

# Finding the origin of heavy elements and exploring the nature of dense matter

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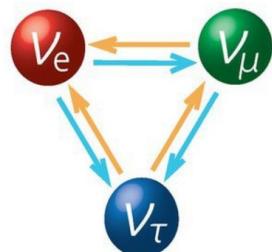
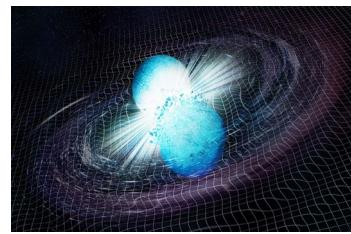
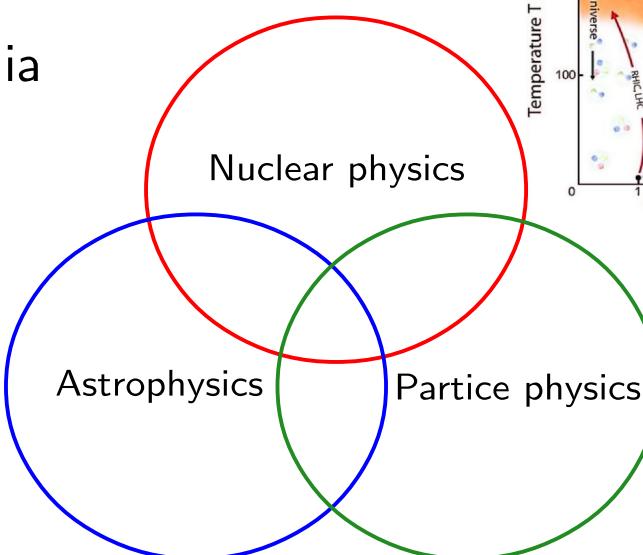
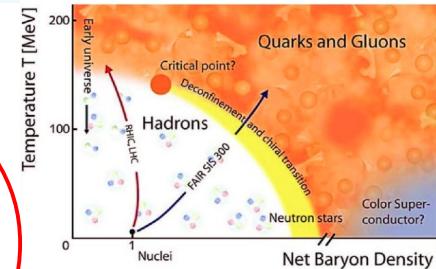
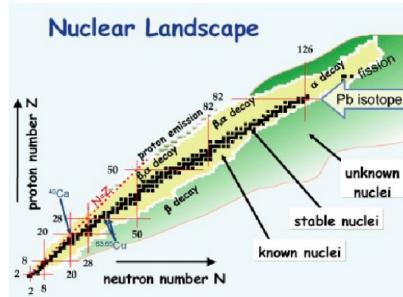


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# Outline

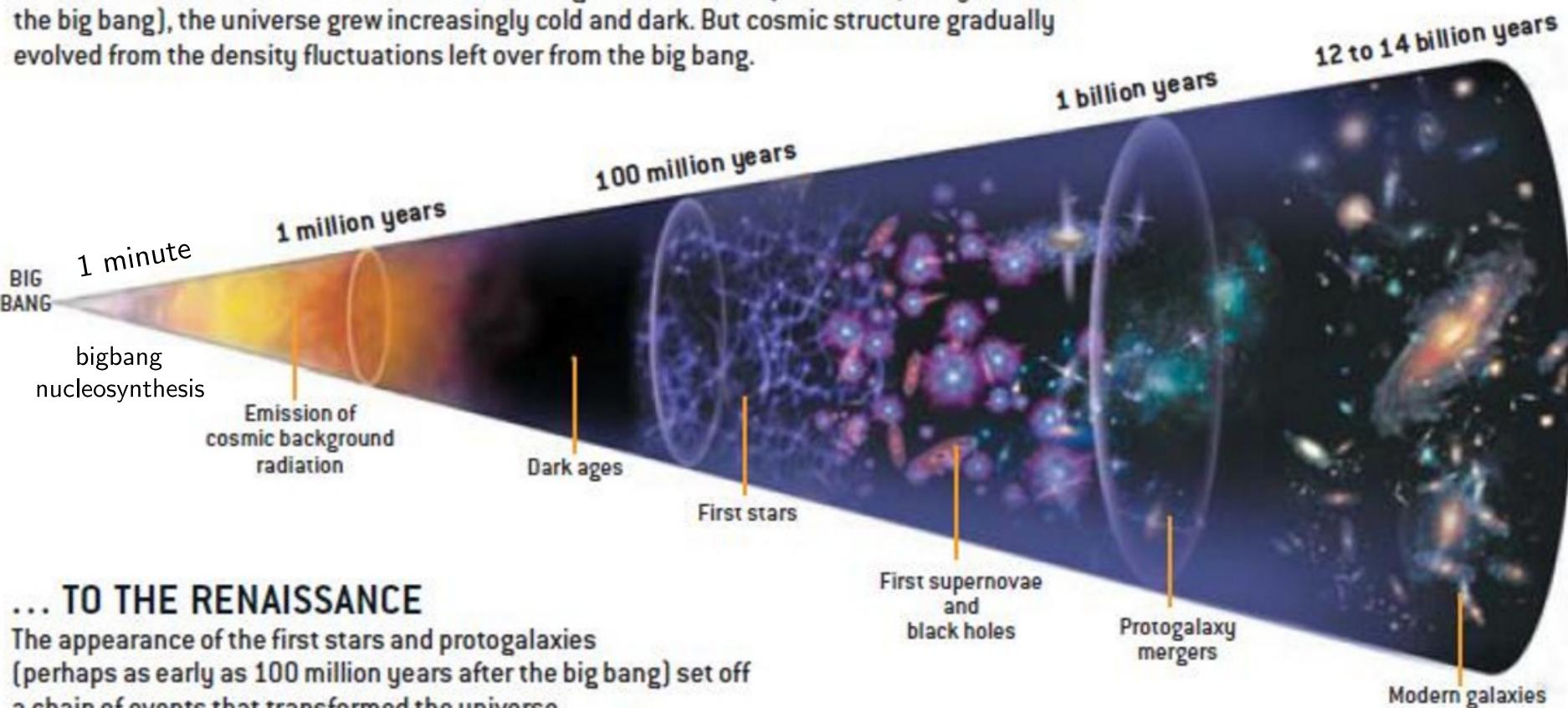
- Introduction
- The origin of heavy elements
  - *r*-process & kilonovae
  - mergers as the main *r*-process sites?
- The nature of dense matter
  - neutrino oscillations in dense media
  - hadron–quark phase transition?
- Summary



# Cosmic evolution, galaxies, and stars

## FROM THE DARK AGES ...

After the emission of the cosmic microwave background radiation (about 400,000 years after the big bang), the universe grew increasingly cold and dark. But cosmic structure gradually evolved from the density fluctuations left over from the big bang.



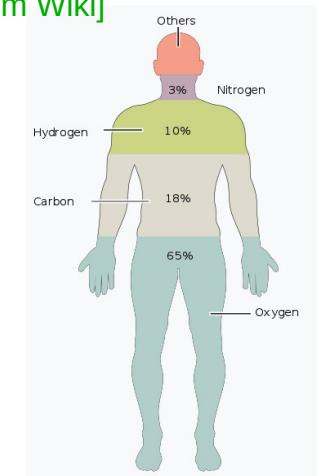
## ... TO THE RENAISSANCE

The appearance of the first stars and protogalaxies (perhaps as early as 100 million years after the big bang) set off a chain of events that transformed the universe.

[adapted from Scientific American]

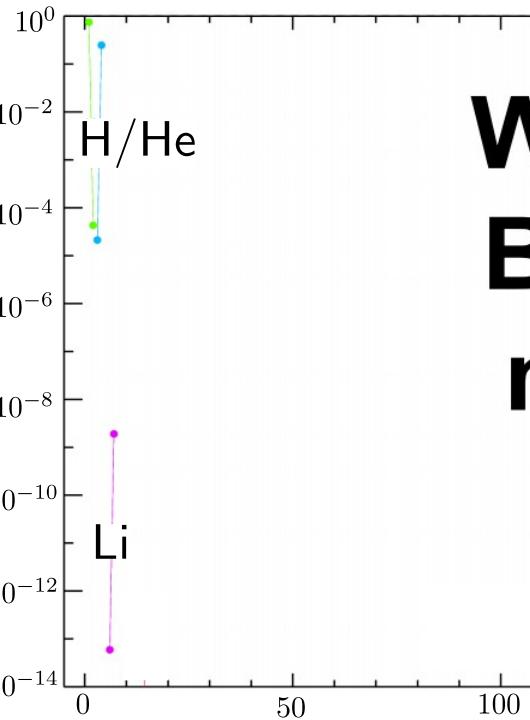
How does the Universe evolve to the present structure we see today?

