{[\mu}\sigma^{\nu]}\mathcal{S}_{\mu\nu} \qquad \grave{S}^<_{LR} = \mathcal{S} - \mathcal{P} + \frac{i}{2}\bar{\sigma}^{[\mu}\sigma^{\nu]}\mathcal{S}_{\mu\nu}$$

WFs with mass corrections

- The L. O. ($\mathcal{O}(1)$) WF :

$$\grave{S}^{<} = 2\pi\bar{\epsilon}(q\cdot n)\delta(q^2-m^2)\left[\not{a}f_V + \boxed{\gamma^5\not{a}\frac{f_A}{2}} + mf_V - \Sigma^{\mu\nu}\epsilon_{\mu\nu\rho\sigma}q_\rho n_\sigma\frac{mf_A}{4q\cdot n}\right],$$

extra mass corrections to the axial charge

$$a_\mu = q_\mu - n_\mu m^2/(q\cdot n)$$

- Up to $\mathcal{O}(\hbar)$ with a global n^μ & $E^\mu = 0$:

$$\begin{aligned}\grave{S}^{<\mu}_{R/L} = 2\pi\Bigg[\delta(q^2-m^2)\Bigg(\left(q^\mu \mp \frac{\hbar\delta_{\mu i}(q\cdot n)\epsilon^{\mu\nu\alpha\beta}n_\nu}{2|q_\perp|^2}q_\alpha\Delta_\beta\right)f_V \pm \left(q^\mu - \frac{n^\mu m^2}{q\cdot n} \mp \frac{\hbar\epsilon^{\mu\nu\alpha\beta}n_\nu}{2q\cdot n}q_\alpha\Delta_\beta\right)\frac{f_A}{2}\Bigg) \\ \mp n^\mu\hbar q\cdot B\frac{\partial\delta(q^2-m^2)}{\partial q^2}\left(f_V \pm \frac{f_A}{2}\right) \boxed{\pm\hbar(q\cdot n)B^\mu\frac{\partial\delta(q^2-m^2)}{\partial q^2}\left(f_V \pm \frac{f_A}{2}\left(1-\frac{m^2}{(q\cdot n)^2}\right)\right)}\Bigg].\end{aligned}$$

CSE/CME in equilibrium

Plus complicated mixing terms

CME with a mass gap

- "Assuming" global equilibrium (with $\mu_A \neq 0$): $f_{\mathrm{eq}R/L}(q\cdot u) = \dfrac{1}{e^{\beta q\cdot u - \bar{\mu}_{R/L}} + 1}$

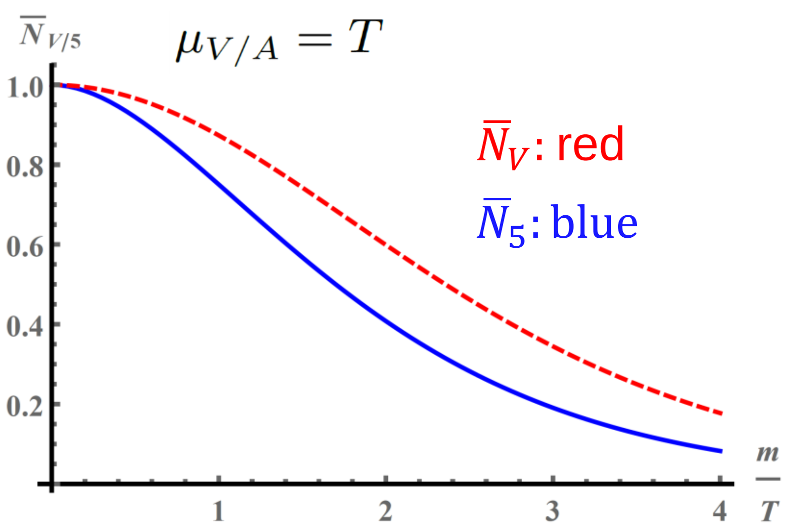
- CSE & CME :

