Dislocation-Mediated Quantum Melting

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Keio University

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- JSPS Kakenhi Grant-in-Aid for Early-Career Scientists (grant no. 18K13502)

Dislocation-Mediated Quantum Melting

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Dislocation-Mediated Quantum Melting

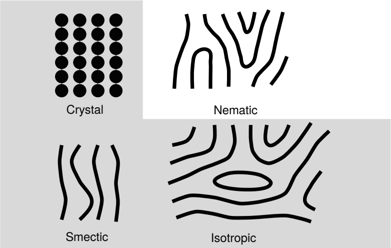

Figure 3 Schematic view of the local stripe order in the various phases discussed in the text. Here, we have assumed that the stripes maintain their integrity throughout, although in reality they must certainly become less and less well def ned as the system becomes increasingly quantum, until eventually they are not the correct variables for describing the important correlations in the system. Heavy lines represent liquid-like stripes, along which the electrons can f ow,

Electronic liquid-crystal phases of a doped Mott insulator

S. A. Kivelson*, E. Fradkin† & V. J. Emery‡

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† *Department of Physics, University of Illinois, Urbana, Illinois 61801-3080, USA*
‡ *Brookhaven National Laboratory, Upton, New York 11973-5000, USA*

The character of the ground state of an antiferromagnetic insulator is fundamentally altered following addition of even a small amount of charge[1]. The added charge is concentrated into domain walls across which a π phase shift in the spin correlations of the host material is induced. In two dimensions, these domain walls are 'stripes' which can be insulating[2,3] or conducting[4–6]—that is, metallic 'rivers' with their own low-energy degrees of freedom. However, in arrays of one-dimensional metals, which occur in materials such as organic conductors[7], interactions between stripes typically drive a transition to an insulating ordered charge-density-wave (CDW) state at low temperatures. Here it is shown that such a transition is eliminated if the zero-point energy of transverse stripe fluctuations is sufficiently large compared to the CDW coupling between stripes. As a consequence, there should exist electronic quantum liquid-crystal phases, which constitute new states of matter, and which can be either high-temperature superconductors or two-dimensional anisotropic 'metallic' non-Fermi liquids. Neutron scattering and other experiments in the copper oxide superconductor $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$ already provide evidence for the existence of these phases in at least one class of materials.

Classical liquid crystals[8] are phases that are intermediate between a liquid and a solid, and spontaneously break the rotation and/or translation symmetries of free space. The proposed electronic liquid crystals are quantum analogues of these phases in which the ground state is intermediate between a liquid, where quantum fluctuations

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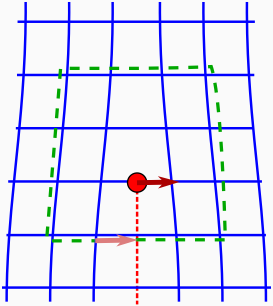
Topological defect associated with translational order

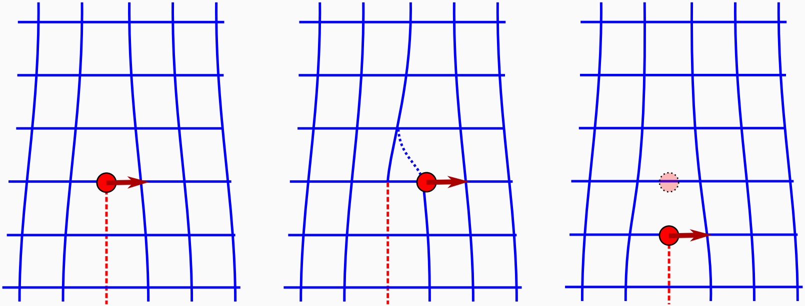


Topological charge: Burgers vector B^a

Outline

- Classical dislocations
  - restricted motion
  - interdependence with disclinations
- Dislocation condensation = quantum melting
  - duality
  - deconfinement of disclinations
- Recent developments
  - critical properties of dislocation condensation
  - relation to fractons
  - superfluids without $U(1)$ breaking

Dislocation motion



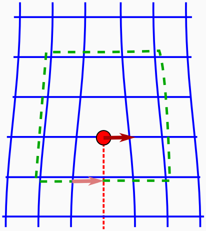
(a) initial dislocation (b) glide motion (c) climb motion

climb motion involves the addition/removal of (interstitial) particles and is suppressed $\leftrightarrow$ particle number conservation