[from Wiki]



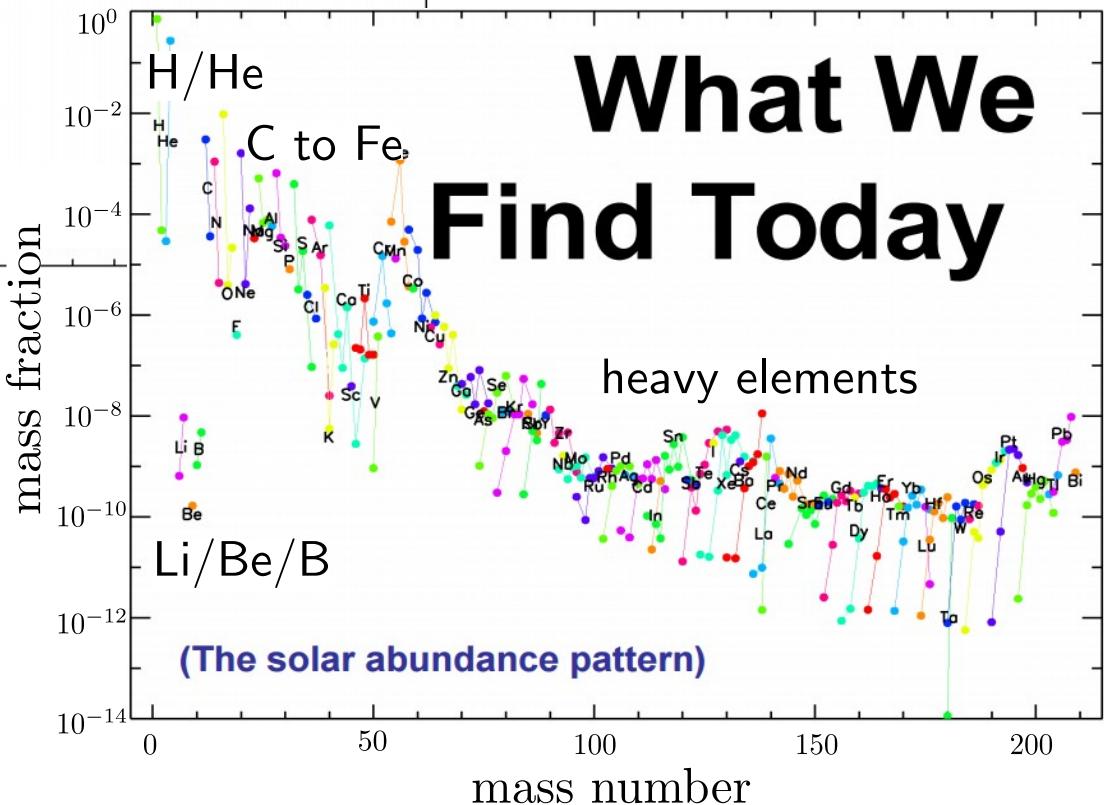
# What the Big Bang made...

mass fraction



[from A. Heger]

# What We Find Today



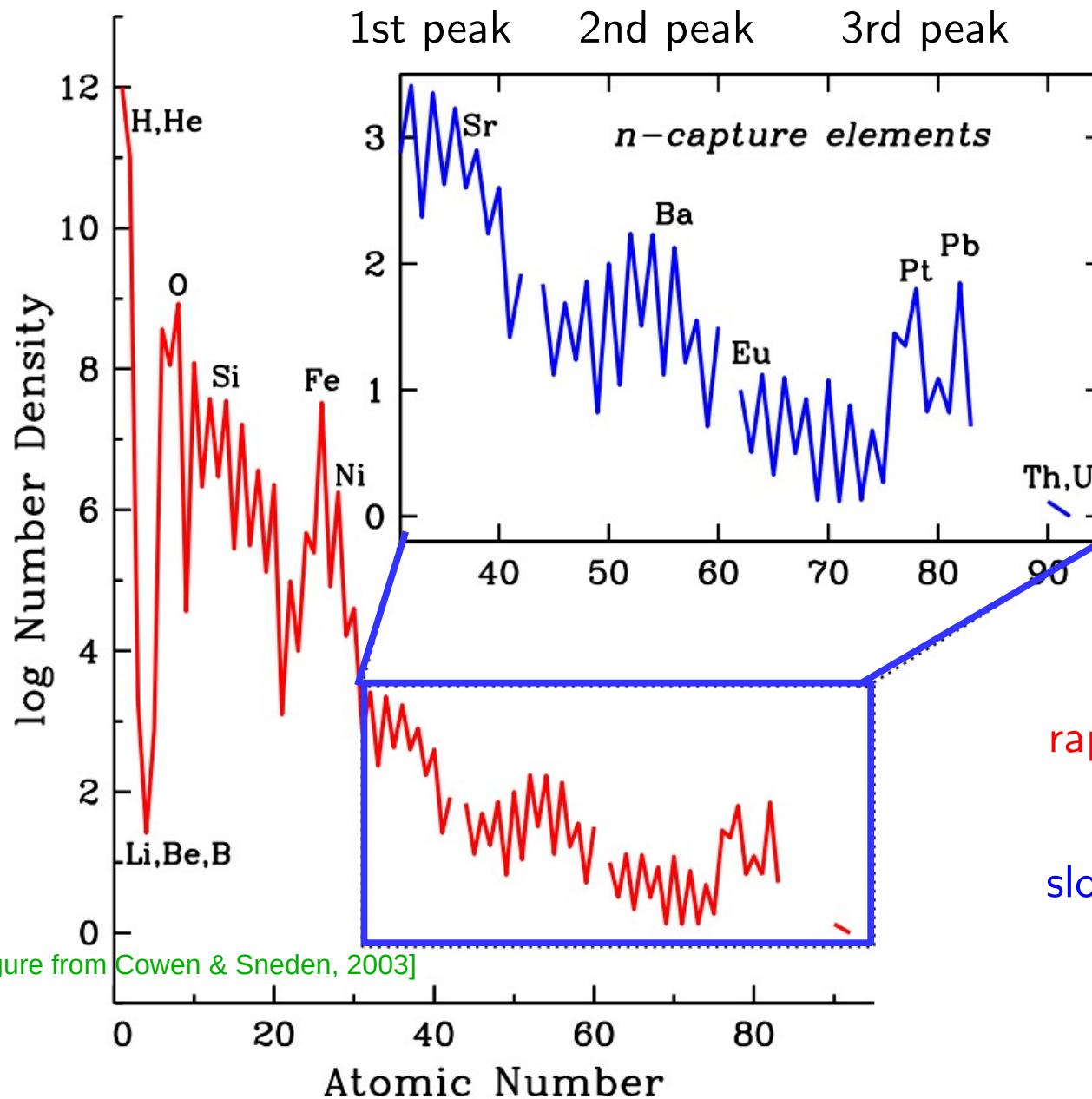
From where were the different atoms/isotopes made in the Universe?

# Nuclear astrophysicist's Periodic Table

The figure shows a periodic table where each element's color represents its primary nucleosynthesis pathway:

