Topological crystalline insulator: from symmetry indicators to material discovery

Department of physics, National Cheng Kung University, Taiwan
（國立成功大學 物理系）
Tay-Rong Chang (張泰榕)

2018/Dec./6

Outline

slides number

1. Topology in condensed matter physics

Basic concepts and properties of topological band structure

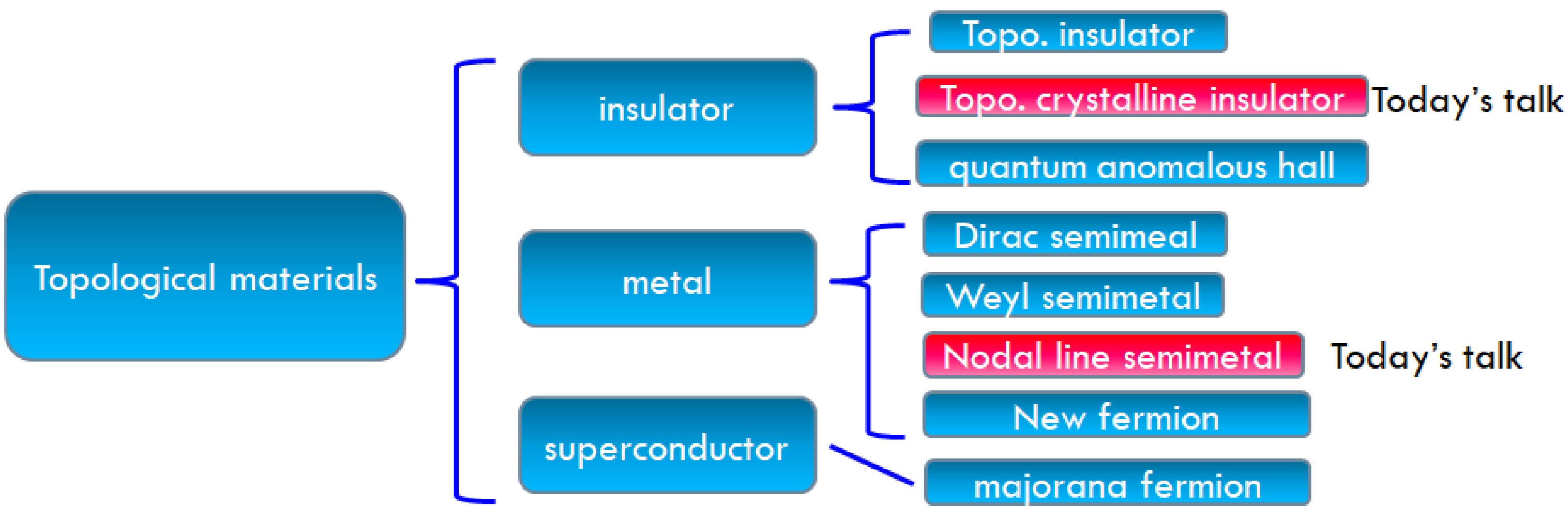
Example: Bi_2Se_3

2. Topological crystalline insulator

Recent prediction: Ca_2As family

3. Type-II nodal line semimetal

Recent prediction: Mg_3Bi_2



Topology in condensed matter physics

Math => real space

≠

=

genus $g=0$ $g=1$ $g=1$

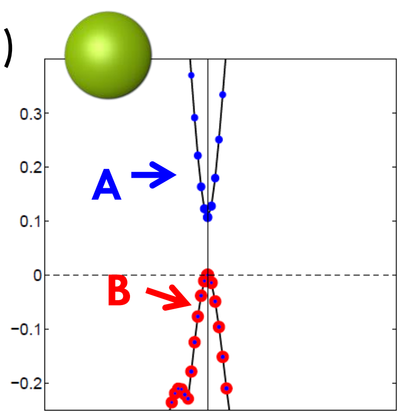
Gauss-Bonnet Theorem:

$$\int_S K_{Gauss}\, ds = 2(1-g)$$

Gauss curvature genus

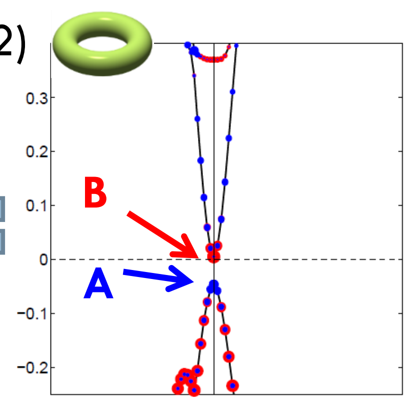
Phys => momentum space

(1)



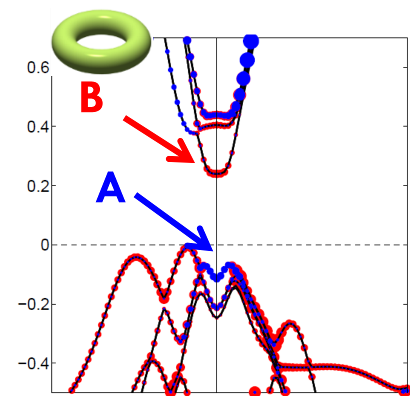
≠

(2)



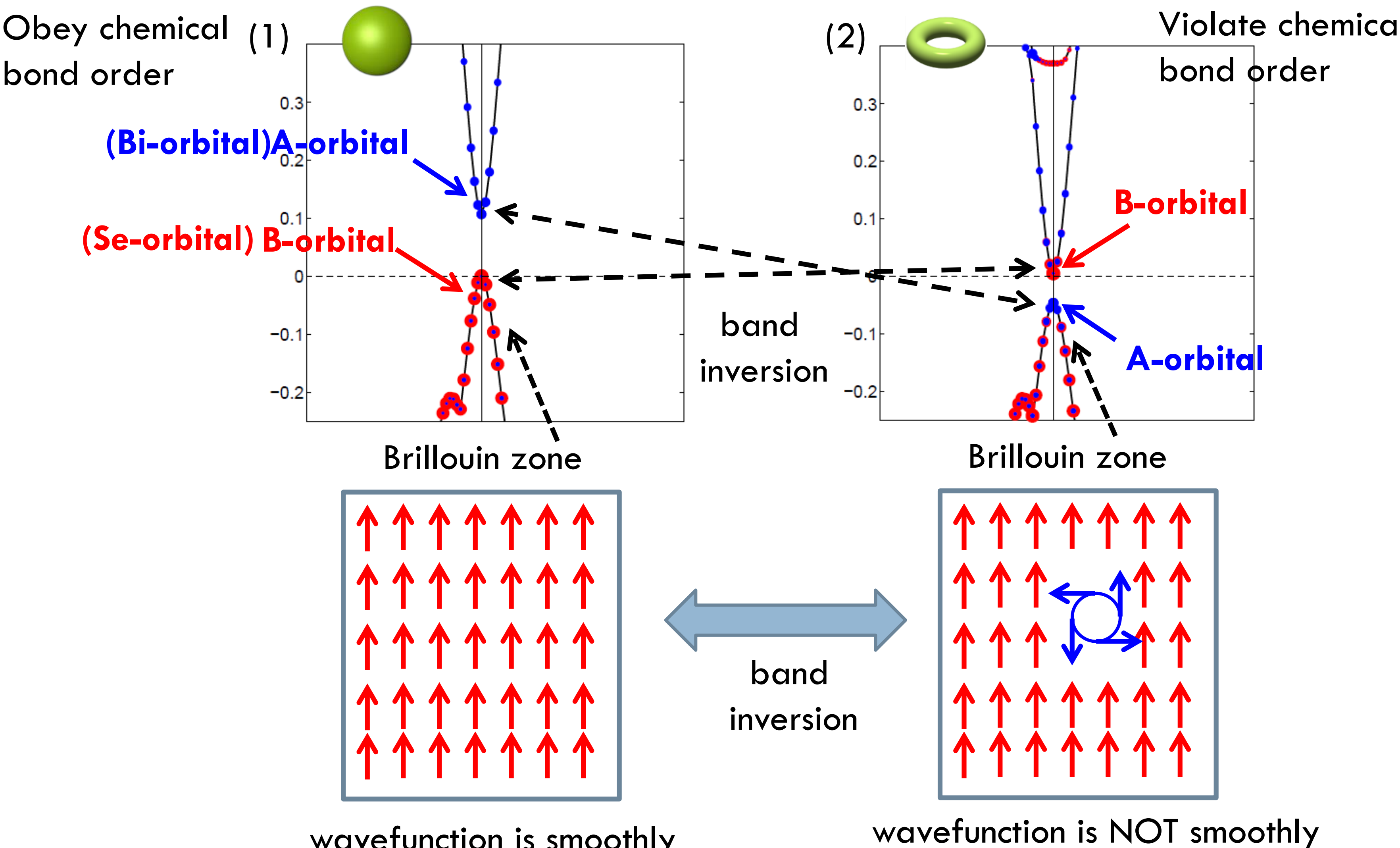
=

(3)



物理雙月刊 藏在邊緣的物理：拓樸材料與拓樸能帶理論
http://pb.ps-taiwan.org/catalog/ins.php?index_m1_id=5&index_id=235

Topology in condensed matter physics



Topology in condensed matter physics

Math => real space

Gauss-Bonnet Theorem:

$$\int_S K_{Gauss}\, ds = 2(1-g)$$

Gauss curvature   genus

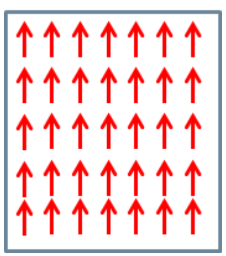
Phys => momentum space

TKNN theory:

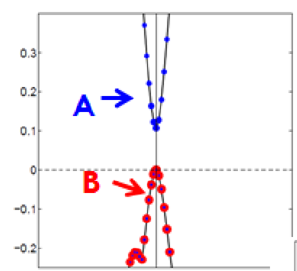
Topological invariant number

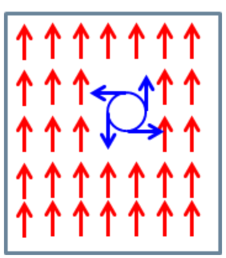
$$\sum_m \frac{1}{2\pi}\int_{BZ} d^2k \nabla \times i\langle u_m | \nabla | u_m \rangle = n$$

Berry curvature   Bloch wavefun

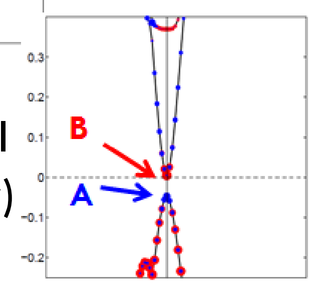


wavefunction is smoothly : $n = 0$ => Topological trivial (normal insulator)

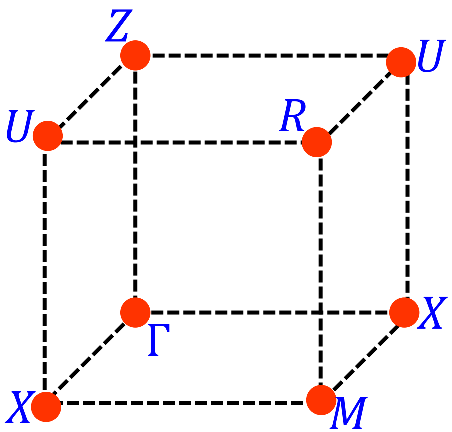




wavefunction is NOT smoothly : $n =$ integer => Topological nontrivial (topological insulator)



