Uncover the mystery of strange metal state in correlated electron systems

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NTU, Nov 3, 2020

In memory of Prof. Pauchy Huang (黃偉彥 教授)

I was Prof. Huang’s Master degree student during 1991-1993 in NTU.

Outlines

- Strange metal phenomena in correlated electron systems
- Strange metal in heavy fermion metals/superconductors

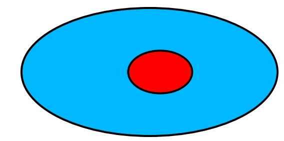
  Heavy-fermion metal: Ge-substituted $YbRh_2Si_2$

  Heavy-fermion superconductors $CeMIn_5$, M=Co, Rh, Ir

  Mechanism: Kondo vs. AF RKKY
- Paramagnetic heavy-fermion metal on frustrated lattice
- Summary

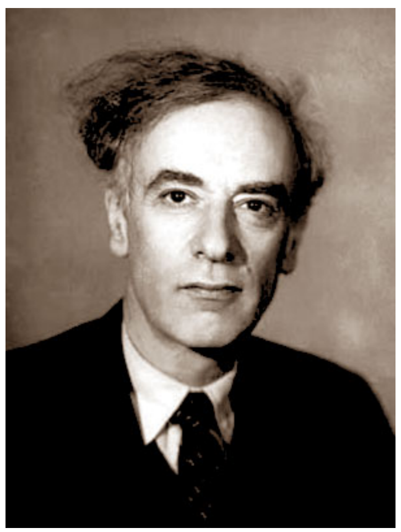
Landou's Fermi -liquid theory: normal metals

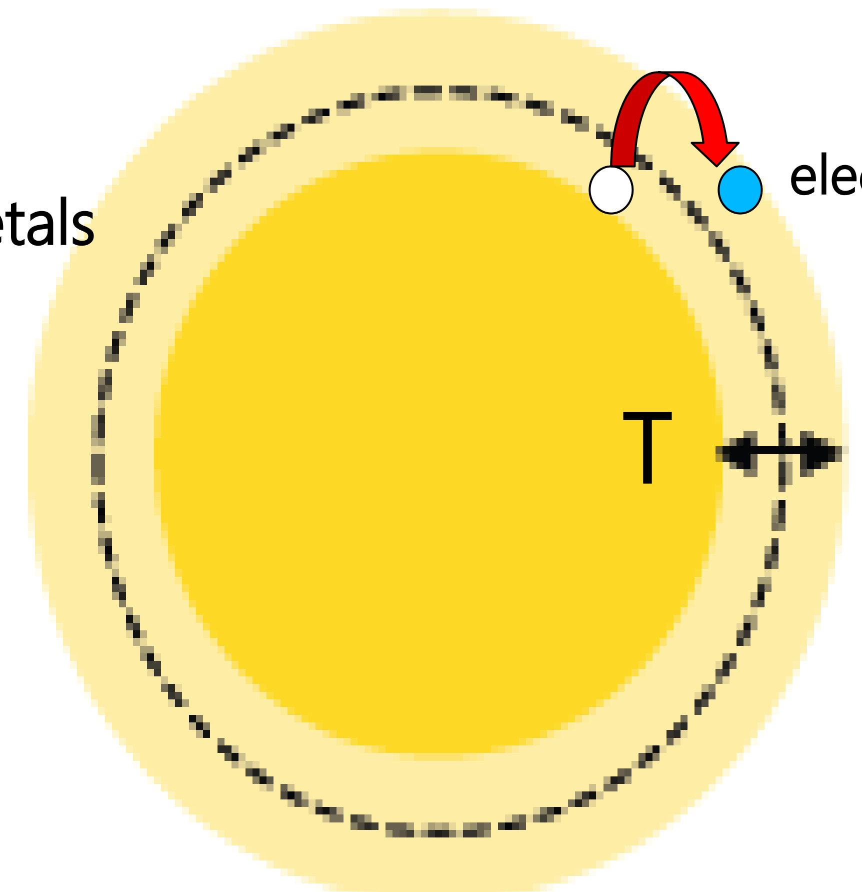
Elementary excitations in fermionic solid state systems: quasiparticles



electrons dressed by density fluctuations

Normal states of most metals





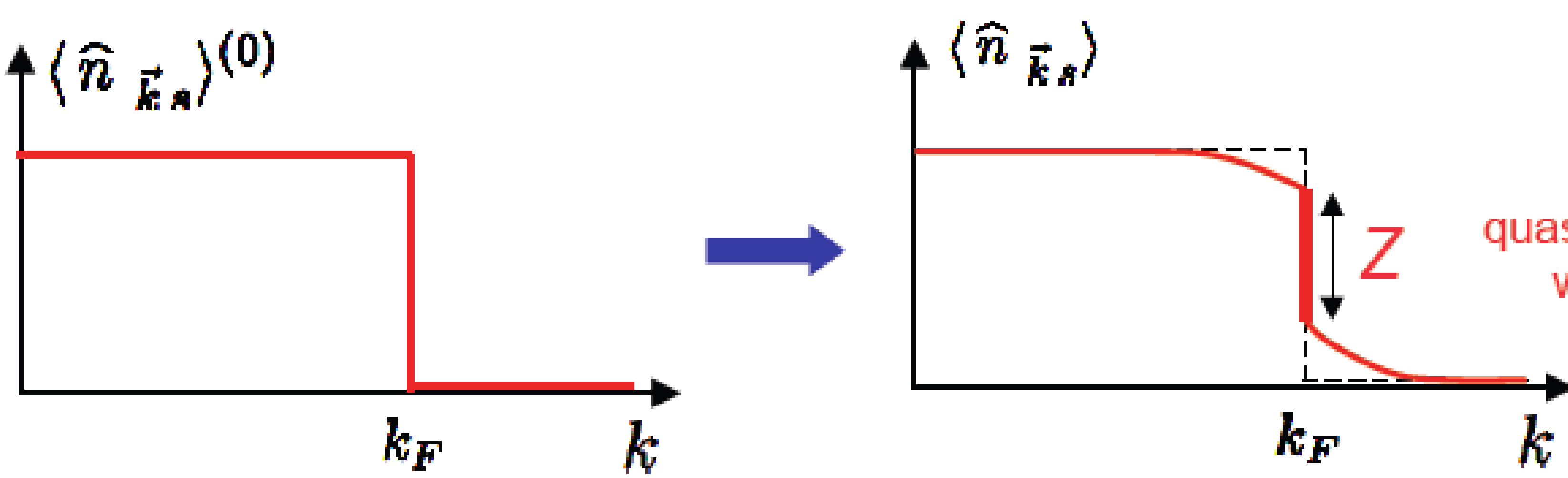
quasi-particles:

weakly interacting
electron-hole pairs

Lev Landou

Enrico Fermi

States of Fermi-liquid described by quasi-particle distributions



$$G(\omega, p) \approx \frac{Z}{\omega + \mu - \epsilon(p)}$$

Fermi liquid theory - physical properties

Electrical resistivity:

$$\rho(T)=\rho(0)+aT^2$$

T^2 -resistivity

specific heat:

$$\delta n_\sigma(\vec{k}) = n_\sigma^{(0)}(T,\vec{k}) - n_\sigma^{(0)}(0,\vec{k})$$

$$C = \frac{\pi^2 k_B^2 N(\epsilon_F)}{3} T$$

T-linear specific heat

compressibility:

$$\kappa = -\frac{1}{\Omega} \frac{\partial \Omega}{\partial p}\bigg|_{T,N}$$

$$\kappa = \frac{3}{2n\epsilon_F} \frac{1}{1+F_0^s} = \frac{1}{n^2} \frac{N(\epsilon_F)}{1+F_0^s}$$

spin susceptibility:

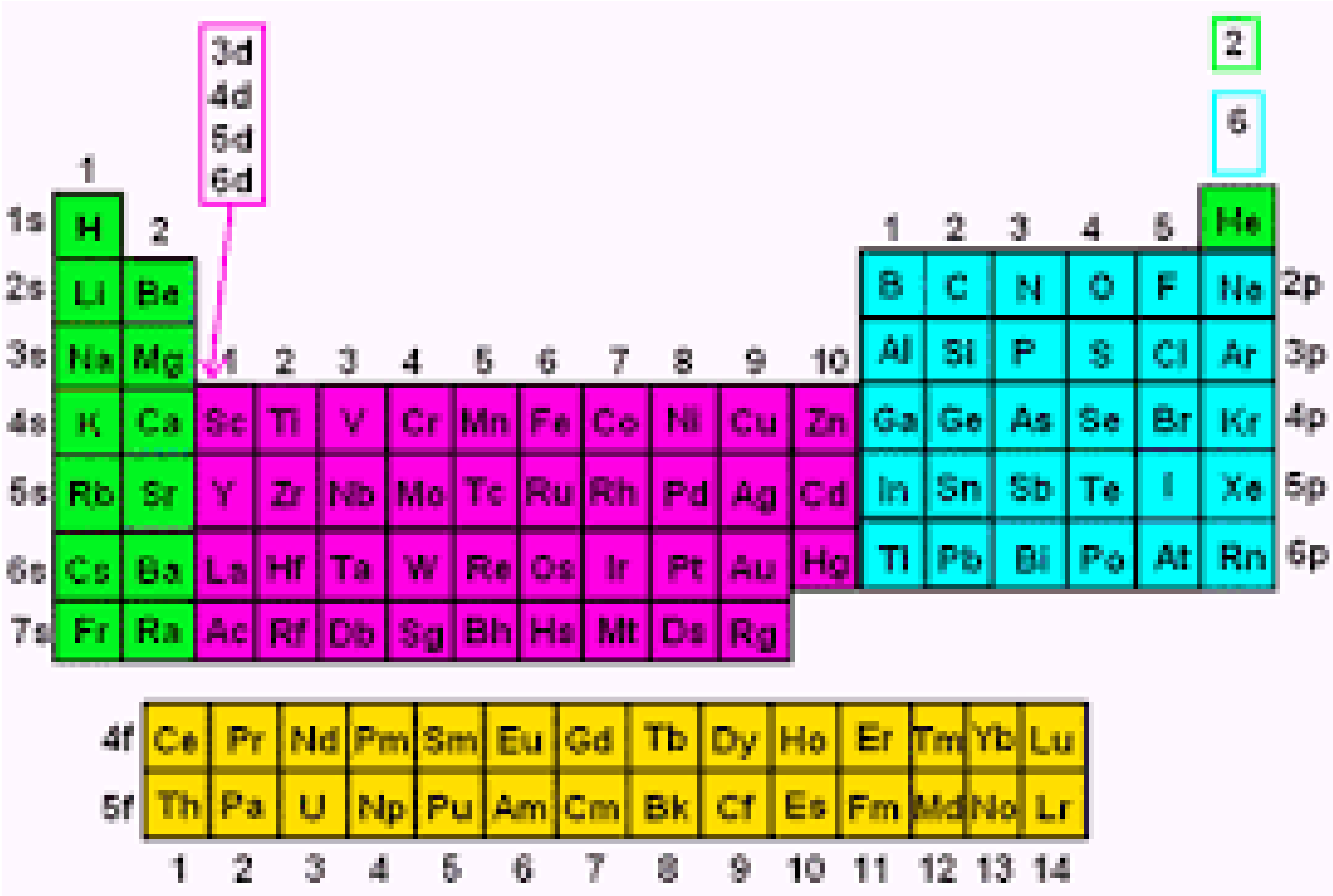
$$\chi = \frac{M}{H}$$

$$\chi = \frac{\mu_B^2 N(\epsilon_F)}{1+F_0^a}$$

Landau parameters: F_l^s

Strongly correlated electron systems

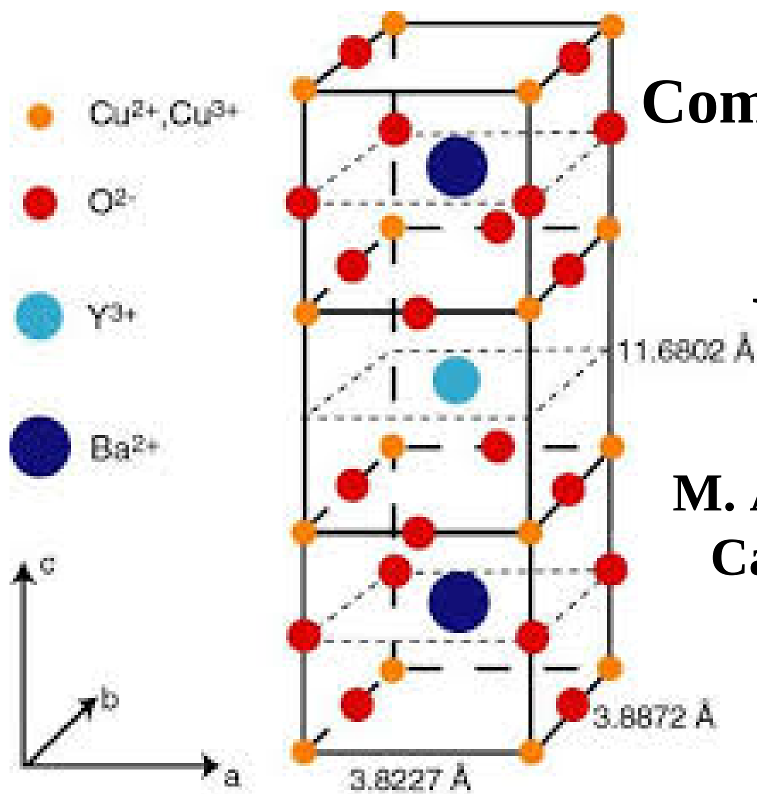
Transition metal compounds



Strongly correlated quantum many-body systems

High-Tc cuprate superconductors

- x=0, Large Coulomb repulsion U--> Mott Insulators +Heisenberg anti-ferromagnet
- x > xc, holes destroy AF order-→ normal Fermi liquid metal

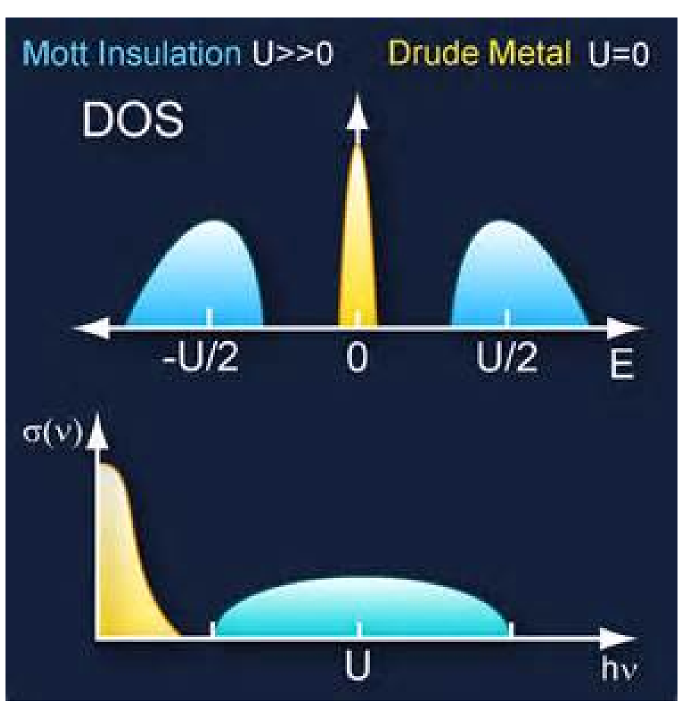


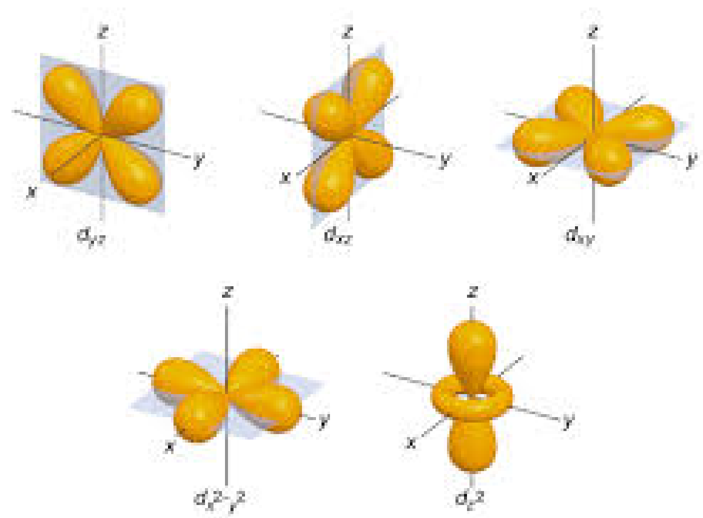
Competing ground state: AF vs. FL

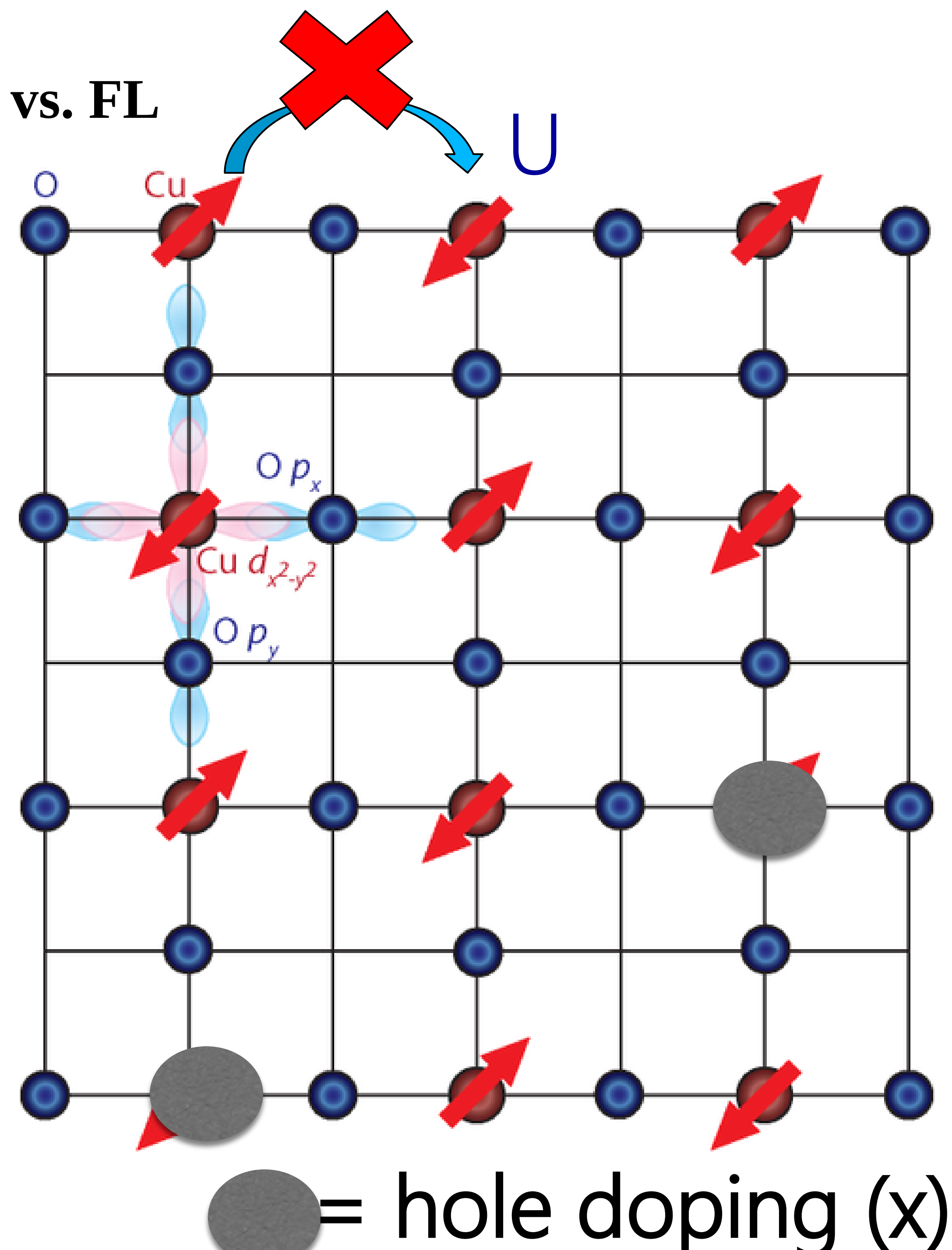
$YBa_2Cu_3O_{7-x}$

M. Ainslie, PhD Thesis, Cambridge U. , 2012

Cu: d-orbitals



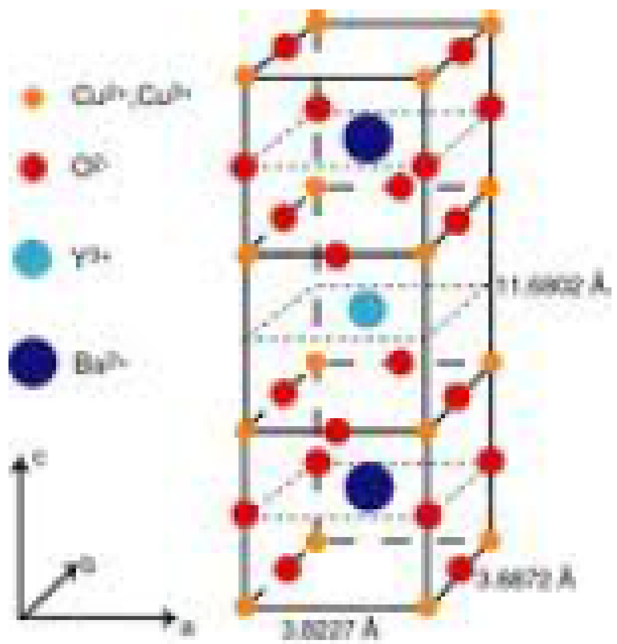




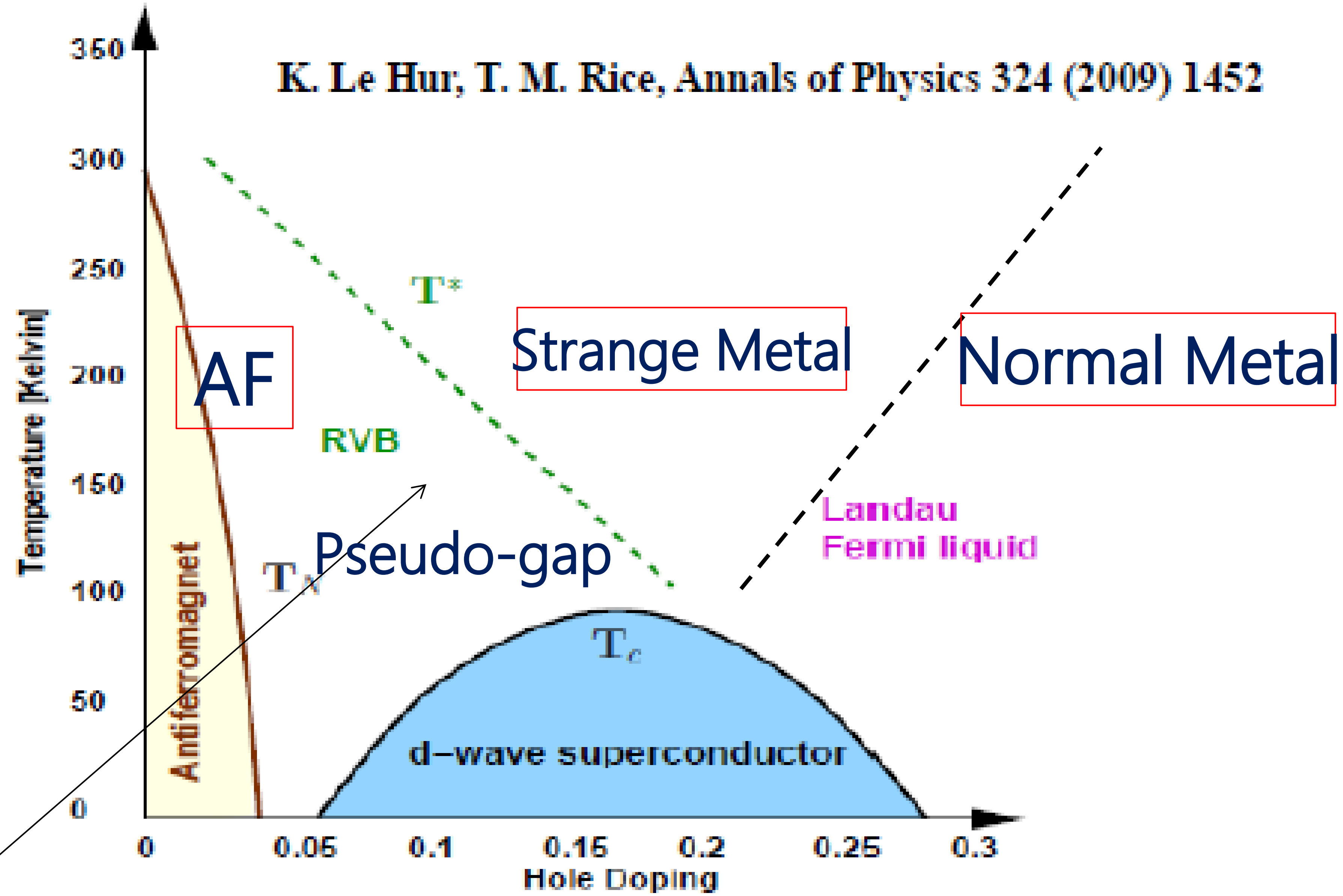
https://www.psi.ch/swissfel/OrigInsTransEN/igp_1024x640%3E_V_11.png

http://qcmd.mpsd.mpg.de/files/qcmd-theme/research/science/Mott/mott-diagr-for-web-2014-dbb-

Phase diagram of high-Tc superconductors



YBCO



$$= \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

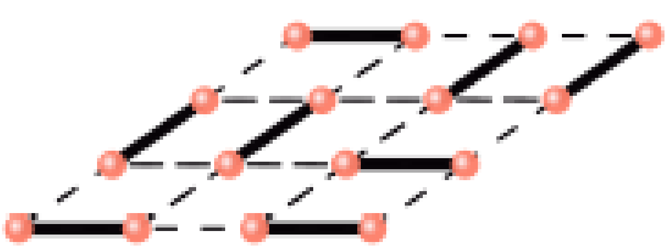


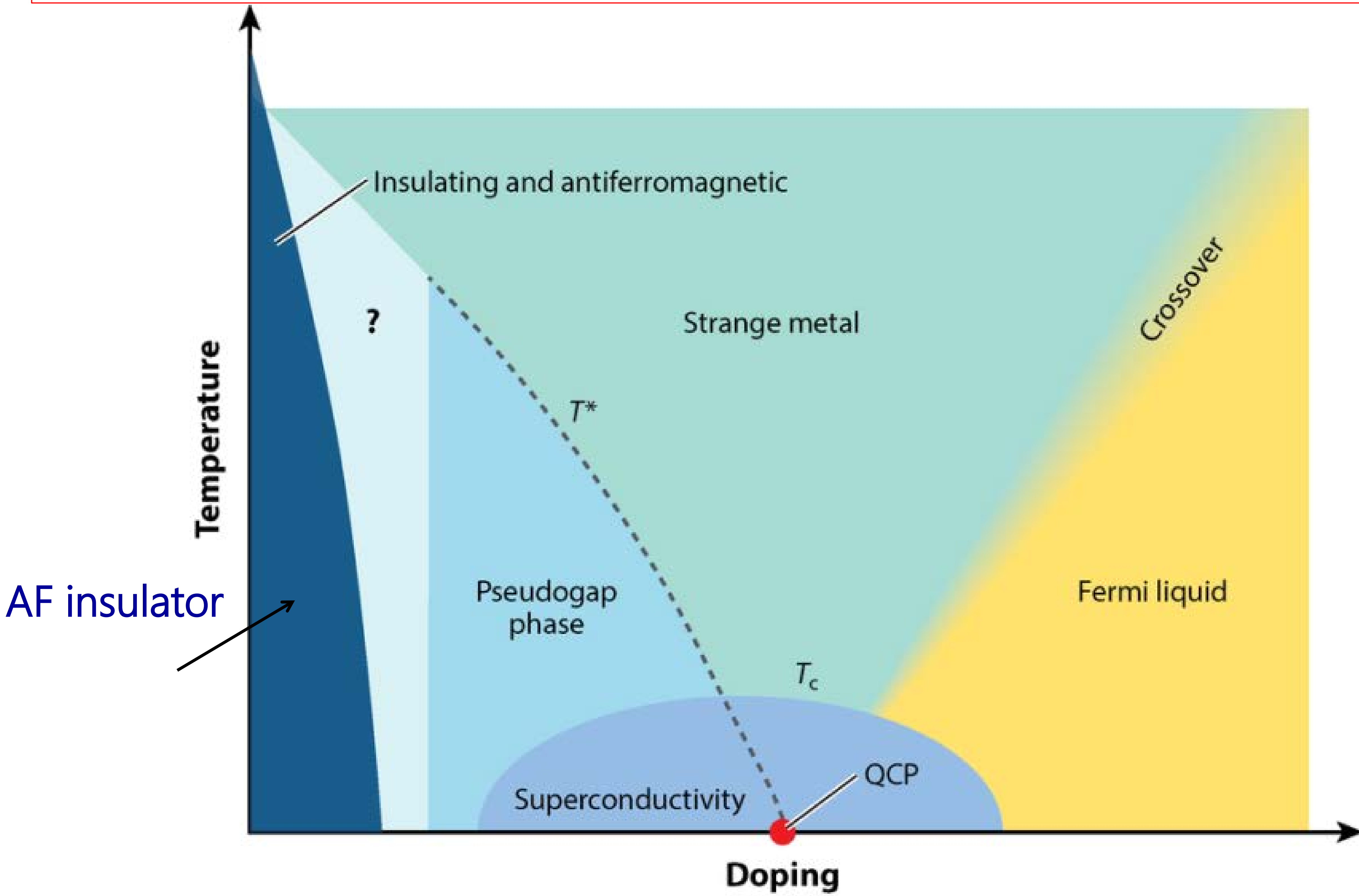
Fig. 1. Schematic phase diagram of high-T_c superconductors.

Highest Tc so far ~ 133K

Even higher Tc ?

https://upload.wikimedia.org/wikipedia/commons/thumb/0/05/Spinon_movin
png/130px-Spinon_moving.png

Strange metal near edge of AF pseudogap and Fermi liquid phases

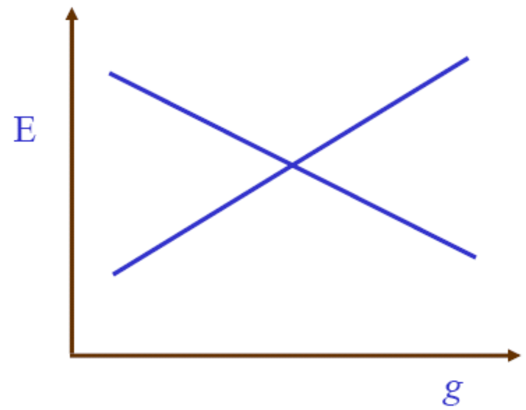


Goldman AM. 2014.
Annu. Rev. Mater. Res. 44:45–63

Quantum phase transitions

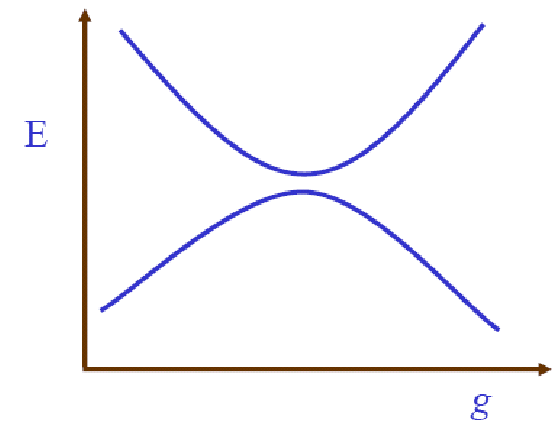
Competing Quantum Ground States

Non-analyticity in ground state properties as a function of some control parameter g



True level crossing: Usually a first-order transition

Sachdev, quantum phase transitions, Cambridge Univ. press, 1999

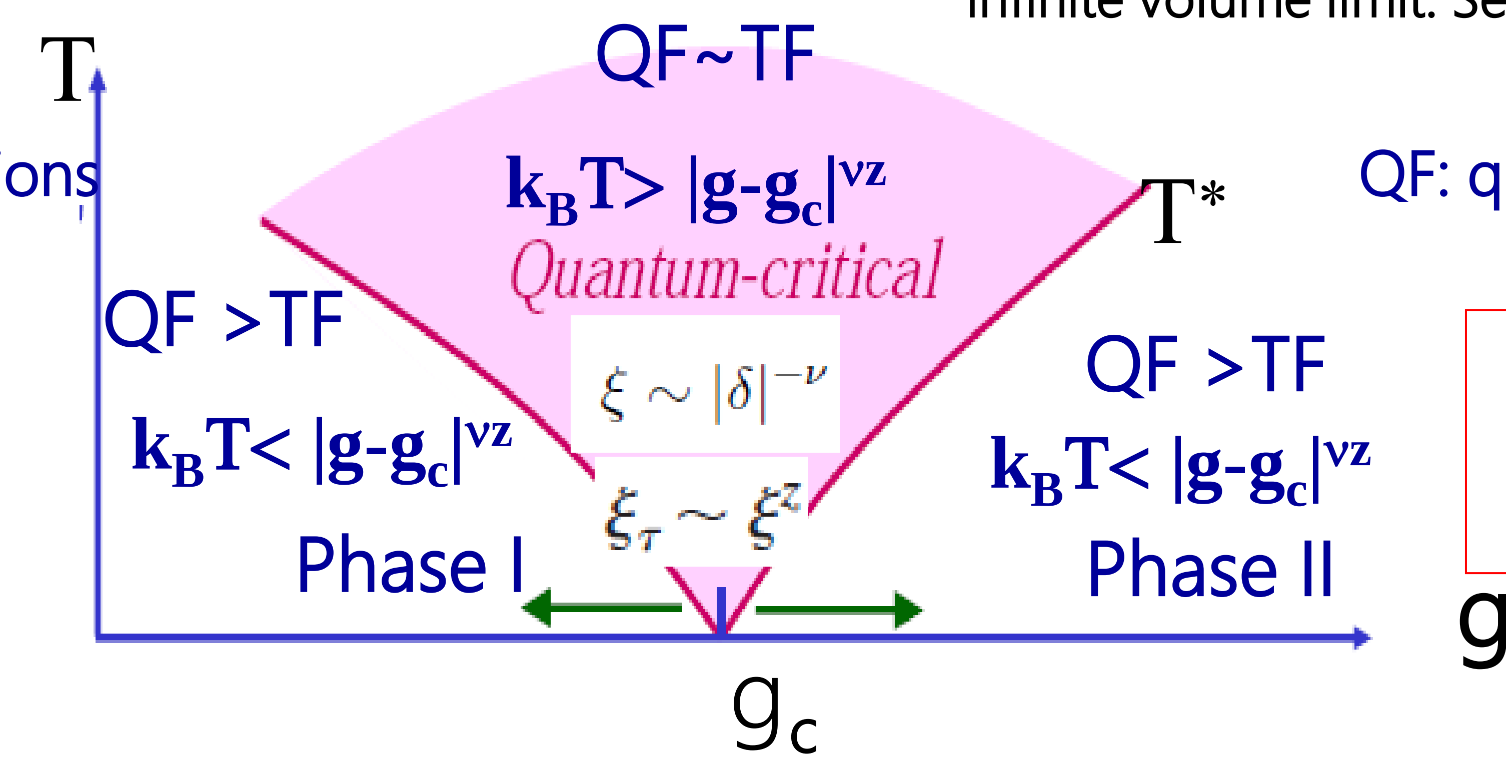


Avoided level crossing which becomes sharp in the infinite volume limit: Second-order transition

TF: thermal fluctuations

QF: quantum fluctuations

$\delta = |g-g_c|$

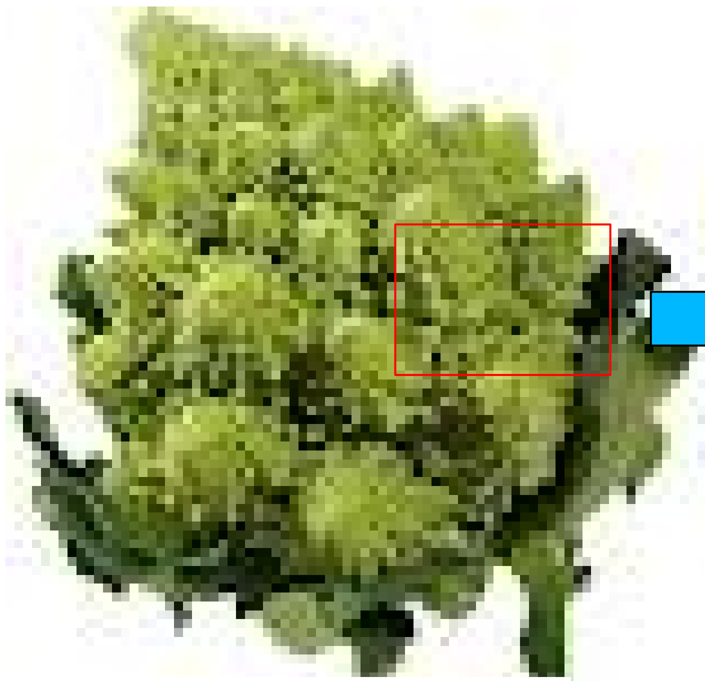


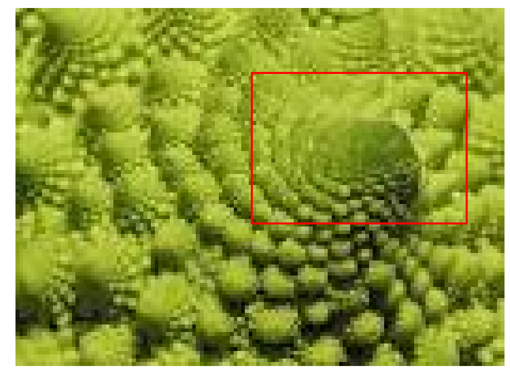
$$\xi \sim |\delta|^{-\nu} \qquad \xi_\tau \sim \xi^z. \longrightarrow \infty$$

- Critical point is a novel state of matter
- Critical excitations control dynamics in the wide quantum-critical region at non-zero temperatures
- Quantum critical region exhibits universal power-law behaviors: Non-Fermi liquid

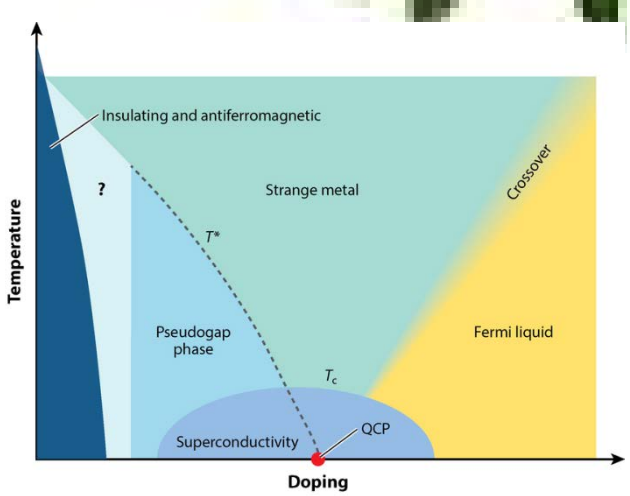
Universal quantum critical behaviours:
Fractal Cauliflower, self-similarity --- Quantum Criticality

$<S(0)\,S(r)> \sim G(r) \sim \exp(-r/\xi)$





$\xi \longrightarrow \infty$ $\xi \sim |\delta|^{-\nu}$



Goldman AM. 2014. Annu. Rev. Mater. Res. 44:45–63

Same correlations at ALL length scale !

Dynamical scaling form near QCP:

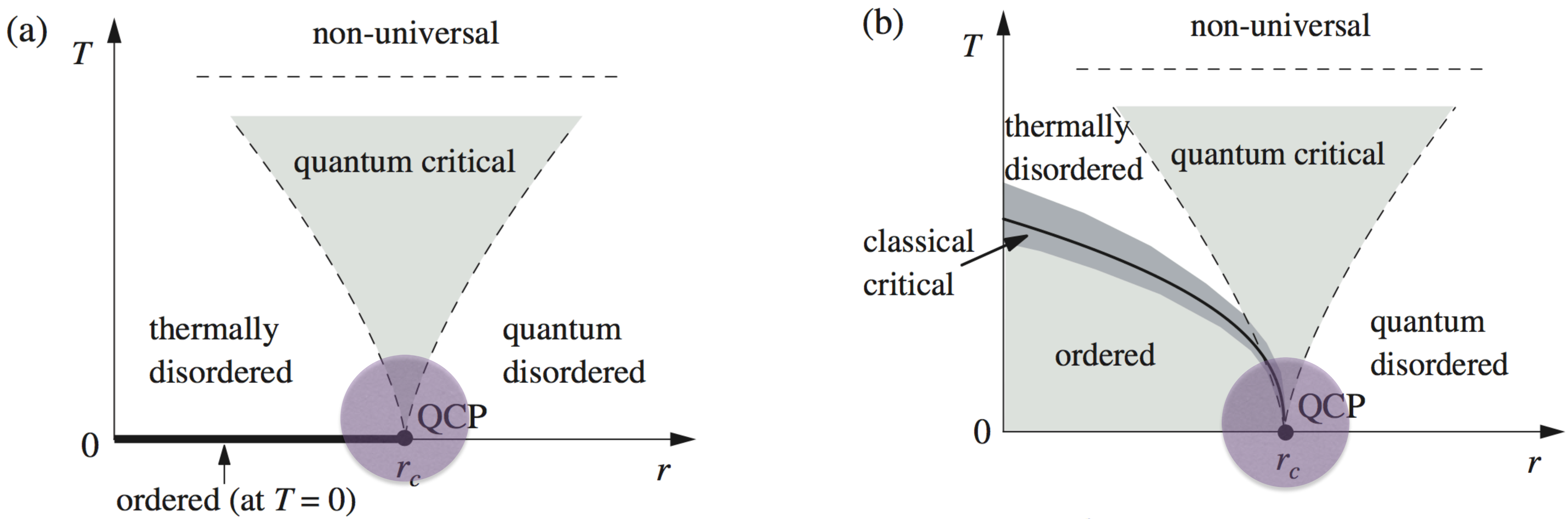
$$\mathcal{O}(k,\omega,K) = \xi^{d_{\mathcal{O}}} O(k\xi,\omega\xi_\tau)$$

Sondhi et al, RMP 1997

$d_{\mathcal{O}}$ scaling dimension of the observable $\mathcal{O}$

close to the critical point, there is no characteristic length scale other than ξ itself and no characteristic time scale other than ξ_τ

Quantum phase transition (QPT) & universal scaling



<S(0) S(r)>~ G(r)~ exp(-r / ξ $\quad \xi \longrightarrow \infty$

Vojta, RPP, 2003

Near QCP r_c

universal scaling

$$\xi \sim |r - r_c|^{-\nu} \;;\; C_V \sim |r - r_c|^{\alpha}$$

$$M \sim |r - r_c|^{-\beta} \;;\; \frac{\partial M}{\partial B} \sim |r - r_c|^{-\gamma}$$

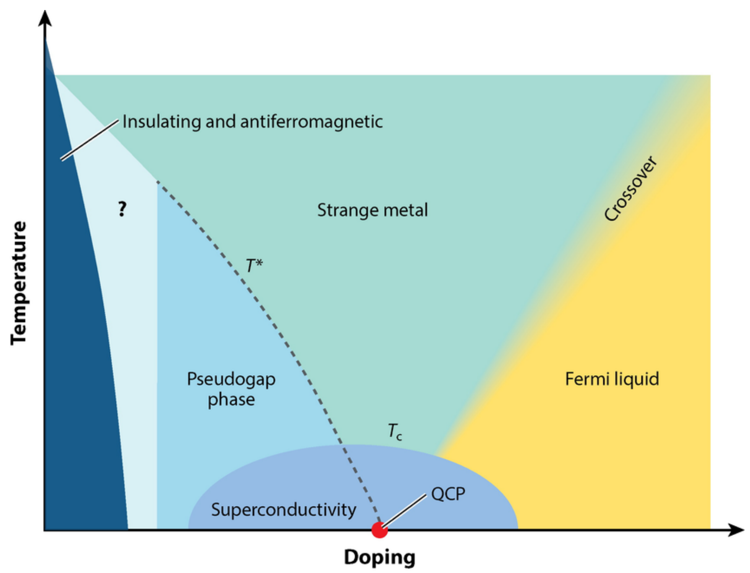
$$\vdots$$

Effective dimension : *d + z*

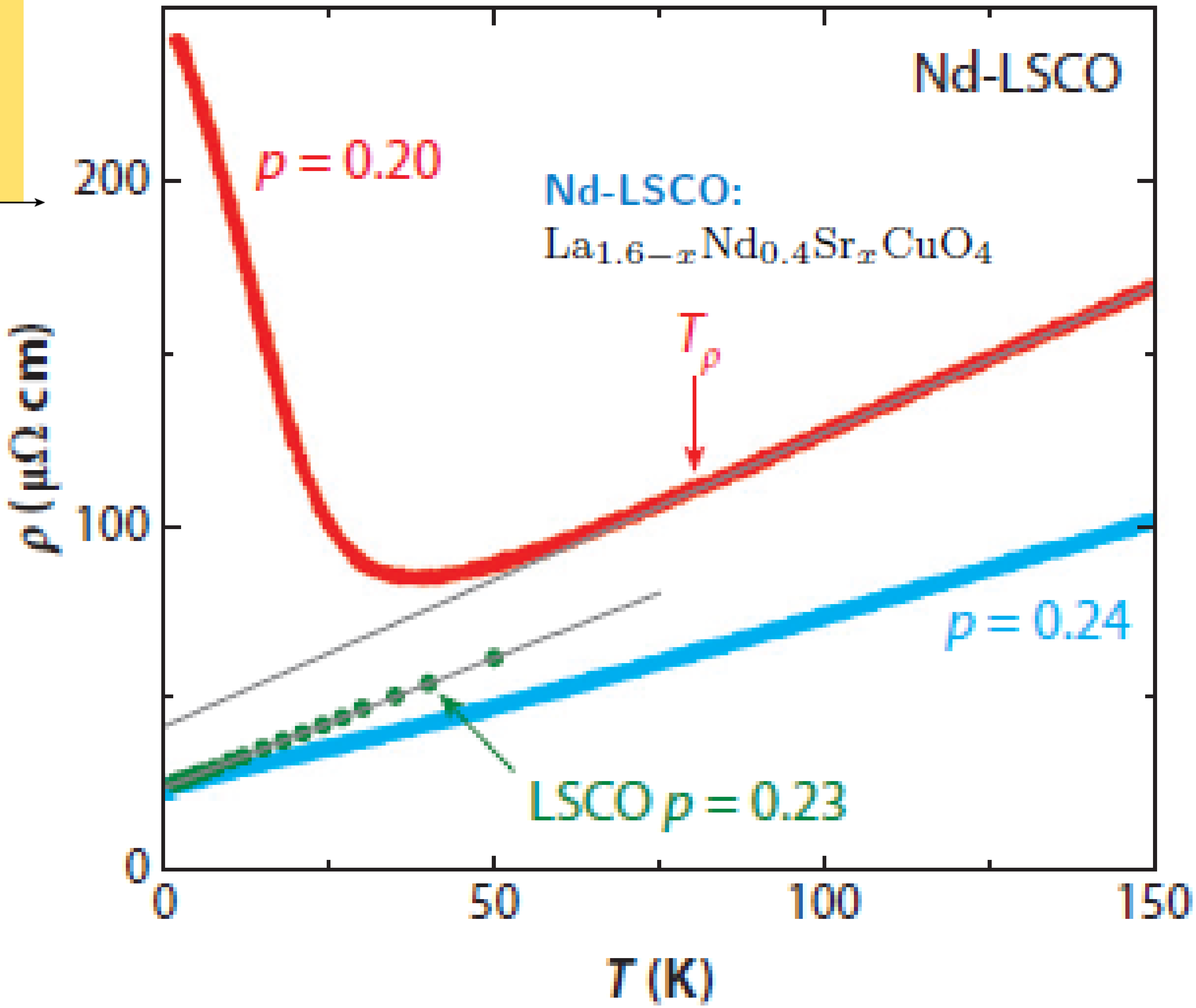
Hyperscalings: for *d+z < 4*

➽ *Relations between various exponents.*

Strange Metal: linear-T resistivity



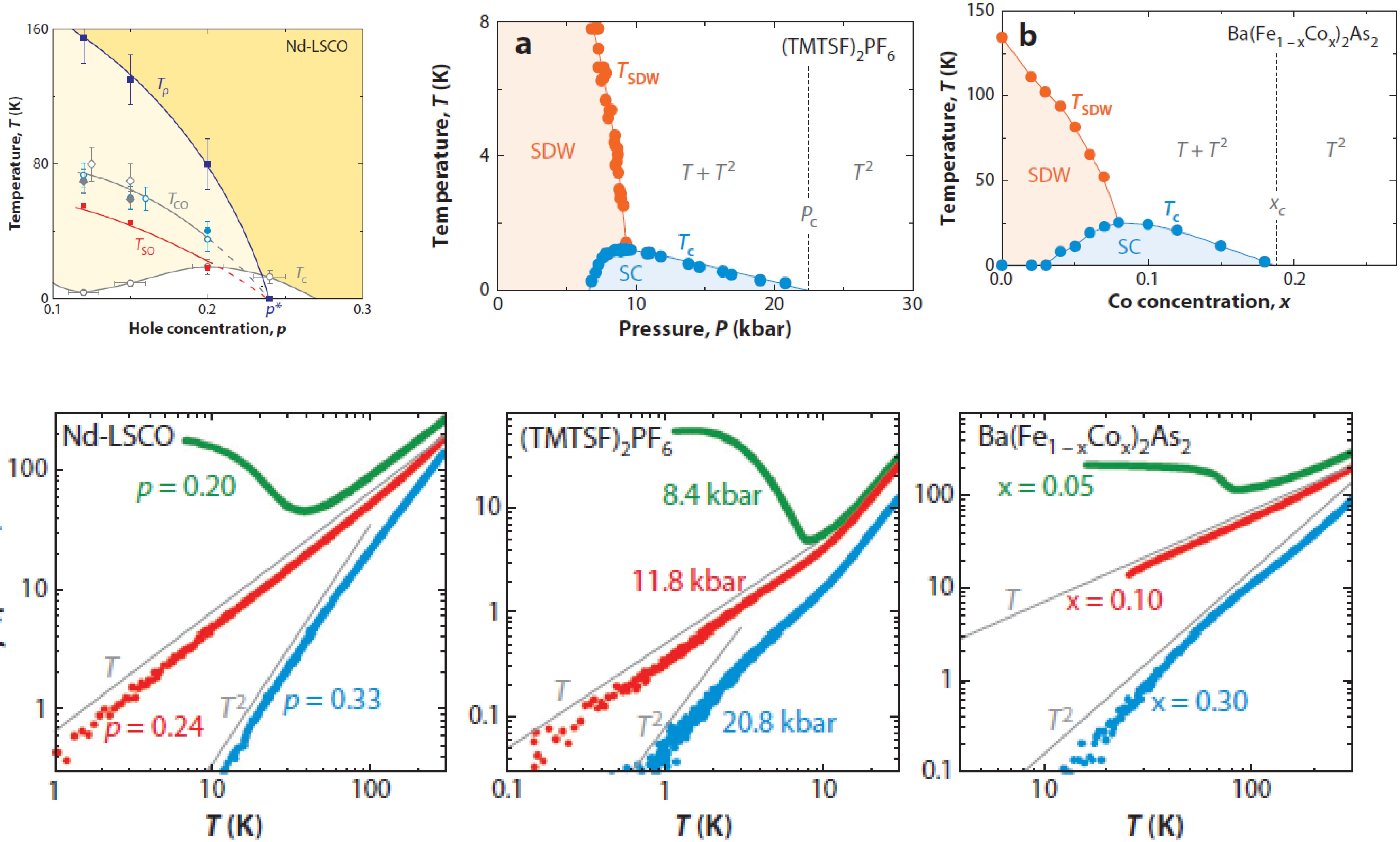
Goldman AM. 2014.
Annu. Rev. Mater. Res. 44:45–63



L. Taillefer, Ann Rev. 2010

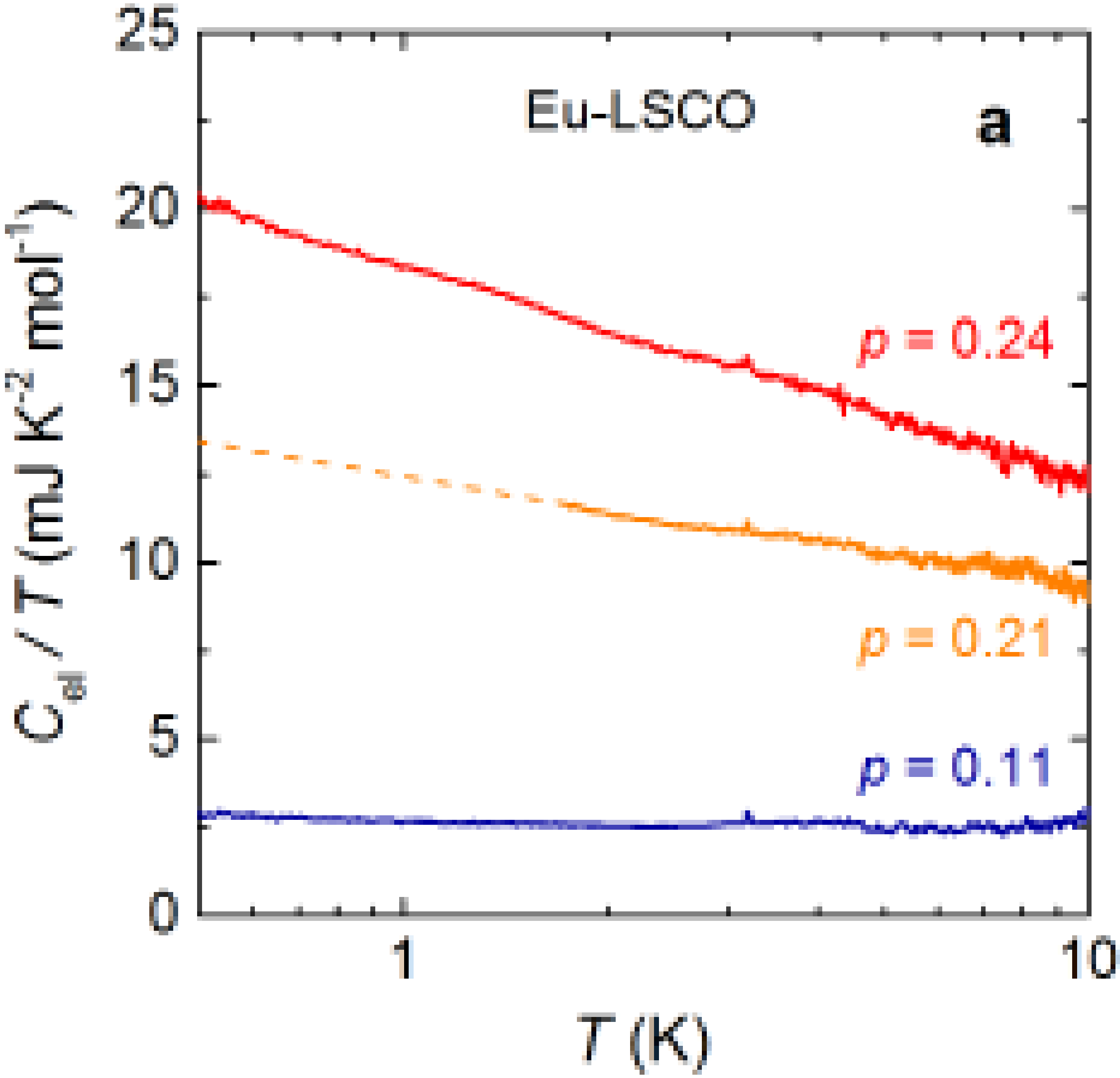
Strange Metal: linear-T resistivity

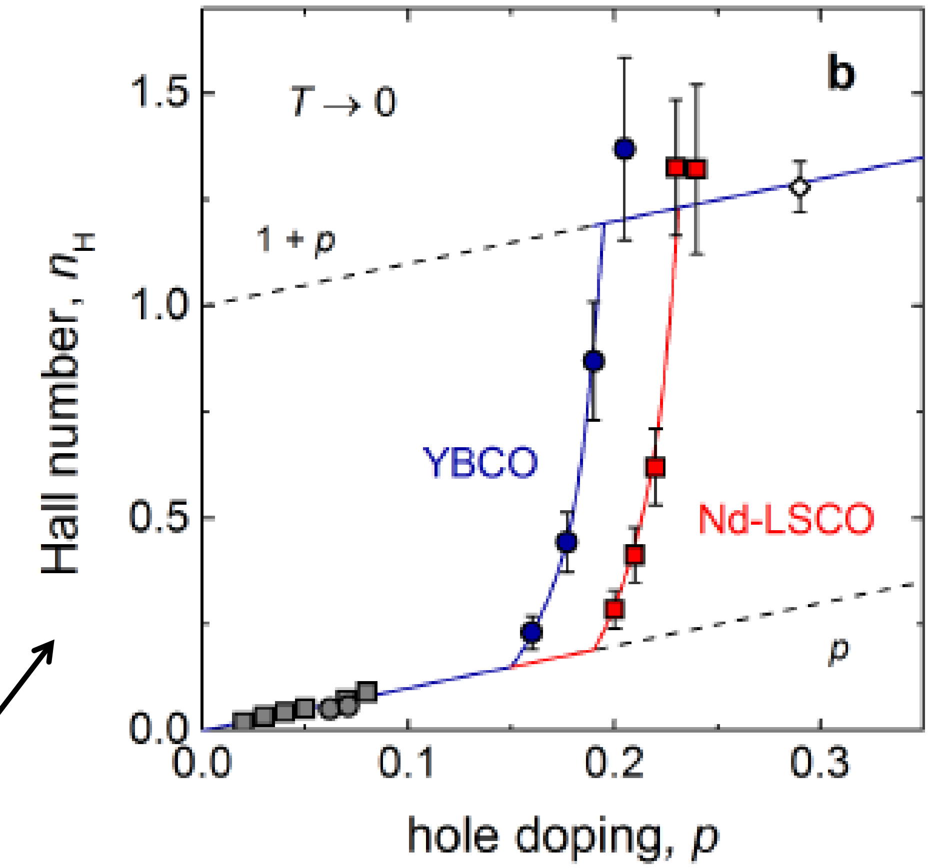
Generic, Ubiquitous across various correlated materials near phase transitions