- Big Bang fusion** (blue): Hydrogen (H) and Helium-3 ( $\text{He}_3$ )
- Dying low-mass stars** (green): Lithium-7 ( $\text{Li}_7$ ), Beryllium-7 ( $\text{Be}_7$ ), Sodium-23 ( $\text{Na}_{23}$ ), Magnesium-25 ( $\text{Mg}_{25}$ ), Potassium-39 ( $\text{K}_{39}$ ), Calcium-40 ( $\text{Ca}_{40}$ ), Strontium-87 ( $\text{Sr}_{87}$ ), Rubidium-87 ( $\text{Rb}_{87}$ ), Cesium-133 ( $\text{Cs}_{133}$ ), Francium-223 ( $\text{Fr}_{223}$ ), Barium-137 ( $\text{Ba}_{137}$ ), and Radium-226 ( $\text{Ra}_{226}$ ).
- Cosmic ray fission** (pink): Lithium-6 ( $\text{Li}_6$ ), Beryllium-6 ( $\text{Be}_6$ ), and Boron-10 ( $\text{B}_{10}$ ).
- Merging neutron stars** (purple): Titanium-46 ( $\text{Ti}_{46}$ ), Vanadium-51 ( $\text{V}_{51}$ ), Chromium-52 ( $\text{Cr}_{52}$ ), Manganese-55 ( $\text{Mn}_{55}$ ), Iron-56 ( $\text{Fe}_{56}$ ), Cobalt-57 ( $\text{Co}_{57}$ ), Nickel-59 ( $\text{Ni}_{59}$ ), Copper-63 ( $\text{Cu}_{63}$ ), Zinc-65 ( $\text{Zn}_{65}$ ), Gallium-67 ( $\text{Ga}_{67}$ ), Germanium-73 ( $\text{Ge}_{73}$ ), Arsenic-75 ( $\text{As}_{75}$ ), Selenium-76 ( $\text{Se}_{76}$ ), Bromine-77 ( $\text{Br}_{77}$ ), and Krypton-81 ( $\text{Kr}_{81}$ ).
- Exploding massive stars** (yellow): Nitrogen-14 ( $\text{N}_{14}$ ), Oxygen-16 ( $\text{O}_{16}$ ), Fluorine-19 ( $\text{F}_{19}$ ), Chlorine-37 ( $\text{Cl}_{37}$ ), and Argon-36 ( $\text{Ar}_{36}$ ).
- Exploding white dwarfs** (grey): Phosphorus-31 ( $\text{P}_{31}$ ), Sulfur-32 ( $\text{S}_{32}$ ), Chlorine-35 ( $\text{Cl}_{35}$ ), Bromine-37 ( $\text{Br}_{37}$ ), Iodine-53 ( $\text{I}_{53}$ ), and Xenon-54 ( $\text{Xe}_{54}$ ).
- Others** (light green): Yttrium-89 ( $\text{Y}_{89}$ ), Zirconium-90 ( $\text{Zr}_{90}$ ), Niobium-91 ( $\text{Nb}_{91}$ ), Molybdenum-92 ( $\text{Mo}_{92}$ ), Technetium-93 ( $\text{Tc}_{93}$ ), Ruthenium-94 ( $\text{Ru}_{94}$ ), Rhodium-95 ( $\text{Rh}_{95}$ ), Palladium-96 ( $\text{Pd}_{96}$ ), Silver-96 ( $\text{Ag}_{96}$ ), Cadmium-97 ( $\text{Cd}_{97}$ ), Indium-99 ( $\text{In}_{99}$ ), Tin-100 ( $\text{Sn}_{100}$ ), Sb-101 ( $\text{Sb}_{101}$ ), Tellurium-102 ( $\text{Te}_{102}$ ), Iodine-127 ( $\text{I}_{127}$ ), and Xenon-131 ( $\text{Xe}_{131}$ ).
- Rare earth metals** (light purple): Lanthanum-139 ( $\text{La}_{139}$ ), Cerium-140 ( $\text{Ce}_{140}$ ), Praseodymium-141 ( $\text{Pr}_{141}$ ), Neodymium-142 ( $\text{Nd}_{142}$ ), Promethium-145 ( $\text{Pm}_{145}$ ), Samarium-146 ( $\text{Sm}_{146}$ ), Europium-152 ( $\text{Eu}_{152}$ ), Gadolinium-154 ( $\text{Gd}_{154}$ ), Thulium-159 ( $\text{Tb}_{159}$ ), Dysprosium-160 ( $\text{Dy}_{160}$ ), Holmium-164 ( $\text{Ho}_{164}$ ), Erbium-166 ( $\text{Er}_{166}$ ), Thulium-168 ( $\text{Tm}_{168}$ ), Ytterbium-170 ( $\text{Yb}_{170}$ ), and Lutetium-174 ( $\text{Lu}_{174}$ ).
- Actinides** (brown): Thorium-232 ( $\text{Th}_{232}$ ), Protactinium-233 ( $\text{Pa}_{233}$ ), Uranium-234 ( $\text{U}_{234}$ ), Neptunium-237 ( $\text{Np}_{237}$ ), and Plutonium-239 ( $\text{Pu}_{239}$ ).

# Elemental abundances in the solar system



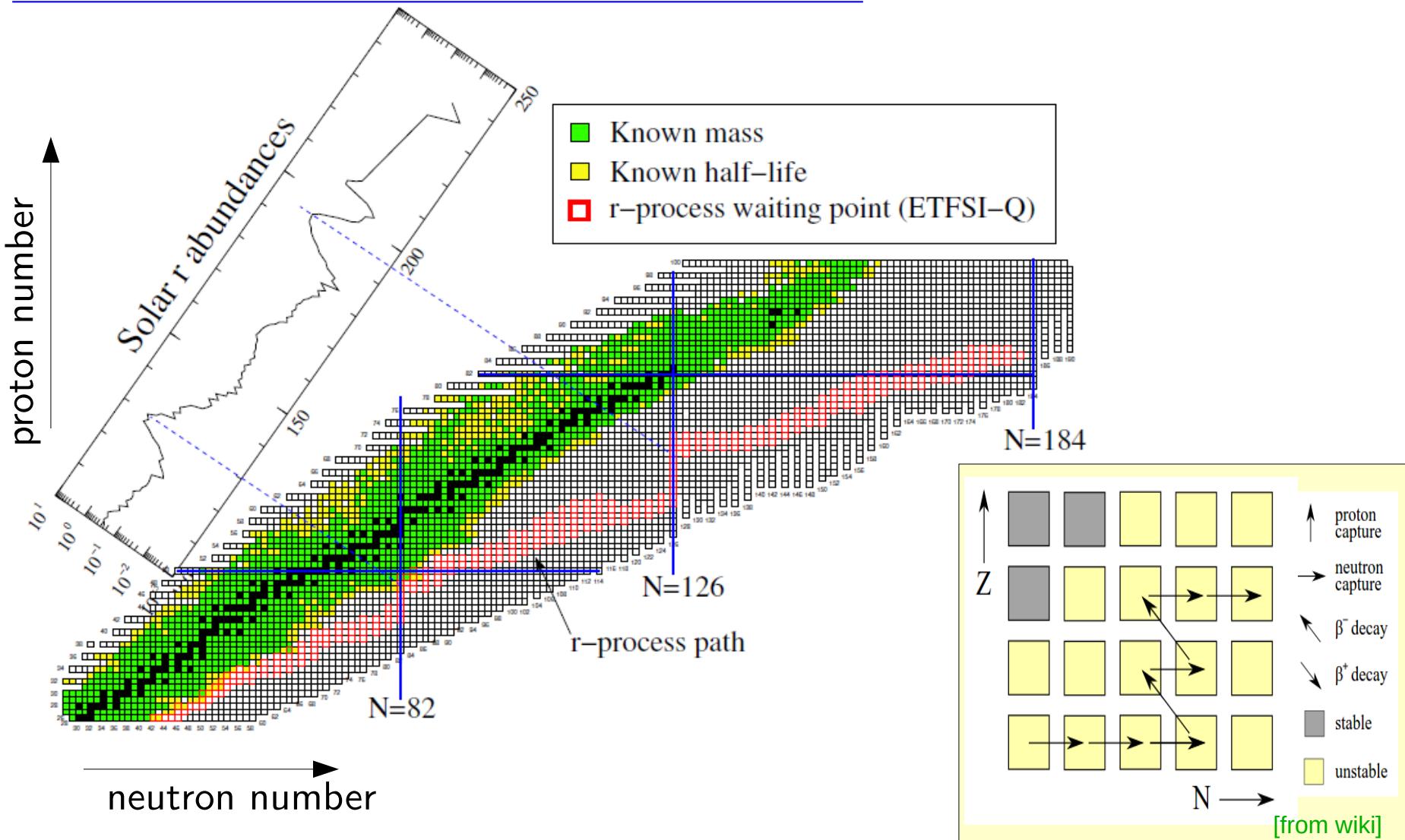
need strong or stable neutron sources in some astrophysical environments

rapid n-capture (*r*-) process:  
 $\tau_n \ll \tau_\beta$

slow n-capture (*s*-) process:  
 $\tau_n \gg \tau_\beta$

[Figure from Cowen & Sneden, 2003]

# Making heavy nuclei with neutron captures



Nuclei with “neutron magic numbers” (i.e., more reluctant to capture neutrons) are more abundant

# The *r*-process and neutron stars

[Casey Reed, PSU]

Ideal condition for rapid neutron captures can be obtained,  
If one can “unbind” (part of) a **neutron star**:

- high density with large amount of neutrons
- compact object  $\leftrightarrow$  short dynamic timescale

$$[\tau_{\text{dyn}} \sim \sqrt{R^3/(GM)}]$$



$$M \sim 1.4M_{\odot}$$

$$R \sim 10 \text{ km}$$

$$\begin{aligned} \rho &\gtrsim 10^{14} \text{ g cm}^{-3} \\ &\gtrsim 95\% \text{ "neutrons"} \end{aligned}$$

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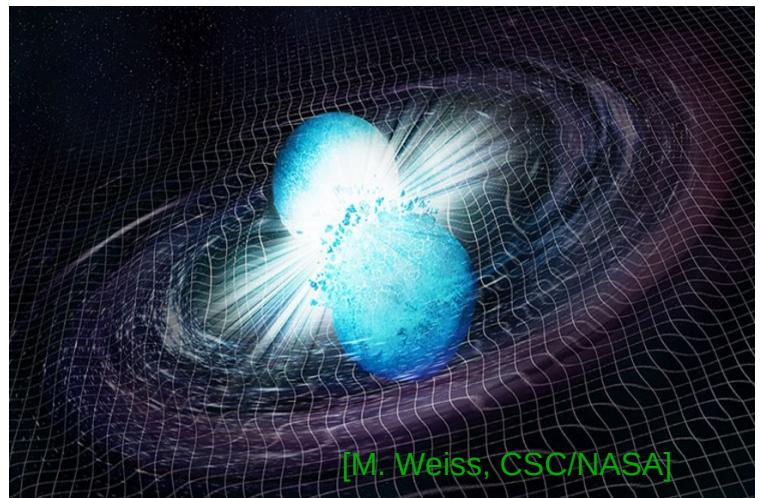
$\gtrsim 95\%$  “neutrons”

