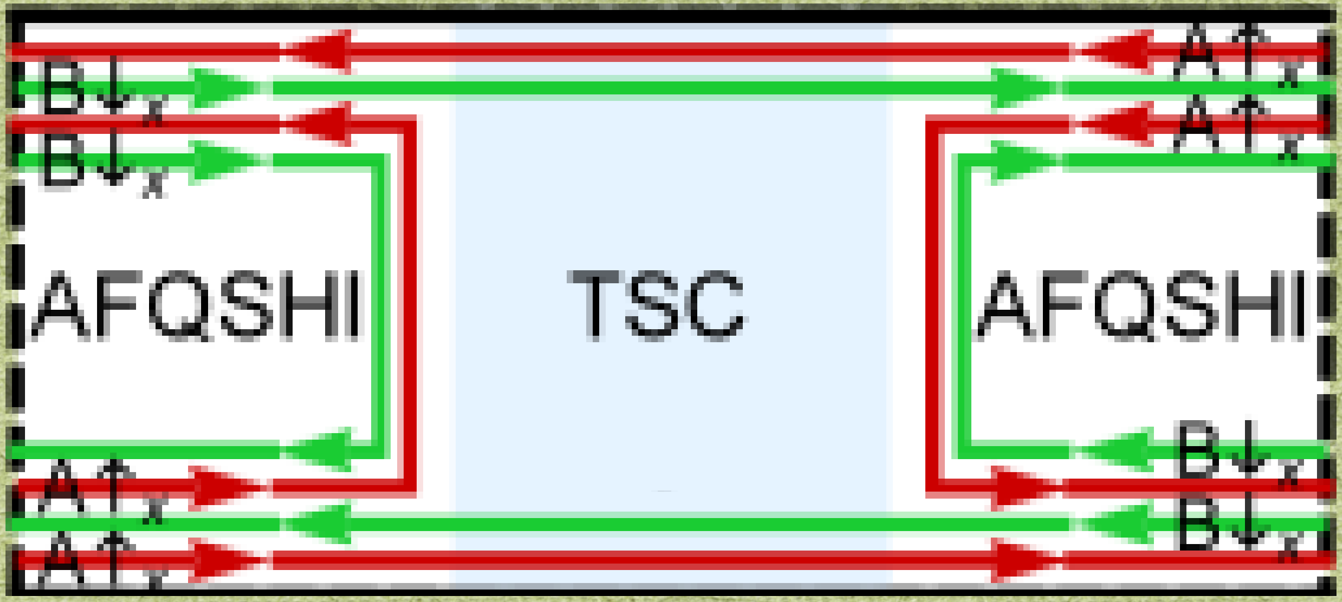
Realization of the Topological Superconductor Phases Protected by Chiral Symmetry



Chiral Matter and Topology Workshop at National Taiwan University
December 6, 2018

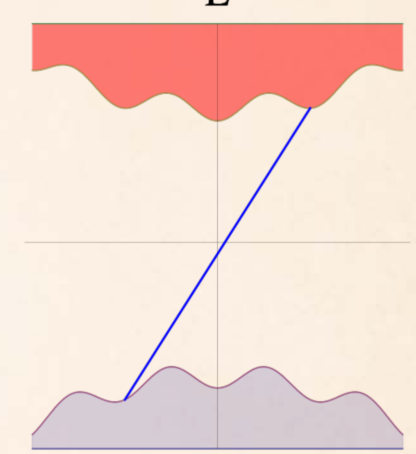
KITS

Ching-Kai Chiu (邱靖凱)
Kavli Institute for Theoretical Sciences
University of Chinese Academy of Sciences

The ten-fold classification of topological phases

Time-Reversal Symmetry **Particle-hole symmetry** **Chiral Symmetry**

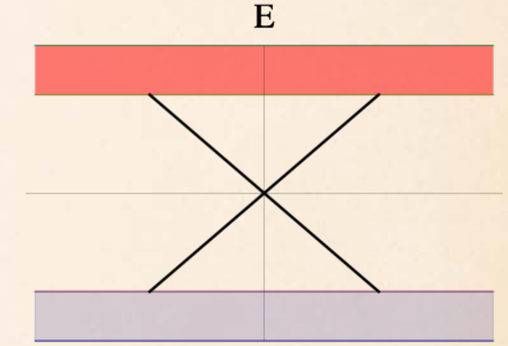
Integer Quantum Hall Effect, Quantum Anomalous Hall Effect
TKNN number



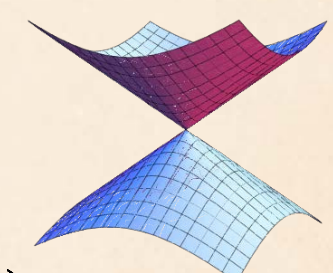
	T	C	S	0d	1d	2d	3d	4d
A	0	0	0	Z	0	Z	0	Z
AIII	0	0	1	0	Z	0	Z	0
BDI	+1	+1	1	Z_2	Z	0	0	0
D	0	+1	0	Z_2	Z_2	Z	0	0
DIII	-1	+1	1	0	Z_2	Z_2	Z	0
AII	-1	0	0	Z	0	Z_2	Z_2	Z
CII	-1	-1	1	0	Z	0	Z_2	Z_2
C	0	-1	0	0	0	Z	0	Z_2
CI	+1	-1	1	0	0	0	Z	0
AI	+1	0	0	Z	0	0	0	Z

Z_2 Time Reversal Symmetric Topological Insulators

2d



3d



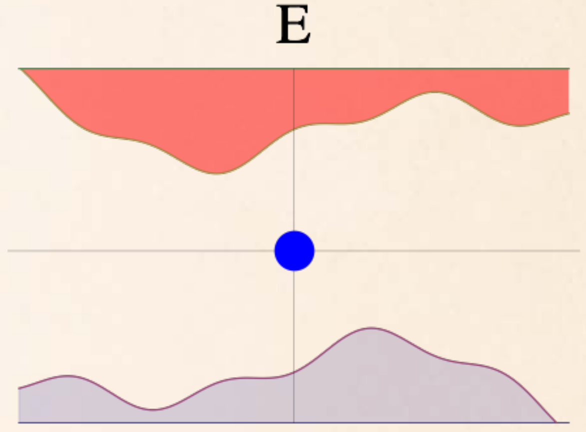
XL Qi, TL Hughes, **Shou-Cheng Zhang**, Physical Review B 78 (19), 195424 (2008)

The ten-fold classification of topological phases

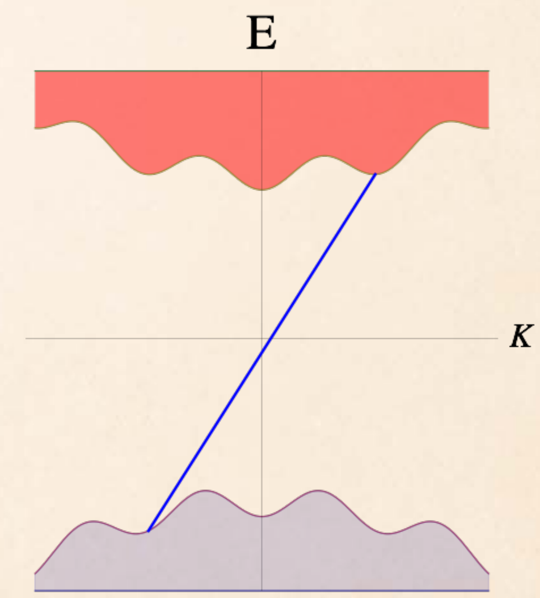
Topological Superconductors

	T	C	S	0d	1d	2d	3d	4d
A	0	0	0	Z	0	Z	0	Z
AIII	0	0	1	0	Z	0	Z	0
BDI	+1	+1	1	Z_2	Z	0	0	0
D	0	+1	0	Z_2	Z_2	Z	0	0
DIII	-1	+1	1	0	Z_2	Z_2	Z	0
AII	-1	0	0	Z	0	Z_2	Z_2	Z
CII	-1	-1	1	0	Z	0	Z_2	Z_2
C	0	-1	0	0	0	Z	0	Z_2
CI	+1	-1	1	0	0	0	Z	0
AI	+1	0	0	Z	0	0	0	Z

Z_2 Topological Superconductors hosting Majorana zero modes



Z Topological Superconductors hosting Majorana chiral edge modes



Science

Report

Evidence for Majorana bound states in an iron-based superconductor

Dongfei Wang[1,2,*], Lingyuan Kong[1,2,*], Peng Fan[1,2,*], Hui Chen[1], Shiyu Zhu[1,2], Wenyao Liu[1,2], Lu Cao[1,2], Yujie Sun[1,3], Shixuan Du[1,3,4], John Schneeloch[5], Ruidan Zhong[5], Genda Gu[5], Liang Fu[6], Hong Ding[1,2,3,4,†], Hong-Jun Gao[1,2,3,4,†]

[1]*Beijing National Laboratory for Condensed Matter Physics and Institute of Physics, Chinese Academy of Sciences (CAS), Beijing 100190, China.*
[2]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China.*
[3]*CAS Center for Excellence in Topological Quantum Computation, University of Chinese Academy of Sciences, Beijing 100190, China.*
[4]*Collaborative Innovation Center of Quantum Matter, Beijing 100190, China.*
[5]*Condensed Matter Physics and Materials Science Department, Brookhaven National Laboratory, Upton, NY 11973, USA.*
[6]*Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.*
↵†Corresponding author. Email: dingh@iphy.ac.cn (H.D.); hjgao@iphy.ac.cn (H.-J.G.)

nature
International journal of science

Letter | Published: 28 March 2018

Quantized Majorana conductance

Hao Zhang, Chun-Xiao Liu, Sasa Gazibegovic, Di Xu, John A. Logan, Guanzhong Wang, Nick van Loo, Jouri D. S. Bommer, Michiel W. A. de Moor, Diana Car, Roy L. M. Op het Veld, Petrus J. van Veldhoven, Sebastian Koelling, Marcel A. Verheijen, Mihir Pendharkar, Daniel J. Pennachio, Borzoyeh Shojaei, Joon Sue Lee, Chris J. Palmstrøm, Erik P. A. M. Bakkers, S. Das Sarma & Leo P. Kouwenhoven

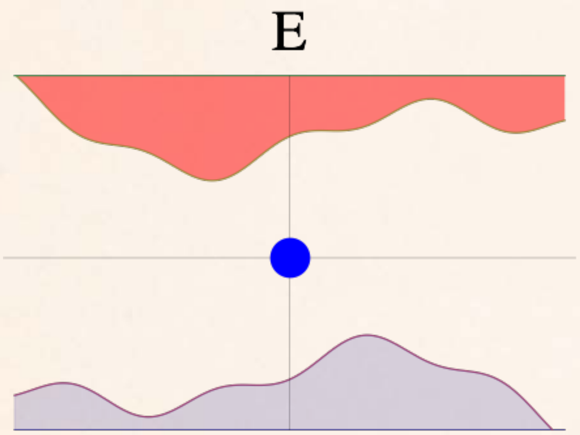
Nature **556**, 74–79 (05 April 2018) | Download Citation

Majorana Workshop at the Kavli Institute for Theoretical Sciences

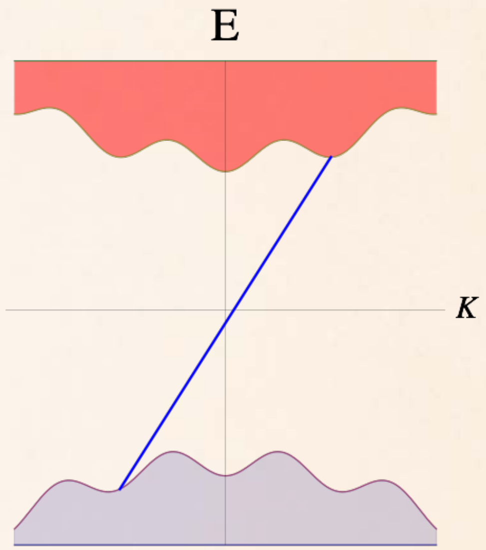
In Beijing, Jan 8th to 11th, 2019

Class D

Z_2 Topological Superconductors hosting Majorana zero modes



Z Topological Superconductors hosting Majorana chiral edge modes



Invited Speakers

Hong Ding, IOP CAS
Hao Zhang, Tsinghua University
Dong Liu, Tsinghua University
Jun-Yi Ge, Shanghai University
Tetsuo Hanaguri, Riken
Tadashi Machida, Riken

Roland Wiesendanger, University of Hamburg
Jinfeng Jia, Shanghai Jiao Tong University
Hao Zheng, Shanghai Jiao Tong University
Hai-Hu Wen, Nanjing University
Donglai Feng, Fudan University
Jun He, Riken

