Lattice field theory and physics beyond the Standard Model

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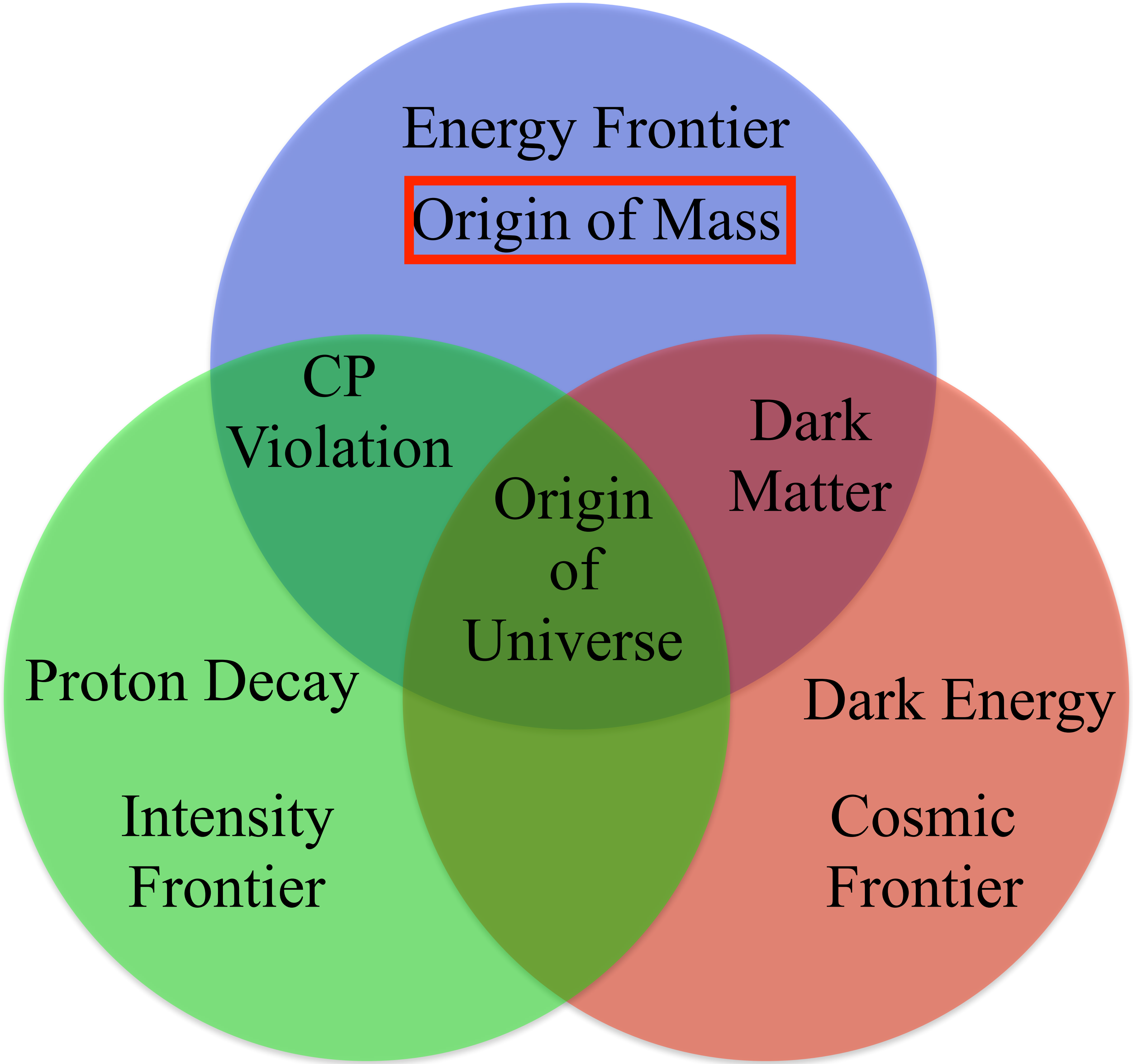
High-energy physics frontiers



Energy Frontier

Origin of Mass

CP Violation

Dark Matter

Origin of Universe

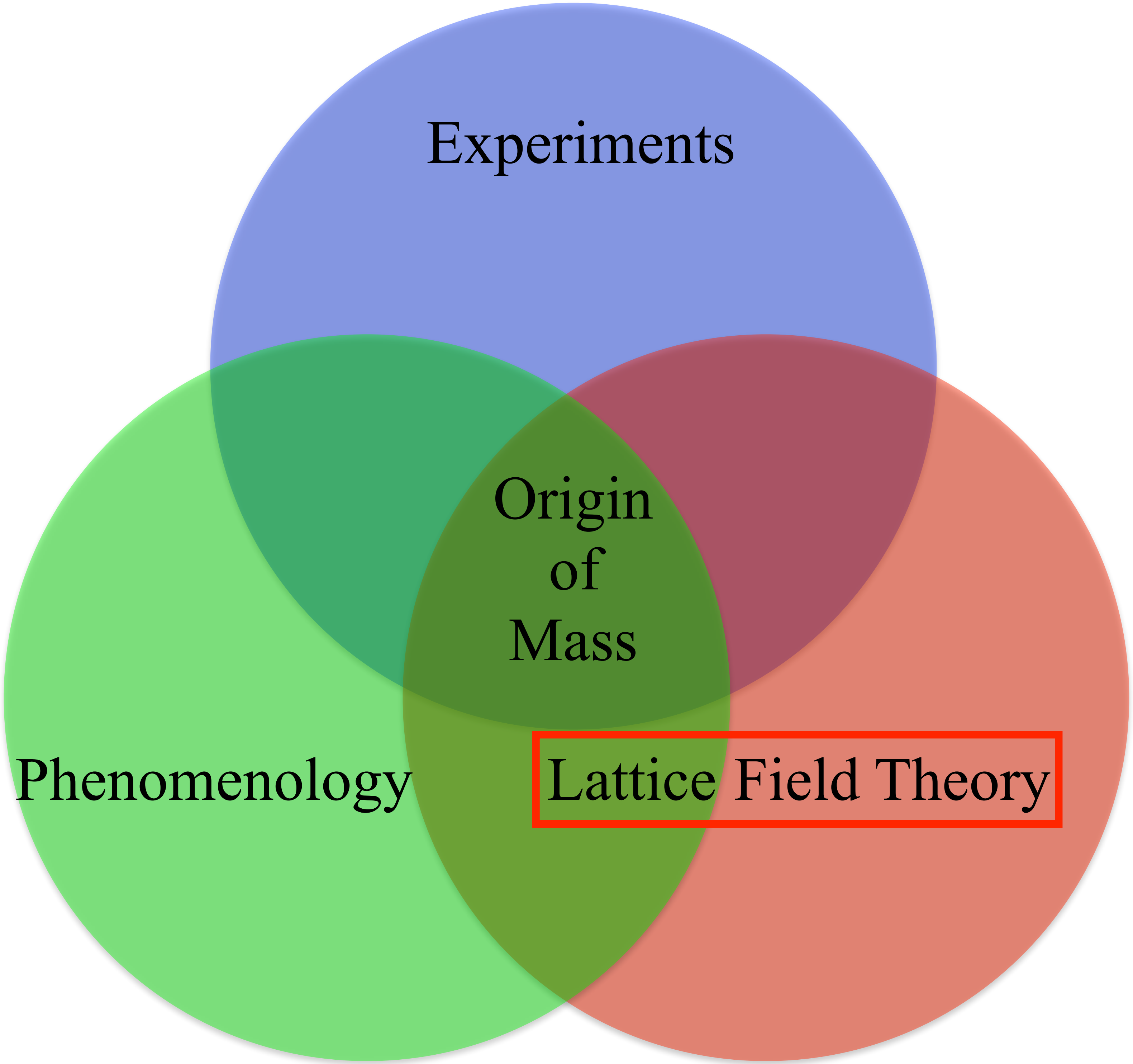
Proton Decay

Dark Energy

Intensity Frontier

Cosmic Frontier

Paths to the origin of mass



Experiments

Origin
of
Mass

Phenomenology

Lattice Field Theory

Outline

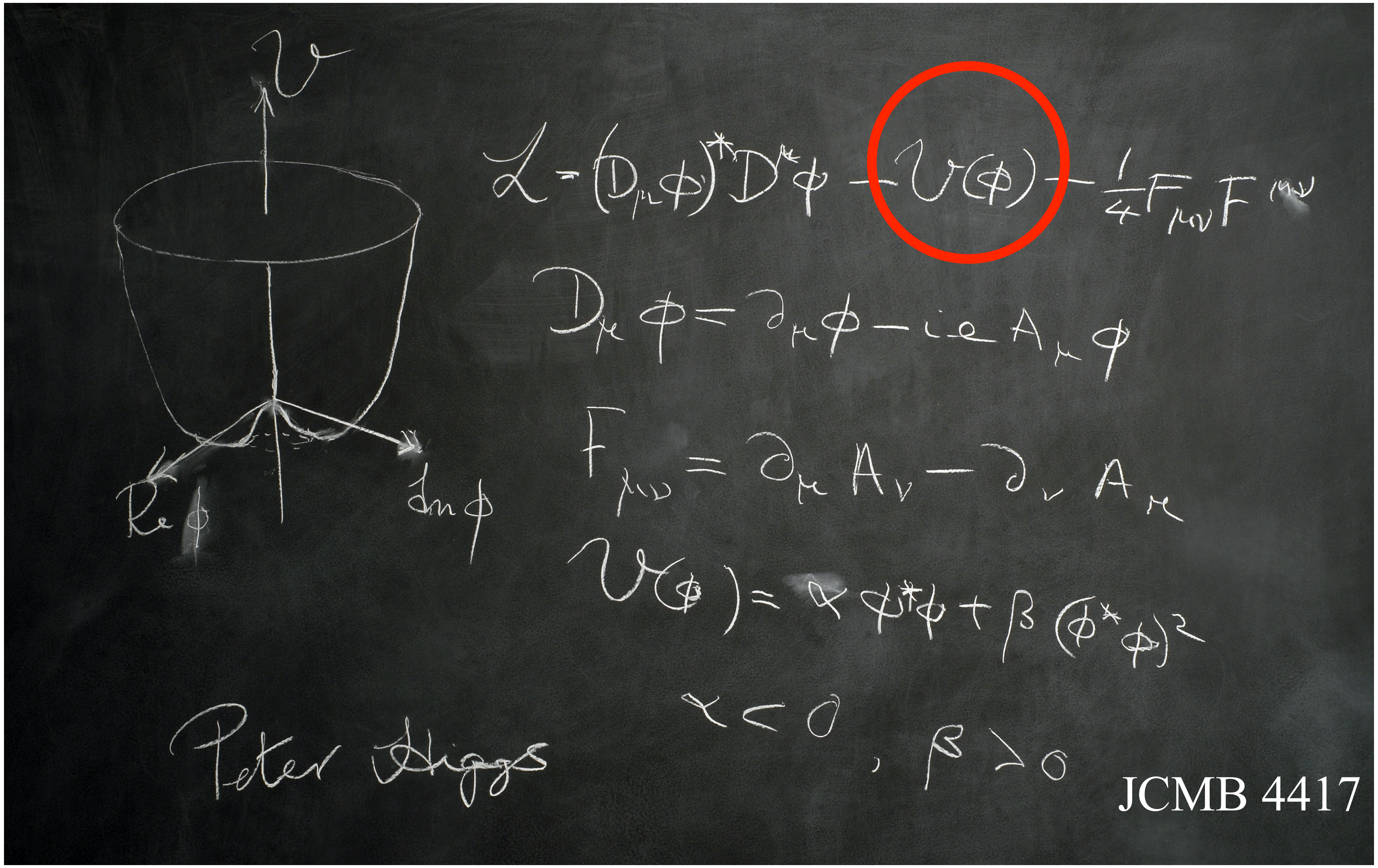
- Introduction: the standard-model Higgs and why it is not enough

- Lattice Field Theory and strong dynamics beyond the SM
  - → The Higgs-Yukawa model
  - → Higgs as a bound state: Composite Higgs

- A new approach inspired by BSM physics and condensed matter theory
  - → Tensor networks

The standard model (SM) Higgs

The standard model Higgs



$$\mathcal{L} = (D_\mu \phi)^* D^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu \phi = \partial_\mu \phi - ie A_\mu \phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

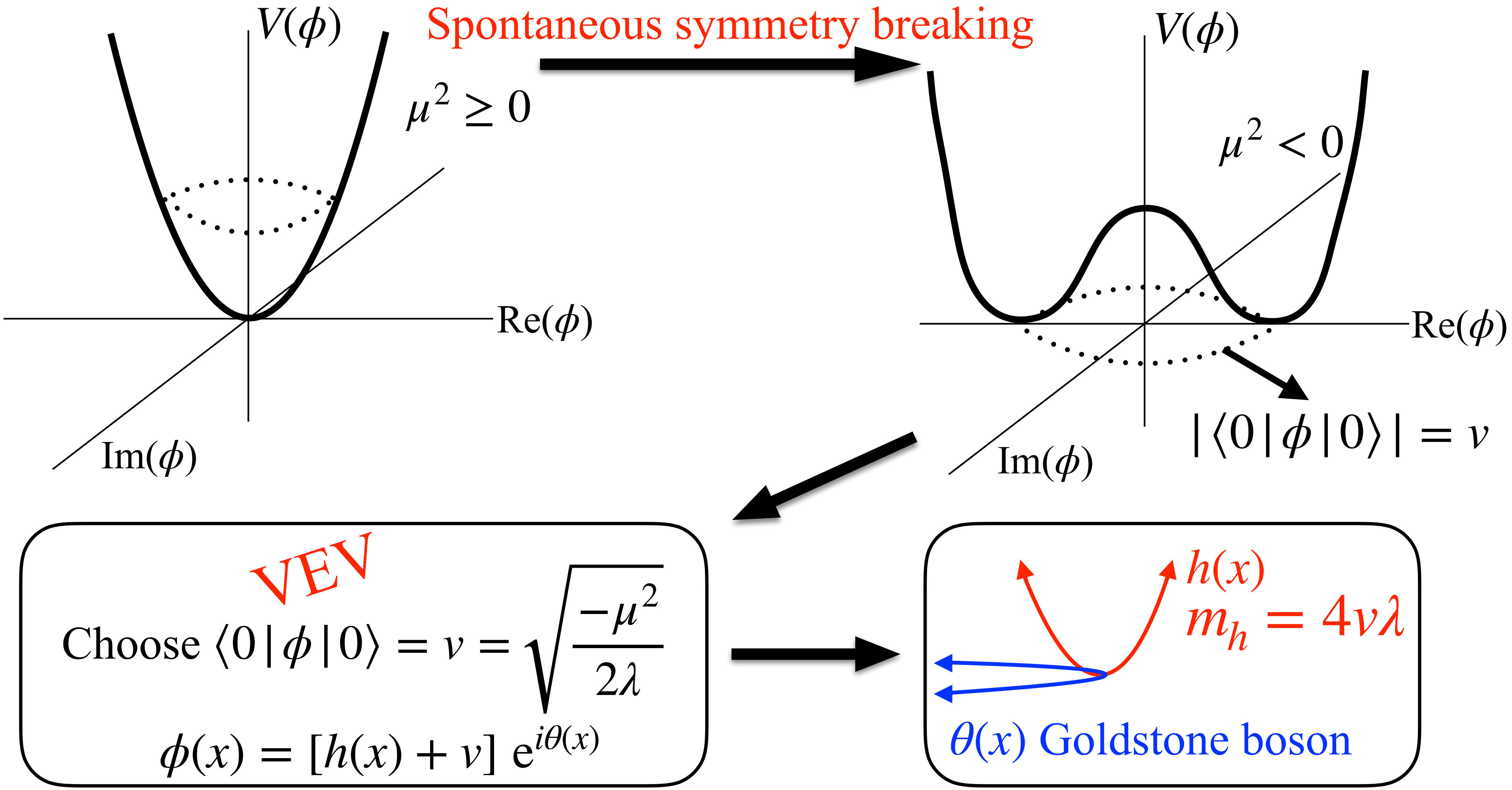
$$V(\phi) = \alpha \phi^* \phi + \beta (\phi^* \phi)^2$$

$$\alpha < 0, \quad \beta > 0$$

- For the standard model: ϕ → complex doublet (4 real scalars)

The standard model Higgs

$V(\phi) = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2$ for illustration