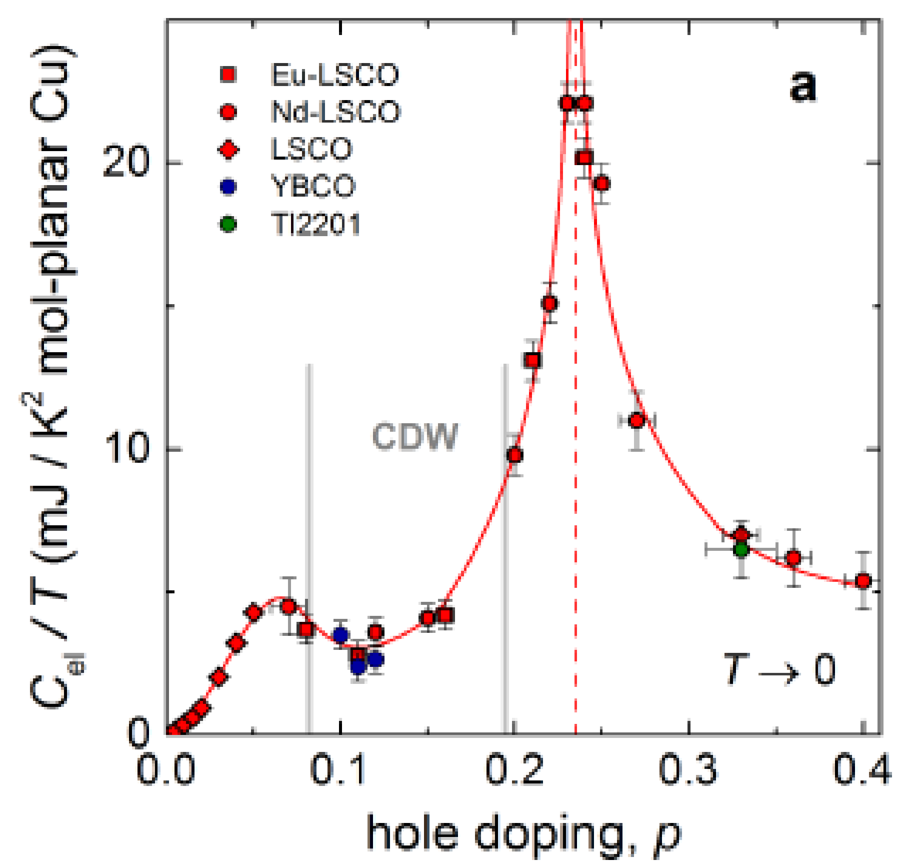
L. Taillefer, Ann Rev. 2010

Strange Metal: T-logarithmic specific heat coefficient Cv/T



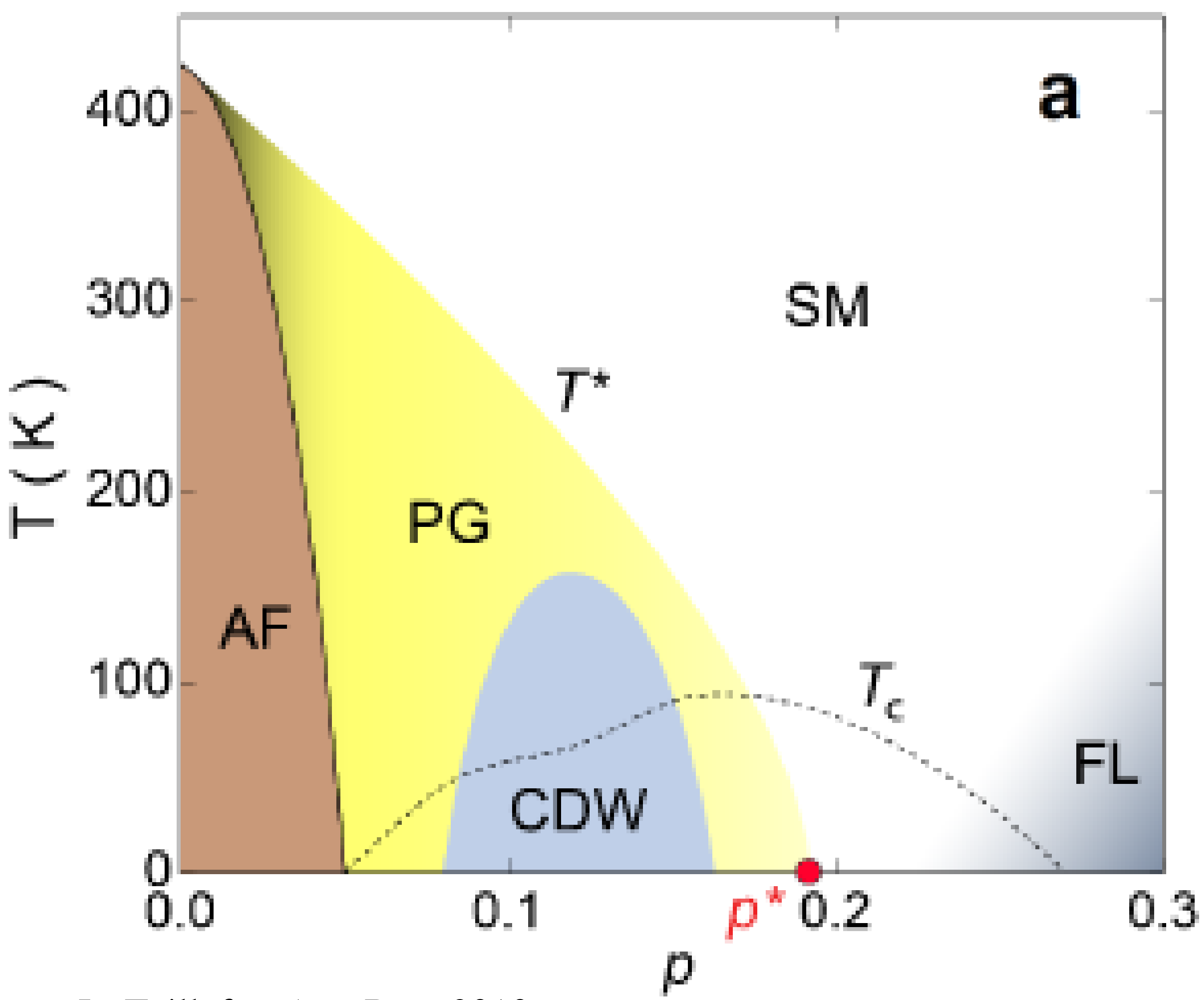


Signature of QCP?

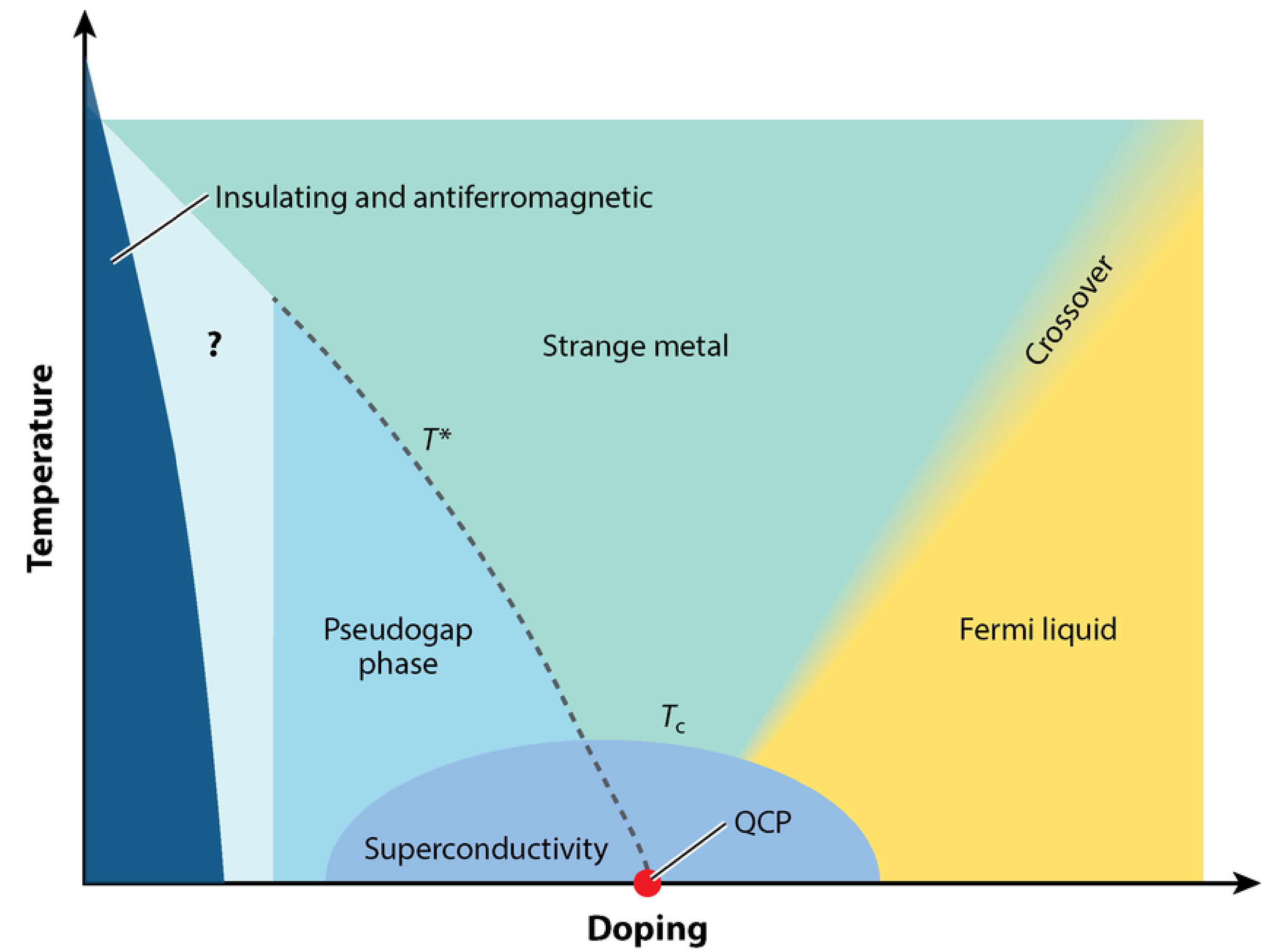


L. Taillefer, Ann Rev. 2010

Strange Metal Behaviours near Quantum Phase Transitions and superconductivity: High-Tc cuprate superconductors



L. Taillefer, Ann Rev. 2010



Goldman AM. 2014.
Annu. Rev. Mater. Res. 44:45–63

Origin of Strange Metal ?

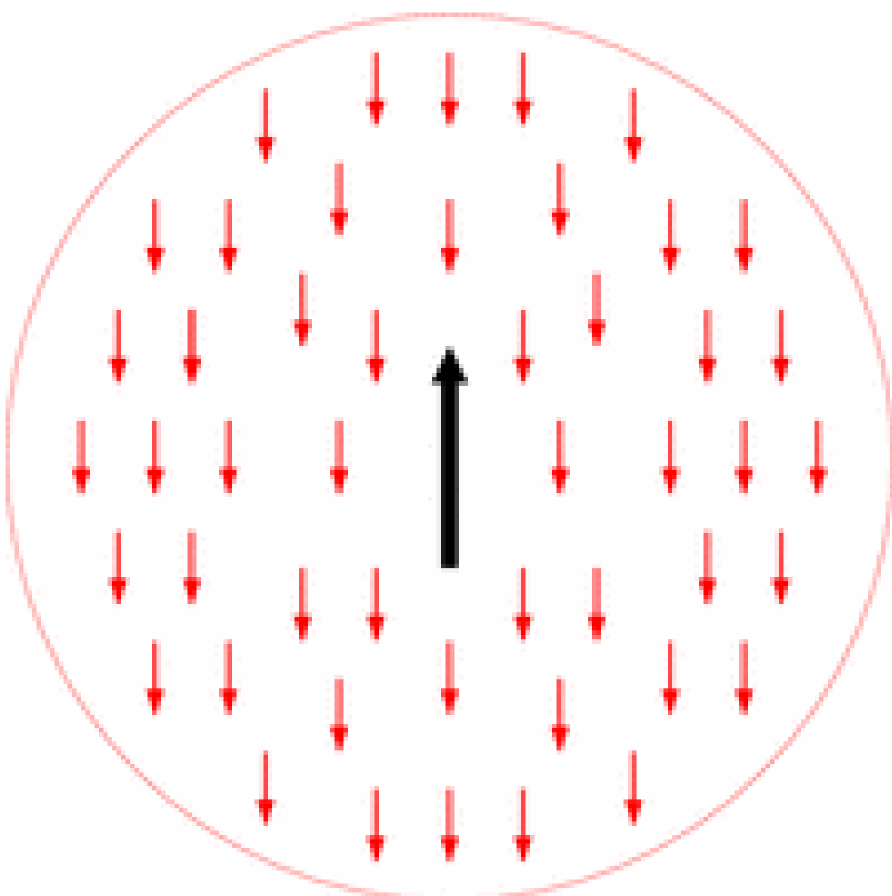
SM phase (new ground state)

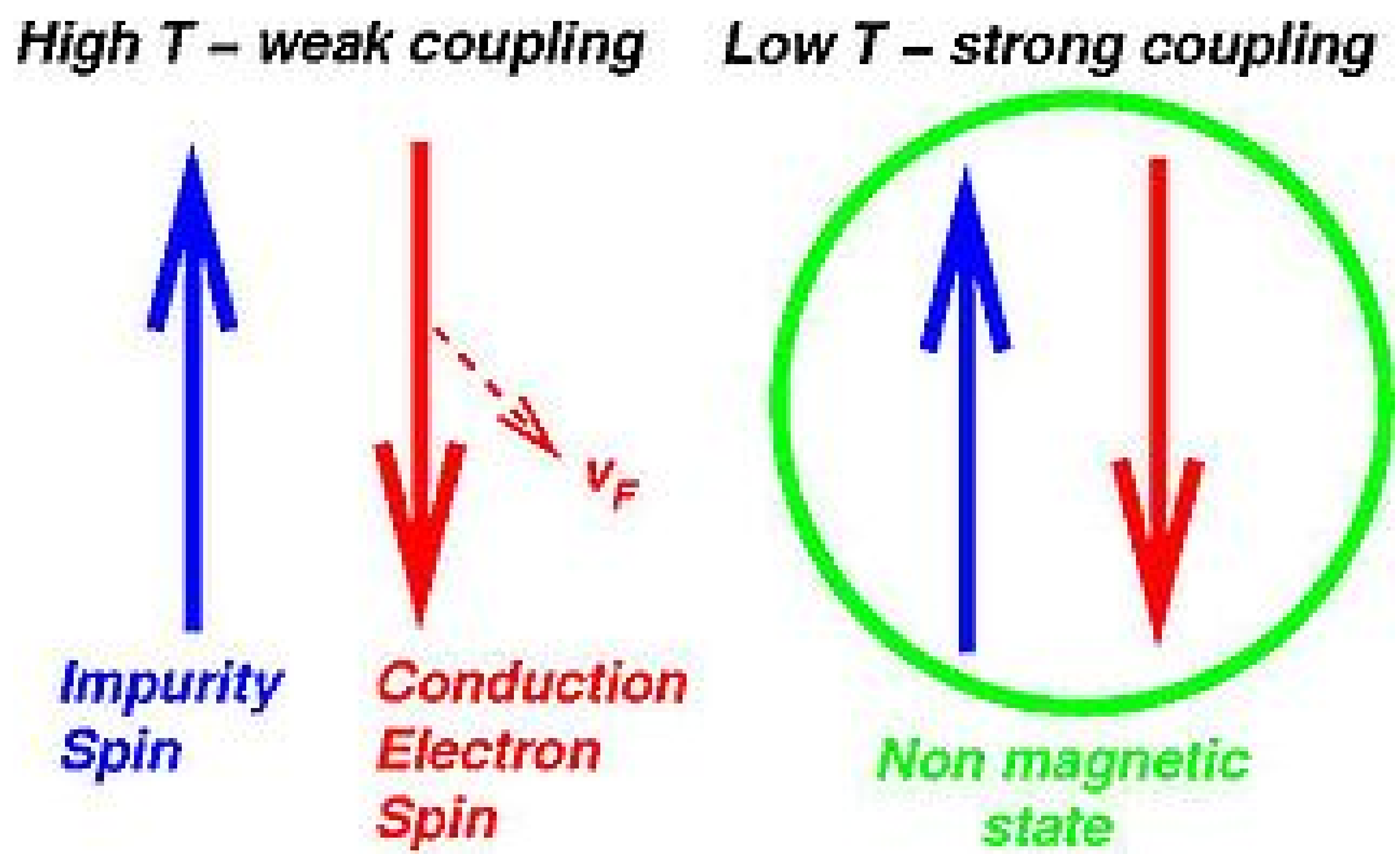
SM region (single QCP)

Debatable Open Question !

Strong correlations--Kondo Effect in metals with magnetic impurity

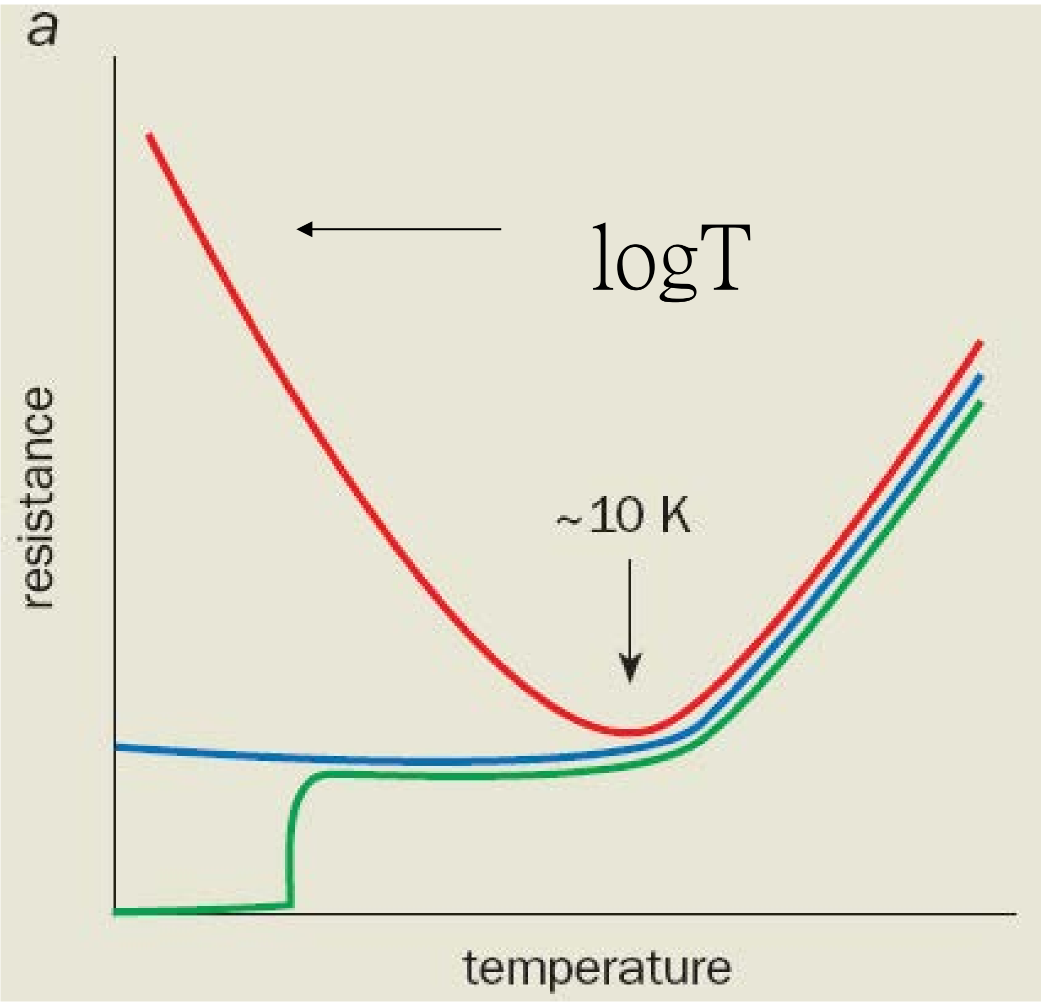
Anti-ferromagnetic spin-exchange between conduction electrons and local impurity spins



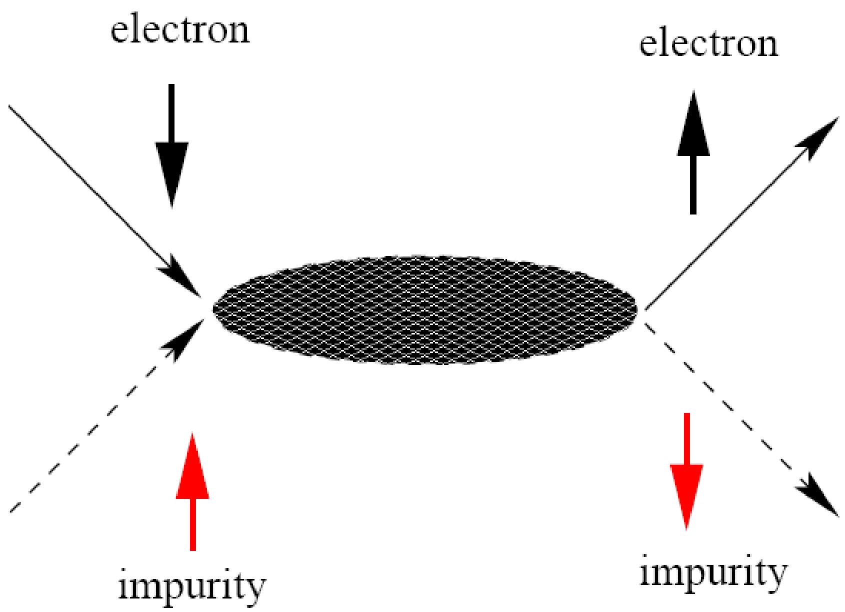


https://upload.wikimedia.org/wikipedia/commons/thumb/e/e2/Kscheme.jpg/320px-Kscheme.jpg

Kondo effect in metals with magnetic impurities



(Glazman *et al. Physics world 2001)*

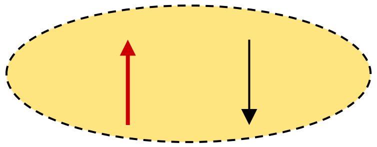


electron-impurity spin-flip scattering

(Kondo, 1964)

For T<Tk (Kondo Temperature), spin-flip scattering off impurities enhances

Kondo singlet $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$

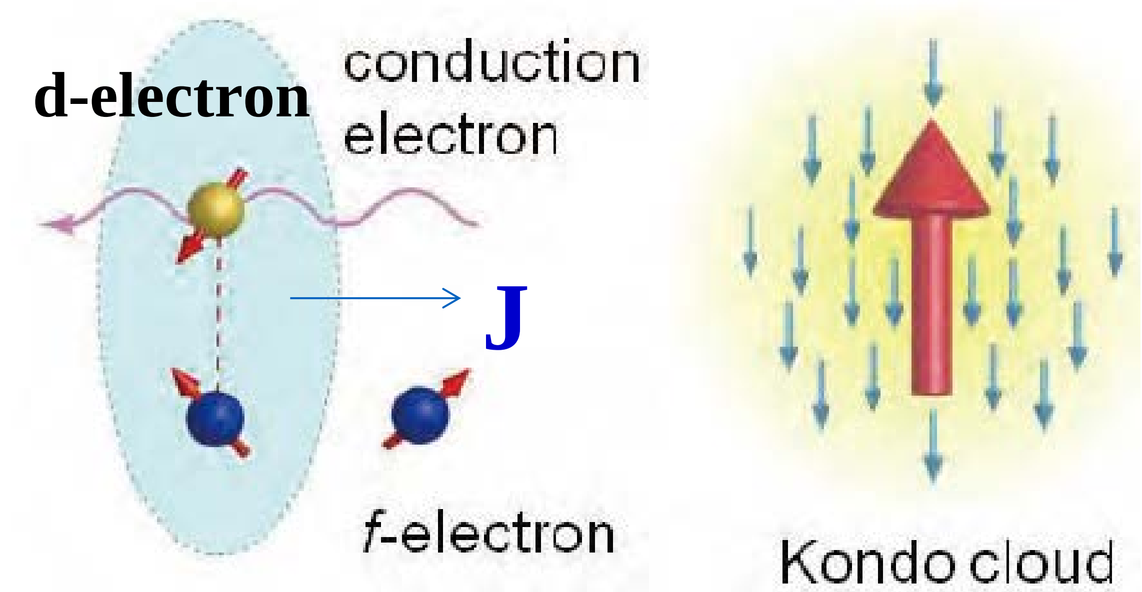


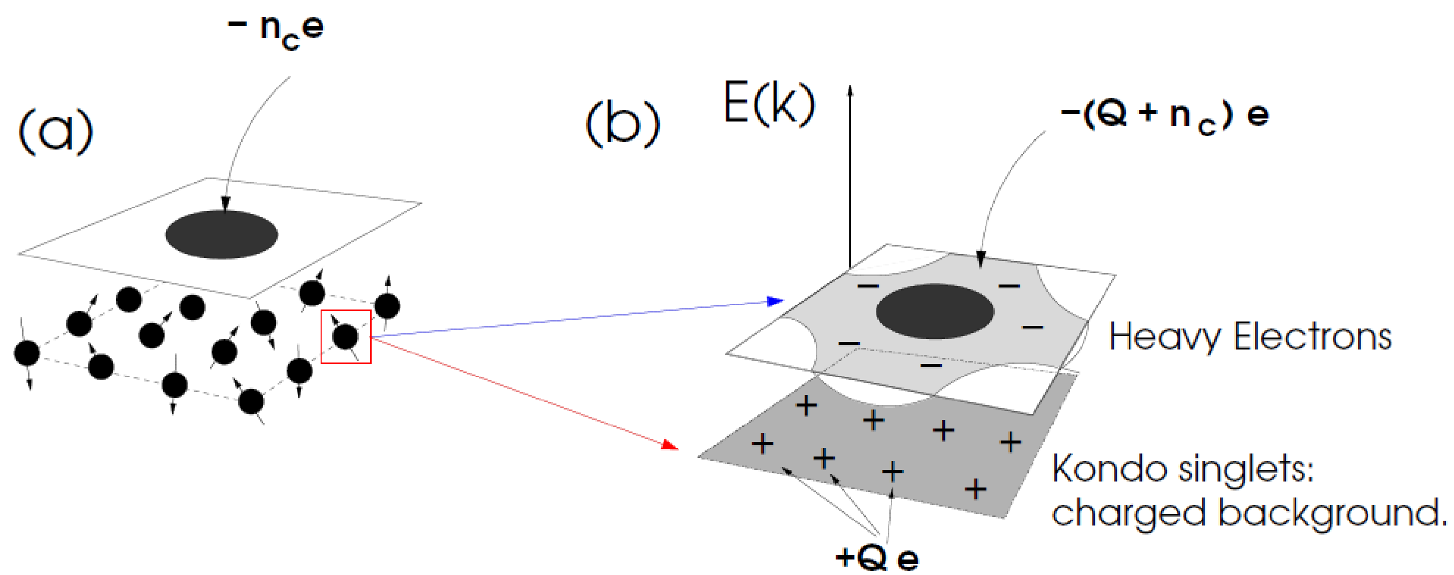
Resistance increases as T is lowered

Kondo effect on a lattice: Kondo lattice

Matsuda, AAPPS Bulletin 2017

P. Coleman, Magnetism and Advanced Magnetic Materials,95-148 (2007).

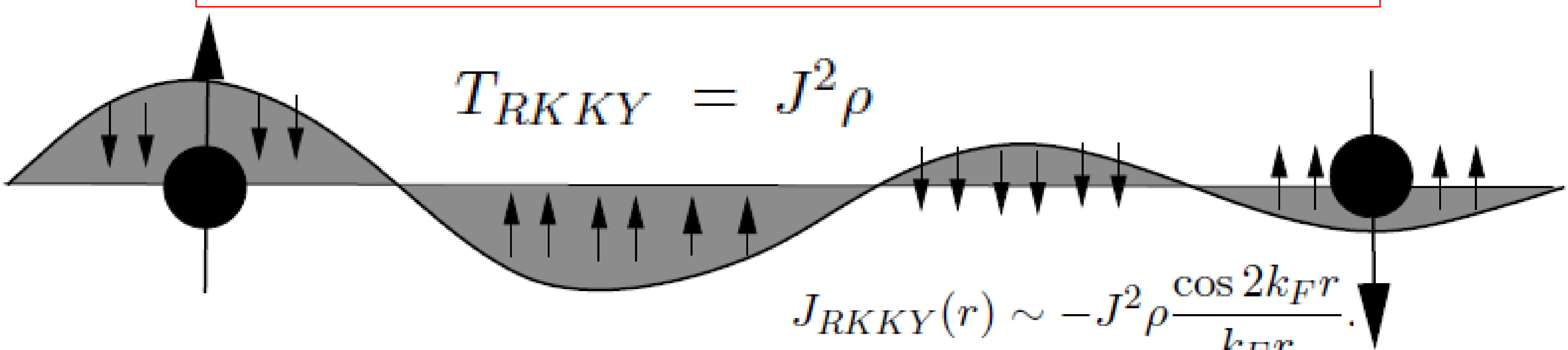




$$T_K = De^{-1/(2J\rho)}$$

heavy fermi-liquid via Kondo effect

Antiferromagnetic RKKY coupling



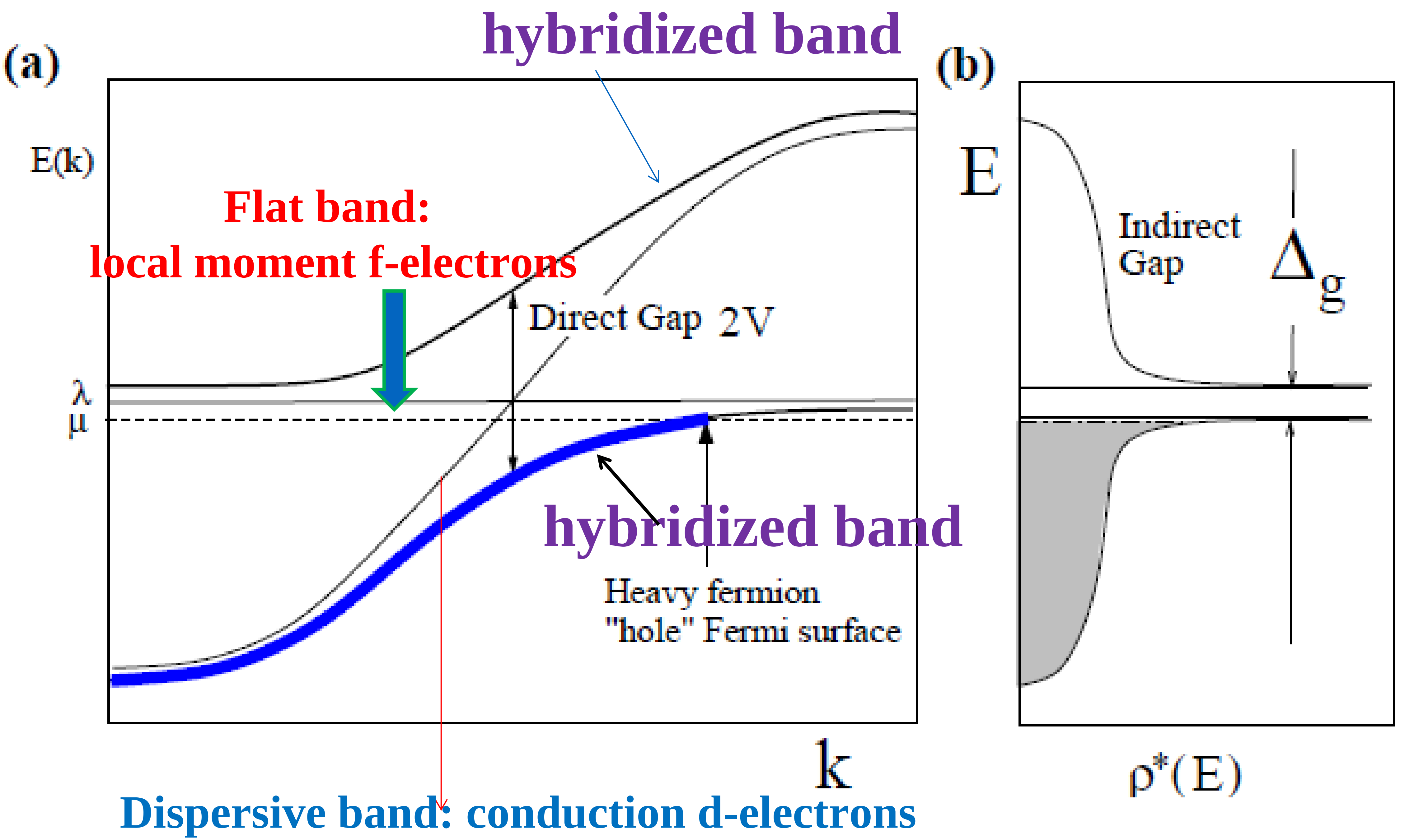
$$T_{RKKY} = J^2\rho$$

$$J_{RKKY}(r) \sim -J^2\rho\frac{\cos 2k_F r}{k_F r}.$$

dilute magnetic metal ions, the oscillatory RKKY → "spin glass".

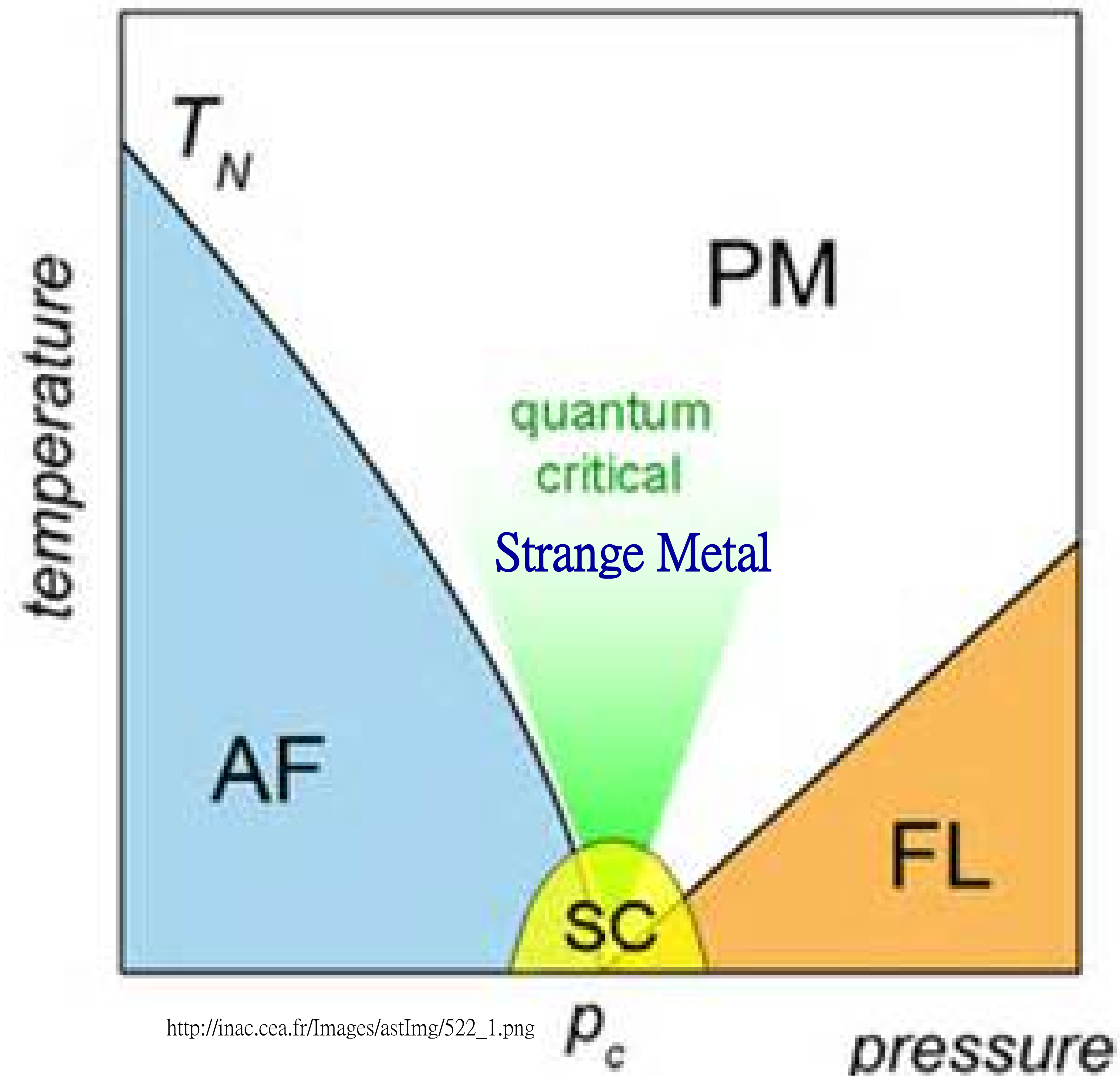
dense systems, the RKKY →ordered antiferromagnetic state

Kondo Hybridization in heavy-fermion systems



P. Coleman, Electrons at the edge of magnetism, Handbook of Magnetism and Advanced Magnetic Materials, Wiley, 2007

Strange Metal Behaviours near Quantum Phase Transitions and superconductivity: Heavy-fermion metals/superconductors



http://inac.cea.fr/Images/astImg/522_1.png

To address the strange metal physics, we must find out :

What are the key quantum critical fluctuations?

In heavy fermion systems, they are:

Bosonic Kondo (charge) fluctuations

Bosonic RVB sin liquid (made of fermionic spinons)

Mechanism of strange metal state near a heavy-fermion quantum critical point

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Collaborators:

Yung-Yeh Chang (NCTU, Taiwan)

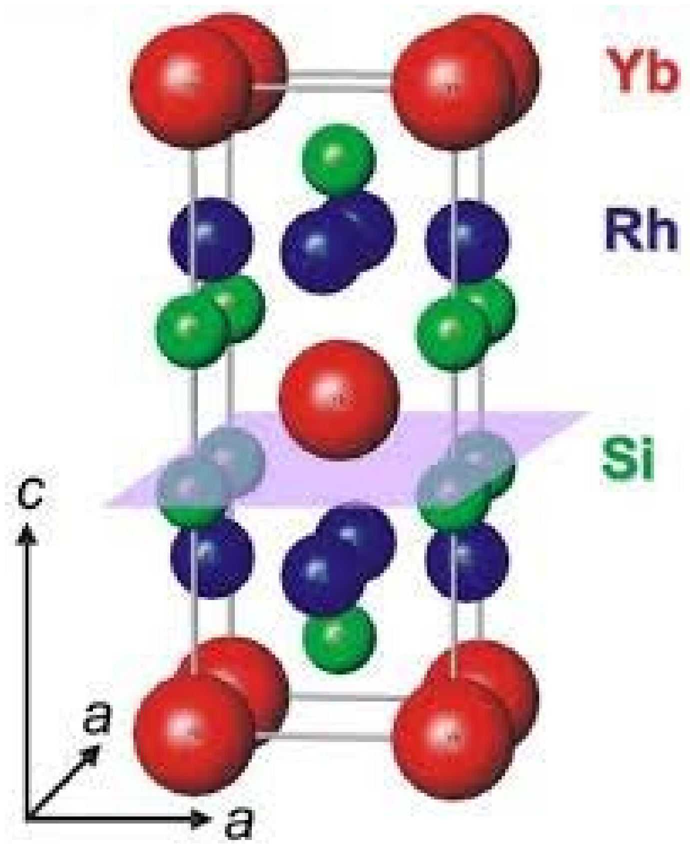
Silke Paschen (TU Vienna, Austria)

PRB 97, 035156 (2018)

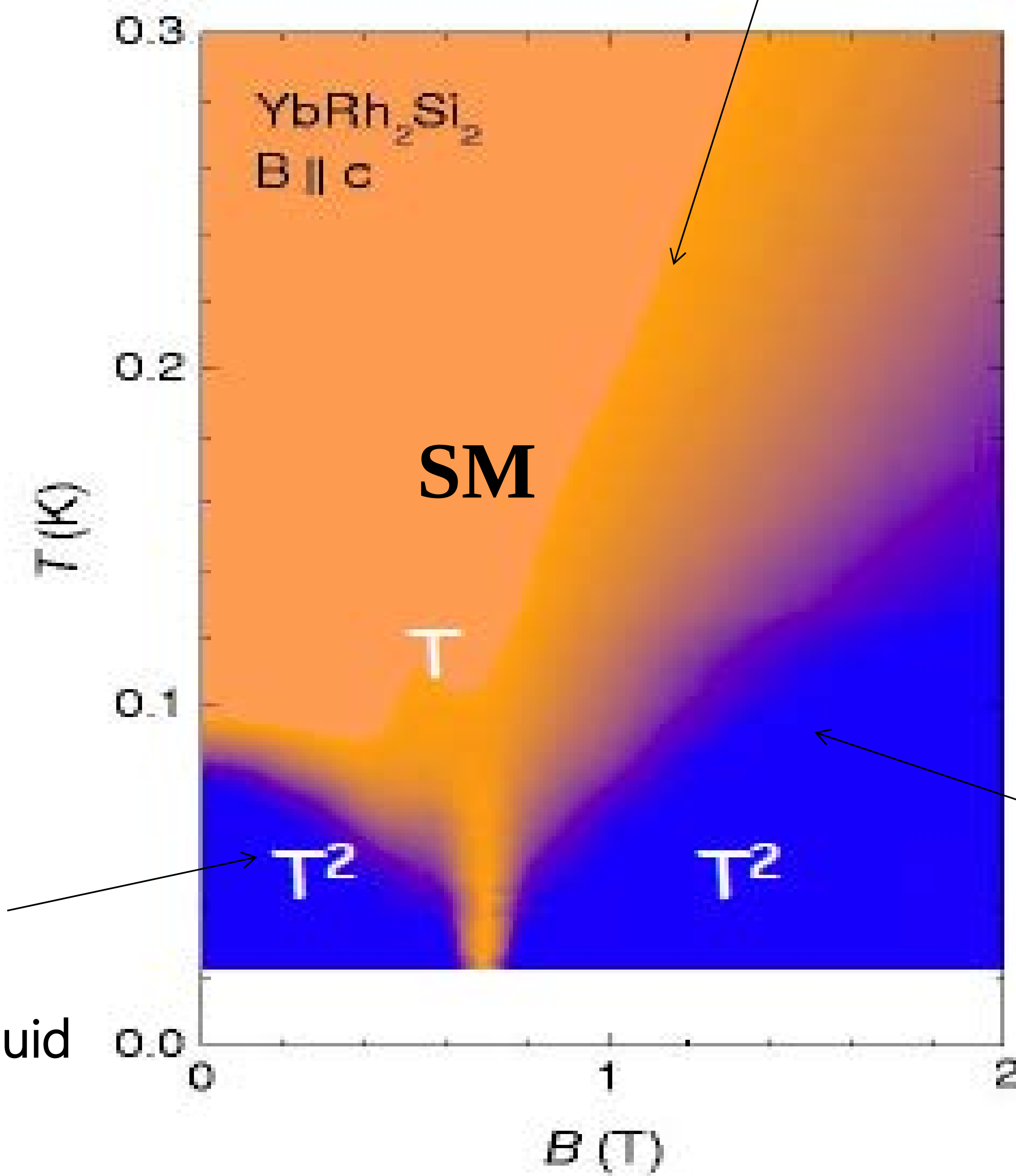
Strange Metal (SM) near a AF quantum critical point (QCP)

heavy fermion Kondo lattice systems YbRh2Si2

Yb: 4f, 5d *Rh:4d*



quantum critical non-Fermi-liquid

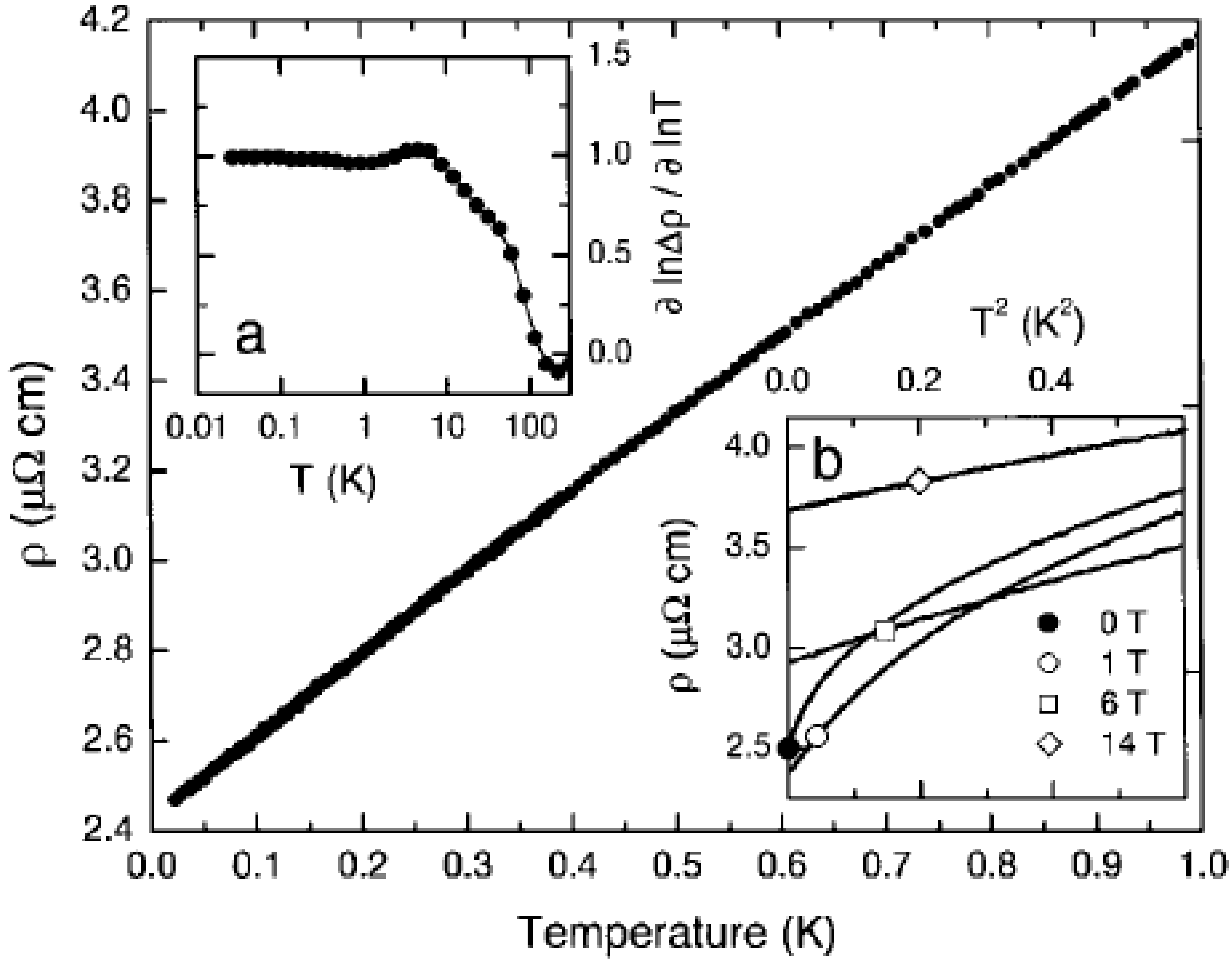


Paramagnetic
heavy Fermi liquid

Anti-ferromagnetic Fermi liquid

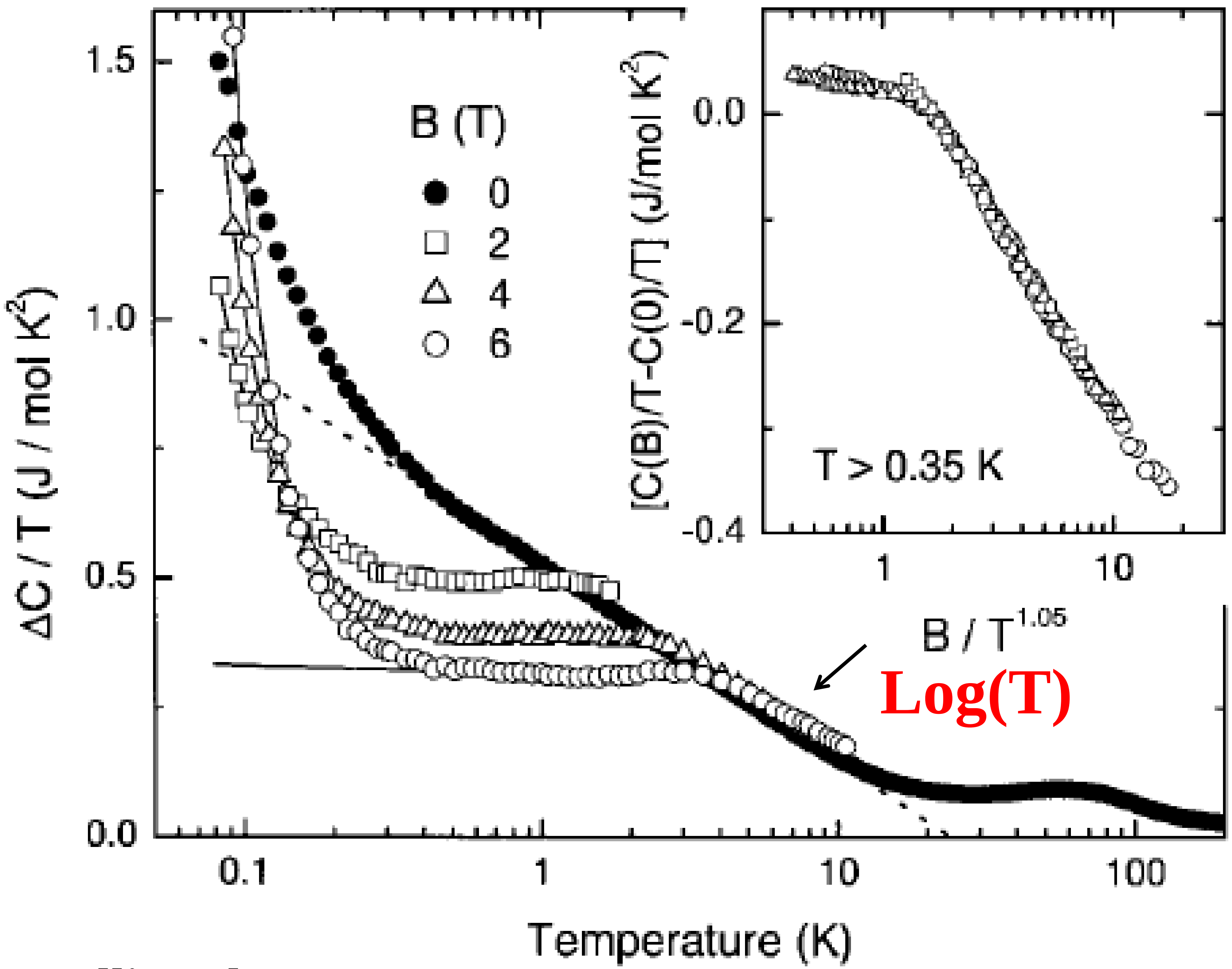
Gegenwart et al (2002)
Custers et al (2003).