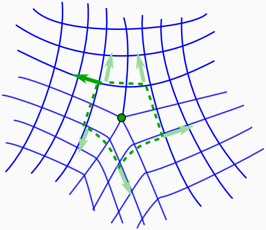
Glide constraint:

"dislocations can only move in the direction of their Burgers vector"

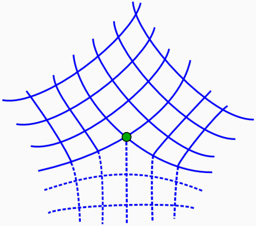
Dislocations and disclinations



(d) dislocation



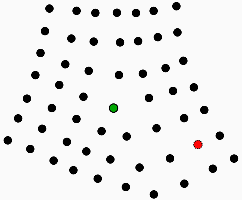
(e) disclination



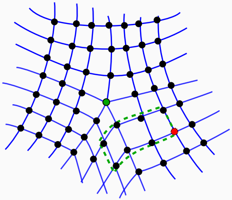
(f) Volterra construction

- dislocation : Burgers vector B^a, torsion
- disclination : Frank scalar Ω, curvature

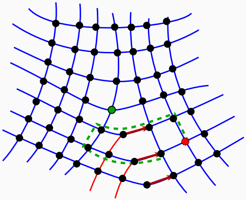
Interdependence of dislocations and disclinations



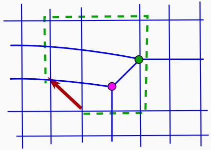
(g) atoms



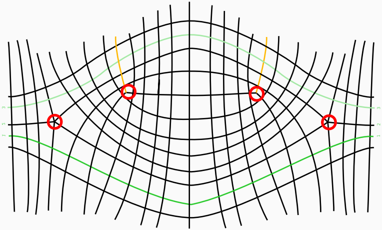
(h) disclination



(i) stack of dislocations



(j) disclination pair



(k) two disclination pairs

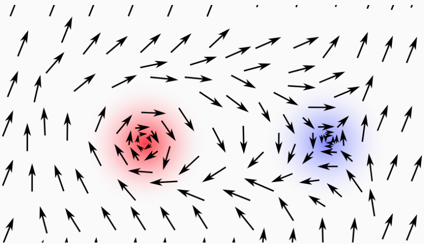
Defect-mediated melting

Nobel Prize in Physics 2016 citation :

"for theoretical discoveries of topological phase transitions ..."

Berezinshkii–Kosterlitz–Thouless melting

Berezinskii 1970-71; Kosterlitz Thouless 1972-73



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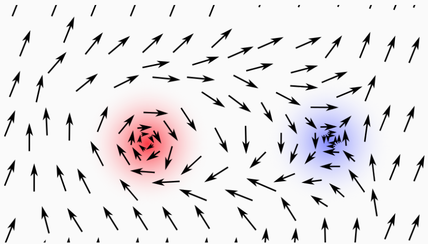
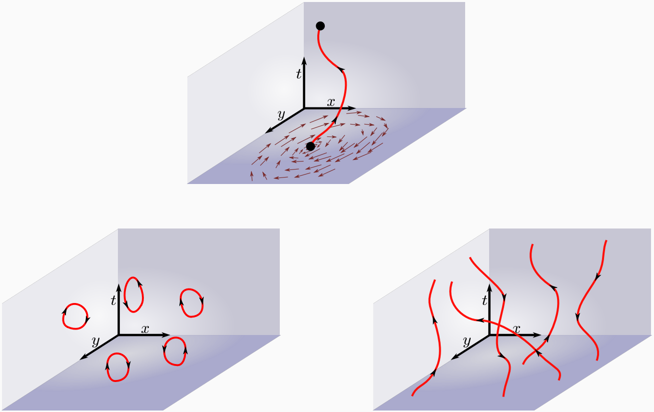


- in 2D, no true long-range order
- higher dimensions: order–disorder defect-unbinding phase transition
- 2+1D superfluid–Bose-Mott insulator quantum phase transition

- D-dim. quantum field theory $\leftrightarrow$ $D+1$-dim. statistical physics, e.g. 2D superfluid–insulator QPT is in the 3D XY universality class.
- Time axis is the additional dimension. Statistical physics of *worldlines*.