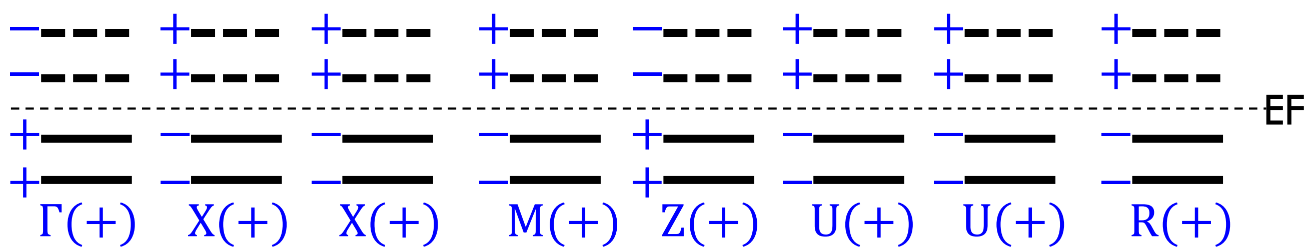
Calculating invariant number



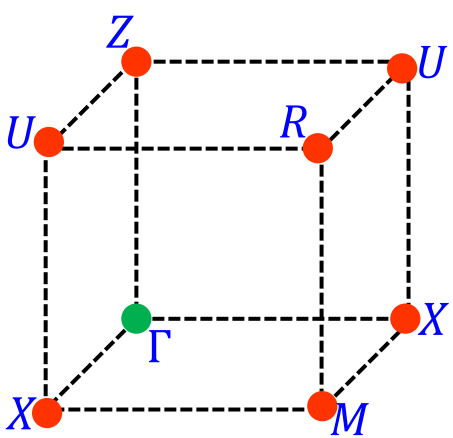
$$\delta_\Gamma \delta_X \delta_M \delta_Y \delta_Z \delta_U \delta_T \delta_R = (-1)^{Z_2}$$

L. Fu, PRB **76**, 045302 (2007)

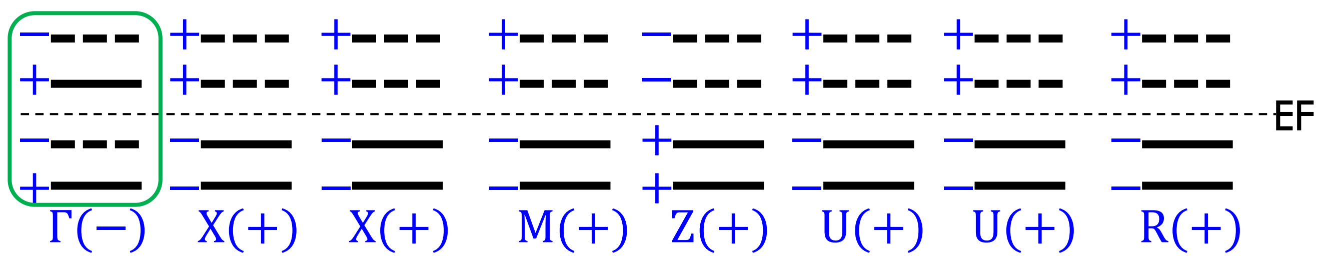
Normal insulator (Z_2=0)



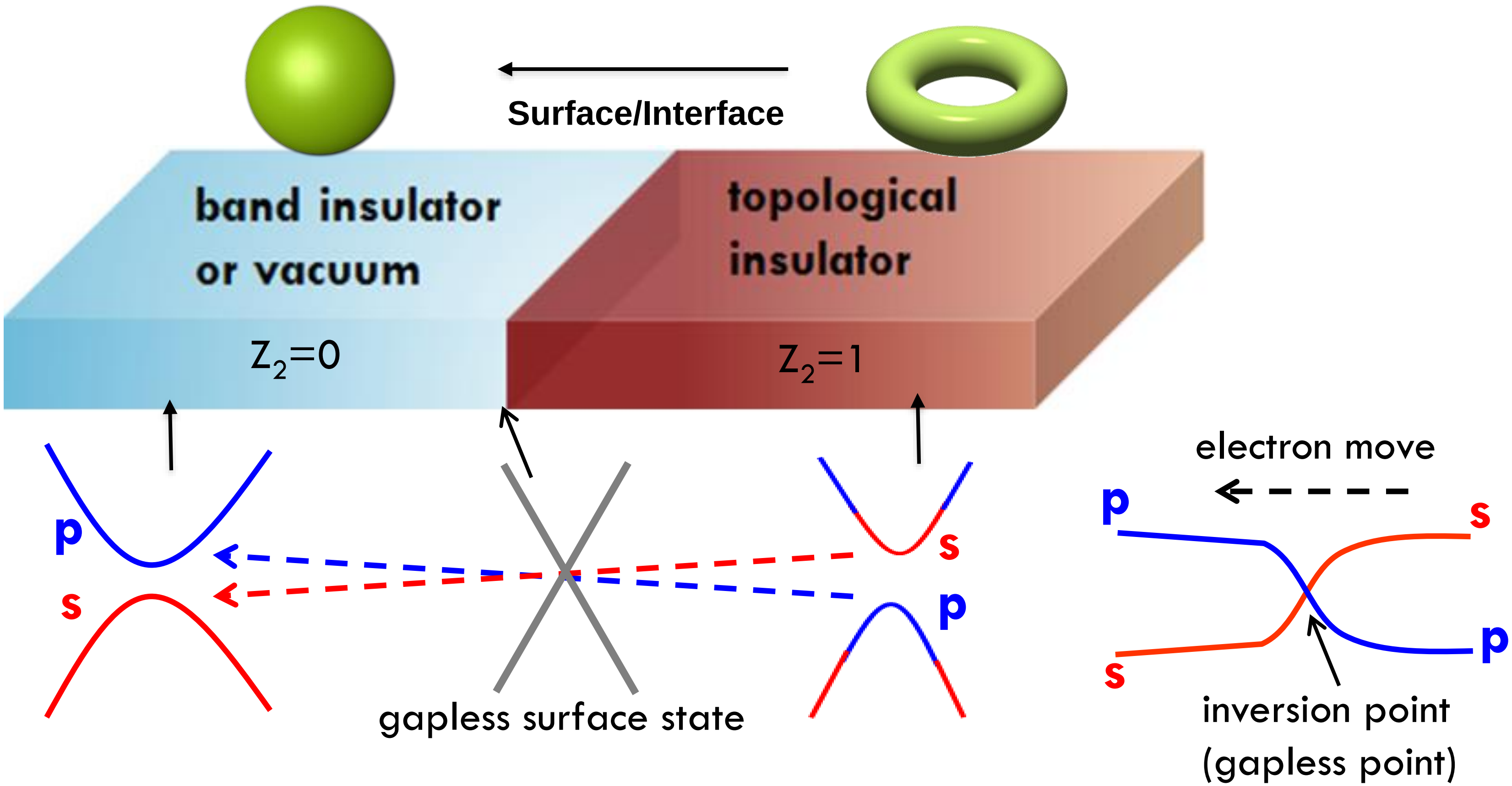
Band inversion at Γ



Topological insulator (Z_2=1)



Topology in condensed matter physics (bulk-edge correspondence)

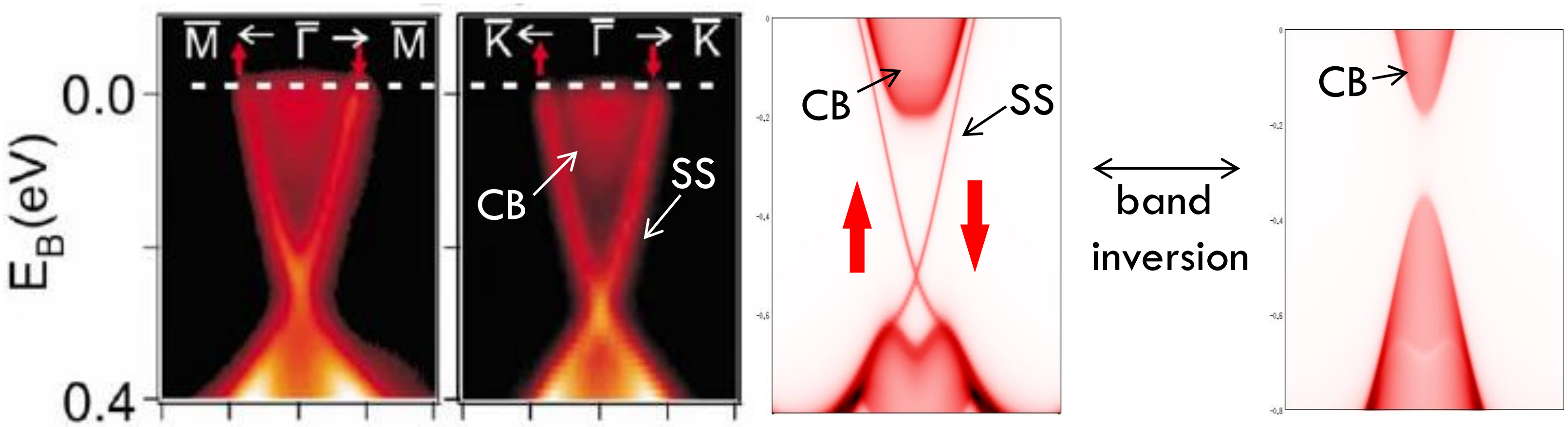


The gapless surface state is the hallmark of topological phase.

M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010)
X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011)

Topological insulator: Bi_2Se_3

ARPES Theory



Bulk: insulating gap

topological Z_2 invariant

odd/even number surface states

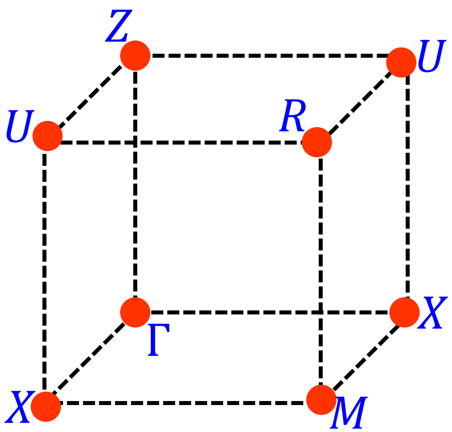
Surface: gapless surface states

spin-momentum locked

Y. Xia et al. Nature Physics **5**, 398 (2009)

D. Hsieh et al. Nature **460**, 1101 (2009)

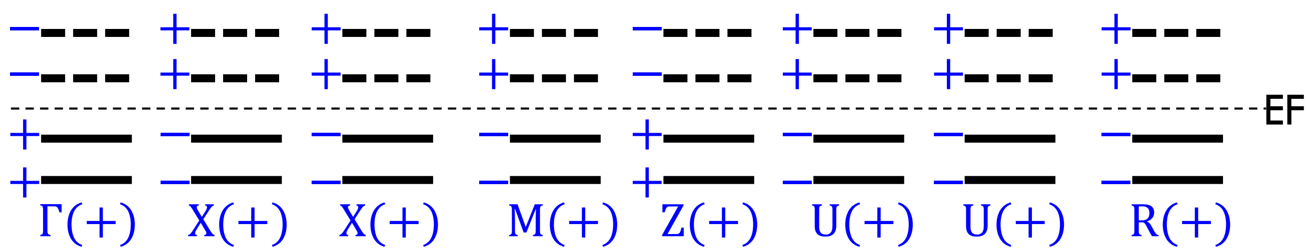
Topological crystalline insulator



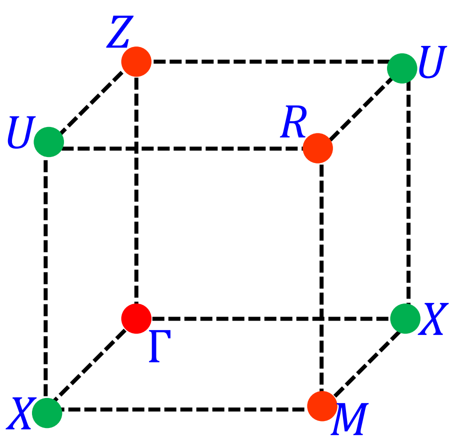
$$\delta_\Gamma \delta_X \delta_M \delta_Y \delta_Z \delta_U \delta_T \delta_R = (-1)^{Z_2}$$

L. Fu, PRB **76**, 045302 (2007)

Normal insulator (Z_2=0)