Spontaneous symmetry breaking

$|\langle 0|\phi|0\rangle| = v$

VEV

Choose $\langle 0|\phi|0\rangle = v = \sqrt{\frac{-\mu^2}{2\lambda}}$

$\phi(x) = [h(x) + v]\, \mathrm{e}^{i\theta(x)}$

$h(x)$

$m_h = 4v\lambda$

$\theta(x)$ Goldstone boson

The standard model Higgs for the origin of masses

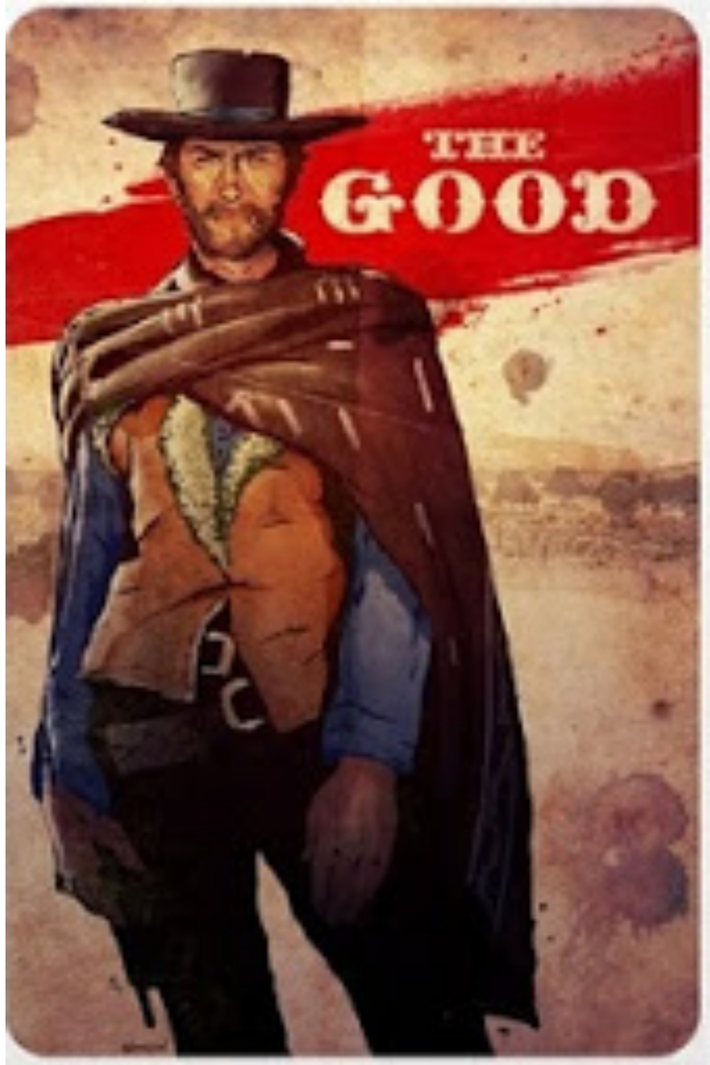
- Coupled to weak gauge bosons *via* $\partial_\mu \phi \to D_\mu \phi$ (coupling g)

  → The weak gauge boson masses $M_{W,Z} \propto gv$

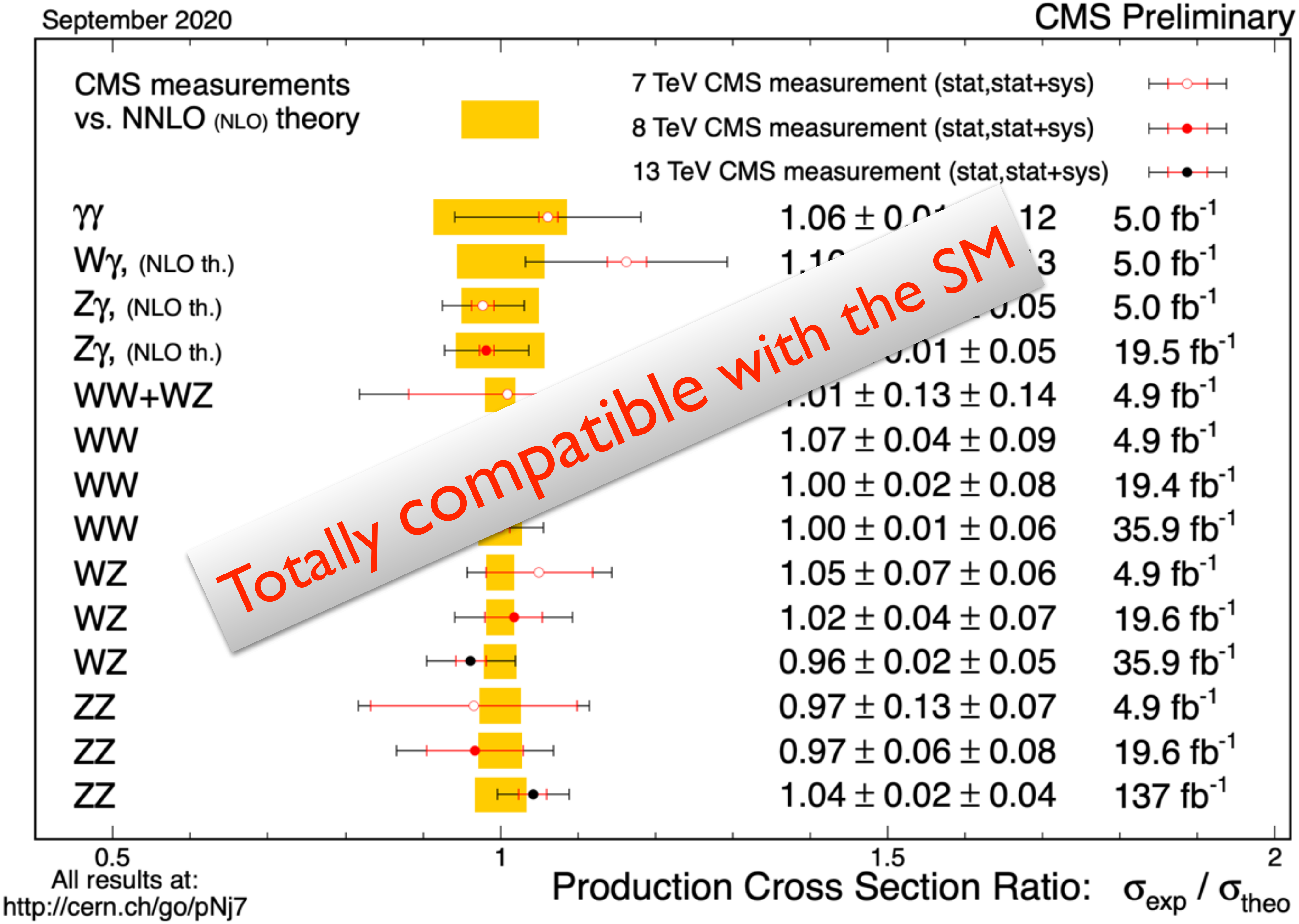
- Coupled to fermions *via* the Yukawa coupling $y\bar{\psi}_L \phi \psi_R + \text{h.c.}$

  → The fermion masses $m_\psi \propto yv$

The good, the bad and the ugly of the standard model

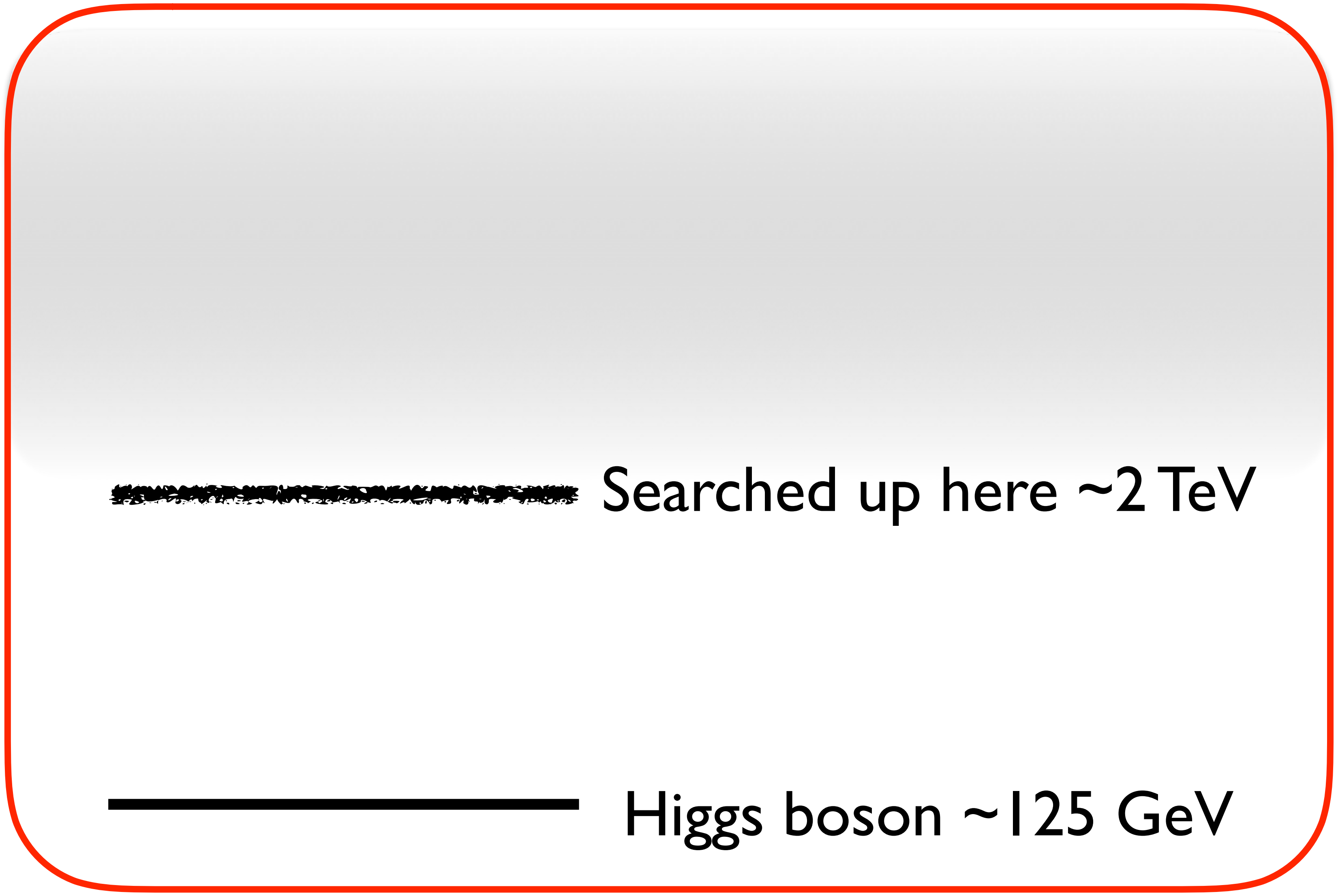

THE
GOOD

What the LHC revealed to us hitherto



Totally compatible with the SM

What the LHC revealed to us hitherto



Searched up here ~2 TeV

Higgs boson ~125 GeV

The Higgs boson is light


THE
BAD

Running coupling in QFT

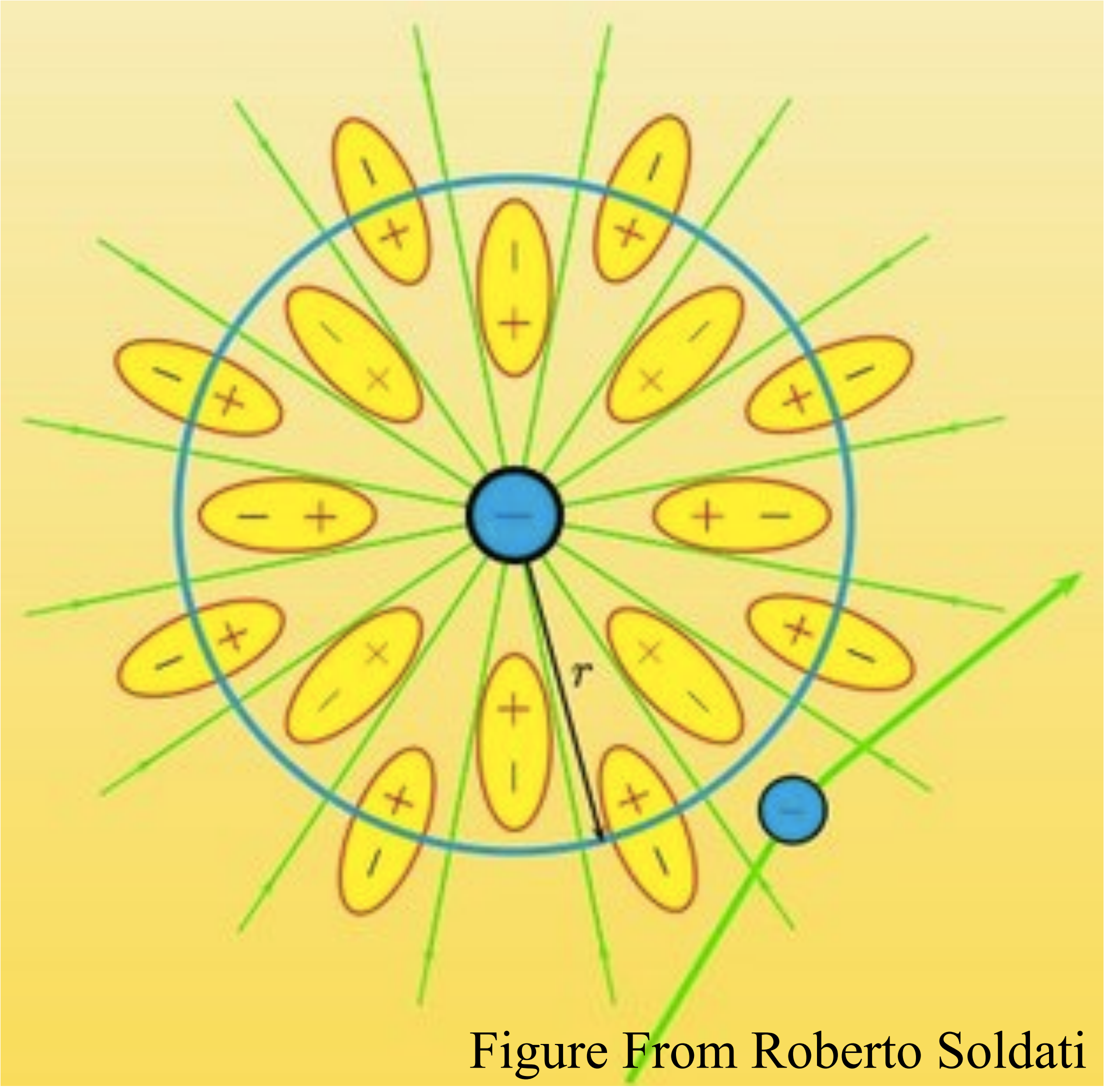
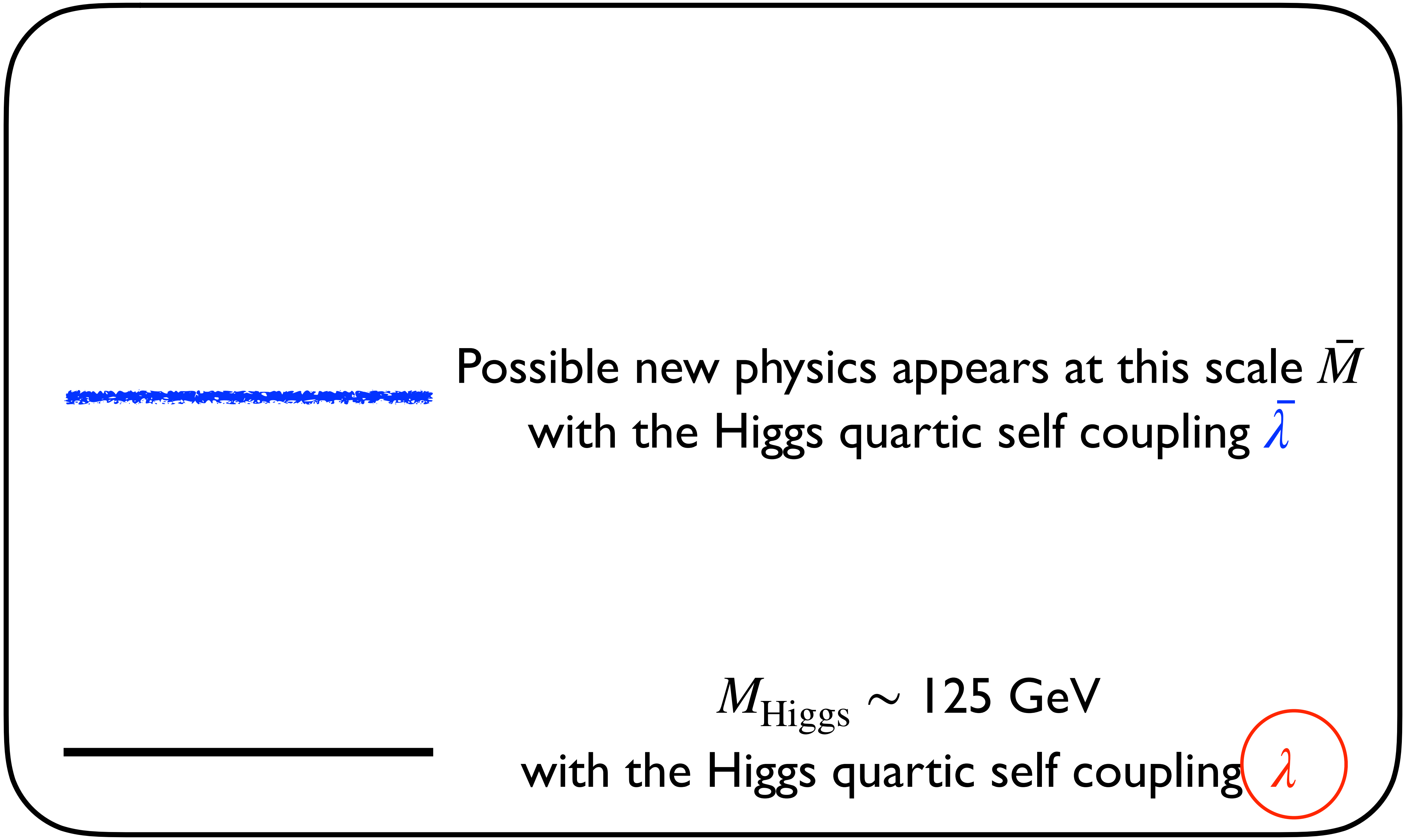