- Unbinding of dislocations = loss of translational order

Berezinskii 1970-71; Kosterlitz Thouless 1972-73

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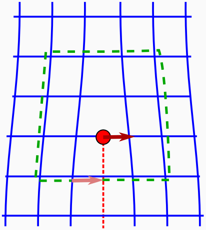
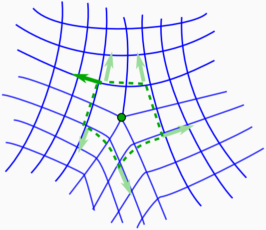
- Two types of topological defects



(a) dislocation – translational



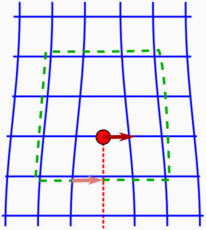
(b) disclination – rotational

Two-dimensional classical melting

- Unbinding of dislocations = loss of translational order

Berezinskii 1970-71; Kosterlitz Thouless 1972-73

- Two types of topological defects



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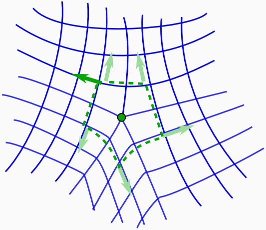
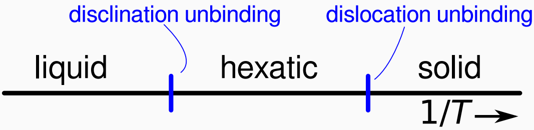
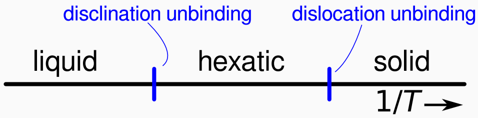


(b) disclination – rotational



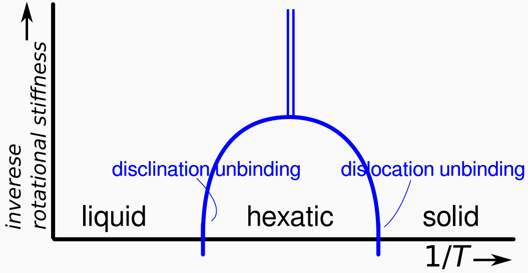
Nelson Halperin 1978-79; Young 1979

- Why is the ordinary solid-to-liquid transition first order?



- Why is the ordinary solid-to-liquid transition first order?
- Simultaneous unbinding

Kleinert 1983



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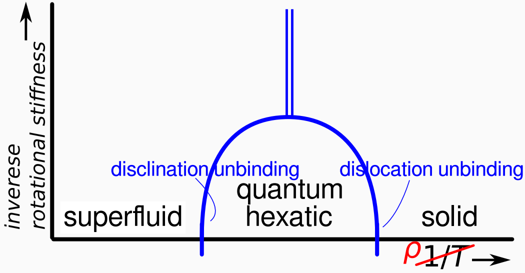
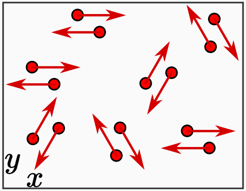
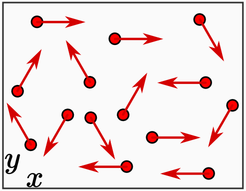


- Towards quantum melting, zero-temperature phase transition

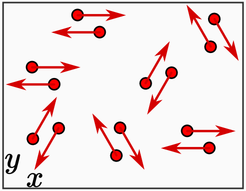
Two-dimensional quantum melting



(a) 2D bound pairs



(b) 2D unbound



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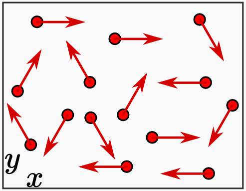
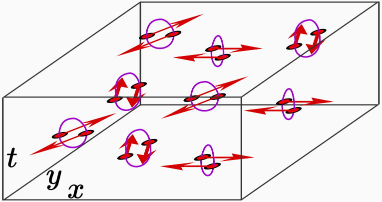
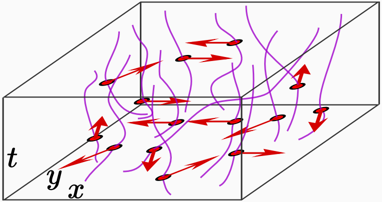