Organizers

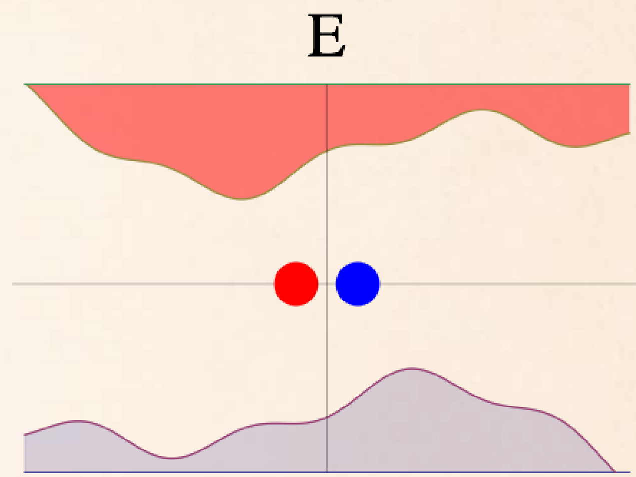
Fuchun Zhang, Ching-Kai Chiu, KITS

Time-reversal symmetry

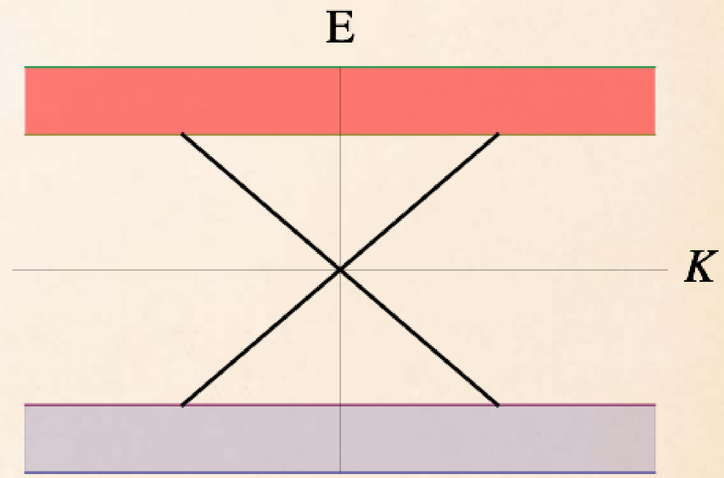
Topological Superconductors

	T	C	S	0d	1d	2d	3d	4d
A	0	0	0	Z	0	Z	0	Z
AIII	0	0	1	0	Z	0	Z	0
BDI	+1	+1	1	Z_2	Z	0	0	0
D	0	+1	0	Z_2	Z_2	Z	0	0
DIII	-1	+1	1	0	Z_2	Z_2	Z	0
AII	-1	0	0	Z	0	Z_2	Z_2	Z
CII	-1	-1	1	0	Z	0	Z_2	Z_2
C	0	-1	0	0	0	Z	0	Z_2
CI	+1	-1	1	0	0	0	Z	0
AI	+1	0	0	Z	0	0	0	Z

Z Time-reversal symmetric topological Superconductors



Z_2 Topological Superconductors hosting Majorana Helical edge modes



Topological Superconductor phases with chiral symmetry

	T	C	S	0d	1d	2d	3d	4d
BDI	+1	+1	1	Z_2	Z	0	0	0
DIII	-1	+1	1	0	Z_2	Z_2	Z	0

Time-reversal symmetry

$$TH(-k)T^{-1} = H(k)$$

Particle-hole symmetry

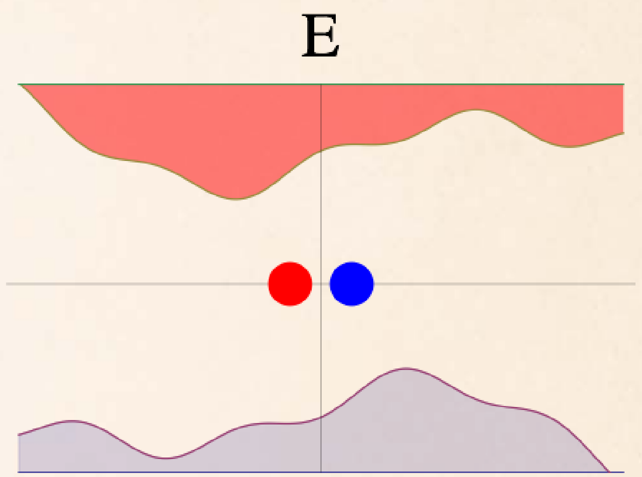
$$CH(-k)C^{-1} = -H(k)$$

Chiral symmetry

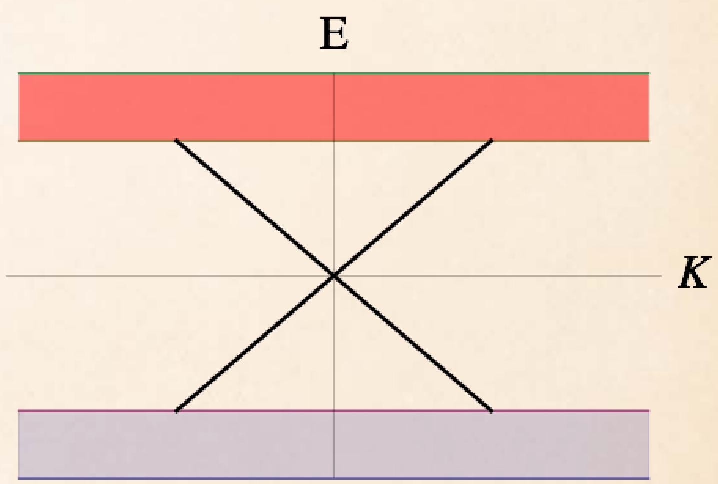
$$SH(-k)S^{-1} = -H(k)$$

Topological Superconductors

Z Time-reversal symmetric topological Superconductors



Z_2 Topological Superconductors hosting Majorana Helical edge modes



Topological Superconductors

Time-Reversal Symmetry **Particle-hole symmetry**

T	C	S

1d	2d	3d

Spinless time-reversal symmetric topological superconductors

1D: Multiple Majorana bound states

BDI	+1	+1	1	Z	0	0

Majorana operator

$$\gamma_i = \gamma_i^\dagger$$

Time reversal symmetry

$$T\gamma_i T^{-1} = \gamma_i$$

Two Majorana coupling

~~$$Ti\gamma_i\gamma_j T^{-1} = -i\gamma_i\gamma_j$$~~

Interaction of four Majorana fermions emerges

$$\gamma_i \gamma_j \gamma_k \gamma_l$$

The interaction preserves all of the symmetries

$$T \gamma_i \gamma_j \gamma_k \gamma_l T^{-1} = \gamma_i \gamma_j \gamma_k \gamma_l$$

This is a well-studied Majorana interacting system

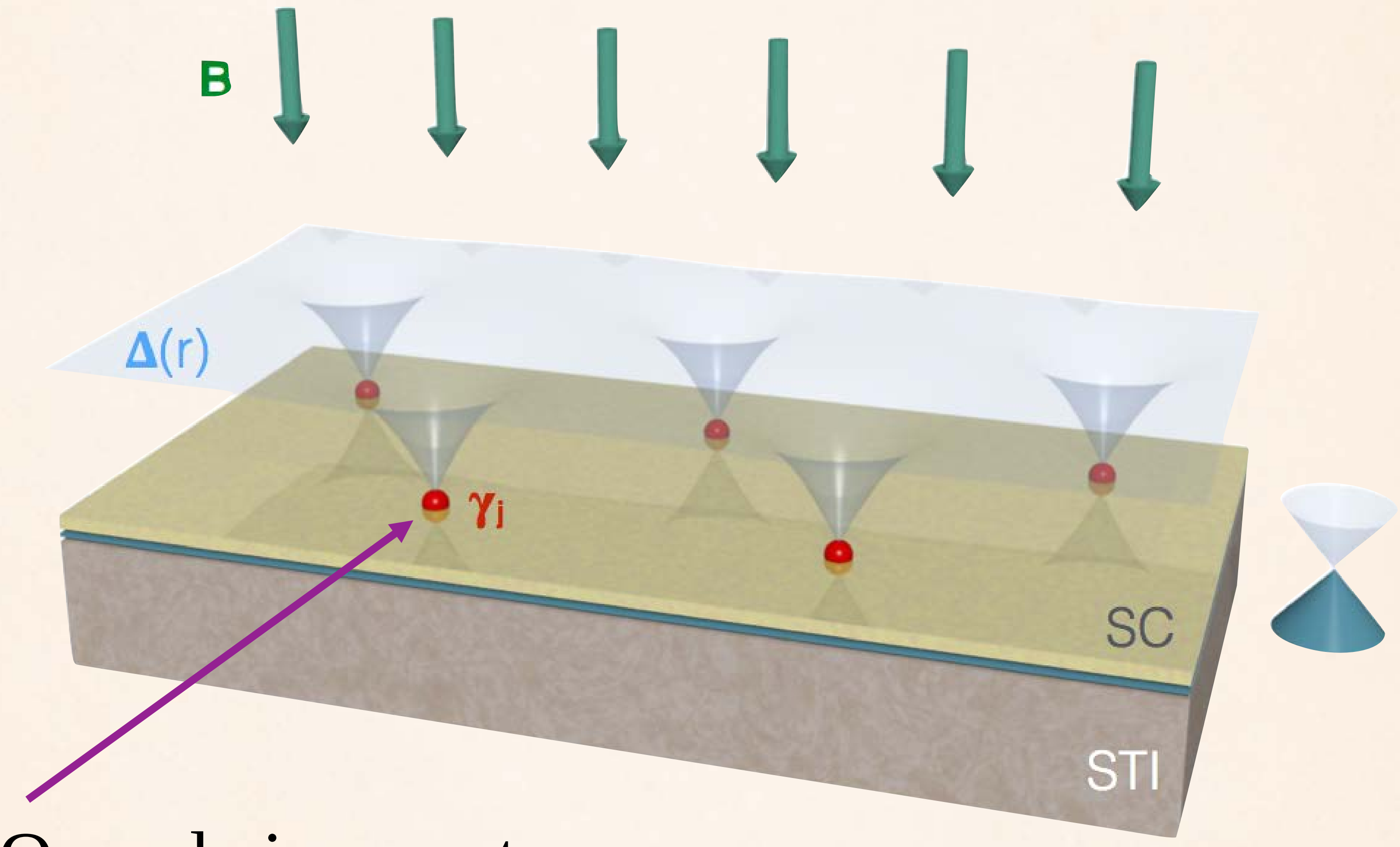
1. Majorana strip order (Y Kamiya, A Furusaki et al, PRB 98 (16), 161409 (2018))
2. Enlarge topological region (HH Hung, CKC et al, Scientific Reports (2017), Jian-Jian Miao, Fuchun Zhang, et al, Scientific Reports (2018))
3. Majorana fermion surface code for universal quantum computation (S Vijay, TH Hsieh, L Fu, PRX 2015)
4. Emergent Supersymmetry (A Rahmani et al, PRL 2015)

The **realization** of the class BDI platform

Setup: Fu-Kane model

L. Fu and C. L. Kane, Phys. Rev. Lett. 100, 096407 (2008).

Abrikosov lattice of vortices in the s-wave superconducting (SC) surface of a strong topological insulator (STI).



A Majorana ZERO mode in a vortex

CK Chiu, DI Pikulin, M Franz, Physical Review B 91 (16), 165402 (2015)

Hamiltonian of the superconducting surface of a STI

$$H_{\mathrm{FK}} = \tau^z(\mathbf{p}\cdot\sigma - \mu) + \Delta_R\tau^x + \Delta_I\tau^y$$

In basis of $\hat{\Psi}_{\mathbf{r}} = (c_{\uparrow\mathbf{r}}, c_{\downarrow\mathbf{r}}, c^\dagger_{\downarrow\mathbf{r}}, -c^\dagger_{\uparrow\mathbf{r}})$

$$\Delta_R = \Delta_0\cos\theta,\ \ \Delta_I = \Delta_0\sin\theta$$

Break time-reversal symmetry

Particle-hole symmetry is automatically preserved.

$C = \tau^y\sigma^y K \qquad C^2 = 1$

J.C.Y. Teo and C.L. Kane, PRB **82**, 115120 (2010)

Class D implies multiple Majorana zero modes are not stable due to Z_2 invariant.

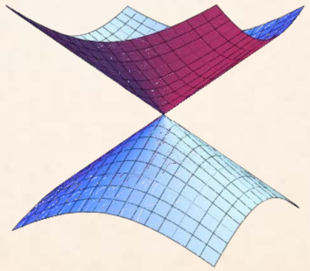
When chemical potential μ vanish, chiral symmetry is preserved.