Figure From Roberto Soldati

Charge screening in Quantum Electrodynamics

Interaction/coupling strength changes with distance (energy scale)

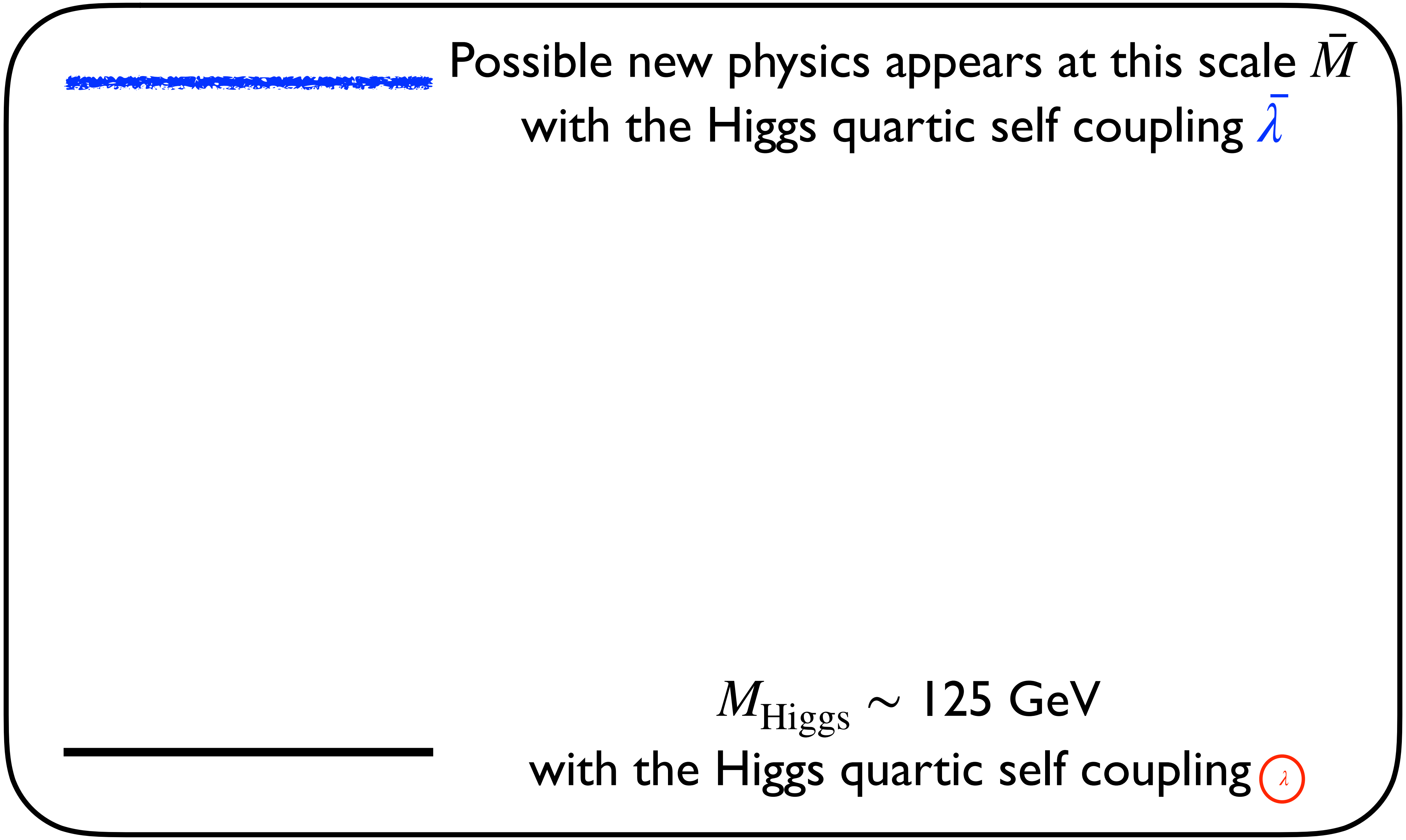
The scalar (Higgs) sector is trivial!



Possible new physics appears at this scale $\bar{M}$
with the Higgs quartic self coupling $\bar{\lambda}$

$M_{\text{Higgs}} \sim$ 125 GeV
with the Higgs quartic self coupling λ

The scalar (Higgs) sector is trivial!



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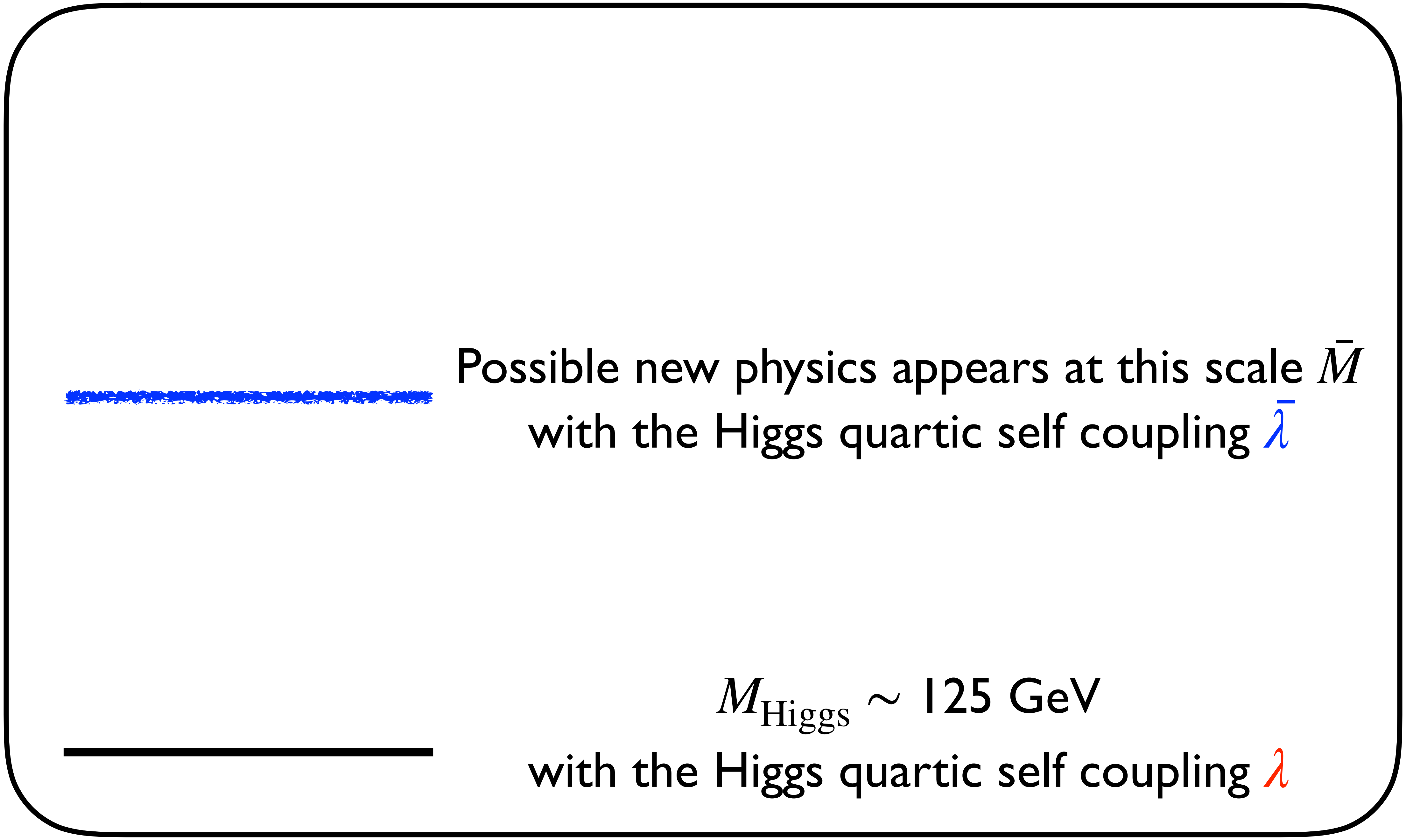

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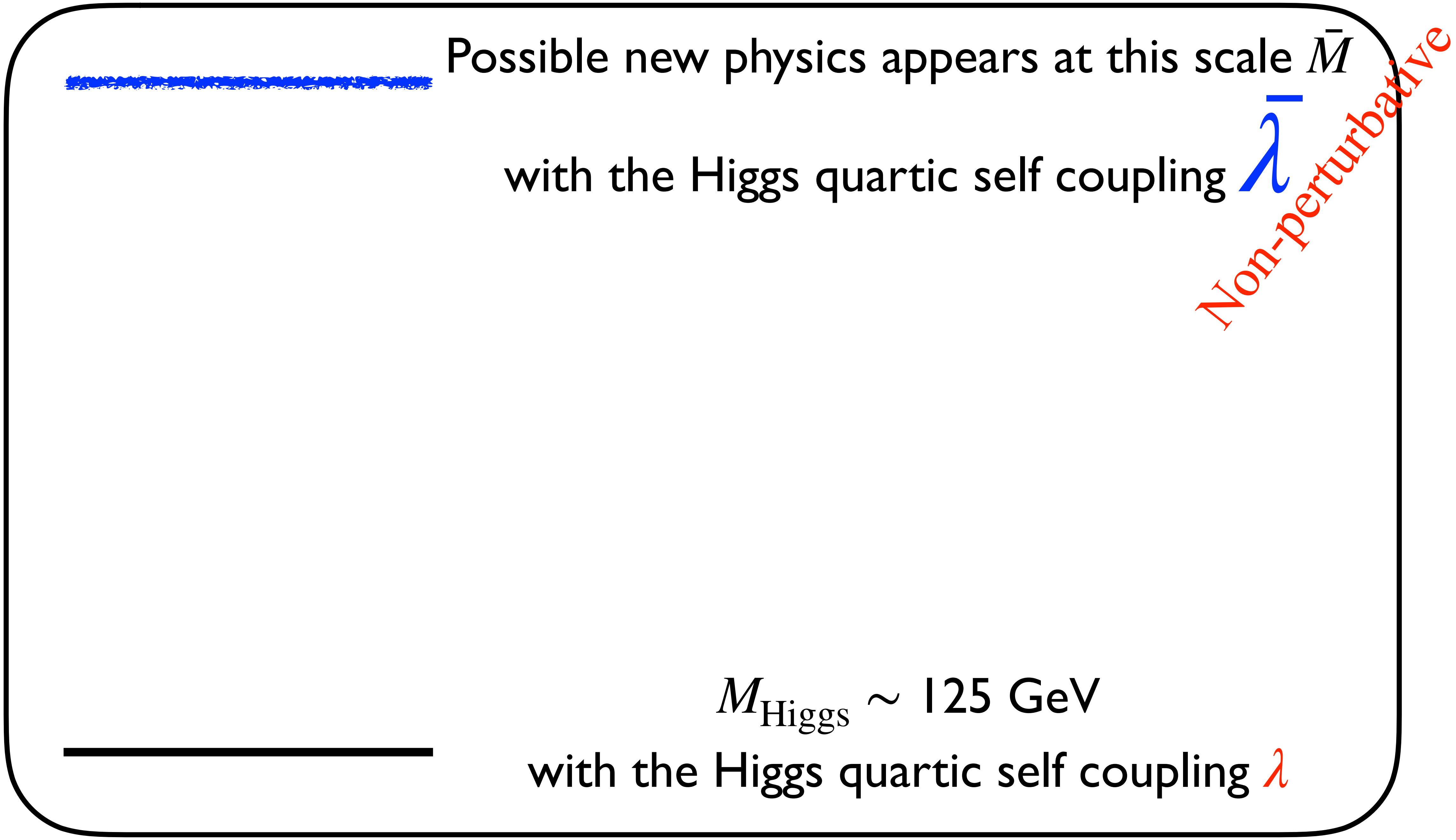


Possible new physics appears at this scale $\bar{M}$

with the Higgs quartic self coupling $\bar{\lambda}$

Non-perturbative

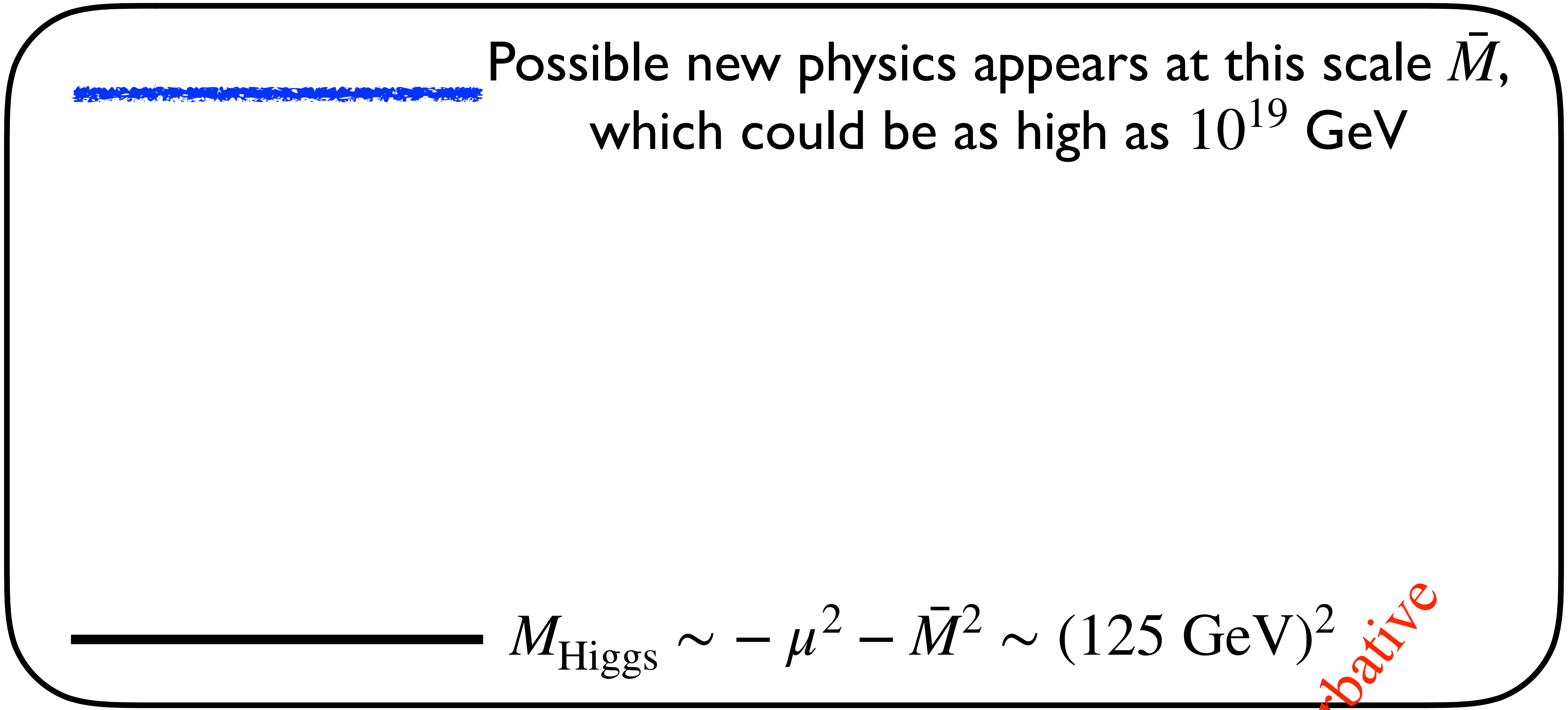
$M_{\text{Higgs}} \sim$ 125 GeV

with the Higgs quartic self coupling λ


THE
UGLY

The Higgs is “unnaturally” light!