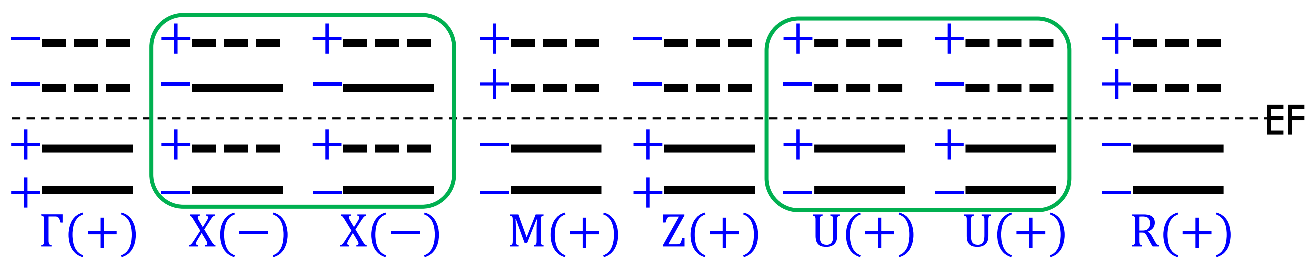


Band inversion at X and U



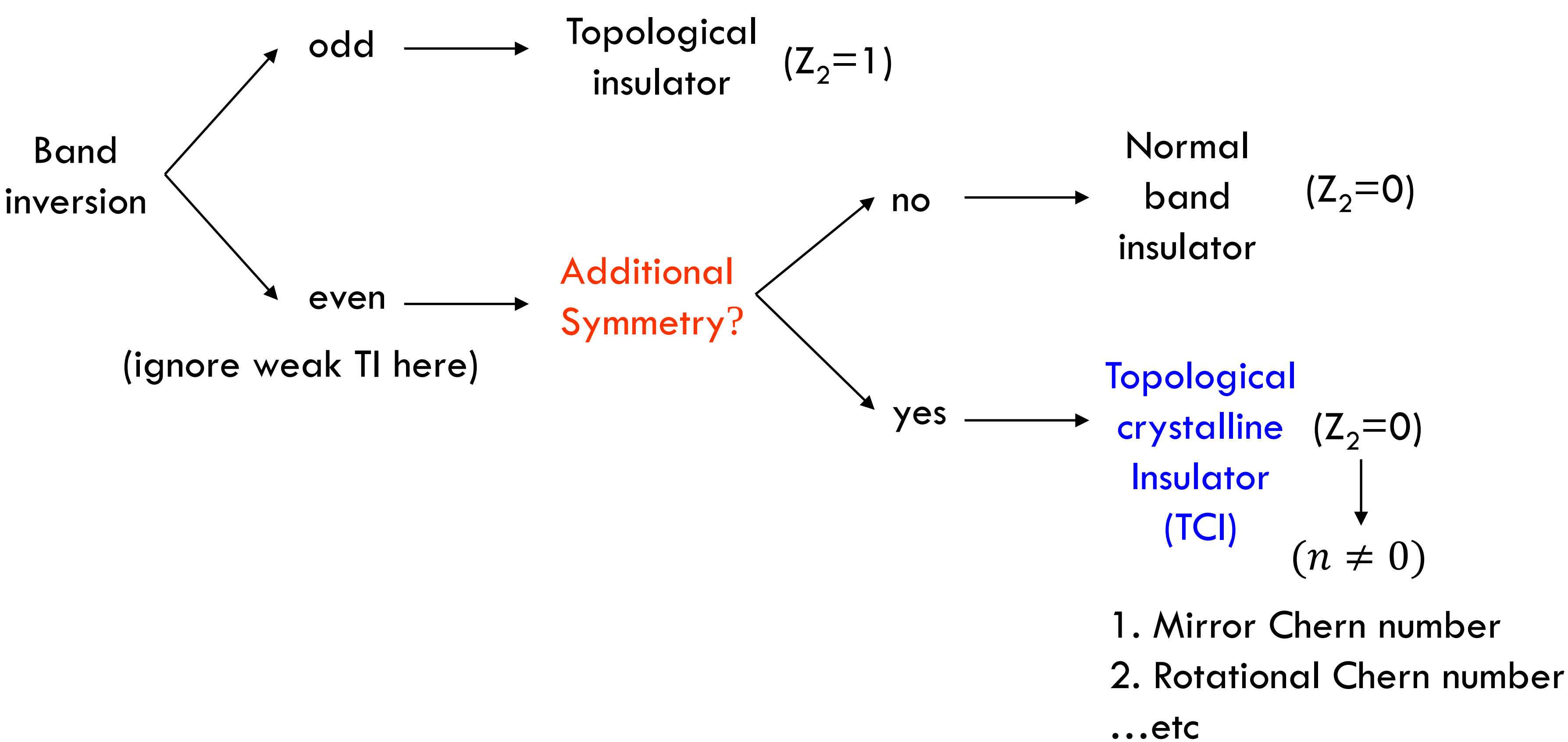
L. Fu, PRL **106**, 106802 (2011)

??? (Z_2=0)



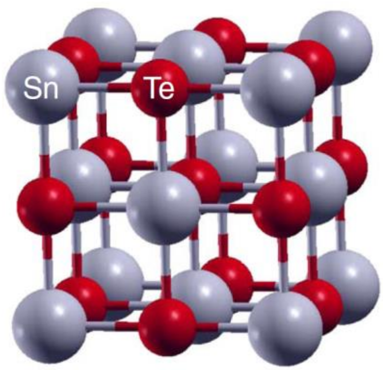
Normal band insulator?

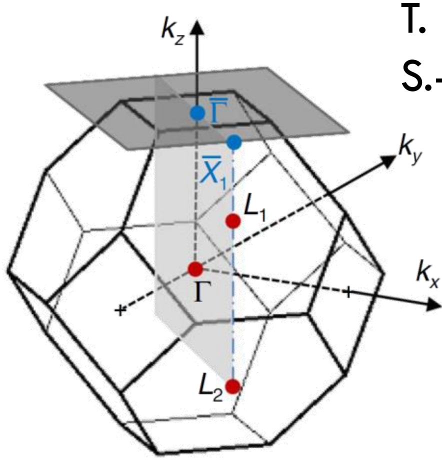
Topological crystalline insulator



SnTe: mirror symmetry

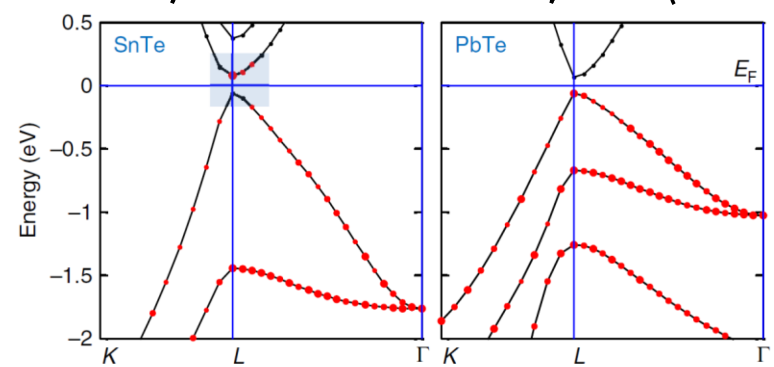
SnTe



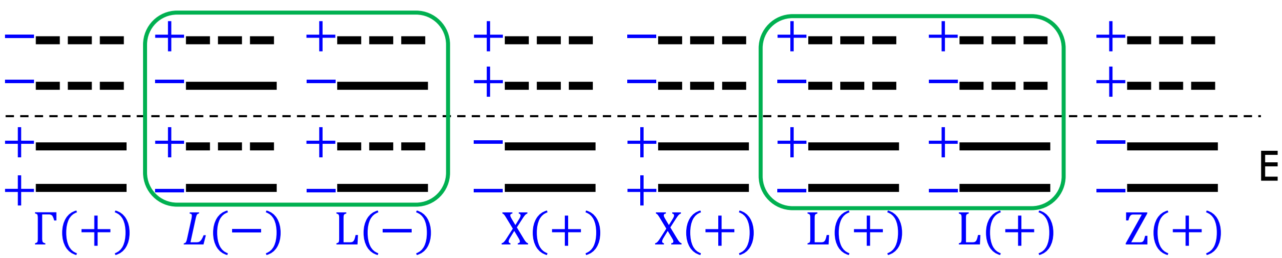


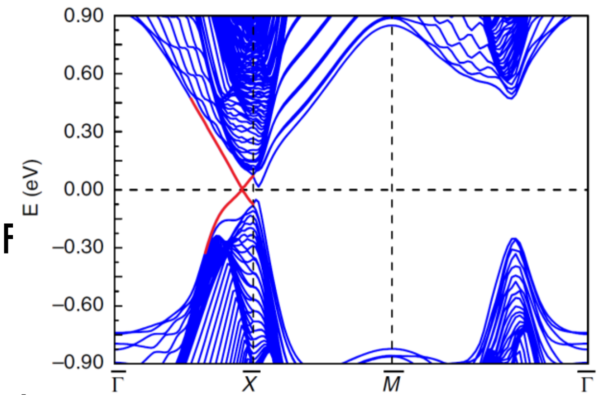
T. H. Hsieh et al., Nature Commun. **3**, 1192 (2012)

S.-Y. Xu et al., Nature Commun. **3**, 982 (2012)



Z_2=0





Even number of times band inversion + additional crystalline symmetry

Mirror Chern number $n_M = (n_{+i} - n_{-i})/2$

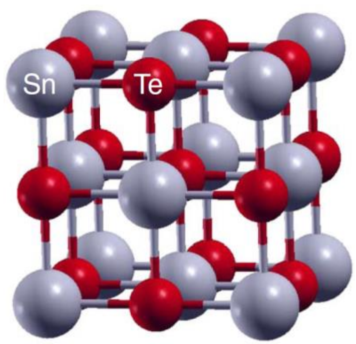
where $n_{\mp i}$ = mirror eigenvalue

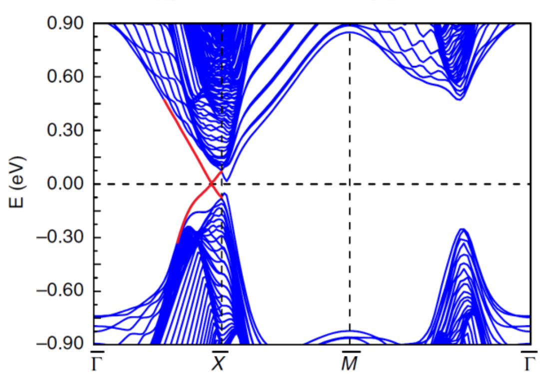
TCI: mirror and glide mirror

T. H. Hsieh et al., Nature Commun. **3**, 1192 (2012)
S.-Y. Xu et al., Nature Commun. **3**, 982 (2012)

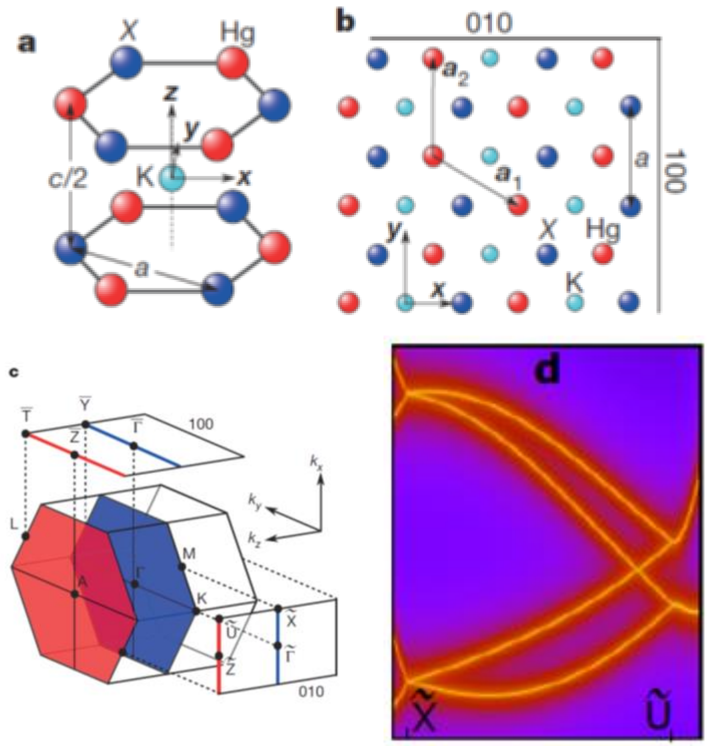
Z. Wang et al., Nature **532**, 189 (2016)
J. Ma et al., Science Advances **3**, e1602415 (2017)