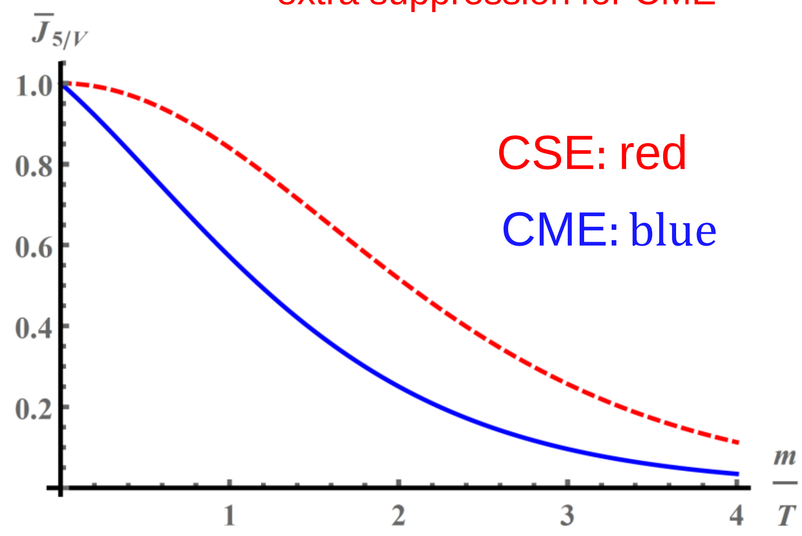
$$J^{\mu}_{B5} = J^{\mu}_{BR} - J^{\mu}_{BL} = \frac{\hbar B^{\mu}}{4\pi^2}\int_0^{\infty} d|\mathbf{q}|\left(f^{(-)}_{\mathrm{eq}R} + f^{(-)}_{\mathrm{eq}L}\right)$$

CSE agrees with Lin & Yang, 18

$$J^{\mu}_{BV} = J^{\mu}_{BR} + J^{\mu}_{BL} = \frac{\hbar B^{\mu}}{4\pi^2}\int_0^{\infty} d|\mathbf{q}|\boxed{\left(1 - \frac{m^4}{E_q^4}\right)}\left(f^{(-)}_{\mathrm{eq}R} - f^{(-)}_{\mathrm{eq}L}\right)$$

extra suppression for CME





Conclusions & outlook

- We have presented a covariant CKT with background fields and collisions for Weyl fermions from the WF formalism.
- Novel 2nd –order non-equilibrium anomalous transport including the viscous corrections on CME/CVE is found.

Outlook :

- Phenomenological applications : HIC, Weyl semimetals, astro, etc.
  - Realistic collisions beyond RTA
  - Dynamical EM fields e.g. D. Rubalka, E. Gorbar, I. Shovkovy, 18

(see Shovkovy and Hatorris' talks)

- Theoretical issues :
  - mass corrections?
  - Perturbative sol. up to $\mathcal{O}(\hbar^2)$: new phenomena? e.g. Hattori & Yin, 16. Gao, Liang, Wang, Wang, 18.
  - side jumps in LL : from weak to strong B fields

Thank you!

Covariant currents and side-jumps

- KB equations : $\mathcal{D}\cdot\grave{S}^{<}=0,\quad q\cdot\grave{S}^{<}=0,$

$$\mathcal{D}_\beta f^{(n)}_q=\Delta_\beta f^{(n)}_q-\mathcal{C}_\beta,$$
$$\Delta_\mu=\partial_\mu+F_{\nu\mu}\partial/\partial q_\nu$$
$$C_\beta[f]=\Sigma^{<}_\beta\bar{f}-\Sigma^{>}_\beta f$$

give WF
$$\begin{cases}2\pi\delta(q^2)\left(q\cdot n\mathcal{D}_\mu-q_\mu n\cdot\mathcal{D}\right)f=-2\epsilon_{\alpha\mu\nu\beta}n^\alpha q^\nu\delta\grave{S}^{<\beta},\\ 2\pi\epsilon_{\alpha\mu\nu\beta}\delta(q^2)n^\alpha q^\nu\mathcal{D}^\beta f=2\left(q\cdot n\delta\grave{S}^{<}_\mu-q_\mu n\cdot\delta\grave{S}^{<}\right).\end{cases}$$

- WF with n^μ :

CVE or non-equilibrium effects

CME in equilibrium

key eq.
$$\boxed{\grave{S}^{<\mu}(q,X)=2\pi\bar{\epsilon}(q\cdot n)\left(q^\mu\delta(q^2)f^{(n)}_q+\boxed{\hbar\delta(q^2)S^{\mu\nu}_{(n)}\mathcal{D}_\nu f^{(n)}_q}+\hbar\epsilon^{\mu\nu\alpha\beta}q_\nu F_{\alpha\beta}\frac{\partial\delta(q^2)}{2\partial q^2}f^{(n)}_q\right)},$$

spin tensor : $S^{\mu\nu}_{(n)}=\dfrac{\epsilon^{\mu\nu\alpha\beta}}{2(q\cdot n)}q_\alpha n_\beta$

side-jump term : magnetization current

Hidaka, Pu, DY, 16

- The full WF has to be frame independent $\Longrightarrow$ $f^{(n)}_q$ is frame dependent

$$(\Lambda^{-1})_\mu{}^\nu\grave{S}'^{<}_\nu(q',X')-\grave{S}^{<}_\mu(q,X)=0$$

- The modified frame transformation : $f^{(n')}_q=f^{(n)}_q+\dfrac{\hbar\epsilon^{\nu\mu\alpha\beta}q_\alpha n'_\beta n_\mu}{2(q\cdot n)(q\cdot n')}\mathcal{D}_\nu f^{(n)}_q$