(b) 2D unbound



(c) 2+1D bound loops



(d) 2D unbound worldlines

2+1D dislocation-mediated quantum melting

- 'Statistical physics'/quantum partition sum of dislocation worldlines
- Role of inverse temperature is played by temporal correlations
- 2D quantum corresponds to 3D classical
- Proliferation of dislocation lines 3D classical

Kleinert 1980s

2+1D dislocation-mediated quantum melting

- 'Statistical physics'/quantum partition sum of dislocation worldlines
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Kleinert 1980s

- Time direction manifestly different from space directions
- Condensation of dislocations = proliferation of worldlines 2+1D quantum

Zaanen Nussinov Mukhin 2004; Cvetkovic Zaanen 2006; AJB *et al.* 2017

Essence: the dislocation condensate is decribed by a collective complex field

$$\Psi^a(x), \qquad a = x, y, \qquad |\Psi^a|^2 \sim \text{ density of 'worldline tangle'} \tag{1}$$

Ginzburg–Landau-type action

$$\mathcal{L}_{\mathrm{GL}} = \sum_{a=x,y} \left(\frac{1}{2}\alpha_a |\Psi^a|^2 + \frac{1}{4}\beta_a |\Psi^a|^4 \right) + \frac{1}{2}\gamma |\Psi^x|^2 |\Psi^y|^2 \tag{2}$$

- Duality mapping, analogous to vortex–boson / Abelian-Higgs duality

Dual gauge field theory of defect-mediated melting

- Duality mapping, analogous to vortex–boson / Abelian-Higgs duality
- Phonons are gauge bosons or stress photons
- Dislocations are shear stress charges
- A solid is a stress vacuum or Coulomb gas of stress charges

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- Duality mapping, analogous to vortex–boson / Abelian-Higgs duality
- Phonons are gauge bosons or stress photons
- Dislocations are shear stress charges
- A solid is a stress vacuum or Coulomb gas of stress charges

- An hexatic is a stress superconductor
- Dual Meissner effect: shear stress is expelled from the liquid crystal

Dual stress effective action

Classical stress energy $$E_{\text{solid}} = \tfrac{1}{2}\sigma^a_m \underbrace{C^{-1}_{mnab}}_{\text{elastic moduli}} \sigma^b_n$$

Quantum stress Lagrangian $$\mathcal{L}_{\text{solid}} = \tfrac{1}{2\rho}(\sigma^a_\tau)^2 + \tfrac{1}{2}\sigma^a_m C^{-1}_{mnab}\sigma^b_n$$

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Stress is conserved $$\partial_\tau \sigma^a_\tau + \partial_m \sigma^a_m = \partial_\mu \sigma^a_\mu = 0$$

Dual stress gauge field $$\sigma^a_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu b^a_\lambda, \qquad a = x, y$$

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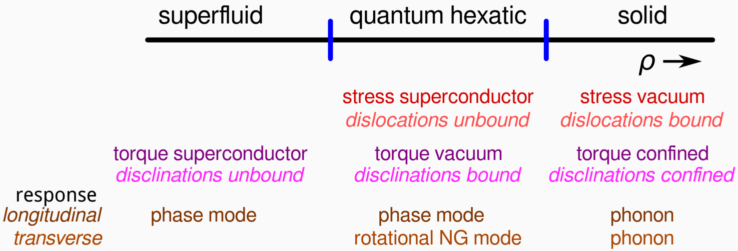
$$\mathcal{L}_{\text{coupling}} = \frac{1}{2}\sum_{a=x,y}|(\partial_\mu - \mathrm{i}b^a_\mu - \mathrm{i}\lambda\epsilon_{\tau\mu a})\Psi^a|^2.$$

Main results

1. Phonons are gauge bosons
2. The disordered solid is a stress superconductor
3. The disordered solid is a real superfluid (longitudinal response)
4. Rotational Goldstone mode deconfines in qu. hexatic (transverse response)
5. Transverse phonon becomes gapped shear mode in quantum hexatic
6. The gapped shear mode is detectable by finite-momentum spectroscopy

AJB *et al.* Phys. Rep. 683, 1 (2017)