T-linear resistivity: YbRh2Si2



O. Trovarelli et al. PRL 2000

Specific heat coefficient: T-logarithmic YbRh2Si2



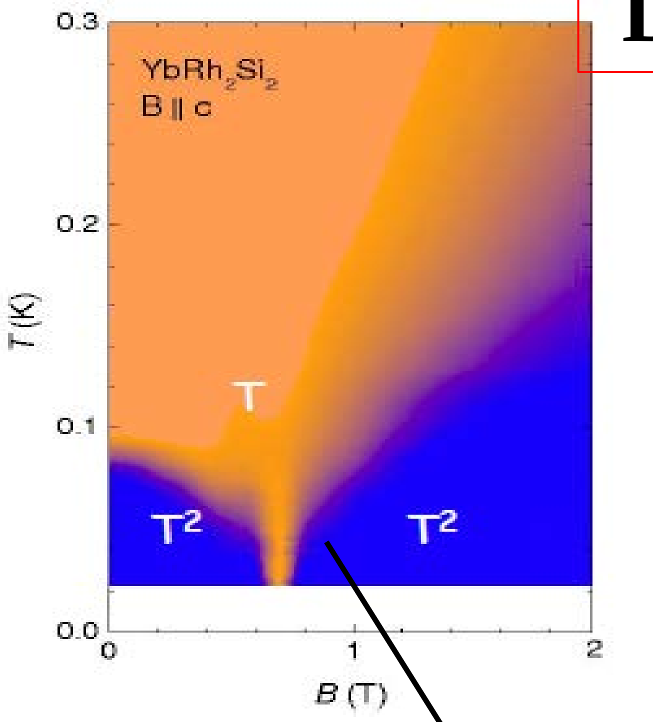
O. Trovarelli et al. PRL 2000

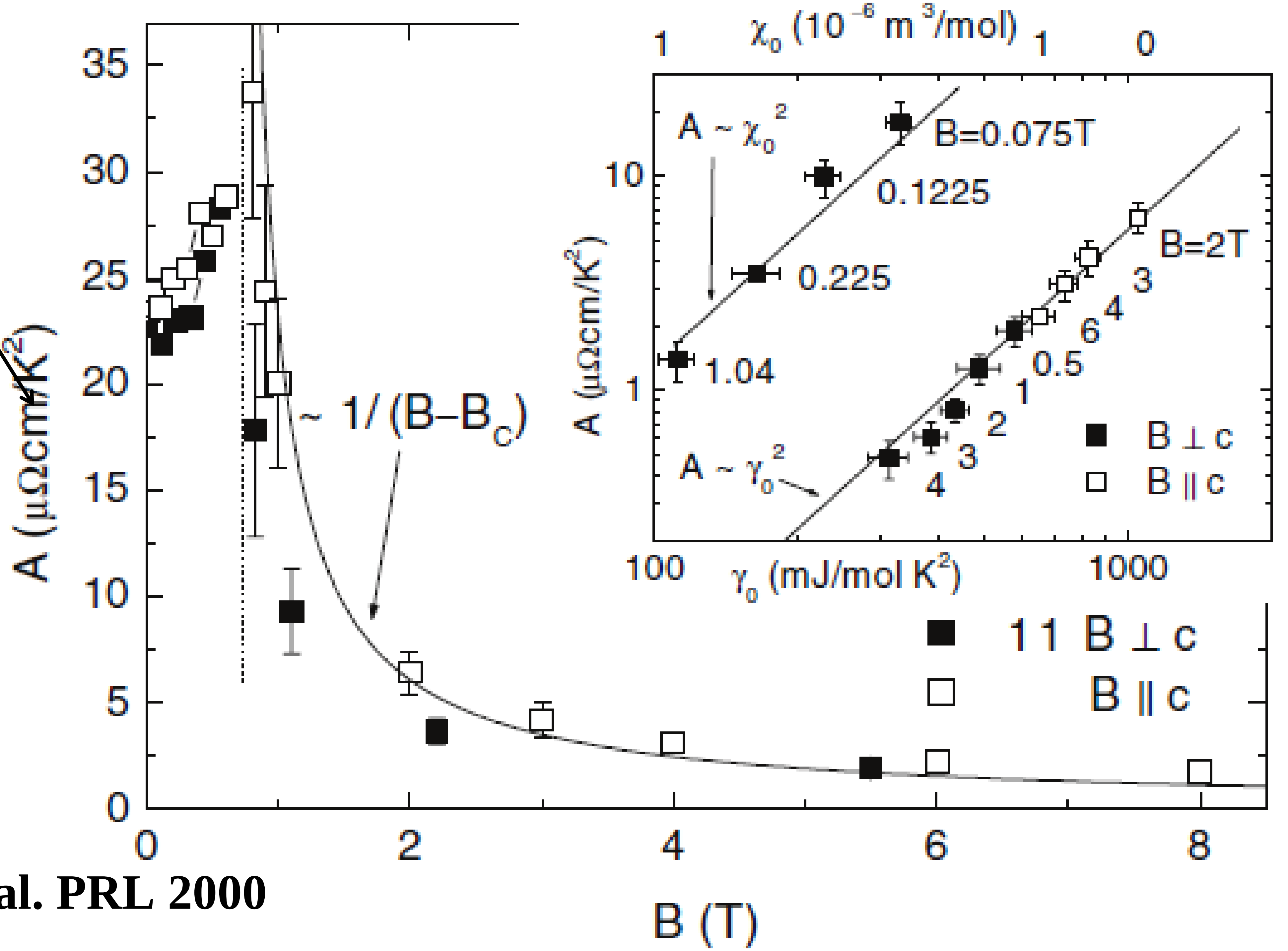
Divergence of A-coefficient and effective mass near QCP

$$\Delta\rho = A(B)T^2$$

$A \sim (m^*)^2 \sim 1/|B - B_c|$

A~ quasiparticle–quasiparticle scattering cross-section

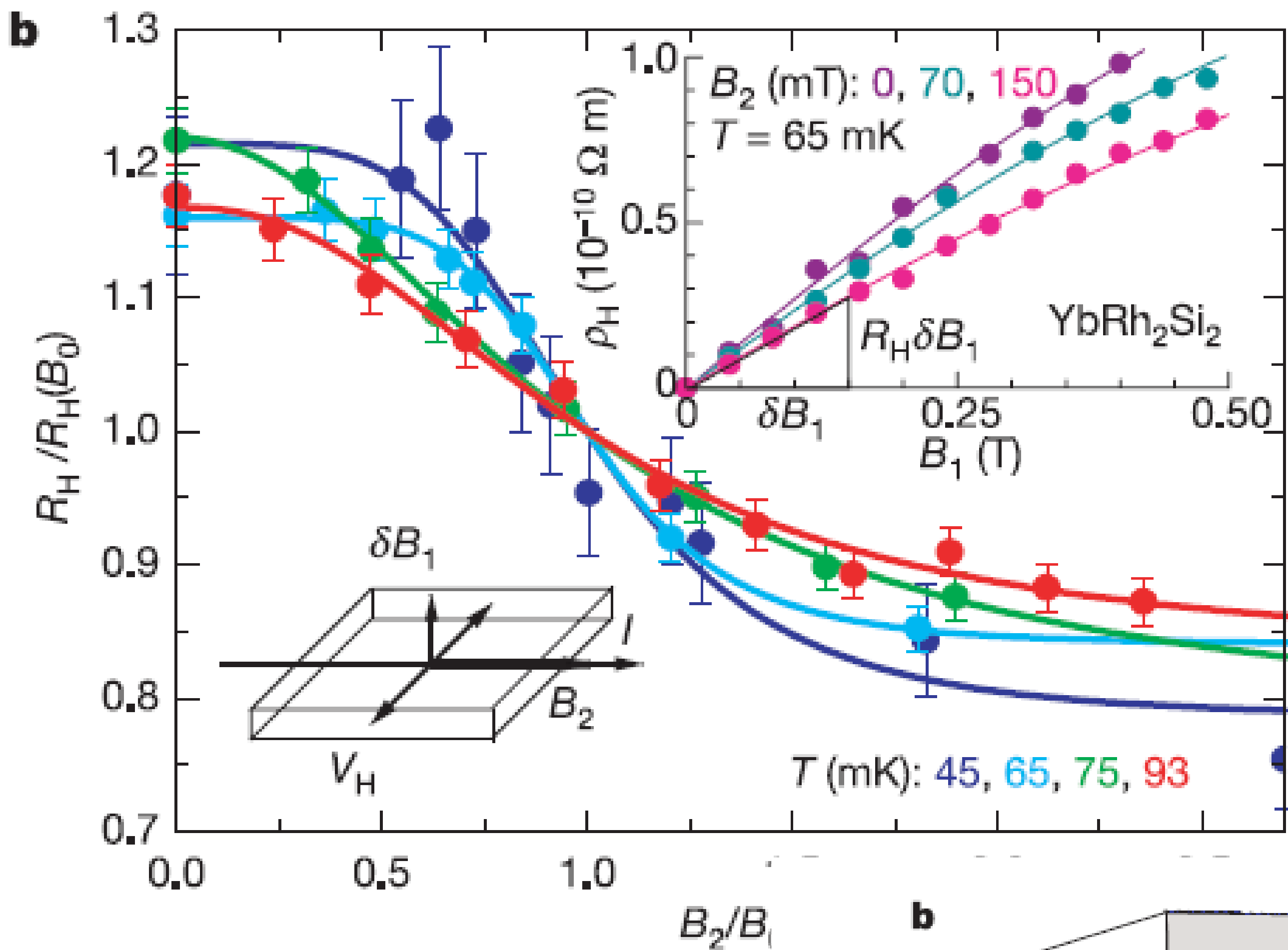




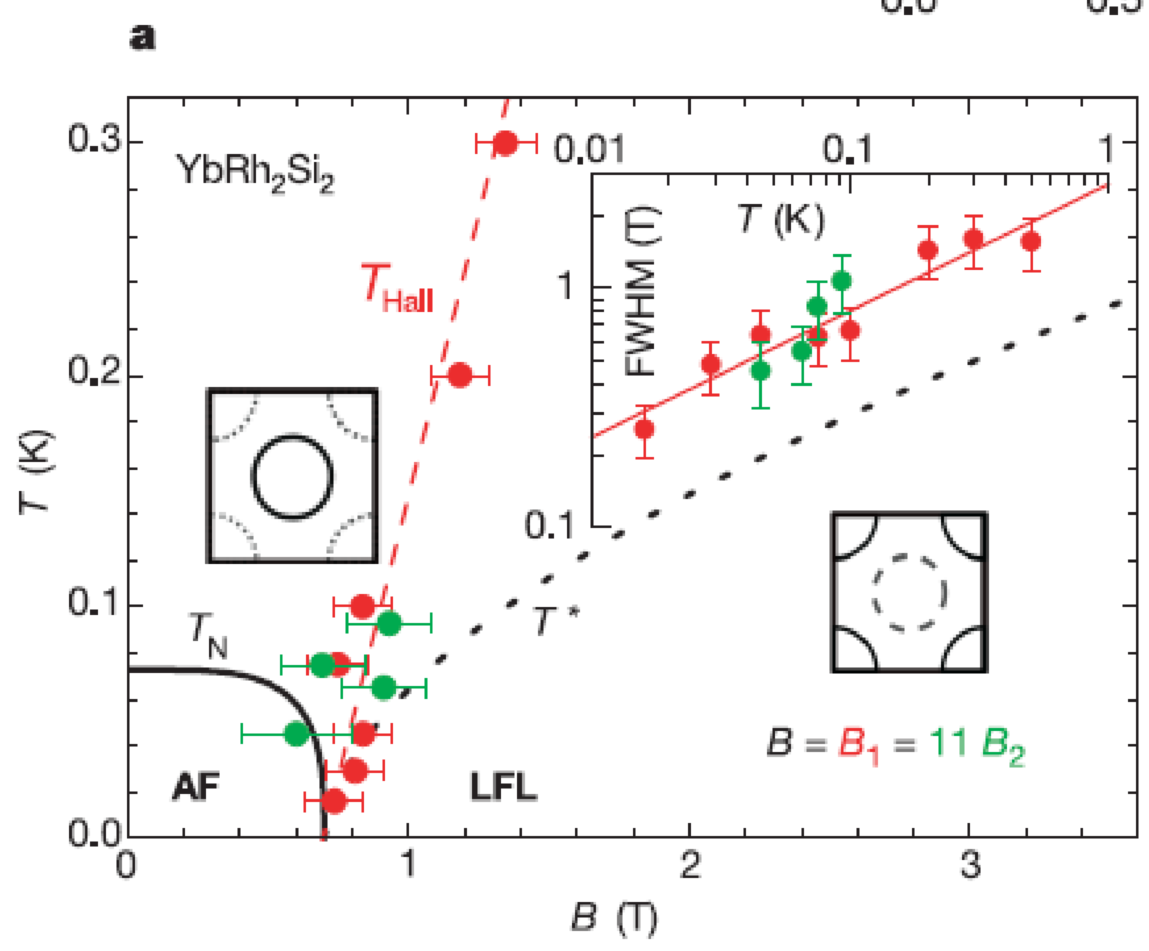
O. Trovarelli et al. PRL 2000

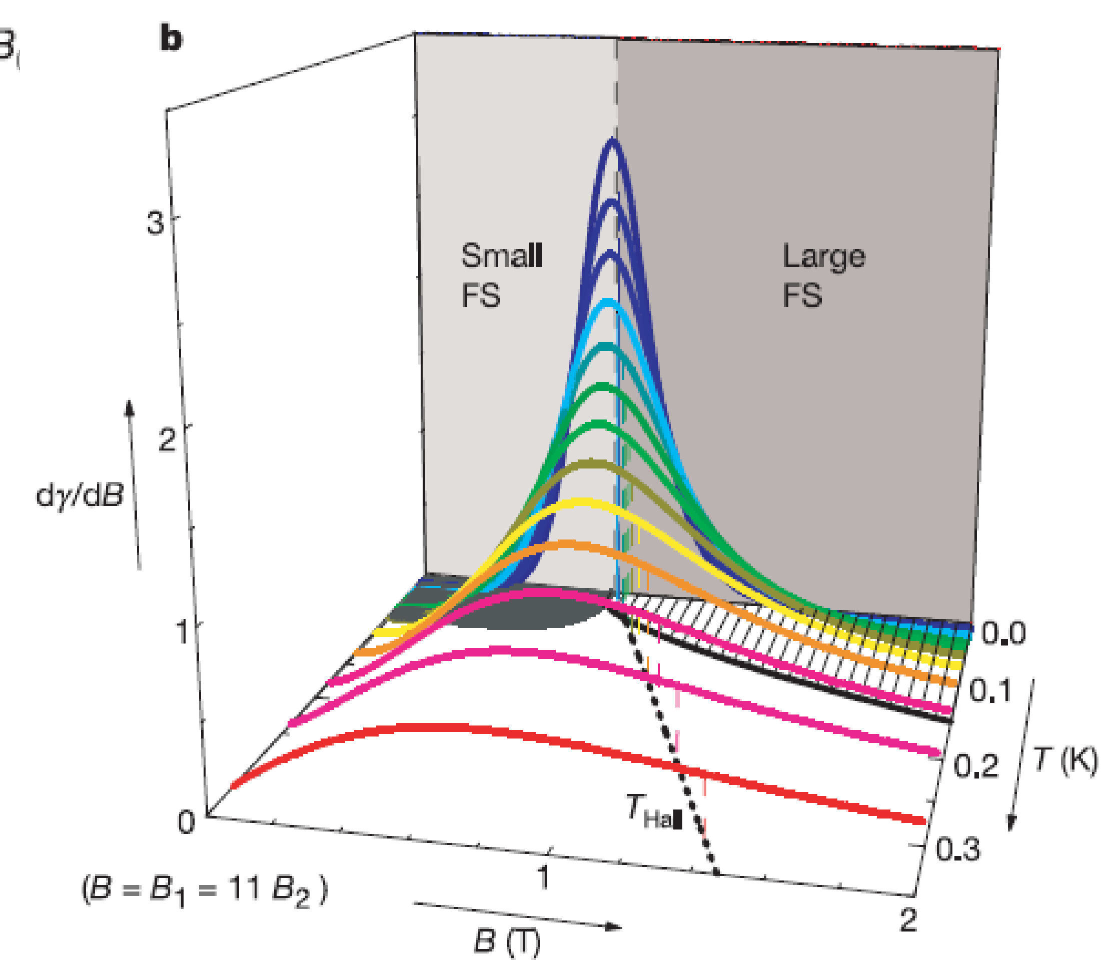
Jump in Fermi surface volume at QCP for T→0 for YRS

S. Paschen et al., Nature (2004)



R_H: Hall coefficient~$1/V_{FS}$

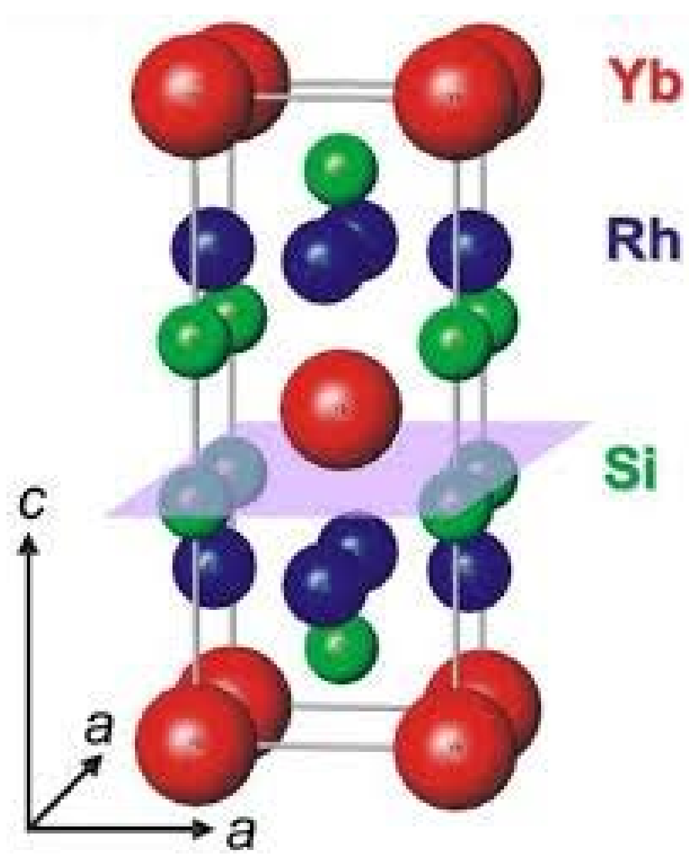




Phase diagram –
Ge-doped YbRh₂Si₂

Heavy fermions :

Yb: 4f, 5d *Rh:4d*



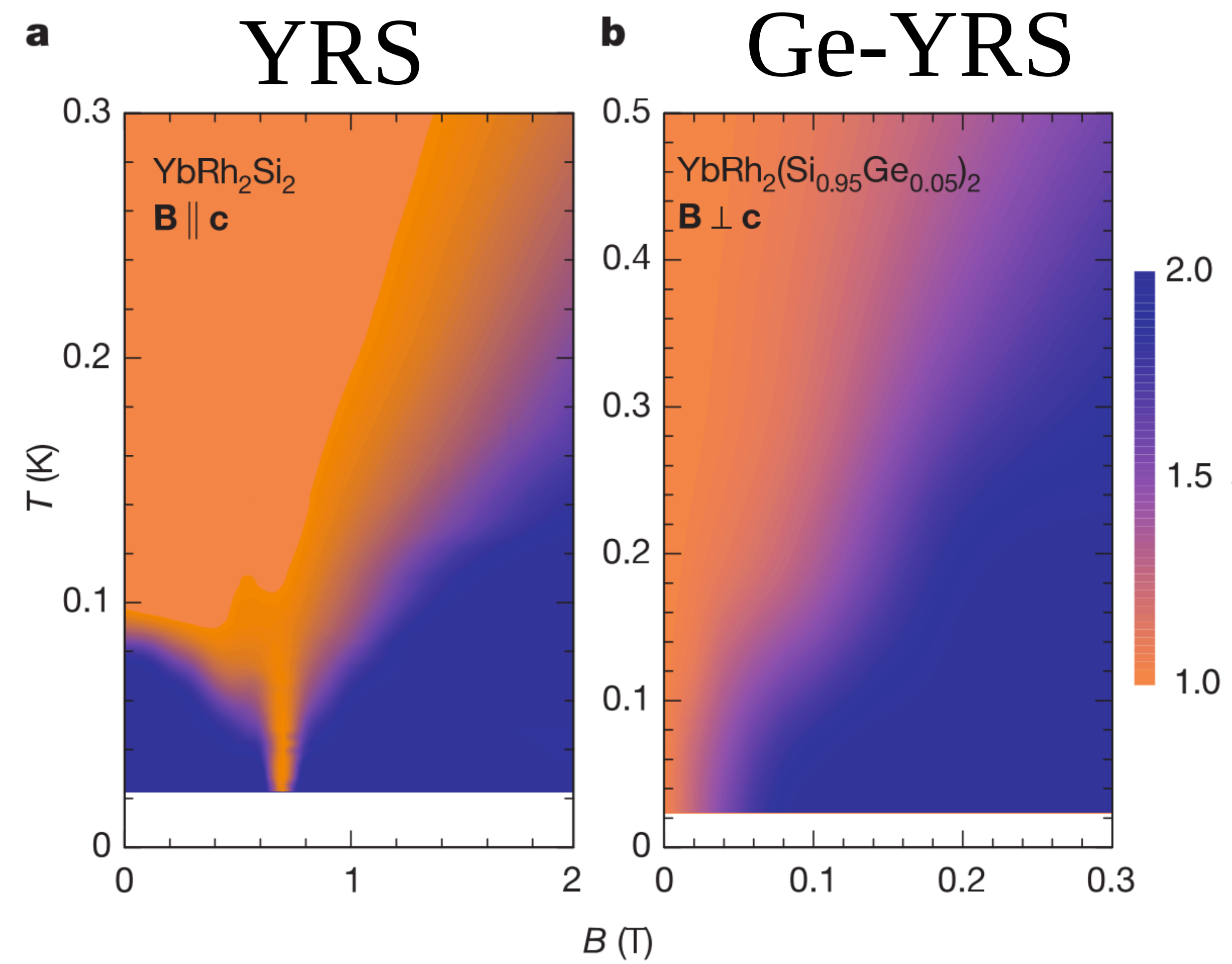
S. Wirth, JPCM, 2012

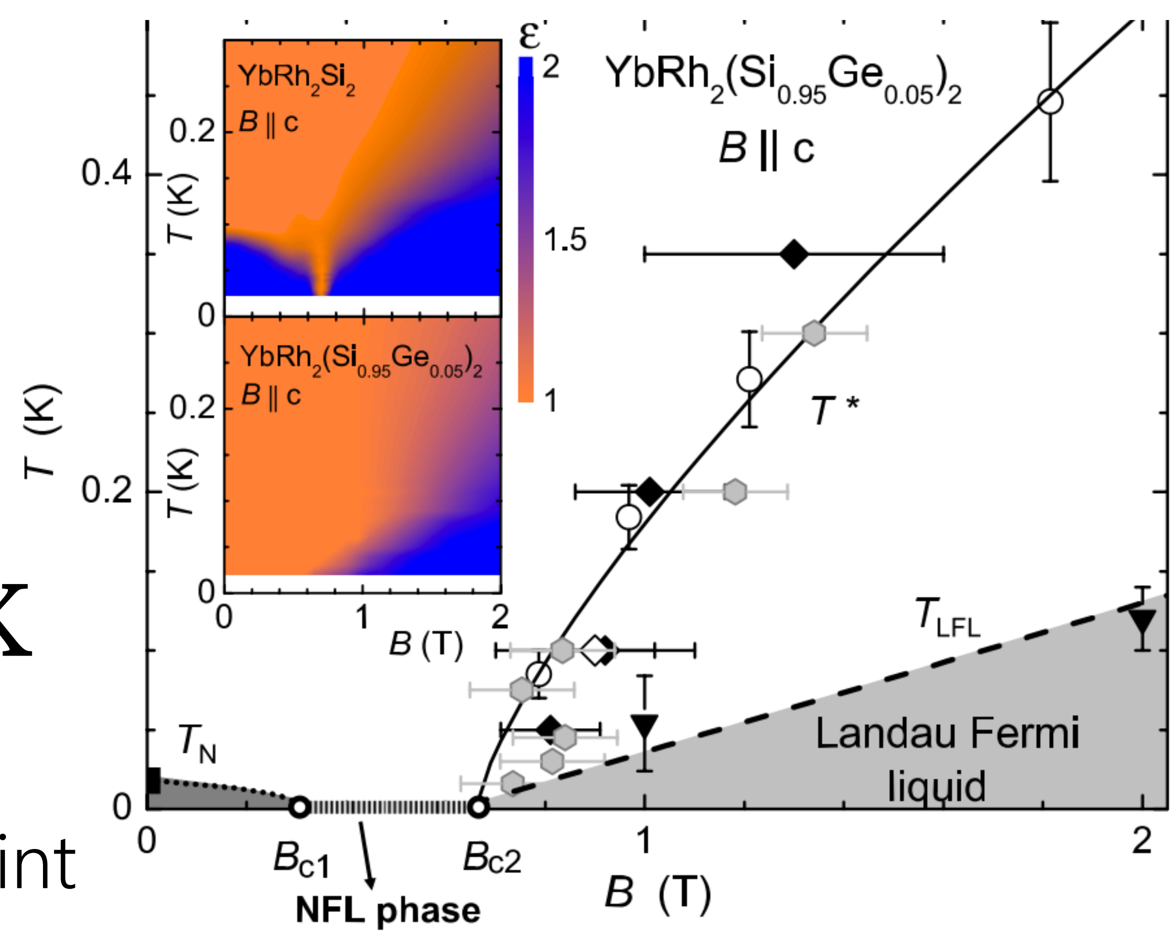
B_{c1}~0.3T

B_{c2}~0.66T

T_N ~18mK

Field-tuned ➸ quantum critical point





Custers, Nature, 2003, Custers, PRL, 2010

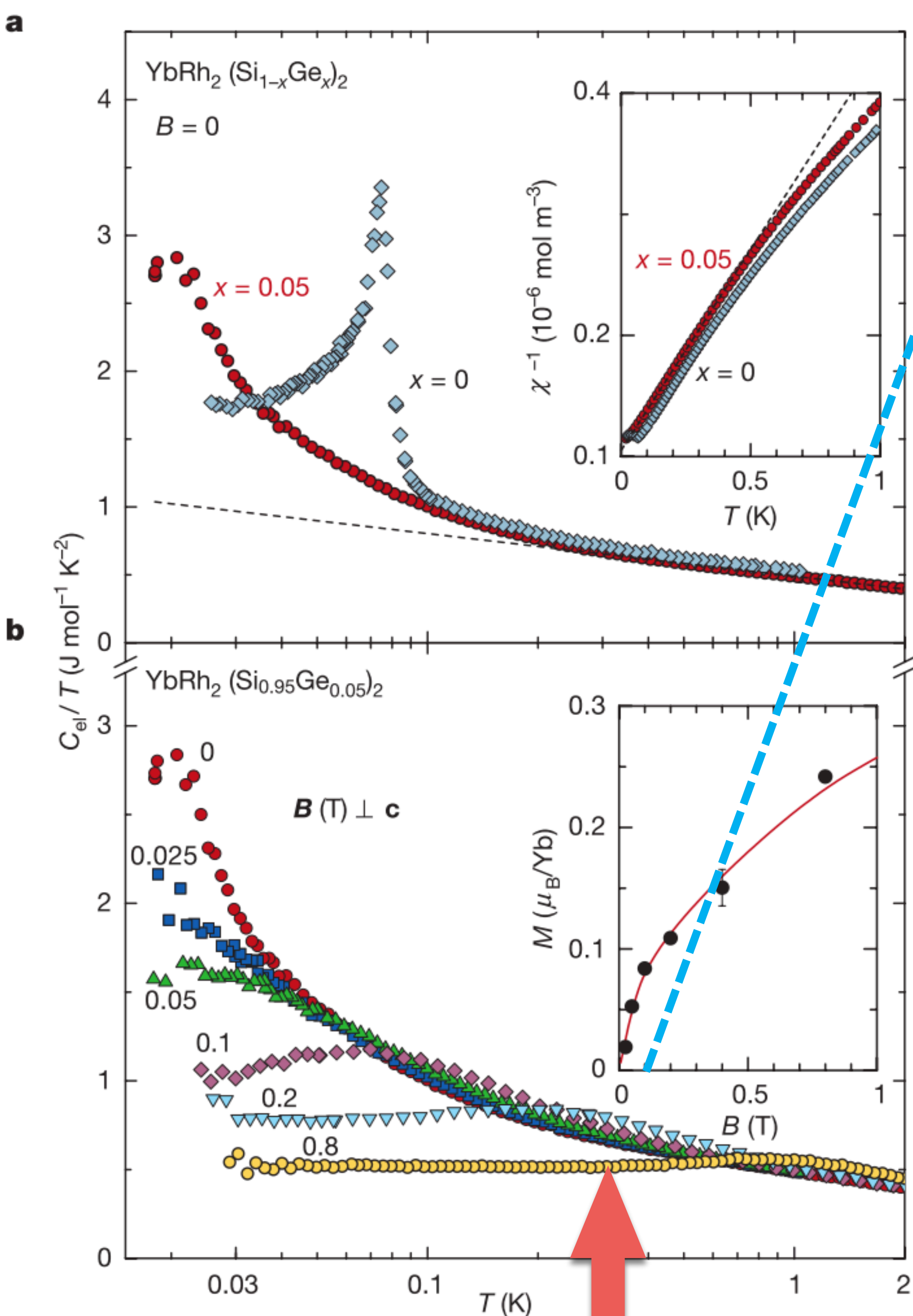
Non-Fermi Liquid Strange Metal Behaviors: Ge-YRS

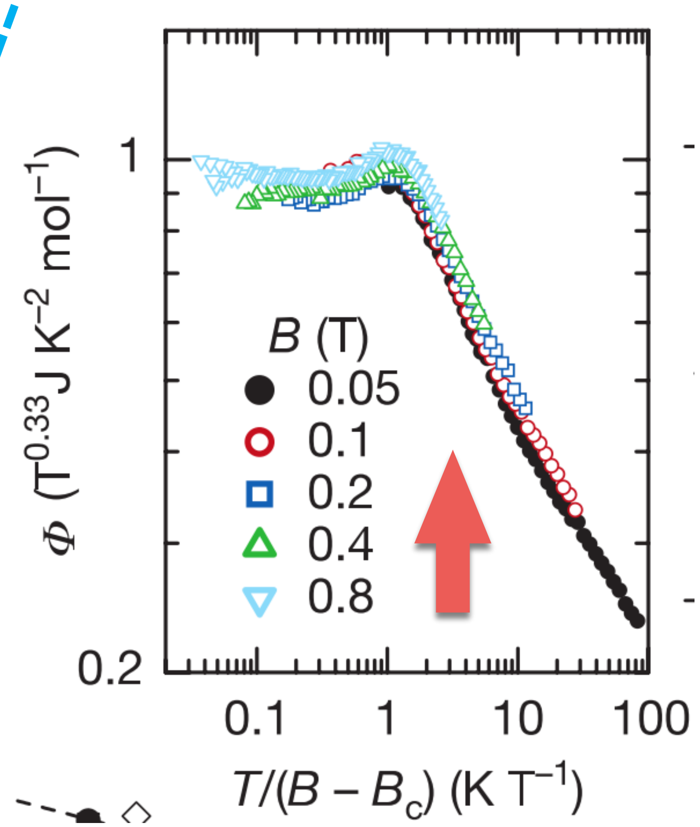
breakup of quasi-particles
spin (f)-charge (d) separation

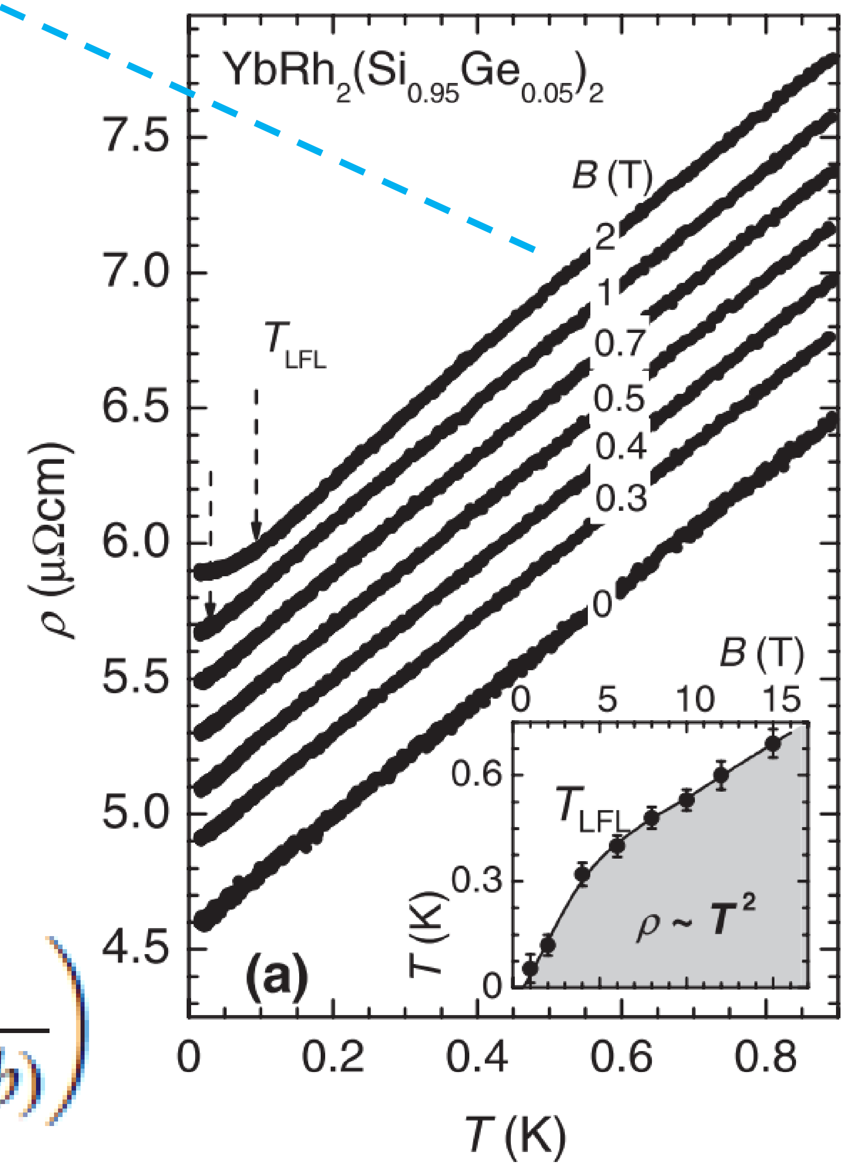
Custers, et al., Nature, 2003
Custers, et al., PRL, 2010

T<0.3K

T<10K







$$\frac{C_V}{T} = \frac{1}{b^{1/3}} \Phi\left(\frac{T}{T_0(b)}\right)$$

$$\Phi(x) \approx (\max(x,\ 1))^{-1/3}$$

Power-law ($T^{-\alpha}$) + ***ln*** (T_0/T) —
Specific heat coefficient

Linear-in-T Resistivity

T<0.3 K

2D SDW $\gamma_{SDW} \propto \ln(1/b)$

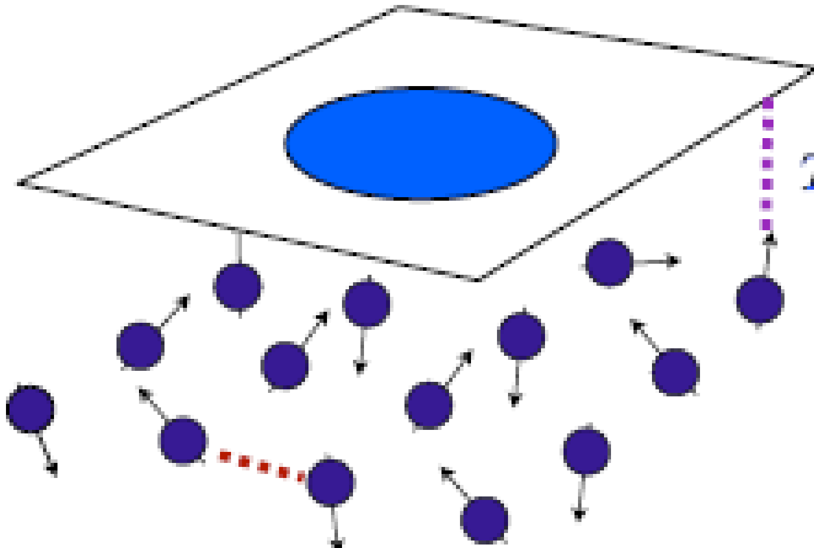
10mK<T<10K

DONIACH'S Hypothesis.

Doniach (1977)

$$H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum_j (\psi^{\dagger}_j \vec{\sigma} \psi_j) \cdot \vec{S}_j$$

Kondo Lattice Model
(Kasuya, 1951)

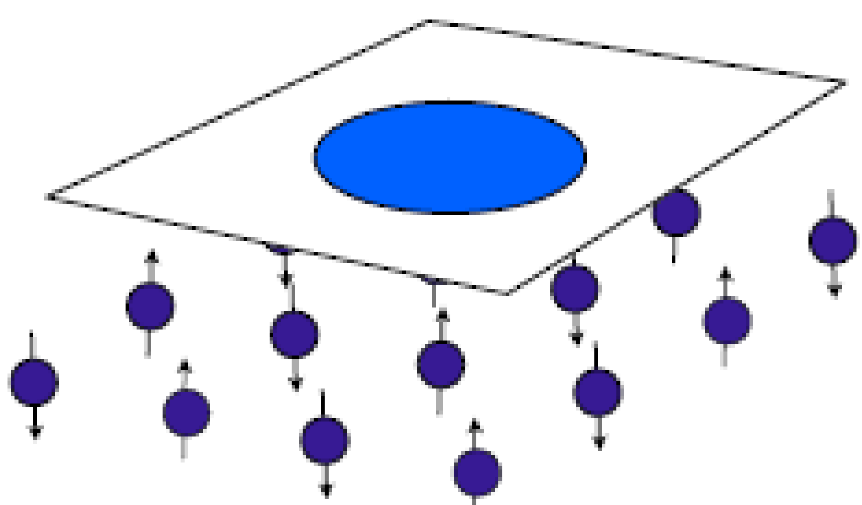


$T_K \sim D \exp\left[-\frac{1}{2J\rho}\right]$

$T_{RKKY} \sim J^2 \rho$

P. Coleman's talk in NCTU, 2016

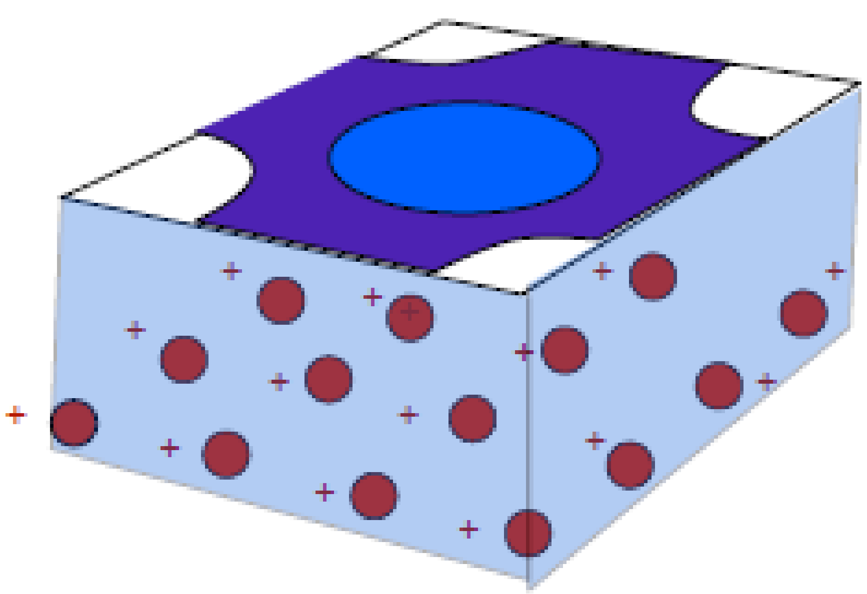
$T_{RKKY} > T_K$



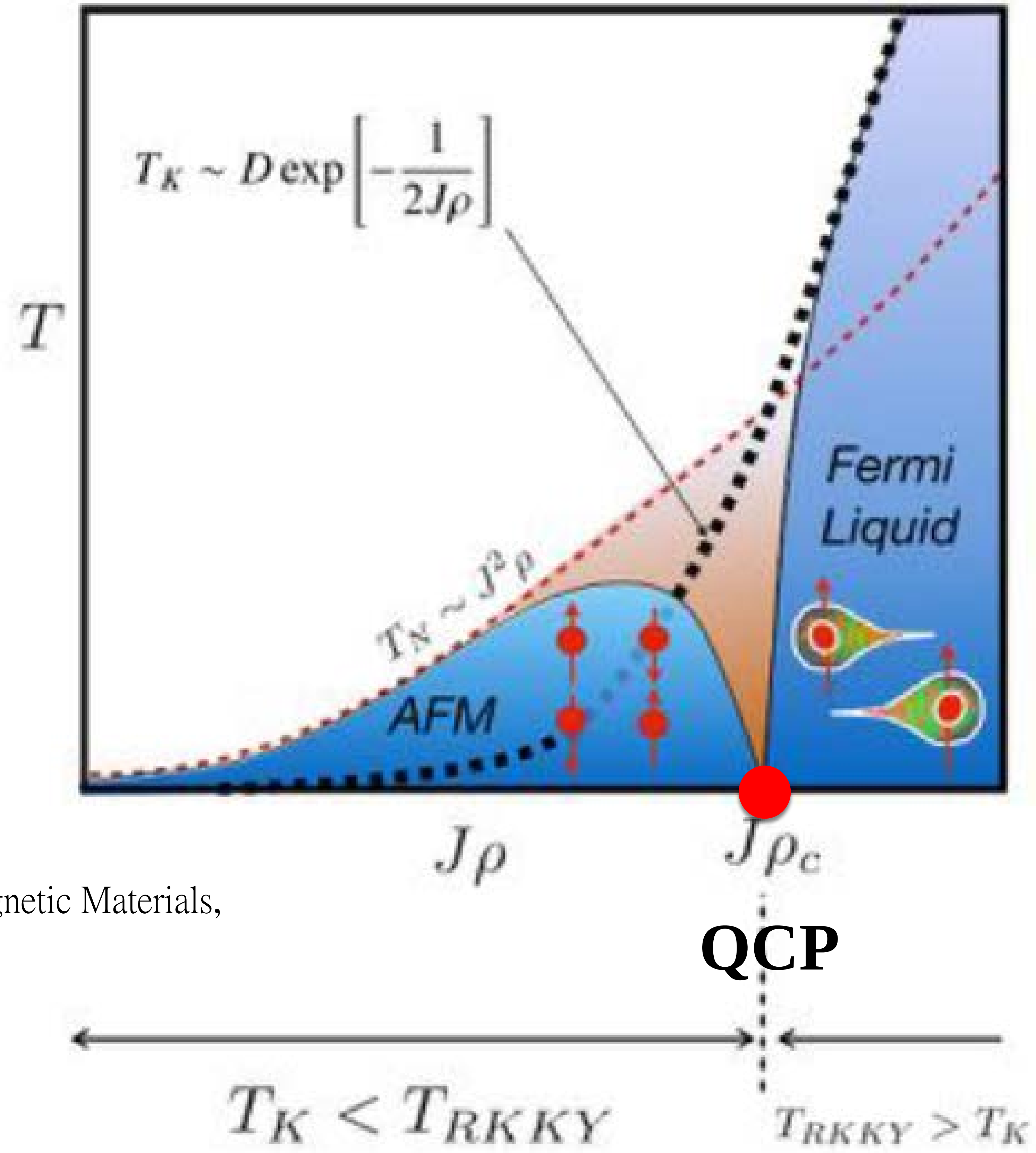
?

$T_{RKKY} < T_K$

Large Fermi surface of composite Fermions



Kondo breakdown and Quantum Criticality in Heavy-fermions

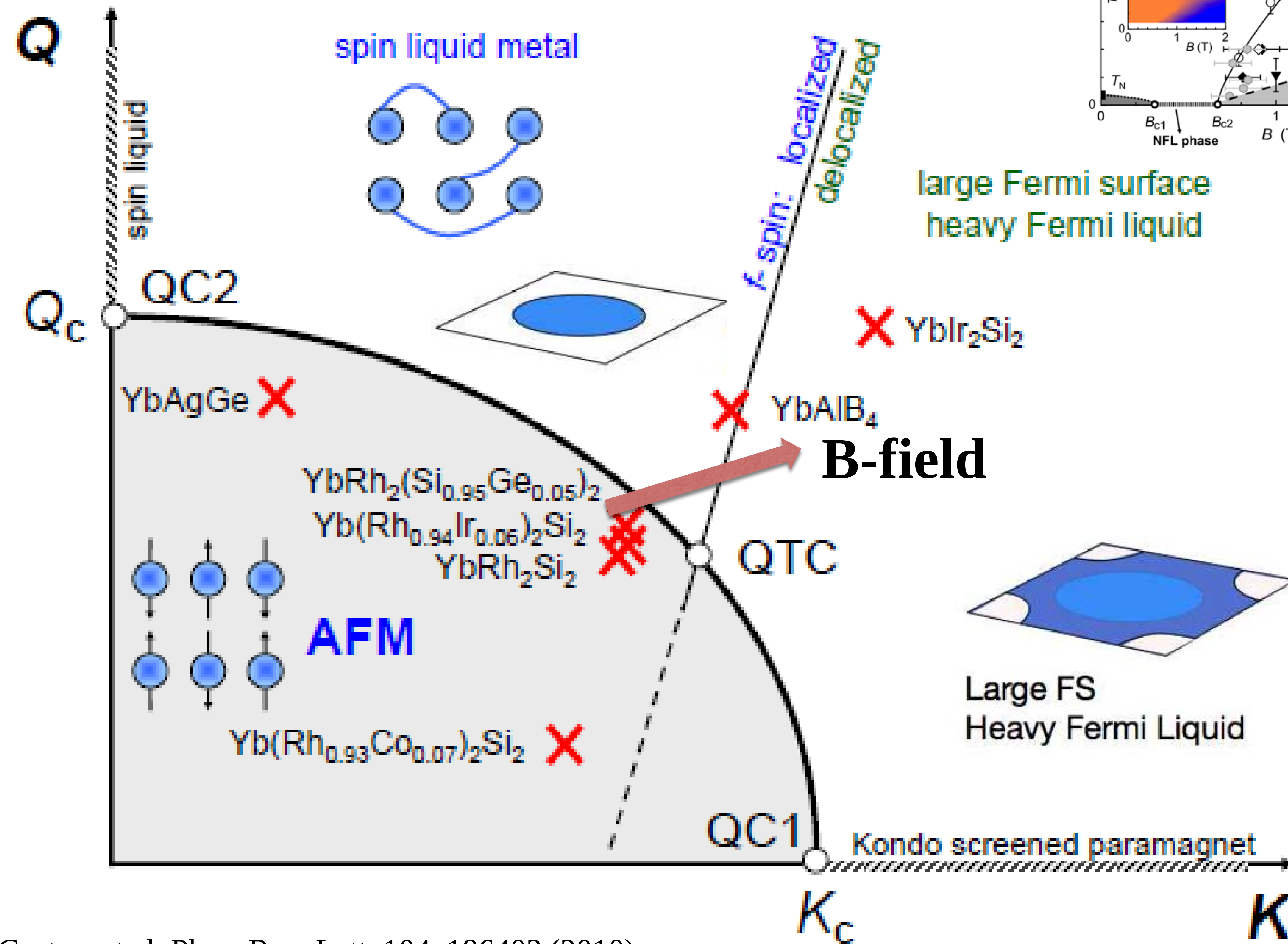


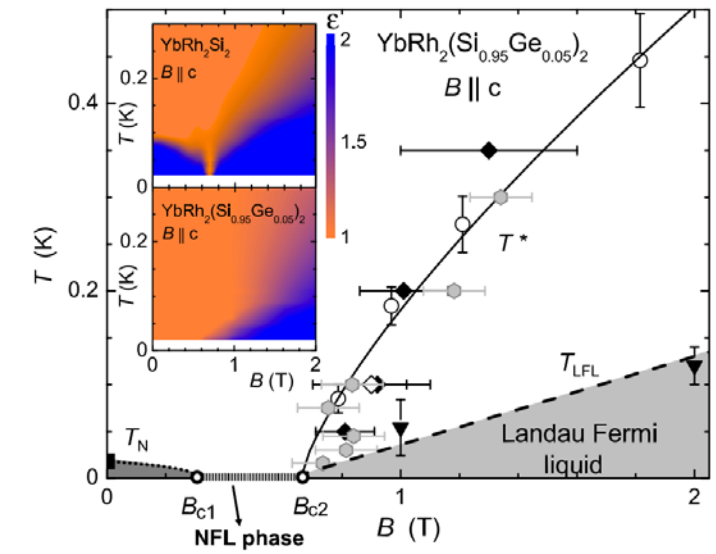
P. Coleman,
Magnetism and Advanced Magnetic Materials,
95-148 (2007).

Doniach phase diagram

Frustrated Kondo lattice

Ge doping induces disorder ~ frustration

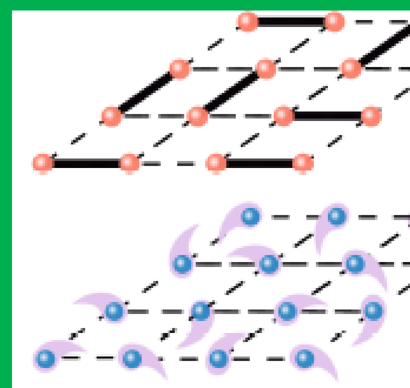




J. Custers et al. Phys. Rev. Lett. 104, 186402 (2010)

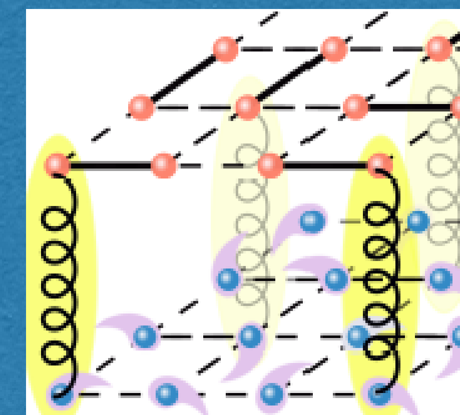
New Kondo breakdown scenario

AF RKKY + disorder induced frustration:
Fractionized Fermi liquid (FL*)
RVB spin-liquid metal

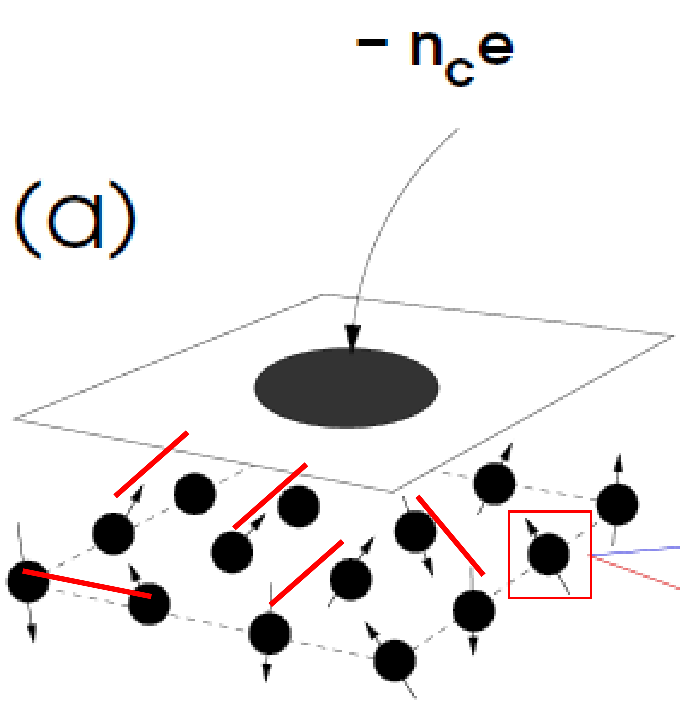


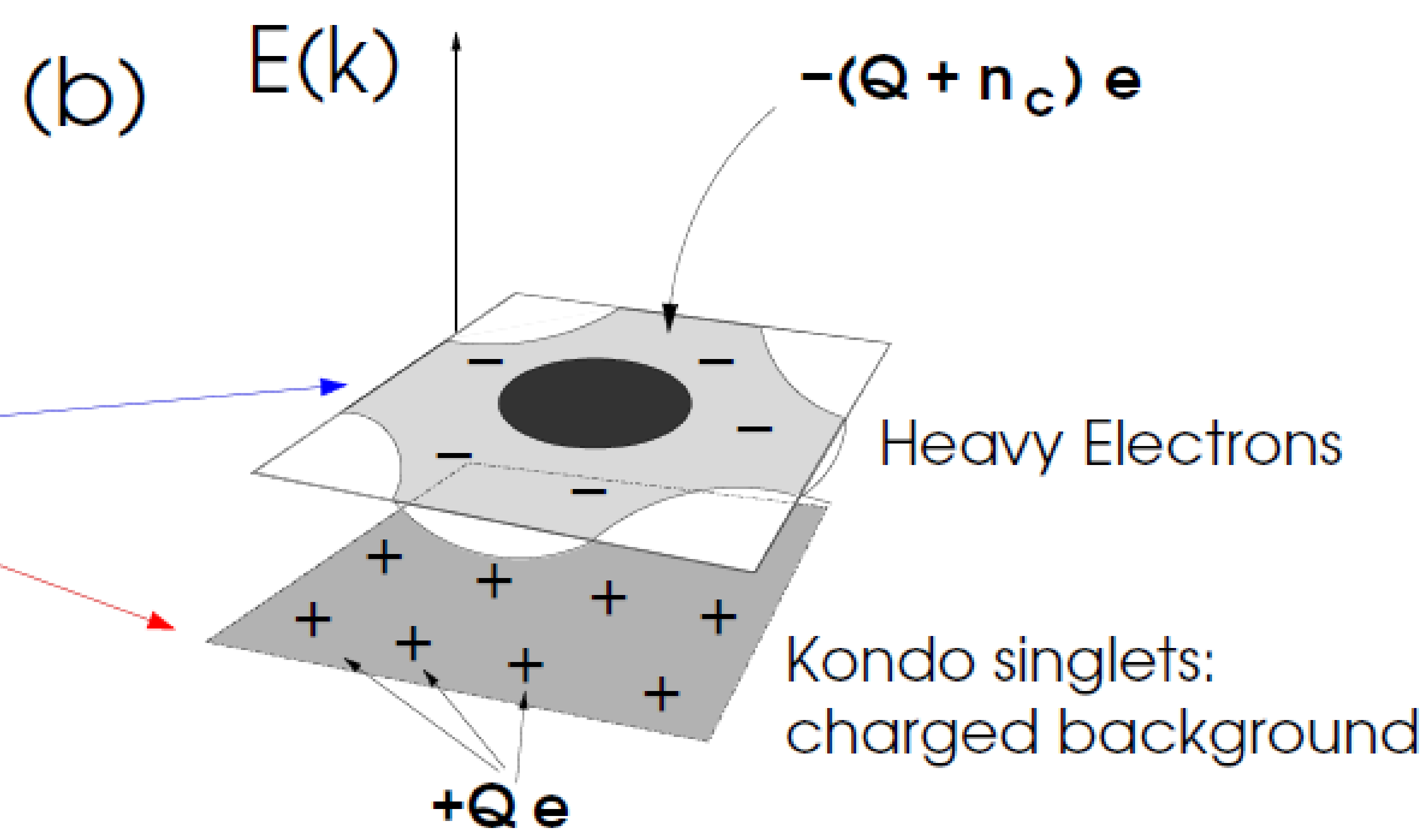
VS.