It's not easy to unbind things from a neutron star. Opportunities are:

(i) a neutron star was born  
(death of massive star)  
→ core-collapse supernovae



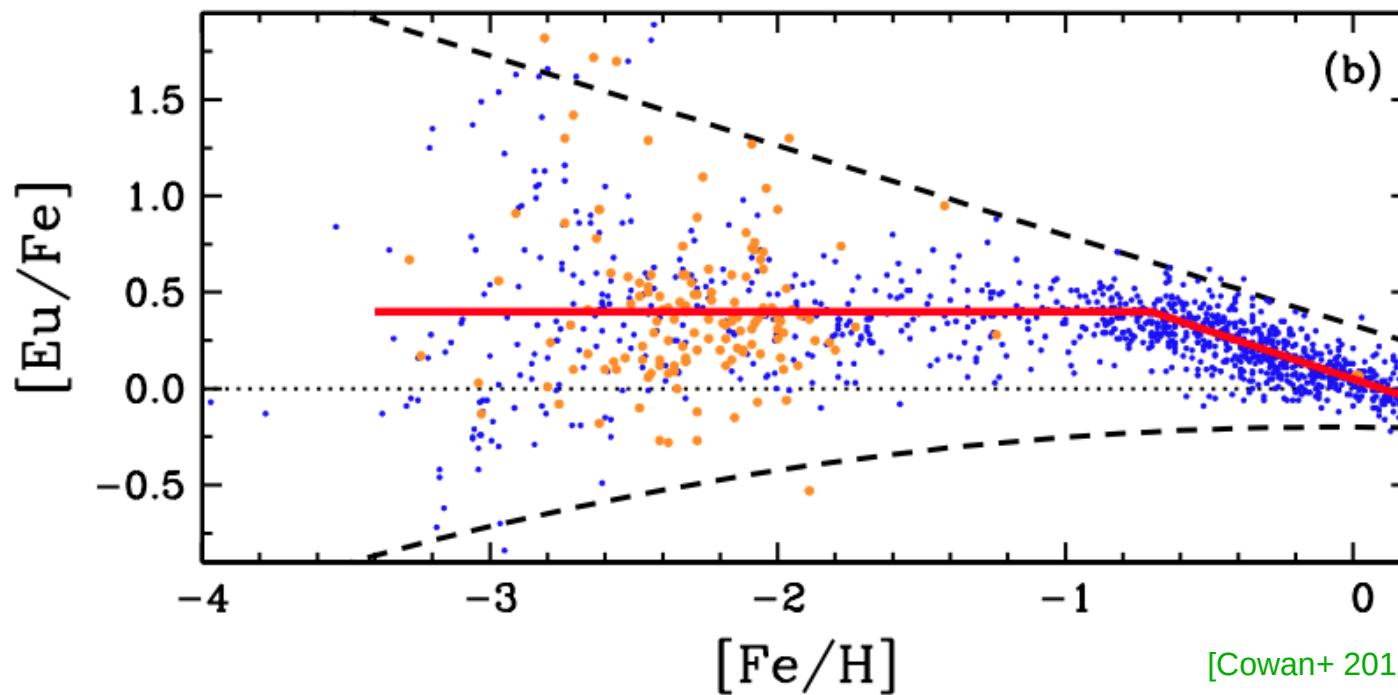
(ii) a neutron star dies  
→ neutron star mergers



## What produce the $r$ -process elements?

$$[A/B] \equiv \log_{10} \left( \frac{n_A^*/n_B^*}{n_A^\odot/n_B^\odot} \right)$$

Observation of Eu abundance for stars with different metalicity



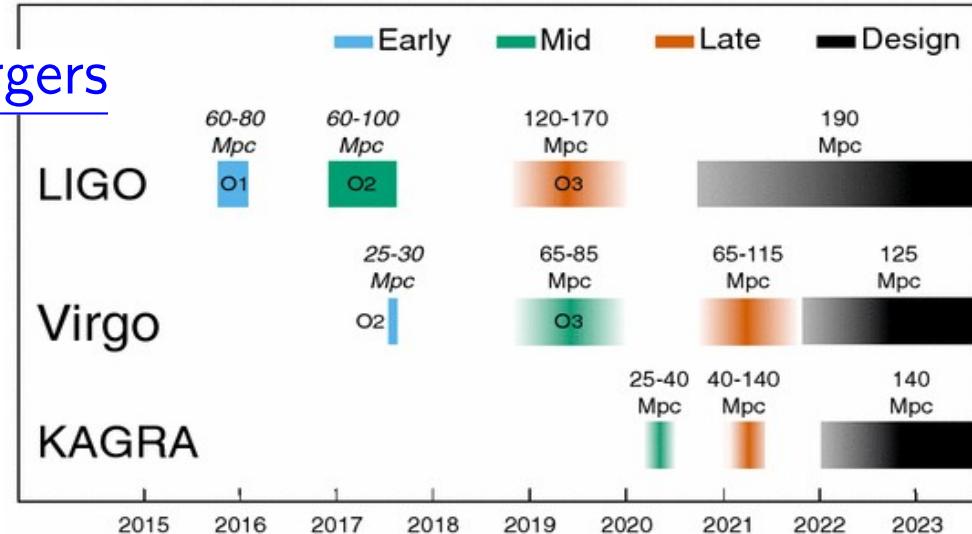
- $r$ -process enrichment at low metalicity ( $[\text{Fe}/\text{H}] \lesssim -3$ )
  - massive stars association?
- large scatters at low metalicity
  - events less frequent than core-collapse supernovae?

A consistent model to trace the (chemical) evolution of the Milky Way is desired

The *r*-process nucleosynthesis and kilonovae

# GW astronomy for NS mergers

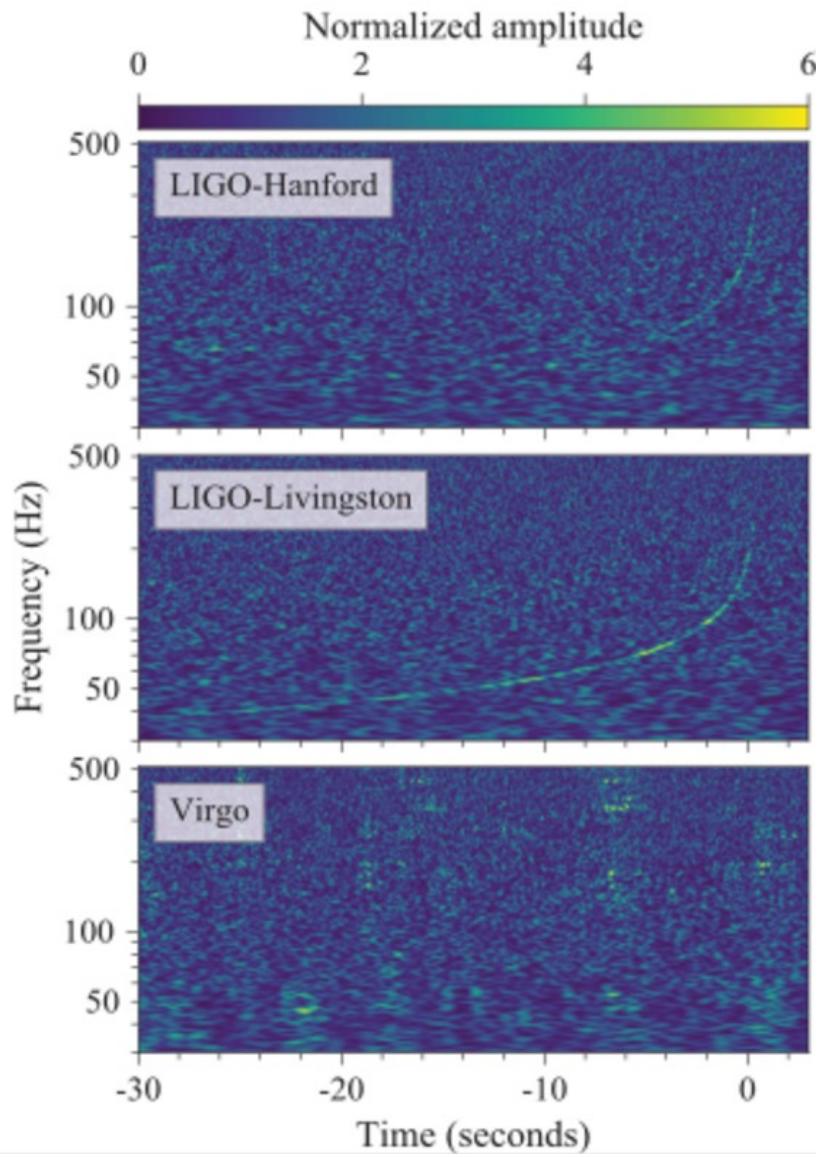
GW170817, still the only event with joint EM observations