$S = \tau^z\sigma^z$

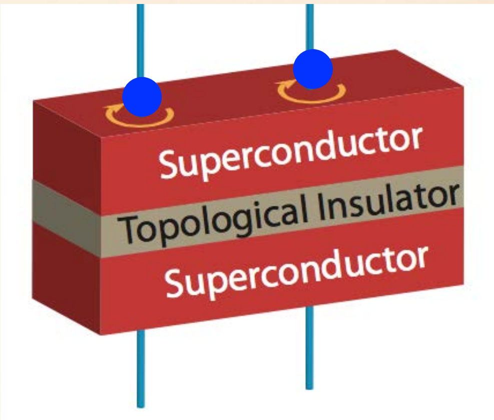
Time reversal symmetry operator T=SC. Class BDI

$T = \tau^x\sigma^x K, \quad T^2 = 1$

M. Cheng et al, PRB **82**, 094504 (2010)



Estimate interaction strength in real superconducting topological insulators



The heterostructure of $NbSe_2$ and Bi_2Te_3

Jin-Feng Jia et al, Phys. Rev. Lett. 112, 217001 (2014)

Microscopic origin of the interaction terms

$$U = \frac{1}{2}\int\int d^2r d^2r' \hat{\rho}(\mathbf{r})V(\mathbf{r}-\mathbf{r}')\hat{\rho}(\mathbf{r}')$$
$$= \frac{1}{2}\sum_{ijkl}\gamma_i\gamma_j\gamma_k\gamma_l\int\int d^2r d^2r' \rho_{ij}(\mathbf{r})V(\mathbf{r}-\mathbf{r}')\rho_{kl}(\mathbf{r}')$$

Where

$$\rho_{ij}(\mathbf{r}) = \Psi_i^\dagger(\mathbf{r})\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\Psi_j(\mathbf{r})$$

The interaction strength $\sim$ from 3 to 0.5 meV (c.f. SC gap = 1meV)

CK Chiu, DI Pikulin, M Franz, Physical Review B 91 (16), 165402 (2015)
HH Hung, CKC et al, Scientific Reports (2017)

2D Topological Superconductors

Time-Reversal Symmetry → T; **Particle-hole symmetry** → C

T	C	S		1d	2d	3d

Time-Reversal symmetry breaking topological superconductors

1D: Majorana Chains (Majorana bound state $FeTe_{1-x}Se_x$)

2D: p+ip topological superconductors (Chiral Majorana edge mode)

D	0	+1	0	Z_2	**Z**	0

Spinfull Time-Reversal Topological Superconductors

1D: Majorana kramers' pair

2D: Helical Majorana edge mode

DIII	-1	+1	1	Z_2	**Z_2**	Z

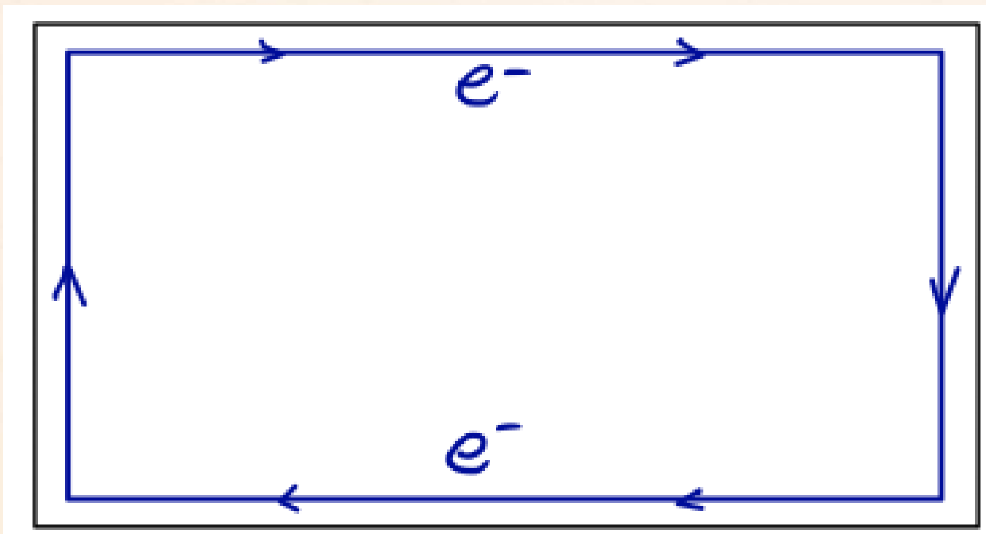
Start from 2D Topological Chern Insulator

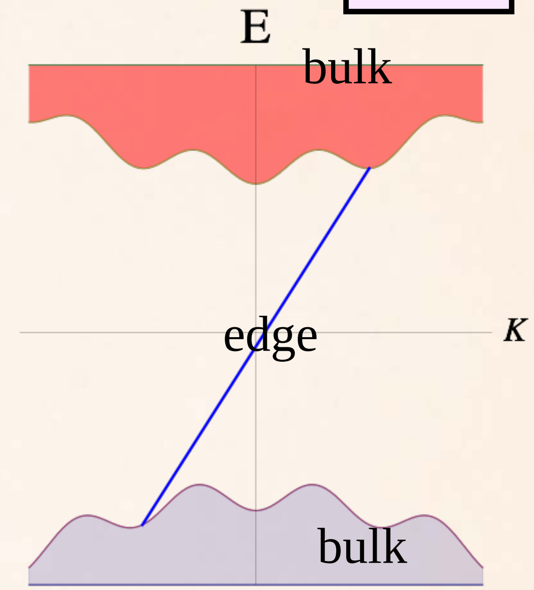
Quantum Anomalous Hall Insulator (QAHI, Chern Insulator)

= Integer Quantum Hall Insulator without magnetic field

Chang, C.-Z., et al., Science 340, 167 (2013)

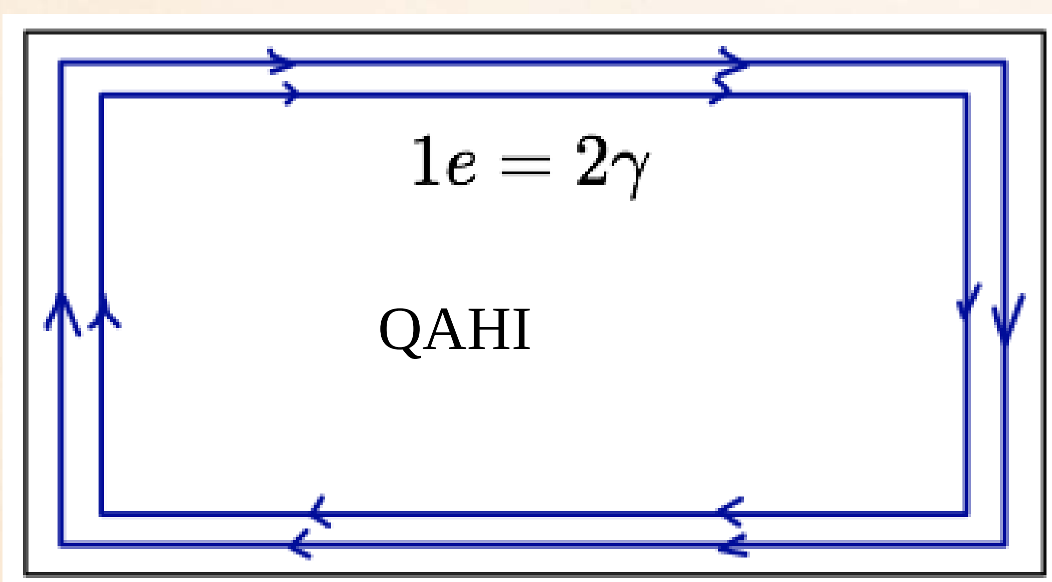
	T	C	S	2d
D	0	+1	0	Z

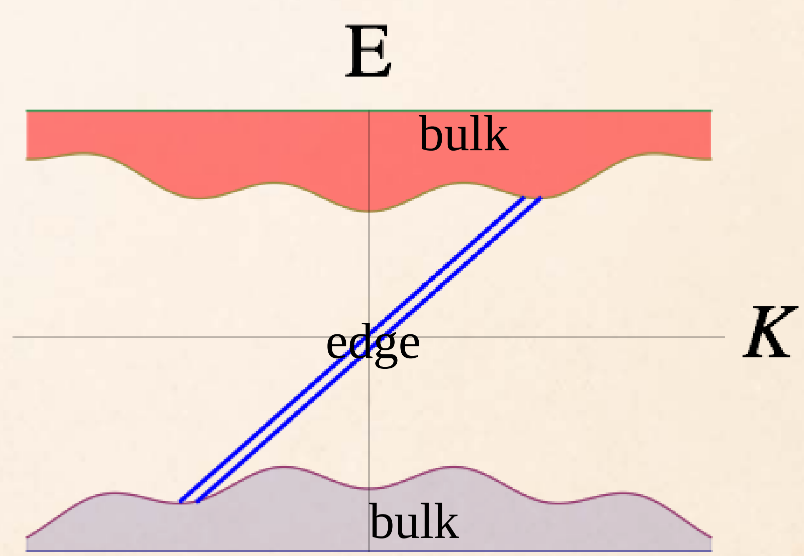




Superconducting Nambu basis (particle-hole)

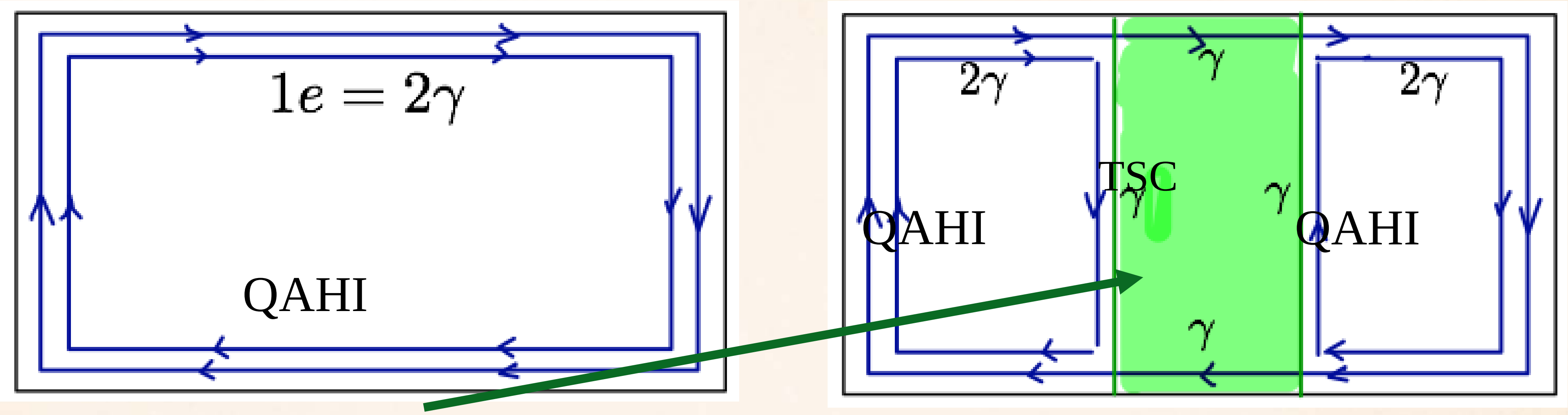
Two Majoranas = One electron $\gamma_1 + i\gamma_2 = C_e$





Single Chiral Majorana Edge Mode

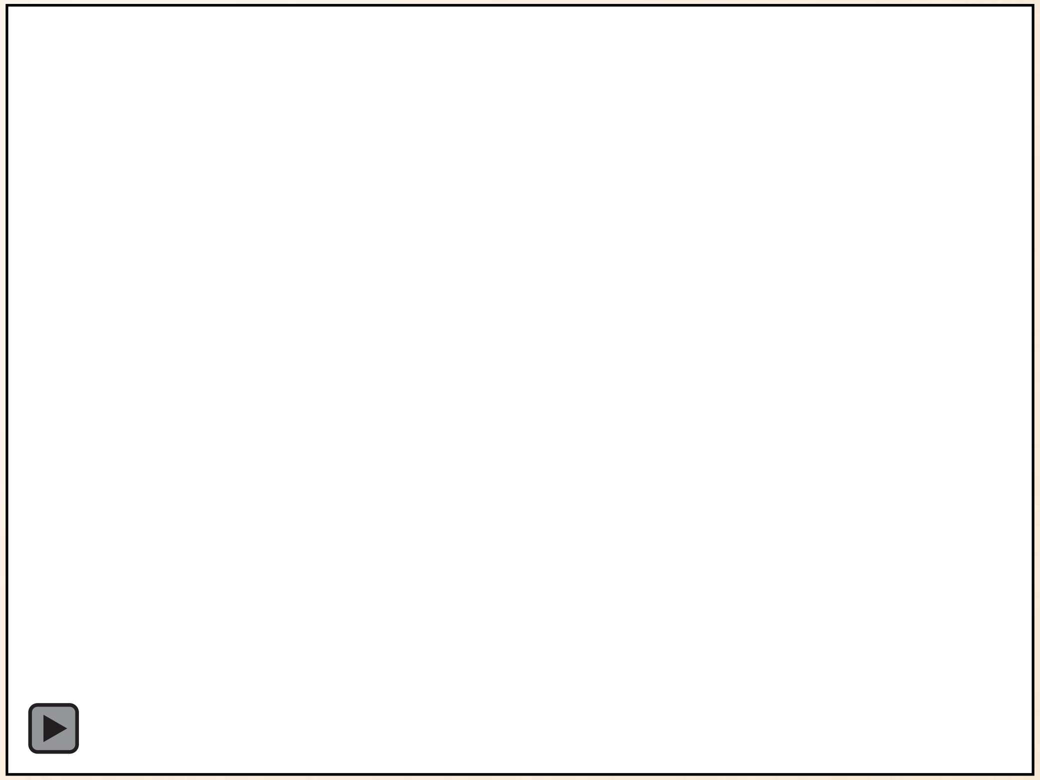
$p + ip$ topological superconductor (TSC)