Disclination deconfinement

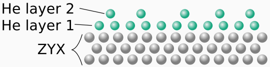
- Displacement field $u^a(x)$
- Rotation field $\omega^{ab} = \partial_a u^b(x) - \partial_b u^a(x)$

		solid	hexatic
Lagrangian	stress	$u^a(\partial_t^2 + \nabla^2)u^a$	$u^a(\partial_t^2 + \nabla^2 + \lvert\Psi\rvert^2)u^a$
	rotation	$\omega^{ab}\nabla^2(\partial_t^2 + \nabla^2)\omega^{ab}$	$\ldots + \omega^{ab}\lvert\Psi\rvert^2(\partial_t^2 + \nabla^2)\omega^{ab}$
propagator	stress	$\frac{1}{\omega^2+q^2}$	$\frac{1}{\omega^2+q^2+\lvert\Psi\rvert^2}$
	rotation	$\frac{1}{q^2(\omega^2+q^2)}$	$\ldots + \frac{\lvert\Psi\rvert^2}{\omega^2+q^2}$
static limit	stress	$\frac{1}{q^2}$	$\frac{1}{q^2+\lvert\Psi\rvert^2}$
	rotation	$\frac{1}{q^4}$	$\ldots + \frac{\lvert\Psi\rvert^2}{q^2}$

For the same reason, rotational Nambu–Goldstone modes are absent in solid, but present in quantum hexatic.

Helium monolayers on *ZYX* exfoliated graphite

S. Nakamura *et al.* PRB 94, 180501(R) (2016)



Helium monolayer experiments

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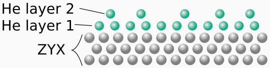

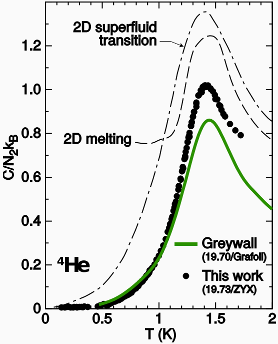

- anomaly in specific heat : BKT-like defect-unbinding transition

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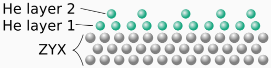

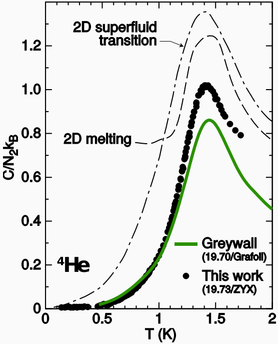

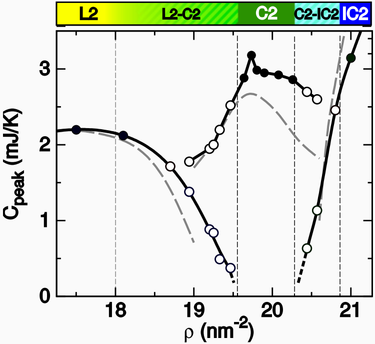

- anomaly in specific heat : BKT-like defect-unbinding transition
- three separate peaks

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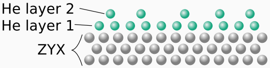

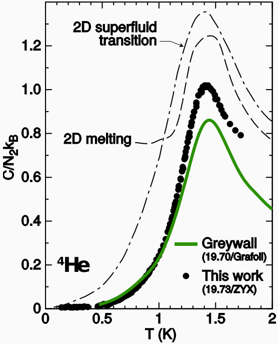

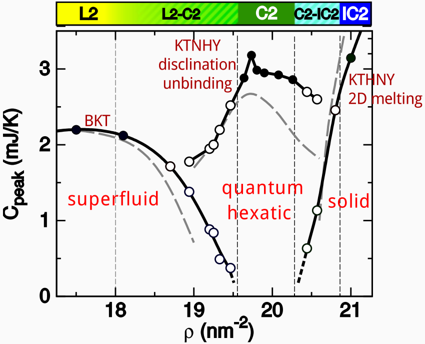

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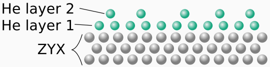