- Left-handed fermions : sign flipping for $\mathcal{O}(\hbar)$ corrections

Manifestation of Lorentz symmetry

- From $a_{\mathbf{p}} \rightarrow e^{-i\phi(p)}a_{\mathbf{p}}$, we may define a scalar distribution function :

scalar

non-scalar

$$\boxed{\check{N}(p',p)} \equiv e^{-i\left(\phi(p)-\phi(p')\right)}\boxed{N(p',p)} \quad \Rightarrow \quad \check{f}(q,X) \equiv \int \frac{d^3\bar{p}}{(2\pi)^3}\check{N}\left(q-\frac{\bar{p}}{2},q+\frac{\bar{p}}{2}\right)e^{-i\bar{p}\cdot X}$$

$$N(p',p) \equiv \langle a^{\dagger}_{\mathbf{p}'}a_{\mathbf{p}}\rangle$$

- The derivation of WF implicitly involves the contribution from anti-fermions.

Key eq. : $\mathrm{Im}\left[c^{\dagger}_{\pm}(q)\sigma^k\frac{\partial}{\partial q_\beta}c_{\pm}(q)\right] = \mp a^{\beta}_{\pm}v^k - \frac{1}{2|\mathbf{q}|}\epsilon^{kj\beta}v_j$ from $c_+(p)c^{\dagger}_+(p)+c_-(p)c^{\dagger}_-(p) = I$

$$\Rightarrow \grave{S}^{<}_{\mu}(q,X) = (2\pi)\theta(q^0)\delta(q^2)\boxed{\left(q_\mu\left(1-\hbar(\partial^{\nu}_q\phi-a^{\nu})\partial_\nu\right)+\hbar\delta_{\mu i}\epsilon_{ijk}\frac{q_j}{2|\mathbf{q}|}\partial_k\right)}\check{f}(q,X)\,,$$

covariant

scalar

- Compare to the previous expression :

$$\grave{S}^{<\mu} = 2\pi\theta(q^0)\delta(q^2)\left(q^\mu+\hbar\delta^{\mu i}\epsilon^{ijk}\frac{q_j}{2|\mathbf{q}|}\partial_k\right)f(q,X)$$

"the origin of side-jumps"

$$\Rightarrow f(q,X) = \check{f}\left(q_\mu, X^\mu\boxed{-\hbar\partial^{\mu}_q\phi(q)+\hbar a^\mu}\right)$$

non-scalar

- Choices of phase field corresponds to the gauge degrees of freedom for the Berry connection.
- The perturbative solution could be uniquely determined by Lorentz symmetry.

Angular momenta for relativistic fermions

- Considering just fermions : $$\mathcal{L} = \bar{\psi}\left(\frac{i\hbar}{2}\gamma^\mu \overleftrightarrow{D}_\mu - m\right)\psi$$
- Canonical EM tensor (from Noether's theorem + EOM) :

$$\bar{T}^{\mu\nu} = T^{\mu\nu} + T_A^{\mu\nu}, \quad T^{\mu\nu} = \frac{i\hbar}{4}\bar{\psi}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi, \quad T_A^{\mu\nu} = \frac{i\hbar}{4}\bar{\psi}\gamma^{[\mu}\overleftrightarrow{D}^{\nu]}\psi$$

- Canonical AM tensor :

review : E. Leader & C. Lorce, 13

$$M_C^{\lambda\mu\nu} = M_{\text{spin}}^{\lambda\mu\nu} + M_{\text{orbit}}^{\lambda\mu\nu},$$

$$M_{\text{spin}}^{\lambda\mu\nu} = \frac{\hbar}{2}\bar{\psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\psi = \boxed{-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\bar{\psi}\gamma_5\gamma_\rho\psi,}$$

related to axial-charge currents

$$M_{\text{orbit}}^{\lambda\mu\nu} = \frac{i\hbar}{2}\bar{\psi}\gamma^\lambda\left(x^\mu\overleftrightarrow{D}^\nu - x^\nu\overleftrightarrow{D}^\mu\right)\psi = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + x^\mu T_A^{\lambda\nu} - x^\nu T_A^{\lambda\mu}$$