SnTe => mirror





KHgX(X=As,Sb,Bi) => glide mirror



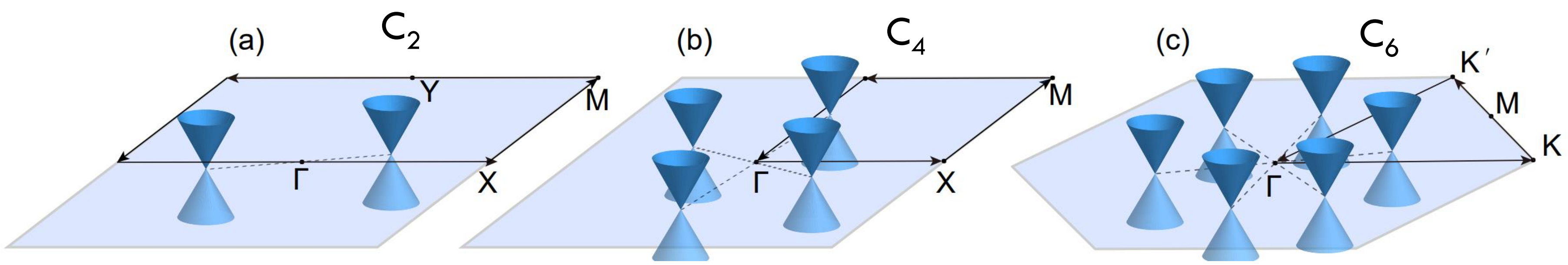
TCI beyond (glide) mirror symmetry ?

TCI: rotational symmetry

New TCI: protected by the N-fold rotational symmetries

Chen Fang and Liang Fu, arXiv:1709.01929 (2017).

C_n = n Dirac points on one surface



Topological invariant number is not convenient to calculate from DFT.

Real material ???

TCI: symmetry indicator

ARTICLE

DOI: 10.1038/s41467-017-00133-2

OPEN

Symmetry-based indicators of band topology in the 230 space groups **(symmetry indicator)**

Hoi Chun Po[1,2], Ashvin Vishwanath[1,2] & Haruki Watanabe[3]

How to implement on first-principle?

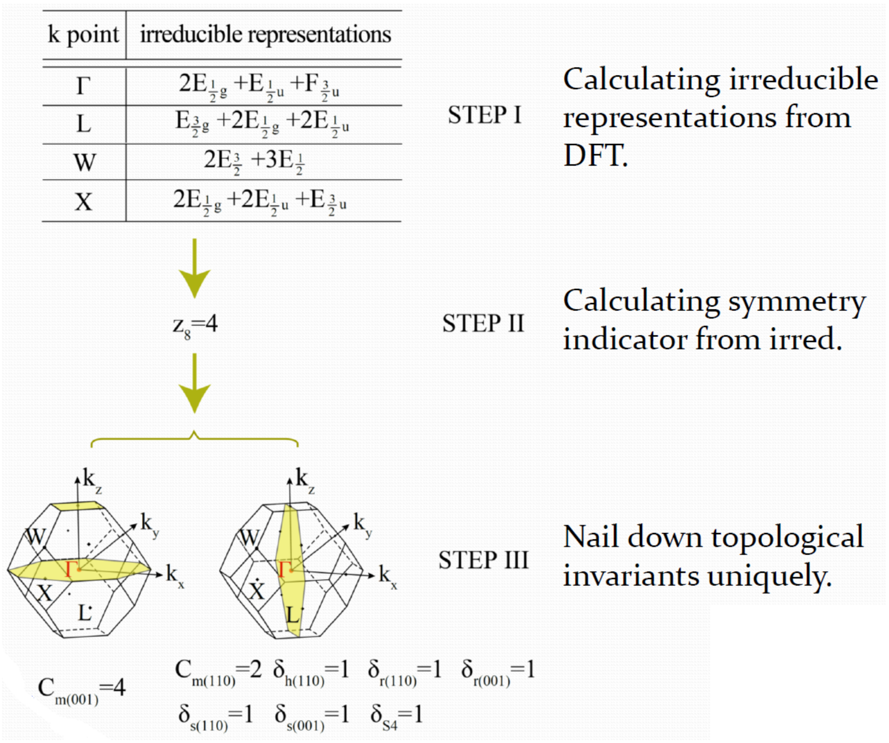
TCI: symmetry indicator

Systematic method (Fu-Kane-like formula) for searching TCI based on symmetry indicator

Zhida Song, Tiantian Zhang, Zhong Fang, Chen Fang:

Nature Communications **9**, 3530 (2018)

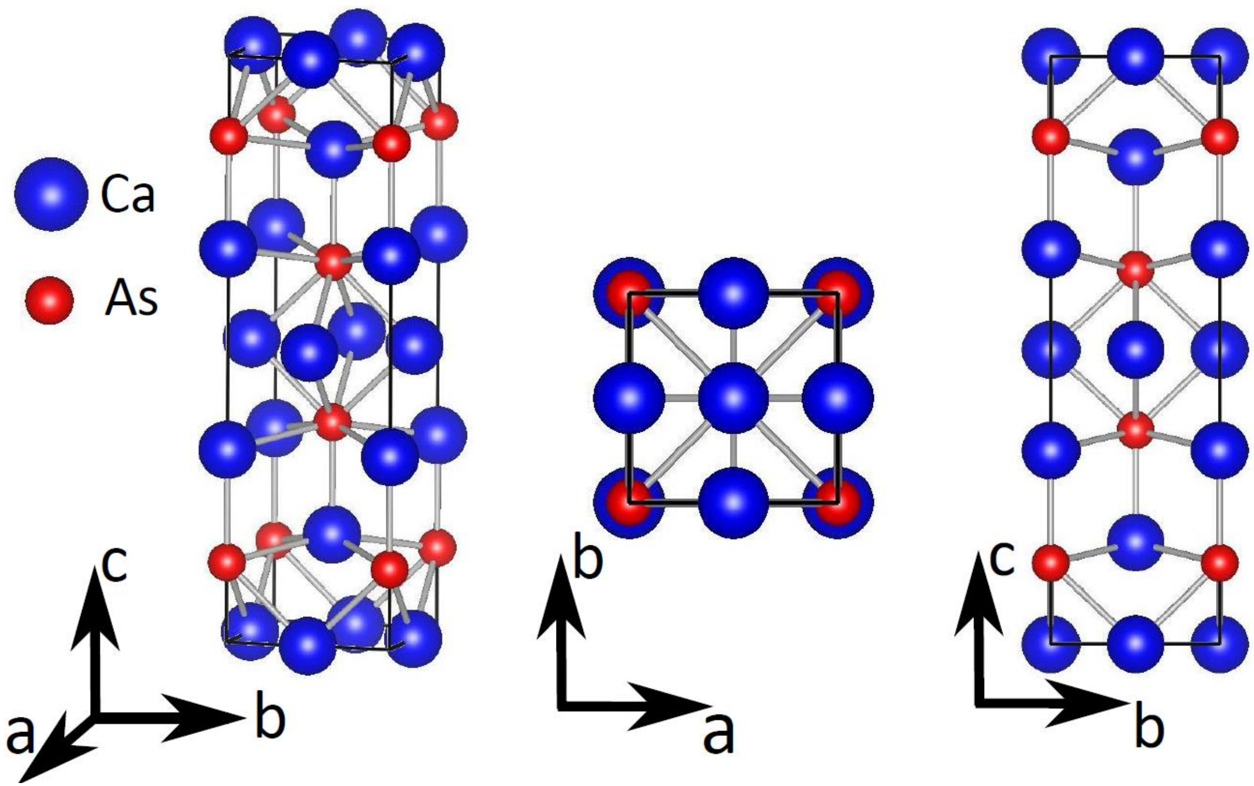
when certain additional symmetry *Y* is present, topological invariants of TCIs protected by symmetry *X* can be inferred by the *Y* -symmetry eigenvalues of energy band.



k point	irreducible representations
Γ	$2E_{\frac{1}{2}g}+E_{\frac{1}{2}u}+F_{\frac{3}{2}u}$
L	$E_{\frac{3}{2}g}+2E_{\frac{1}{2}g}+2E_{\frac{1}{2}u}$
W	$2E_{\frac{3}{2}}+3E_{\frac{1}{2}}$
X	$2E_{\frac{1}{2}g}+2E_{\frac{1}{2}u}+E_{\frac{3}{2}u}$

TCI: candidate material Ca_2As

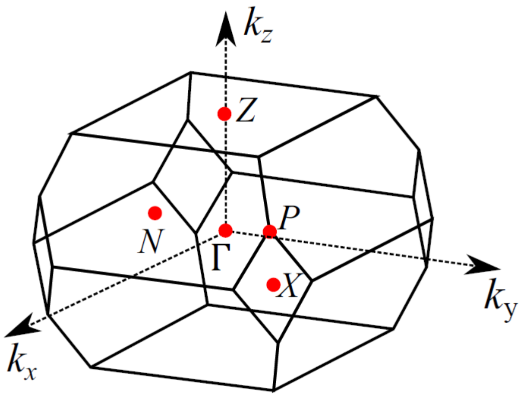
Ca_2As: body-centered tetragonal lattice

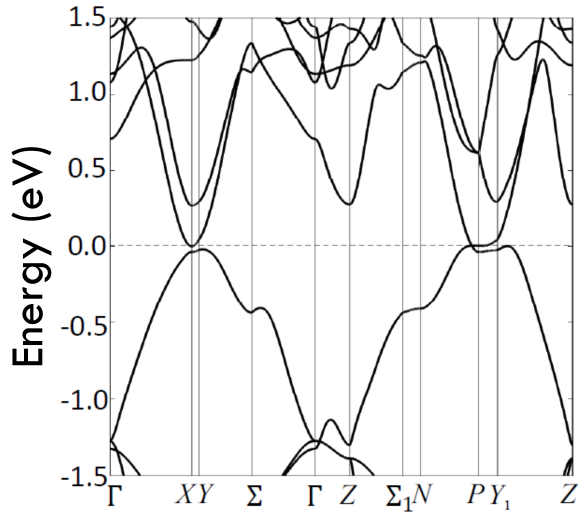


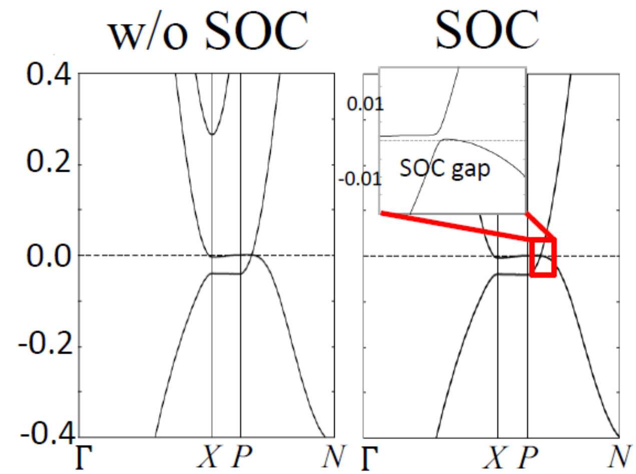
I4/mmm (#139)

Symmetry operation
- Time-reversal
- Inversion
- M(100)
- M(010)
- M(001)
- M(110)
- M(1-10)
- C_4(001)
- C_2(100)
- C_2(010)
- C_2(001)
- C_2(110)
- C_2(1-10)