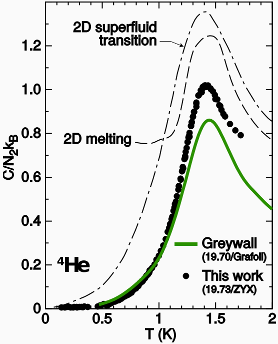

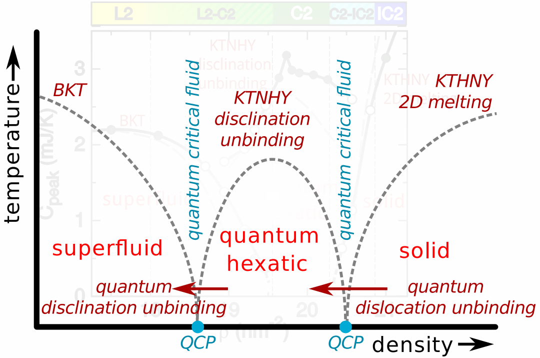

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- three separate peaks

Effective field theory (Ginzburg–Landau)

$$\mathcal{L}_{\mathrm{solid}} = \frac{1}{2}(\epsilon_{\mu\kappa\lambda}\partial_\kappa b^a_\lambda) C^{-1}_{\mu\nu ab}(\epsilon_{\nu\rho\sigma}\partial_\rho b^a_\sigma),$$

$$\mathcal{L}_{\mathrm{GL}} = \sum_{a=x,y}\left(\frac{1}{2}\alpha_a|\Psi^a|^2 + \frac{1}{4}\beta_a|\Psi^a|^4\right) + \frac{1}{2}\gamma|\Psi^x|^2|\Psi^y|^2,$$

$$\mathcal{L}_{\mathrm{coupling}} = \frac{1}{2}\sum_{a=x,y}|(\partial_\mu - \mathrm{i}b^a_\mu - \mathrm{i}\lambda\epsilon_{\tau\mu a})\Psi^a|^2.$$

Simplified to

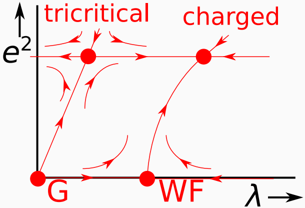
$$\mathcal{L}_{\mathrm{solid}} = \frac{1}{2}(\nabla\times\mathbf{b}^x)^2 + \frac{1}{2}(\nabla\times\mathbf{b}^y)^2,$$

$$\mathcal{L}_{\mathrm{GL}} = \frac{1}{2}m^2(|\Psi^x|^2 + |\Psi^y|^2) + \frac{1}{4}\lambda(|\Psi^x|^4 + |\Psi^y|^4) + \frac{1}{2}g|\Psi^x|^2|\Psi^y|^2,$$

$$\mathcal{L}_{\mathrm{coupling}} = \frac{1}{2}|(\nabla - \mathrm{i}e\mathbf{b}^x)\Psi^x|^2 + \frac{1}{2}|(\nabla - \mathrm{i}e\mathbf{b}^x)\Psi^y|^2$$

- Only m^2 and λ^2: $O(2)$ Wilson-Fisher theory
- with e: charged $O(2)$, Abelian-Higgs model
- with g: $O(2)\times O(2)$, two-component BEC

Abelian-Higgs $d = 3$

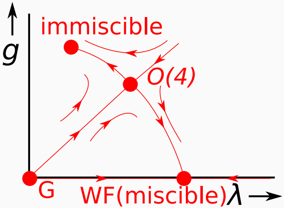


shown with FRG down to $N = 2$:

G. Fejos & T. Hatsuda

PRD **93**, 121702 (2016) **96**, 056018 (2017)

Two-component BEC $d = 3$



ε-expansion

Ceccarelli *et al.*

PRA **92**, 024513 (2016) **93**, 033647 (2017)

Work in progress (with Gergely Fejos):

- FRG for charged $O(2) \times O(2)$ in $d = 3$, charged fixed points
- Influence of stress gauge field dynamics
- Influence of glide constraint
- Quantum critical exponent for specific heat

Relation to fracton physics

Fractons: objects/particles with spatially restricted dynamics

PHYSICAL REVIEW LETTERS **120,** 195301 (2018)

Editors' Suggestion | Featured in Physics

Fracton-Elasticity Duality

Michael Pretko and Leo Radzihovsky

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(Received 30 November 2017; published 7 May 2018)

Motivated by recent studies of fractons, we demonstrate that elasticity theory of a two-dimensional quantum crystal is dual to a fracton tensor gauge theory, providing a concrete manifestation of the fracton phenomenon in an ordinary solid. The topological defects of elasticity theory map onto charges of the tensor gauge theory, with disclinations and dislocations corresponding to fractons and dipoles, respectively. The transverse and longitudinal phonons of crystals map onto the two gapless gauge modes of the gauge theory. The restricted dynamics of fractons matches with constraints on the mobility of lattice defects. The duality leads to numerous predictions for phases and phase transitions of the fracton system, such as the existence of gauge theory counterparts to the (commensurate) crystal, supersolid, hexatic, and isotropic fluid phases of elasticity theory. Extensions of this duality to generalized elasticity theories provide a route to the discovery of new fracton models. As a further consequence, the duality implies that fracton phases are relevant to the study of interacting topological crystalline insulators.