An **s-weve superconductor** on the top of Quantum Anomalous Hall Insulator (QAHI)

X.-L. Qi, T. L. Hughes, S.-C. Zhang, Phys. Rev. B 82, 184516 (2010)

Edge and bulk spectrum



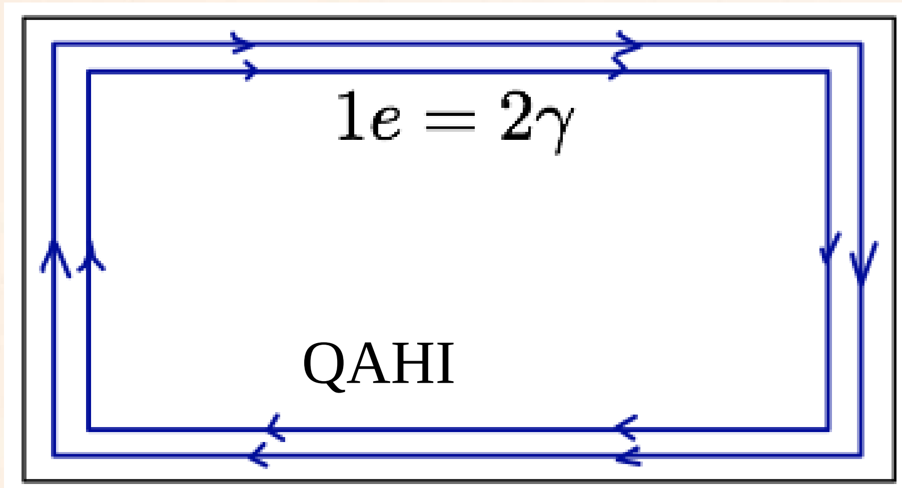
Two Majoranas = One electron

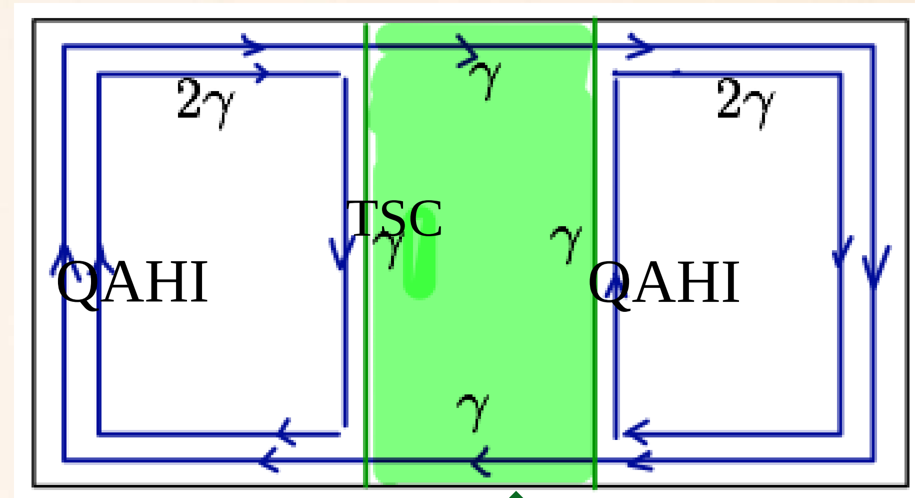
Δ is the superconducting gap

Start with $\Delta = 0$

Single Chiral Majorana Edge Mode

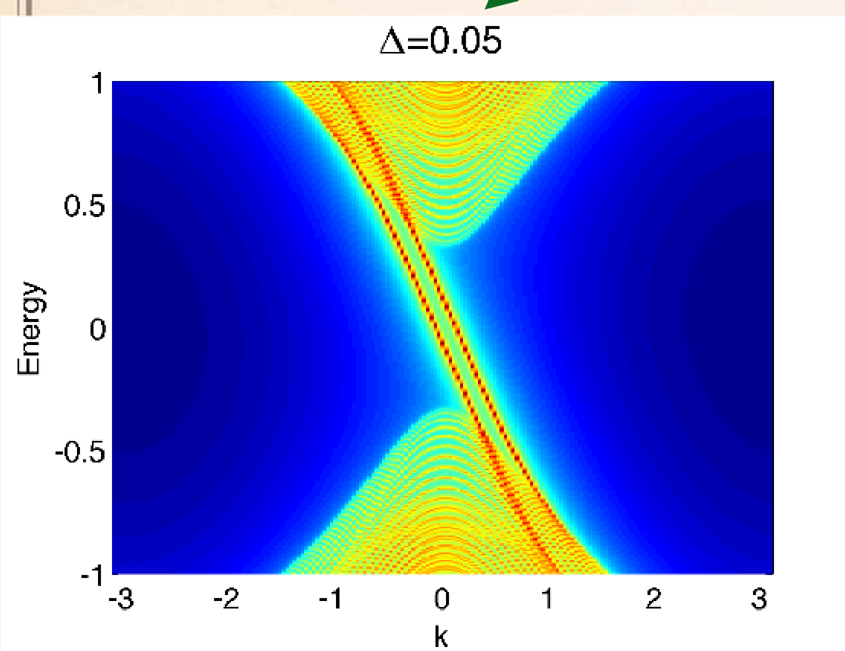
$p + ip$ topological superconductor (TSC)