Ca_2As: symmetry indicator







Step-1

Counting the irred. rep. number of each band

Tetragonal body-centred	87 $\boxed{(139,}$ 140)[a]	$n_{\frac{1}{2}}^{+}$	$n(E^{\Gamma}_{\frac{1}{2}g}) + n(E^{M}_{\frac{1}{2}g}) + n(E^{X}_{\frac{1}{2}g}) + 2n(E^{N}_{\frac{1}{2}g}) + n(E^{P}_{\frac{1}{2}})$ [b]
		$n_{\frac{1}{2}}^{-}$	$n(E^{\Gamma}_{\frac{1}{2}u}) + n(E^{M}_{\frac{1}{2}u}) + n(E^{X}_{\frac{1}{2}u}) + 2n(E^{N}_{\frac{1}{2}u}) + n(E^{P}_{\frac{3}{2}})$
		$n_{\frac{3}{2}}^{+}$	$n(E^{\Gamma}_{\frac{3}{2}g}) + n(E^{M}_{\frac{3}{2}g}) + n(E^{X}_{\frac{1}{2}g}) + 2n(E^{N}_{\frac{1}{2}g}) + n(E^{P}_{\frac{3}{2}})$
		$n_{\frac{3}{2}}^{-}$	$n(E^{\Gamma}_{\frac{3}{2}u}) + n(E^{M}_{\frac{3}{2}u}) + n(E^{X}_{\frac{1}{2}u}) + 2n(E^{N}_{\frac{1}{2}u}) + n(E^{P}_{\frac{1}{2}})$

The symmetry indicator of # 139 is (Z_2, Z_8)

z_2	All SGs with S_4	$\sum_{\mathbf{K}} \frac{1}{2} n_{\mathbf{K}}^{\frac{3}{2}} - \frac{1}{2} n_{\mathbf{K}}^{\frac{1}{2}} \mod 2$ [b]
z_8	83, 87, 123, 124, 127, 128, $\boxed{139,}$ 140, 221, 225, 226, 229	$\frac{3}{2} n_{\frac{3}{2}}^{+} - \frac{3}{2} n_{\frac{3}{2}}^{-} - \frac{1}{2} n_{\frac{1}{2}}^{+} + \frac{1}{2} n_{\frac{1}{2}}^{-} \mod 8$ [d]

$(Z_2, Z_8) = (0,4)$ for Ca2As

Ca_2As: symmetry indicator

	$(\mathcal{Z}_2\mathcal{Z}_8)$
Ca_2As and Ca_2Sb	(04)

Step-2

I4/mmm (#139)

		mirror			glide mirror			rotation									
$\mathbb{Z}_{2,8}$	weak	$m^{001}_{(4)}$	$m^{1\bar{1}0}_{(2)}$	$m^{100}_{(2)}$	$g^{001}_{\frac{1}{2}\frac{1}{2}0}$	$g^{1\bar{1}0}_{\frac{1}{2}\frac{1}{2}0}$	$g^{100}_{0\frac{1}{2}\frac{1}{2}}$	2^{001}	2^{100}	2^{110}	4^{001}	i	2_1^{001}	2_1^{100}	2_1^{110}	4_2^{001}	$\bar{4}^{001}$
00	000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
00	000	0	2	2	0	$\mathbb{1}$	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0
00	000	4	0	2	0	0	$\mathbb{1}$	0	$\mathbb{1}$	0	$\mathbb{1}$	0	0	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$
00	000	4	2	0	0	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$
04	000	0	0	2	0	0	$\mathbb{1}$	0	$\mathbb{1}$	0	$\mathbb{1}$	0	0	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$
04	000	0	2	0	0	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$
04	000	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04	000	4	2	2	0	$\mathbb{1}$	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0
12	111	2	0	0	0	0	$\mathbb{1}$	0	$\mathbb{1}$	1	0	1	$\mathbb{1}$	0	0	$\mathbb{1}$	1
12	111	2	2	2	0	$\mathbb{1}$	0	0	0	0	0	1	$\mathbb{1}$	$\mathbb{1}$	1	$\mathbb{1}$	1
12	111	$\bar{2}$	0	2	0	0	0	0	0	1	$\mathbb{1}$	1	$\mathbb{1}$	$\mathbb{1}$	0	0	0
12	111	$\bar{2}$	2	0	0	$\mathbb{1}$	$\mathbb{1}$	0	$\mathbb{1}$	0	$\mathbb{1}$	1	$\mathbb{1}$	0	1	0	0

Ca_2As: symmetry indicator

	$(\mathbb{Z}_2\mathbb{Z}_8)$
Ca_2As and Ca_2Sb	(04)

Step-3

I4/mmm (#139)

$\mathbb{Z}_{2,8}$	weak	$m^{001}_{(4)}$	$m^{1\bar{1}0}_{(2)}$	$m^{100}_{(2)}$	$g^{001}_{\frac{1}{2}\frac{1}{2}0}$	$g^{1\bar{1}0}_{\frac{1}{2}\frac{1}{2}0}$	$g^{100}_{0\frac{1}{2}\frac{1}{2}}$	2^{001}	2^{100}	2^{110}	4^{001}	i	2^{001}_1	2^{100}_1	2^{110}_1	4^{001}_2	$\bar{4}^{001}$
00	000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
00	000	0	2	2	0	1	1	0	1	1	0	0	0	1	1	0	0
00	000	4	0	2	0	0	1	0	1	0	1	0	0	1	0	1	1
00	000	4	2	0	0	1	0	0	0	1	1	0	0	0	1	1	1
04	000	0	0	2	0	0	1	0	1	0	1	0	0	1	0	1	1
04	000	0	2	0	0	1	0	0	0	1	1	0	0	0	1	1	1
04	000	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04	000	4	2	2	0	1	1	0	1	1	0	0	0	1	1	0	0
12	111	2	0	0	0	0	1	0	1	1	0	1	1	0	0	1	1
12	111	2	2	2	0	1	0	0	0	0	0	1	1	1	1	1	1
12	111	$\bar{2}$	0	2	0	0	0	0	0	1	1	1	1	1	0	0	0
12	111	$\bar{2}$	2	0	0	1	1	0	1	0	1	1	1	0	1	0	0

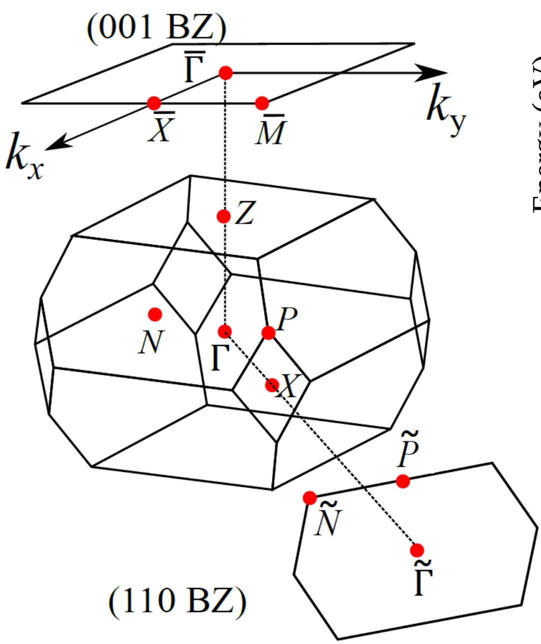
	$(\nu_0;\nu_1\nu_2\nu_3)$	$n_{4(001)}$	$n_{2(100)}$	$n_{2(001)}$	$n_{2(110)}$	$n_{\mathcal{M}(1\bar{1}0)}$	$n_{\mathcal{M}(001)}$	$n_{\mathcal{M}(100)}$	$\mathcal{I}$
Ca_2As	(0;000)	1	0	0	1	2	0	0	1

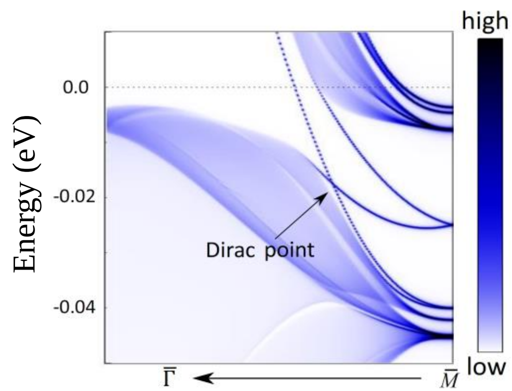
mirror Chern number

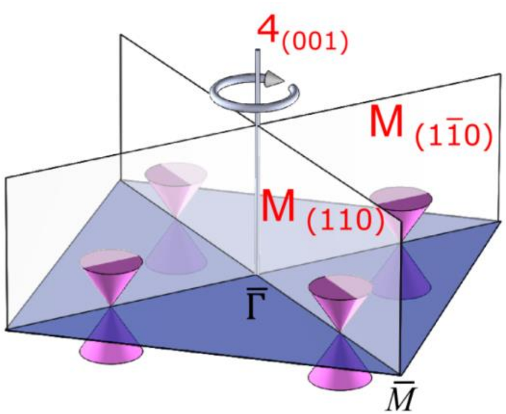
Topological surface states

I4/mmm (#139)

$\mathbb{Z}_{2,8}$	weak	$m^{001}_{(4)}$	$m^{1\bar{1}0}_{(2)}$	$m^{100}_{(2)}$	$g^{001}_{\frac{1}{2}\frac{1}{2}0}$	$g^{1\bar{1}0}_{\frac{1}{2}\frac{1}{2}0}$	$g^{100}_{0\frac{1}{2}\frac{1}{2}}$	2^{001}	2^{100}	2^{110}	4^{001}	i	2^{001}_1	2^{100}_1	2^{110}_1	4^{001}_2	$\bar{4}^{001}$
04	000	0	2	0	0	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$





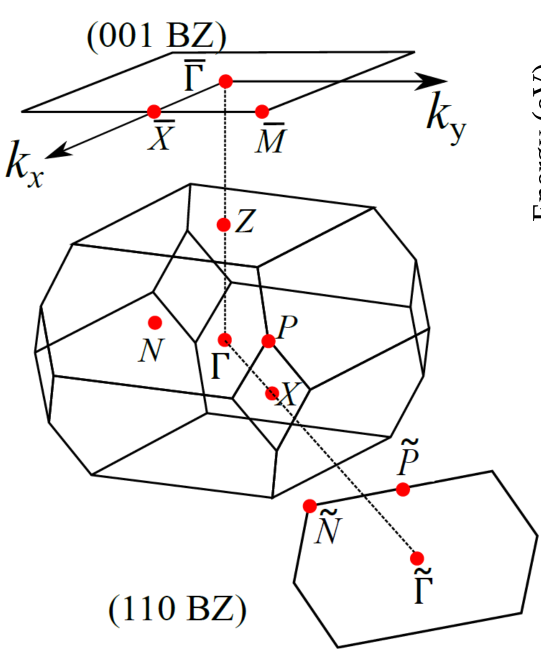


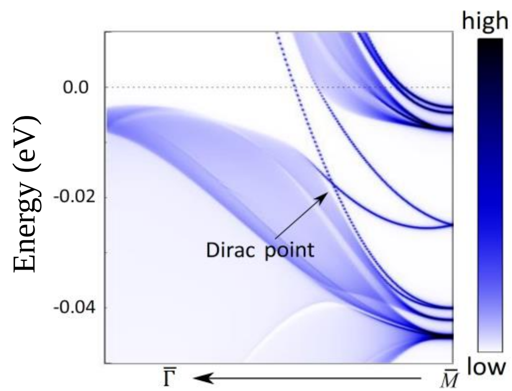
C_4
+
Mirror

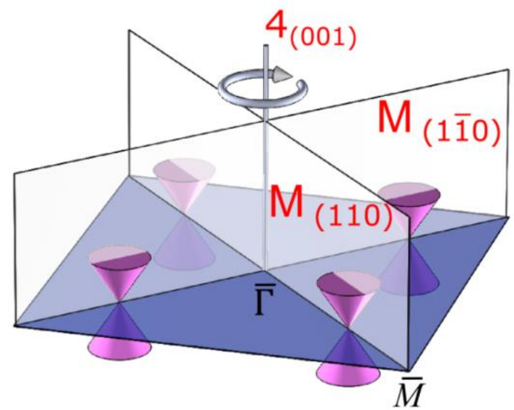
Topological surface states

I4/mmm (#139)