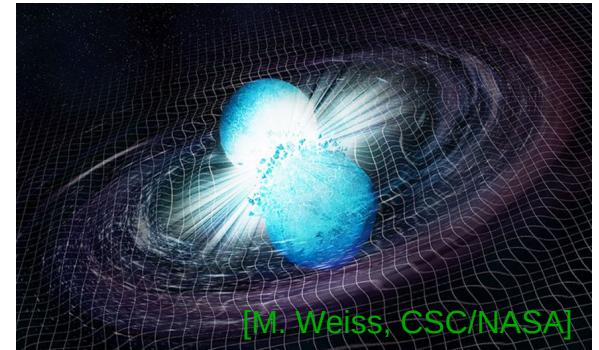
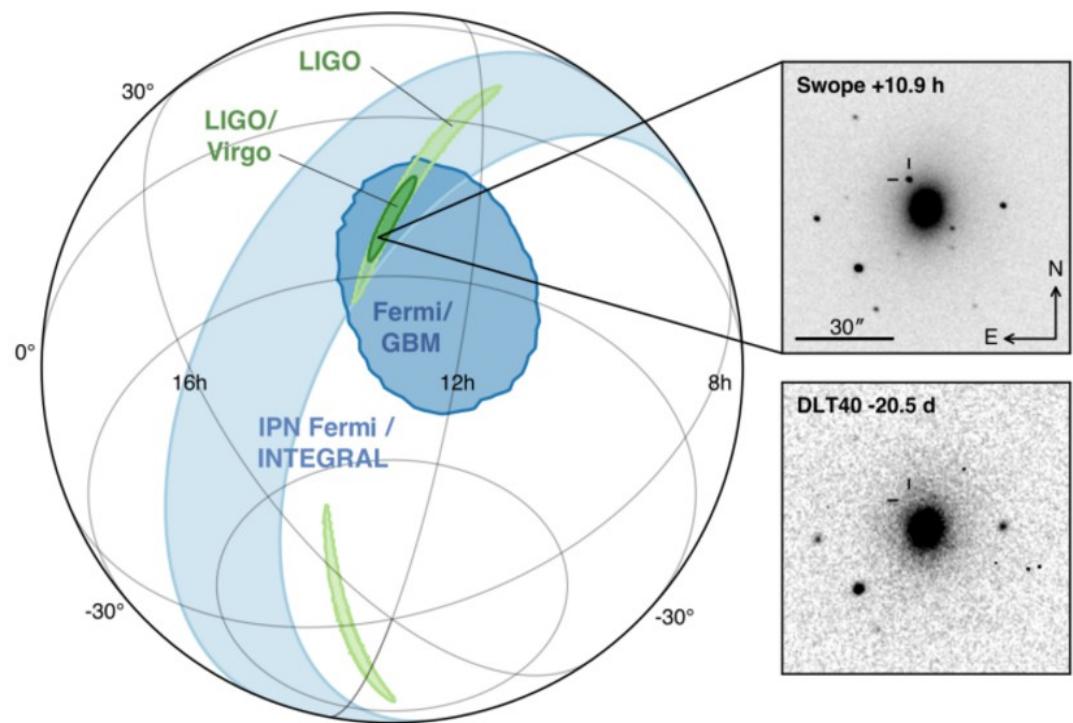
[Abbott+ (LIGO/Virgo Collaborations), Living Rev. Relativ. (2018) 21:3]

Epoch	2015 – 2016	2016 – 2017	2018 – 2019	2020+	2024+
Planned run duration	4 months	9 months	12 months	(per year)	(per year)
Expected BNS range/Mpc	LIGO 40–80 Virgo – KAGRA –	80–120 20–65 –	120–170 65–85 –	190 65–115 –	190 125 140
Achieved BNS range/Mpc	LIGO 60–80 Virgo – KAGRA –	60–100 25–30 –	–	–	–
Estimated BNS detections	0.05–1	0.2–4.5	1–50	4–80	11–180
Actual BNS detections	0	1	7 candidates (GW190425 published)		

# Multi-messenger observations of GW170817

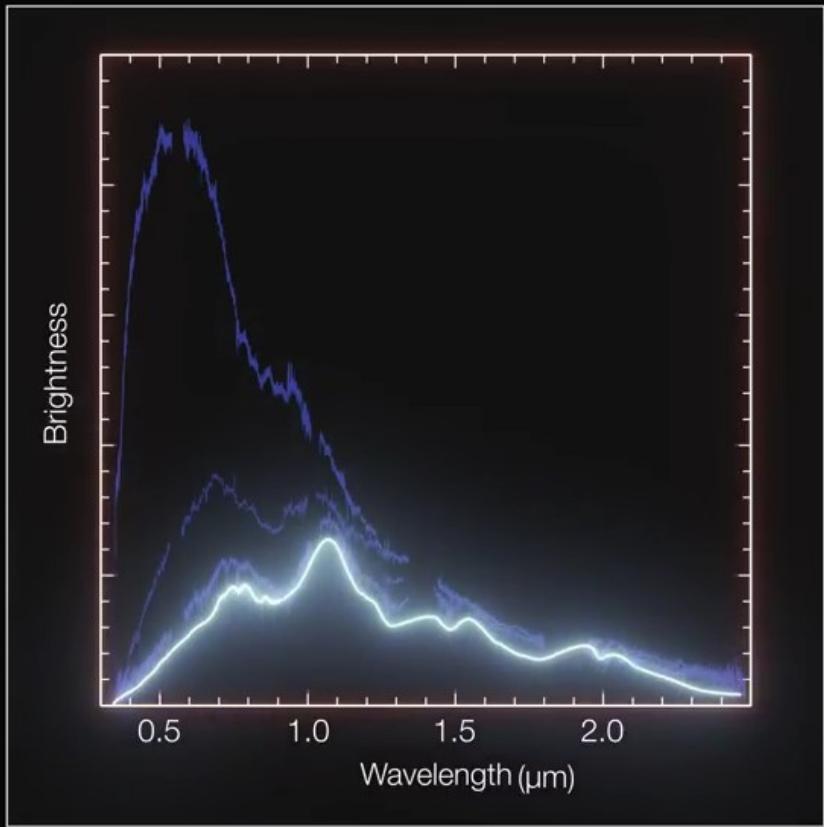


[from LIGO/Virgo]



[M. Weiss, CSC/NASA]

# The GW170817 kilonova (AT2017gfo)



Time: +4.1 days

[From ESO]



*Cataclysmic Collision Artist's illustration of two merging neutron stars. The narrow beams show the bursts of gamma rays that are shown here. The clouds glow with visible light and other wavelengths depicted. The clouds glow with visible light and other wavelengths*

# Astronomers Confirm Origin of Universe's Heaviest Elements in Neutron Star Mergers

Oct 17, 2017 by News Staff / Source

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OCTOBER 16, 2017

## Published in

Astronomy

## Tagged as

Gold

Gravitational

waves

Origin of Universe's heavy elements, ranging from gold to uranium, has finally been confirmed, after a gravitational wave source was [seen](#) and [heard](#) for the first time ever by an international collaboration of astronomers and astrophysicists.

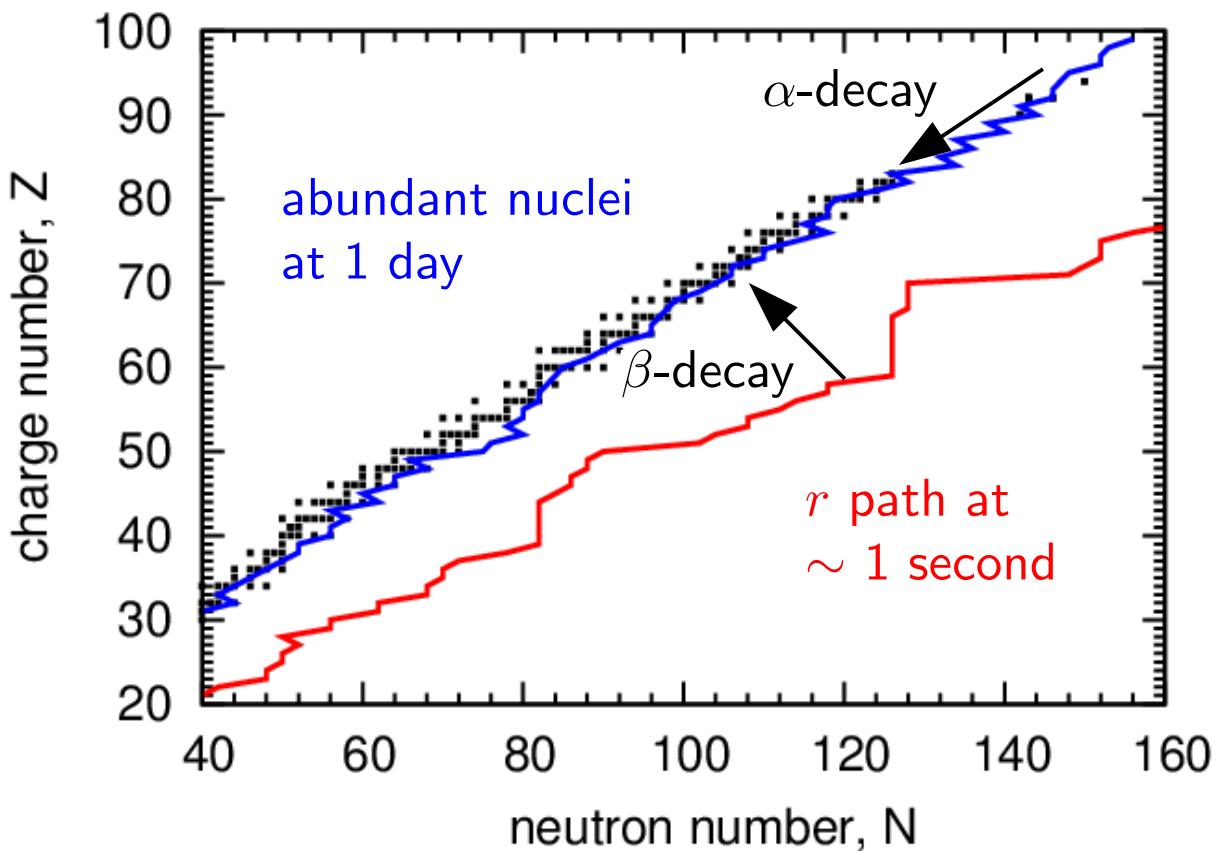
## Astronomers strike cosmic gold, confirm origin of precious metals in neutron star mergers