Kondo effect



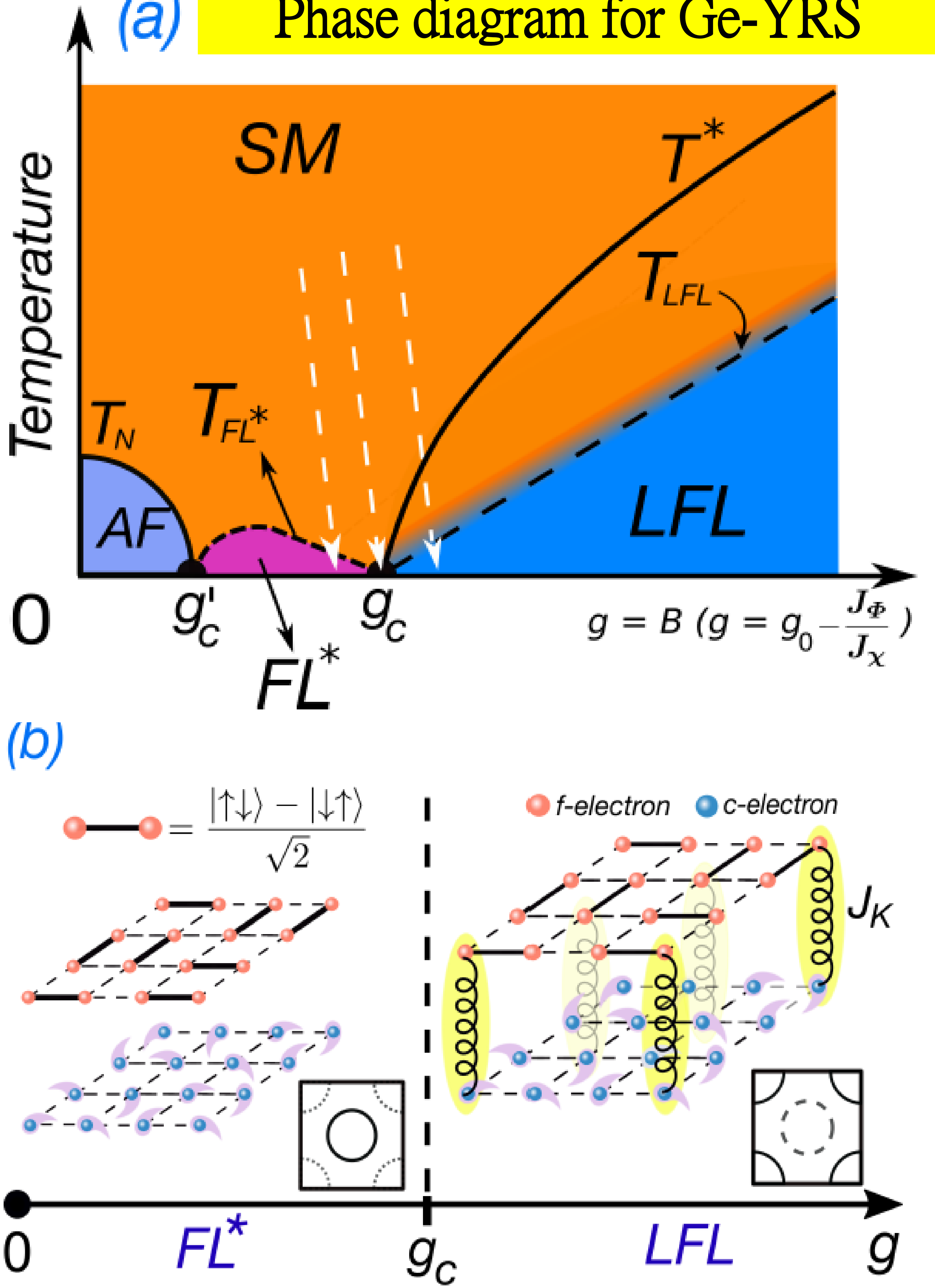
Coleman et al., J Low Temp Phys 161, No1-2, 182-202 (2010)





P. Coleman,
Magnetism and Advanced Magnetic Materials,
95-148 (2007).

Phase diagram for Ge-YRS



Large-N (Sp(N)) Mean-field Kondo-Heisenberg Model

$$H_0 = \sum_{\langle i,j\rangle;\sigma} \left[t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + h.c.\right] - \sum_{i\sigma} \mu\, c^\dagger_{i\sigma} c_{i\sigma},$$ *(conduction electrons)*

$$H_\lambda = \sum_{i,\sigma} \lambda \left[f^\dagger_{i\sigma} f_{i\sigma} - 2S\right]$$ *(localized electrons)*

$$H_J = \sum_{\langle i,j\rangle} J_H \mathbf{S}^{imp}_i \cdot \mathbf{S}^{imp}_j = \sum_{\langle i,j\rangle;\alpha,\beta} \left[\Phi_{ij} \mathcal{J}^{\alpha\beta} f_{i\alpha} f_{j\beta} + h.c.\right] + \sum_{\langle i,j\rangle} N \frac{|\Phi_{ij}|^2}{J_H},$$

$$H_K = J_K \sum_i \mathbf{S}^{imp}_i \cdot \mathbf{s}^c = \sum_{i,\ \sigma} \left[\left(c^\dagger_{i\sigma} f_{i\sigma}\right) \chi_i + h.c.\right] + \sum_i N \frac{|\chi_i|^2}{J_K}.$$

$$\sigma, \alpha, \beta \in \{-\tfrac{N}{2}, \cdots, \tfrac{N}{2}\}$$

$$N \rightarrow \infty$$

$$\mathcal{J}^{\alpha\beta} = \mathcal{J}_{\alpha\beta} = -\mathcal{J}^{\beta\alpha}$$

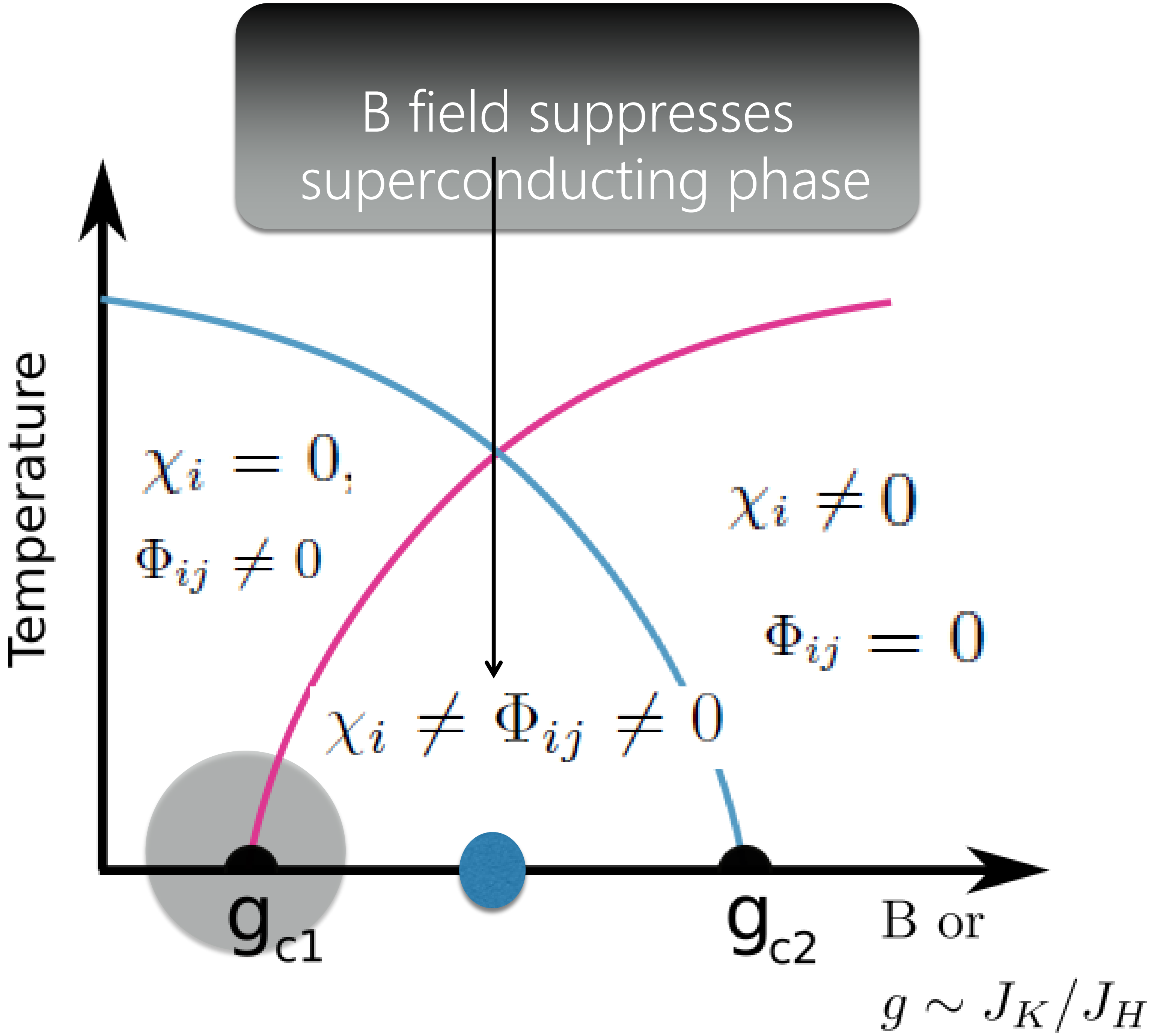
generalization of the $SU(2)$ antisymmetric tensor

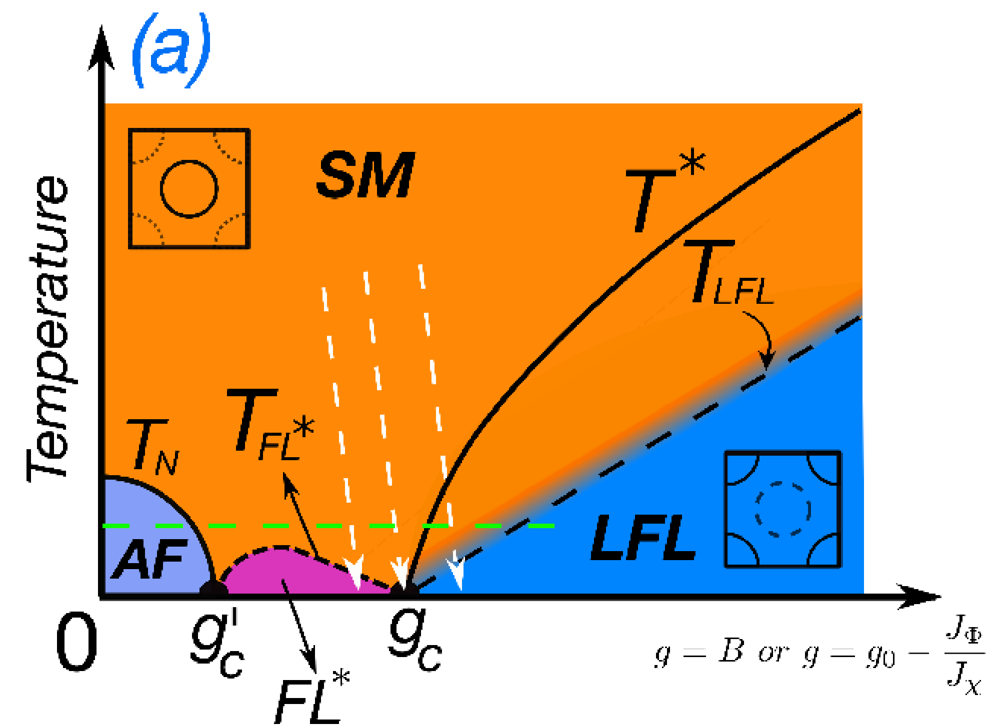
$$\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = -\epsilon_{\beta\alpha} = i\sigma_2$$

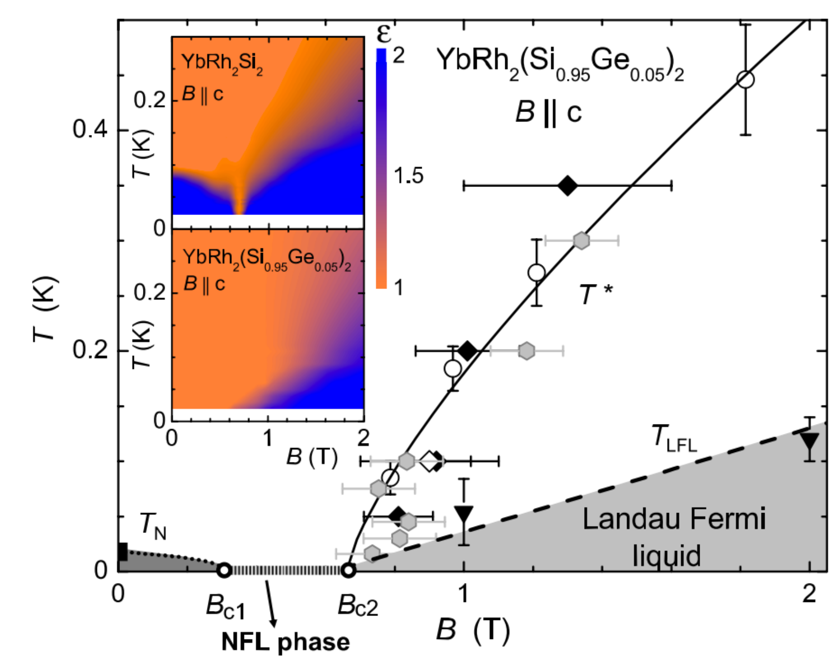
$\Phi_{ij} \equiv \langle \frac{J_H}{N} \sum_{\alpha,\beta} \mathcal{J}_{\alpha\beta} f^{\alpha\dagger}_i f^{\beta\dagger}_j \rangle$ RVB spin-singlet bond (Characterize the spin-liquid)

$\chi_i \equiv \langle \frac{J_K}{N} \sum_\sigma f^\dagger_{i\sigma} c_\sigma \rangle$ Kondo hybridisation (Characterize the Kondo phase)

Proposed phase diagram







Fractionalized Fermi-liquid (FL*)	V.S.	Kondo Heavy Fermi Liquid

Senthil, PRL, 2003
Custers, Nature, 2003
Custers, PRL, 2010

Effective Action—Mean-field

$$\Phi_{ij} \longrightarrow \Phi_{ij} + \hat{\tilde{\Phi}}_{ij} \qquad \Phi_{ij},\ \chi_i : \text{uniform values}$$

$$\chi_i \longrightarrow \chi_i + \hat{\tilde{\chi}}_i \qquad \hat{\tilde{\Phi}}_{ij},\ \hat{\tilde{\chi}}_i : \text{fluctuations}$$

$$S_{eff} = S_0 + S_\chi + S_Q + S_4 + S_K + S_J + S_G$$

$$S_0 = \int dk \sum_{\sigma=\uparrow\downarrow} c^\dagger_{k\sigma} \left(-i\omega + \epsilon_c(\mathbf{k})\right) c_{k\sigma} + f^\dagger_{k\sigma} \left(-\frac{i\omega}{\Gamma} + \lambda\right) f_{k\sigma}$$

$$S_\chi = \int dk \sum_{\sigma=\uparrow\downarrow} \left[\chi_{\mathbf{k}} f^\dagger_{k\sigma} c_{k\sigma} + h.c.\right] + \sum_i \int d\tau |\chi_i|^2 / J_K,$$

$$S_\Phi = \int dk \sum_{\alpha\beta} \left[\Phi_{\mathbf{k}} \epsilon_{\alpha\beta} f^\alpha_k f^\beta_{-k} + h.c.\right] + \sum_{\langle i,j \rangle} \int d\tau |\Phi_{ij}|^2 / J_H$$

Effective action—amplitude (Gaussian) fluctuation

Beyond Ginzburg-Landau theory of phase transitions

$$S_G = \int dk \left[\hat{\tilde{\chi}}_k^\dagger \ \left(-G_\chi^{b\,-1}(\omega,\, \mathbf{k})\right) \hat{\tilde{\chi}}_k + \hat{\tilde{\Phi}}_k^\dagger \ \left(-G_\Phi^{b\,-1}(\omega,\, \mathbf{k})\right) \hat{\tilde{\Phi}}_k \right]$$

quasi-2d:
d=z+η, z=2,
0<η<<1

$$G^b_{\chi(\Phi)}(\omega,\, \mathbf{k}) \ = \ \frac{2(\epsilon_{\chi(\Phi)}(\mathbf{k})+m_{\chi(\Phi)})}{(i\omega_{\chi(\Phi)})^2-(\epsilon_{\chi(\Phi)}(\mathbf{k})+m_{\chi(\Phi)})^2}$$

$$S_K = J_\chi \sum_{\sigma=\uparrow\downarrow} \int dkdk' \left[(c_{k\sigma}^\dagger f_{k'\sigma}) \hat{\tilde{\chi}}_{k+k'}^\dagger + h.c. \right],$$

$$S_J = J_\Phi \sum_{\alpha,\beta=\uparrow\downarrow} \int dkdk' \left[\epsilon_{\alpha\beta} \hat{\tilde{\Phi}}_k f_{k'}^\beta f_{k+k'}^\alpha + h.c. \right],$$

boson-fermion Yukawa coupling

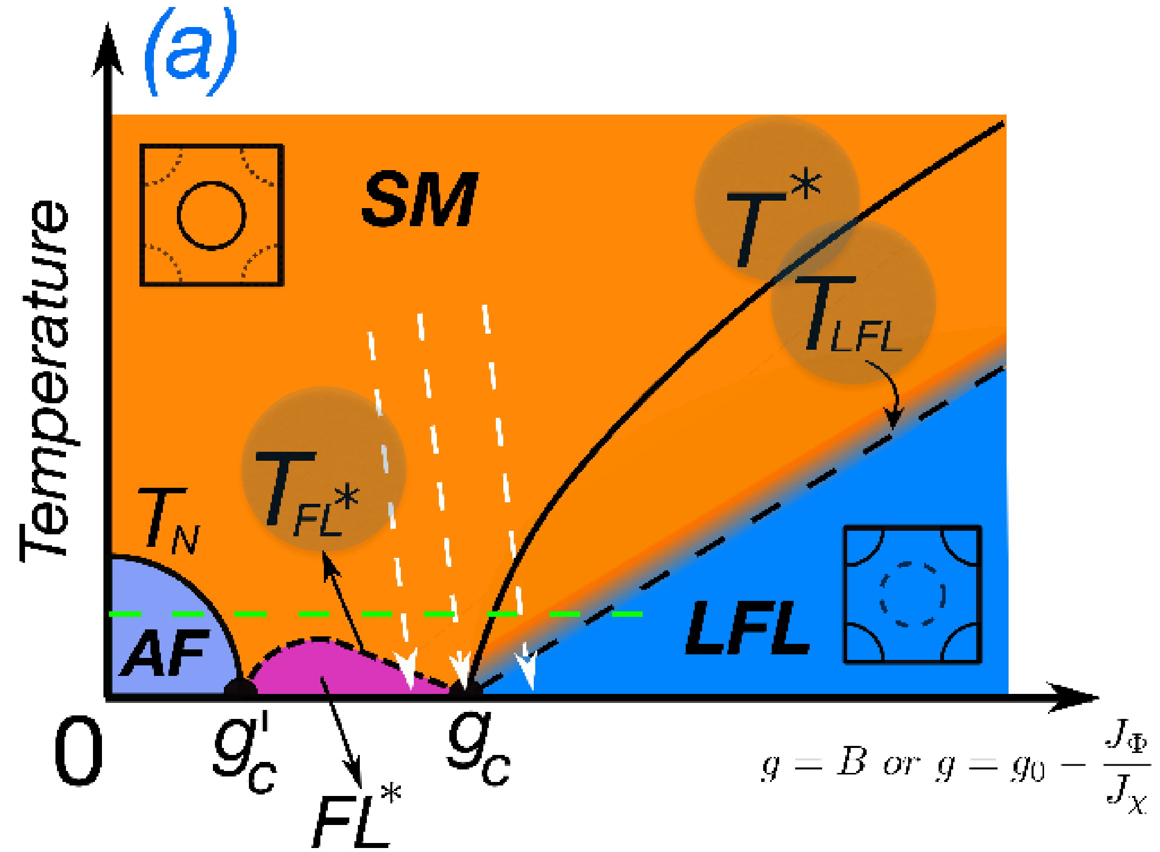
new scaling!

$$S_4 = \frac{u_\chi}{2} \int dk_1 dk_2 dk_3 \hat{\tilde{\chi}}_{k_1}^\dagger \hat{\tilde{\chi}}_{k_2}^\dagger \hat{\tilde{\chi}}_{k_3} \hat{\tilde{\chi}}_{-k_1-k_2-k_3}$$

$$+ \frac{u_\Phi}{2} \int dk_1 dk_2 dk_3 \hat{\tilde{\Phi}}_{k_1}^\dagger \hat{\tilde{\Phi}}_{k_2}^\dagger \hat{\tilde{\Phi}}_{k_3} \hat{\tilde{\Phi}}_{-k_1-k_2-k_3}$$

☞ B suppresses J_Φ but keep J_χ nearly at J_χ*

Crossover scales



T_{FL*} : $J_\Phi > J_\chi$*, J_χ is marginal.

$$T_{FL^*} \sim N_0^2\, |J_\chi|^5\, |g - g_c|$$

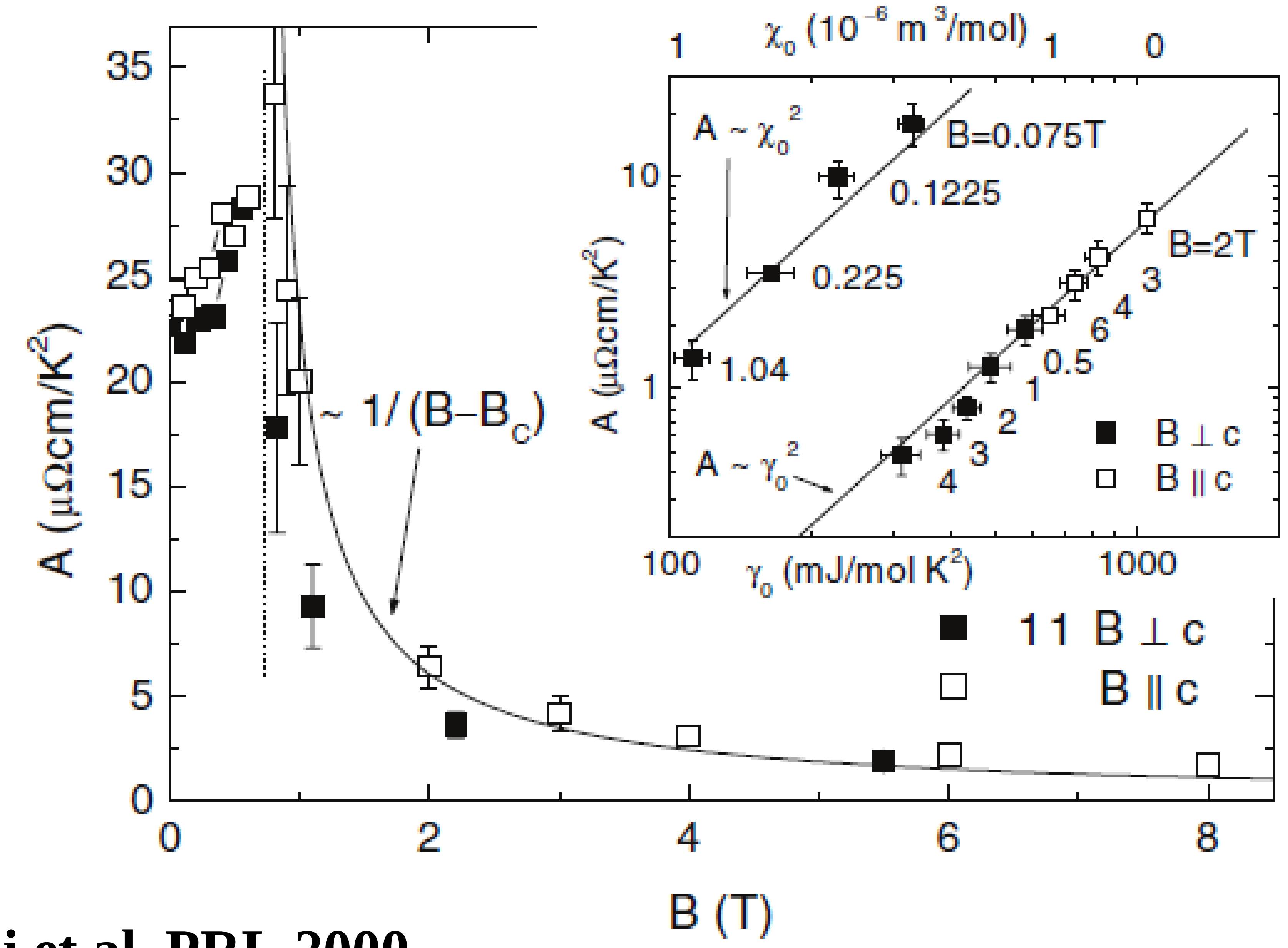
T_{LFL} : $J_\Phi < J_\chi$*, J_χ is marginal, χ is relevant relative to marginal J_χ

$$T_{LFL} \sim N_0^2\, J_\chi\, |\chi|^4\, |g - g_c|$$

T* : χ (J_K) is marginal but J_χ is irrelevant.

$$T^* \sim N_0^2\, |\chi|^5\, |g - g_c|^{1-\eta/z}$$

Divergence of A-coefficient in FL phase



O. Trovarelli et al. PRL 2000

theory prediction:

$$A \propto \xi^2 \sim |g - g_c|^{-2\nu} \sim |g - g_c|^{-1}$$

Specific heat coefficient $\gamma\left(\frac{T}{T_{LFL}}\right)$

Critical bosonic RVB fluctuations

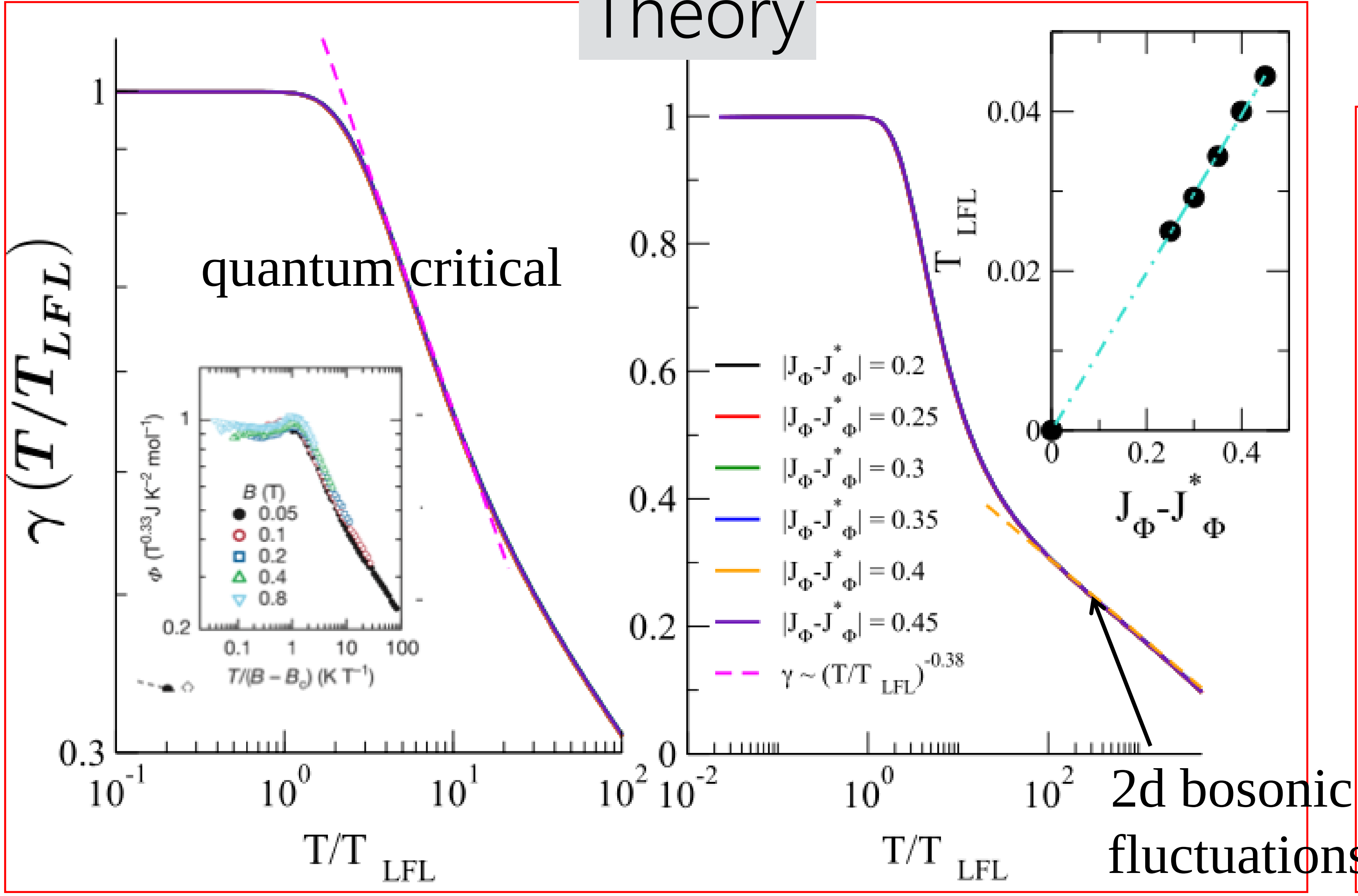
$$S_G = \int dk \left[\hat{\hat{\Phi}}_k^\dagger \left(-G_\Phi^{b\,-1}(\omega, \mathrm{k})\right) \hat{\hat{\Phi}}_k \right]$$

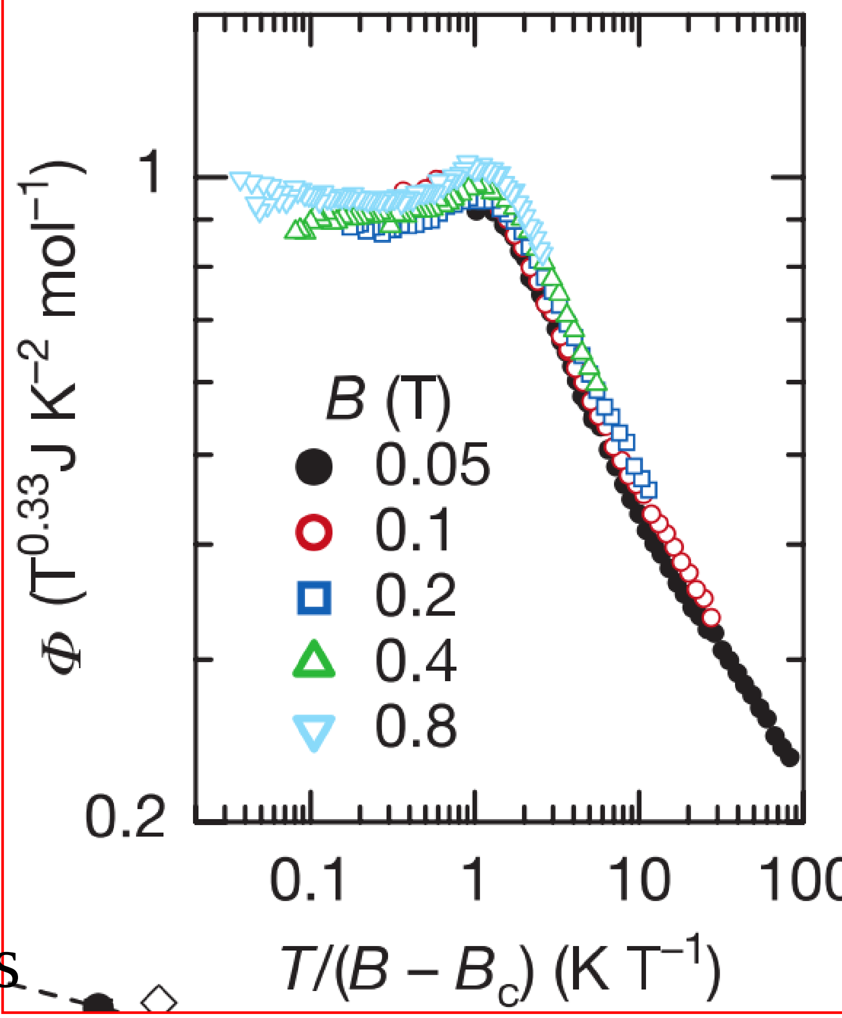
$$\gamma\left(\bar{T}\right) = -\frac{r^{-\bar{\alpha}}\,\bar{T}^{\eta/2}}{4} \int_{1/\bar{T}}^{\Lambda/\bar{T}} dx \, \frac{x^{2+\eta/2}}{\sinh^2(x/2)}$$

$$\bar{\alpha} = \eta^2 + 3\eta/2 \approx 0.32 \qquad \bar{T} \equiv \frac{T}{T_{LFL}}$$

fitting parameter: $\eta = 0.18$

Specific heat coefficient $\gamma\left(\frac{T}{T_{LFL}}\right)$





$$\frac{C_V}{T} = \frac{1}{b^{1/3}}\Phi\left(\frac{T}{T_0(b)}\right)$$

$$\Phi(x) \approx (\max(x,\ 1))^{-1/3}$$

anomalous exponent

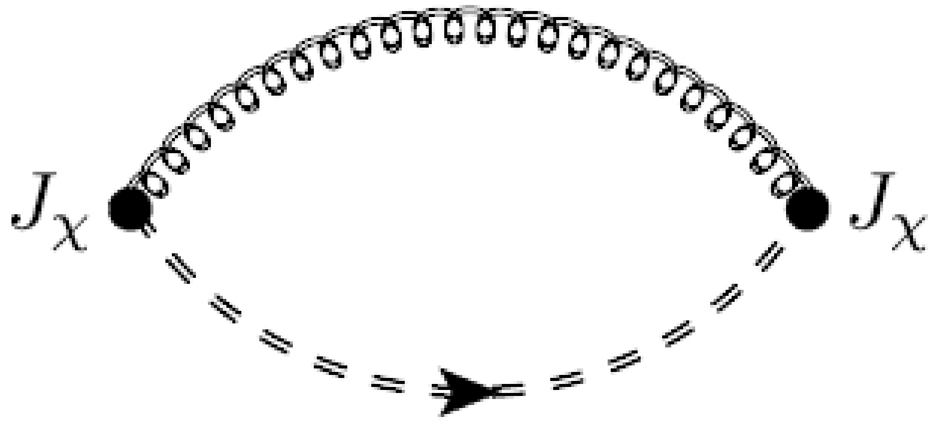
$$\Phi(B, T) = (B - B_c)^{0.33} C_{el}/T,$$

$$T_{LFL} \sim |J_\phi - J_\phi^*| \sim |B - B_c|$$

Linear-T Resistivity

Critical Kondo fluctuations (bosonic charge)

Conduction electron T-matrix



$T_{\mathbf{kk}}(\omega^+)$

$$J_\chi \sum_{\sigma=\uparrow\downarrow} \int dk dk' \left[(c^\dagger_{k\sigma} f_{k'\sigma}) \hat{\chi}^\dagger_{k+k'} + h.c. \right]$$

$$\tau^{-1}(\omega) = -\frac{c_{imp}}{2} \sum_{\mathbf{k}} \mathrm{Im}\, T_{\mathbf{kk}}(\omega^+).$$

$$\sigma(T) = -\frac{2e^2}{3} \int \frac{d\mathbf{k}}{(2\pi)^3} v_k^2\, \tau(k)\, \frac{\partial f}{\partial \epsilon_k}$$

Linear-T Resistivity

Conductivity

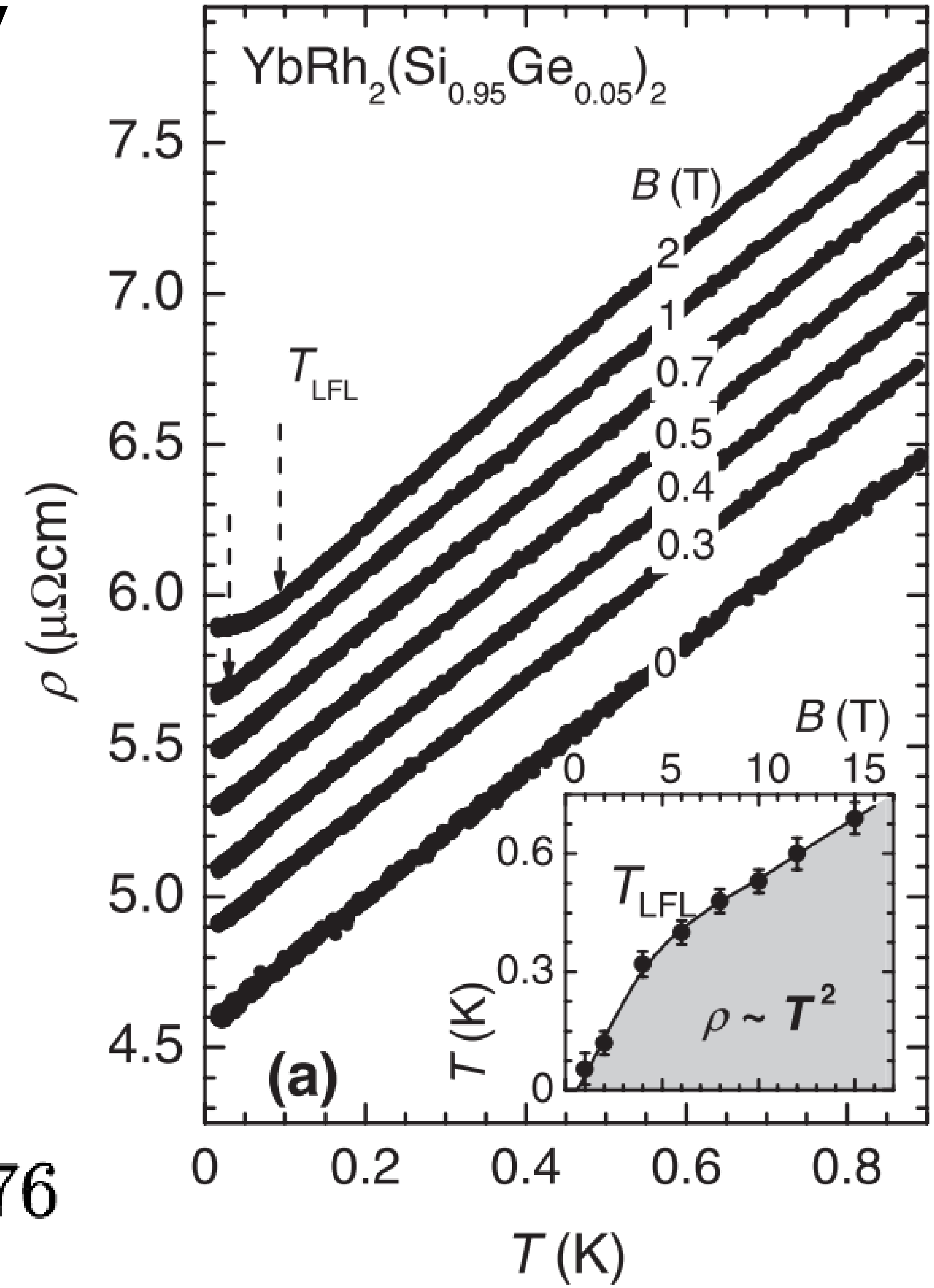
$$\sigma(T) = -\frac{2e^2}{3}\int \frac{d\mathbf{k}}{(2\pi)^3} v_k^2\, \tau(k)\, \frac{\partial f}{\partial \epsilon_k}$$

τ(k) : life-time of c-electrons.

T-linear Resistivity: $\rho(T) \sim a + bT$

$$\text{Theory} \;:\; \frac{\rho(T) - \rho(T=0)}{\rho(T=0)} \sim 1.76$$

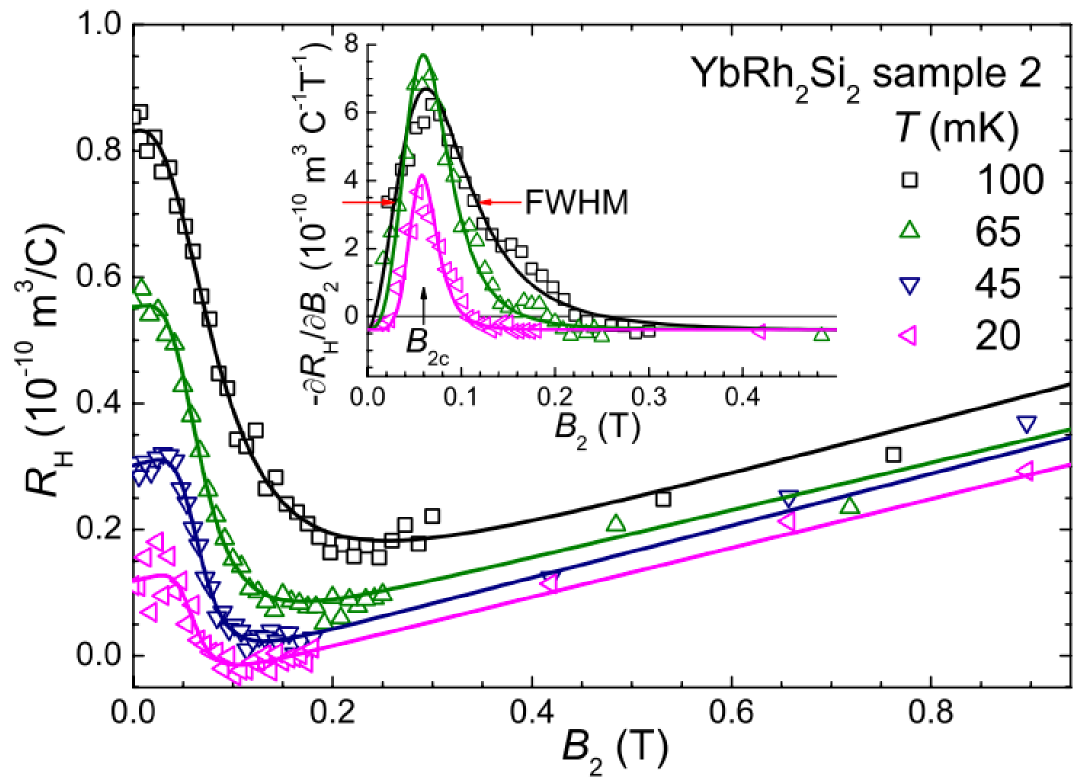
$$\text{Experiment} \;:\; \frac{\rho(T) - \rho(T=0)}{\rho(T=0)} \sim 0.44$$



Custers, et al., PRL, 2010

Jump in Fermi surface volume

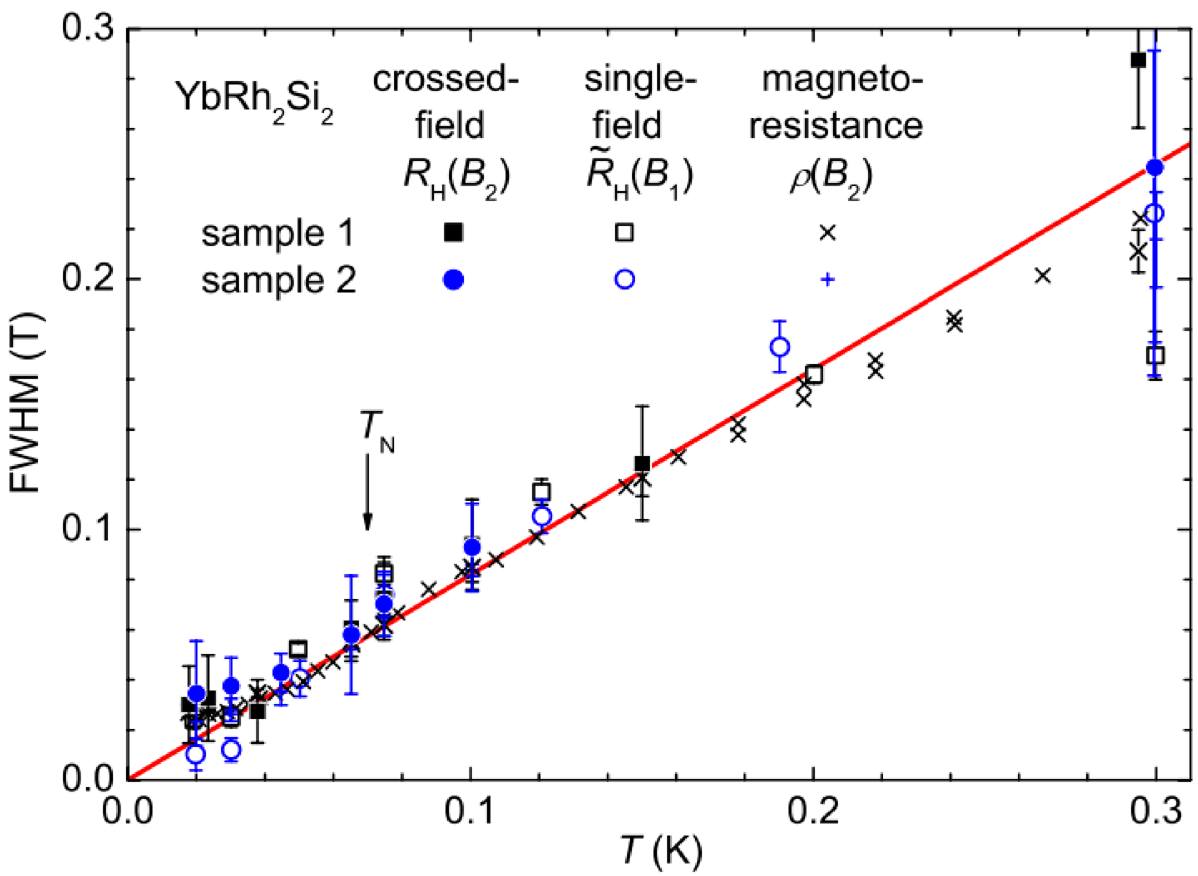
$$G_c(\boldsymbol{k},\omega) = \frac{1}{\omega - \epsilon_c(\boldsymbol{k}) - \Sigma_c(\boldsymbol{k},\omega)}$$



T = 0

$$\epsilon_F^* = \frac{{k_F^*}^2}{2m} \equiv \epsilon_F + \frac{|\chi|^2}{\bar{\lambda}}$$

$$\begin{cases} \chi = 0, & \text{if } g < g_c \\ \chi \neq 0, & \text{if } g > g_c \end{cases}$$



T > 0

$\mathrm{Im}\Sigma_c = \alpha T$ α : constant

Friedeman et al., PNAS, 2010

Strange superconductivity near heavy-fermion quantum critical point:

application for CeMIn5 (M= Rh, Co)

PHYSICAL REVIEW B **99**, 094513 (2019)

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T. K. Lee (Academia Sinica)

Acknowledgement:

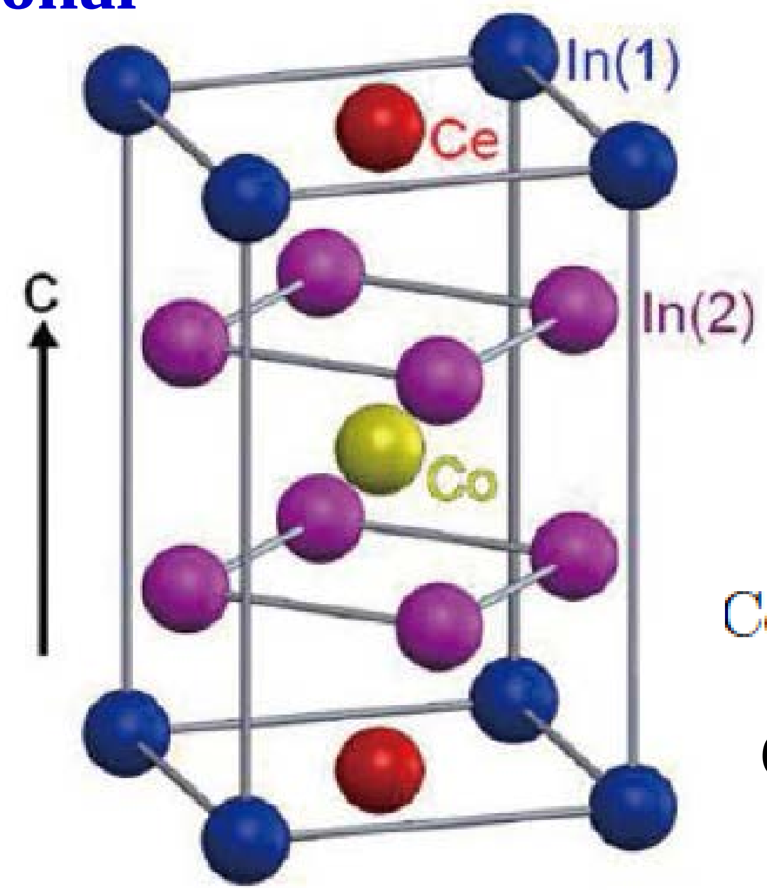
J. D. Thompson (LANL), Piers Coleman (Rutgers)

CeCoIn5: Lattice Structure

Tetragonal

CeIn3

CoIn2



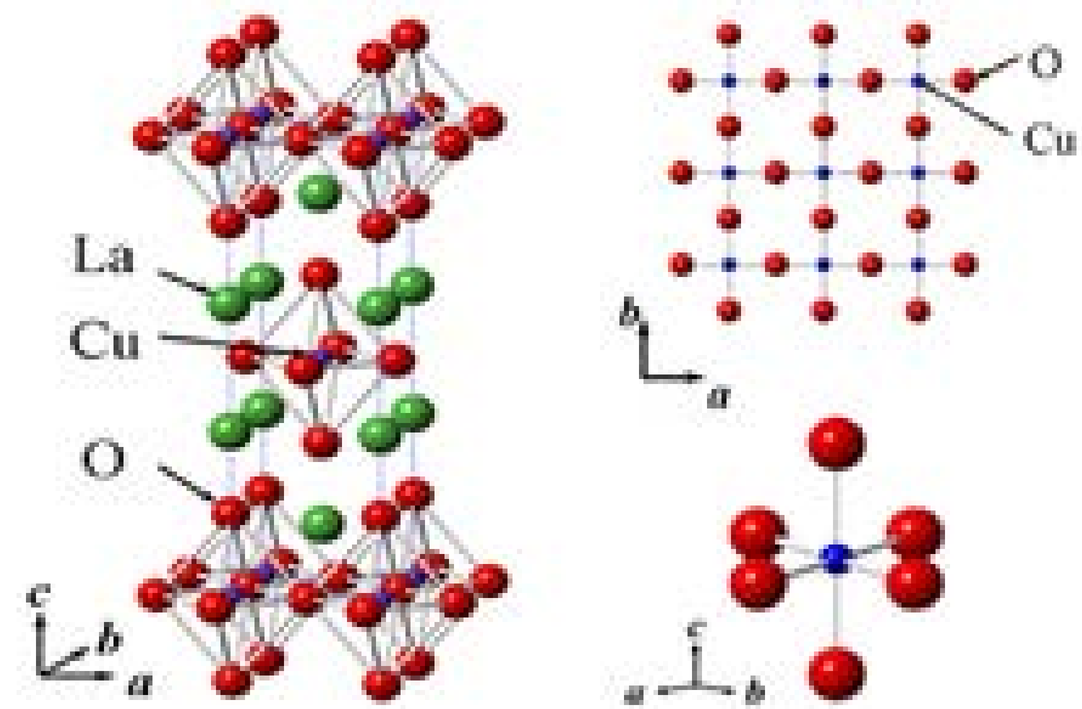
$a=b=4.6\text{Å}$, $c=7.51\text{Å}$

Ce^{3+} $(6s^2 4f^1 5d^1)$

Co^{2+} ($3d^7$)

Matsuda, AAPPS Bulletin 2017

Tetragonal



- **Discovered in 2001 by Fisk et al., heavy-fermion analogue of cuprate (LaCu2O4) superconductor**
- **quasi-2D structure + proximity to magnetic order, favorable for unconventional superconductivity**

- **local-moment 4f electron on Ce + itinerant 5d (Ce) and 3d (Co) electrons**

Kondo hybridization between f and d electrons, Anti-ferromagnetism (Ce)
Superconductivity at the boarder (quantum critical point QCP) of anti-ferromagnetism

Tc ~ 2.3K

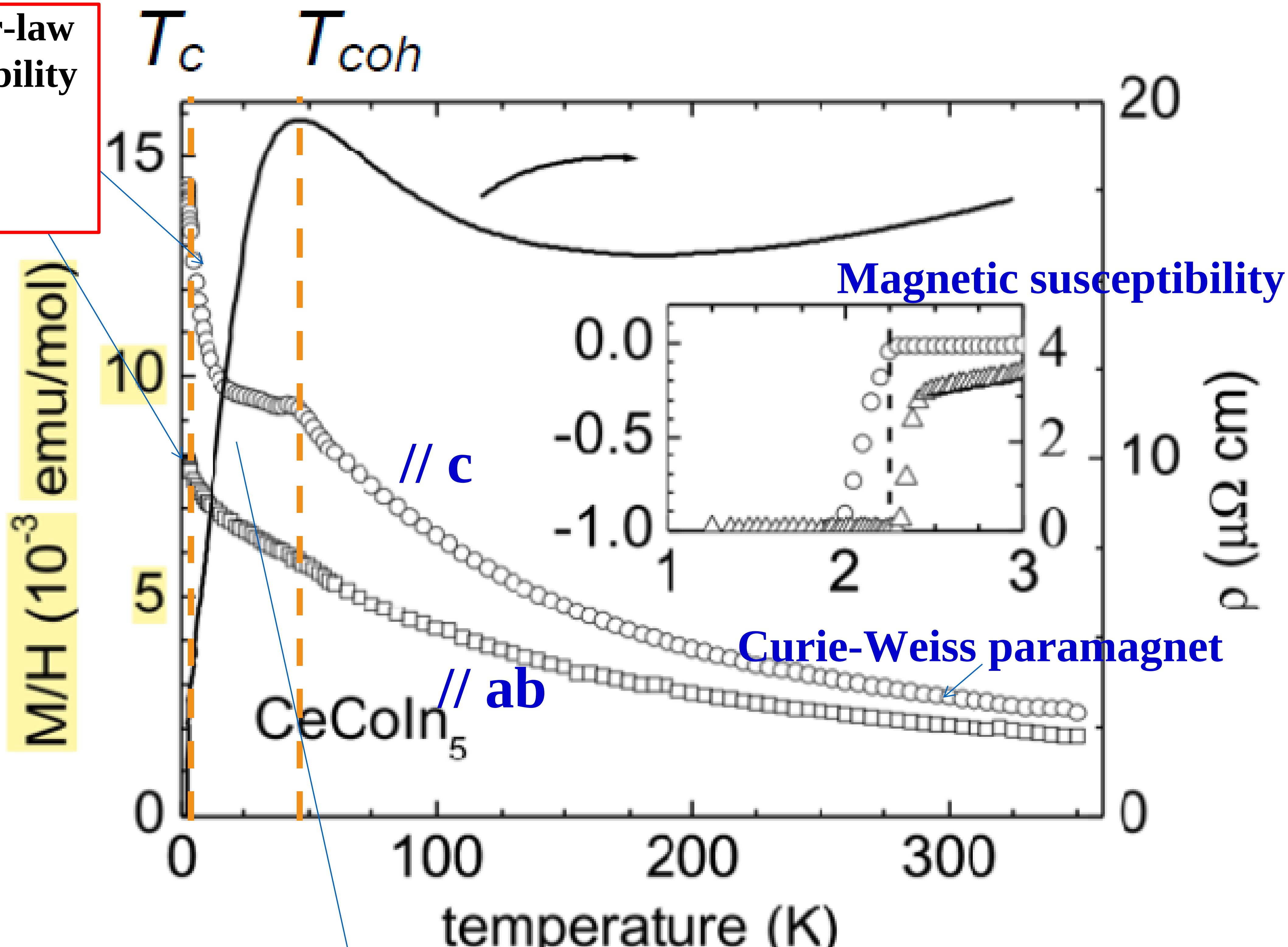
strange superconductivity in CeCoIn5:

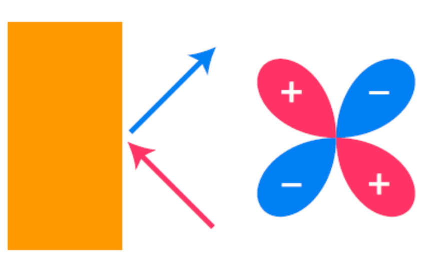
Non-Fermi liquid (strange metal) normal state

anomalous power-law magnetic susceptibility

$$\chi(T) \propto T^{-\alpha}$$

$$0 < \alpha < 1$$



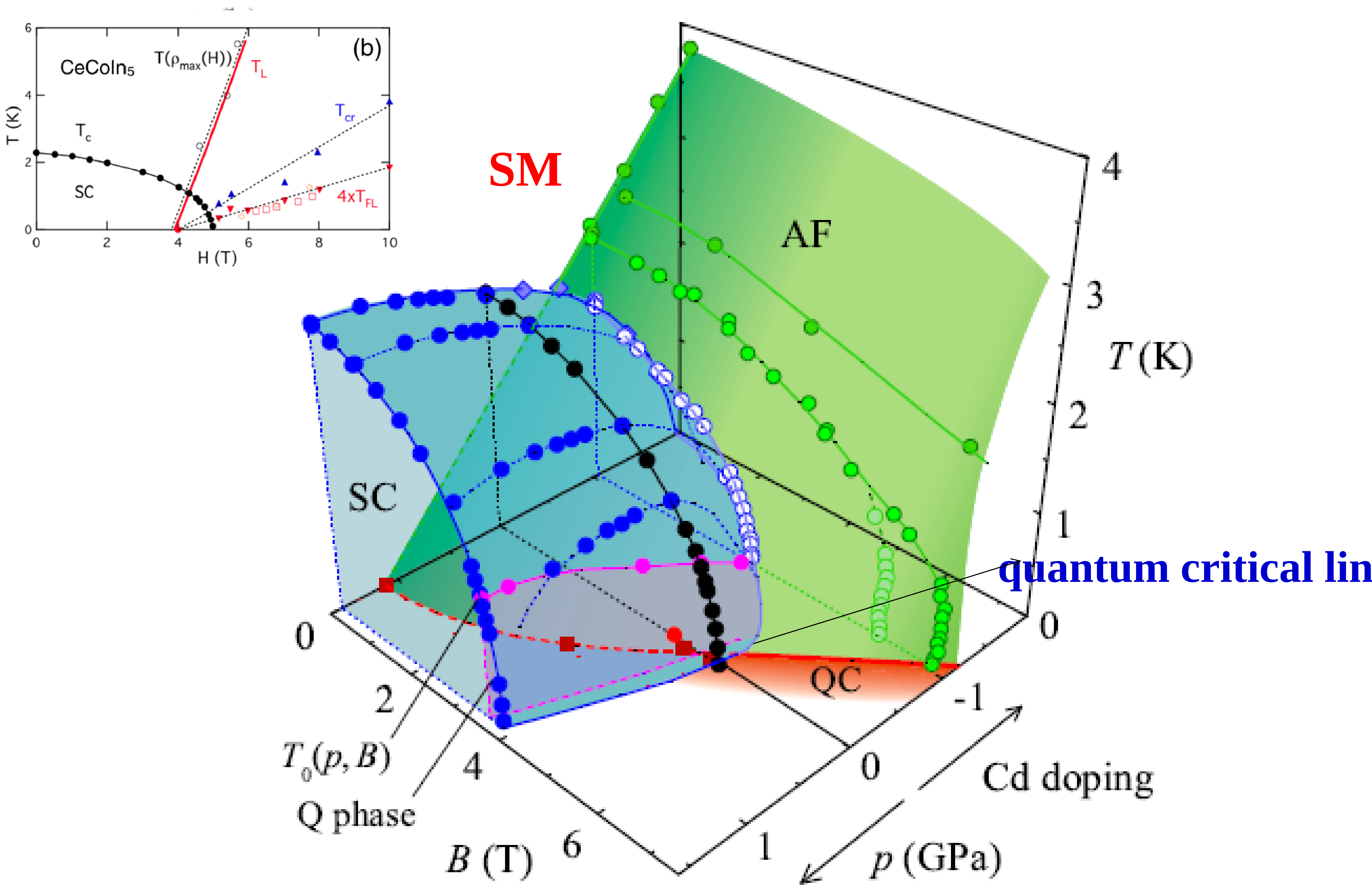


d_{x2-y2} wave gap

C. Petrovic, JPCM. 13, L337-L342 (2001).

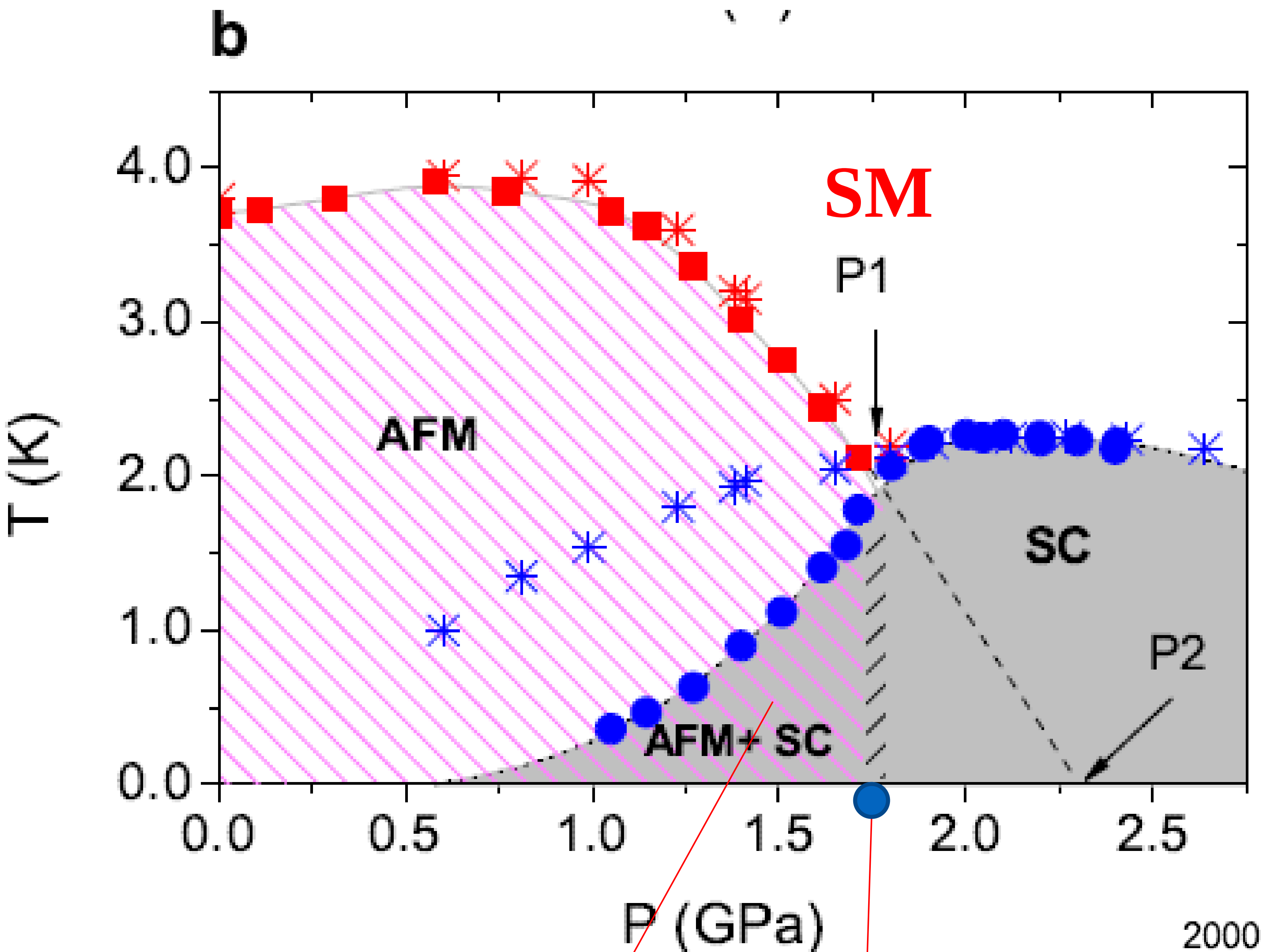
linear-in-T resistivity $\rho(T) \propto T$.