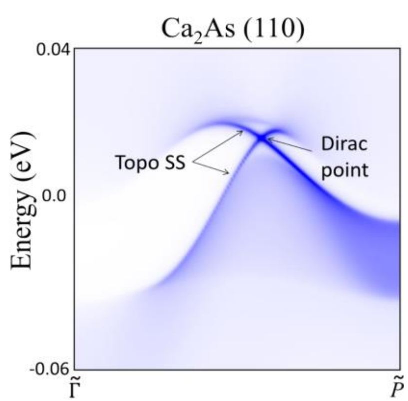
$\mathbb{Z}_{2,8}$	weak	$m^{001}_{(4)}$	$m^{1\bar{1}0}_{(2)}$	$m^{100}_{(2)}$	$g^{001}_{\frac{1}{2}\frac{1}{2}0}$	$g^{1\bar{1}0}_{\frac{1}{2}\frac{1}{2}0}$	$g^{100}_{0\frac{1}{2}\frac{1}{2}}$	2^{001}	2^{100}	2^{110}	4^{001}	i	2^{001}_{1}	2^{100}_{1}	2^{110}_{1}	4^{001}_{2}	$\bar{4}^{001}$
04	000	0	2	0	0	1	0	0	0	1	1	0	0	0	1	1	1

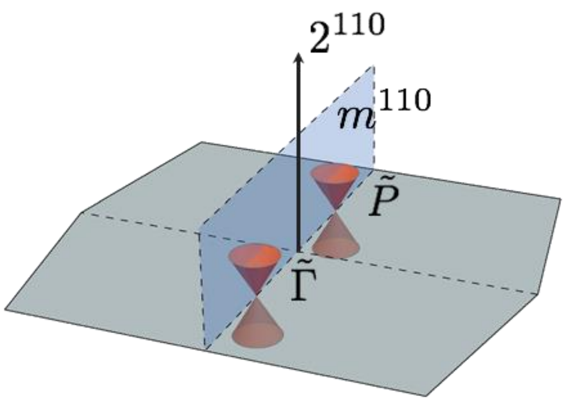






C_4
+
Mirror

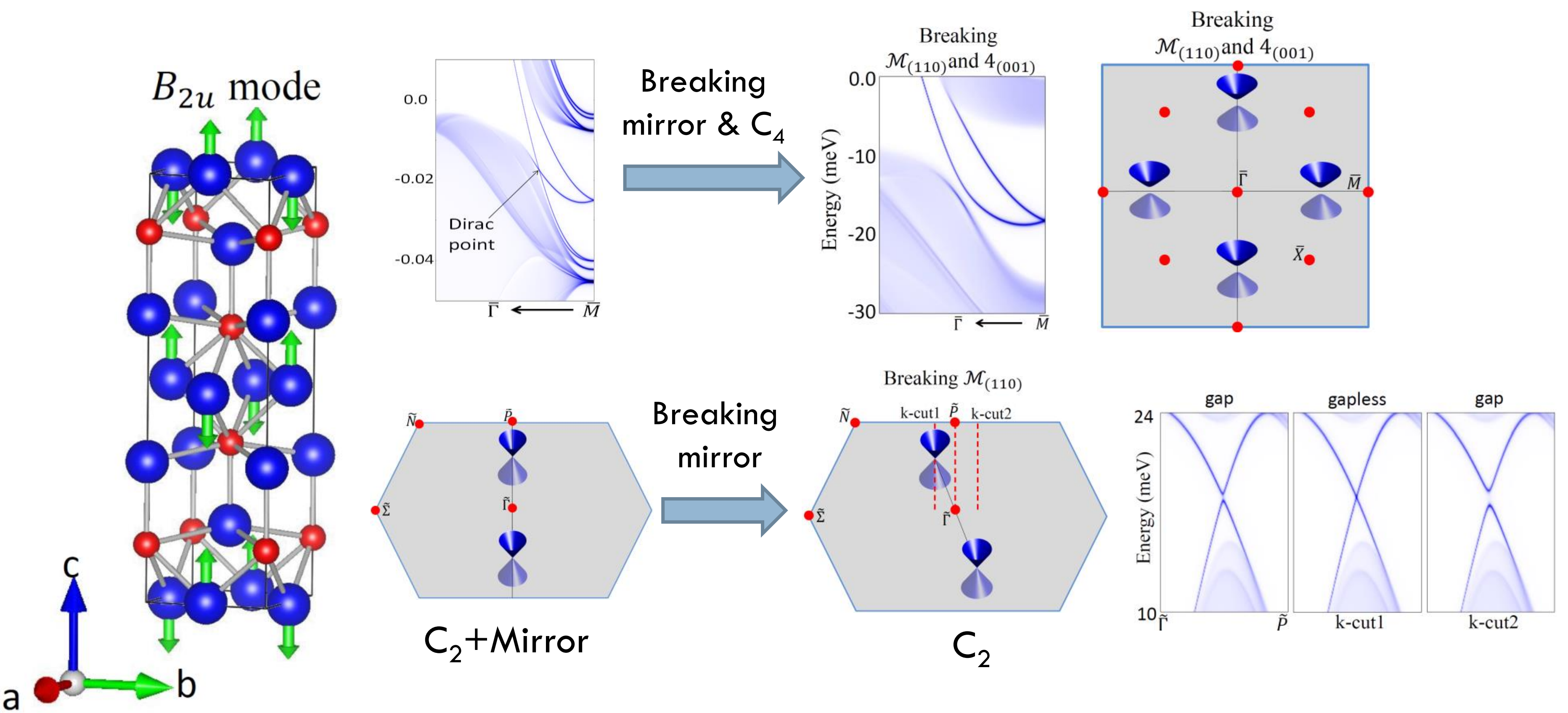




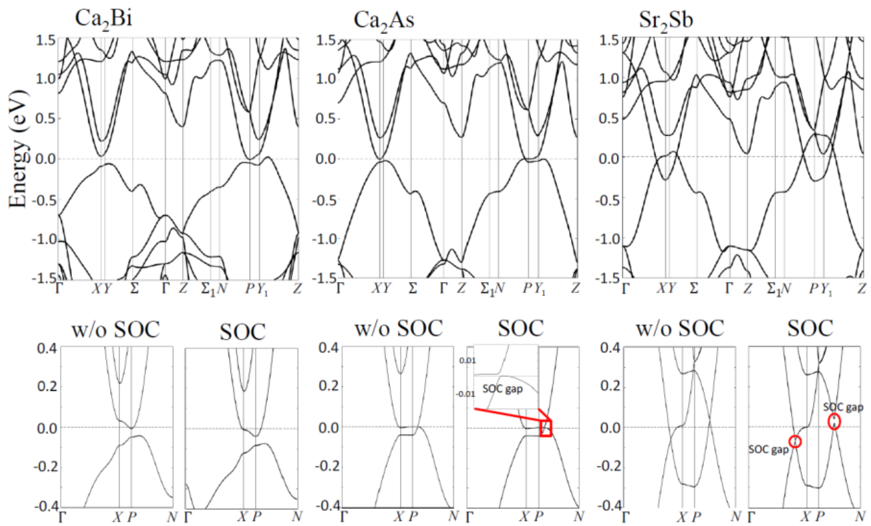
C_2
+
Mirror

TCI: rotational symmetry protected

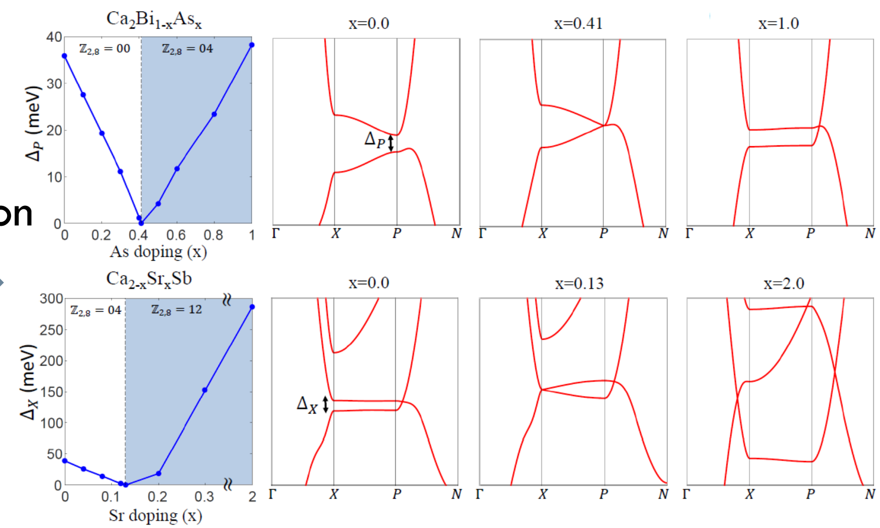
$\mathbb{Z}_{2,8}$	weak	$m^{001}_{(4)}$	$\cancel{m^{1\bar{1}0}_{(2)}}$	$m^{100}_{(2)}$	$g^{001}_{\frac{1}{2}\frac{1}{2}0}$	$g^{1\bar{1}0}_{\frac{1}{2}\frac{1}{2}0}$	$g^{100}_{0\frac{1}{2}\frac{1}{2}}$	2^{001}	2^{100}	$\boxed{2^{110}}$	$\cancel{4^{001}}$	i	2^{001}_1	2^{100}_1	2^{110}_1	4^{001}_2	$\bar{4}^{001}$
04	000	0	$\cancel{2}$	0	0	1	0	0	0	$\boxed{1}$	$\cancel{1}$	0	0	0	1	1	1



Topological phase transition



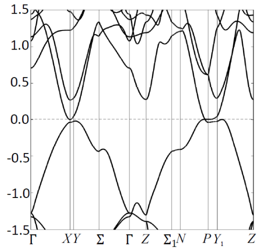
Phase transition



	$\mathbb{Z}_{2,8}$	weak	$m^{001}_{(4)}$	$m^{1\bar{1}0}_{(2)}$	$m^{100}_{(2)}$	$g^{001}_{\frac{1}{2}\frac{1}{2}0}$	$g^{1\bar{1}0}_{\frac{1}{2}\frac{1}{2}0}$	$g^{100}_{0\frac{1}{2}\frac{1}{2}}$	2^{001}	2^{100}	2^{110}	4^{001}	i	2^{001}_1	2^{100}_1	2^{110}_1	4^{001}_2	$\bar{4}^{001}$	
Ca_2Bi	00	000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Normal insulator
	00	000	0	2	2	0	$\mathbb{1}$	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0	
	00	000	4	0	2	0	0	$\mathbb{1}$	0	$\mathbb{1}$	0	$\mathbb{1}$	0	0	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$	
	00	000	4	2	0	0	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	
	04	000	0	0	2	0	0	$\mathbb{1}$	0	$\mathbb{1}$	0	$\mathbb{1}$	0	0	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$	
Ca_2As	04	000	0	2	0	0	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	TCI
	04	000	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	04	000	4	2	2	0	$\mathbb{1}$	$\mathbb{1}$	0	$\mathbb{1}$	$\mathbb{1}$	0	0	0	$\mathbb{1}$	$\mathbb{1}$	0	0	
Sr_2Sb	12	111	2	0	0	0	0	$\mathbb{1}$	0	$\mathbb{1}$	1	0	1	$\mathbb{1}$	0	0	$\mathbb{1}$	1	TCI+Weak TI
	12	111	2	2	2	0	$\mathbb{1}$	0	0	0	0	0	1	$\mathbb{1}$	$\mathbb{1}$	1	$\mathbb{1}$	1	
	12	111	$\bar{2}$	0	2	0	0	0	0	0	1	$\mathbb{1}$	1	$\mathbb{1}$	$\mathbb{1}$	0	0	0	
	12	111	$\bar{2}$	2	0	0	$\mathbb{1}$	$\mathbb{1}$	0	$\mathbb{1}$	0	$\mathbb{1}$	1	$\mathbb{1}$	0	1	0	0	