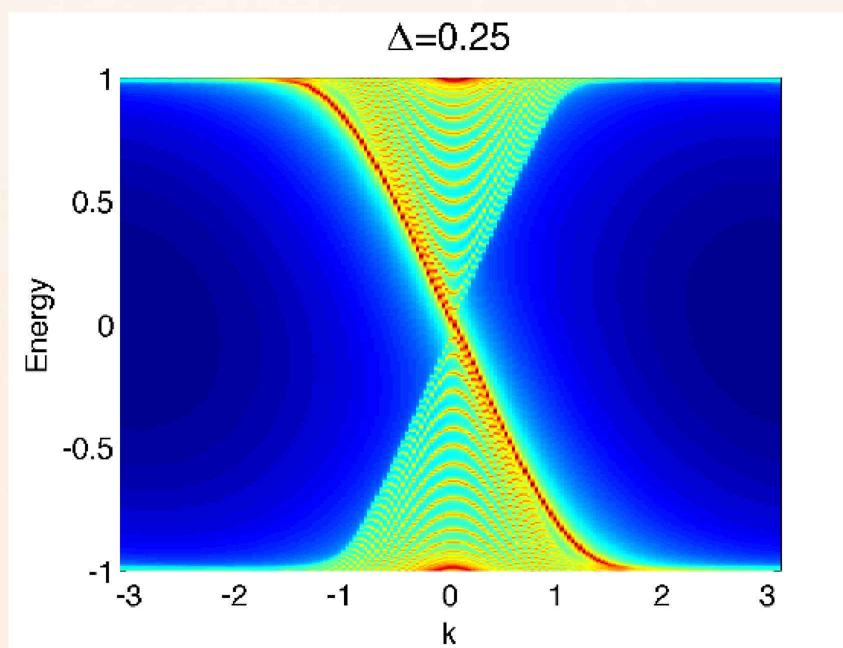


Green Region

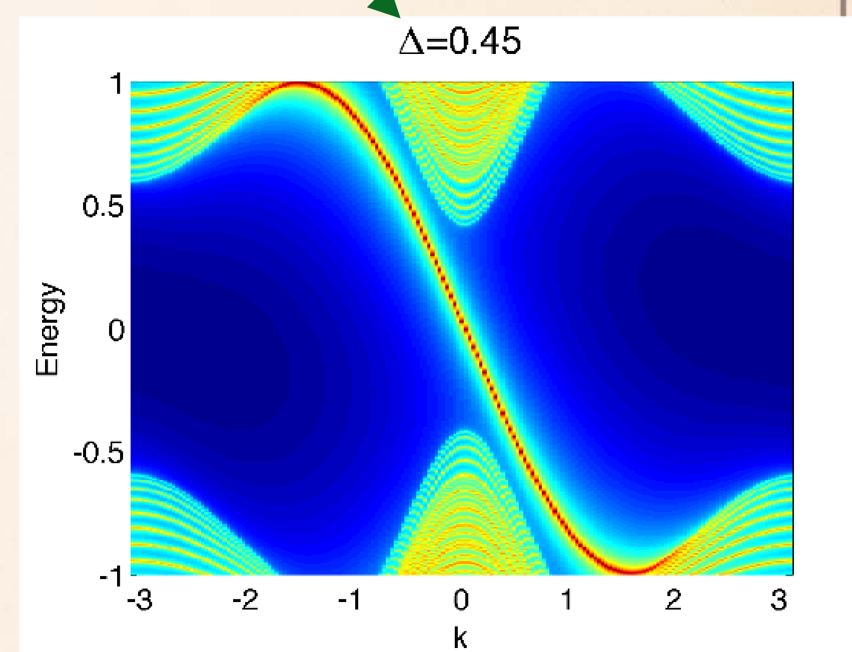
Topological Phase Transition



One electron mode =
Two Majorana modes



Bulk Gap closing

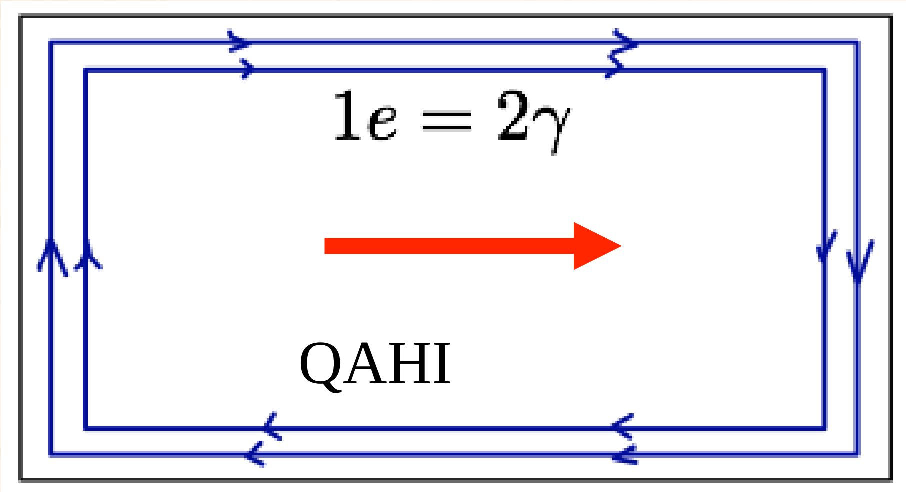


Single Chiral Majorana mode

Two-terminal Conductance

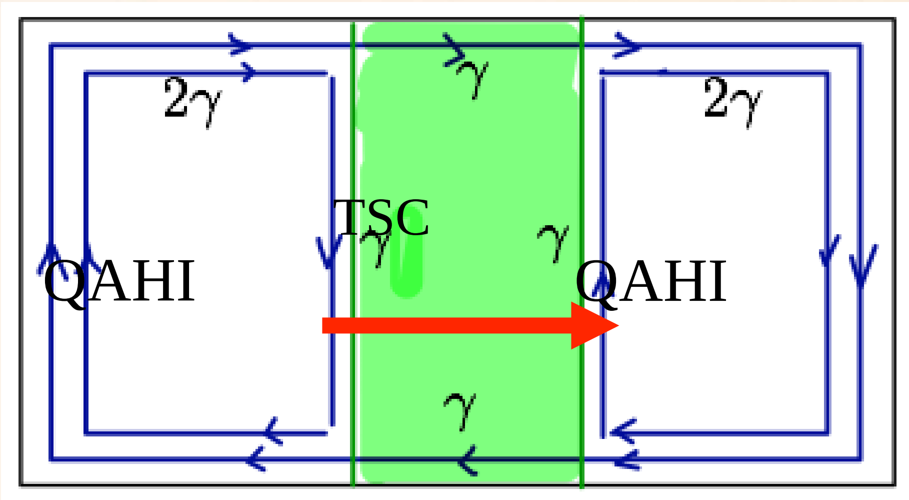
$p + ip$ topological superconductor

Lead 1



Lead 2

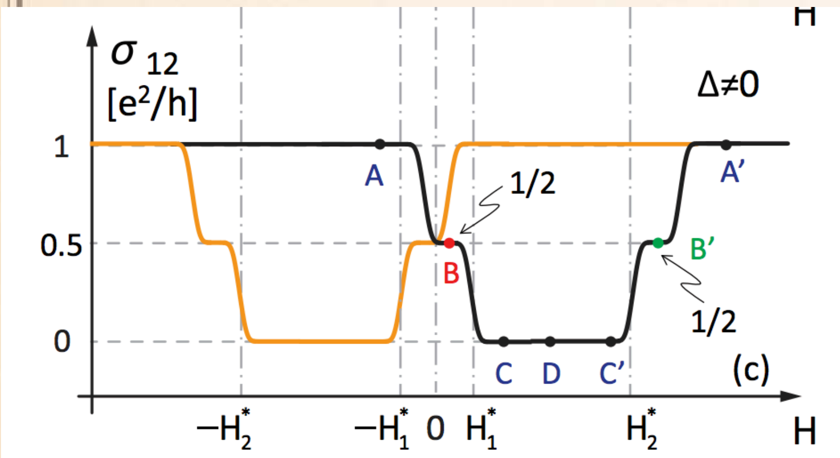
Lead 1

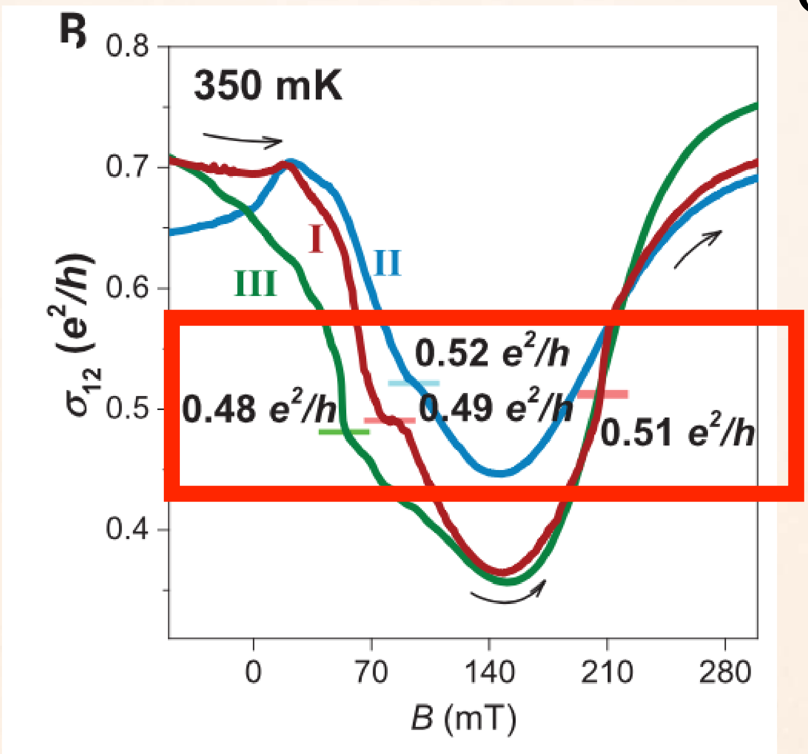


Lead 2

Longitudinal Conductance $\sigma_{12} = e^2/h$

Longitudinal Conductance $\sigma_{12} = e^2/2h$





Majorana is a half electron

J. Wang, Q. Zhou, B. Lian, S.-C. Zhang, Phys. Rev. B 92, 064520 (2015).

Q. L. He, L. Pan, A. L. Stern, E. C. Burks, X. Che, G. Yin, J. Wang, B. Lian, Q. Zhou, E. S. Choi, K. Mu- rata, X. Kou, Z. Chen, T. Nie, Q. Shao, Y. Fan, S.-C. Zhang, K. Liu, J. Xia, and K. L. Wang, Science 357, 294 (2017)

2D Topological Superconductors

Time-Reversal Symmetry **Particle-hole symmetry**

T	C	S

1d	2d	3d

Time-Reversal symmetry breaking topological superconductors

1D: Majorana Chains (Majorana bound state $FeTe_{1-x}Se_x$)

2D: p+ip topological superconductors (Chiral Majorana edge mode)

D	0	+1	0	Z_2	Z	0

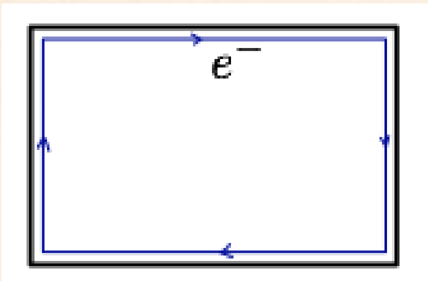
Spinfull Time-Reversal Topological Superconductors

1D: Majorana kramers' pair

2D: Helical Majorana edge mode

DIII	-1	+1	1	Z_2	Z_2	Z

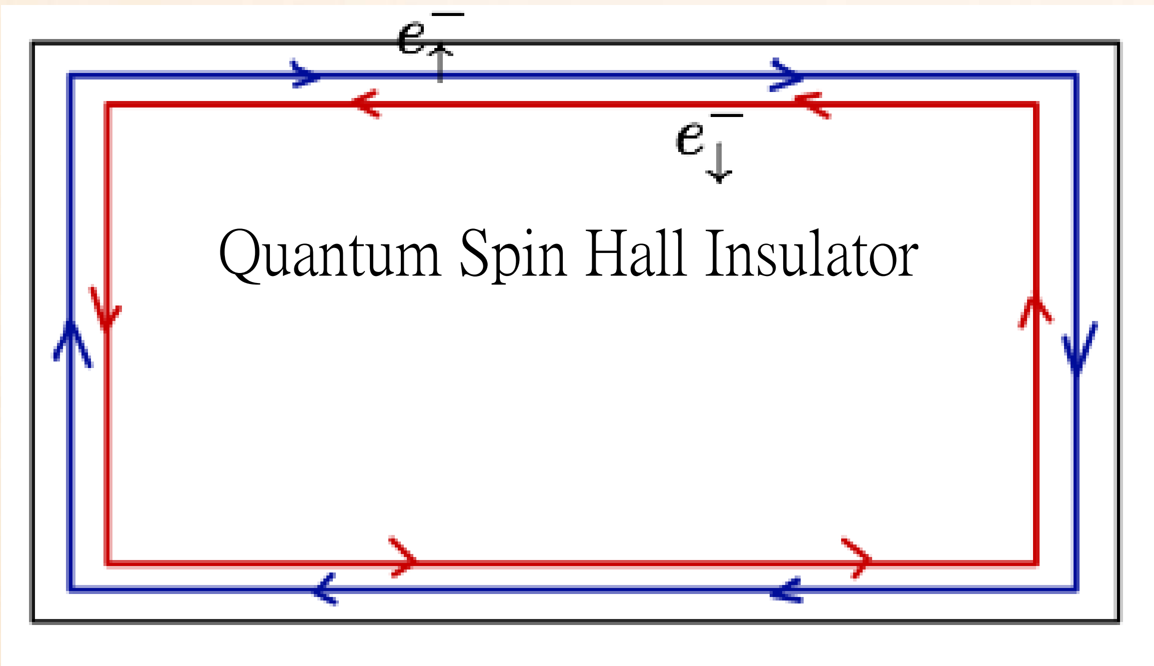
Time Reversal Symmetry



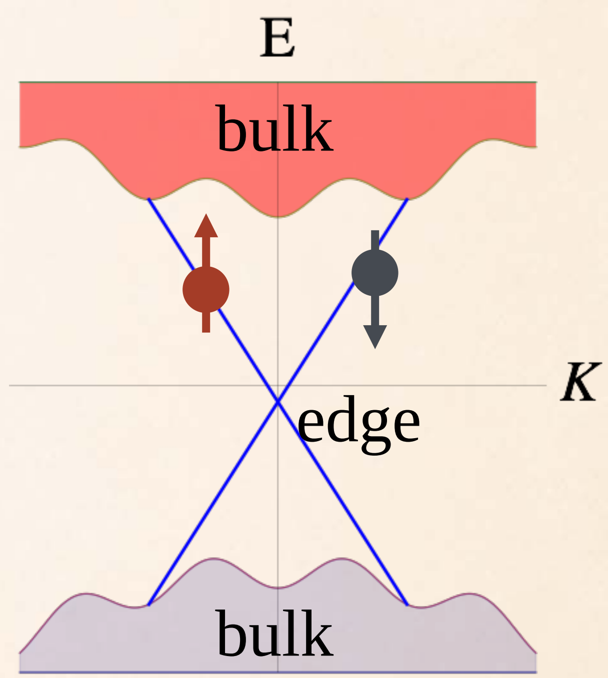
Integer Quantum Hall Effect in 1980
Chiral Electron Mode

Time Reversal Symmetry

Quantum Spin Hall Effect in 2005 ~2007
Helical Electron Mode protected by time reversal symmetry

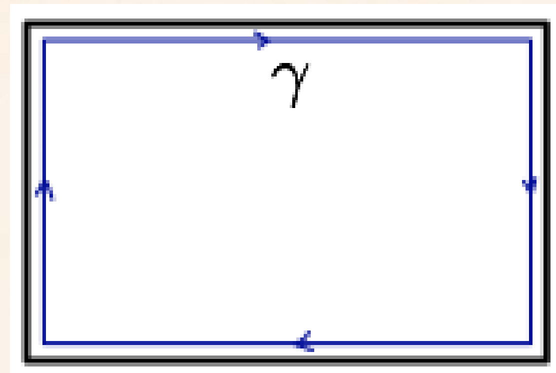


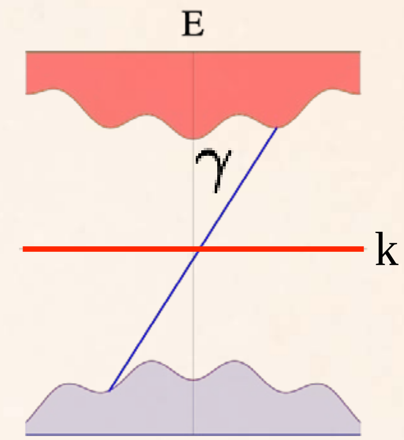
$$Te_\downarrow^- = e_\uparrow^-$$



	T	C	S	2d
A	0	0	0	Z
AII	-1	0	0	Z_2

Time Reversal Symmetry

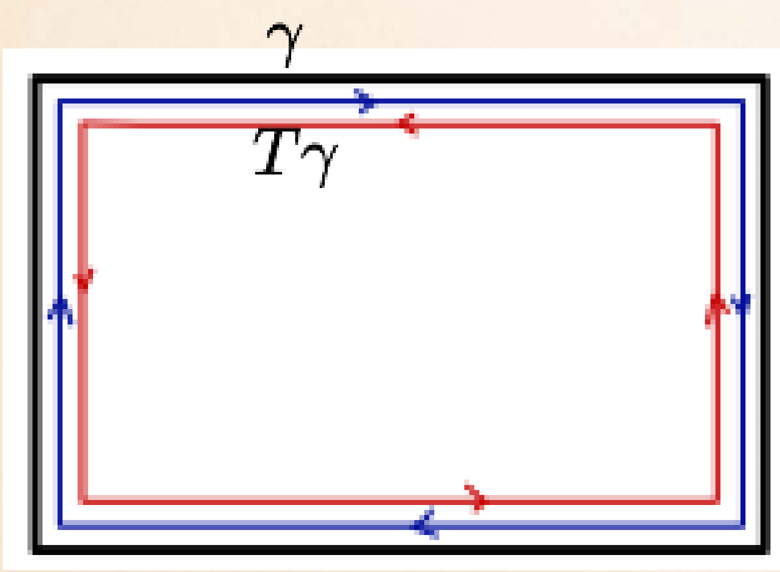


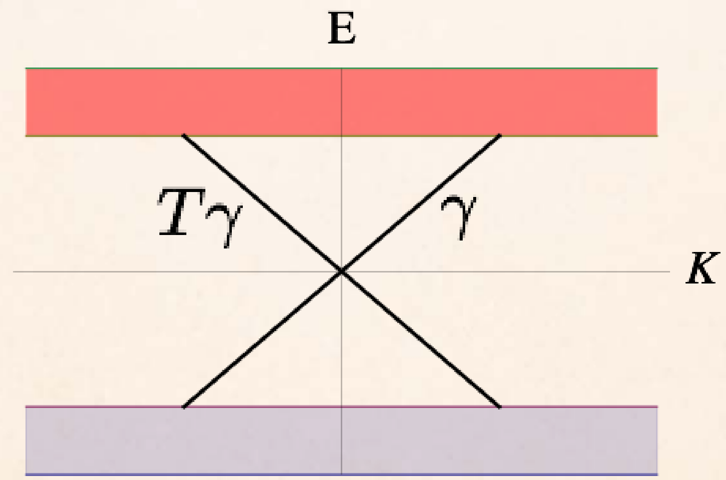


Chiral Topological Superconductor in 2016
Chiral Majorana Mode

Time Reversal Symmetry

Helical Topological Superconductor
Helical Majorana Mode





	T	C	S	2d
D	0	+1	0	Z
DIII	-1	+1	1	Z_2

Proposals to realize Helical Majorana edge mode

π Phase difference

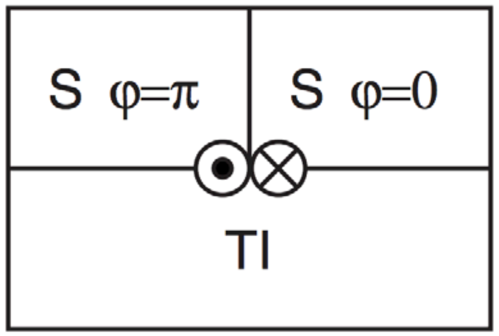


FIG. 6. Helical Majorana fermions at a linear Josephson junction with phase difference π on the surface of a topological insulator.