Global Phase Diagrams of CeCoIn5



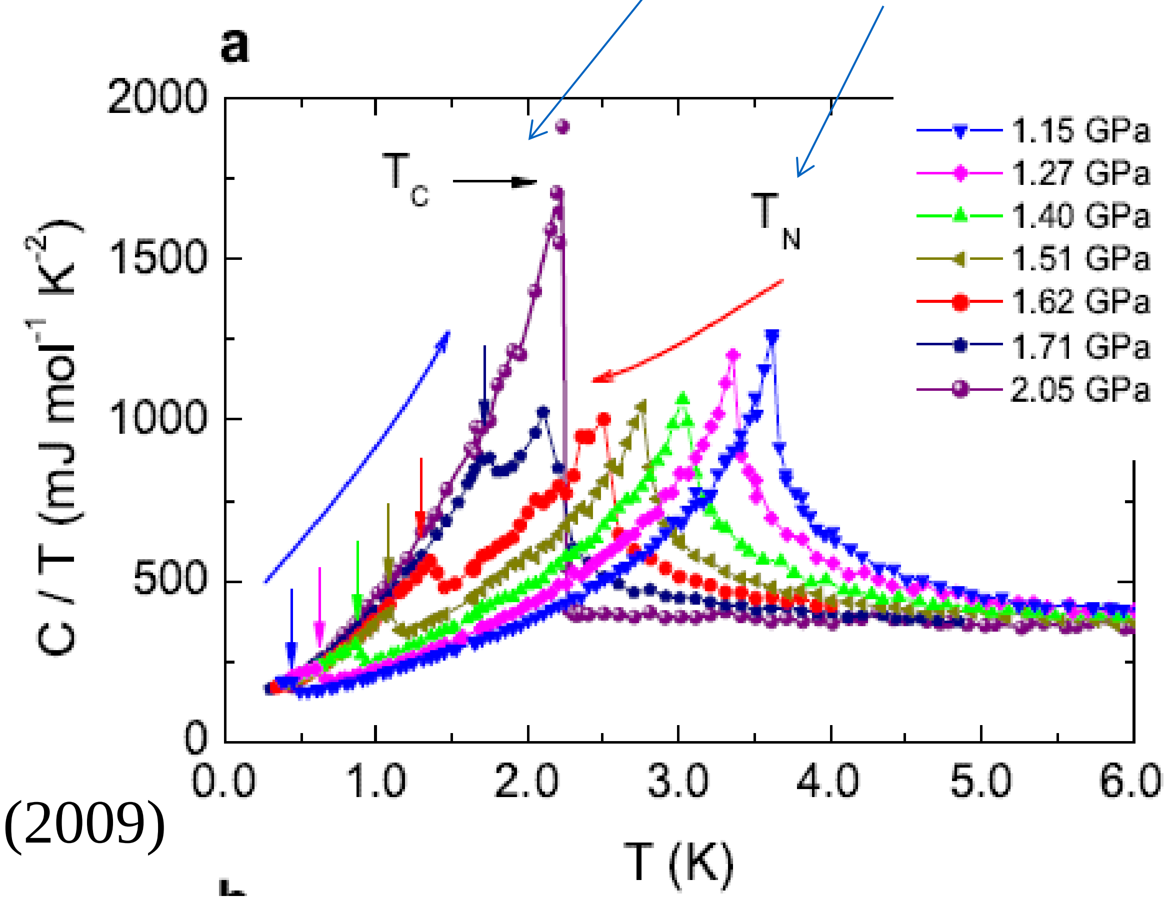
J.D. Thompson *et al.* Phys. Rev. Lett. 106, 087003 (2011)

Phase diagram for CeRhIn5



P2: QCP in the absence of SC

2 peaks: Tc and T_N



Spiral (incommensurate) spin order (SDW)

ordered moment of 0.48-0.65μ_B/Ce

$q_M = (1/2, 1/2, 0.297)$

W. Bao, PRB 2000

Co-existing AFM+SC

First-order transition

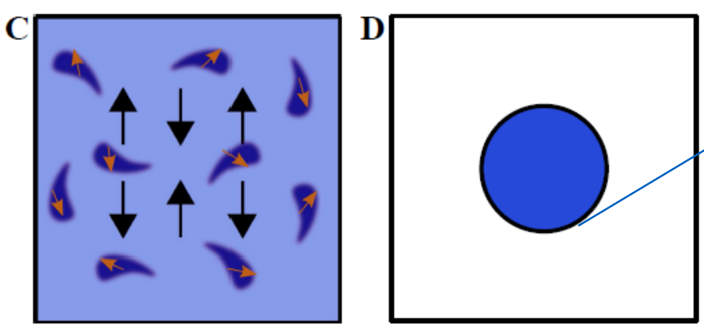
J.D. Thompson *et al*. New Jouranl of Physics 11, 055062 (2009)

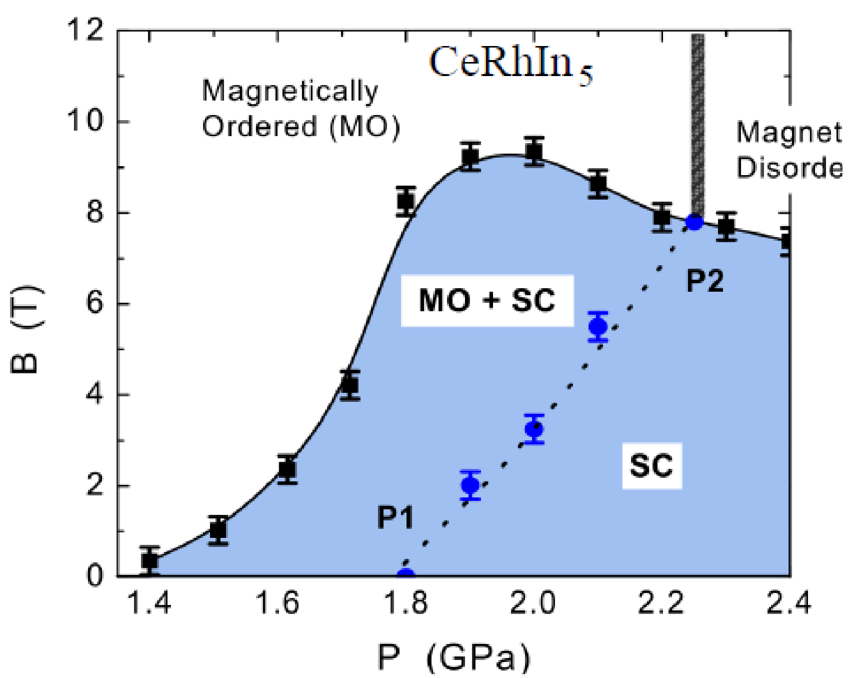
Kondo breakdown QCP for $CeRhIn_5$

A Drastic Change of the Fermi Surface at a Critical Pressure in $CeRhIn_5$: dHvA Study under Pressure

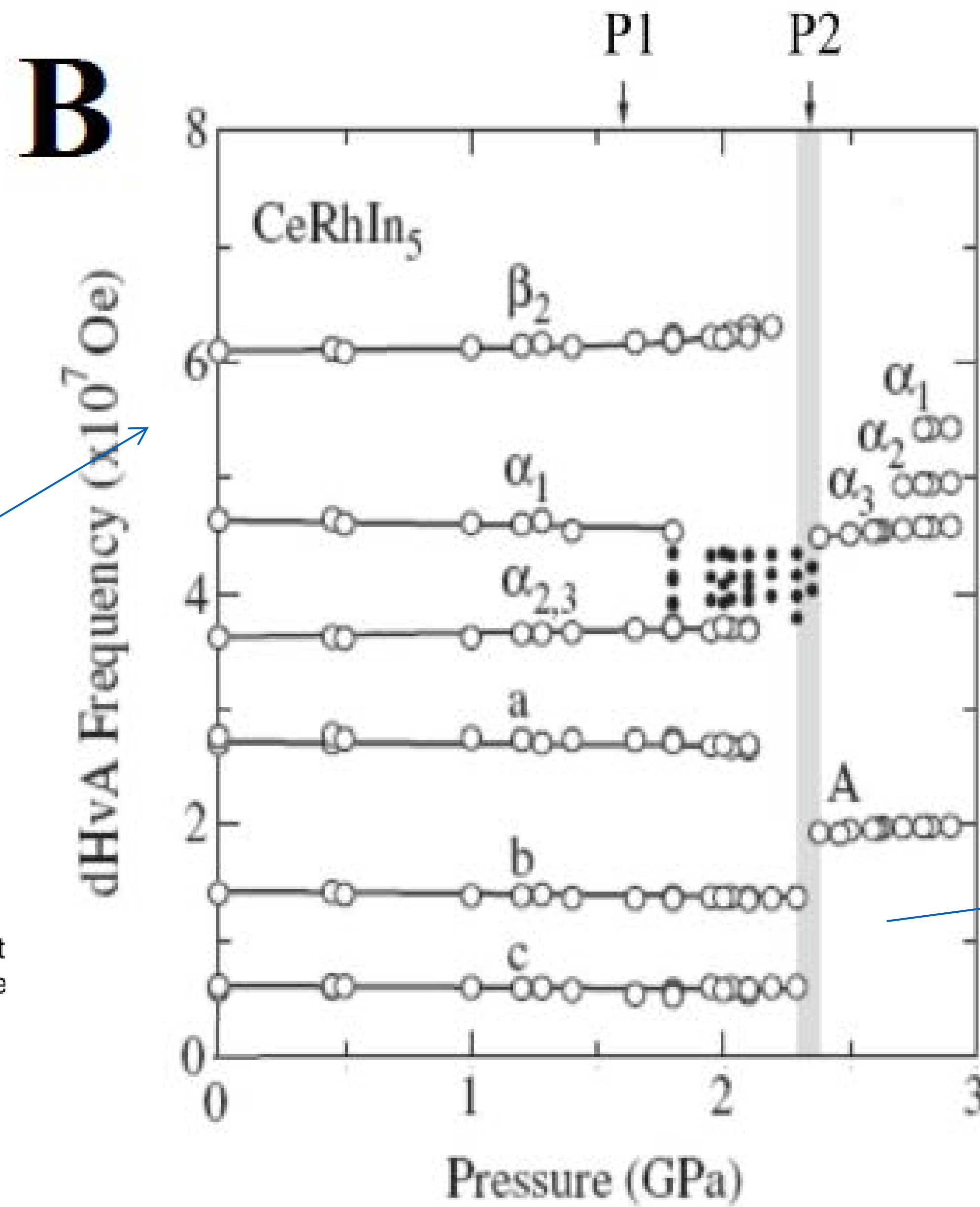
H. Shishido, R. Settai, H. Harima, and Y. Ōnuki, J. Phys. Soc. Jpn **74**, 1103 (2005) .

Small Fermi surface Kondo destruction)

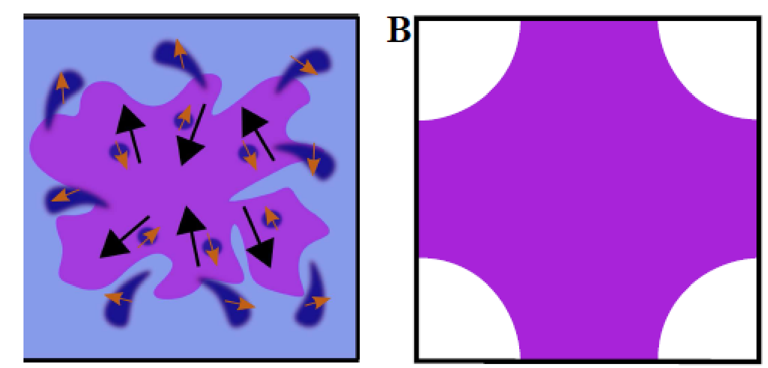




T. Park, *et al.*, *Nature* **440**, 65 (2006).



Large Fermi surface (Kondo)



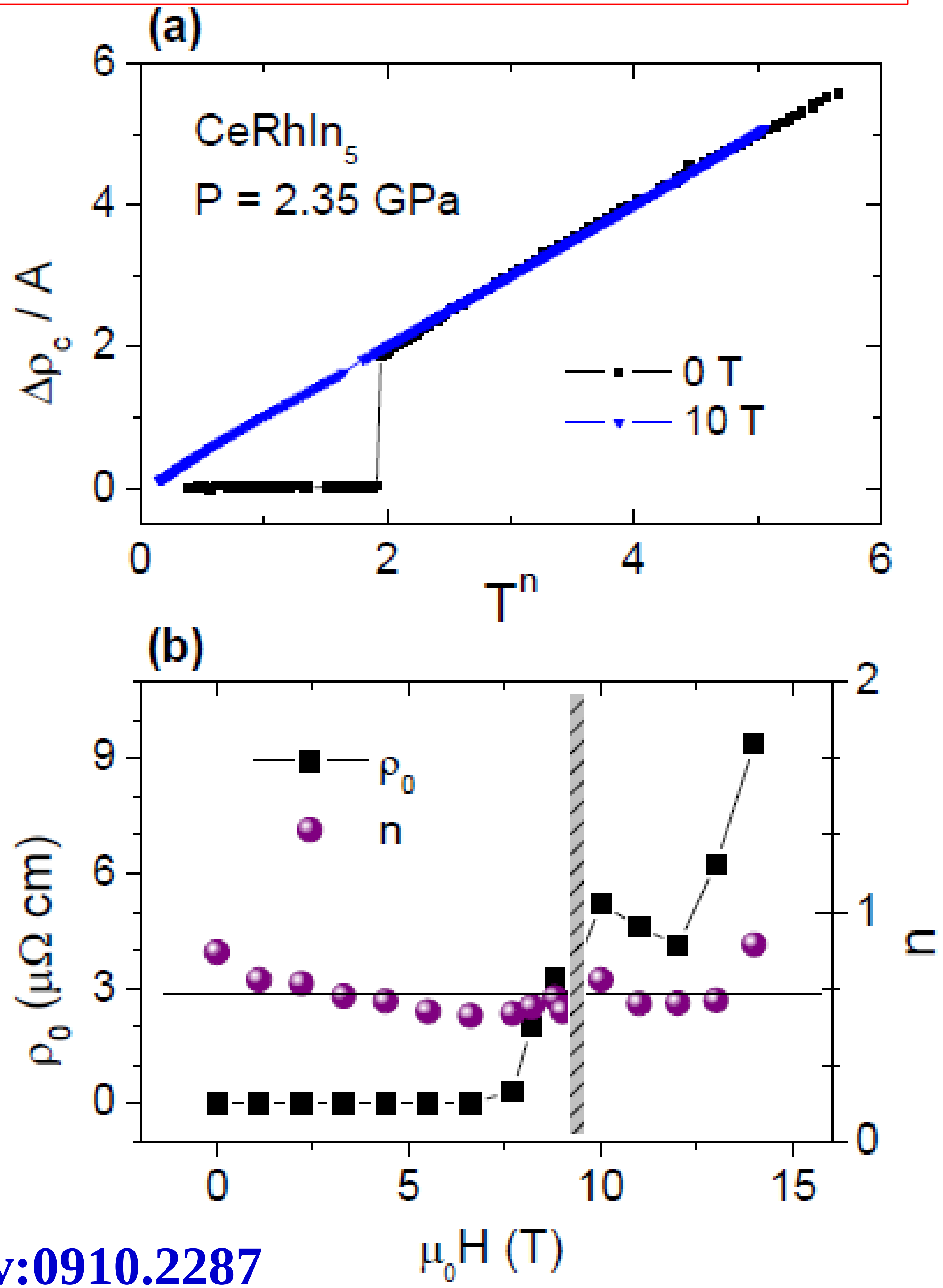
Q. Si et al. Science 329, 1161 (2010)

$$V_{\rm osc} = A \sin\left(\frac{2\pi F}{H} + \phi\right),$$

dHvA frequency $F\ (= \hbar S_{\rm F}/2\pi e)$

S_F cross-sectional area of Fermi surface

Sub-linear-T resistivity (non-Fermi liquid)



J. Thompson et al. arXiv:0910.2287

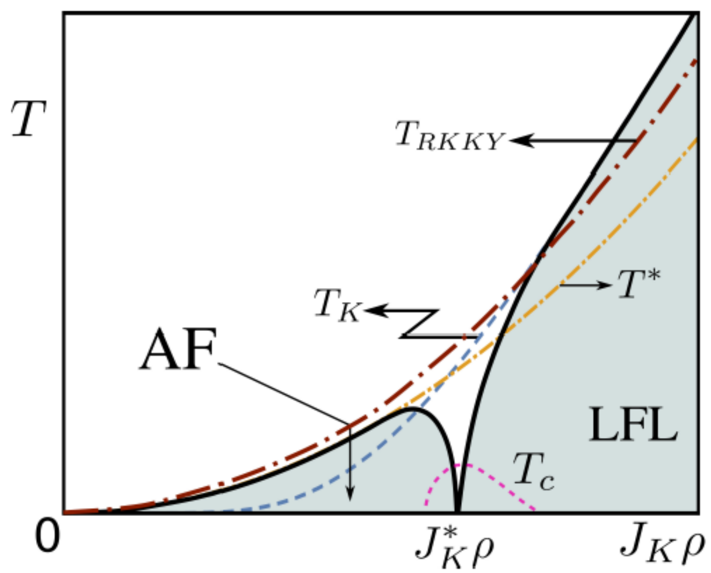
Open issues on mechanism of strange superconductivity in CeMIn5

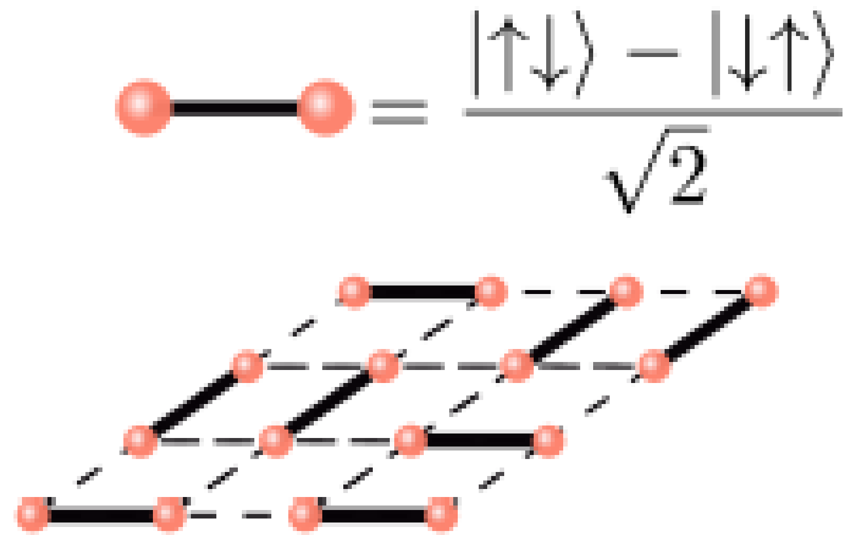
- How do (f) electrons incorporate in the superconducting state?

- How does a strange metal turn into a superconductor?

- What are the links among SM, Kondo coherence, superconductivity, and QCP?

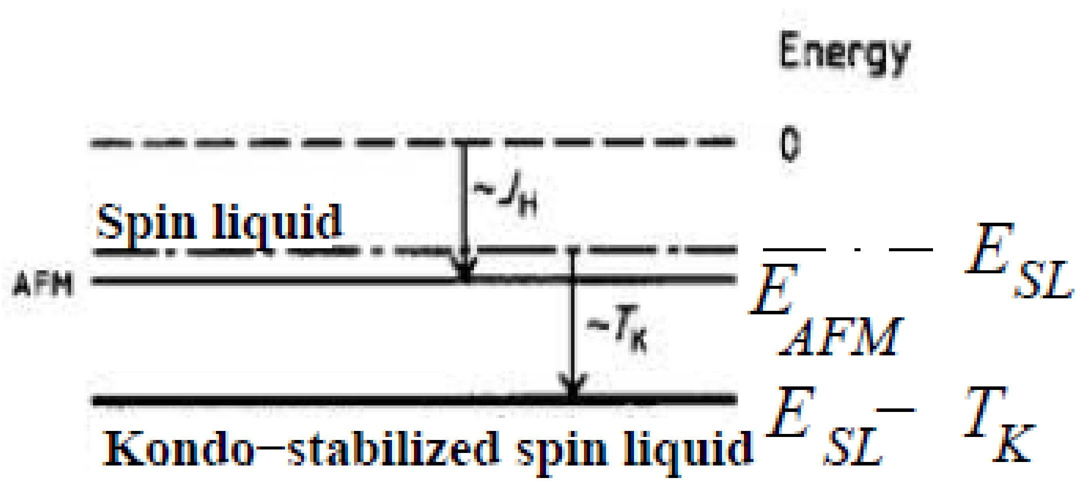
Anderson's RVB spin-liquid for cuprate supeconductors



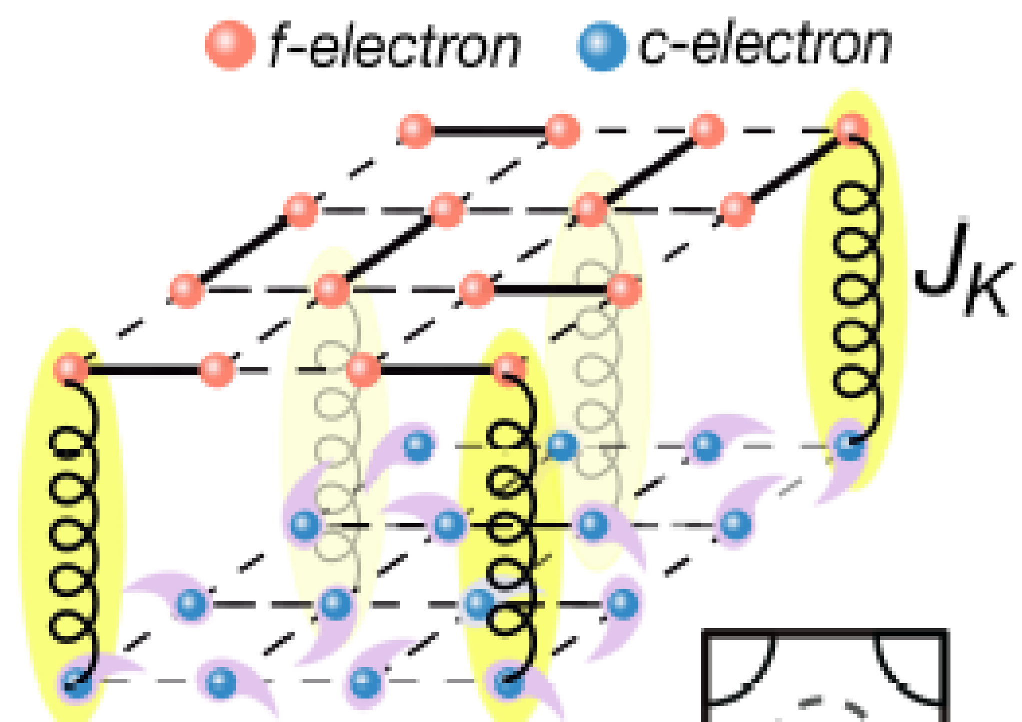


Resonating Valence Bond (RVB) spin-liquid

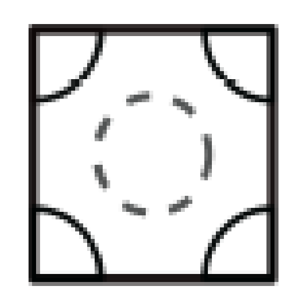
Kondo stablized spin-liquid close to magnetic instability (phase transition)

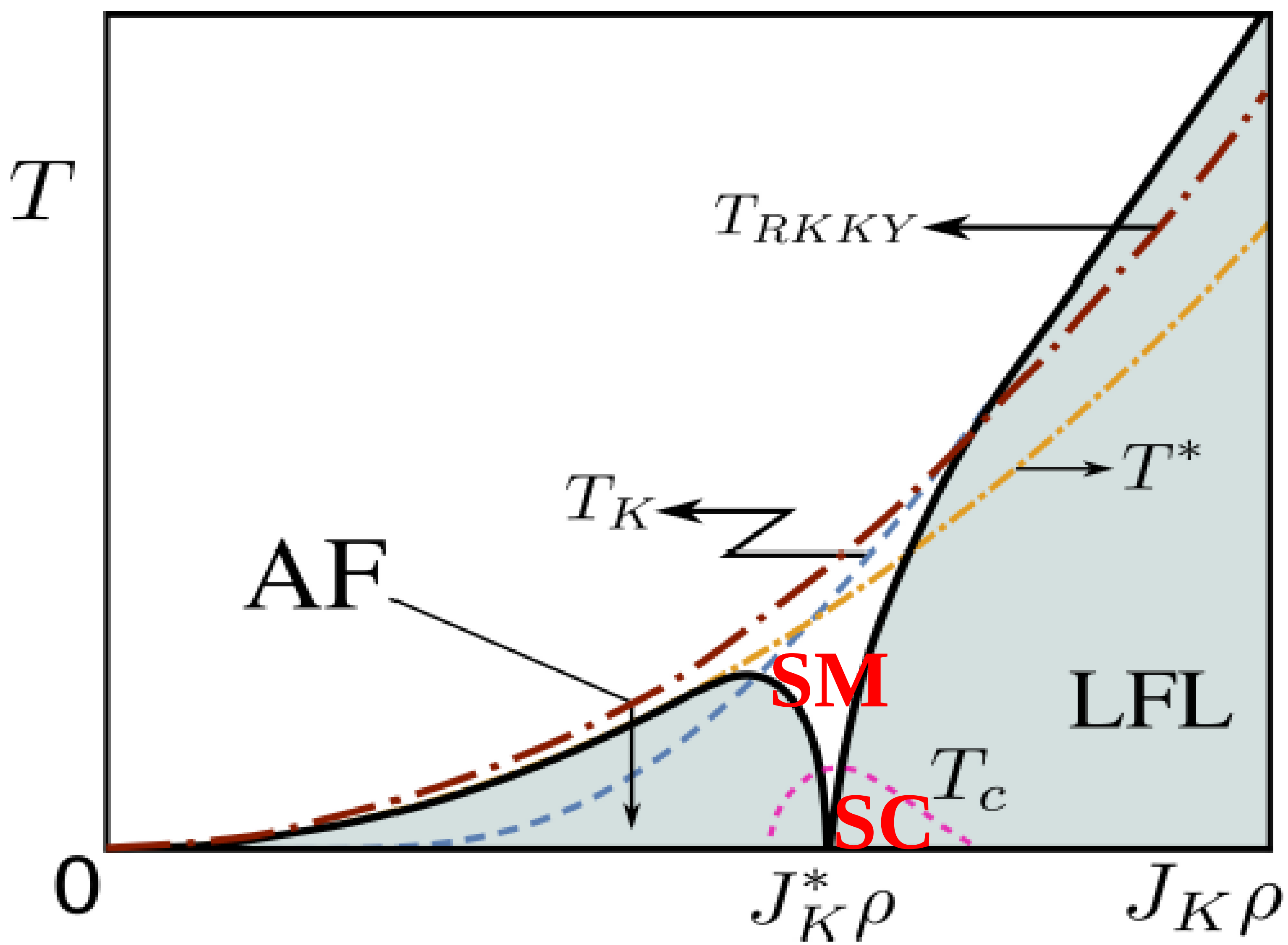


Escape of RVB singlets into conduction sea
→Bose condensing Cooper pairing-
→superconductivity **Andrei, Coleman JPCM 1989**

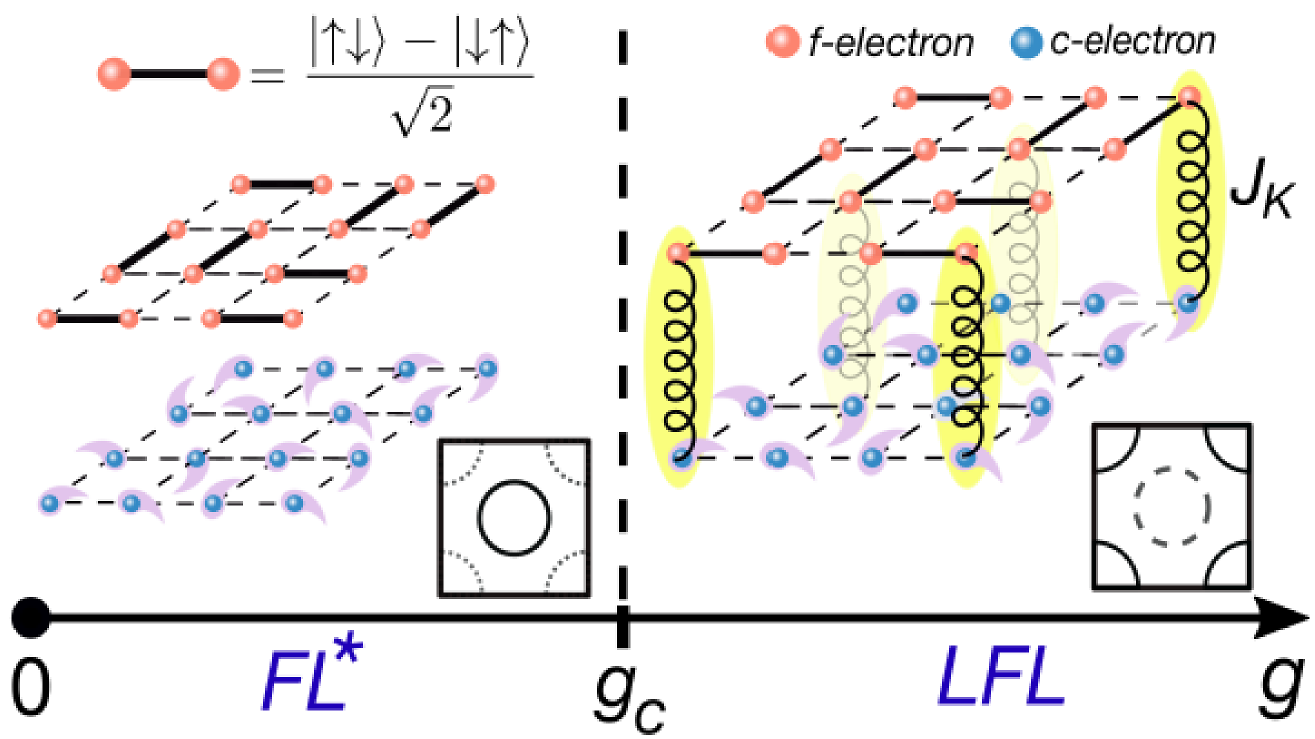


inter-layer proximity



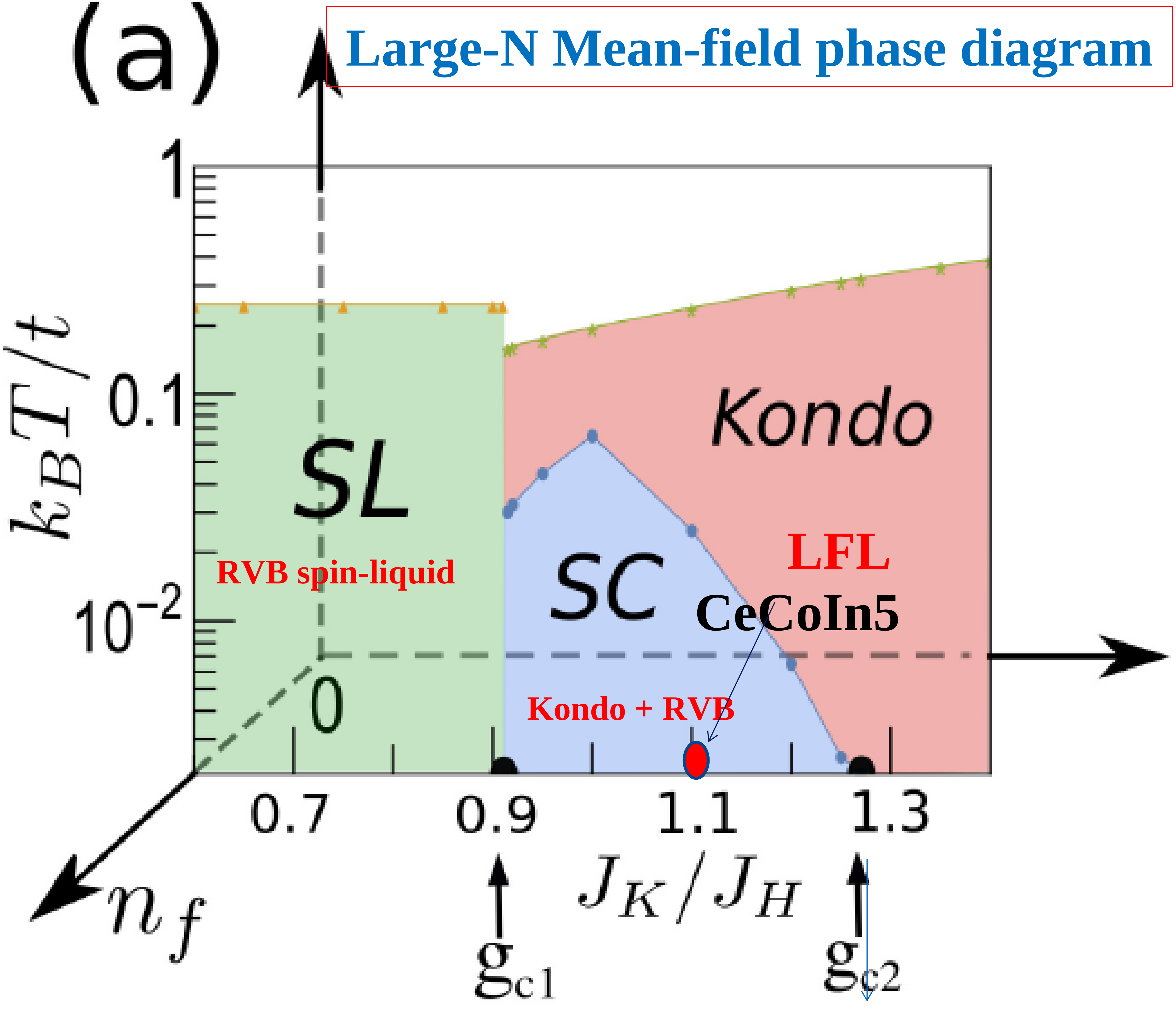


T
T_{RKKY}
T^*
T_K
AF
SM
LFL
T_c
SC
0
$J_K^*\rho$
$J_K\rho$



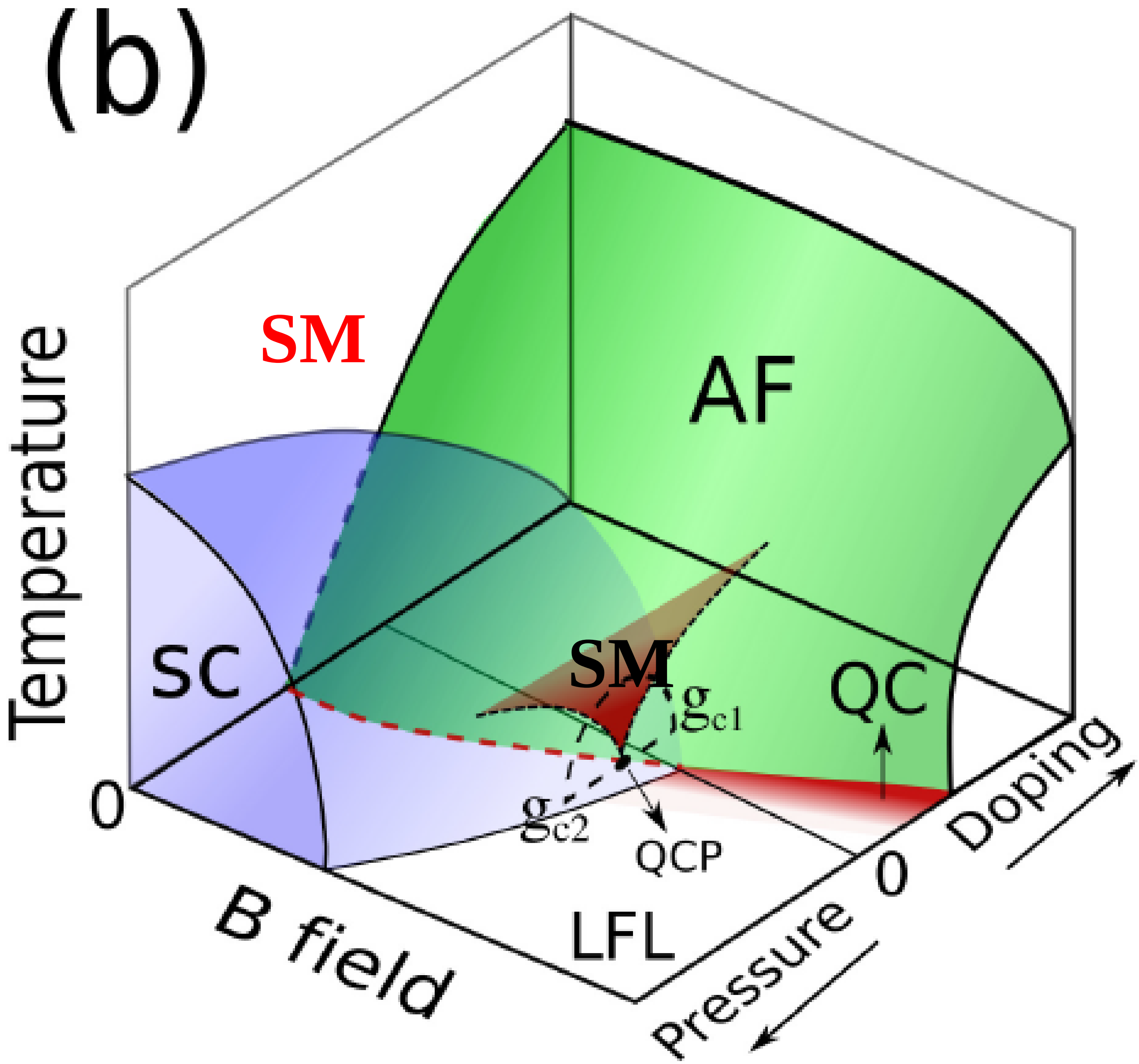
$= \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$
f-electron
c-electron
J_K
0
FL^*
g_c
LFL
g

Large-N Mean-field phase diagram

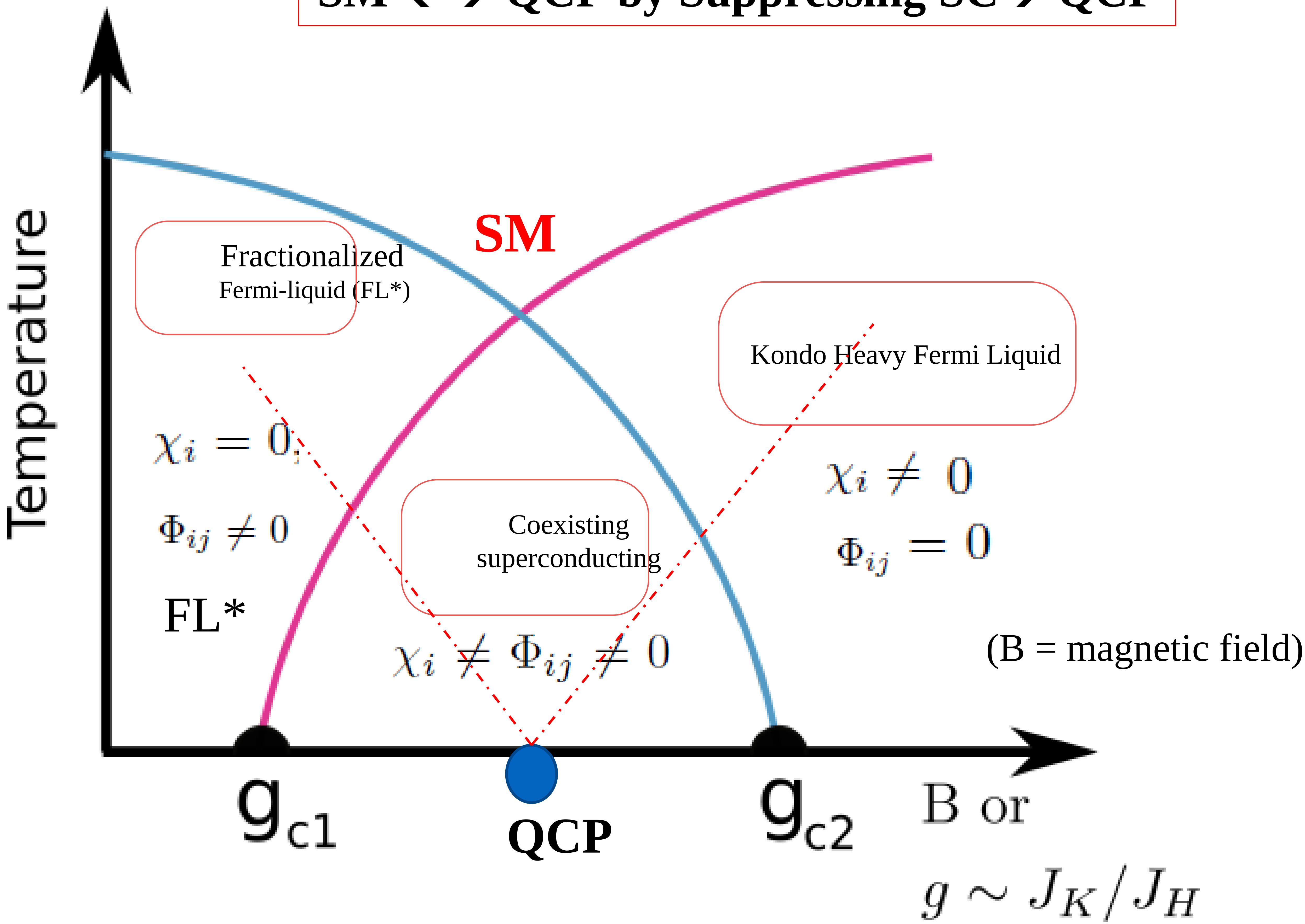


Superconductivity = co-existence btw Kondo and RVB spin-liquid

Strange metal (SM), superconductivity and quantum criticality



SM ←→ QCP by Suppressing SC→ QCP



Effective field theory beyond mean-field

Effective Lagrangian — Mean-field

Y. Chang *et al* *PRB 2018*

$$\Phi_{ij} \longrightarrow \Phi_{ij} + \hat{\tilde{\Phi}}_{ij} \qquad \Phi_{ij},\ \chi_i : \text{uniform values}$$

$$\chi_i \longrightarrow \chi_i + \hat{\tilde{\chi}}_i \qquad \hat{\tilde{\Phi}}_{ij},\ \hat{\tilde{\chi}}_i : \text{fluctuations}$$

$$S_{eff} = S_0 + S_\chi + S_Q + S_4 + S_K + S_J + S_G$$

$$S_0 = \int dk \sum_{\sigma=\uparrow\downarrow} c^\dagger_{k\sigma} \left(-i\omega + \epsilon_c(\mathbf{k})\right) c_{k\sigma} + f^\dagger_{k\sigma} \left(-\frac{i\omega}{\Gamma} + \lambda\right) f_{k\sigma}$$

$$S_\chi = \int dk \sum_{\sigma=\uparrow\downarrow} \left[\chi_{\mathbf{k}} f^\dagger_{k\sigma} c_{k\sigma} + h.c.\right] + \sum_i \int d\tau |\chi_i|^2 / J_K,$$

$$S_\Phi = \int dk \sum_{\alpha\beta} \left[\Phi_{\mathbf{k}} \epsilon_{\alpha\beta} f^\alpha_k f^\beta_{-k} + h.c.\right] + \sum_{\langle i,j\rangle} \int d\tau |\Phi_{ij}|^2 / J_H$$

Effective action beyond mean-field —amplitude (Gaussian) fluctuation

$$S_G = \int dk \left[\hat{\hat{\chi}}_k^\dagger \left(-G_\chi^{b\,-1}(\omega, \mathbf{k}) \right) \hat{\hat{\chi}}_k + \hat{\hat{\Phi}}_k^\dagger \left(-G_\Phi^{b\,-1}(\omega, \mathbf{k}) \right) \hat{\hat{\Phi}}_k \right]$$

$$G^b_{\chi(\Phi)}(\omega, \mathbf{k}) = \frac{2(\epsilon_{\chi(\Phi)}(\mathbf{k}) + m_{\chi(\Phi)})}{(i\omega_{\chi(\Phi)})^2 - (\epsilon_{\chi(\Phi)}(\mathbf{k}) + m_{\chi(\Phi)})^2}$$

Competition Kondo (S_k) vs. RVB (S_J)

$$S_K = J_\chi \sum_{\sigma=\uparrow\downarrow} \int dk dk' \left[(c_{k\sigma}^\dagger f_{k'\sigma}) \hat{\hat{\chi}}_{k+k'}^\dagger + h.c. \right],$$

$$S_J = J_\Phi \sum_{\alpha,\beta=\uparrow\downarrow} \int dk dk' \left[\epsilon_{\alpha\beta} \hat{\hat{\Phi}}_k f_{k'}^\beta f_{k+k'}^\alpha + h.c. \right],$$

$$S_4 = \frac{u_\chi}{2} \int dk_1 dk_2 dk_3 \hat{\hat{\chi}}_{k_1}^\dagger \hat{\hat{\chi}}_{k_2}^\dagger \hat{\hat{\chi}}_{k_3} \hat{\hat{\chi}}_{-k_1-k_2-k_3}$$

$$+ \frac{u_\Phi}{2} \int dk_1 dk_2 dk_3 \hat{\hat{\Phi}}_{k_1}^\dagger \hat{\hat{\Phi}}_{k_2}^\dagger \hat{\hat{\Phi}}_{k_3} \hat{\hat{\Phi}}_{-k_1-k_2-k_3}$$

Cooper instability: RG analysis near g_{c1} and g_{c2}

$$S_{eff} = S_0 + S_\chi + S_\Phi + S_G + S_K + S_J + S_4 + S_{sc}$$

Composite Co $\Delta \sim \langle \chi^\dagger \chi^\dagger \Phi \rangle$

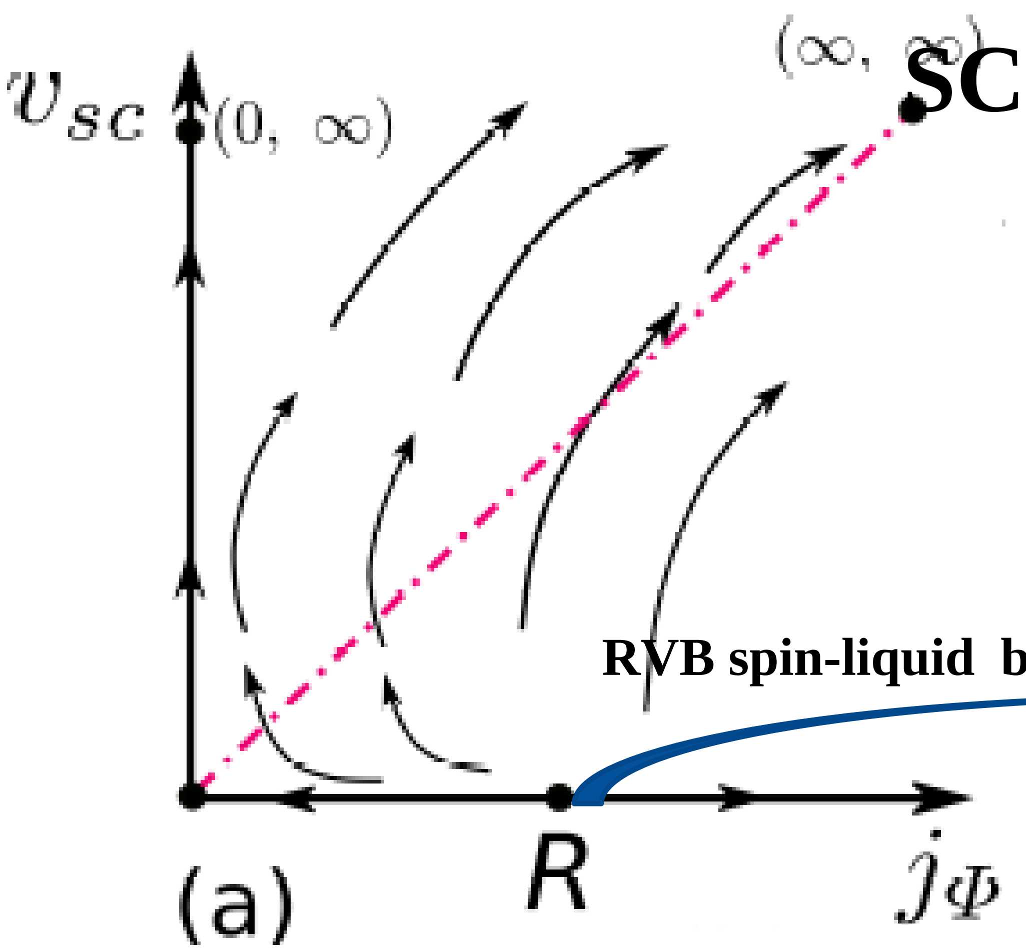
via higher order collaborations btw Kondo and RVB

$$S_{sc} = \begin{cases} v_{sc} \sum_{(\alpha,\beta)=\uparrow,\downarrow} \int dk_1 dk_2 dk_3 \left[\chi^\dagger_{k_1} \chi^\dagger_{k_2} \epsilon^{\alpha\beta} c^\dagger_{k_3,\alpha} c^\dagger_{-k_1-k_2-k_3,\beta} + h.c. \right] & (\text{near } g_{c1}) \,, \\ v_{sc} \sum_{(\alpha,\beta)=\uparrow\downarrow} \int d^d k \, d^d k' \left[\hat{\tilde{\Phi}}_k \epsilon^{\alpha\beta} c^\dagger_{\alpha,k'} c^\dagger_{\beta,k+k'} + h.c. \right] & (\text{near } g_{c2}). \end{cases}$$