by University of California - Berkeley

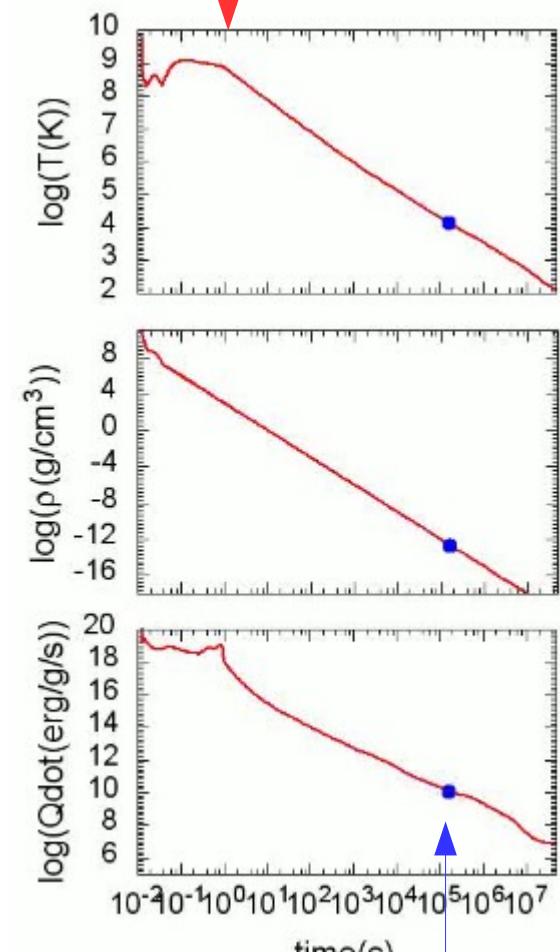
## r-process and kilonovae

The radioactive nuclei decaying back to the valley of stability keep injecting energy into the expanding ejecta can power EM transients peaked at  $\sim$  days

[Li & Paczynsky 1998, Metzger+ 2010,...]



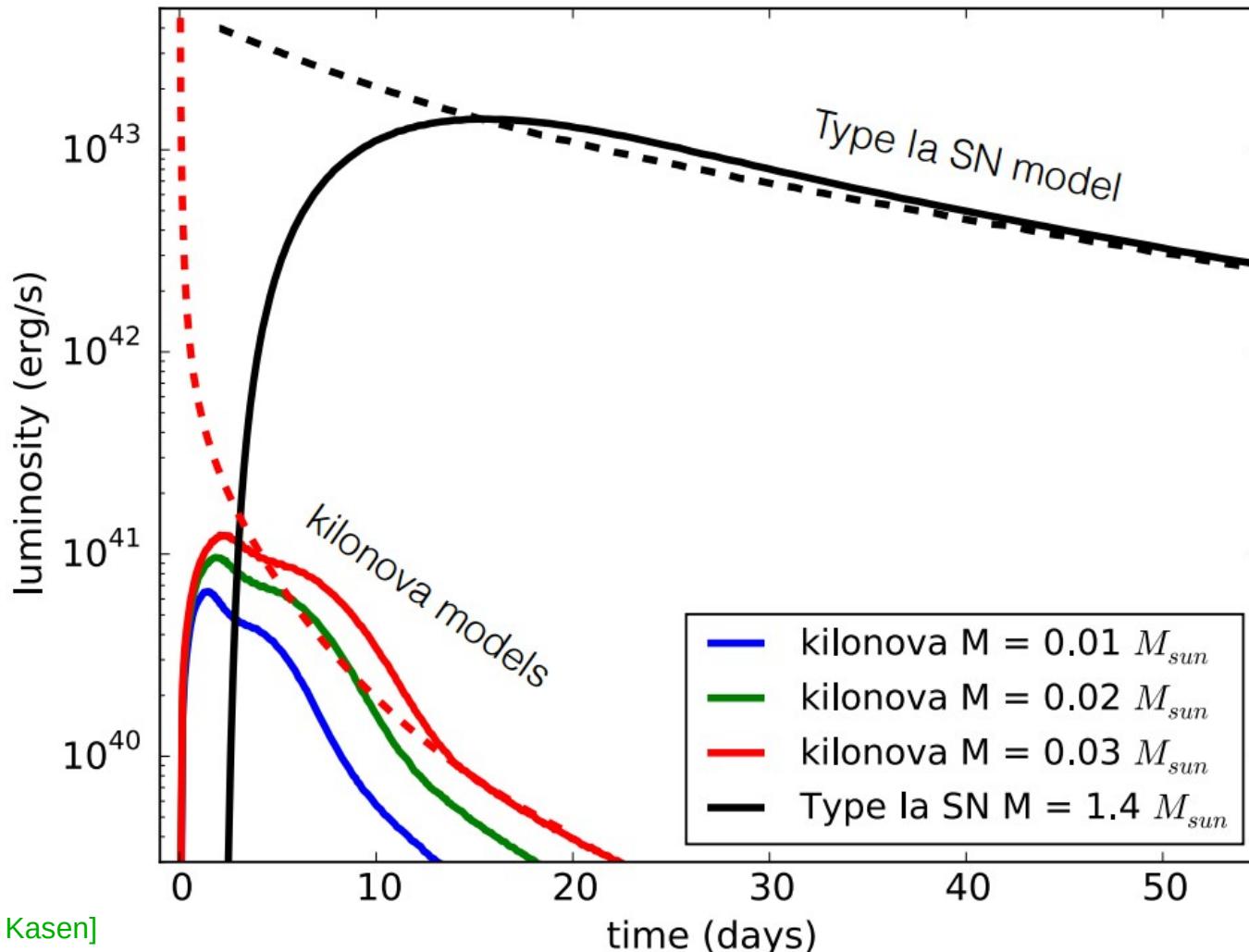
*r*-process ends here



we see kilonova here

## *r*-process and kilonovae

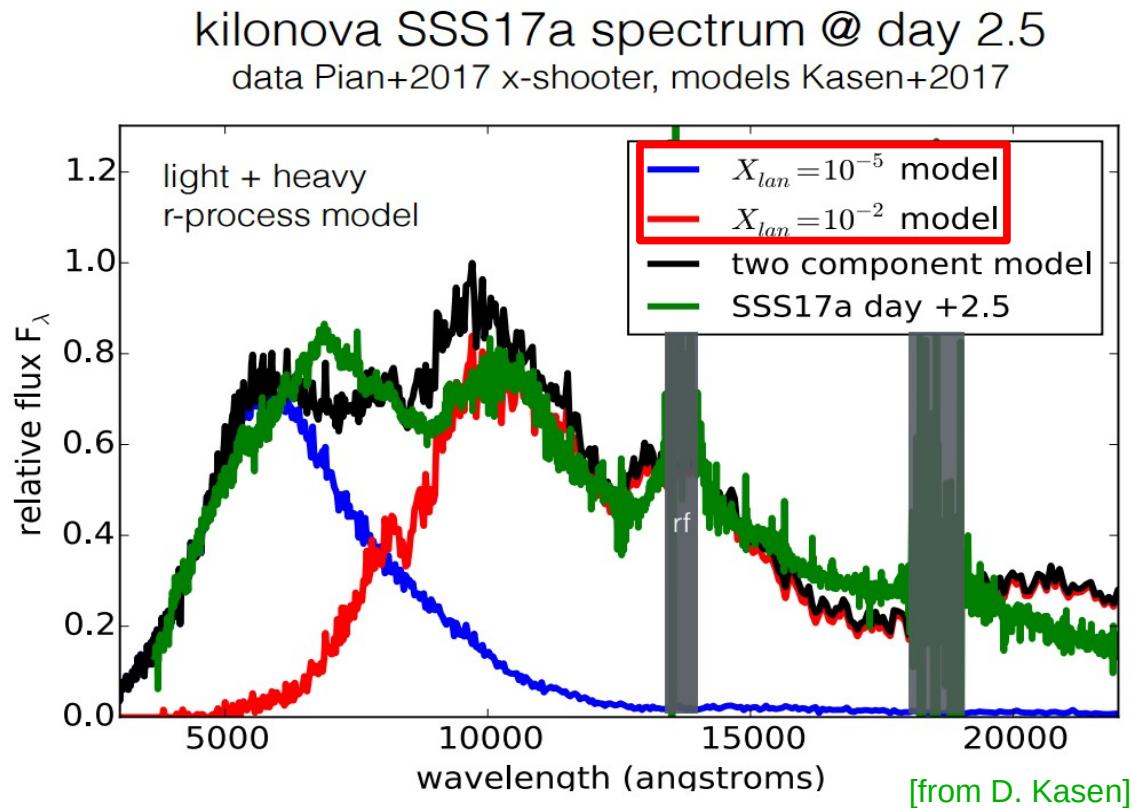
- more massive ejecta  $\rightarrow$  brighter but later peak light
- faster ejecta  $\rightarrow$  brighter and earlier peak light