$V(\phi) = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2$ for illustration



Possible new physics appears at this scale $\bar{M}$, which could be as high as 10^{19} GeV

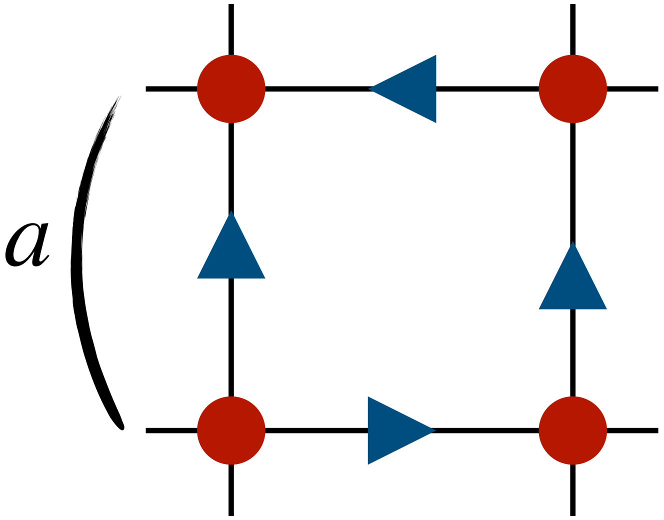
$M_{\text{Higgs}} \sim -\mu^2 - \bar{M}^2 \sim (125\ \text{GeV})^2$

Huge cancellation!

Non-perturbative

Lattice field theory

Basic ingredients



● Matter fields (fermions and scalars)

▶ Gauge fields

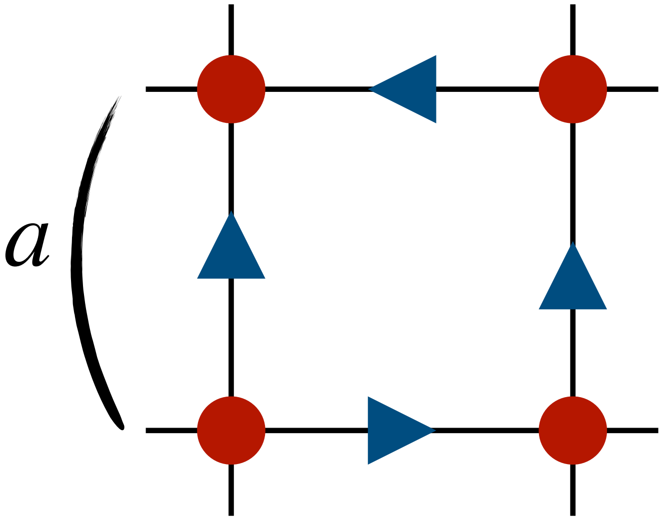
In finite 4-dimensional volume

$$
\begin{aligned}
Z &= \int DU\, D\phi D\bar{\psi} D\psi \; \mathrm{e}^{-S_U} \mathrm{e}^{-S_\phi} \mathrm{e}^{-a^4 \sum_x \bar{\psi}_x \mathcal{M}[U,\phi] \psi_x} \\
&= \int DU\, D\phi D\bar{\psi} D\psi \; \det(\mathcal{M}[U,\phi]) \; \mathrm{e}^{-S_U} \mathrm{e}^{-S_\phi}
\end{aligned}
$$

➡ Monte-Carlo simulations with importance sampling

➡ *c.f.* Partition function in statistical physics

The continuum limit



● Matter fields (fermions and scalars)

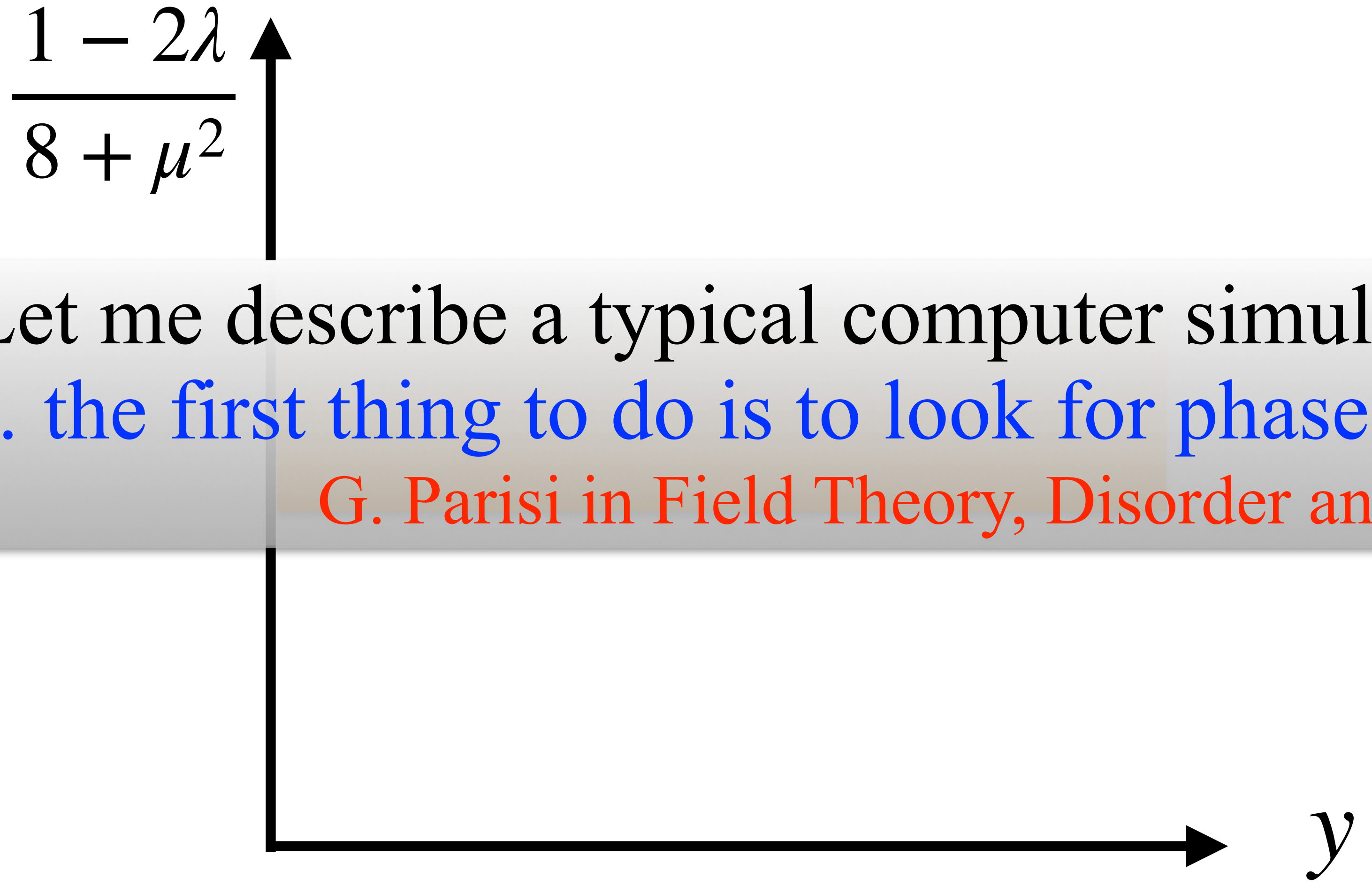
▶ Gauge fields

- $a \to 0$ means $am \to 0$ or $\xi/a \to \infty$
  ➡ m : typical low-energy mass scale
  ➡ ξ : typical long-distance length scale (correlation length)
- The continuum limits are at 2nd-order phase transition points
  ➡ Critical phenomena are crucial for lattice field theory

New physics in the Higgs-Yukawa theory?

The Higgs-Yukawa sector of the SM…

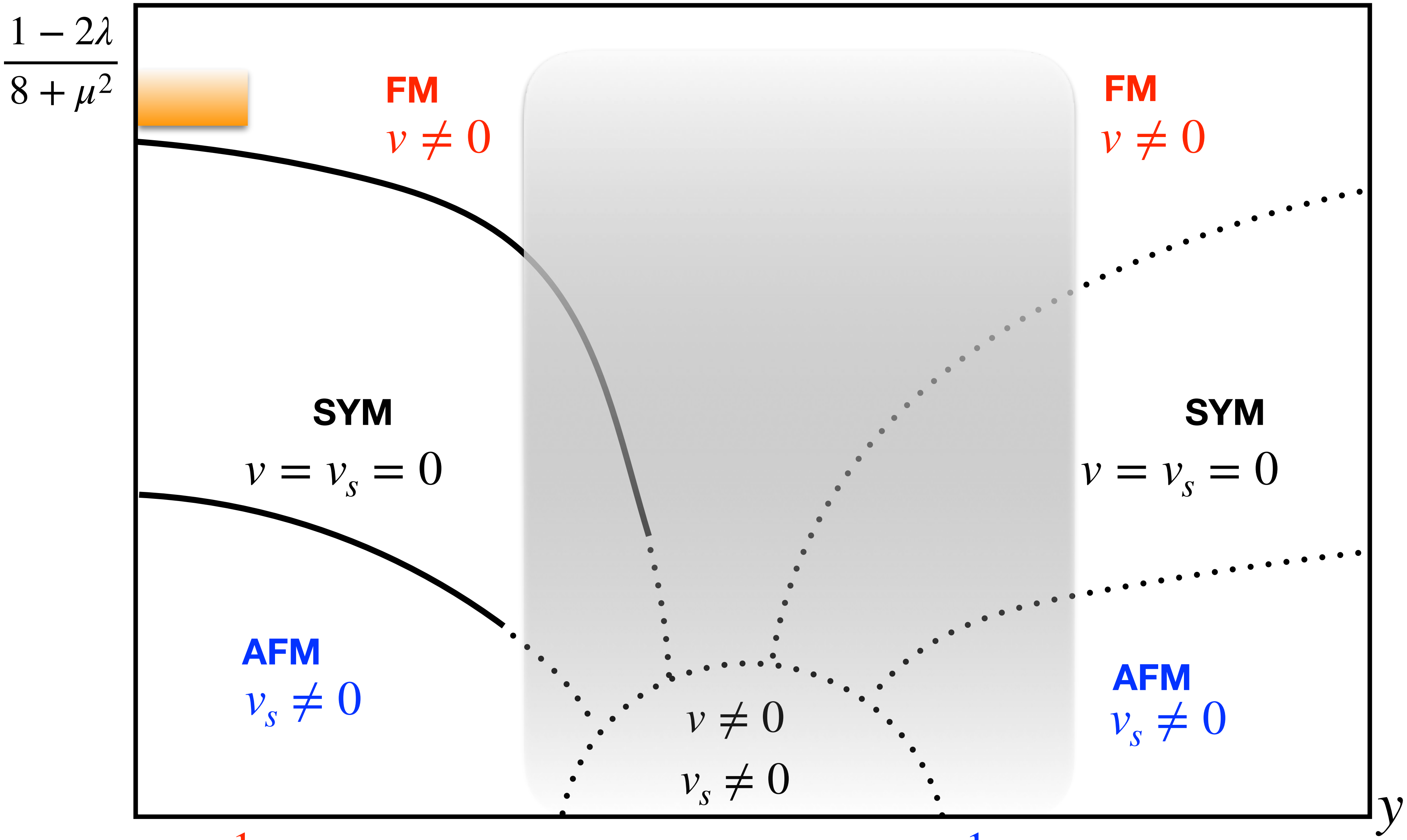
$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \text{ with } y\ (\bar{\psi}_L\phi\psi_R + \mathrm{h.c.}) \text{ at fixed } \lambda$$



> "Let me describe a typical computer simulation:
> … the first thing to do is to look for phase transition."
> G. Parisi in Field Theory, Disorder and Simulations

is a small corner on the phase diagram

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \text{ with } y\ (\bar{\psi}_L \phi \psi_R + \mathrm{h.c.}) \text{ at fixed } \lambda$$



$$v = \frac{1}{V_4} \sum_x \langle 0 | \phi(x) | 0 \rangle \qquad v_s = \frac{1}{V_4} \sum_x \eta(x) \langle 0 | \phi(x) | 0 \rangle$$

It is a challenging yet important task

$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$ with $y\,(\bar{\psi}_L \phi \psi_R + \mathrm{h.c.})$ at fixed λ

- Possible new fixed point?

  D.Y.-J.Chu, K.Jansen, B.Knippschild, CJDL, JHEP01 (2019) 110

  J.Bulava *et al.*, AHEP 2013 (2013) 875612

- Possible new four-fermion condensate?

  S.Catterall and D.Schaich, PRD96 (2017)

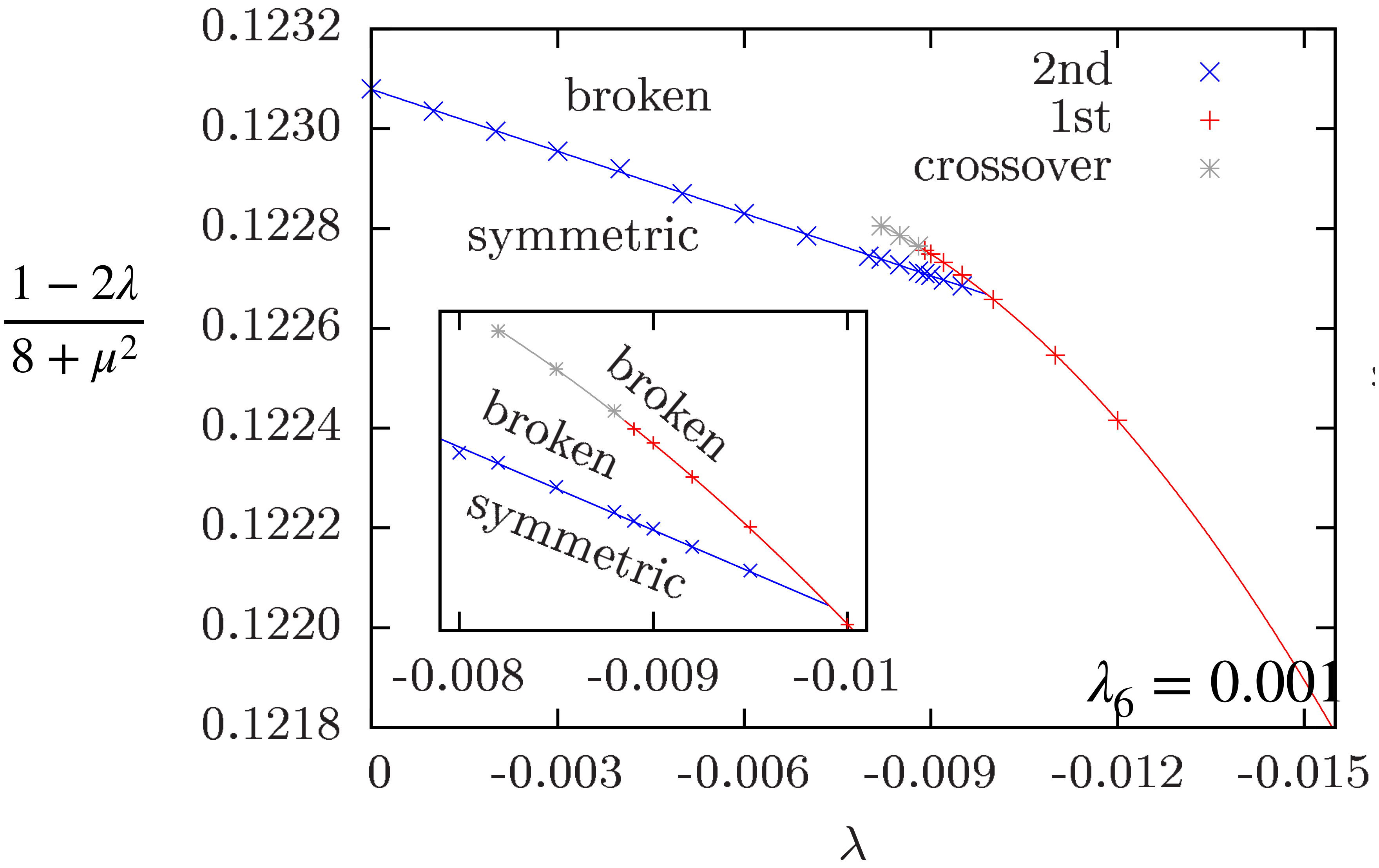
- Possible first-order phase transition?