Near g_{c2} : phase diagram of CeCoIn5 (Kondo dominated)

$$j'_\Phi = -\frac{d}{2} j_\Phi + 4 j_\Phi^3 + v_{sc}^2 j_\Phi,$$

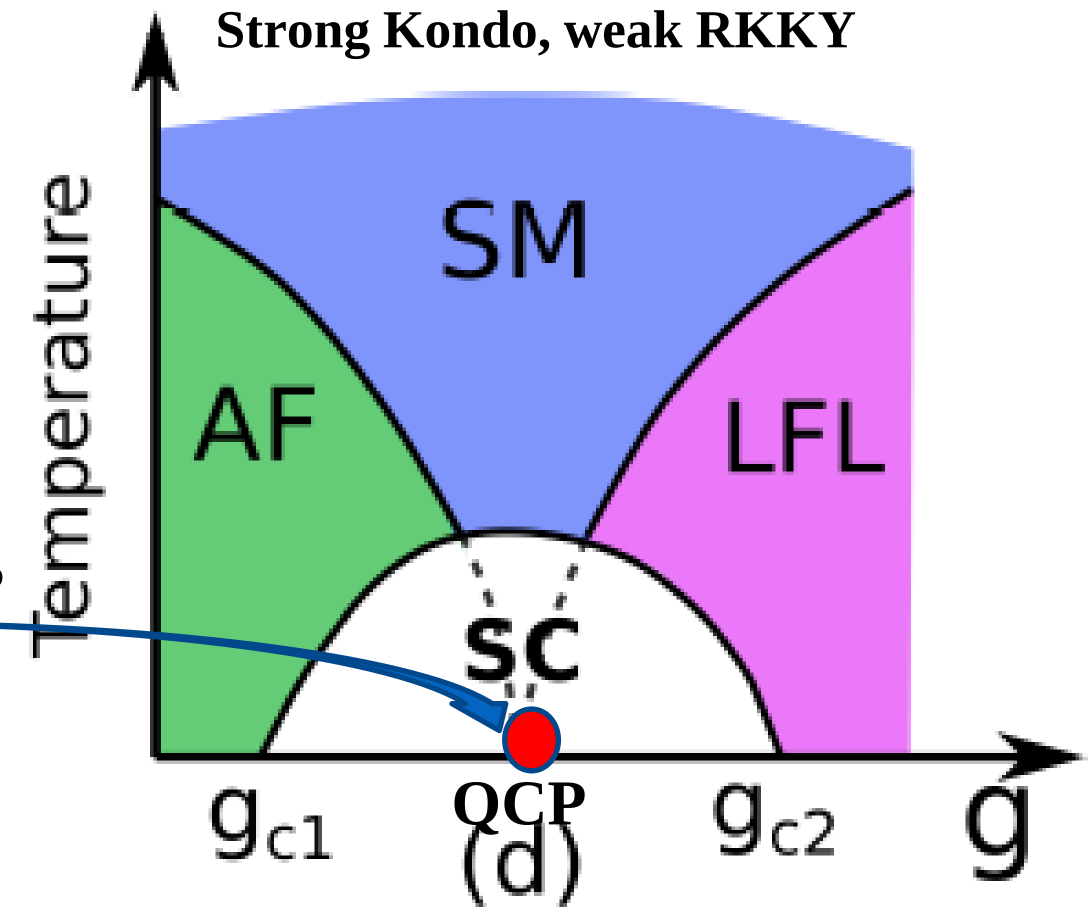
$$v'_{sc} = \left(z - \frac{d}{2}\right) v_{sc} + v_{sc}^3$$

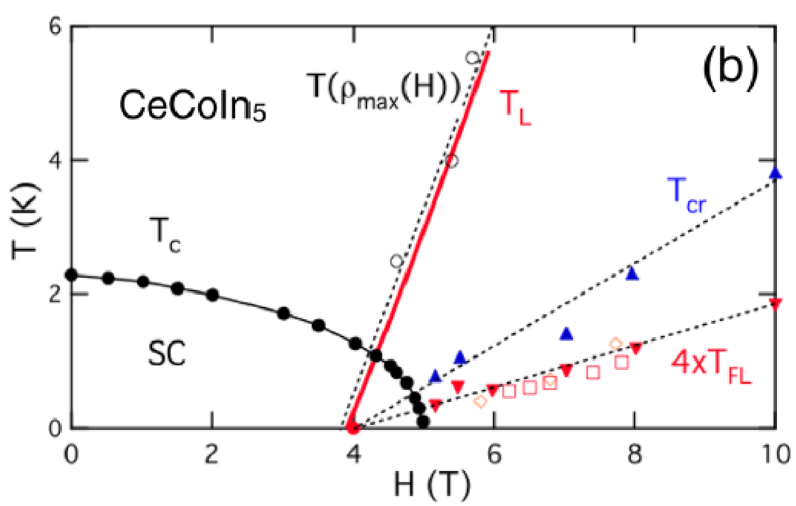


RVB spin-liquid breakdown QCP

CeCoIn5

Strong Kondo, weak RKKY





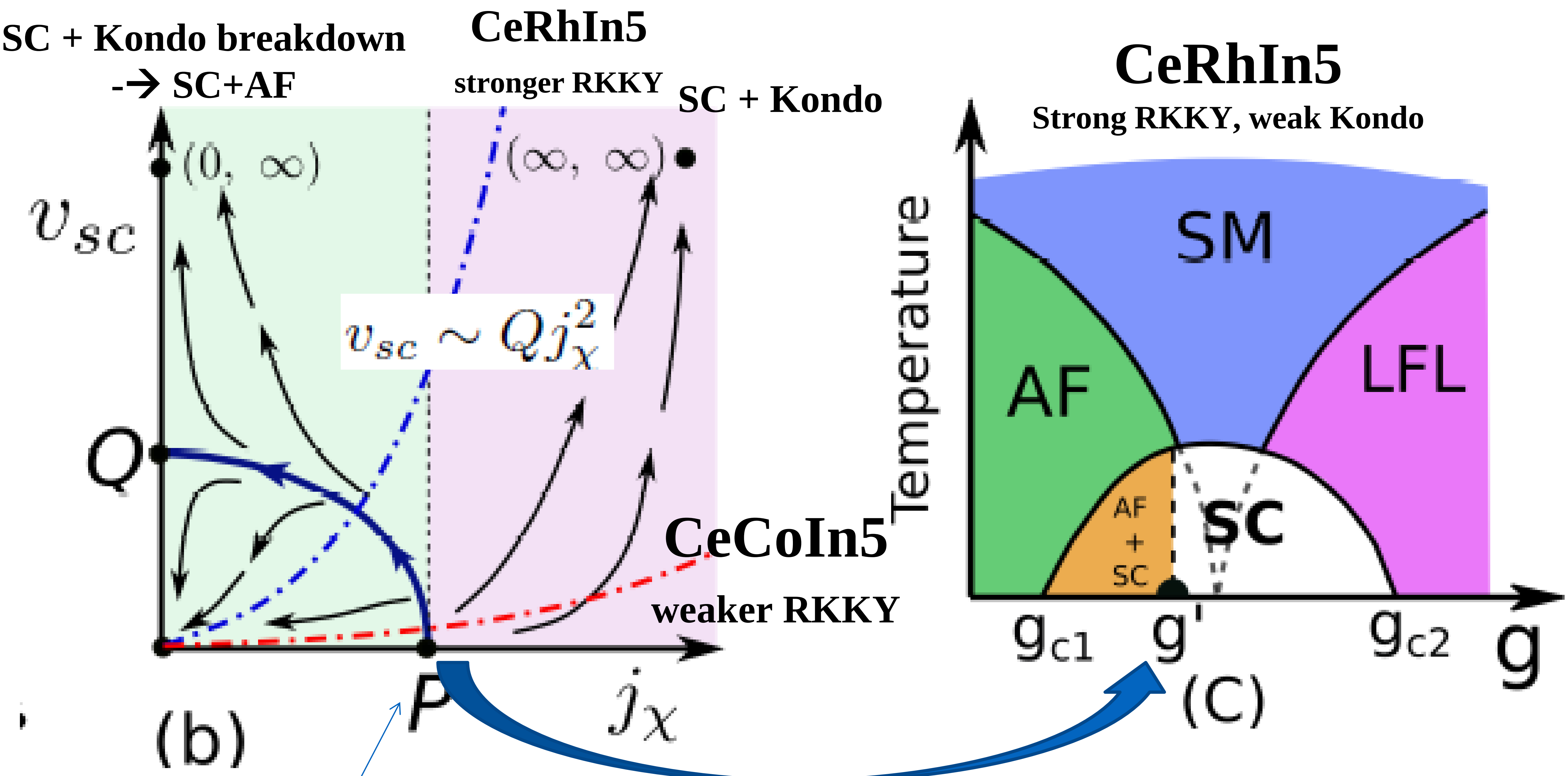
linear crossover (via RVB breakdown under RG)

$$T_{FL^*} \propto |g - g_c|^{\nu z} \sim |g - g_c|$$

Near g_{c1} : phase diagram of CeRhIn5 and CeCoIn5

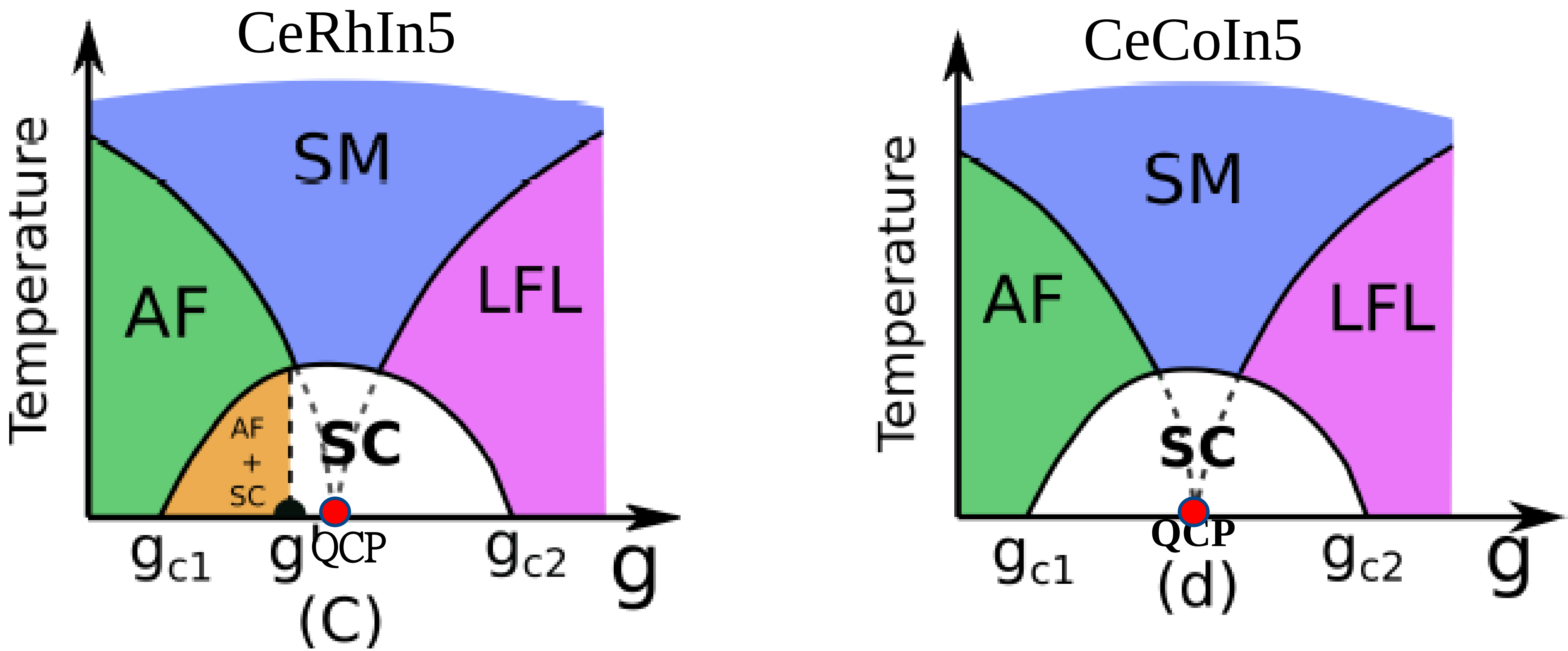
(strong AF RKKY limit)

$$j'_\chi = \left(-\frac{\epsilon}{2}\right) j_\chi + \frac{1}{2} j_\chi^3, \qquad v'_{sc} = -\epsilon v_{sc} + j_\chi^2 v_{sc} + 12 v_{sc}^3,$$



Kondo breakdown QCP (g)

strange superconductivity near heavy-fermion quantum critical point



Outstanding puzzles:

- **How do the f-electrons incorporate in the superconducting state? Kondo?**
- **How does superconductivity emerge from the strange-metallic (SM) normal state?**
- **What are the links among SM, Kondo coherence, superconductivity, and QCP?**

Strange metal state in paramagnetic Kondo lattice: dynamical large-N Fermionic multichannel pseudo fermion approach (arXiv: 2005.03427)

Chung-Hou Chung (仲崇厚)

Department of Electrophysics, NCTU, Hsinchu, Taiwan

Collaborators

Jiangfan Wang (NCTU, Taiwan & IoP, CAS, China)

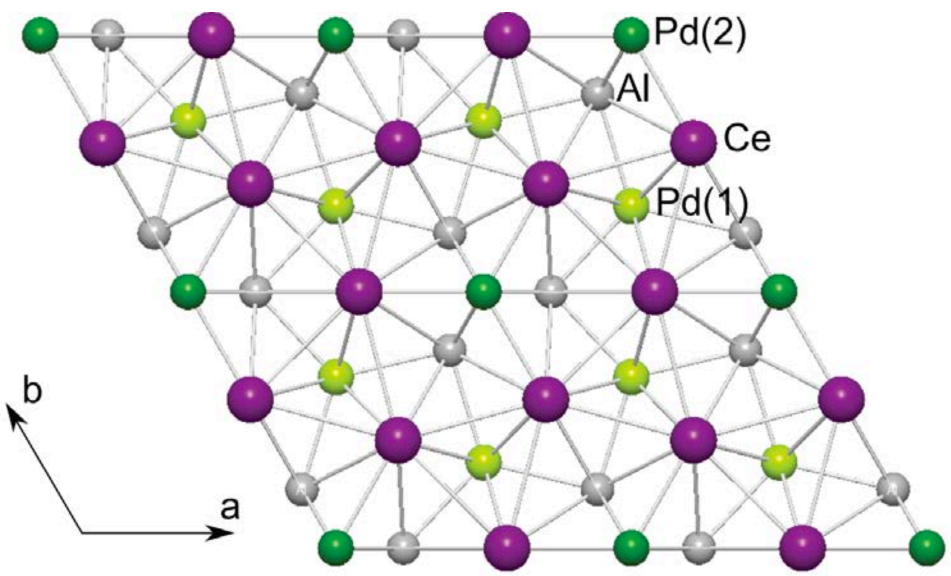
Yung-Yeh Chang (NCTS & NCTU, Taiwan)

Strange metal phase: new!

CePdAl under B, p

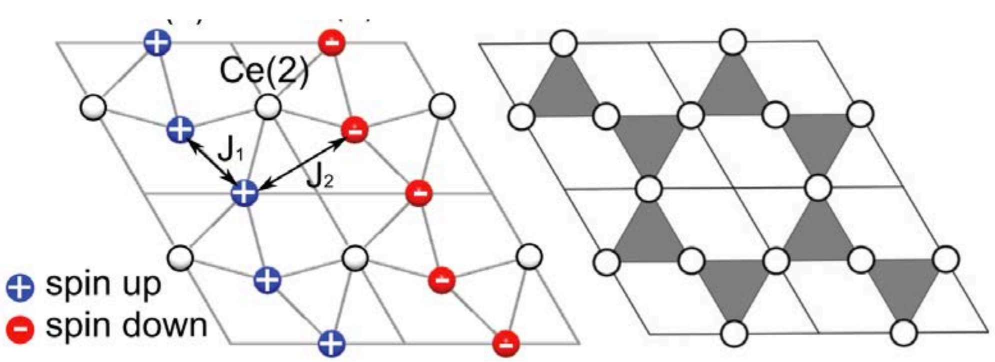
Paramagnetic heavy-fermion metal

Crystal structure: Kagome Kondo lattice



Ce: 5d, 4f

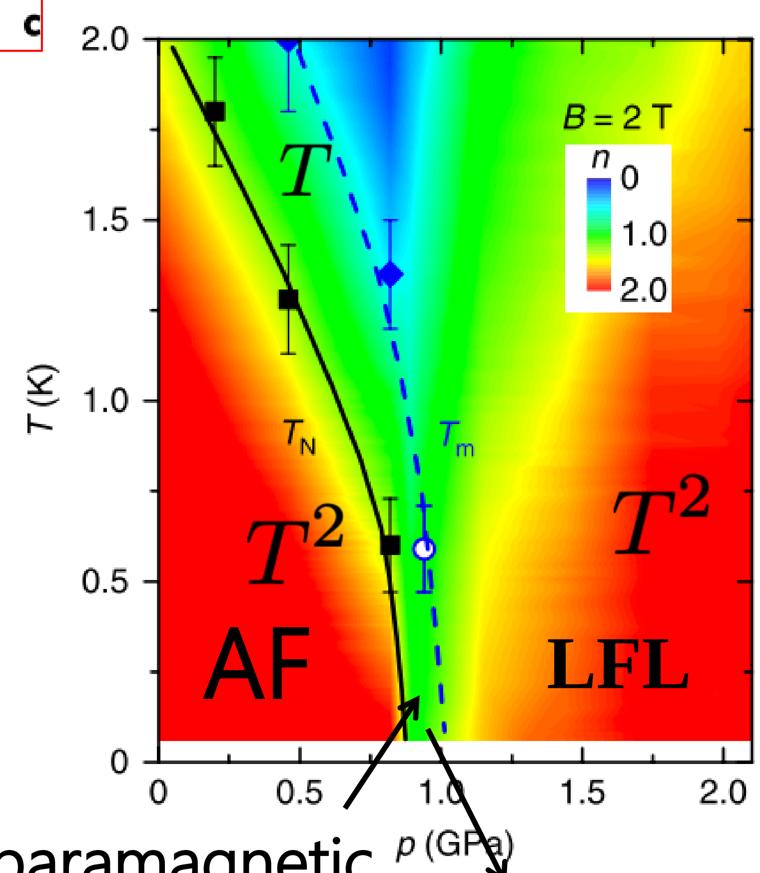
Peijie Sun et al., Nature Phys., 2019

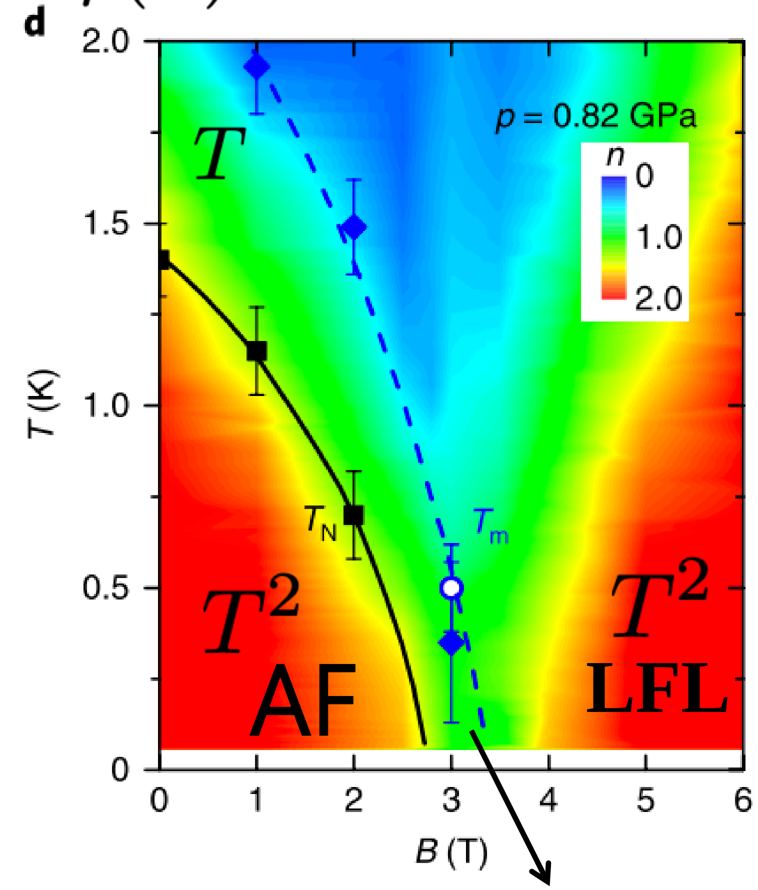


H. v. Lohneysen et al., PRB, 2014

Peijie Sun et al., PRB, 2018

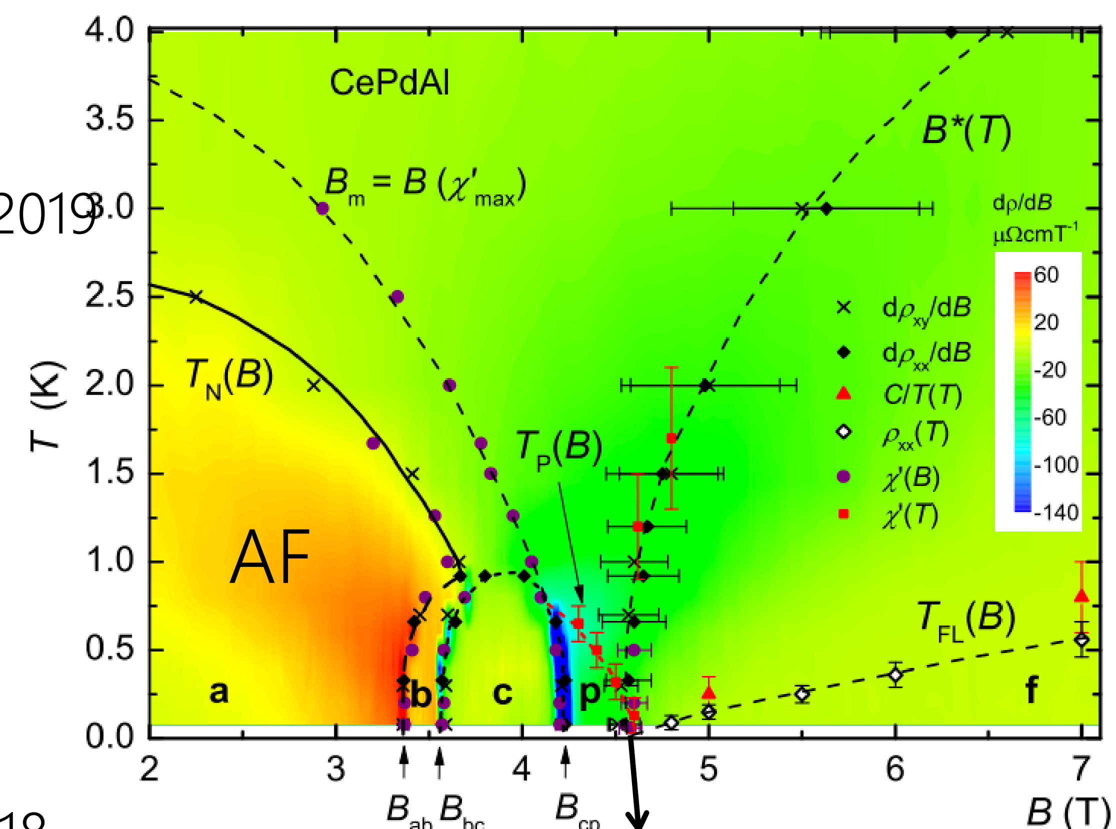
T-quasi-linear Resistivity $\rho(T) \sim T^n$ 1<m<2





paramagnetic spin-liquid

quantum critical strange metal phase



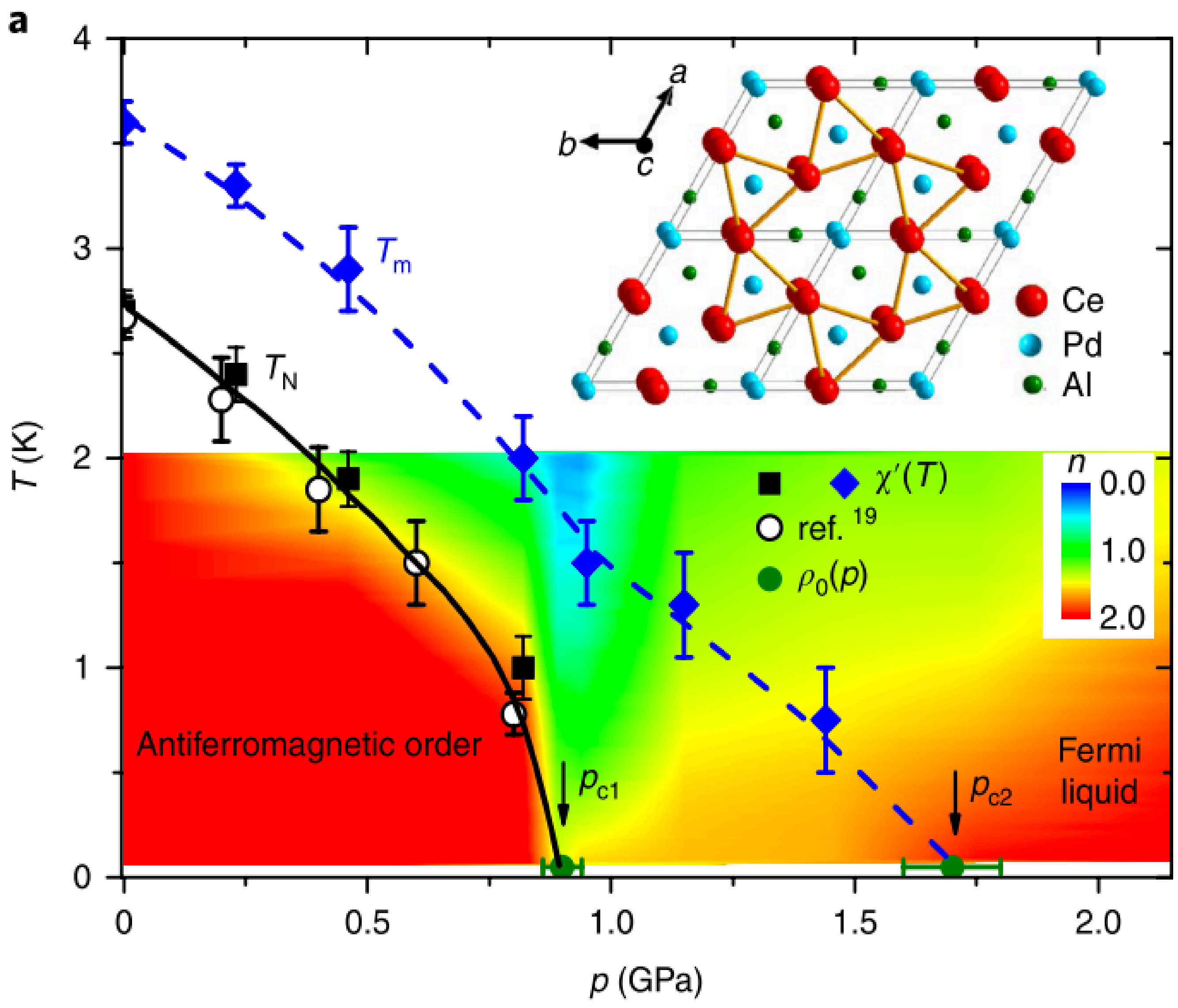
FL

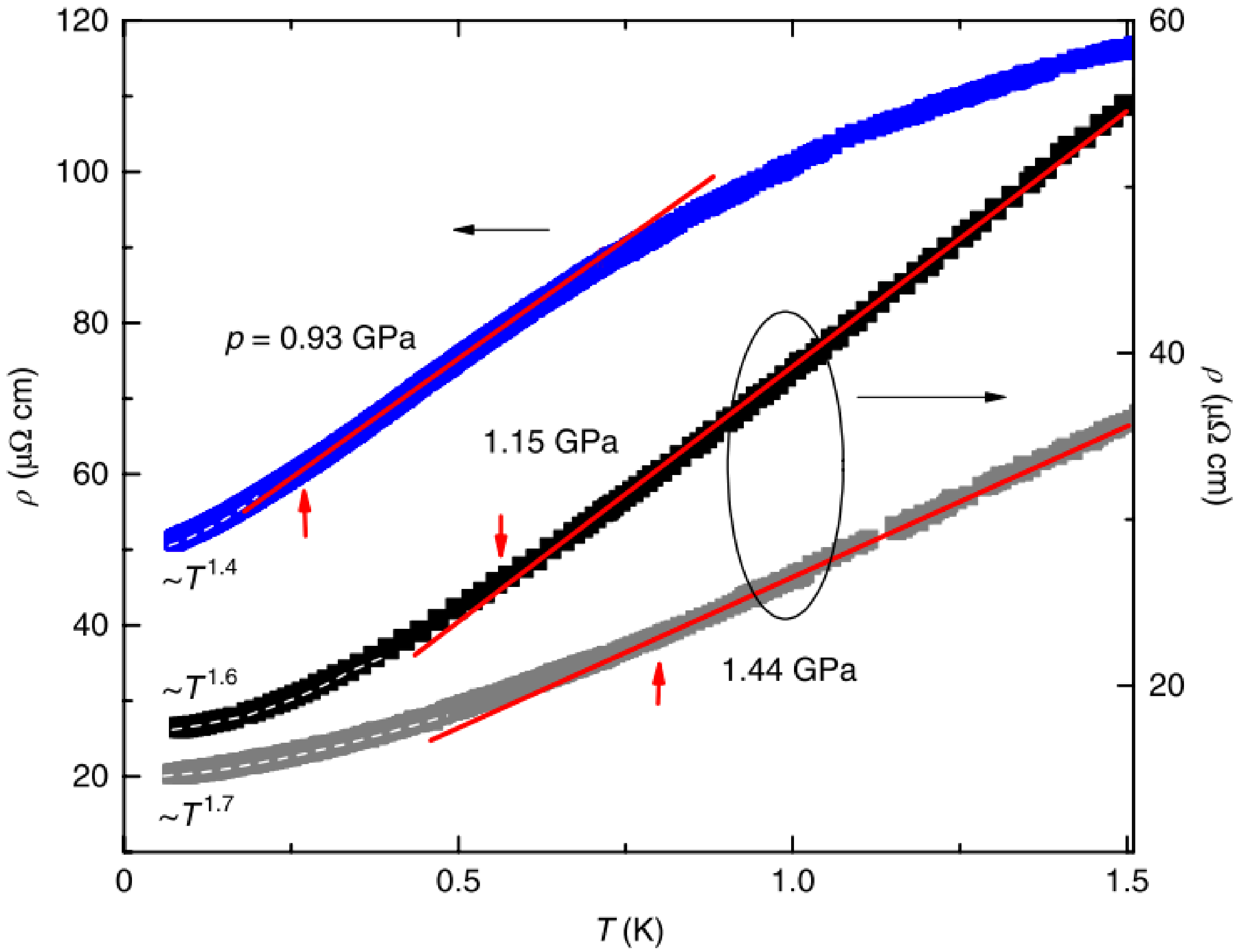
FS reconstruction /Kondo breakdown

Non-Fermi liquid strange metal resistivity

B=0 under pressure

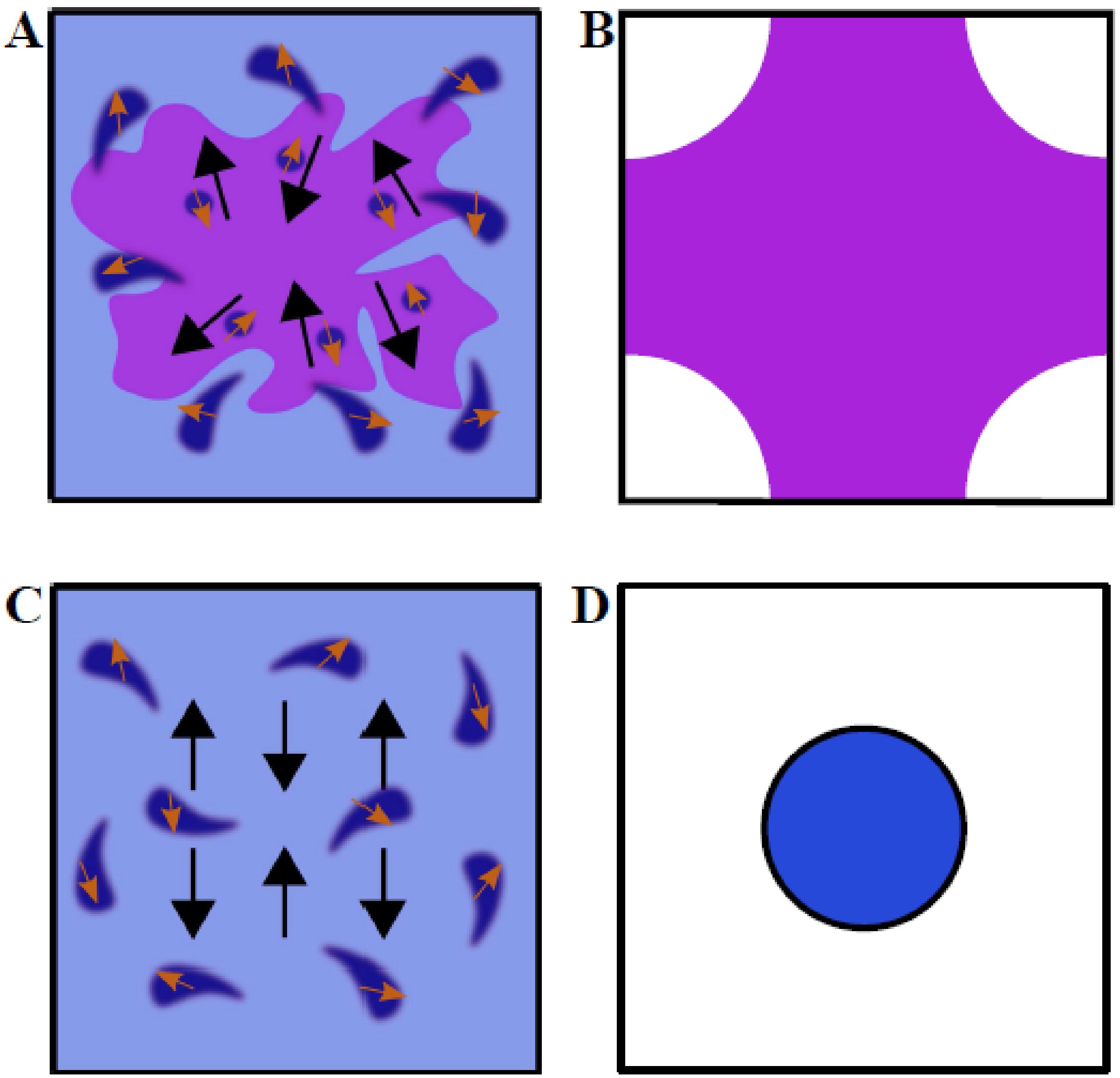
Peijie Sun et al., Nature Phys, 2019



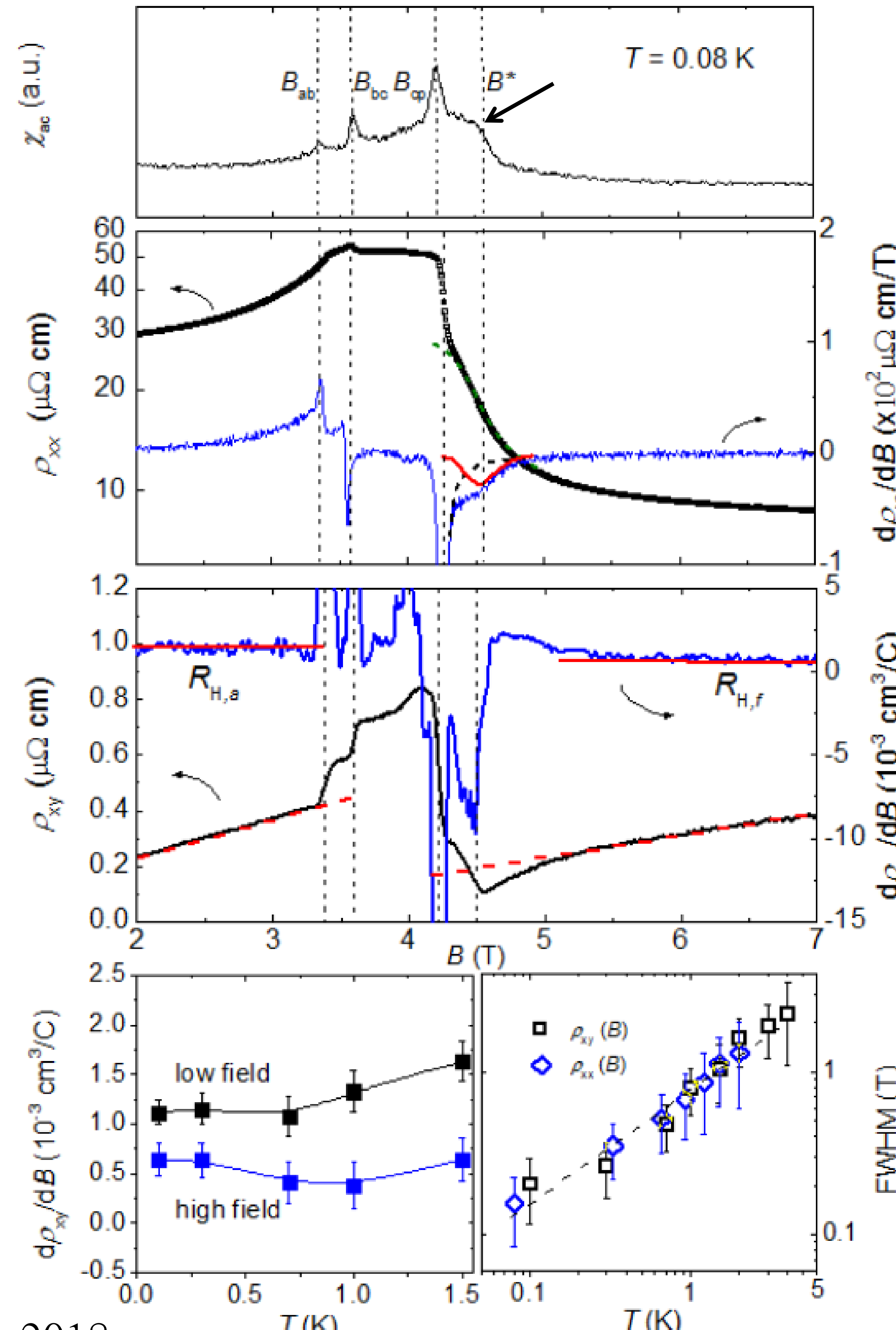


Kondo breakdown and Fermi surface crossover-line B*(T)

Sharp jump in Fermi surface volume at Kondo breakdown QCP



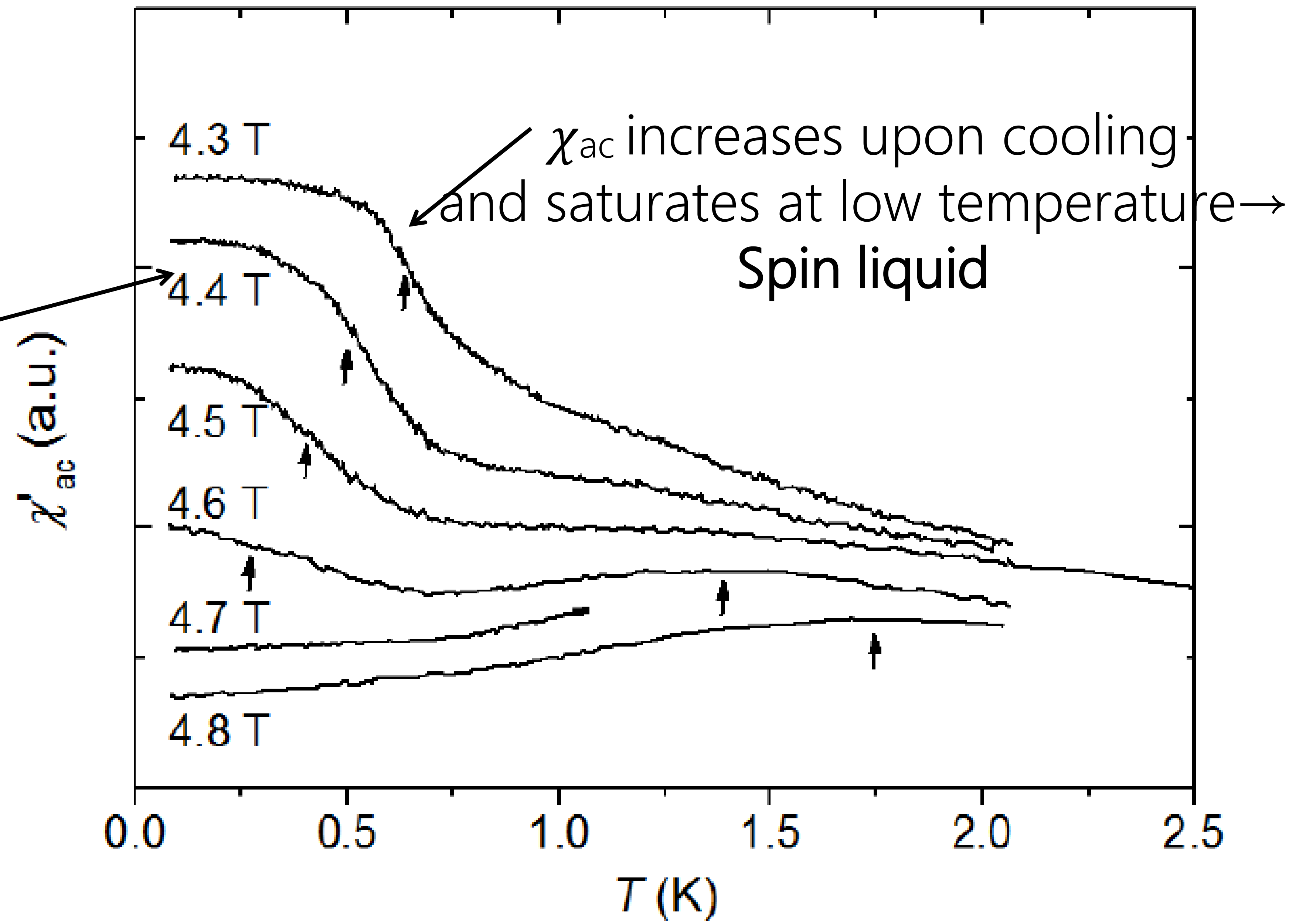
Q. Si et. al. Science, 329, 1161 (2010)



Peijie Sun et al., PRB, 2018

No pressure, paramagnetic fermionic metallic spin-liquid (state "P")

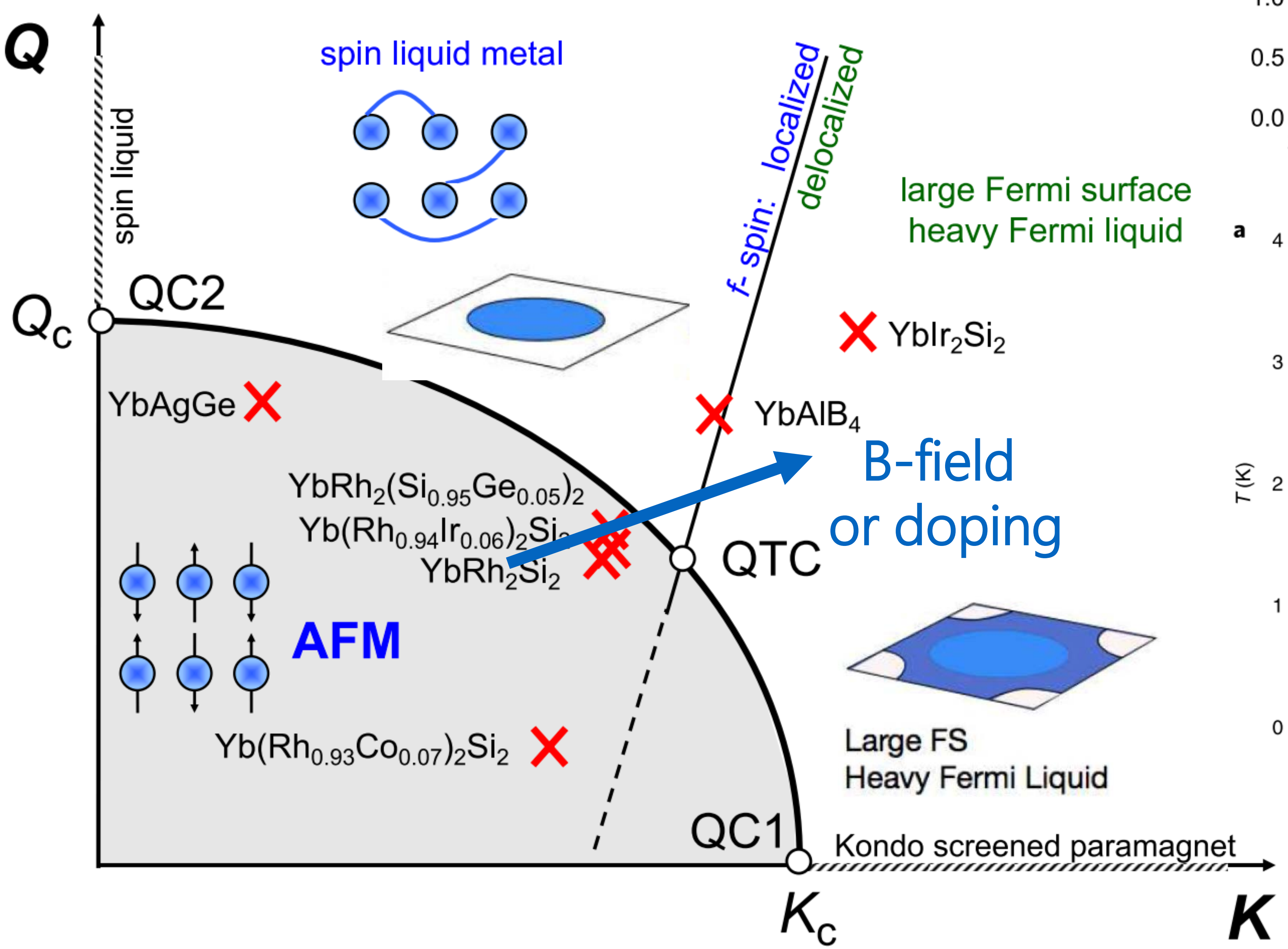
Pauli spin susceptibility T→0
→ Fermionic excitations



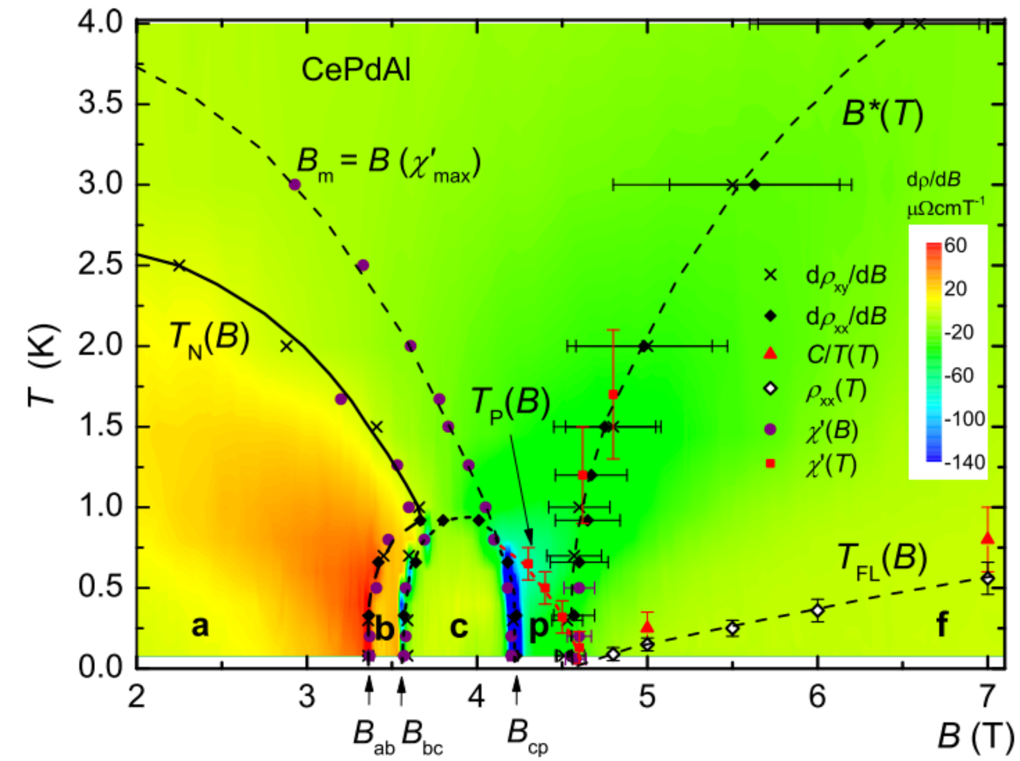
Peijie Sun et al., PRB, 2018

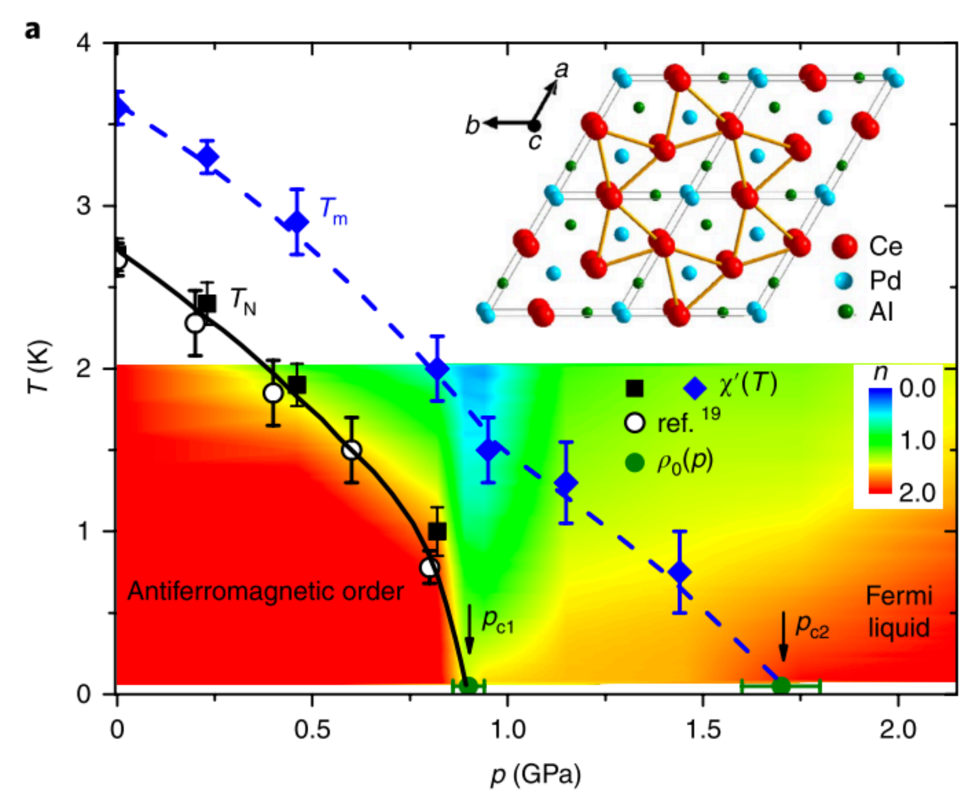
Frustrated Kondo Lattice

J. Custers et al., PRL, 2010



Peijie Sun et al., PRB, 2018

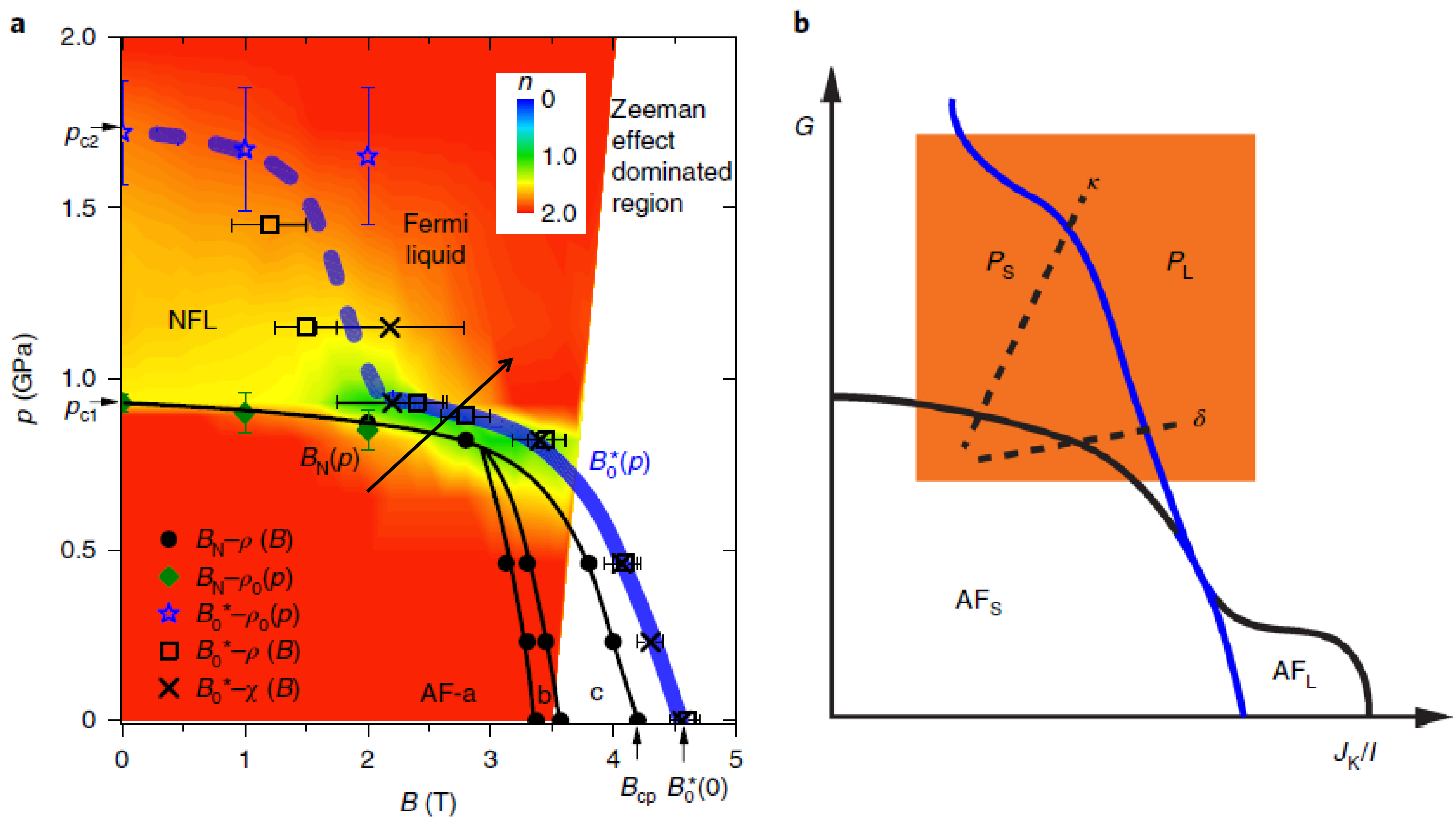




Peijie Sun et al., Nature, Phys. 2019

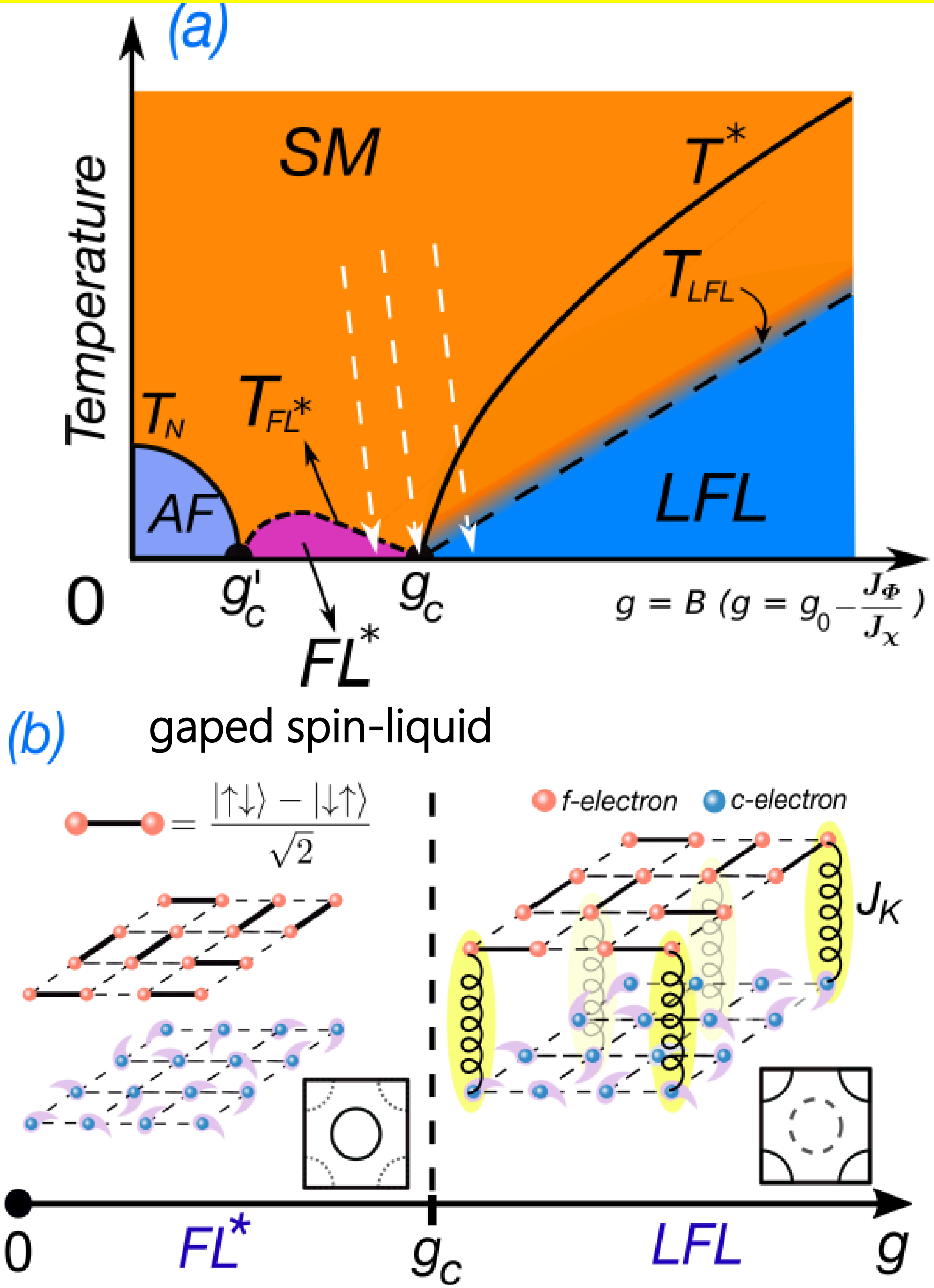
Frustrated Kondo Lattice

Quantum phase transition between a paramagnetic spin-liquid NFL phase and a heavy FL phase