- Conservation (in global equilibrium) : $\partial_\mu \bar{T}^{\mu\nu} = 0, \quad \partial_\lambda M_C^{\lambda\mu\nu} = 0.$
- In relation to the Belinfante one :

$$M_C^{\lambda\mu\nu} = M_B^{\lambda\mu\nu} + \partial_\beta V^{[\beta\lambda][\mu\nu]},$$

(using EOM)

$$M_B^{\lambda\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$$

Angular momenta in Wigner functions

- Phase-space distributions in terms of WFs : DY, 18

$$T^{\mu\nu}(q,X) = \Pi^{\{\nu}\grave{S}^{<\mu\}}_{V}(q,X), \quad T^{\mu\nu}_{A}(q,X) = \Pi^{[\nu}\grave{S}^{<\mu]}_{V}(q,X),$$

$$M^{\lambda\mu\nu}_{\text{spin}}(q,X) = -\hbar\epsilon^{\lambda\mu\nu\rho}\grave{S}^{<}_{5\rho}(q,X),$$

$$M^{\lambda\mu\nu}_{\text{orbit}}(q,X) = X^{\mu}\bar{T}^{\lambda\nu}(q,X) - X^{\nu}\bar{T}^{\lambda\mu}(q,X) + \boxed{\hbar\big(\partial^{\mu}_{q}\nabla^{\nu} - \partial^{\nu}_{q}\nabla^{\mu}\big)\grave{S}^{<\lambda}_{V}(q,X),}$$

quantum corrections on OAM

$$\nabla_{\mu} = \partial_{\mu} + F_{\nu\mu}\partial^{\nu}_{q} - \frac{\hbar^2}{24}(\partial_{\rho}\partial^{\rho}_{q})^2 F_{\nu\mu}\partial^{\nu}_{q} + \mathcal{O}(\hbar^4),$$

$$\Pi_{\mu} = q_{\mu} + \frac{\hbar^2}{12}\partial_{\rho}\partial^{\rho}_{q}F_{\nu\mu}\partial^{\nu}_{q} + \mathcal{O}(\hbar^4).$$

- Pauli-Lubanski (pseudo)vector : $W^{\mu}_{C/B}(q,X) \equiv -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\Pi_{\nu}(M_{C/B})_{\alpha\beta}(q,X)$
- AM-tensor density : $M^{\lambda\mu\nu}_{C}(X) = -\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}J_{5\rho}(X) + \Big(X^{\mu}\bar{T}^{\lambda\nu}(X) - X^{\nu}\bar{T}^{\lambda\mu}(X)\Big)$
- Conservation of AM : nontrivial interplay between the spin and quantum corrections from OAM
- How to understand the dynamics of spins in a relativistic rotating fluid? (try a chiral fluid with massless fermions first)

AM conservation in global equilibrium

- Global equilibrium (no collisions $\omega^\mu \neq 0,\ B^\mu \neq 0.$) : $\partial_\mu J^\mu_{V/5} = \partial_\mu T^{\mu\nu} = 0,$
- Conservation of canonical EM & AM tensors :

$$\partial_\mu \bar{T}^{\mu\nu} = 0, \quad \partial_\lambda M_C^{\lambda\mu\nu} = 0. \Longrightarrow \underbrace{\boxed{-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_\lambda J_{5\rho}}}_{\text{spin}} + \underbrace{\boxed{2T_A^{\mu\nu}}}_{\text{orbit}} = 0$$

Florkowski, et.al. 17
Thermodynamic approach

- Massive fermions : $T_A^{\mu\nu} = 0 \Longrightarrow$ spin alone is conserved
- Weyl fermions : $\boxed{T_A^{\mu\nu} = \frac{\hbar}{2}N_A\left(\omega^\mu u^\nu - \omega^\nu u^\mu\right)}$ from side-jumps

DY, 18

$$M_{\rm spin}^{\lambda\mu\nu}(X) = \frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\left(N_A u_\rho + \boxed{\hbar\sigma_{BA}B_\rho + \hbar\sigma_{\omega A}\omega_\rho}\right)$$

CSE & CVE

- $\mathcal{O}(\hbar)$: spin-orbit cancellation
- Higher orders : we need higher-order WFs.
- Near local equilibrium :

local torque even without EM fields

$$\partial_\lambda M_C^{\lambda\mu\nu} = X^{[\mu}F^{\nu]\rho}J_{V\rho} - \frac{u_\rho}{\tau_R}X^{[\mu}\delta T^{\rho\nu]} - \boxed{\frac{\hbar}{4}\partial_\lambda\left(X^{[\mu}\epsilon^{\nu]\lambda\alpha\beta}\frac{u_\alpha \delta J_{5\perp\beta}}{\tau_R}\right)}$$