DOI: 10.1103/PhysRevLett.120.195301

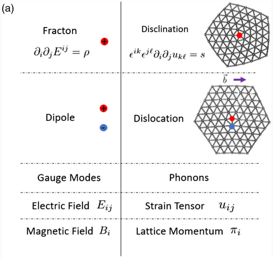

FIG 1 (a) The Fracton-Elasticity Dictionary: Excitations and

Superfluid sound

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Duality in 2 + 1D quantum elasticity: superconductivity and quantum nematic order

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Received 18 September 2003

> a superfluid is an elastic medium having lost its rigidity against shear stresses due to the presence of a dual dislocation condensate,

to arrive at a complete characterization. We emphasize that both viewpoints are equally correct. All one can discern in the scaling limit is the universal definition, as given by Landau:

> a superfluid is a state of matter characterized by a low energy spectrum which is exhausted by a propagating, massless compression mode.

Since both 'states' are hydrodynamically indistinguishable they are actually the same state: by general principle it has to be possible to adiabatically continue the Bose-gas superfluid into our 'order' superfluid. These are just limiting 'microscopic' descriptions of the same entity. Among others, this also implies a 'don't worry' theorem regarding the problem that interstitials cannot be incorporated in the field theory. Because of continuity, it has to be that these can be 'smoothly' inserted in the theory, driving the superfluid away from the order-asymptote which is not a singular limit.

To complete the argument we still have to demonstrate that in this irrotational fluid a genuine Meissner effect should occur when electromagnetic fields are coupled in.

Superfluid sound

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"Superfluid Goldstone mode arises only when $U(1)$ particle conservation symmetry is broken, i.e. when the glide constraint is relaxed."

Pretko & Radzihovsky arXiv:1808.05616
Kumar & Potter arXiv:1808.05621

Redundant Goldstone modes: Superfluid breaks boosts and particle conservation. Solid breaks boosts, translations and rotations.

Nicolis, Brauner, Watanabe, Hidaka, . . .

Summary

- Condensation of topological defects as symmetry-restoring phase transition
- Topological defects in solids study case for restricted mobility
  - dislocations: glide constraint
  - disclination: confinement
- Possibly novel critical behaviour
- Nature of superfluid sound

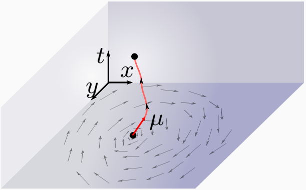
Collaborators:

Jan Zaanen	Leiden	Robert-Jan Slager	Dresden
Jaakko Nissinen	Aalto	Vladimir Cvetkovic	
Kai Wu		Zohar Nussinov	St. Louis
Ke Liu	Munich	Gergely Fejos	Keio U

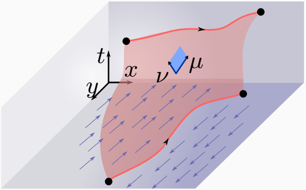
Assumptions and limitations

- Zero temperature
- Ginzburg–Landau $\rightarrow$ only near the phase transition
- London limit, phase fluctuations only
- Maximal crystalline correlations (collective physics only)
- No interstitials
- No disclinations
- Bosons only but 4-He and 3-He experiments similar
- Isotropic solid only

3D quantum liquid crystals



dislocation worldline



dislocation worldsheet

- phonons are now two-form gauge fields $b_{\mu\nu}$
- quantum versions of columnar, smectic and nematic liquid crystals

Goldstone modes		solid	columnar	smectic	nematic
phonons	2+1D	2 / 0	–	1 / 0	0 / 1
rotational	3+1D	3 / 0	2 / 0	1 / 1	0 / 3

AJB, J. Nissinen, K. Wu, J. Zaanen, Physical Review B 96, 165115 (2017)