Peijie Sun et al., Nature, Phys. 2019

RG Phase diagram for Ge-YRS



YY Chang et al PRB 2018

Fermionic Multichannel dynamical large-N 2D Kondo-Heisenberg (KH) Lattice model

P. Coleman et al., PRL 2018

$$H = H_0 + H_f + H_J + H_K$$

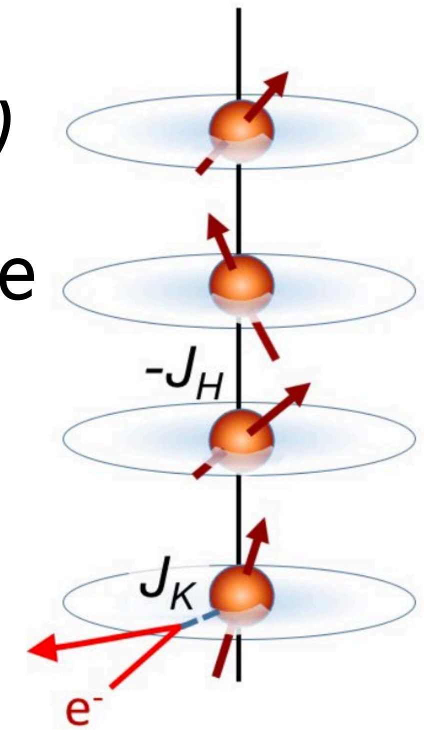
$$H_0 = \sum_{i,\boldsymbol{P},\alpha a} \left[\varepsilon_{\boldsymbol{P}} c^\dagger_{i\alpha a}(\boldsymbol{P}) c_{i\alpha a}(\boldsymbol{P})\right],$$ *(conduction electrons)*

square lattice

$$H_f = \sum_{i,\alpha} \lambda_i \left[f^\dagger_{i\alpha} f_{i\alpha} - 2S\right]$$ *(localized electrons)*

$$H_K = J_K \sum_i \boldsymbol{S}^f_i \cdot \boldsymbol{s}^c_i = -\frac{J_K}{N} \sum_{i,j} \sum_{\alpha\beta,ab} \left(f^\dagger_{i\beta} c_{i\beta a}\right)\left(c^\dagger_{j\alpha b} f_{j\alpha}\right)$$

$$H_J = J_H \sum_{ij} \boldsymbol{S}^f_i \cdot \boldsymbol{S}^f_j = -\frac{J_H}{N} \sum_{ij} \sum_{\alpha\beta} \left[\mathrm{sgn}(\alpha) f^\dagger_{i\alpha} f^\dagger_{j,-\alpha}\right] \left[\mathrm{sgn}(\beta) f_{i,-\beta} f_{j\beta}\right]$$



$$\begin{cases} i = 1, 2, \cdots, \mathcal{N}_s & \text{Sites} \\ \alpha = \pm 1, \cdots, \pm N/2 & \text{Spin} \\ a = 1, 2, \cdots, K & \text{Channel} \end{cases} \qquad \begin{cases} K > 2S & \text{Overscreened} \\ K = 2S & \text{Fully screened} \\ K < 2S & \text{Underscreened} \end{cases}$$

$$c_{i\alpha} \rightarrow c_{i\alpha a}$$

$$\kappa \equiv \frac{K}{N} : \text{fixed}$$

Hubbard-Stratonovich transformation & order parameters

$$H_K \to \frac{1}{\sqrt{2}} \sum_{ia\alpha} \chi_{ia} f^\dagger_{i\alpha} c_{ia\alpha} + h.c. + \frac{|\chi_{ia}|^2}{J_K}$$

$$H_J \to \frac{1}{\sqrt{2}} \sum_{ij\alpha} \Delta_{ij} \tilde{\alpha} f_{i\alpha} f_{j,-\alpha} + h.c. + \frac{|\Delta_{ij}|^2}{J_H}$$

χ, Δ

Bosonic fields

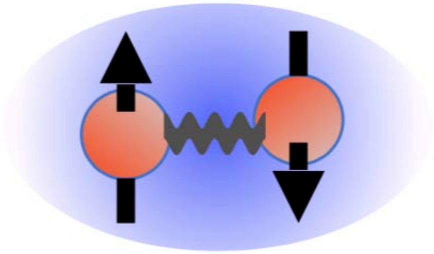
Sp(N) sym.

Mean-field order parameters

RVB: resonating valence bond

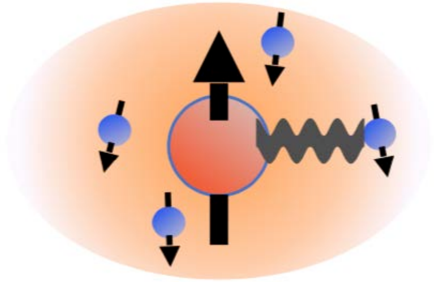
Fermionic RVB spin-singlet bond

$$\Delta_{ij} = -\frac{J_H}{N} \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$$



Bosonic Kondo correlation

$$\chi_{ia} = -\frac{J_K}{\sqrt{N}} \langle c^\dagger_{i\alpha a} f_{i\alpha} \rangle$$



Dynamical large-N self consistent NCA equations

local bath approximation ~DMFT

$$G_\chi(\nu) = \left[-\frac{1}{J_K} - \Sigma_\chi(\nu)\right]^{-1},$$

$$G_f(\omega) = \frac{2}{\pi\gamma(\omega)} E_K\left[-\frac{16\Delta^2}{\gamma(\omega)\gamma(-\omega)}\right]$$

$$\Sigma_\chi(\nu) = \sum_\omega G_f(\omega+\nu) G_{c0}(\omega),$$

$$\Sigma_f(\omega) = -\kappa \sum_\nu G_\chi(\nu) G_{c0}(\omega-\nu)$$

$$\gamma(\omega) \equiv i\omega + \lambda - |x|^2 G_{c0}(\omega) - \Sigma_f(\omega)$$

$$E_K(z) \equiv \int_{y=0}^{\pi/2} (1 - z\sin^2 y)^{-1/2}$$

Large-N limit:

$$\Sigma_c(\omega) = -\frac{1}{N}\sum_\nu G_\chi(\nu) G_f(\omega+\nu)$$

$$\propto \mathcal{O}\left(\frac{1}{N}\right) \to 0 \text{ as } N \to \infty$$

Saddle-point eqs.

$$\frac{\partial F}{\partial \Delta} = \frac{\partial F}{\partial x} = \frac{\partial F}{\partial \lambda} = 0$$

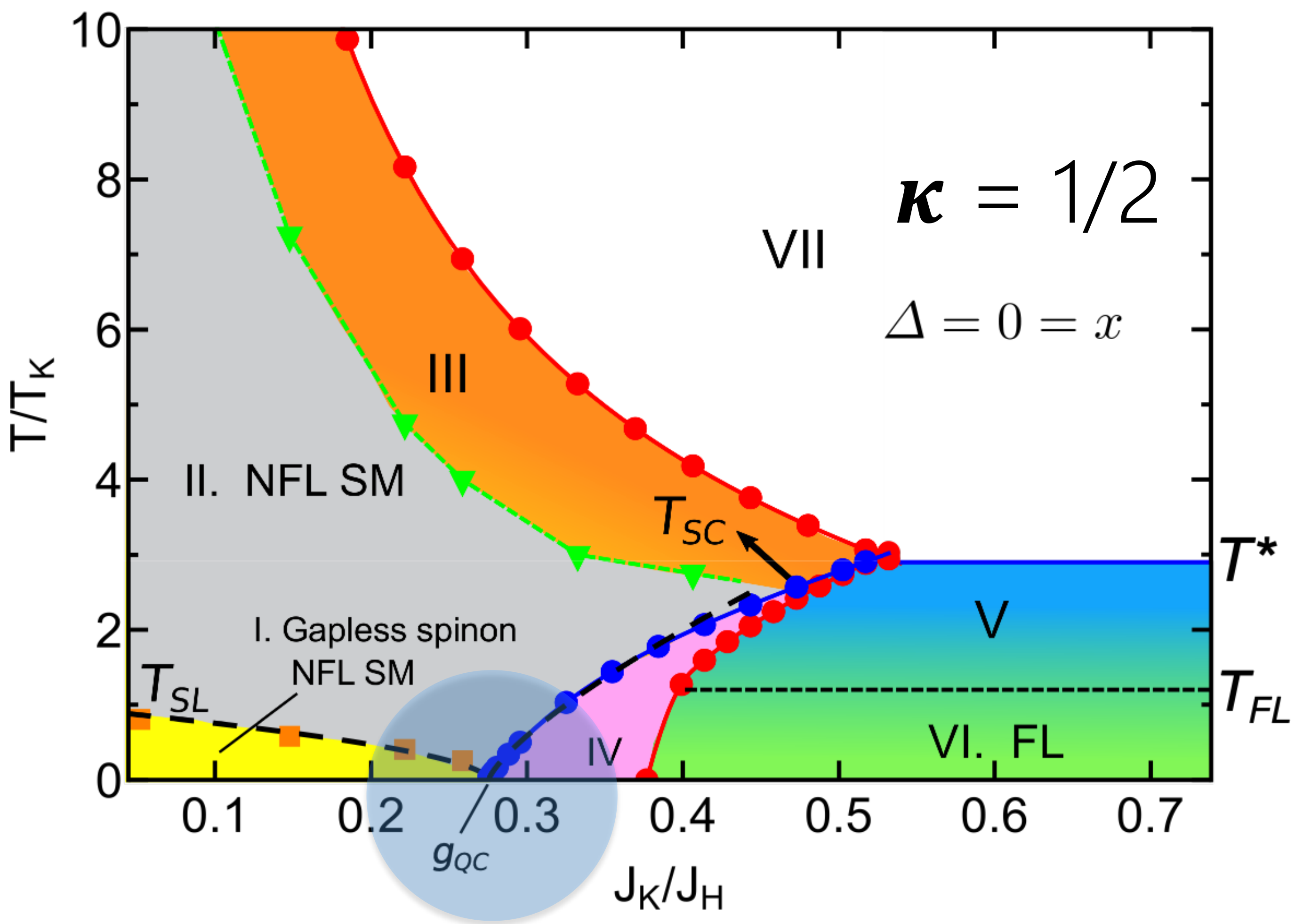
F: free energy

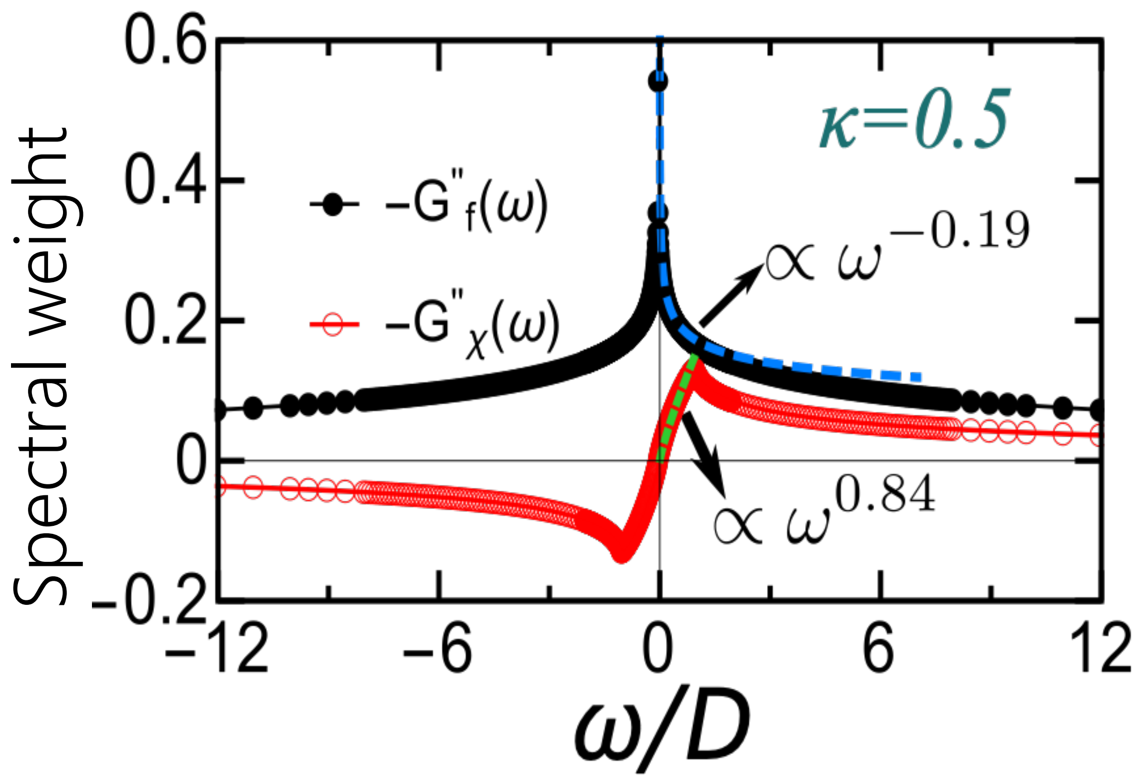
$$\kappa = -\frac{1}{\pi}\int_\omega n_F(\omega) G''_f(\omega),$$

$$\frac{1}{J_H} = \frac{1}{2\pi^2\Delta^2}\int_\omega n_F(\omega) E''_K\left[\frac{-16\Delta^2}{\gamma(\omega)\gamma(-\omega)}\right],$$

$$\frac{1}{J_K} = \frac{2}{\pi}\int_\omega n_F(\omega) \mathrm{Im}\left[G_f(\omega) G_{c0}(\omega)\right]$$

Quantum phase transition & critical spin liquid strange metal phase for **κ = ½** (S=1/2 in Sp(2)=SU(2) limit)



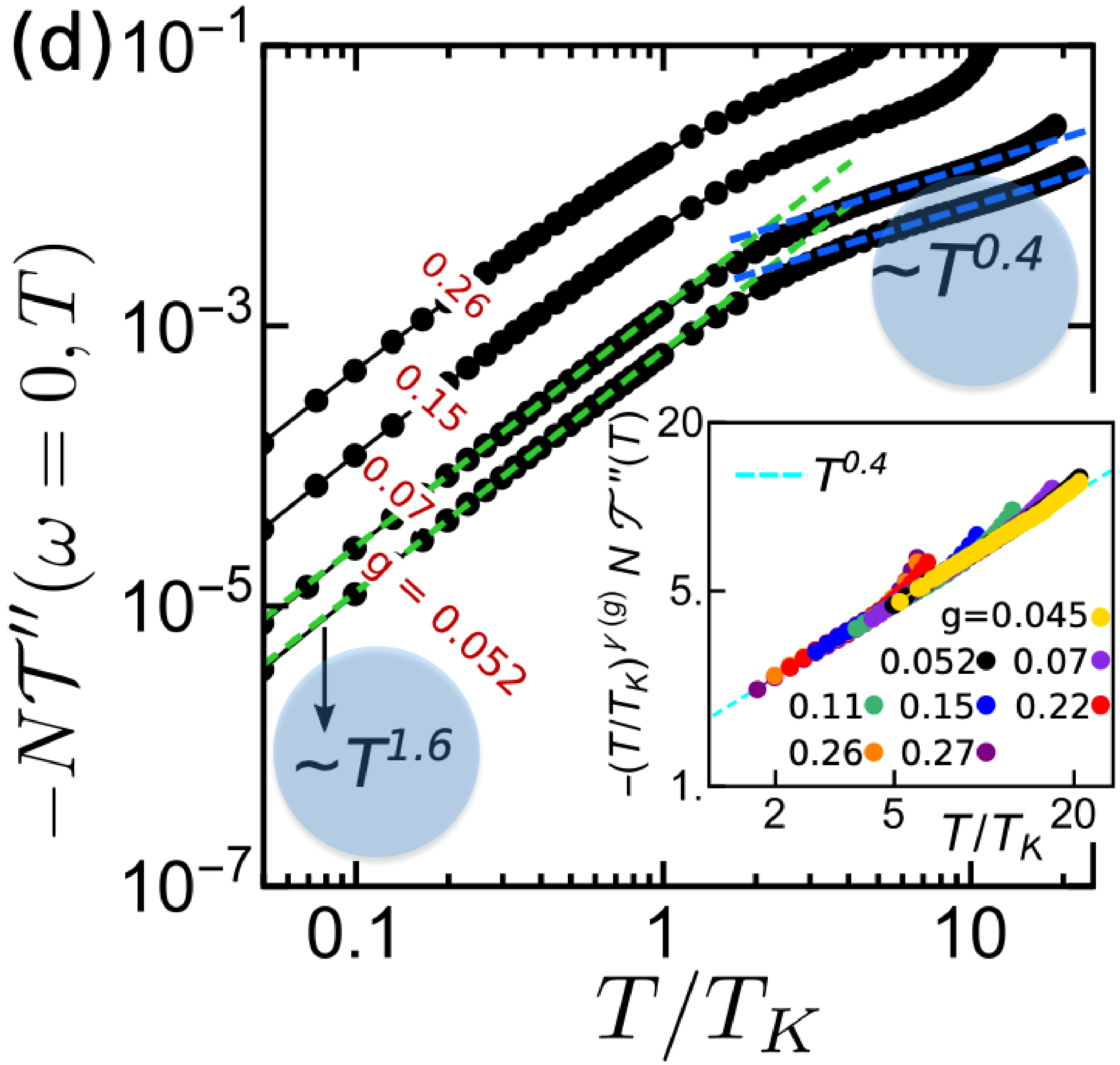


- Particle-hole symmetry for **κ** = 1/2.
- **Region I.: Quantum critical strange metal phase:**
  Spinons and holons show critical (gapless) power-law spectral functions.
- *g_{QC}* becomes a QCP (continuous transition):
- Region II. NFL SM becomes truly quantum critical region

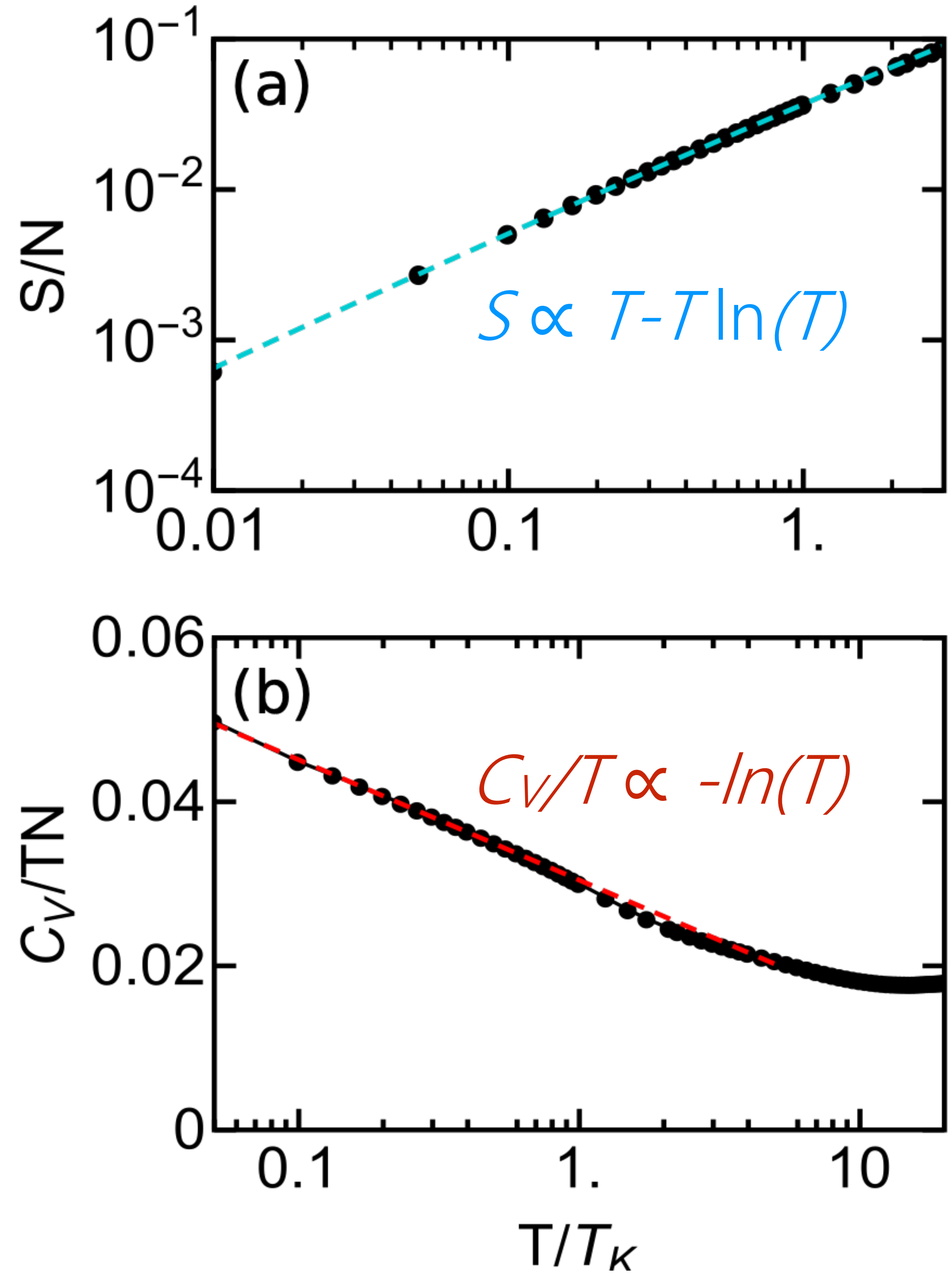
NFL properties of critical spin liquid

The critical spinon and holon give rise to NFL behavior in various observables

Power-law *T*-matrix & scaling

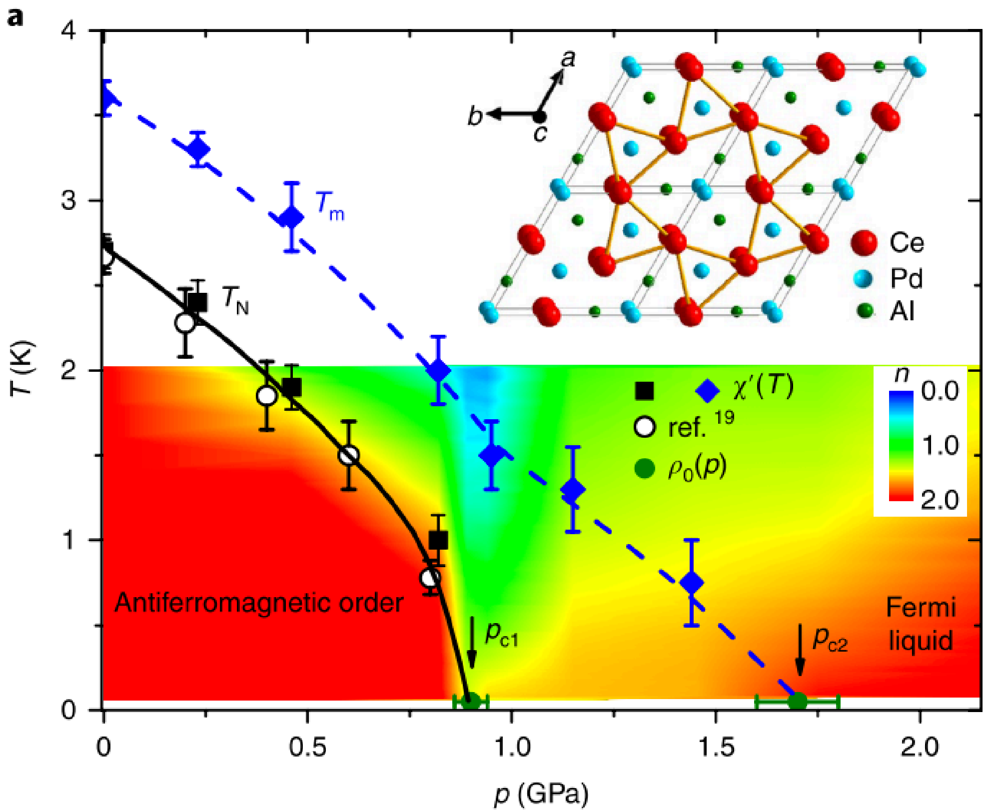


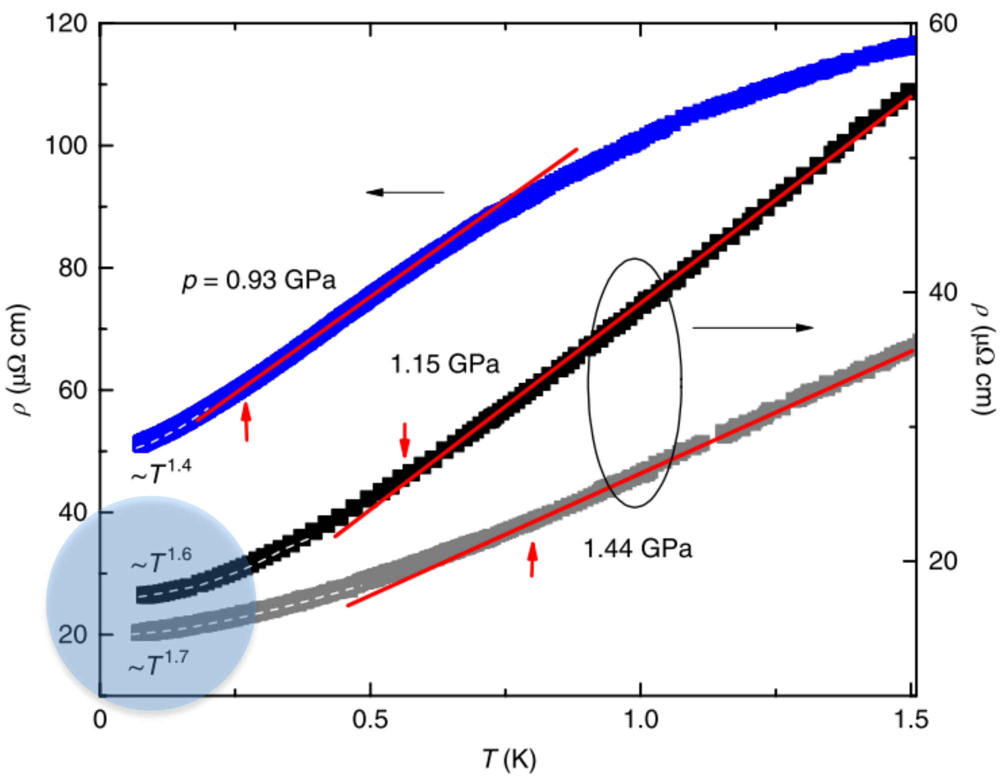
Entropy & specific heat coefficient

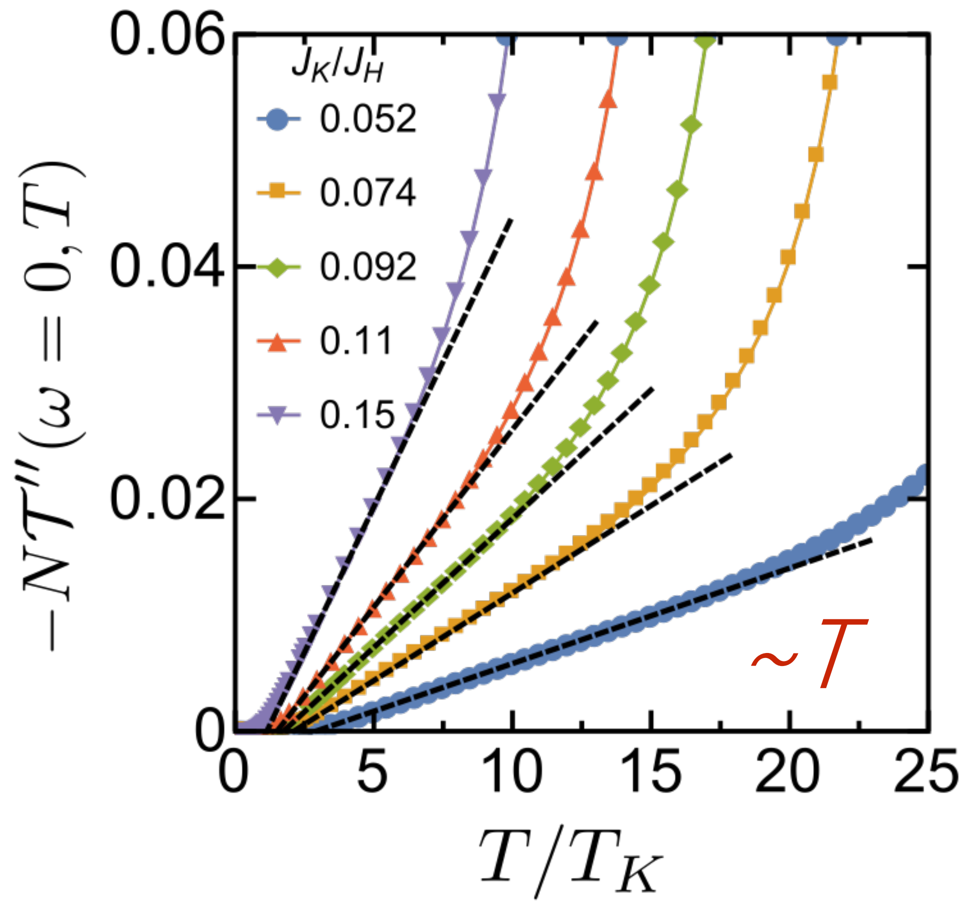


NFL SM resistivity

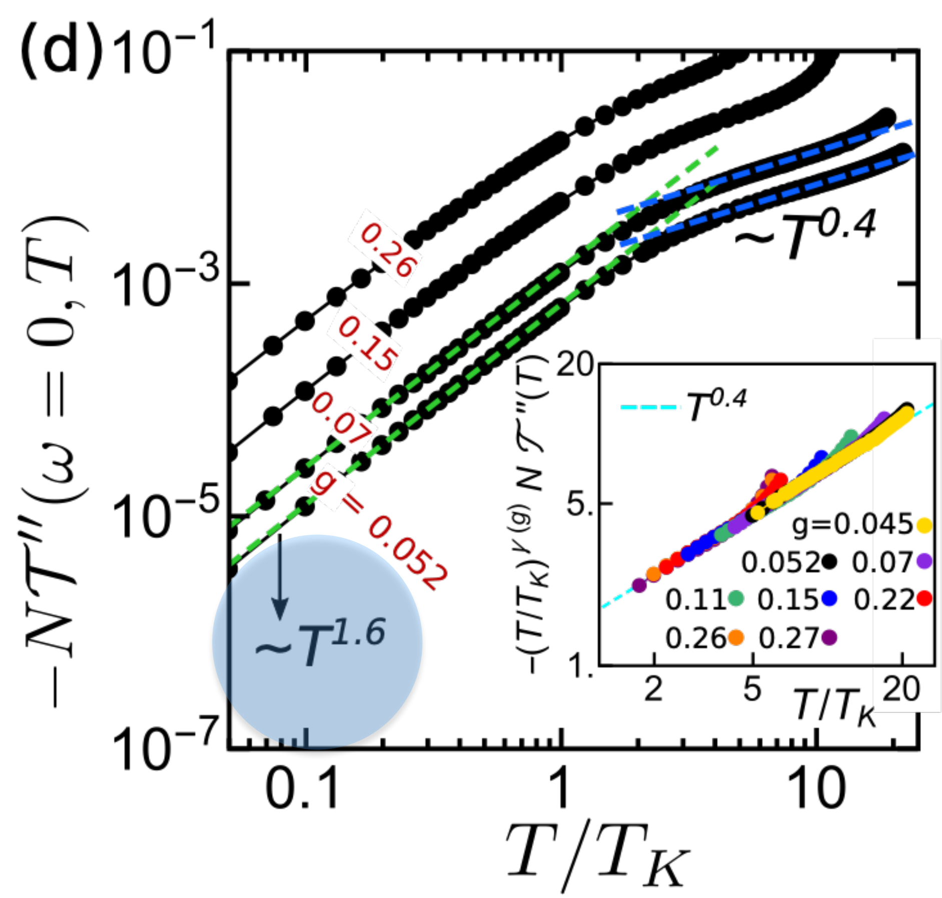
Peijie Sun et al., Nature, 2019







κ = 0.3



κ = 1/2

Summary

- Strange metal state are generic non-Fermi liquid properties in correlated electron systems near quantum phase transitions

- Kondo in competition with RVB spin-liquid provides an excellent description on the mechanism of strange metal behaviors observed in quasi-2D heavy-fermion metals and superconductors

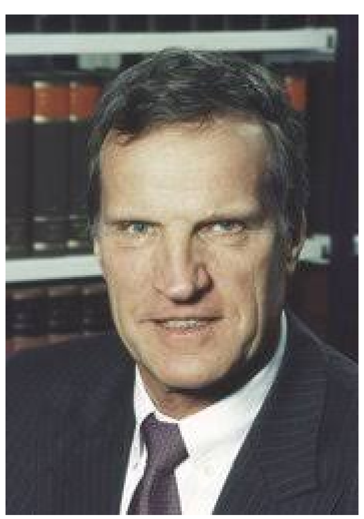
- Critical Kondo (bosonic charge) fluctuations lead to T-linear resistivity

- Critical bosonic RVB spin-liquid fluctuations (made of fermionic spinons) lead to T-logarithmic singularity in specific heat coefficient

Acknowledgement

Experimentalists

Joe Thompson, LANL



Frank Steglich, Max-Planck, Dresden

Theorists

Qimiao Si
Rice U.

Piers Coleman,
Rutgers U.

Stefan Kirchner
Zhejiang U.

Matthias Vojta,
TU Dresden

Phenomenological Theory by Landau

energy functional:

$$E = E_0 + \sum_{\vec{k},\sigma} \epsilon_\sigma(\vec{k})\delta n_\sigma(\vec{k}) + \frac{1}{2\Omega}\sum_{\vec{k},\vec{k}'}\sum_{\sigma,\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\delta n_\sigma(\vec{k})\delta n_{\sigma'}(\vec{k}')$$

deviation from ground state

$$\delta n_\sigma(\vec{k}) = n_\sigma(\vec{k}) - n_\sigma^{(0)}(\vec{k})$$

spin index $\sigma = \pm 1$

ground state distribution

$$n_\sigma^{(0)}(\vec{k}) = \Theta(k_F - |\vec{k}|)$$

filled Fermi sea

effective quasiparticle spectrum:

$$\tilde{\epsilon}_\sigma(\vec{k}) = \frac{\delta E}{\delta n_\sigma(\vec{k})} = \epsilon_\sigma(\vec{k}) + \frac{1}{\Omega}\sum_{\vec{k}',\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\delta n_{\sigma'}(\vec{k}')$$

bare quasiparticle spectrum:

$$\epsilon_\sigma(\vec{k}) = \frac{\hbar^2 \vec{k}^2}{2m^*}$$

effective mass

Fermi velocity:

$$\frac{1}{\hbar}\vec{\nabla}_{\vec{k}}\epsilon_\sigma(\vec{k})\Big|_{k_F} = \vec{v}_F = \frac{\hbar\vec{k}_F}{m^*}$$

density of states at ϵ_F:

$$N(\epsilon_F) = \frac{1}{\Omega}\sum_{\vec{k},\sigma}\delta(\epsilon_\sigma(\vec{k}) - \epsilon_F) = \frac{k_F^2}{\pi^2\hbar v_F} = \frac{m^* k_F}{\pi^2\hbar^2}$$

Change of carrier density at the pseudogap critical point of a cuprate superconductor

S. Badoux[1], W. Tabis[2,3], F. Laliberté[2], G. Grissonnanche[1], B. Vignolle[2], D. Vignolles[2], J. Béard[2], D. A. Bonn[4,5], W. N. Hardy[4,5], R. Liang[4,5], N. Doiron-Leyraud[1], Louis Taillefer[1,5] & Cyril Proust[2,5]

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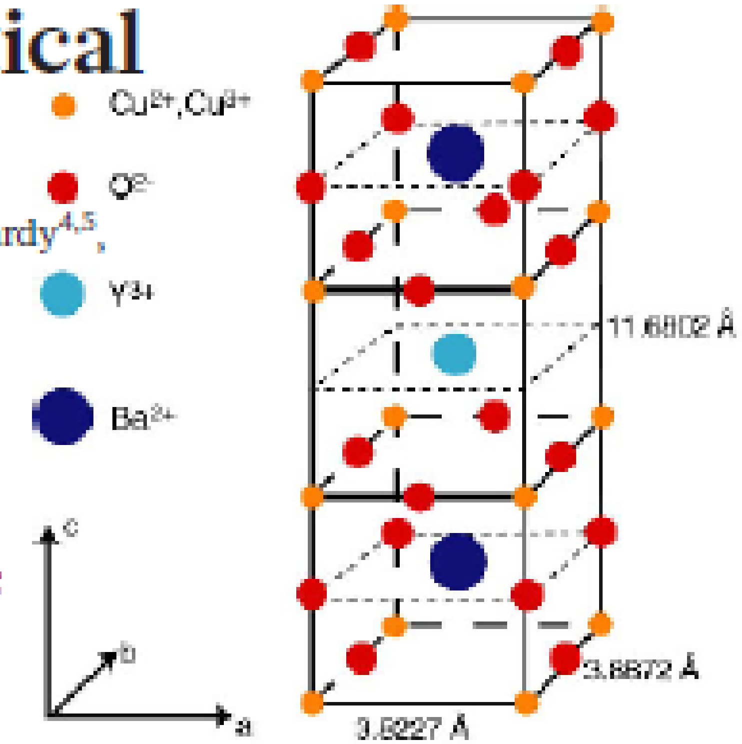
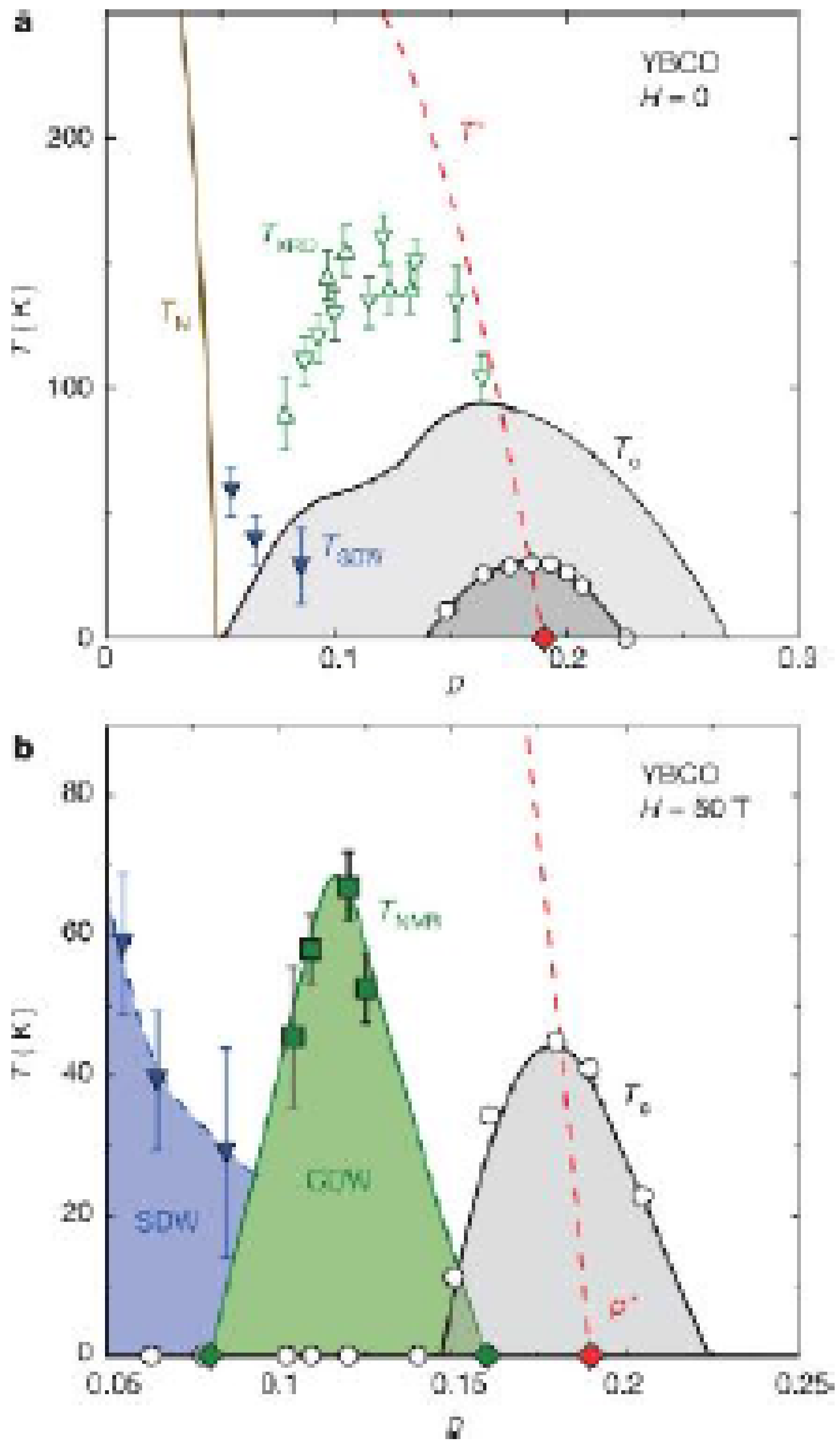
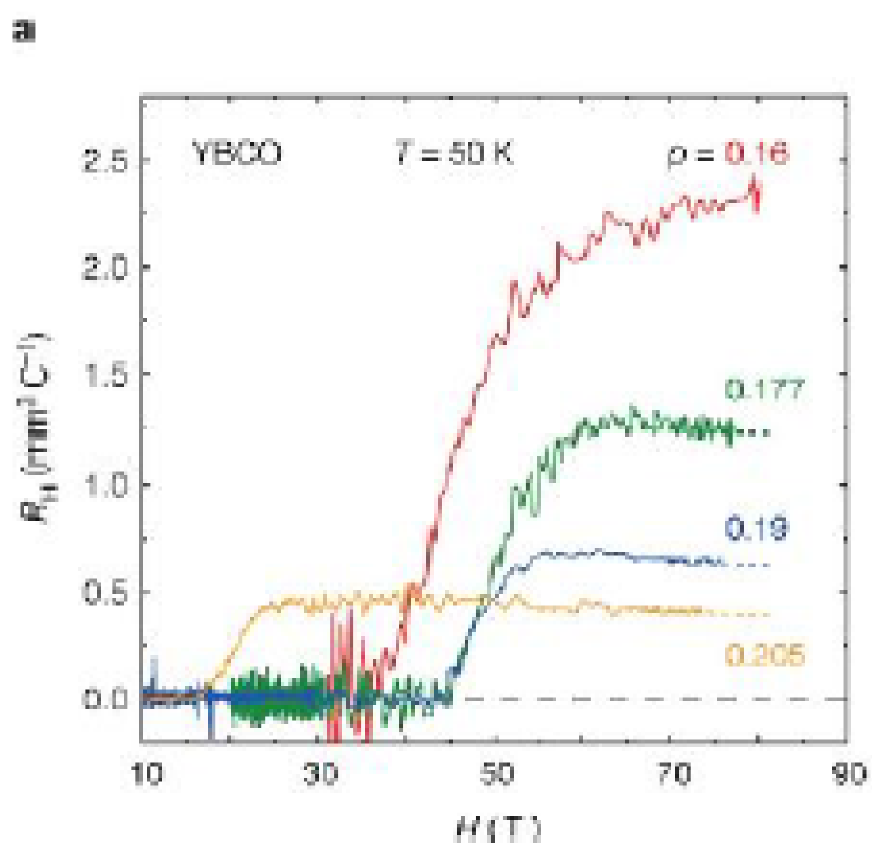
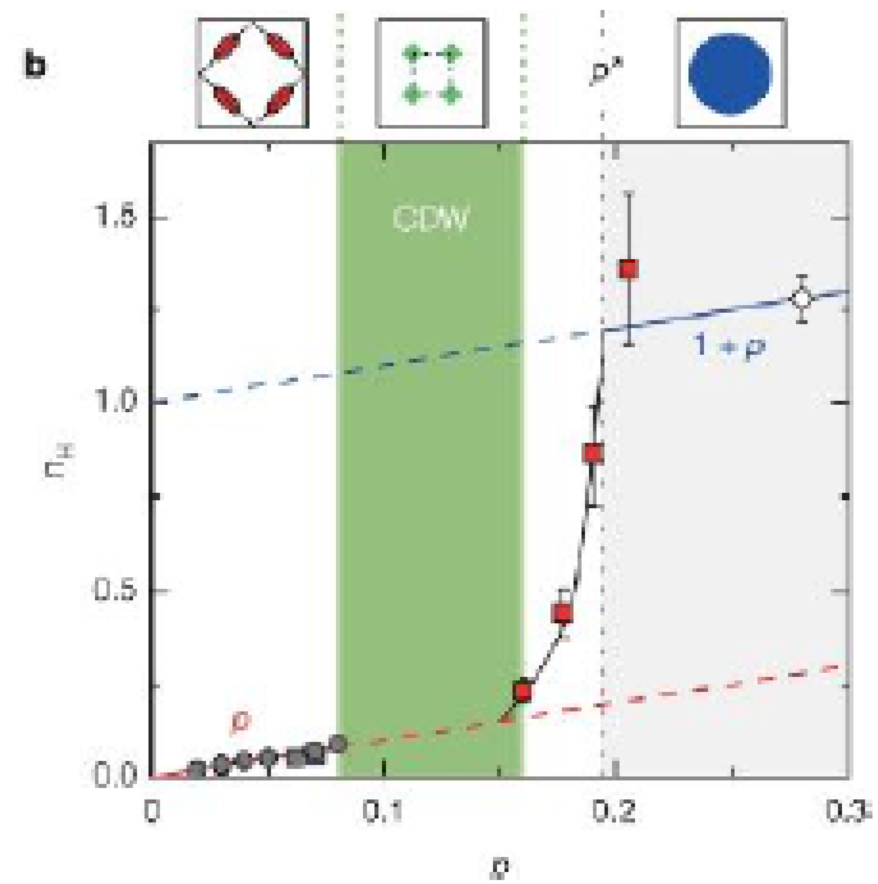


$YBa_2Cu_3O_{6+x}$



S. Sachdev's Onsarger Prize Talk APS March Meeting 2018

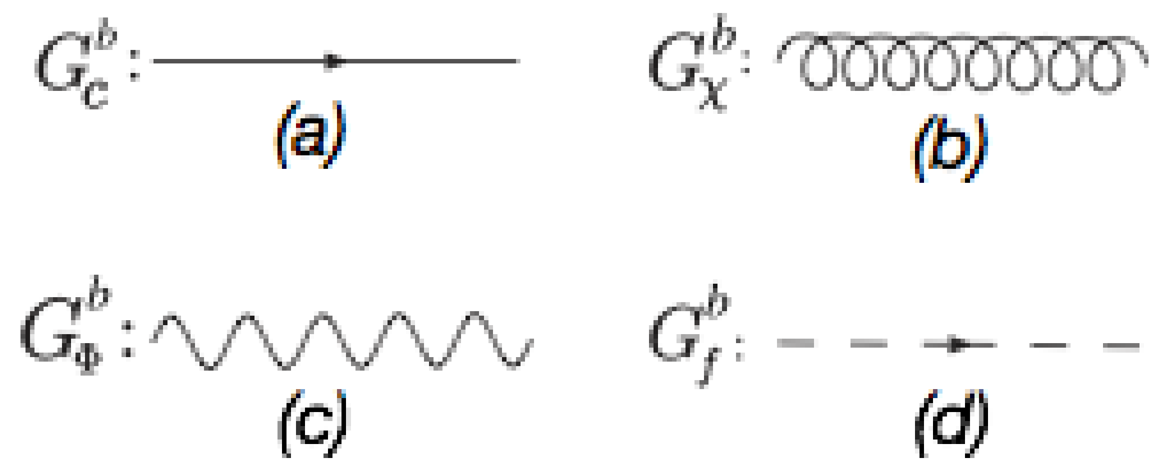




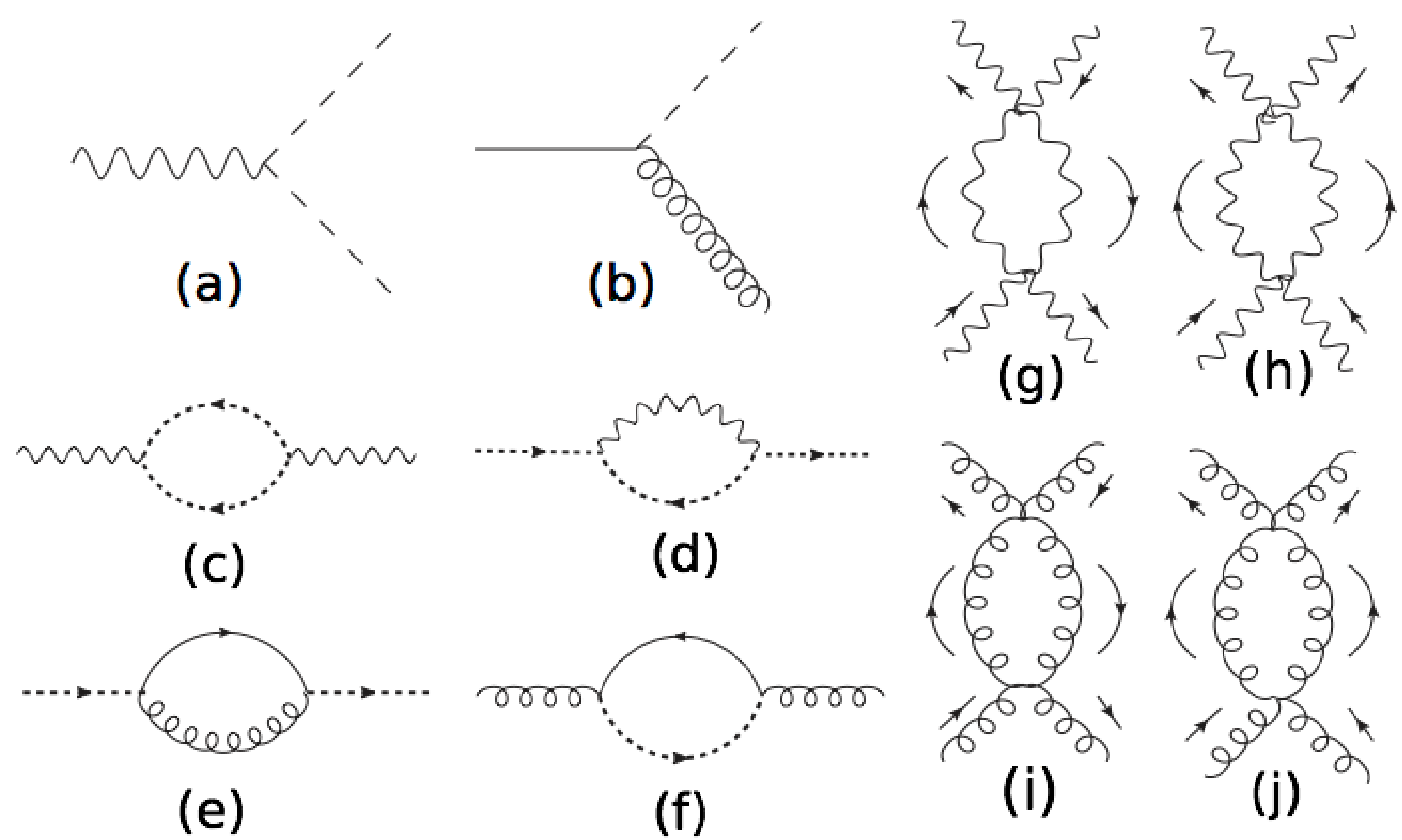
Perturbative renormalization group (RG)