  A.Hasenfratz, K.Jansen, Y.Shen, NPB394 (1993)

- Other directions (*e.g.* 2HDM)?

New physics as new interactions?

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \lambda_6(\phi^\dagger\phi)^3 \text{ with } y\,(\bar\psi_L\phi\psi_R + \mathrm{h.c.})$$

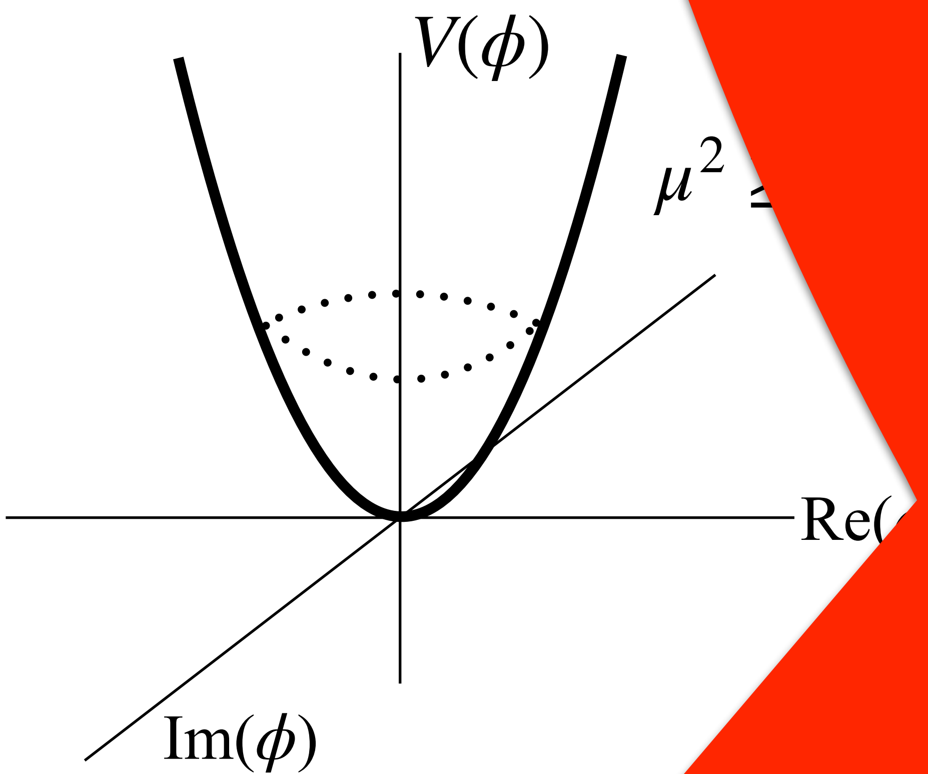


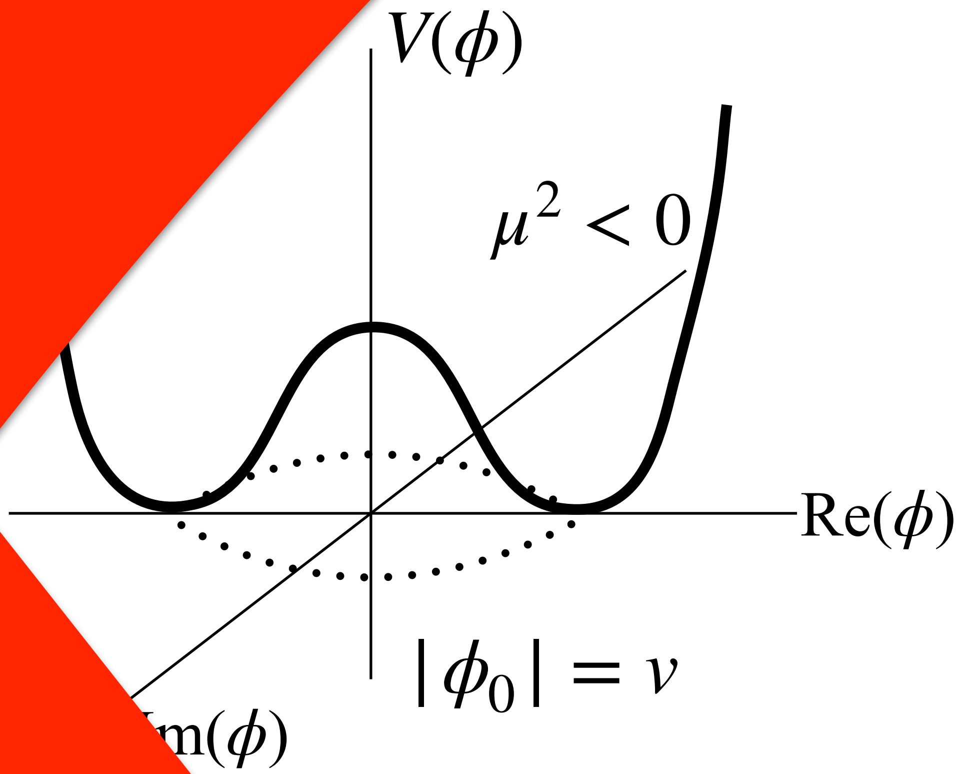
D.Y.-J.Chu, K.Jansen, B.Knippschild, CJDL, A.Nagy, PLB 744 (2015) 146

The Higgs boson as a bound state

The st ndard model H

$V(\phi)$ $+ \lambda(\phi^*\phi)^2$ fo



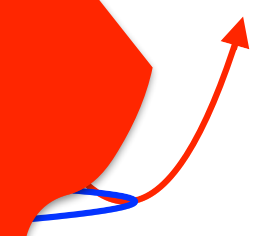


VEV

Choose $\langle 0 | \phi$

2λ

$\phi(x) = [$ $\theta(x)$



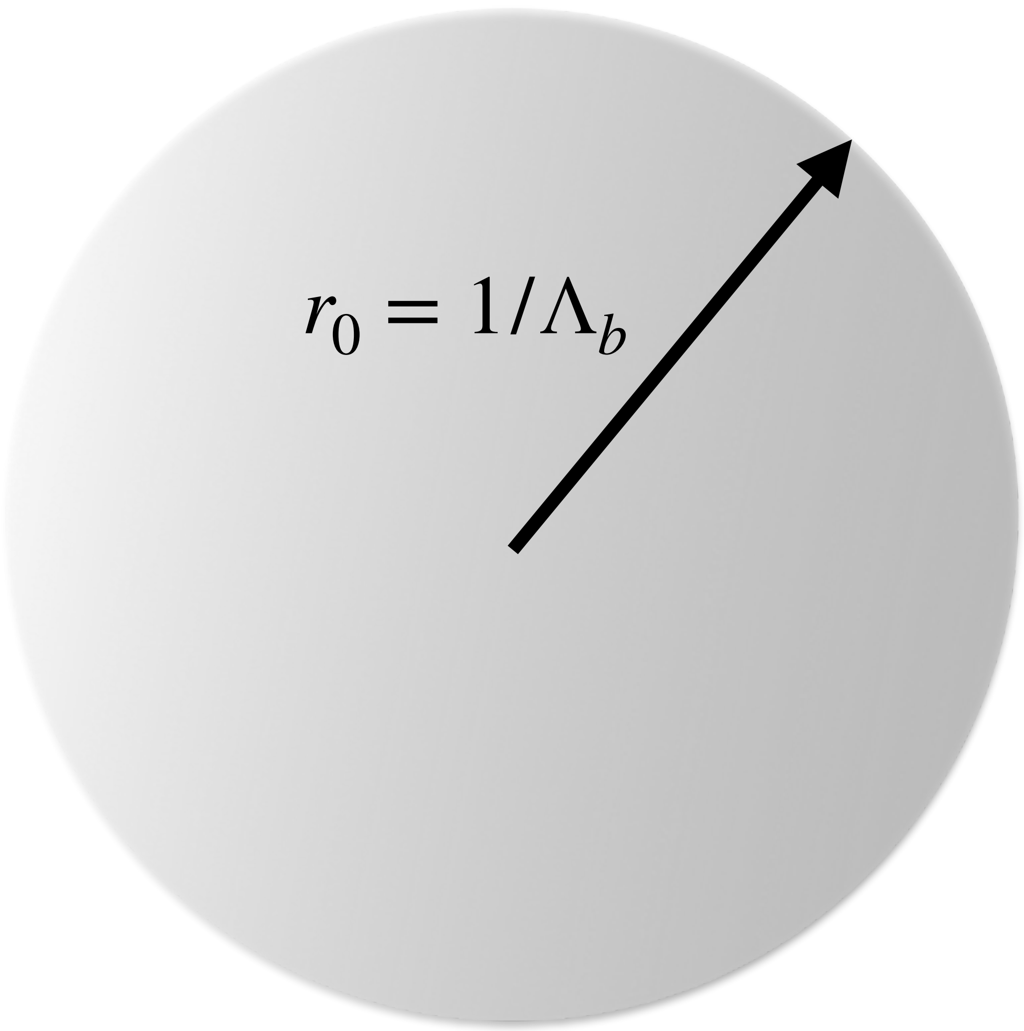
$h(x)$

$m_h = 4v\lambda$

$\theta(x)$ Goldstone boson

Different paths to electroweak symmetry breaking

Self-interacting scalars replaced by strongly-interacting fermions and gauge bosons



- Coupling ⤴ as energy ⤵

  ⬇

  Bound state formed at low energy

Opposite to λ in $V(\phi)$!

- Typical bound-sate mass Λ_b

  ⬇

  Typical bound-sate size $r_0 = 1/\Lambda_b$

- Not yet seen experimentally

  ⬇

  2×10^3 GeV $\lesssim \Lambda_b < 10^4$ GeV

Two issues: I. The light Higgs

Q: Why is the Higgs so light, $M_{\text{Higgs}} \ll \Lambda_b$?

A: Resort to spontaneous symmetry breaking

- → Global symmetry breaking like QCD (Composite Higgs)
- → Scale-invariance breaking unlike QCD (Dilaton Higgs)

Two issues: II. The SM fermion mass

The need to suppress flavour-changing neutral-current processes…

$\Lambda_f \sim 10^7$ GeV

High to suppress FCNC

$\sim$ 10 - 100 GeV

mass

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{f}f$$

FCNC

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}$$

Two issues: II. The SM fermion mass

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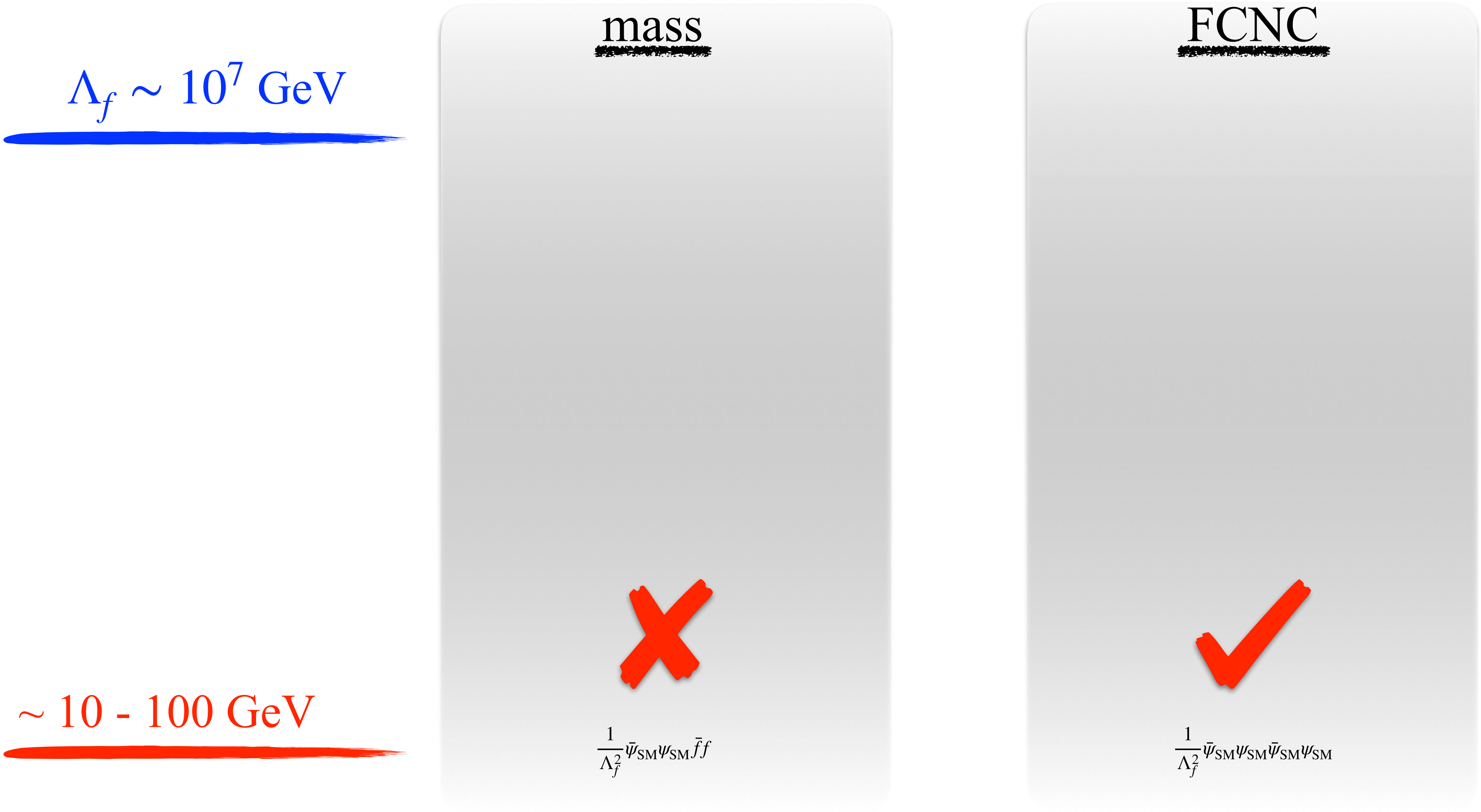

$\Lambda_f \sim 10^7$ GeV

$\sim$ 10 - 100 GeV

mass

✗

$\frac{1}{\Lambda_f^2}\bar{\psi}_{\rm SM}\psi_{\rm SM}\bar{f}f$

FCNC

✓

$\frac{1}{\Lambda_f^2}\bar{\psi}_{\rm SM}\psi_{\rm SM}\bar{\psi}_{\rm SM}\psi_{\rm SM}$

Two issues: II. The SM fermion mass

The need to suppress flavour-changing neutral-current processes…

$\Lambda_f \sim 10^7$ GeV

High to suppress FCNC

~ 10 - 100 GeV

mass

$$\frac{1}{\Lambda_f^2}\bar{\Psi}_{\rm SM}\Psi_{\rm SM}\bar{f}f$$

FCNC

$$\frac{1}{\Lambda_f^2}\bar{\Psi}_{\rm SM}\Psi_{\rm SM}\bar{\Psi}_{\rm SM}\Psi_{\rm SM}$$

Two issues: II. The SM fermion mass

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$\Lambda_f \sim 10^7$ GeV

$(1/\Lambda_f^2) \rightarrow 1/\Lambda_f$

mass

$$\frac{1}{\Lambda_f}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{f}f$$

FCNC

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}$$

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Two issues: II. The SM fermion mass