Conclusion (TCI)

(1) Searching TCI by using symmetry indicator for the first time

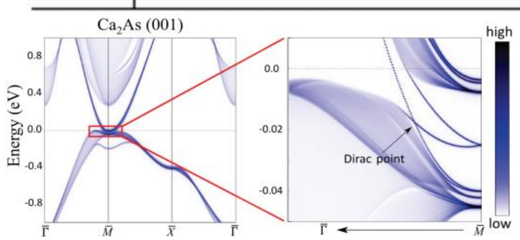


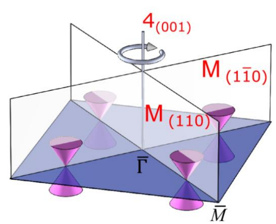
Ca_2As $(Z_2,Z_8)=(0,4)$

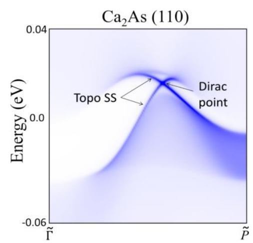
Tetragonal body-centred	87 (139, 140)[a]	$n^+_{\frac{1}{2}}$	$n(E^{\Gamma}_{\frac{1}{2}g}) + n(E^{M}_{\frac{1}{2}g}) + n(E^{X}_{\frac{1}{2}g}) + 2n(E^{N}_{\frac{1}{2}g}) + n(E^{P}_{\frac{1}{2}})$[b]
		$n^-_{\frac{1}{2}}$	$n(E^{\Gamma}_{\frac{1}{2}u}) + n(E^{M}_{\frac{1}{2}u}) + n(E^{X}_{\frac{1}{2}u}) + 2n(E^{N}_{\frac{1}{2}u}) + n(E^{P}_{\frac{3}{2}})$
		$n^+_{\frac{3}{2}}$	$n(E^{\Gamma}_{\frac{3}{2}g}) + n(E^{M}_{\frac{3}{2}g}) + n(E^{X}_{\frac{1}{2}g}) + 2n(E^{N}_{\frac{1}{2}g}) + n(E^{P}_{\frac{3}{2}})$
		$n^-_{\frac{3}{2}}$	$n(E^{\Gamma}_{\frac{3}{2}u}) + n(E^{M}_{\frac{3}{2}u}) + n(E^{X}_{\frac{1}{2}u}) + 2n(E^{N}_{\frac{1}{2}u}) + n(E^{P}_{\frac{1}{2}})$

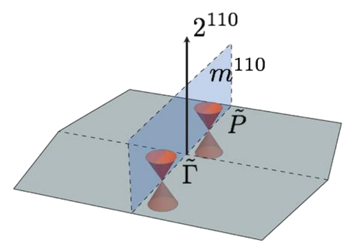
(2) Topological surface states are protected by multi topological invariant numbers

$\mathbb{Z}_{2,8}$	weak	$m^{001}_{(4)}$	$m^{1\bar{1}0}_{(2)}$	$m^{100}_{(2)}$	$g^{001}_{\frac{1}{2}\frac{1}{2}0}$	$g^{1\bar{1}0}_{\frac{1}{2}\frac{1}{2}0}$	$g^{100}_{0\frac{1}{2}\frac{1}{2}}$	2^{001}	2^{100}	2^{110}	4^{001}	i	2^{001}_1	2^{100}_1	2^{110}_1	4^{001}_2	$\bar{4}^{001}$

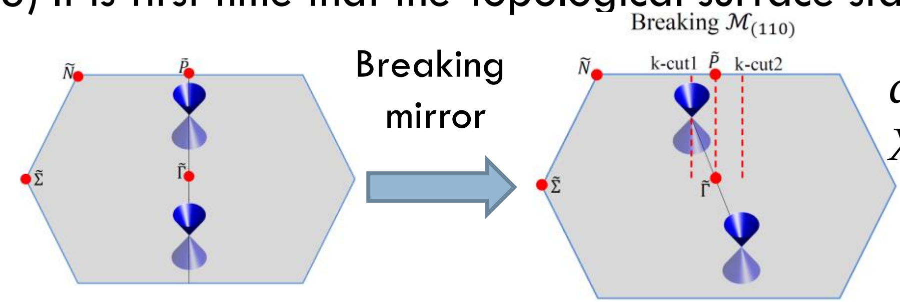








(3) It is first time that the Topological surface states are protected by rotational symmetry

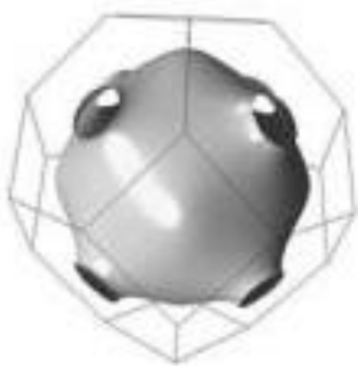


arXiv:1805.05215 (2018)

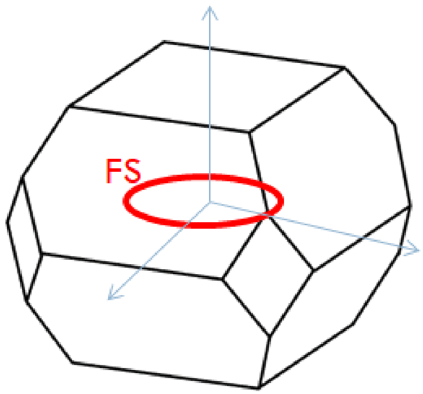
X. Zhou et al., Phys. Rev. B ***98****, 241104(R) (2018)*

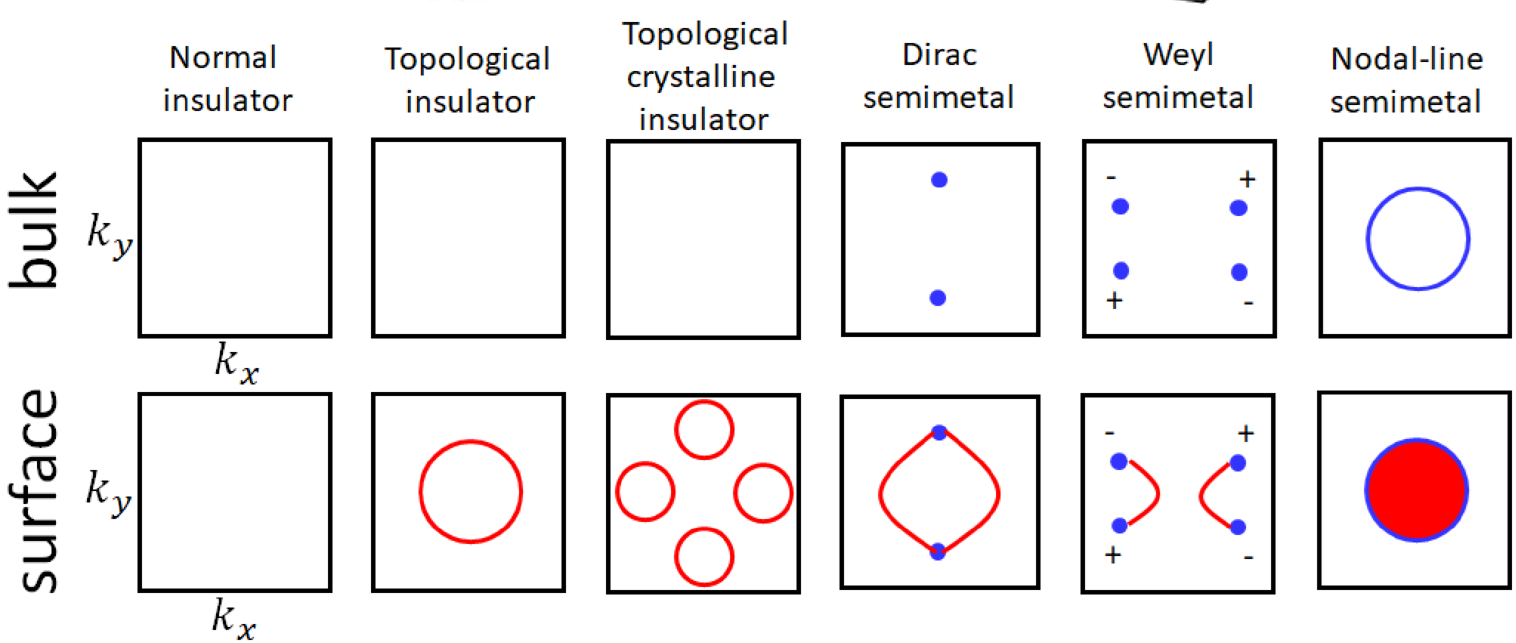
Nodal-line semimetal (節線半金屬)