C.-X. Liu and B. Trauzettel, Phys. Rev. B 83, 220510 (2011).

J. C. Y. Teo and C. L. Kane, Phys. Rev. B 82, 115120 (2010).

Exotic Superconducting Pairing

$$\Delta(p_x + p_y i)c_\uparrow c_\uparrow$$
$$\Delta(p_x - p_y i)c_\downarrow c_\downarrow$$

M. Sato and S. Fujimoto, Phys. Rev. B 79, 094504 (2009).

S. Nakosai, Y. Tanaka, and N. Nagaosa, Phys. Rev. Lett. 108, 147003 (2012).

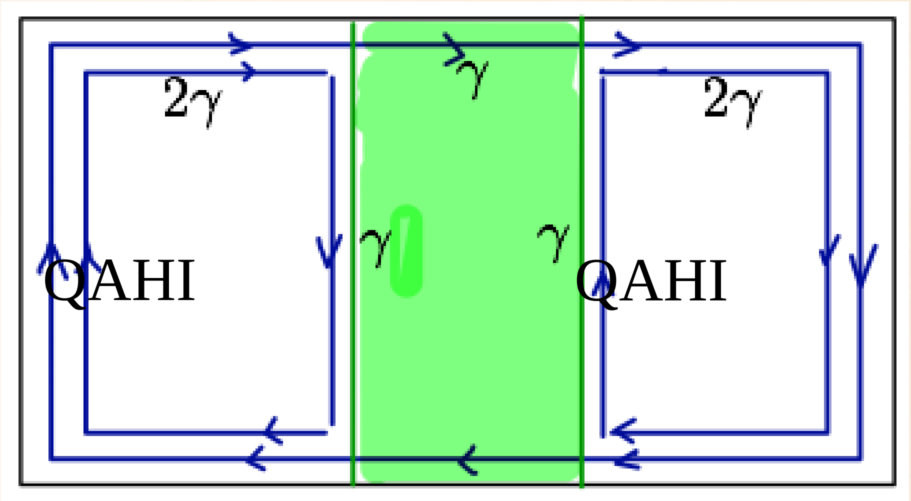
J. Wang, Y. Xu, and S.-C. Zhang, Phys. Rev. B 90, 054503 (2014).

J. Wang, Phys. Rev. B 94, 214502 (2016)

Idea

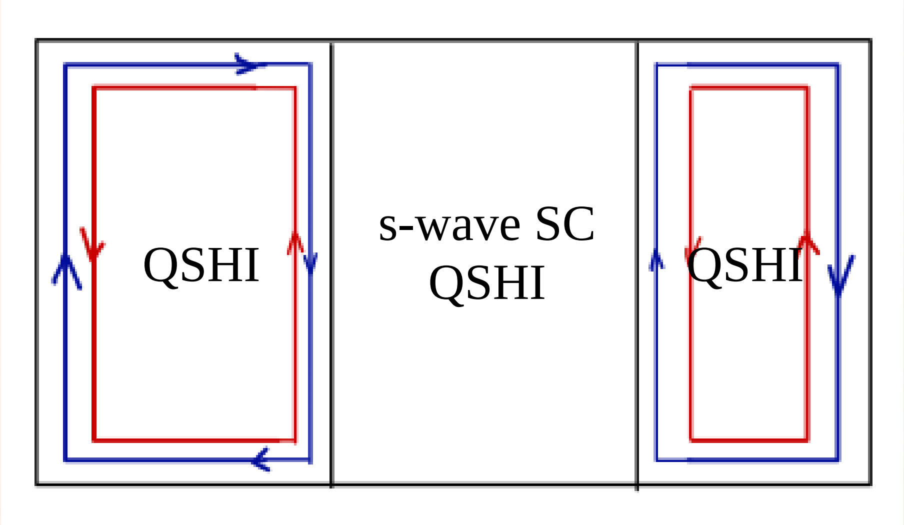
Quantum Anomalous Hall Insulator: QAHI

S-wave superconductor



Chiral Majorana edge mode

Quantum Spin Hall Insulator: QSHI



Helical Majorana edge mode?

Quantum Spin Hall Insulator with S-wave SC

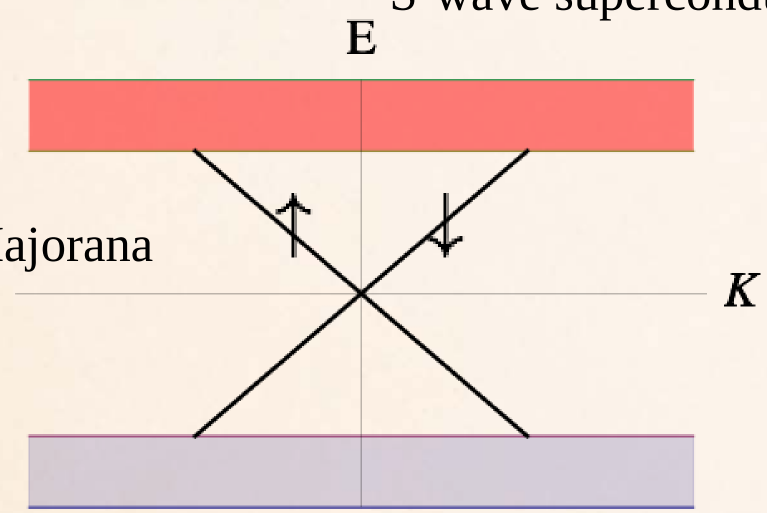
Yingyi Huang, **CKC**, Physical Review B 98 (8), 081412 (R)(2018)

Quantum Spin Hall Insulator (QSHI)

$$\begin{pmatrix} C_\uparrow^\dagger & C_\downarrow^\dagger & C_\uparrow & C_\downarrow \end{pmatrix} \begin{pmatrix} k & 0 & 0 & \Delta \\ 0 & -k & -\Delta & 0 \\ 0 & -\Delta & k & 0 \\ \Delta & 0 & 0 & -k \end{pmatrix} \begin{pmatrix} C_\uparrow \\ C_\downarrow \\ C_\uparrow^\dagger \\ C_\downarrow^\dagger \end{pmatrix} \qquad E_\pm = \pm\sqrt{k^2 + \Delta^2}$$

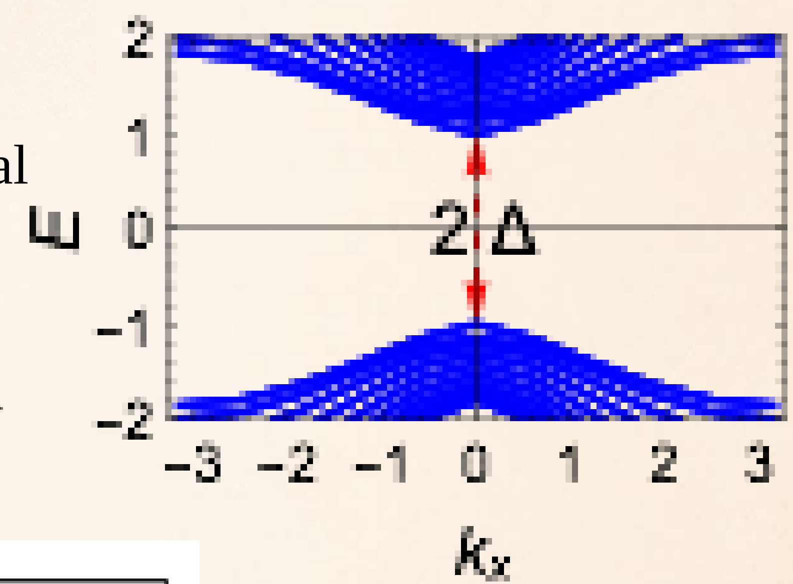
S-wave superconductivity destroys the helical electron mode

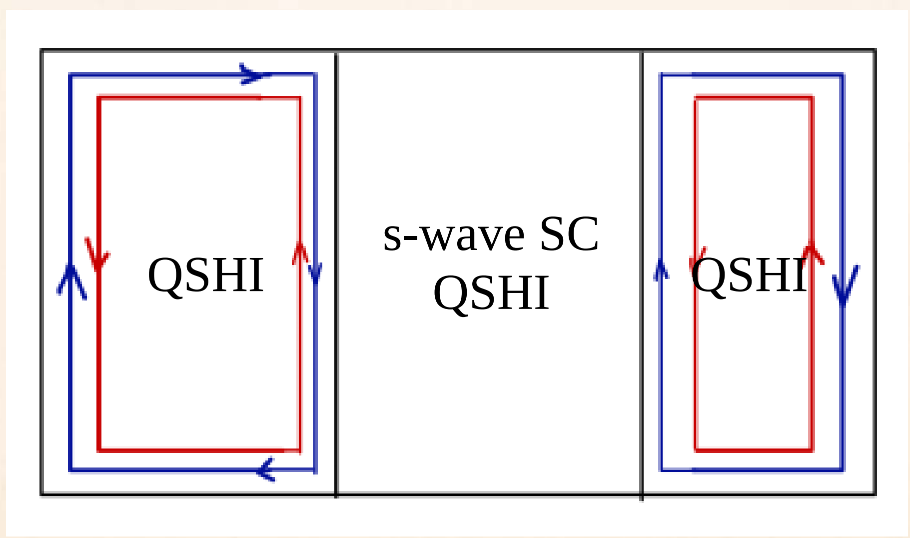
Electron, not Majorana



s-wave pairing potential

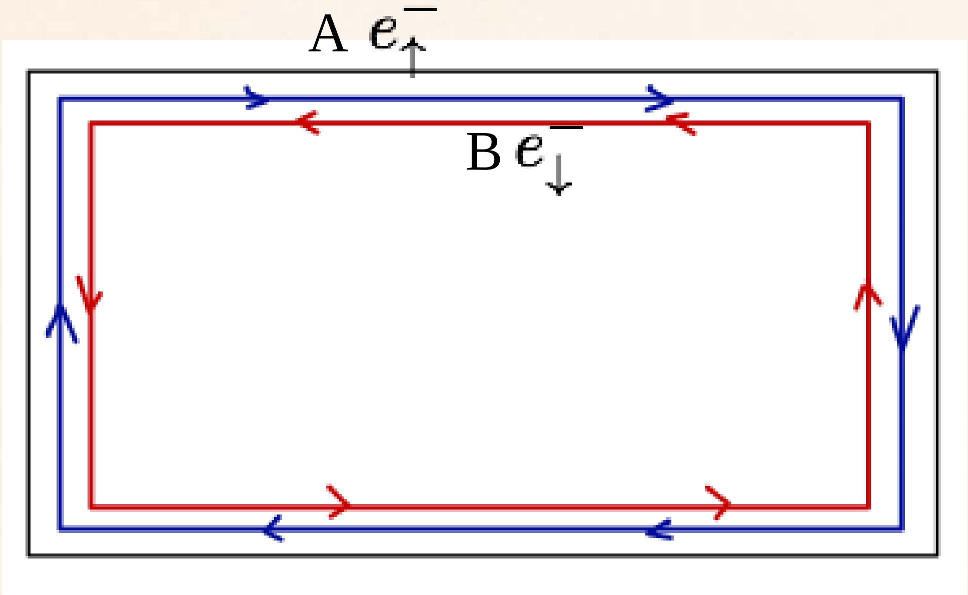
$$\Delta^* c_\uparrow^\dagger c_\downarrow^\dagger + \Delta c_\downarrow c_\uparrow$$