The need to suppress flavour-changing neutral-current processes…

$\Lambda_f \sim 10^7$ GeV

$\sim$ 10 - 100 GeV

mass

$$\frac{1}{\Lambda_f}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{f}f$$

FCNC

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}$$

Two issues: II. The SM fermion mass

The need to suppress flavour-changing neutral-current processes…

mass

FCNC

$\Lambda_f \sim 10^7$ GeV

$1/\Lambda_f \to (1/\Lambda_f)^2$

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{f}f$$

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}\bar{\psi}_{\mathrm{SM}}\psi_{\mathrm{SM}}$$

$\sim$ 10 - 100 GeV

Two issues: II. The SM fermion mass

The need to suppress flavour-changing neutral-current processes…

$\Lambda_f \sim 10^7$ GeV

$\sim$ 10 - 100 GeV

mass

✓

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\rm SM}\psi_{\rm SM}\bar{f}f$$

FCNC

✓

$$\frac{1}{\Lambda_f^2}\bar{\psi}_{\rm SM}\psi_{\rm SM}\bar{\psi}_{\rm SM}\psi_{\rm SM}$$

Two issues: II. The SM fermion masses

- Dramatically alter the suppression of $\bar{\psi}_{\rm SM}\psi_{\rm SM}\bar{f}f$ from $1/\Lambda_f^2$ to $1/\Lambda_f$

- Need power-law scaling behaviour in $\bar{f}f$

- The system is at criticality for a large range of interaction strength

→ *c.f.* Berezinskii-Kosterlitz-Thouless phase transition

The Higgs boson as a bound state: Composite Higgs models

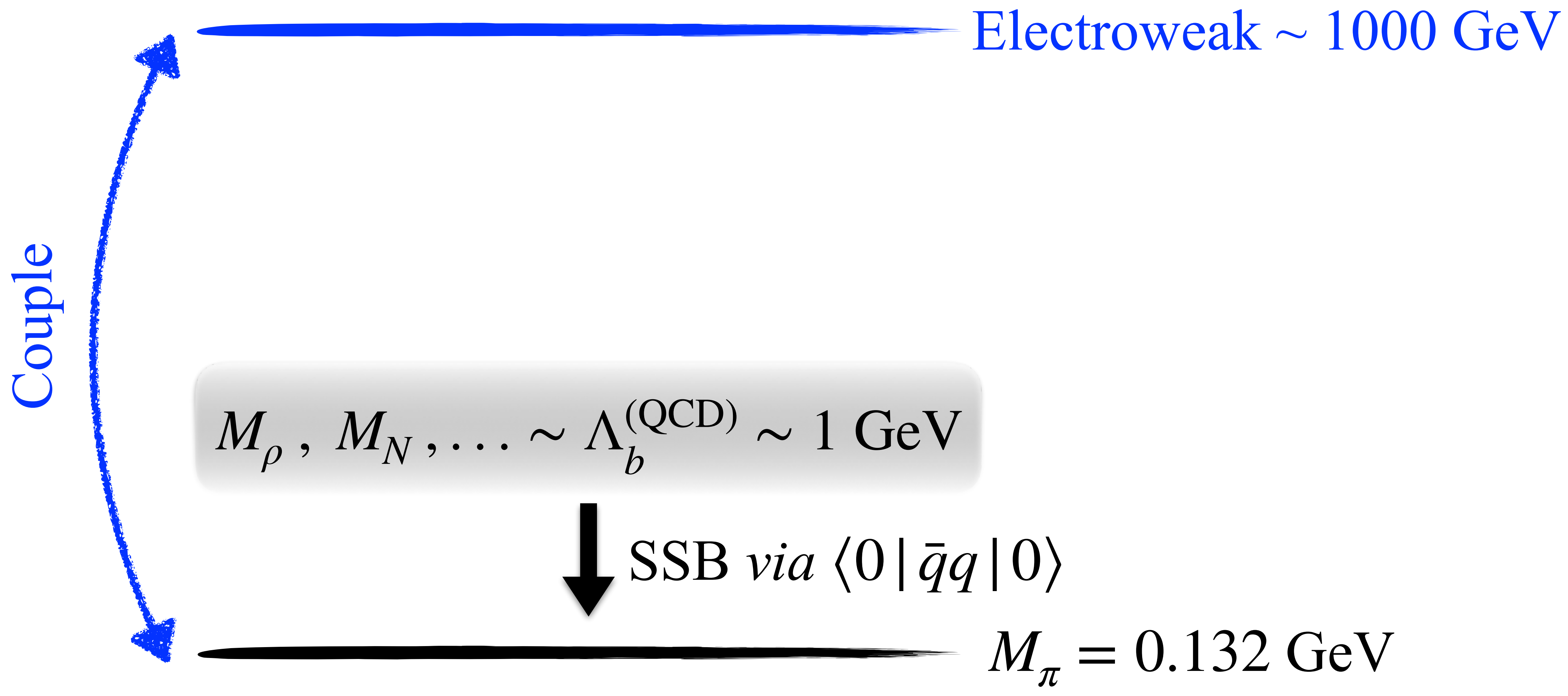
Lesson from Quantum Chromodynamics

$$M_\rho\,,\; M_N\,, \ldots \sim \Lambda_b^{(\mathrm{QCD})} \sim 1\ \mathrm{GeV}$$

$\Big\downarrow$ SSB *via* $\langle 0\,|\,\bar{q}q\,|\,0\rangle$

$$M_\pi = 0\ \mathrm{GeV}$$

Lesson from Quantum Chromodynamics



Electroweak ~ 1000 GeV

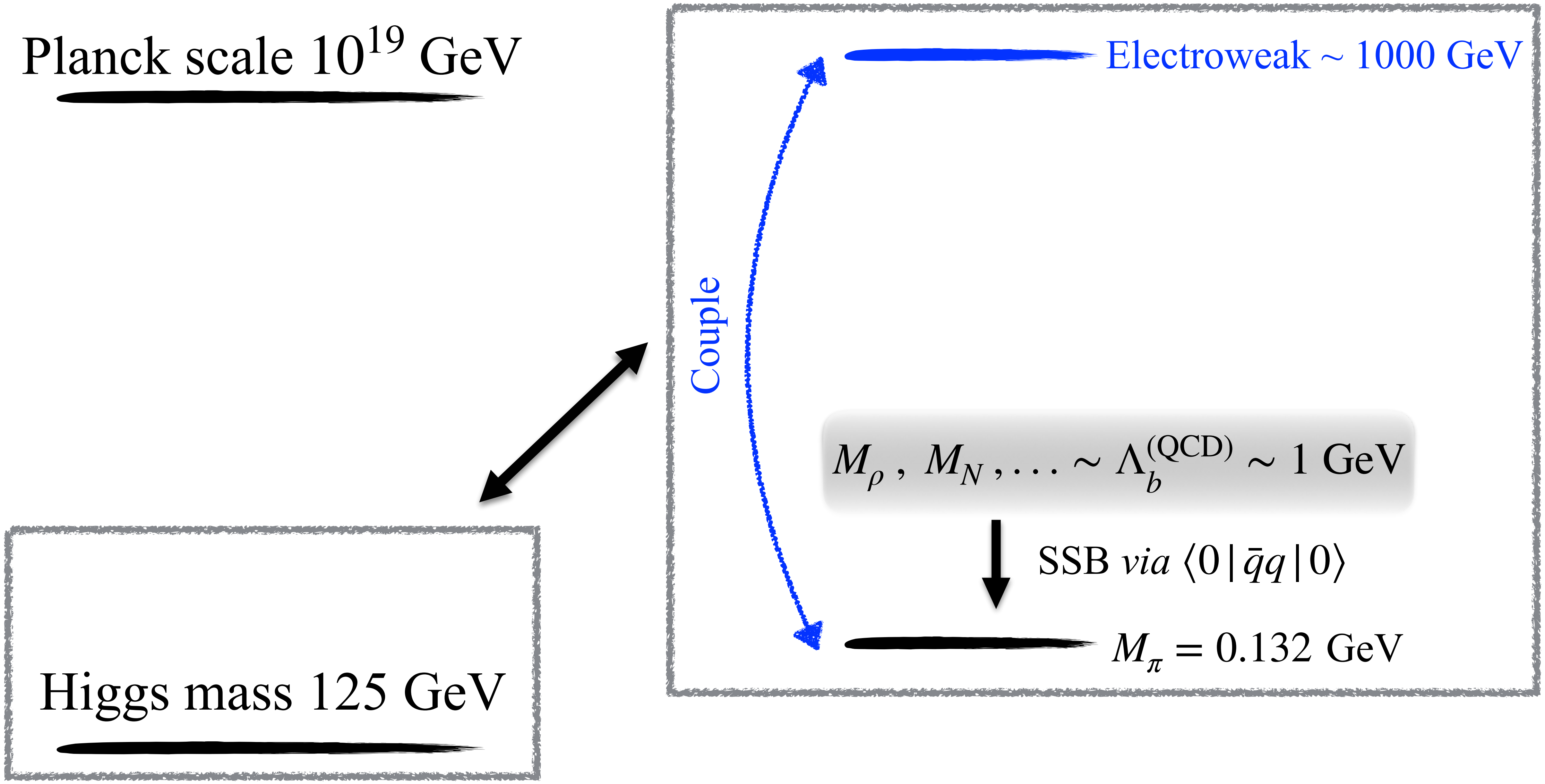
Couple

$M_\rho\,,\ M_N\,,\ldots \sim \Lambda_b^{(\mathrm{QCD})} \sim 1\ \mathrm{GeV}$

SSB *via* $\langle 0\,|\,\bar{q}q\,|\,0\rangle$

$M_\pi = 0.132\ \mathrm{GeV}$

Lesson from Quantum Chromodynamics



Planck scale 10^{19} GeV

Electroweak ~ 1000 GeV

Couple

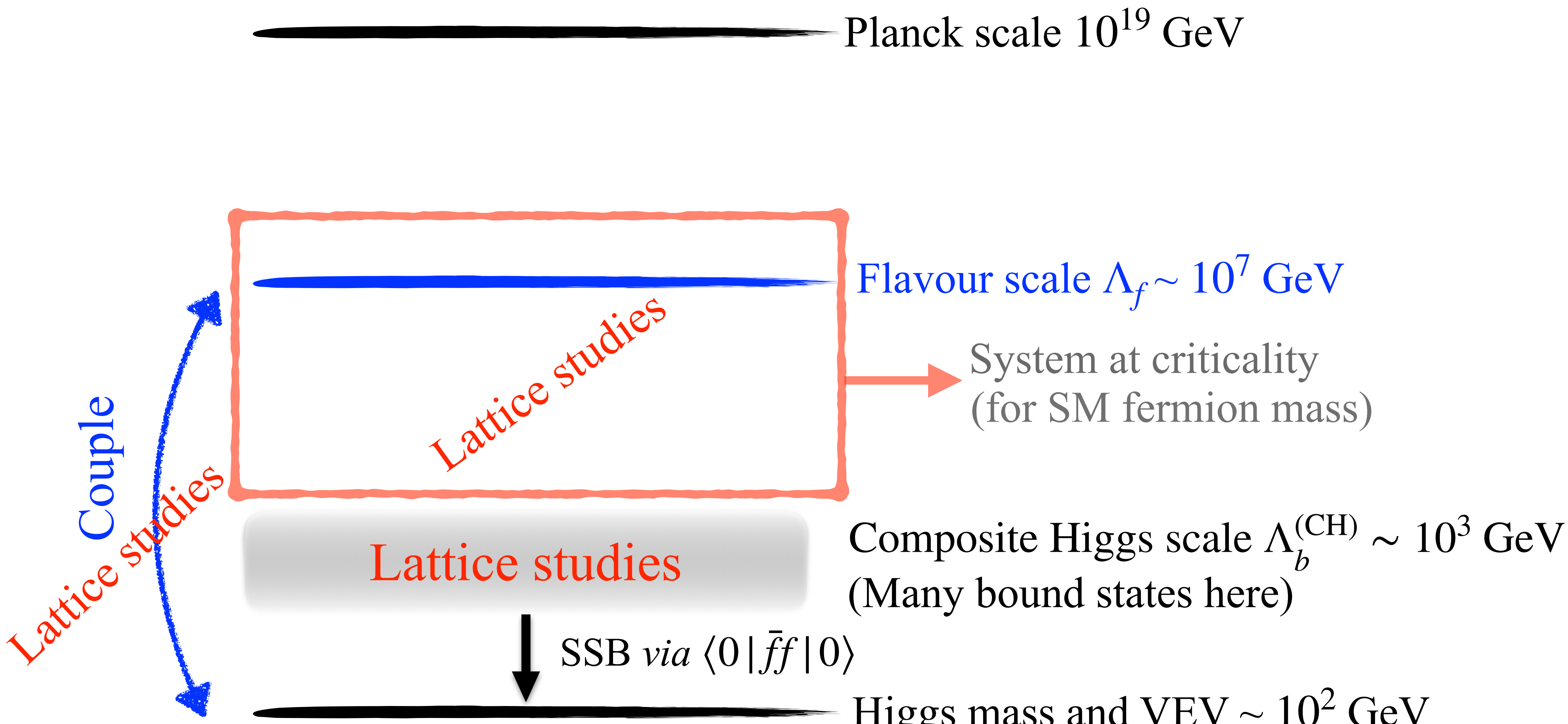
$M_\rho\,,\ M_N\,,\ldots \sim \Lambda_b^{(\mathrm{QCD})} \sim 1$ GeV

SSB *via* $\langle 0\,|\,\bar{q}q\,|\,0\rangle$

$M_\pi = 0.132$ GeV

Higgs mass 125 GeV

Composite Higgs models



Planck scale 10^{19} GeV

Flavour scale $\Lambda_f \sim 10^7$ GeV

Lattice studies

System at criticality
(for SM fermion mass)

Couple

Lattice studies

Lattice studies

Composite Higgs scale $\Lambda_b^{(CH)} \sim 10^3$ GeV
(Many bound states here)

SSB *via* $\langle 0|\bar{f}f|0\rangle$

Higgs mass and VEV ~ 10^2 GeV

Composite Higgs models

- Higgs is light because of the SSB for the global symmetry

Similar to QCD

- SM fermion masses: *via* mixing with the hybrid baryons ffF

Fermions in different representations of the gauge group, different from QCD

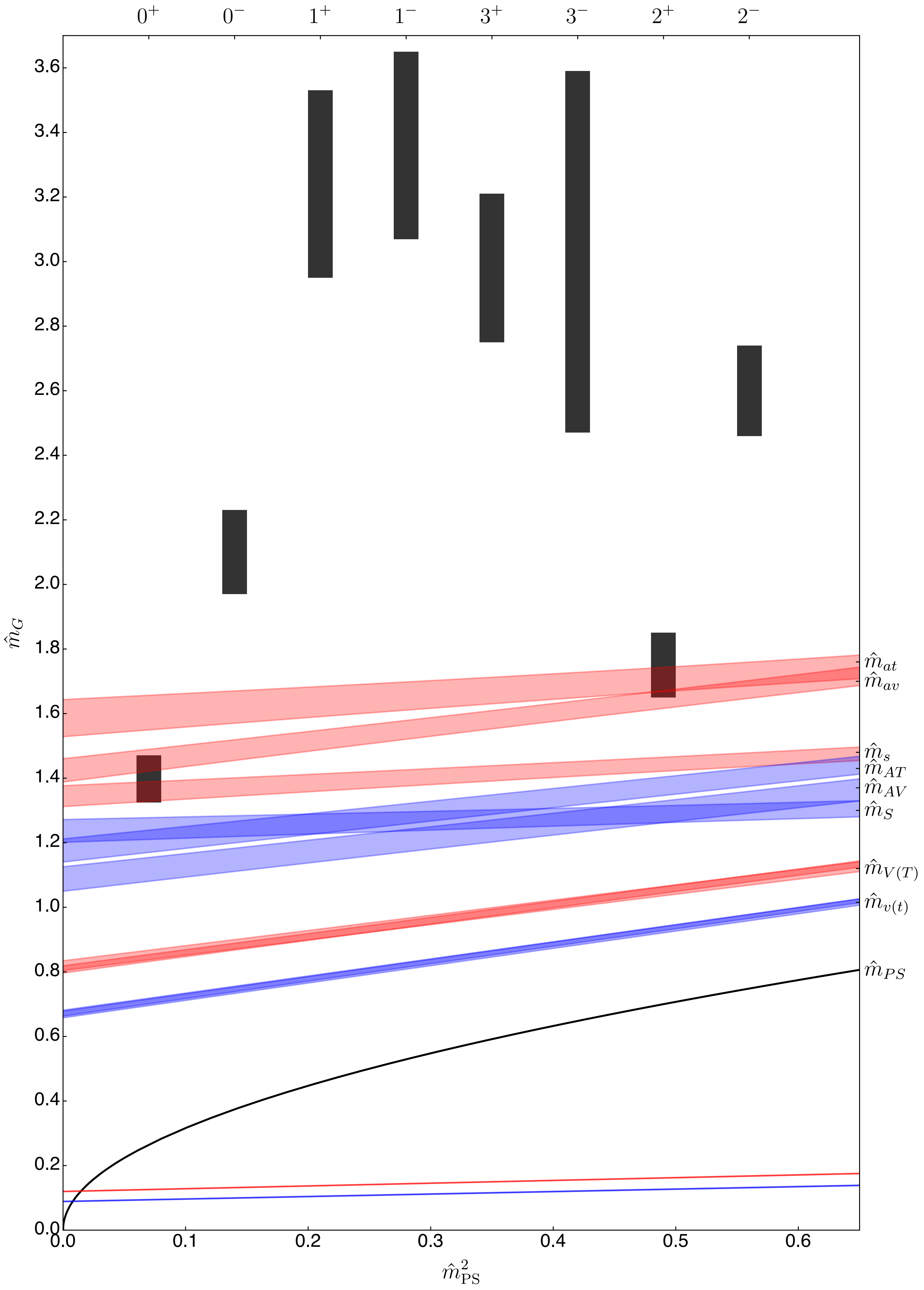
- The theory is at/close to criticality to enhance effects of $(\psi_{\mathrm{SM}} ffF)$

Different from QCD

- Recent new direction in the community: spectrum studies hitherto

Sp(4) gauge theory spectrum

Hsinchu-Pusan-Swansea collaboration,
PRD101 (2020)



Technique for the future:
Tensor networks and Hamiltonian formalism

(Matrix Product States)

Logic flow

No more path integrals as we go back to the canonical formalism

Hamiltonian formalism for LFT

↓

Quantum spin model

↓

MPS & variational method for obtaining the ground state

↓

Compute correlators and other quantities

Why?