# Evidence of the *r*-process

[Barnes+ 2013, Tanaka+ 2013]

Presence of lanthanides & actinides (with *f*-shell valance electrons)  
↔ large opacity and spectral peak at IR → signature of the *r*-process



$$M_{ej,tot} \sim 0.05 M_\odot$$

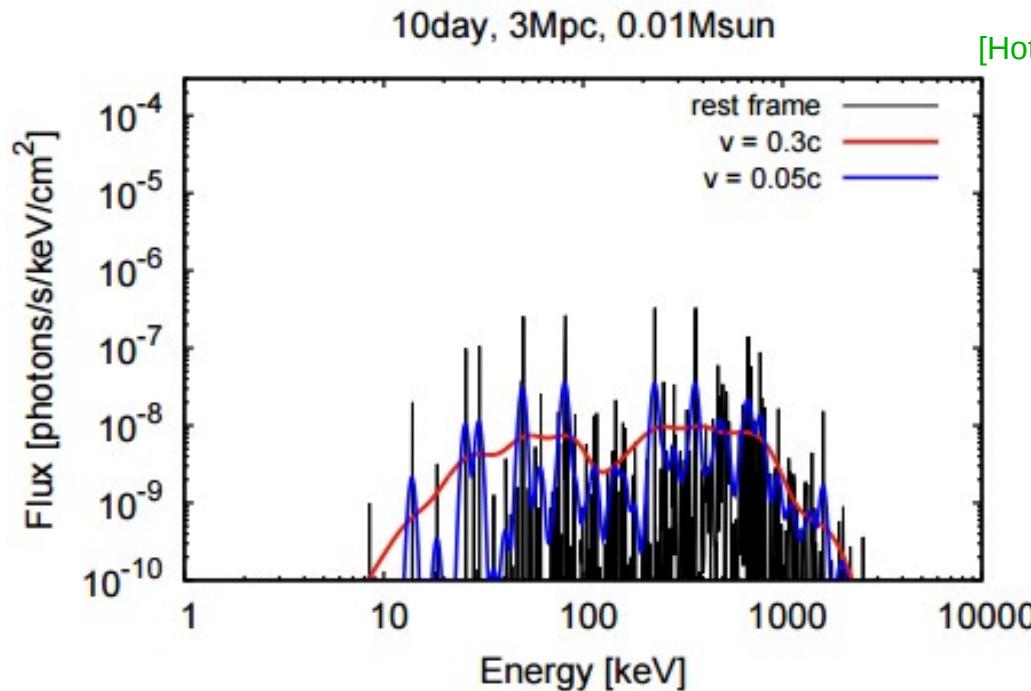
(see also analysis with multi-band lightcurves,  
e.g., Kilpatrick+,  
Cowperthwaite+,...)

- no direct evidence of individual elements Au, Pt, U, etc.
- uncertainties in opacity modeling, ejecta morphology,...  
(see e.g., Kawaguichi+, Wollaeger+, Waxman+,...)

## Definitive proof of existence of heavy nuclei?

Spectral analysis around peak-luminosity ( $\lesssim 10$  days) can only tell the information about lanthanides but not specific or heavier elements.

How about  $\gamma$ -rays? also difficult due to Doppler broadening and blending



[Hotokezaka+ MNRAS 459 (2016) 35]  
(see also Korobkin+2019)

(we may have better chances to catch the decay  $\gamma$ -ray lines from BNS merger remnants  $\sim 10^5$  yr old)

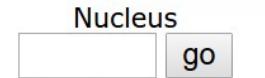
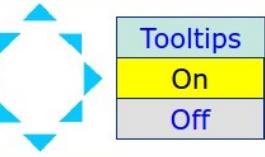
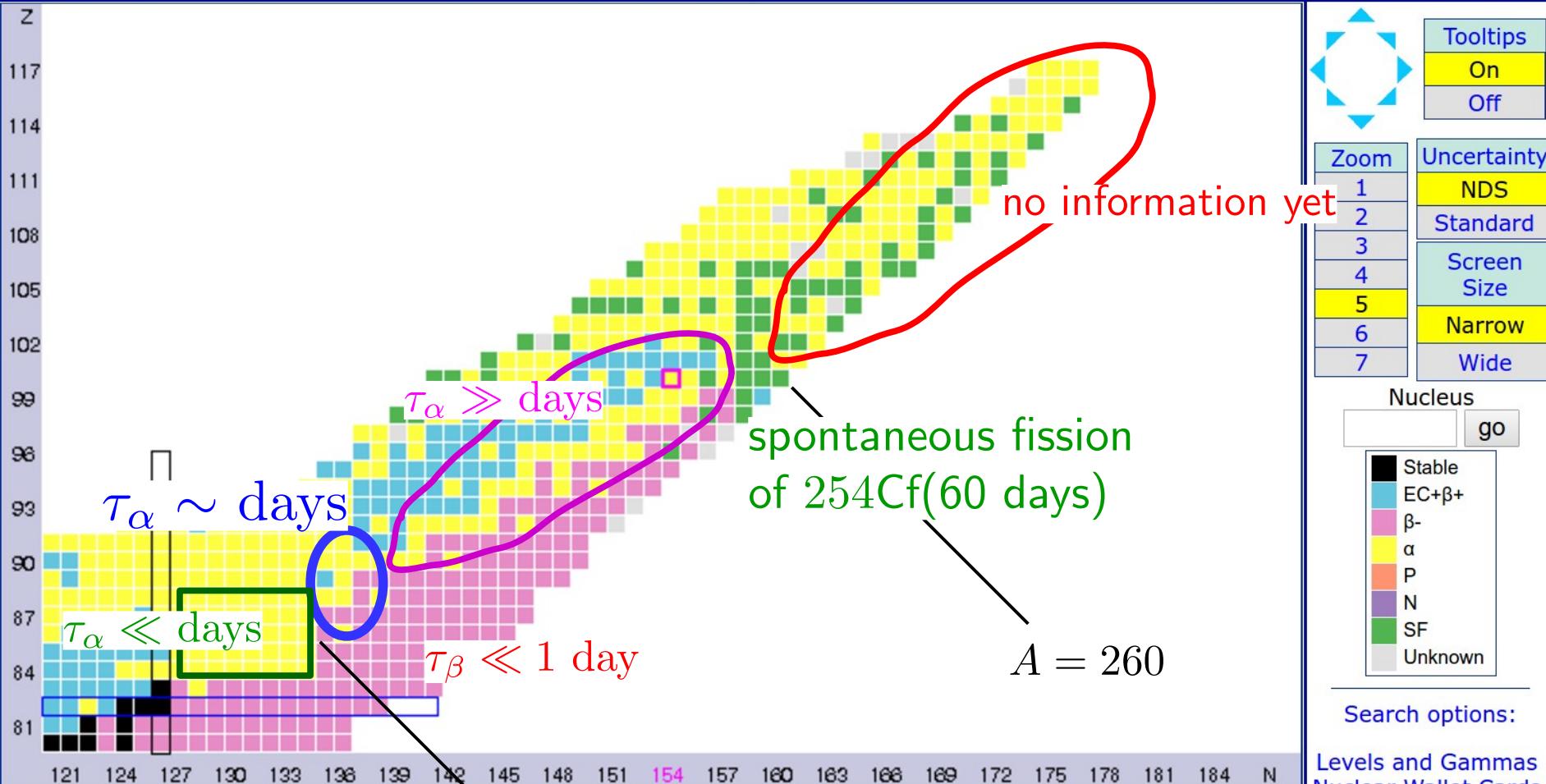
[MRW, Banerjee, Metzger+, ApJ 880 (2019) 23, arXiv:1905.03793]