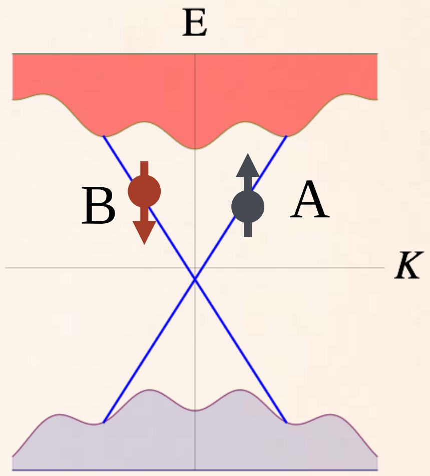


Antiferromagnetic Quantum Spin Hall Insulator

Z. Wang, H. Zhang, D. Liu, C. Liu, C. Tang, C. Song, Y. Zhong, J. Peng, F. Li, C. Nie, et al., Nature Materials 15, 968 (2016)
R. S. K. Mong, A. M. Essin, and J. E. Moore, Phys. Rev. B 81, 245209 (2010)



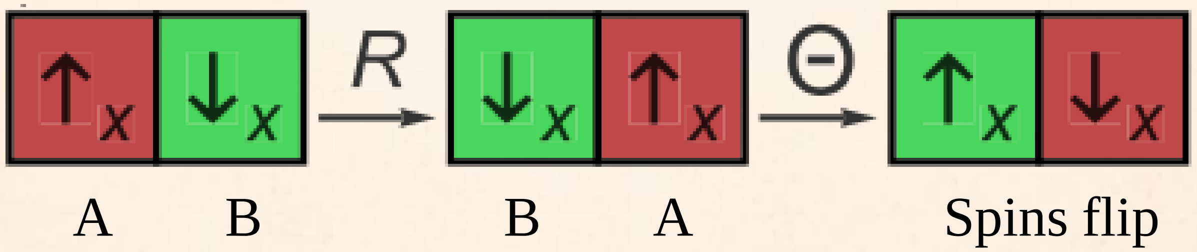
A helical electron mode



$$\Delta(A_\uparrow A_\downarrow + B_\uparrow B_\downarrow)$$

Antiferromagnetic order — Different spins located at different atoms

Effective time reversal symmetry (Use crystalline symmetry)



Model of the Antiferromagnetic Quantum Spin Hall Insulator

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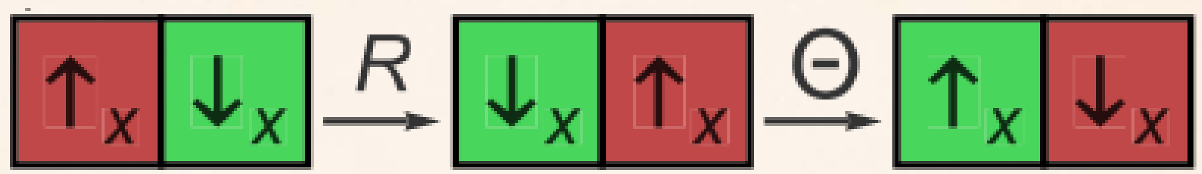


A B B A Spins flip

$$R = \tau_x$$

orbital

$$\Theta = i\sigma_y K$$

spin

Effective time reversal symmetry operator

$$\tilde{\Theta} = i\tau_x\sigma_y K$$

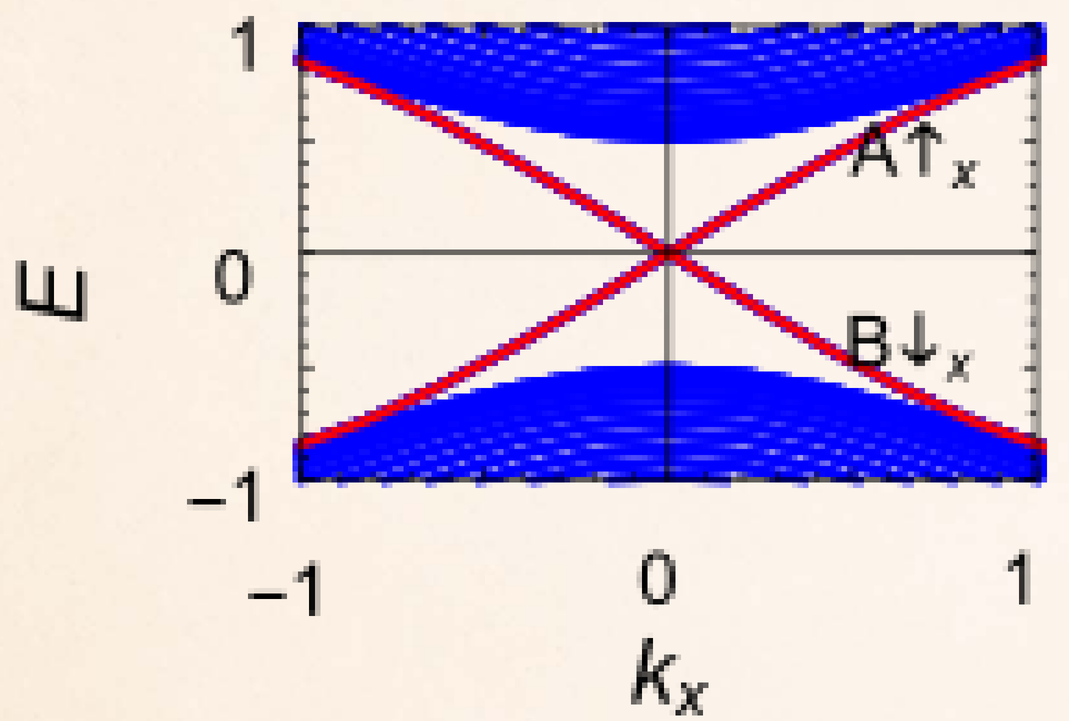
Hamiltonian

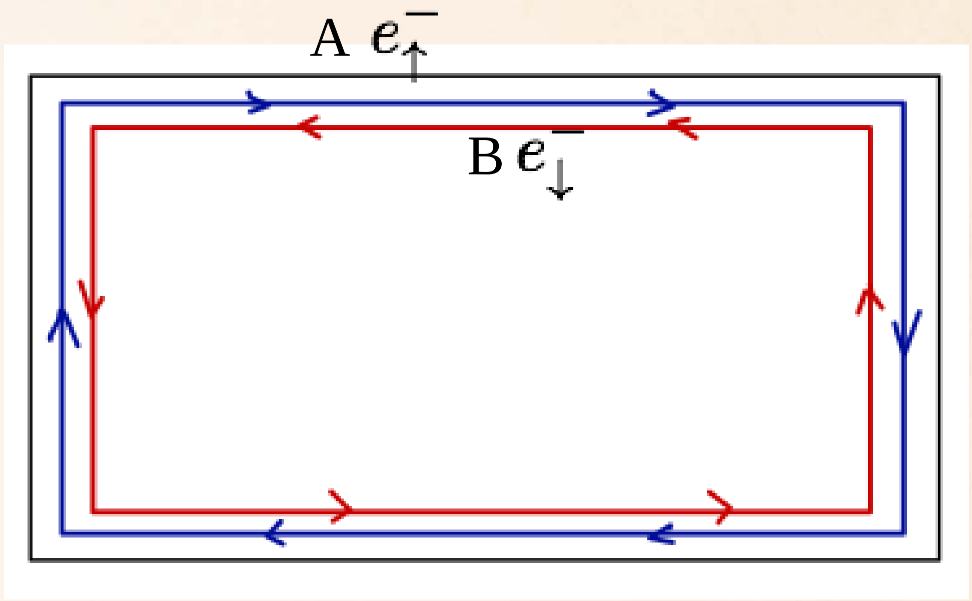
$$H_{\mathrm{AII}}(\vec{k}) = (m_0 + \cos k_x + \cos k_y)\tau_z\sigma_z + \sin k_x \tau_0\sigma_x + \sin k_y \tau_0 \sigma_y$$

Time-reversal symmetry

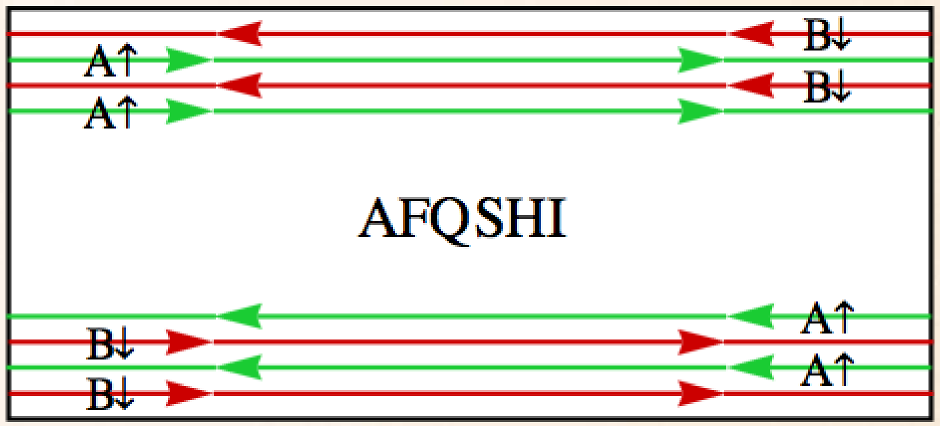
$$\tilde{\Theta} H_{\mathrm{AII}}(-\vec{k})\tilde{\Theta}^{-1} = H_{\mathrm{AII}}(\vec{k})$$

Helical Electron Edge Mode





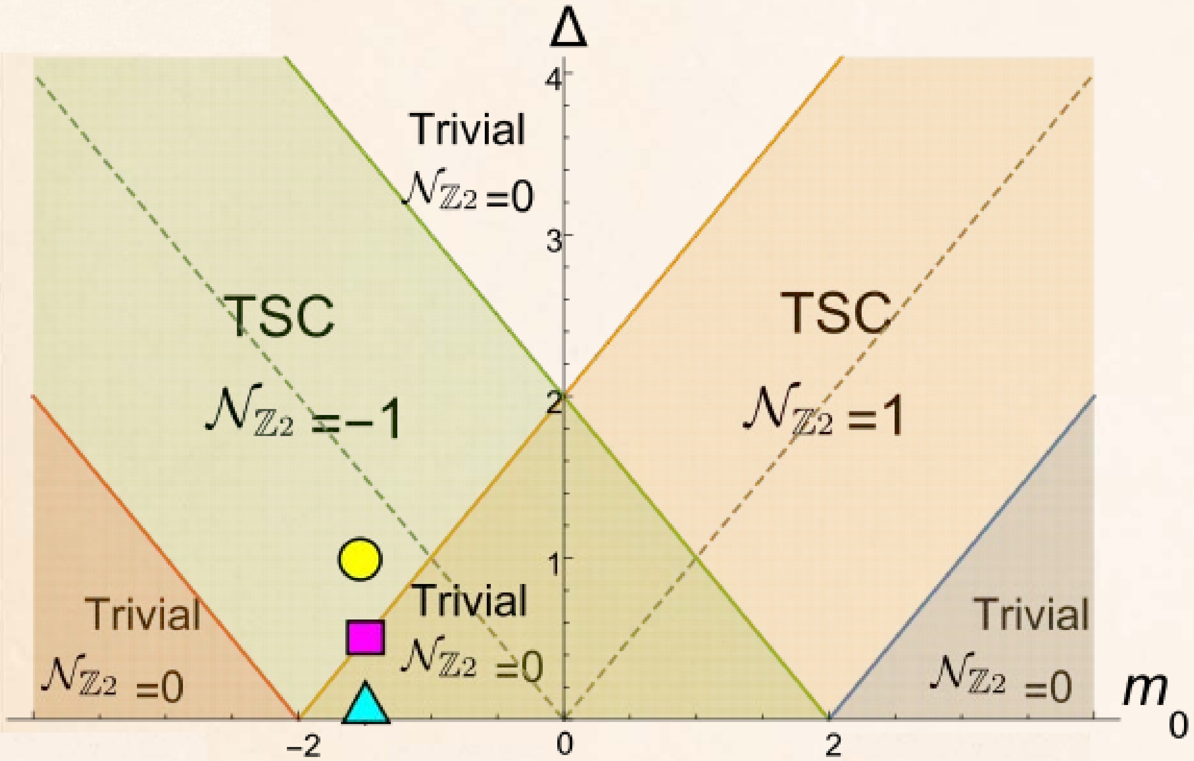
Two Helical Majorana modes



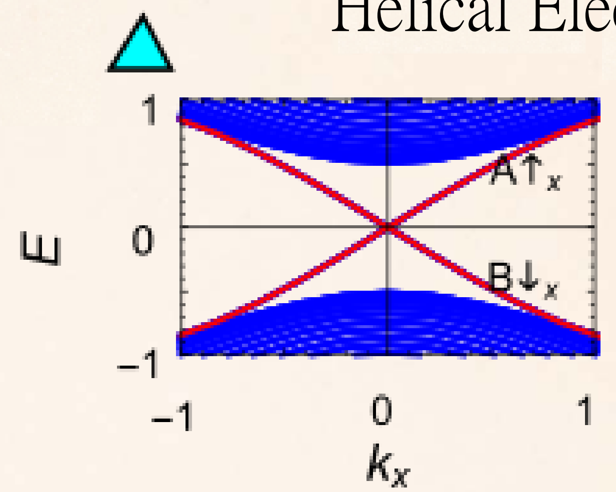
Non-Trivial

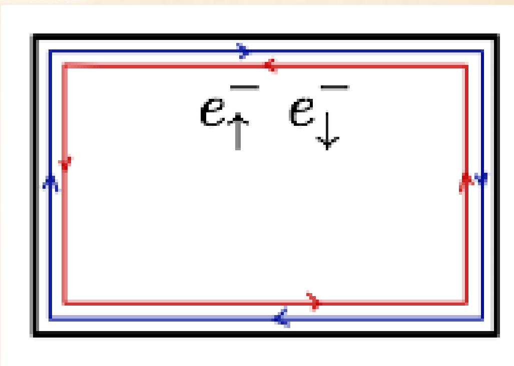
	T	C	S	2d
DIII	-1	+1	1	Z_2

Introduce s-wave superconductivity $\Delta(A_\uparrow A_\downarrow + B_\uparrow B_\downarrow)$