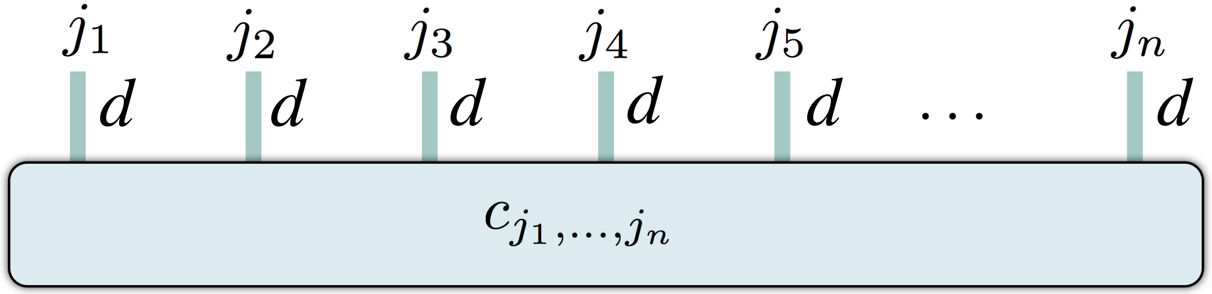
- Mapping field theories onto quantum spin models offers new insights

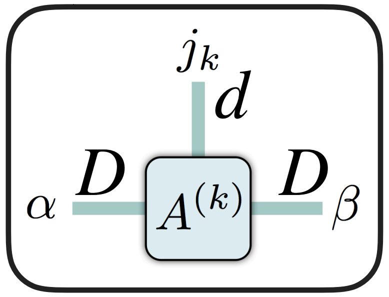
- Possible formulation for quantum computers

Matrix product states in a nutshell

S. White, 1992; M.B. Hasting, 2004; F. Verstraeten and I. Cirac, 2006; …

$$|\psi\rangle = \sum_{j_1,\ldots,j_n=1}^{d} c_{j_1,\ldots,j_n} |j_1,\ldots,j_n\rangle = \sum_{j_1,\ldots,j_n=1}^{d} c_{j_1,\ldots,j_n} |j_1\rangle \otimes \cdots \otimes |j_n\rangle$$

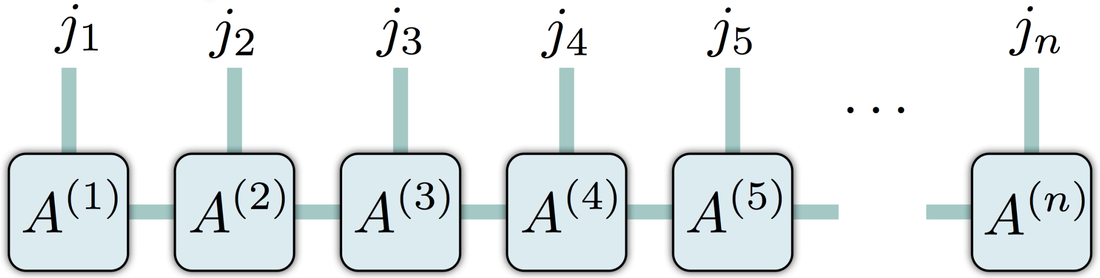




$$O(d^n) \downarrow O(ndD^2)$$

Entanglement-based argument for choosing D

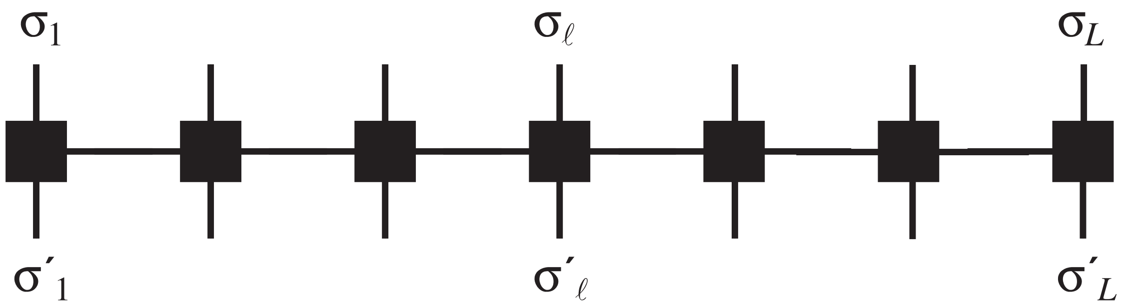
Bond dim



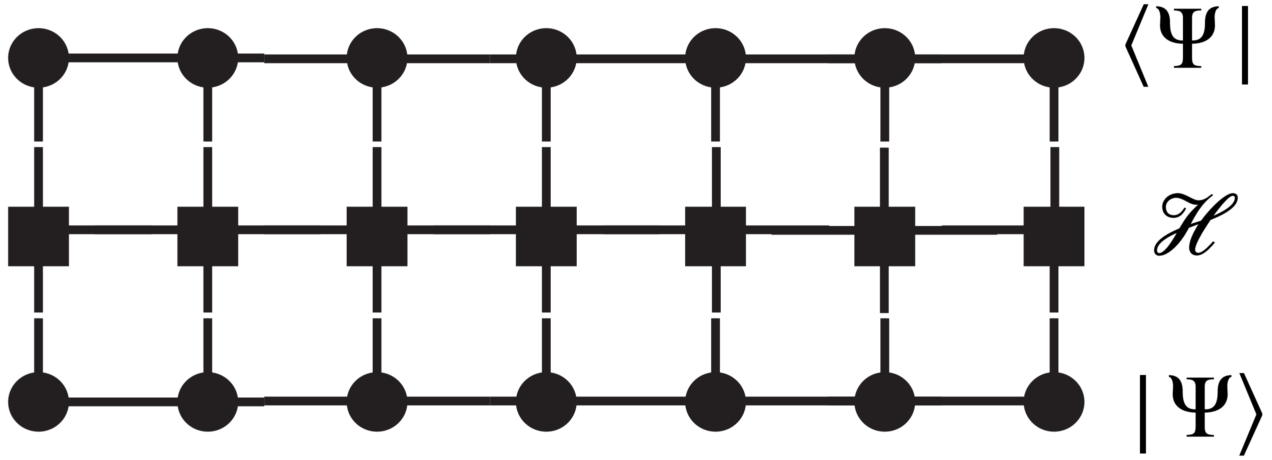
$$c_{j_1,\ldots,j_n} = \sum_{\alpha,\ldots,\omega}^{D} A^{(1)}_{\alpha;j_1} A^{(2)}_{\alpha,\beta;j_2} \ldots A^{(n)}_{\omega;j_n} = A^{(1)}_{j_1} A^{(2)}_{j_2} \ldots A^{(n)}_{j_n}$$

The Hamiltonian and matrix elements

$\mathscr{H}$:



$\langle\Psi|\mathscr{H}|\Psi\rangle$:

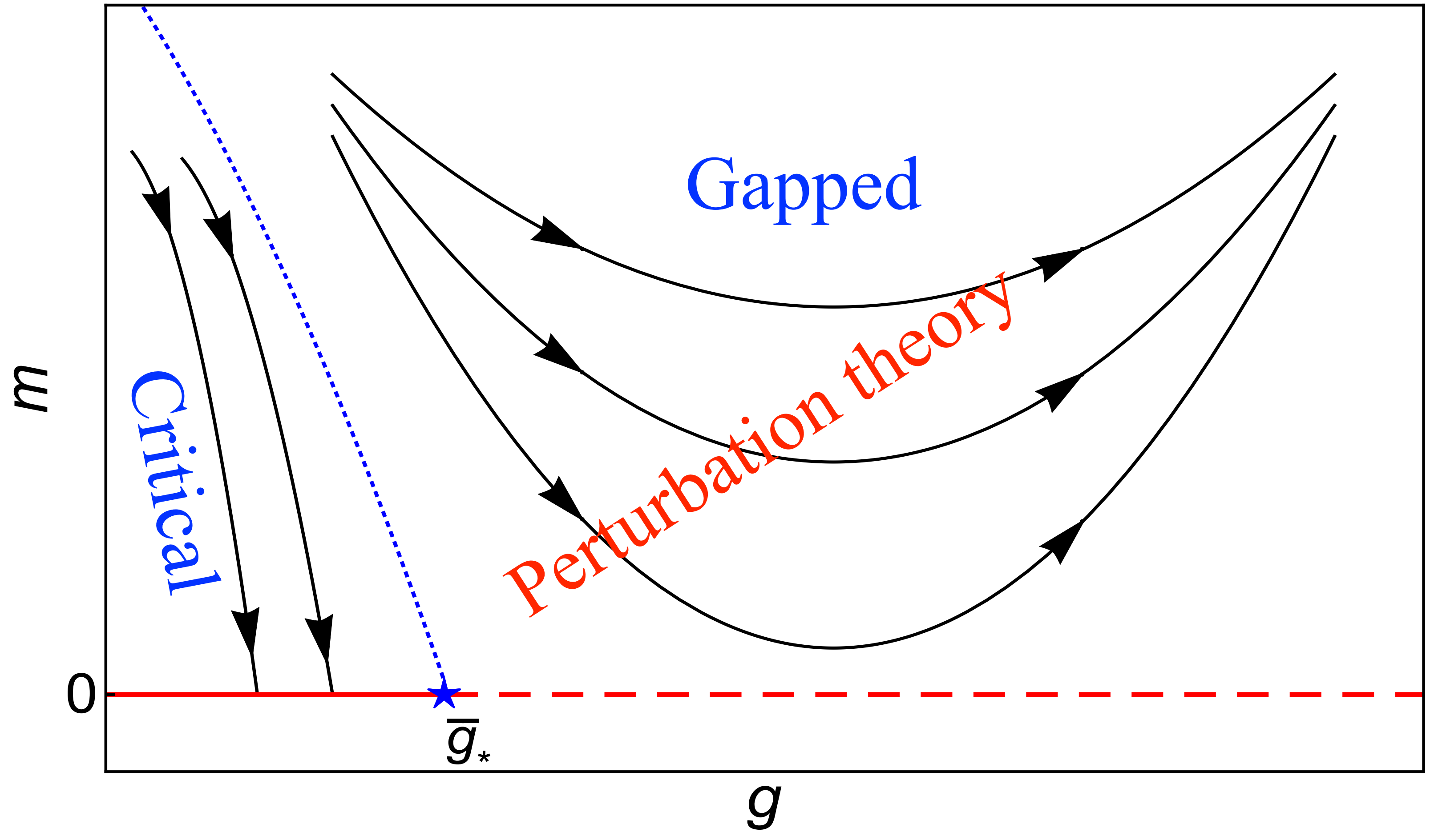


Variational search for the ground state

Application to the Thirring model

1+1 dimensional massive Thirring model

$$S_{\mathrm{Th}}\left[\psi,\bar{\psi}\right]=\int d^2x\left[\bar{\psi}i\gamma^\mu\partial_\mu\psi-m_0\bar{\psi}\psi-\frac{g}{2}\left(\bar{\psi}\gamma_\mu\psi\right)^2\right]$$



A critical phase → Expect BKT transitions.

Phase structure of the Thirring model

1+1 dimensional QFT

↓

1 dimensional XXZ quantum spin chain with constant and staggered B-field

↓

Commonly-studied correlators in QFT can be turned into spin correlates

↓

Look at $C(r)$: exponential/power law in the gapped/critical phase

Well… but this is challenging….

A string correlator in the spin model helps…

$$C_{\text{string}}(r) = \langle 0 | \sigma_0^z \sigma_1^z \sigma_2^z \cdots \sigma_r^z | 0 \rangle$$

$r \to \infty$

$C_{\text{string}}(r) \to k \neq 0$

Gapped

$C_{\text{string}}(r) \to 0$

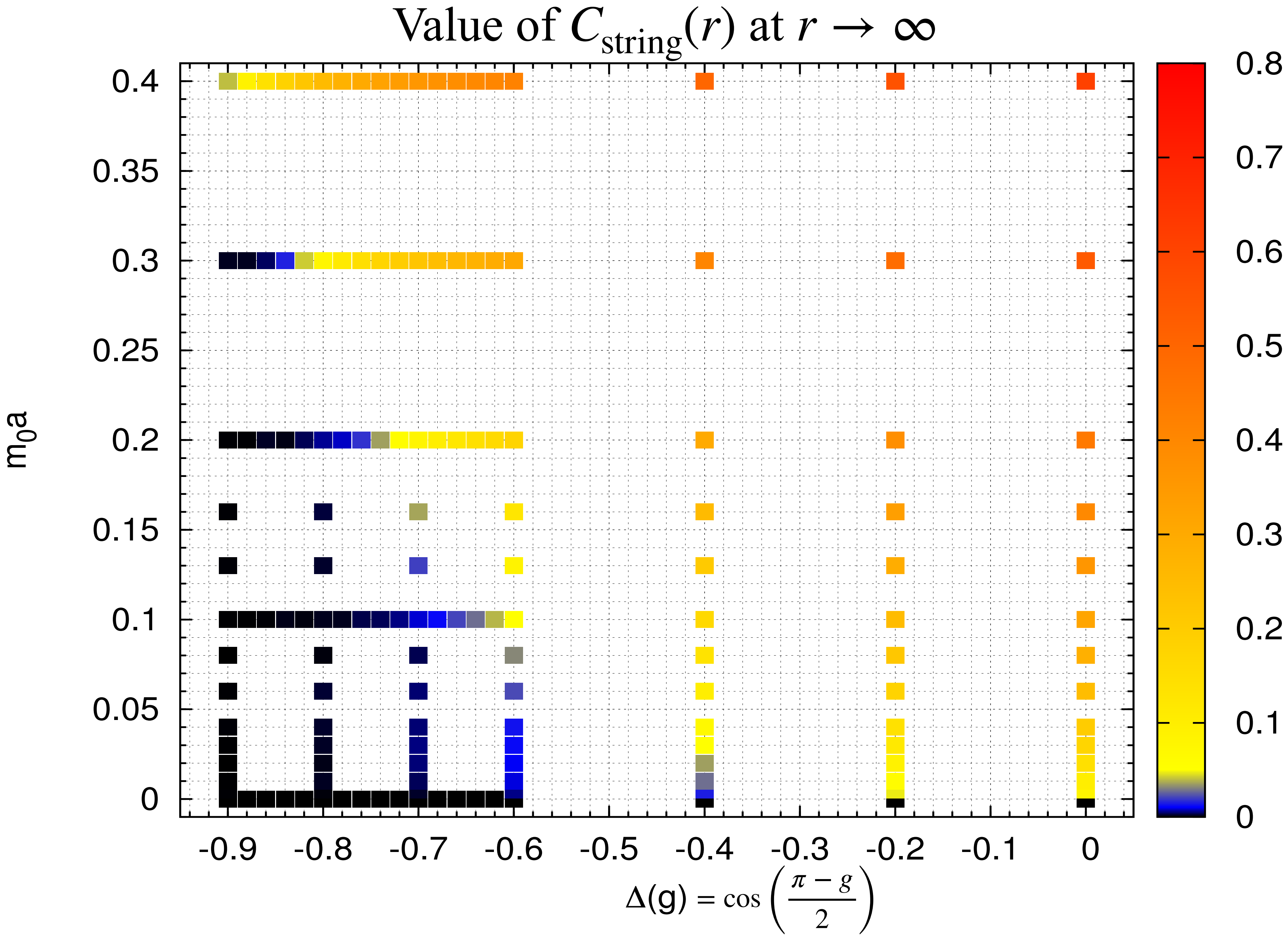
Critical

This corresponds to $\bar{\psi}(x)\psi(x)$ at each lattice site

NOT a commonly studied quantity in QFT

Probing phase structure with $C_{\text{string}}(r)$

M.C.Banuls, K.Cichy, Y.-J.Kao, CJDL, Y.-P.Lin, D.T.-L.Tan, PRD100 (2019)