Normal metal: 2D Fermi surface



Topological metal: 1D Nodal-line

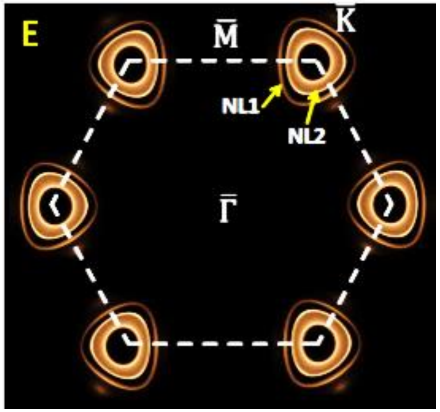


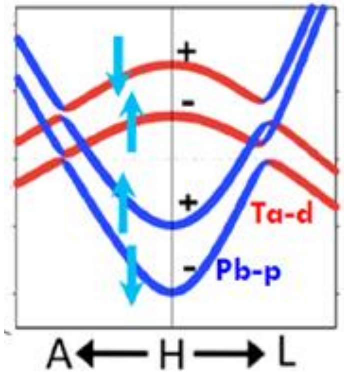


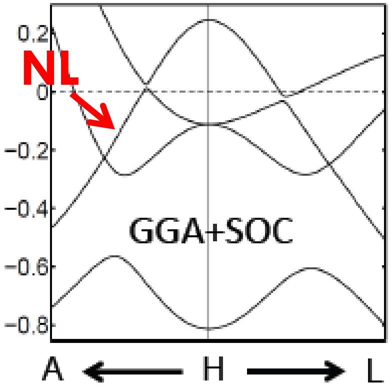
Nodal-line semimetal

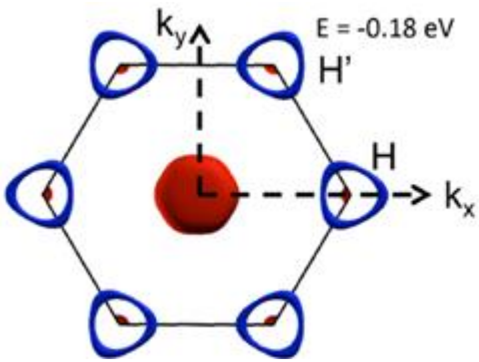
$PbTaSe_2$

$TlTaSe_2$





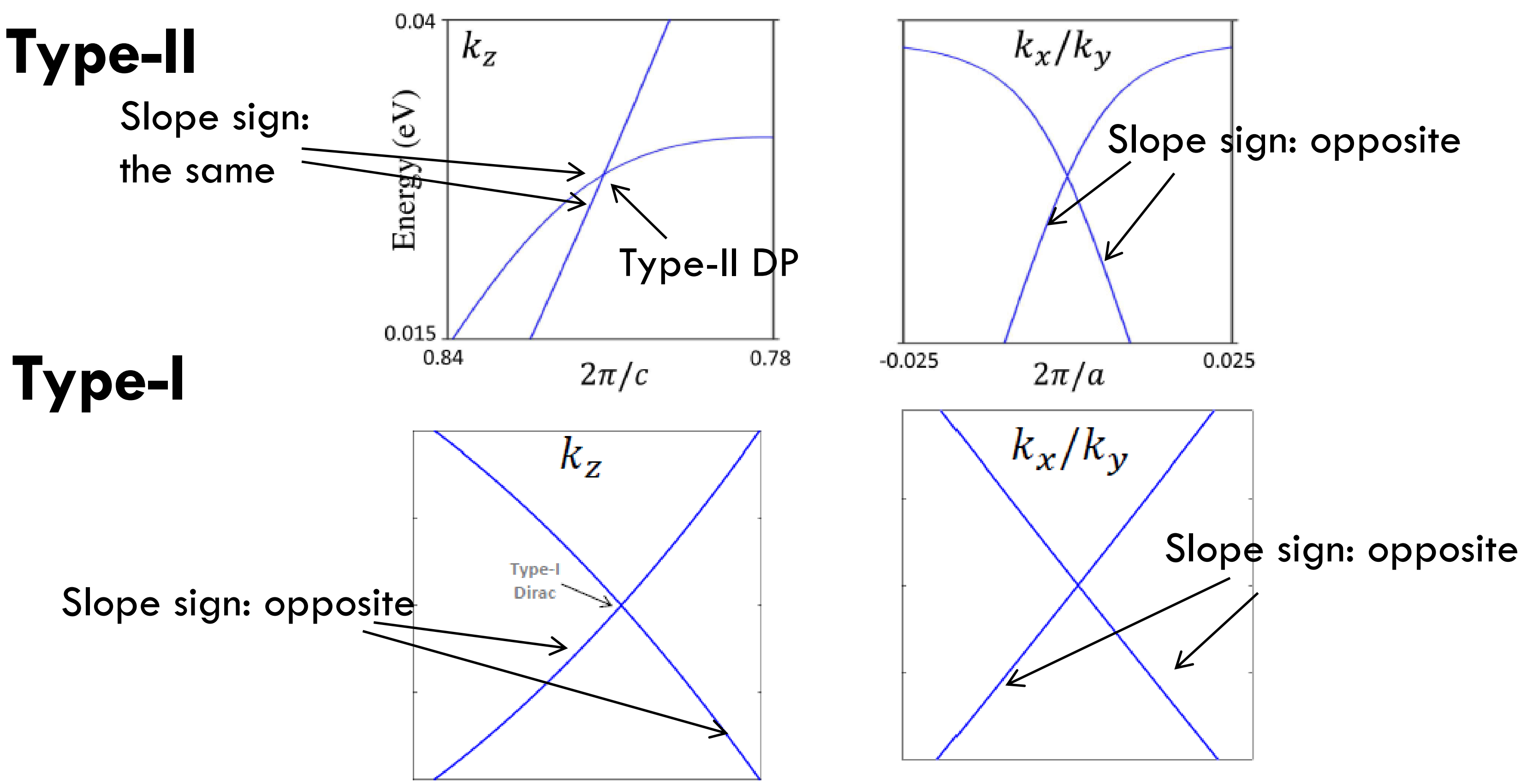




G. Bian et al., Nature Commun. ***7****, 10556 (2016)*

G. Bian et al., Phys. Rev. B ***93****, 12113(R) (2016)*

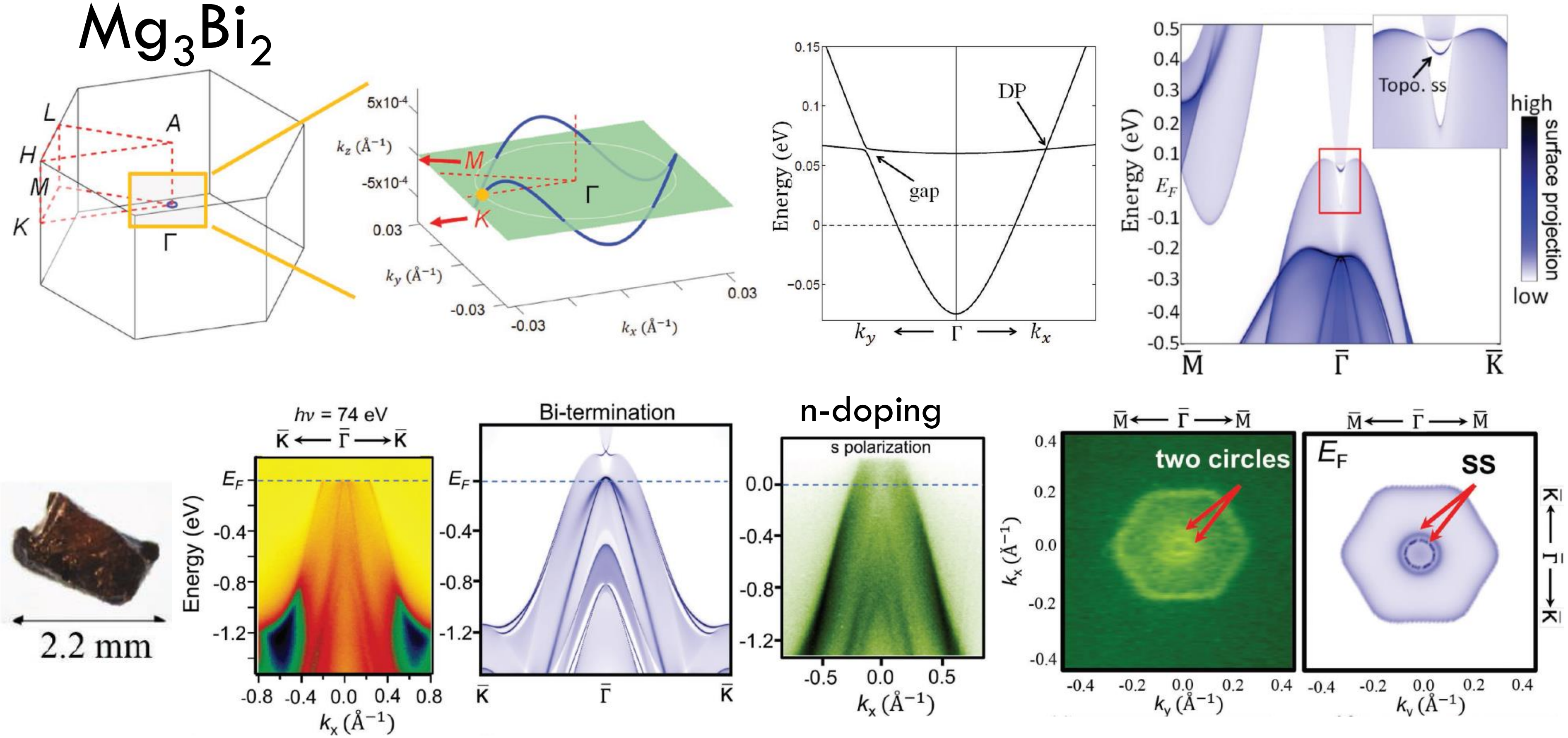
Type-I vs Type-II



*Type-II Weyl: A. A. Soluyanov et al.,Nature volume **527**, 495 (2016)*
*Type-II Dirac: T.-R. Chang et al., Phys. Rev. Lett. **119**, 026404 (2017)*

Type-II Nodal-line semimetal

Mg_3Bi_2

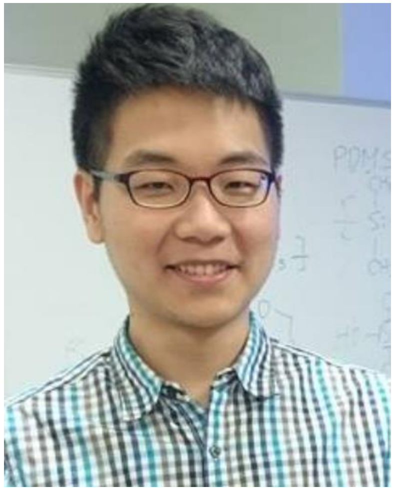


T.-R. Chang et al., Advanced Science (2018), DOI: 10.1002/advs.201800897

Acknowledgements (TCI)

Dr. Xiaoting Zhou
(NCKU)

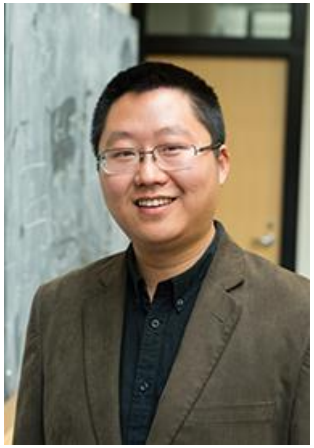
Chuang-Han Hsu
(NUS)



Prof. Hsin Lin
(Academia Sinica)

Dr. Su-Yang Xu
(MIT)

Prof. Liang Fu
(MIT)



X. Zhou et al., Phys. Rev. B ***98****, 241104(R) (2018)*

MOST 科技部 Ministry of Science and Technology

MOST(Taiwan) Young Scholar Fellowship:
MOST Grant for the Columbus Program

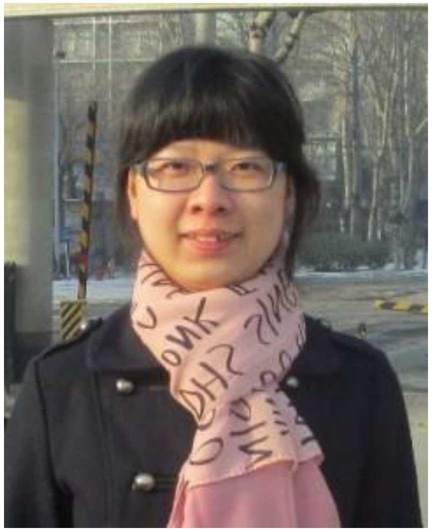
國立成功大學 National Cheng Kung University 1931

NCTS
National Center for Theoretical Sciences
Physics Division 國家理論科學研究中心 物理組

Acknowledgements (Nodal-line)

Prof. R. J. Cava
(Princeton U.)

Prof. Weiwei Xie
(Louisiana State U.)



Prof. Guang Bian
(University of Missouri)

T.-R. Chang et al., Advanced Science (2018), DOI: 10.1002/advs.201800897

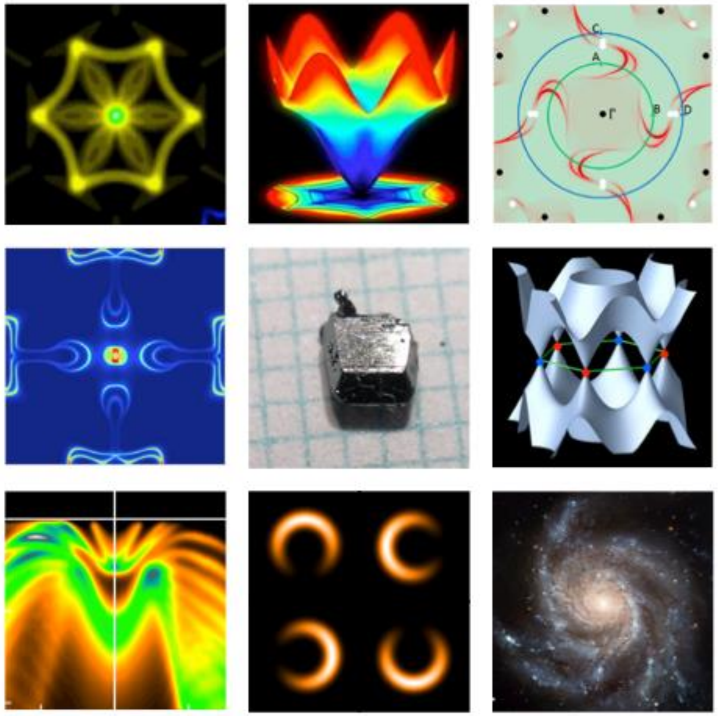
MOST 科技部
Ministry of Science and Technology

MOST(Taiwan) Young Scholar Fellowship:
MOST Grant for the Columbus Program

國立成功大學
National Cheng Kung University
1931

NCTS
National Center for Theoretical Sciences
Physics Division 國家理論科學研究中心 物理組

To see a world in a grain of sand … —William Blake



Thank you !