Wave-function + coupling constant renormalizations

□□ Bare Green functions



Feynman diagrams ➠
(one-loop)



RG equations and RG flows

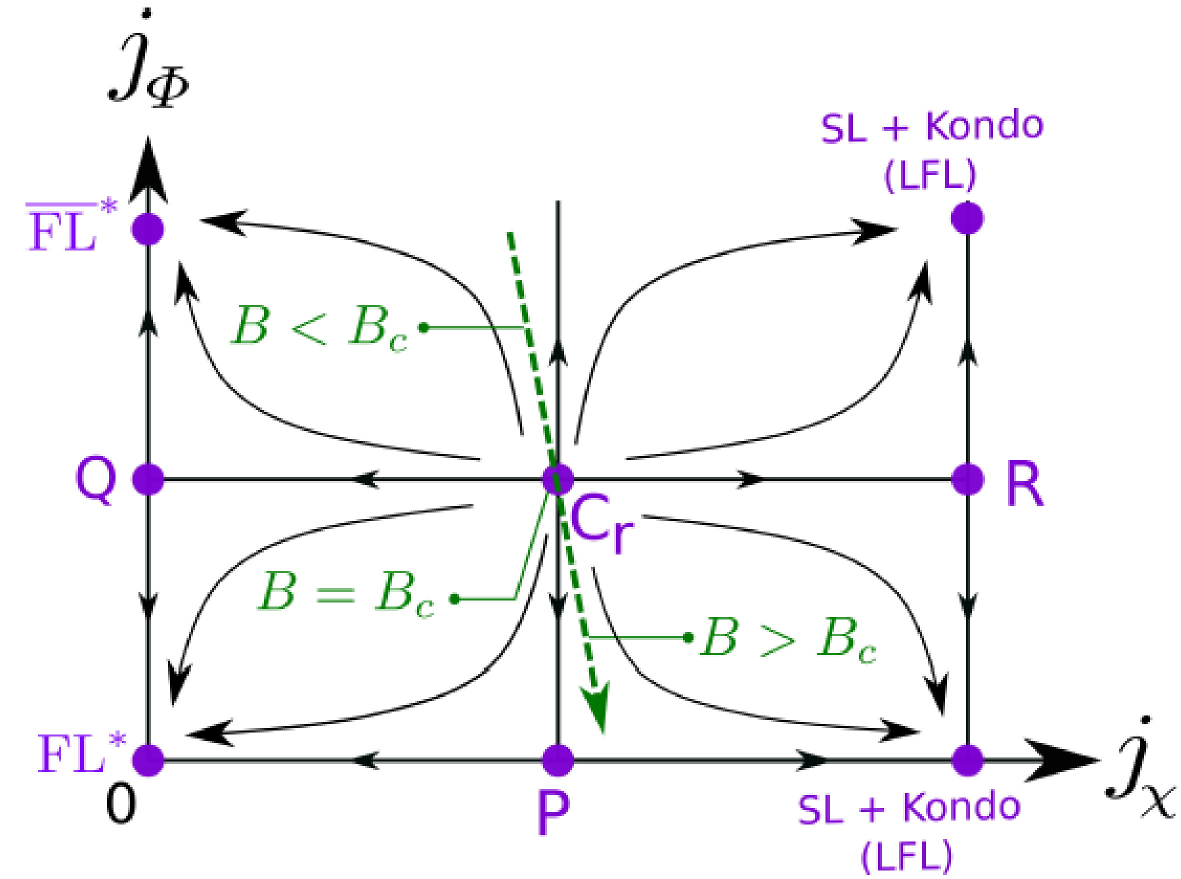
quasi-2d: d=z+η, z=2, 0<η<<1

Near P, relative to $J_\chi = J_\chi^*$ fixed :

$$\frac{d\tilde{j}_\chi}{dl} = -\left(\frac{\eta}{2} + \frac{d}{4} - 2(j_\Phi^*)^2\right)\tilde{j}_\chi + \frac{1}{2}\tilde{j}_\chi^3$$

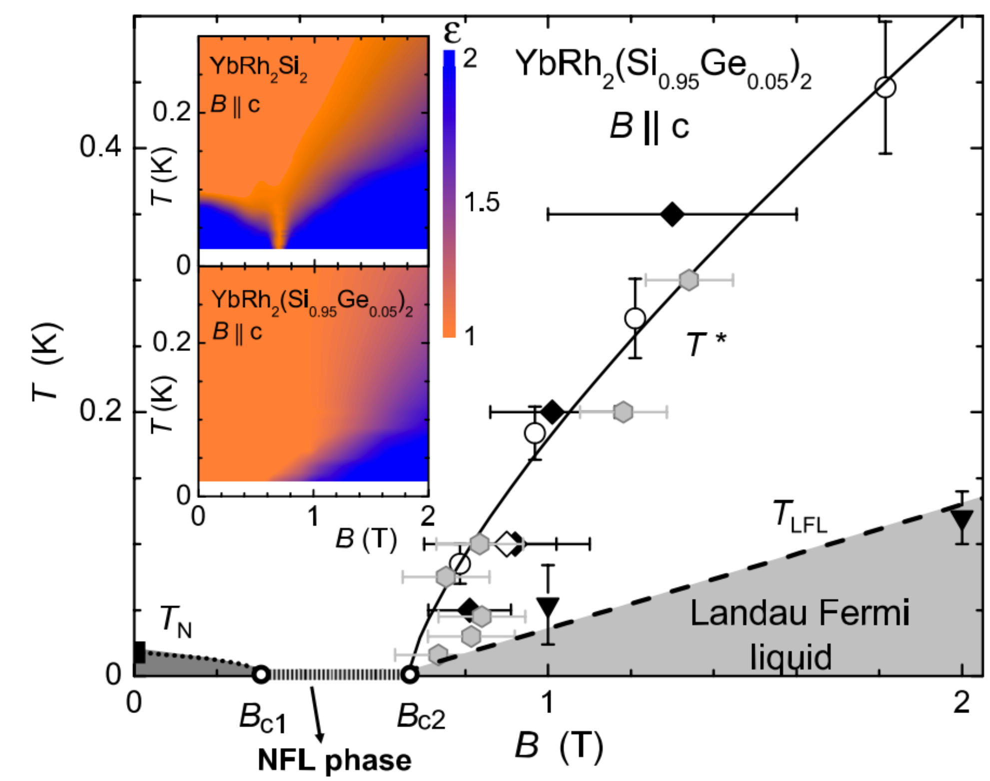
Near Q, relative to $J_\Phi = J_\Phi^*$ fixed :

$$\frac{d\tilde{j}_\Phi}{dl} = \left(-\frac{d}{2} + \frac{\eta}{2}\right)\tilde{j}_\Phi + 4\,\tilde{j}_\Phi^3$$



Correlation length ξ: $\xi \sim |g - g_c|^{-\nu}$

Crossover scale T_{LFL}: $T_{LFL} \sim |g - g_c|^{z\nu}$



RG relative to fixed $J_\chi \Rightarrow$ $\nu = 1/z$

Specific heat coefficient $\gamma\left(\frac{T}{T_{LFL}}\right)$

The Gaussian fluctuation of RVB singlets dominate the specific heat.

rescaling of T
(Hertz-Millis theory)

$$T_l = T_{l=0}e^{zl}$$

Millis, PRB, 1993

$$\bar{E} = \sum_{\mathbf{k}} \frac{\epsilon_\Phi(\mathbf{k})}{e^{\beta\epsilon_\Phi(\mathbf{k})} - 1}$$

$$= \int_{m_\Phi}^{\Lambda} d^d\mathbf{k} \frac{\epsilon_\Phi(\mathbf{k})}{e^{\beta\epsilon_\Phi(\mathbf{k})} - 1}$$

$$= W(J_\Phi) \int_{m_\Phi}^{\Lambda} d\epsilon_\Phi \frac{\epsilon_\Phi^{1+\eta/2}}{e^{\beta\epsilon_\Phi} - 1},$$

m_Φ is strongly relevant, $m_\Phi \sim O(1)$. $e^l \sim \xi$ ➔ $T_l = T_{l=0}/T_{\mathrm{LFL}}$

$$C_V \equiv \frac{\partial\bar{E}}{\partial T_l} = W(J_\Phi) \int d\epsilon_\Phi \frac{\beta^2 \epsilon_\Phi^{2+\eta/2} e^{\beta\epsilon_\Phi}}{(e^{\beta\epsilon_\Phi} - 1)^2}$$

$$= \frac{W(J_\Phi)}{4} \left(\frac{T}{T_{LFL}}\right)^{1+\eta/2} \int_{T_{LFL}/T}^{T_{LFL}\Lambda/T} dx \frac{x^{2+\eta/2}}{\sinh^2(x/2)}$$

Anomalous Scaling in Free Energy and Hyperscaling Violation.

The Gaussian fluctuation of RVB singlets dominate the Gaussian Free energy (spin)

$$F = -T\,W_\Phi \int_0^\Lambda \epsilon^{\eta/2}\, d\epsilon \left[\ln\left(1 - e^{-\beta(\epsilon + m_\Phi)}\right)\right]$$

upon rescaling $k \to k/b$ and $T \to T b^z$ $(b \equiv e^l)$

$$b \sim \xi \sim r^{-\nu}$$

$$F(r, T) = r^{\nu(d+z)-\nu\Delta}\, F_r(1, T/r^{\nu z})$$

$$r \equiv (J_\Phi - J_\Phi^*)/J_\Phi^*$$

anomalous scaling dimension $\Delta = 4\eta + 2\eta^2$

$$W_\Phi(j_\chi(b), j_\Phi(b), \lambda(b)) = b^\Delta\, W_\Phi(J_\chi, J_\Phi, \lambda)$$

hyperscaling violation due to boson-fermion coupling:

$$\gamma \equiv C_V/T$$

$$\gamma \propto r^{-\bar{\alpha}},\ \bar{\alpha} = \nu(z - d + \Delta)$$

Conventional Hyperscaing $\bar{\alpha} = \nu(z - d)$

Specific heat coefficient $\gamma\left(\frac{T}{T_{LFL}}\right)$

$$S_G = \int dk \left[\hat{\tilde{\Phi}}_k^\dagger \; (-G_\Phi^{b\,-1}(\omega, \mathrm{k})) \; \hat{\tilde{\Phi}}_k \right]$$

$$\gamma\left(\bar{T}\right) = -\frac{r^{-\bar{\alpha}}\,\bar{T}^{\eta/2}}{4} \int_{1/\bar{T}}^{\Lambda/\bar{T}} dx\, \frac{x^{2+\eta/2}}{\sinh^2(x/2)}$$

$$\bar{T} \equiv \frac{T}{T_{LFL}}$$

$$\bar{\alpha} = \eta^2 + 3\eta/2 \approx 0.32$$

fitting parameter: $\eta = 0.18$

Open issues

- **Microscopic mechanism of SM (NFL) properties**

  Due to QCP? What are the competing states?

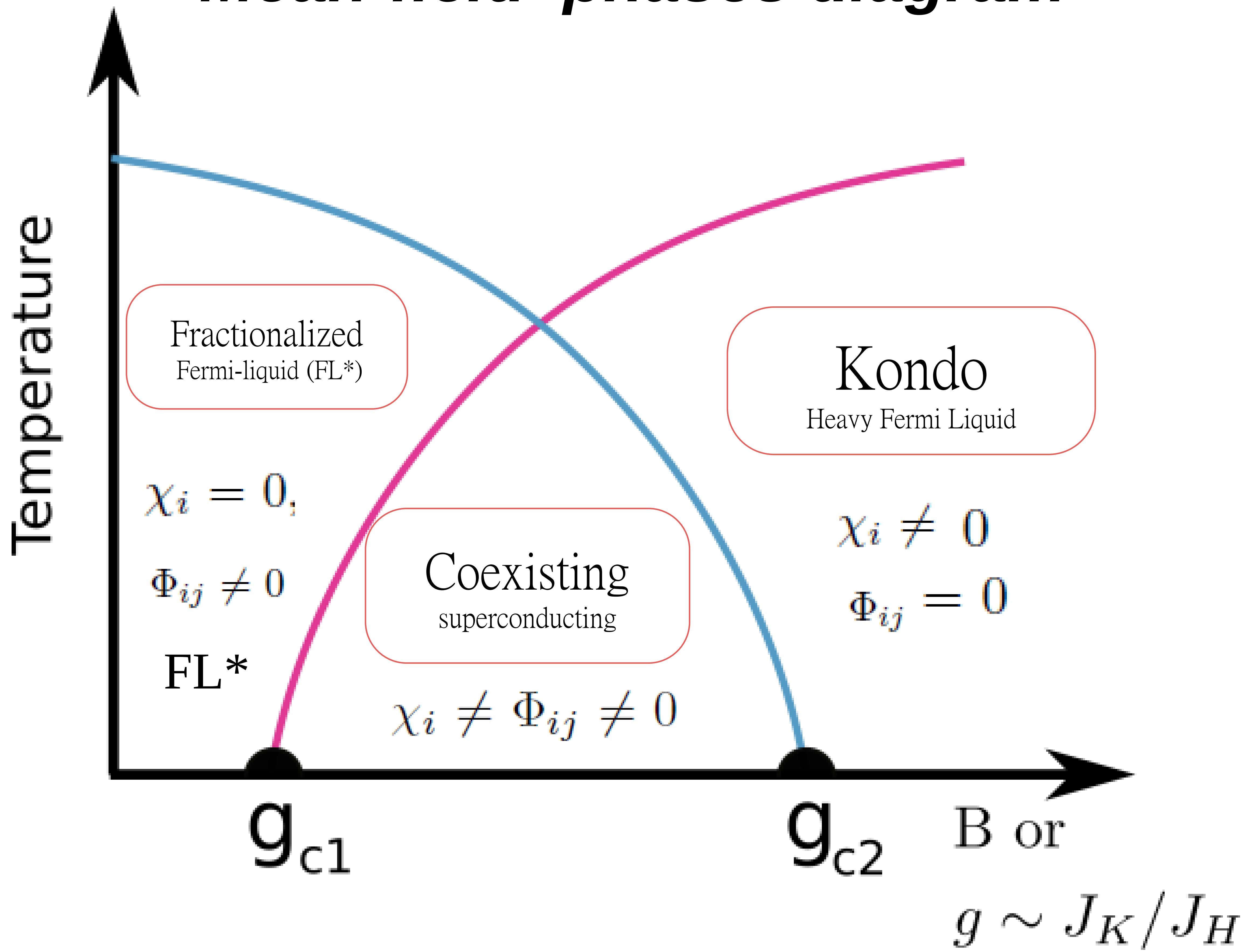
  Nature of the transition?

- **How to explain exotic scaling behaviors in SM state?**

$$\frac{C_V}{T} = \frac{1}{b^{1/3}} \Phi\left(\frac{T}{T_0(b)}\right)$$

- **Role of magnetic field?**

Mean-field phases diagram



(B = magnetic field)

Summary

- **SM in Ge-YRS can be explained by a quantum critical region due to a single QCP at gc within Kondo breakdown scenario.**

- **The magnetic field mainly suppresses the RVB term, while the Kondo term stays nearly critical. YRS has spatial dimension d = 2 + η, $\eta \rightarrow 0$.**

- **Remarkable agreements between our theory and experiments on Ge-YRS.**

  The specific heat is dominated by the RVB (spinon) fluctuation
  Kondo fluctuation contributes to the electrical (charge) transport.

- **Hyperscaling violation**
  Anomalous exponent in specific heat coefficient is explained

- **The dynamical ω/T scaling exists even for $d+z > 4$ due to the Kondo breakdown**

Intertwine between dynamics and thermodynamics

Sondhi et al, RMP 1997

Partition function (thermodynamics)

$$[x_i, p_j] = i\hbar\,\delta_{i,j}$$

$$Z(\beta) = \sum_n \langle n | e^{-\beta H} | n \rangle,$$

$$\langle O \rangle = \frac{1}{Z(\beta)} \mathrm{Tr}\; O e^{-\beta H}.$$

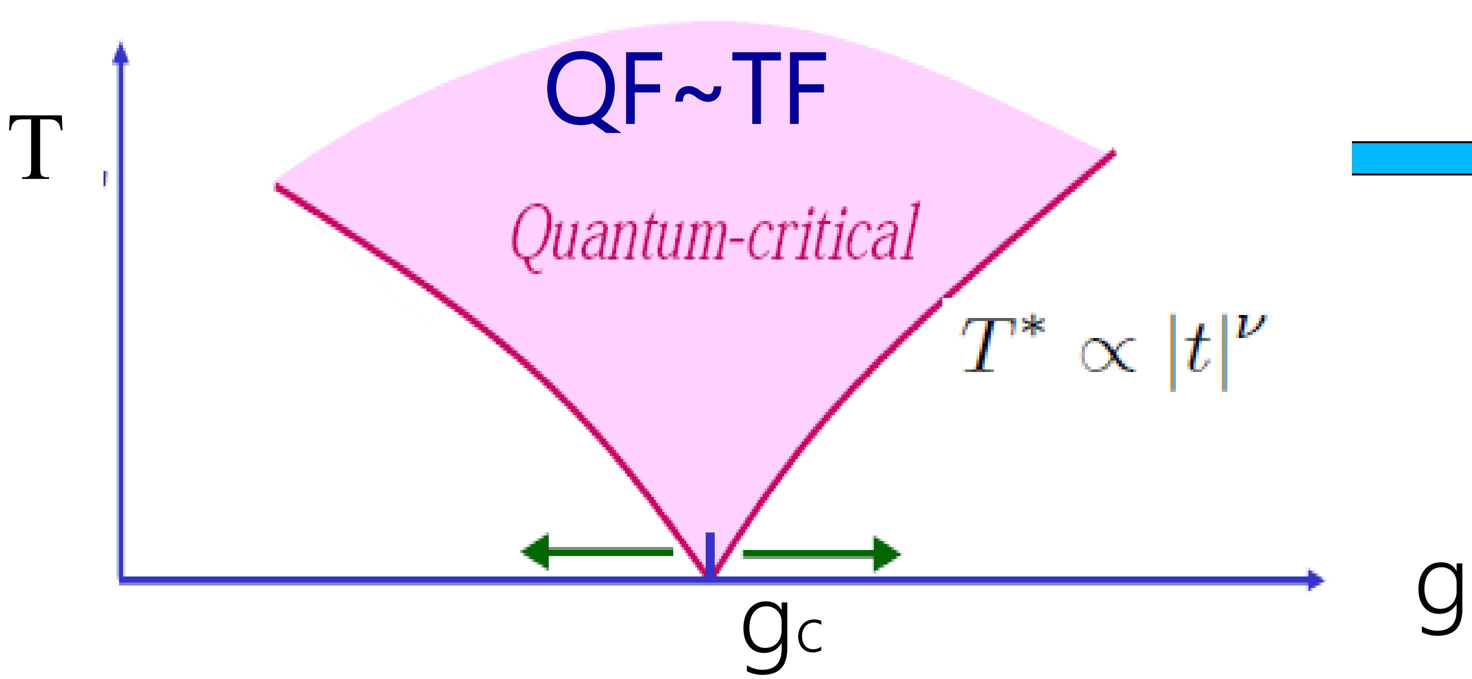
Imaginary-time Feynmann's path integral (dynamics)

$$Z(\beta) = \sum_n \sum_{m_1, m_2, \ldots, m_N} \langle n | e^{-\frac{1}{\hbar}\delta\tau H} | m_1 \rangle \langle m_1 | e^{-\frac{1}{\hbar}\delta\tau H} | m_2 \rangle \langle m_2 | \ldots | m_N \rangle \langle m_N | e^{-\frac{1}{\hbar}\delta\tau H} | n \rangle.$$

Imaginary-time $\mathcal{T} = -i\hbar\beta$

$\langle S(0)\, S(r) \rangle \sim G(r) \sim \exp(-r/\xi)$

Power-law divergent correlation lengths ξ



$$\xi \sim |\delta|^{-\nu} \longrightarrow \infty$$

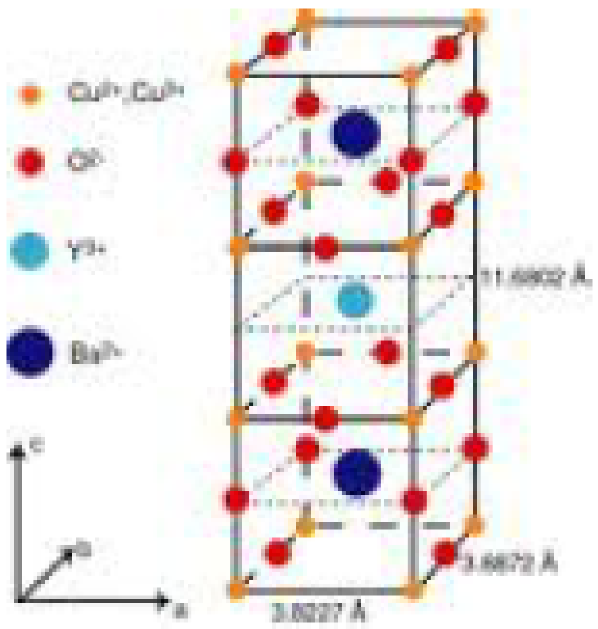
$$\xi_\tau \sim \xi^z.$$

t= $\delta \equiv$ g-gc $\longrightarrow 0$

correlation length exponent ν.

dynamical scaling exponent, z.

Phase diagram of high-Tc superconductors



YBCO

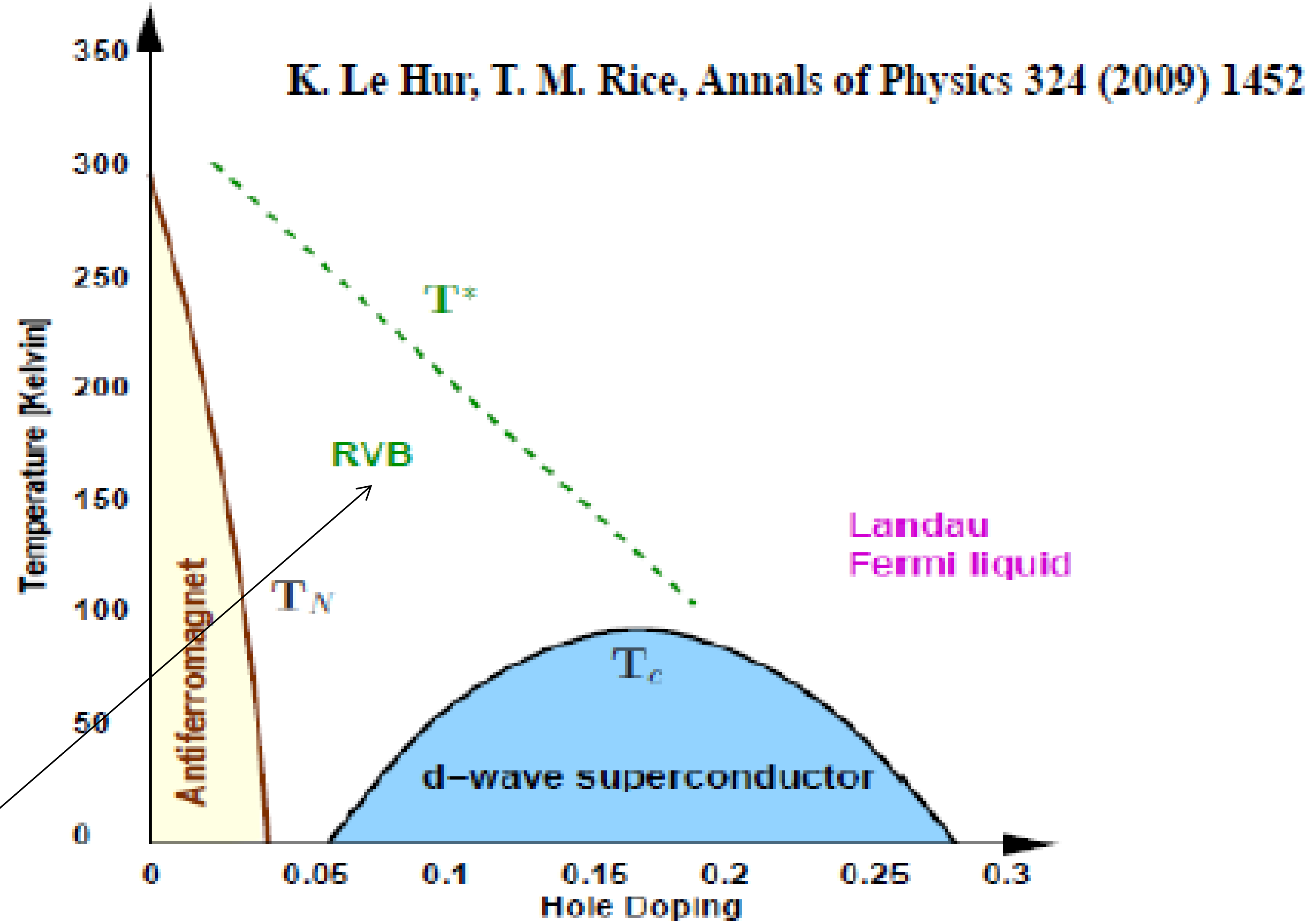
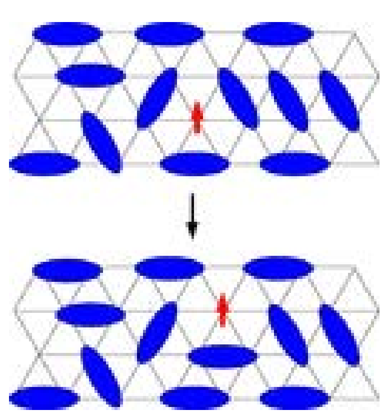


Fig. 1. Schematic phase diagram of high-T_c superconductors.



Highest Tc so far ~ 133K

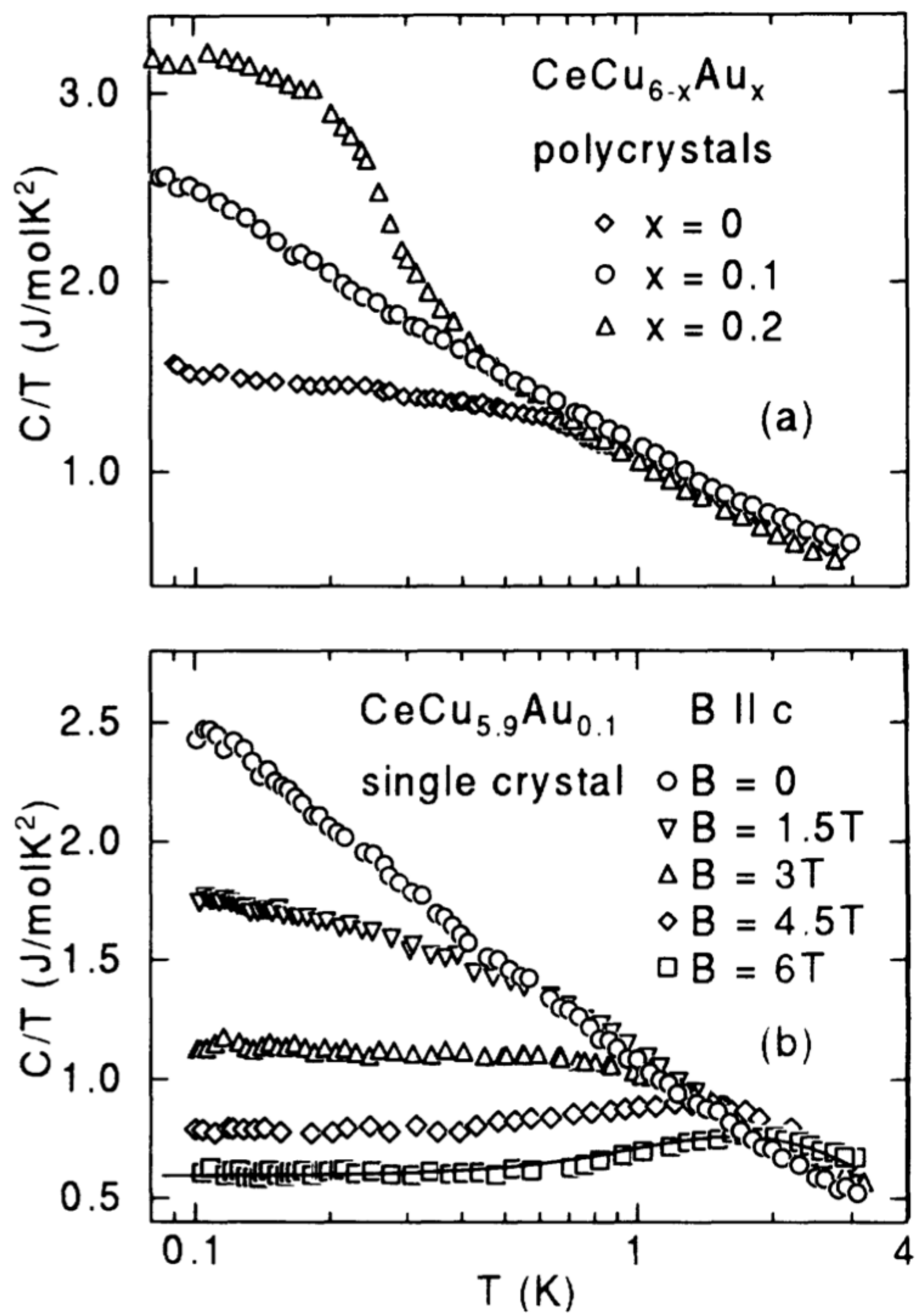
Even higher Tc ?

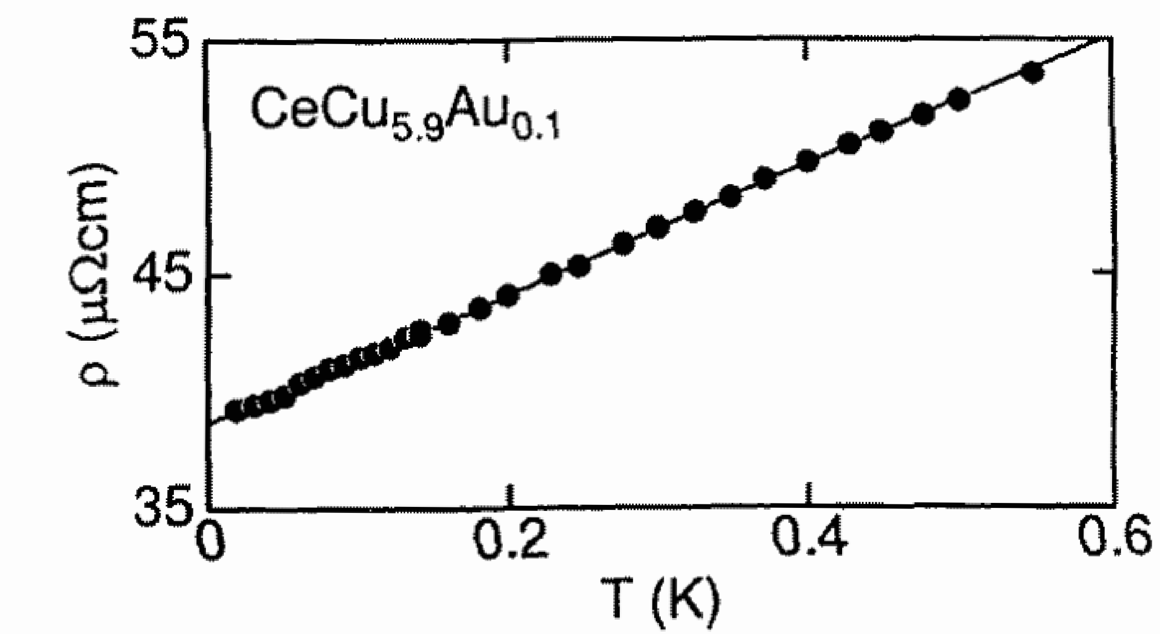
https://upload.wikimedia.org/wikipedia/commons/thumb/0/05/Spinon_moving.png/130px-Spinon_moving.png

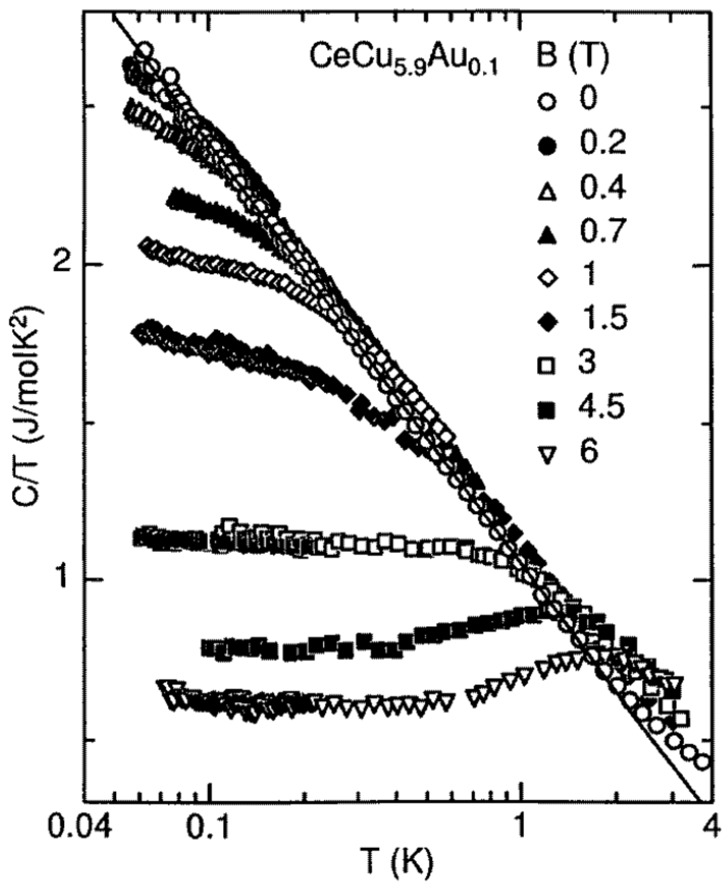
NFL SM behaviors in other heavy-fermion compounds

$CeCu_{6-x}Au_x$

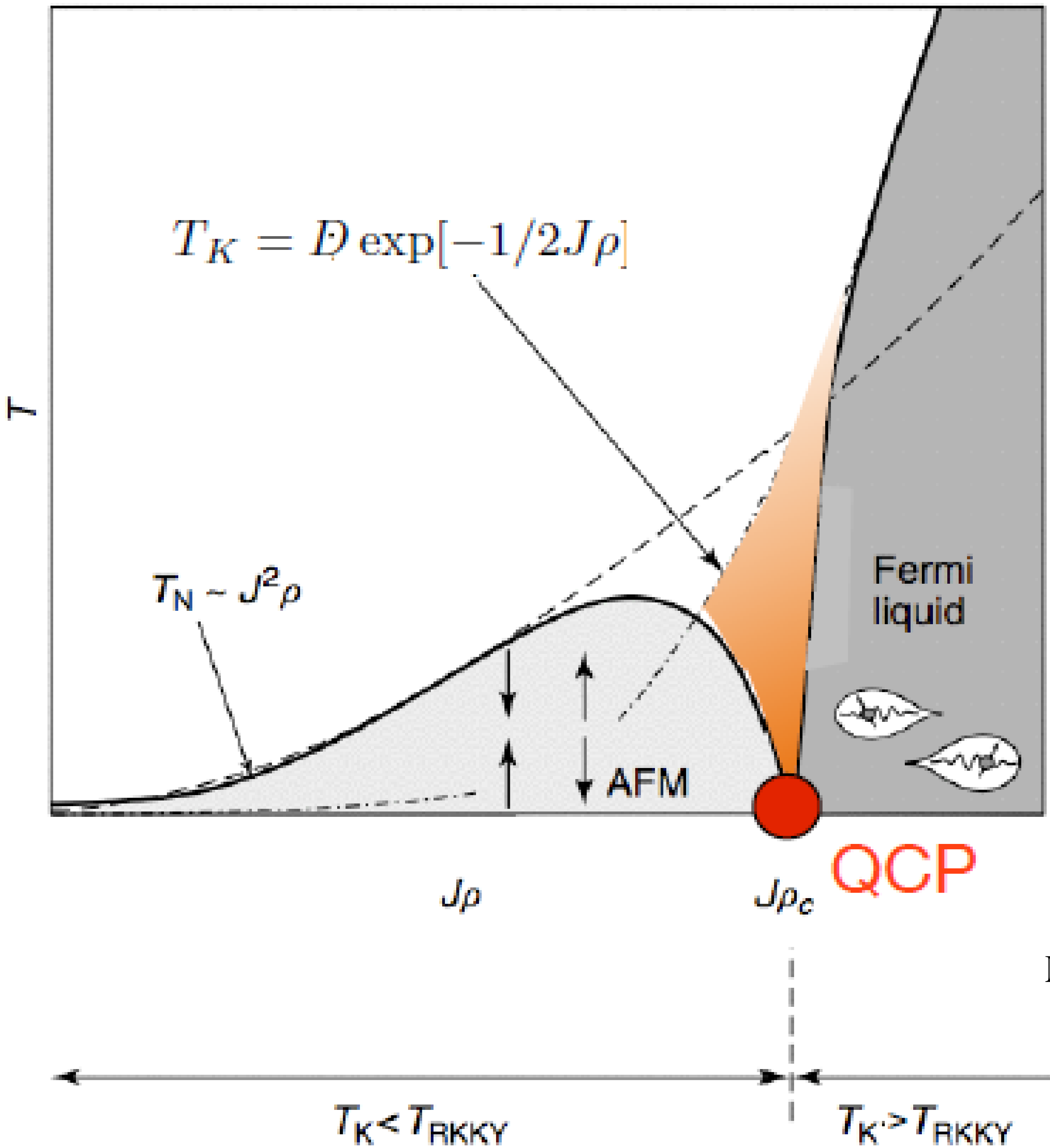
QPT by doping







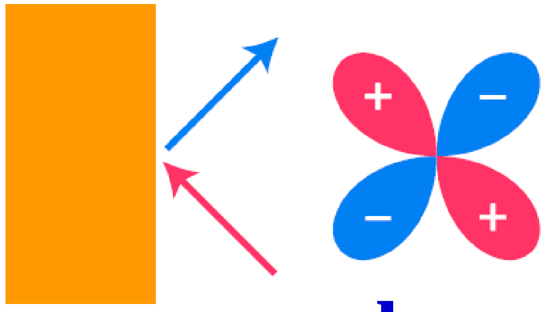
H. v. Löhneysen, PRL, 1994
H. v. Löhneysen, JPCM, 1996



Strange Metal

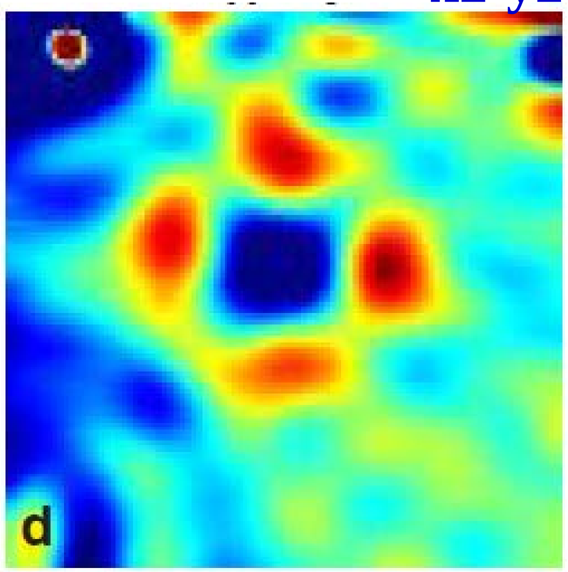
P. Coleman,
Magnetism and Advanced Magnetic Materials,
95-148 (2007).

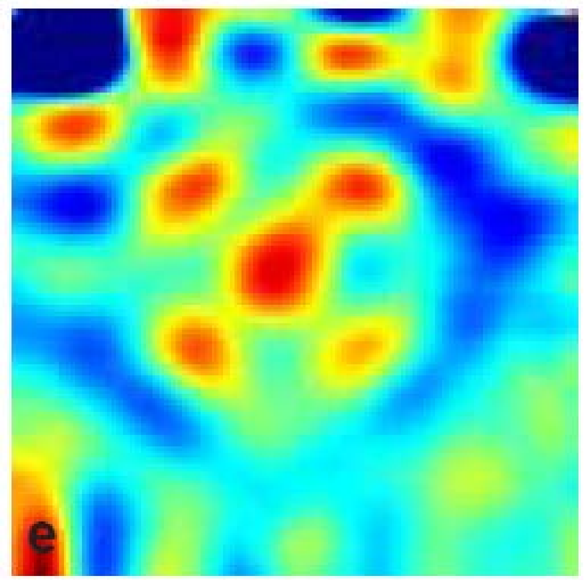
CeCoIn5: d-wave nodal superconducting quasi-particle scattering

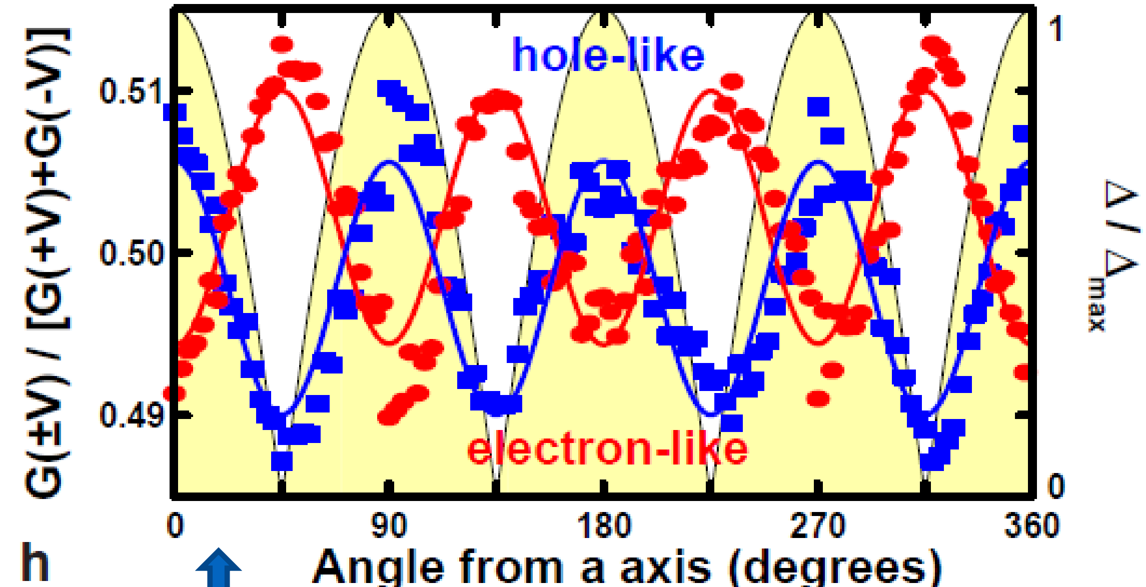


d_{x2-y2} **wave gap**

STM QPI V-shaped $\bar{N}(E) \propto E$ **nodal gap**

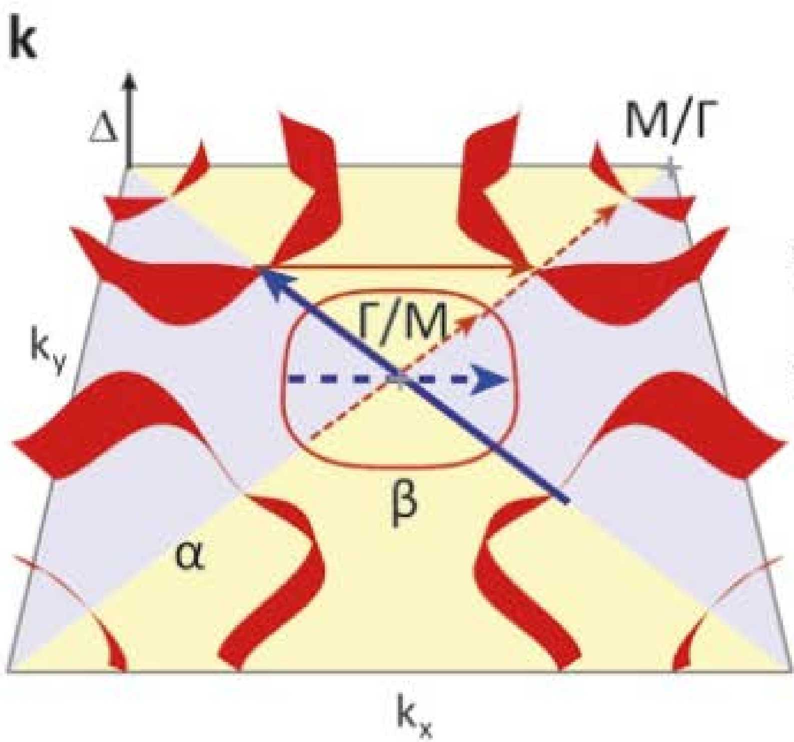


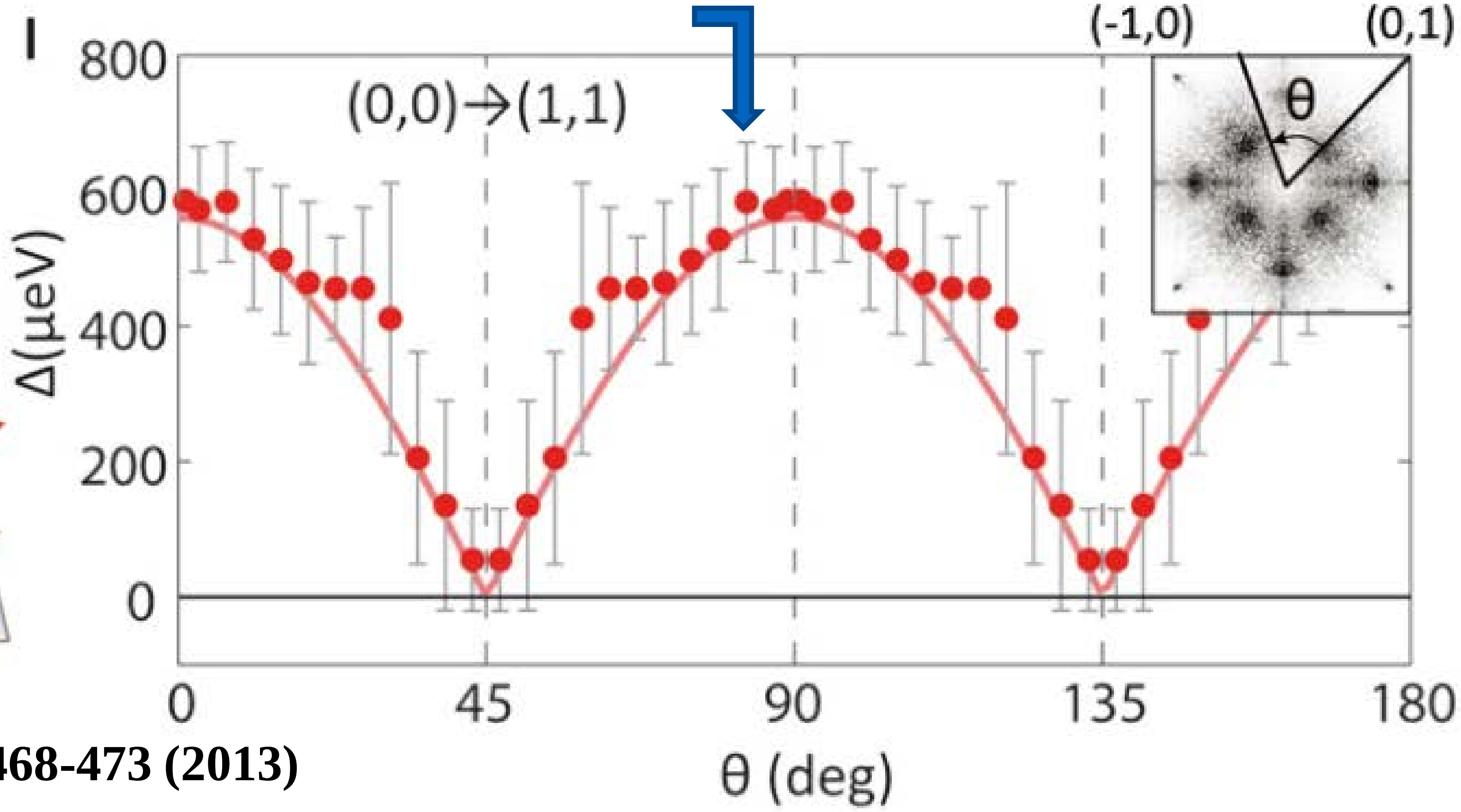




<u>Ali Yazdani</u> et al. *Nature Physics* 9, 474–479 (2013))

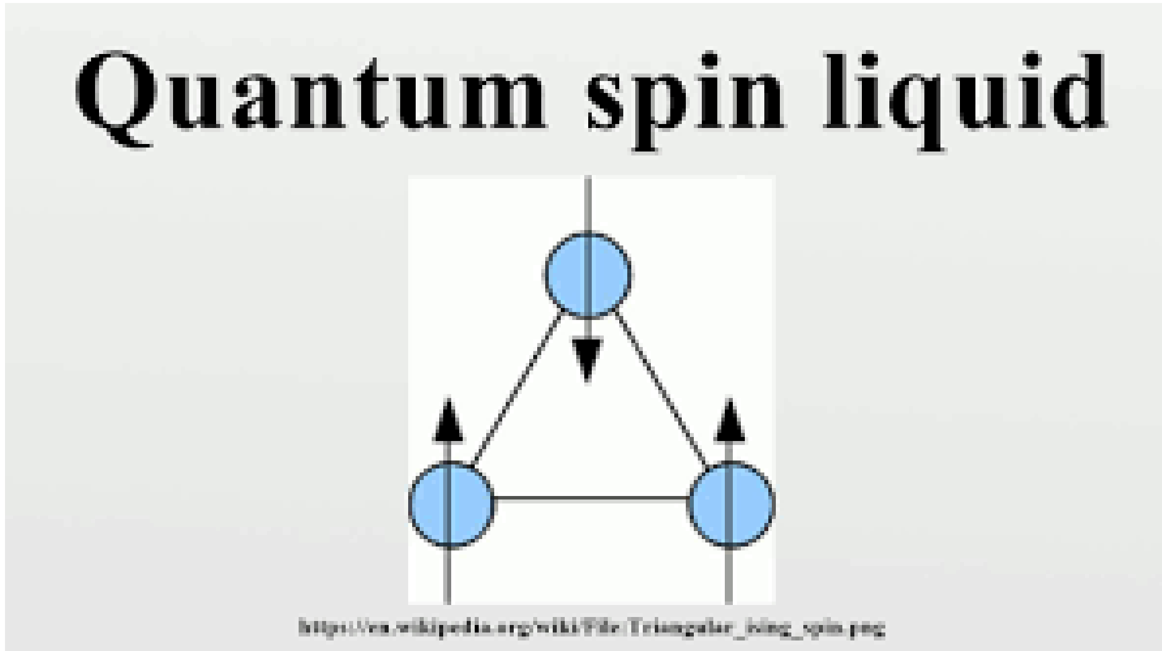
d_{x2-y2} **wave gap**



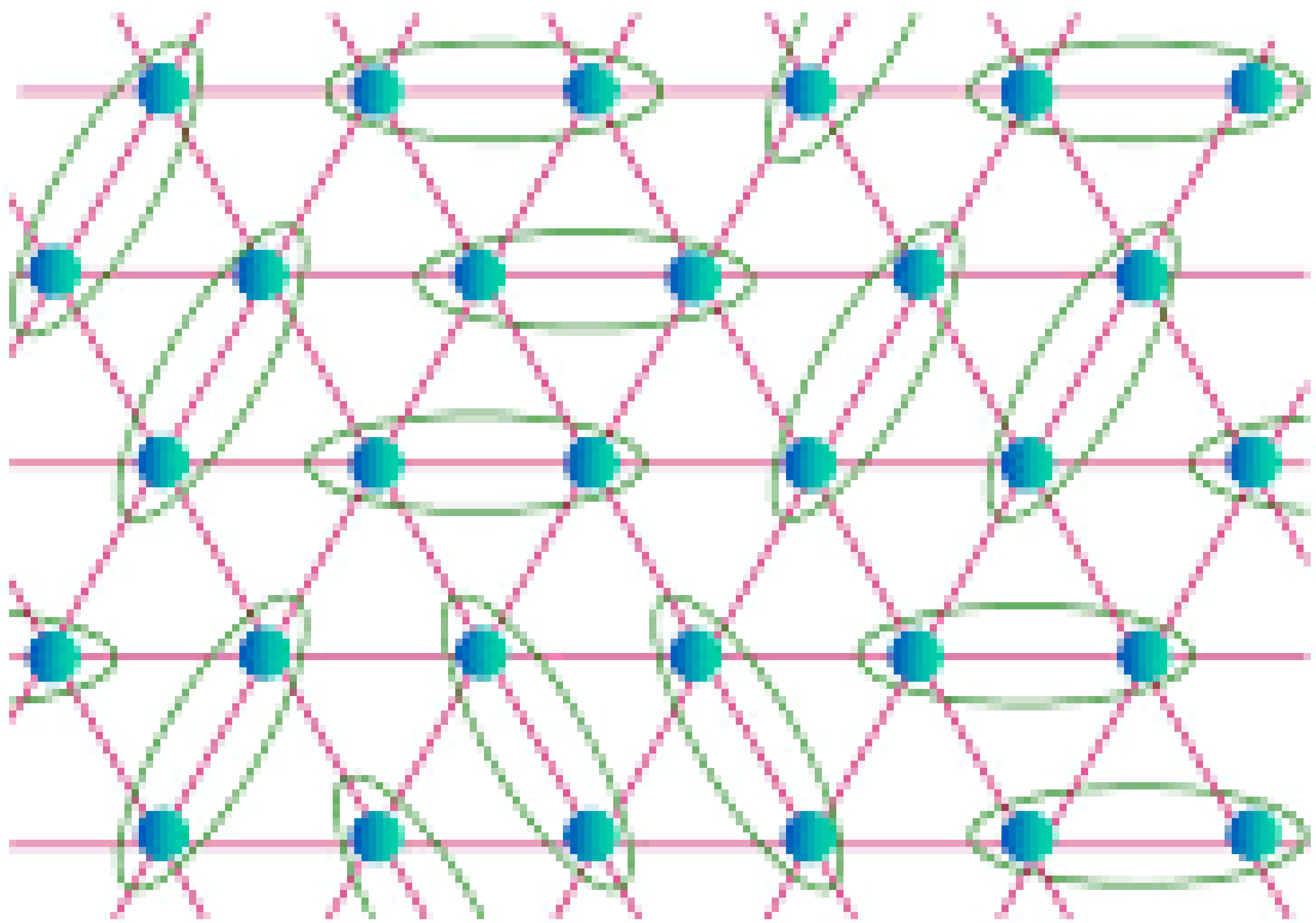


J.C. Davis et al., Nature Physics 9, 468-473 (2013)

Anderson’s RVB spin liquid



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



$\frac{\omega}{T}$ -scaling

Vojta, RPP, 2003

$$\mathcal{O}(g = g_c, \mathbf{k} = 0, \omega, T) = T^{-d_{\mathcal{O}}/z} O(\frac{\omega}{T})$$

usually exists at d+z < 4 G-L theory

The Green function

$$G_c(\boldsymbol{k}, \omega) = \frac{1}{\omega - \epsilon_c(\boldsymbol{k}) - \Sigma_c(\boldsymbol{k}, \omega)}$$

$$(\mathrm{Im}\Sigma_c = \alpha T)$$

ω/T scaling:

$$G_c(k_F, \omega) = \frac{1}{\omega - i\,\mathrm{Im}\Sigma(k_F, \omega)} = \frac{1}{\omega - i\,\alpha T} = T^{-1} g\left(k_F, \frac{\omega}{T}\right)$$

Even for d+z > 4,

ω/T still exists due to boson-fermion interactions.