Probing phase structure with $C_{\text{string}}(r)$

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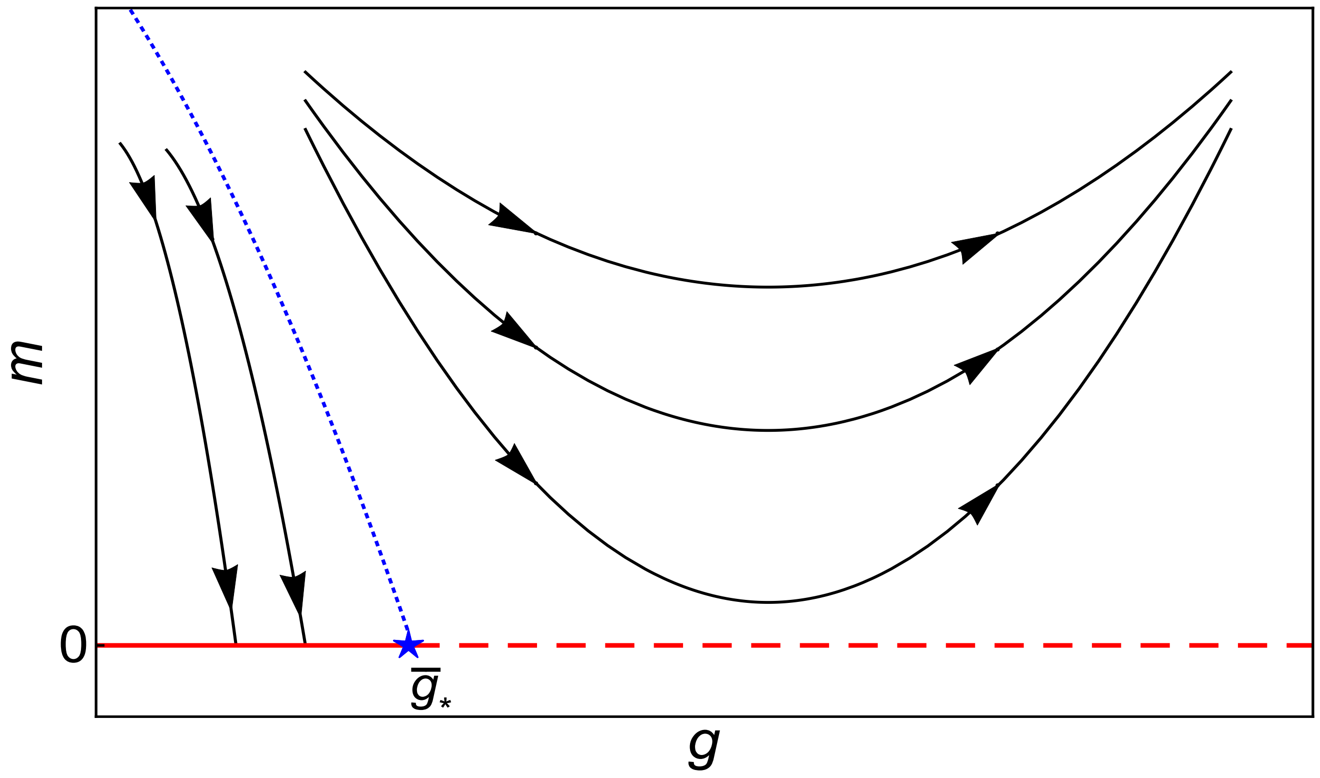
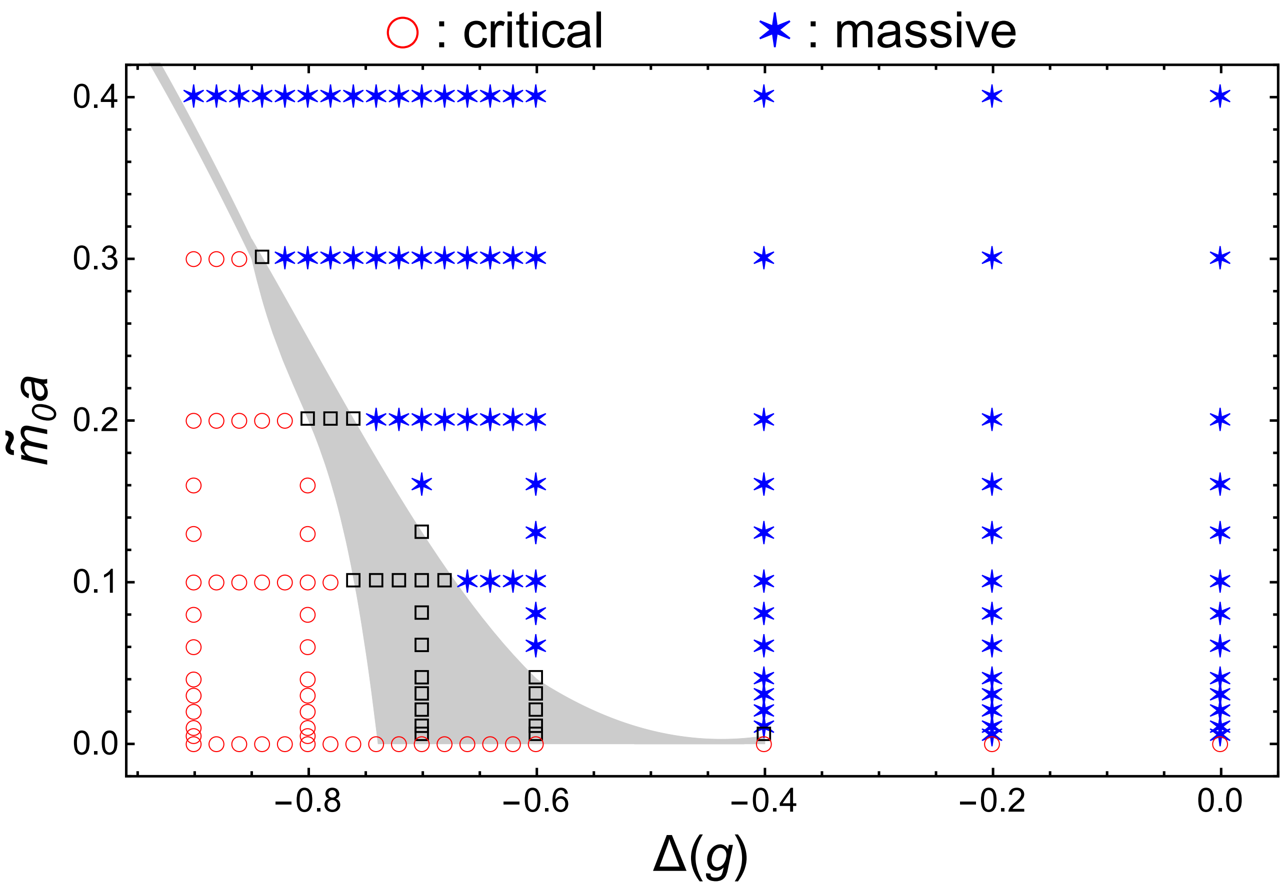


Perturbation theory (qualitative)



Simulation with MPS

Conclusion and outlook

- Strong dynamics can play an important role in BSM physics

- Lattice Field Theory is a powerful method in this subject

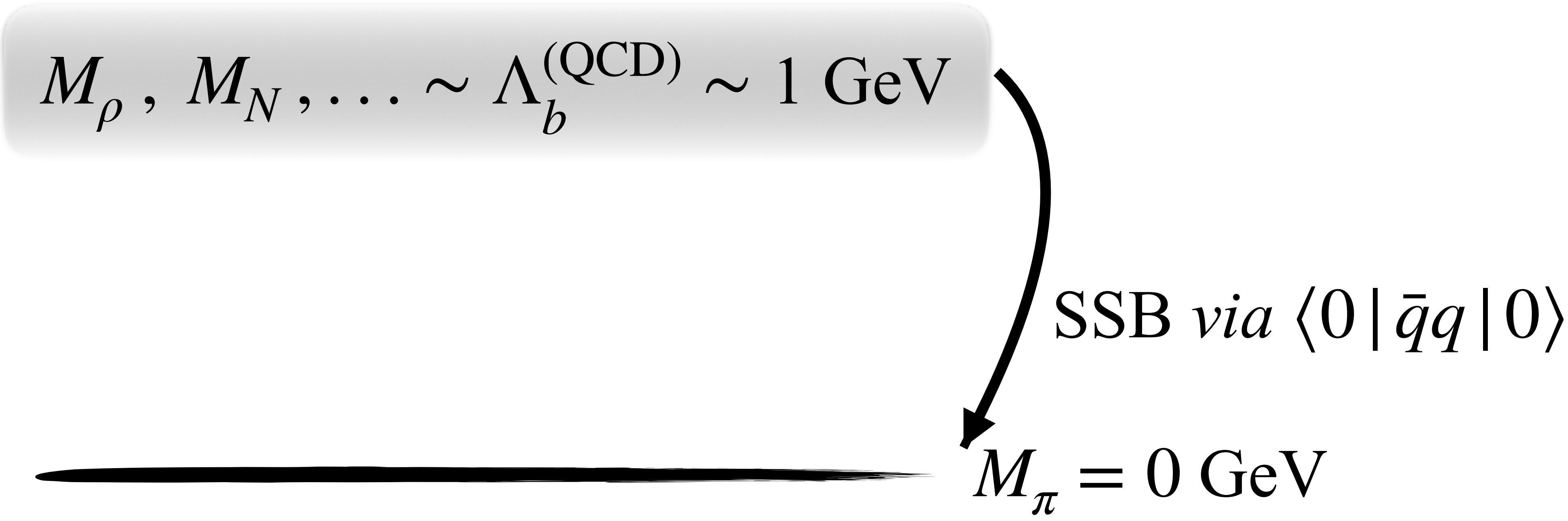
- New approaches inspired by BSM physics and condensed matter theory

Thank you all for your attention!

Backup slides

The Higgs boson as a bound state: Dilaton Higgs

Lesson from Quantum Chromodynamics

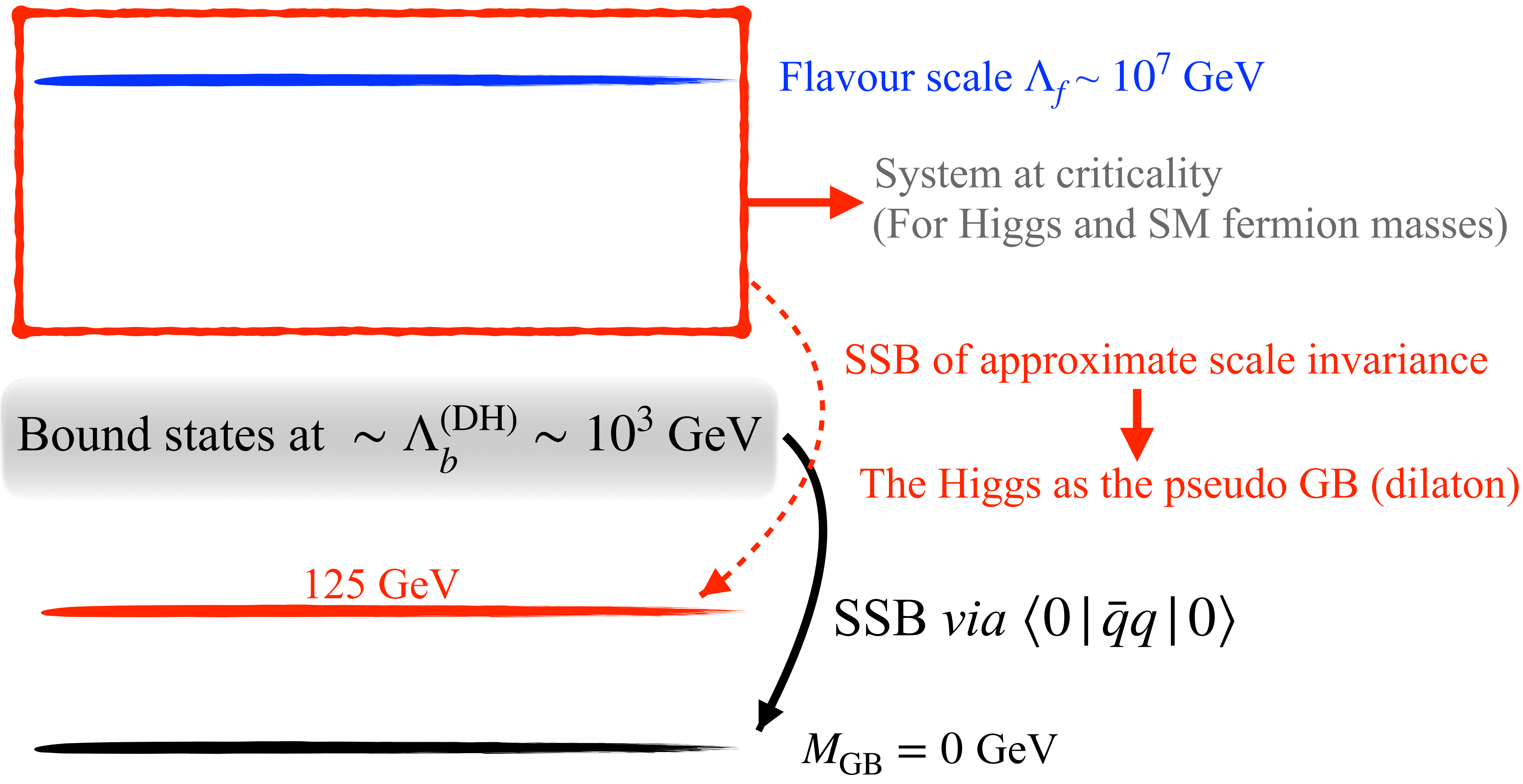


$M_\rho\,,\ M_N\,,\ldots \sim \Lambda_b^{(\mathrm{QCD})} \sim 1\ \mathrm{GeV}$

SSB *via* $\langle 0\,|\,\bar{q}q\,|\,0\rangle$

$M_\pi = 0\ \mathrm{GeV}$

Introducing (approximate) scale invariance



Flavour scale $\Lambda_f \sim 10^7$ GeV

System at criticality
(For Higgs and SM fermion masses)

SSB of approximate scale invariance

Bound states at $\sim \Lambda_b^{(\mathrm{DH})} \sim 10^3$ GeV

The Higgs as the pseudo GB (dilaton)

125 GeV

SSB *via* $\langle 0|\bar{q}q|0\rangle$

$M_{\mathrm{GB}} = 0$ GeV

Looking for viable candidate theories

- SU(3) gauge theories with various fermion contents

A.Hasenfratz, C.Rebbi, O.Witzel, PRD101 (2020)
T.-W.Chiu, PRD99 (2019)
Z.Fodor, J.Kuti, K.Holland, S.Mondal, D.Nogradi, PRD94 (2016)
CJDL, K.Ogawa, A.Ramos, JHEP12 (2015)
T.DeGrand, Y.Shamir, B.Svetitsky, PRD82 (2010)
A.Deuzeman, M.P.Lombardo, T.N.Da Silva, E.Pallante, PLB720 (2013)
T.Appelquist, G.Fleming, E.Neil, PRL100 (2008)
⋮

- SU(2) gauge theories with various fermion contents

⋮

What does a viable spectrum look like

LSD Collaboration, PRD 99, 2019