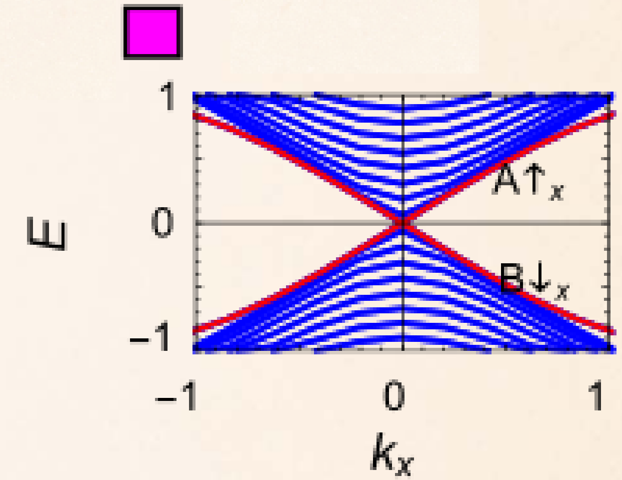


Helical Electron Edge Mode

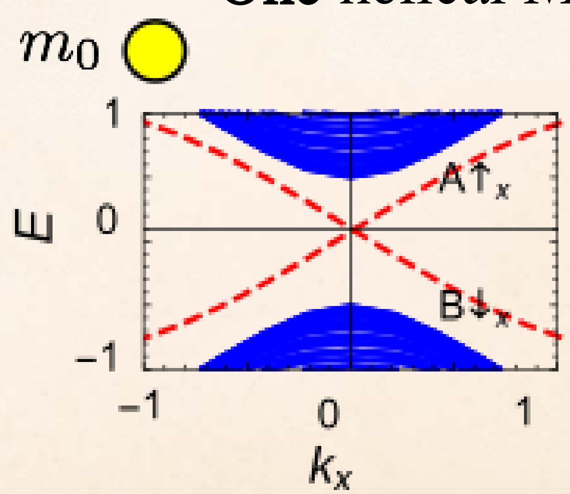


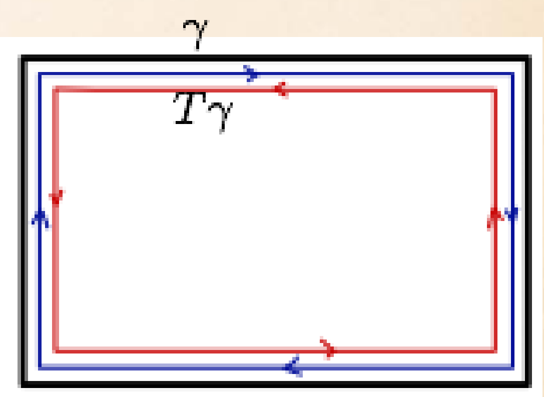


Topological Phase Transition



One helical Majorana Edge Mode





How to observe the Helical Majorana Edge Mode

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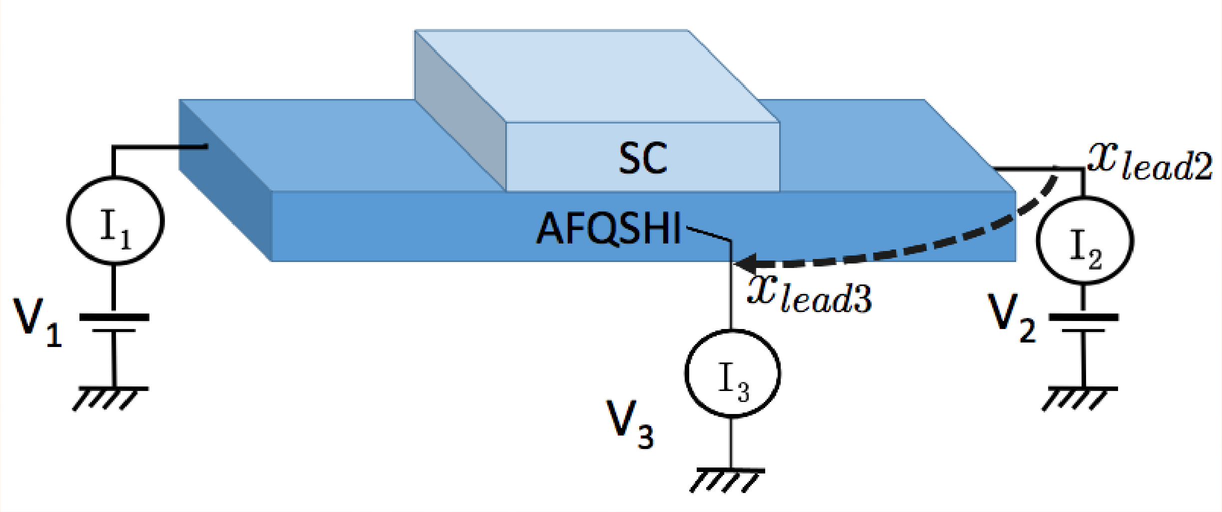
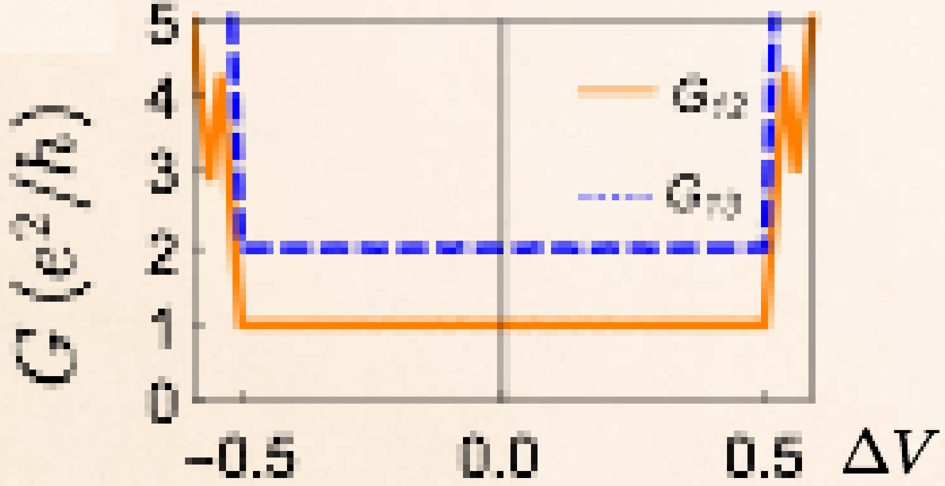
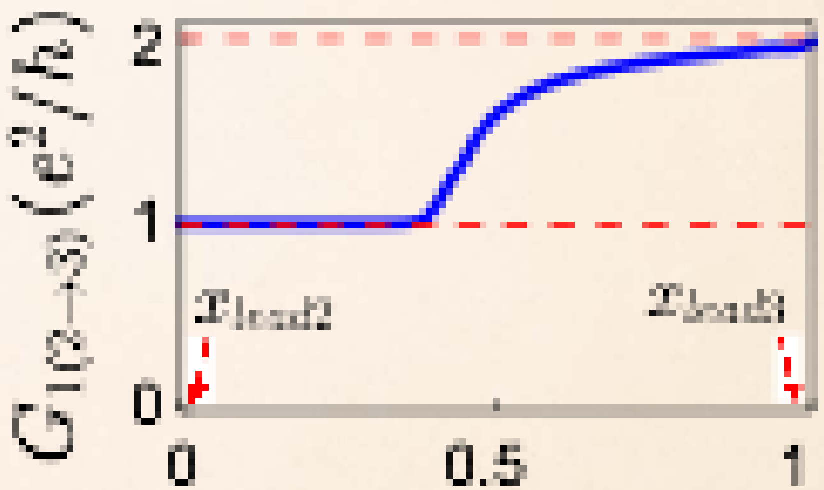


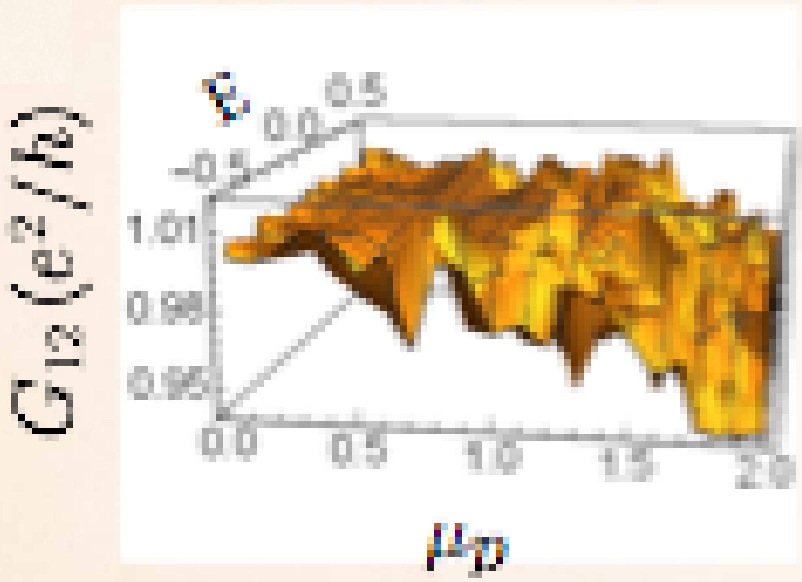
Two-terminal conductance $G_{12} = e^2/h$ and $G_{13} = 2e^2/h$



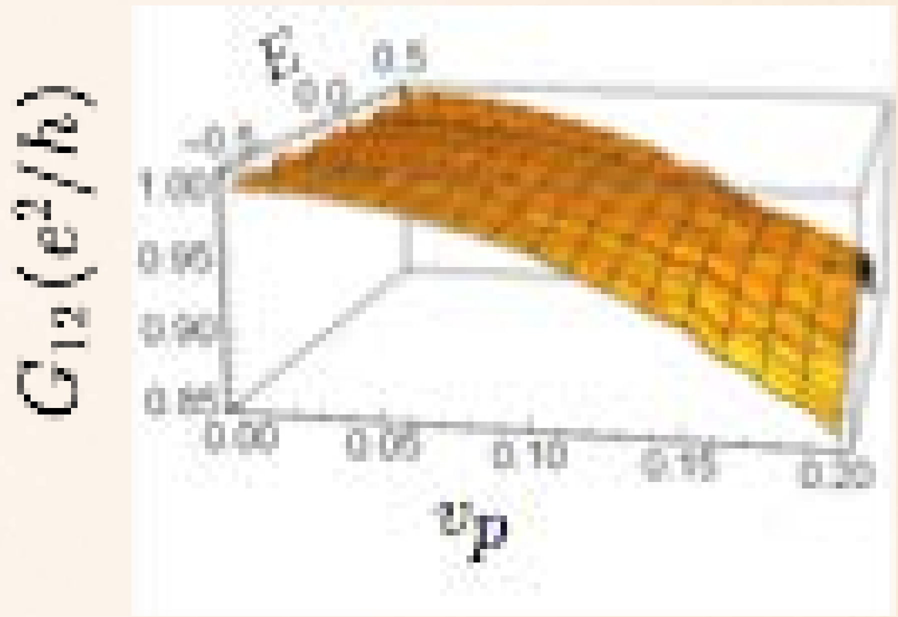


Disorders breaks the effective time-reversal symmetry

Potential difference disorder



Disorder destroys helical electron mode



The helical Majorana edge mode can be present in weak disorders

Conclusion

- A Majorana zero mode is trapped in a vortex on the superconducting Dirac cone surface. When the chemical potential is at the neutral point of the Dirac cone, the chiral symmetry emerges; the Majorana interactions dominate.
- An **antiferromagnetic** quantum spin Hall insulator with s-wave superconductivity can host a helical Majorana edge mode.
- When this topological superconductor is sandwiched by the spin Hall insulator (I-S-I), $\mathbf{e^2/h}$ conductance measurement is the primary step to reveal this topological state.
- The helical Majorana edge mode is under the effective time-reversal symmetry protection. The Majorana mode can survive only in weak disorders.

Thank you !