# Trans-lead nuclei?

## Chart of Nuclides

Click on a nucleus for information

Color code	Half-life	Decay Mode	$Q_{\beta^-}$	$Q_{EC}$	$Q_{\beta^+}$	$S_n$	$S_p$	$Q_a$	$S_{2n}$	$S_{2p}$	$Q_{2\beta^-}$	$Q_{2EC}$	$Q_{ECp}$
$Q_{\beta^-n}$	BE/A	(BE-LDM Fit)/A	$E_{1st \ ex. \ st.}$	$E_{2+}$	$E_{3-}$	$E_{4+}$	$E_{4+}/E_{2+}$	$\beta_2$	$B(E2)_{42}/B(E2)_{20}$	$\sigma(n,\gamma)$	$\sigma(n,F)$	235U FY	239Pu FY



- Stable
- EC+ $\beta^+$
- $\beta^-$
- $\alpha$
- P
- N
- SF
- Unknown

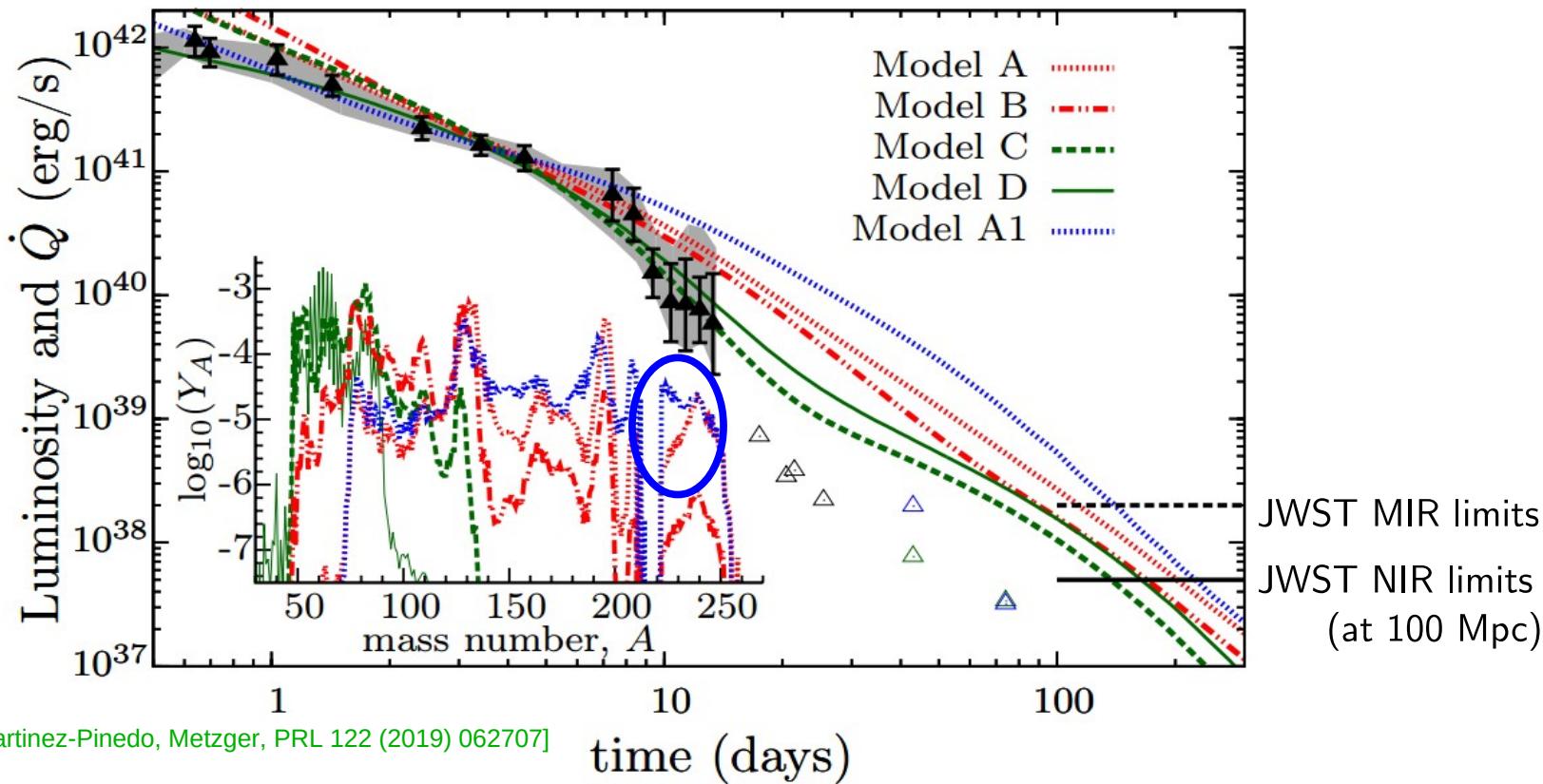
Search options:

Levels and Gammas  
Nuclear Wallet Cards

# Definitive proof of existence of heavy nuclei?

Recently, we showed that a precise measurement of the late-time lightcurve can offer diagnostic of heavy element composition

e.g., if  $\gtrsim 10^{-3}$  mass fraction of Ra and Rn gets produced



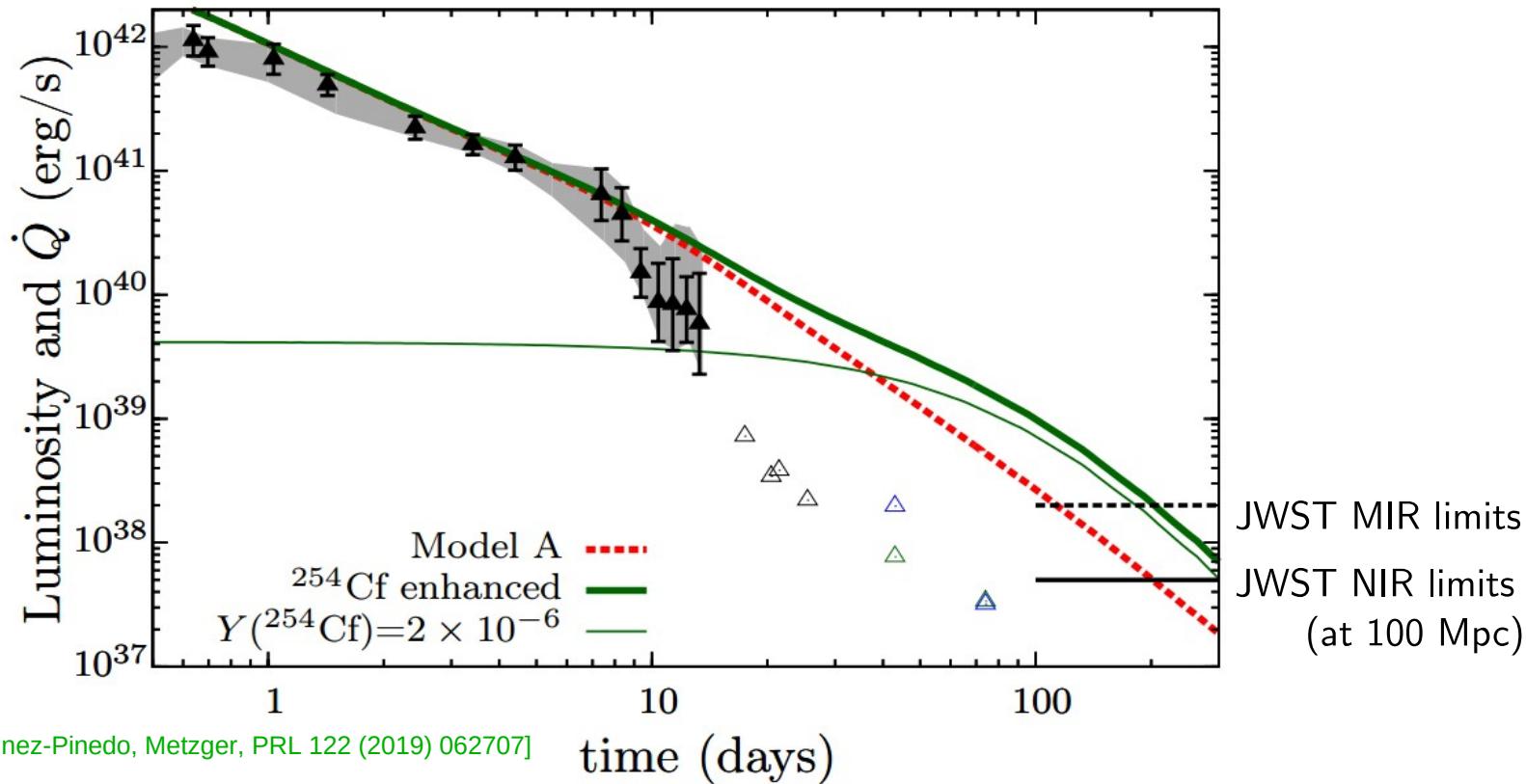
[MRW, Barnes, Martinez-Pinedo, Metzger, PRL 122 (2019) 062707]

(energy release by  $\alpha$ -decay of Ra or Rn)  $\gg$  (that from beta decays)

# Definitive proof of existence of heavy nuclei?

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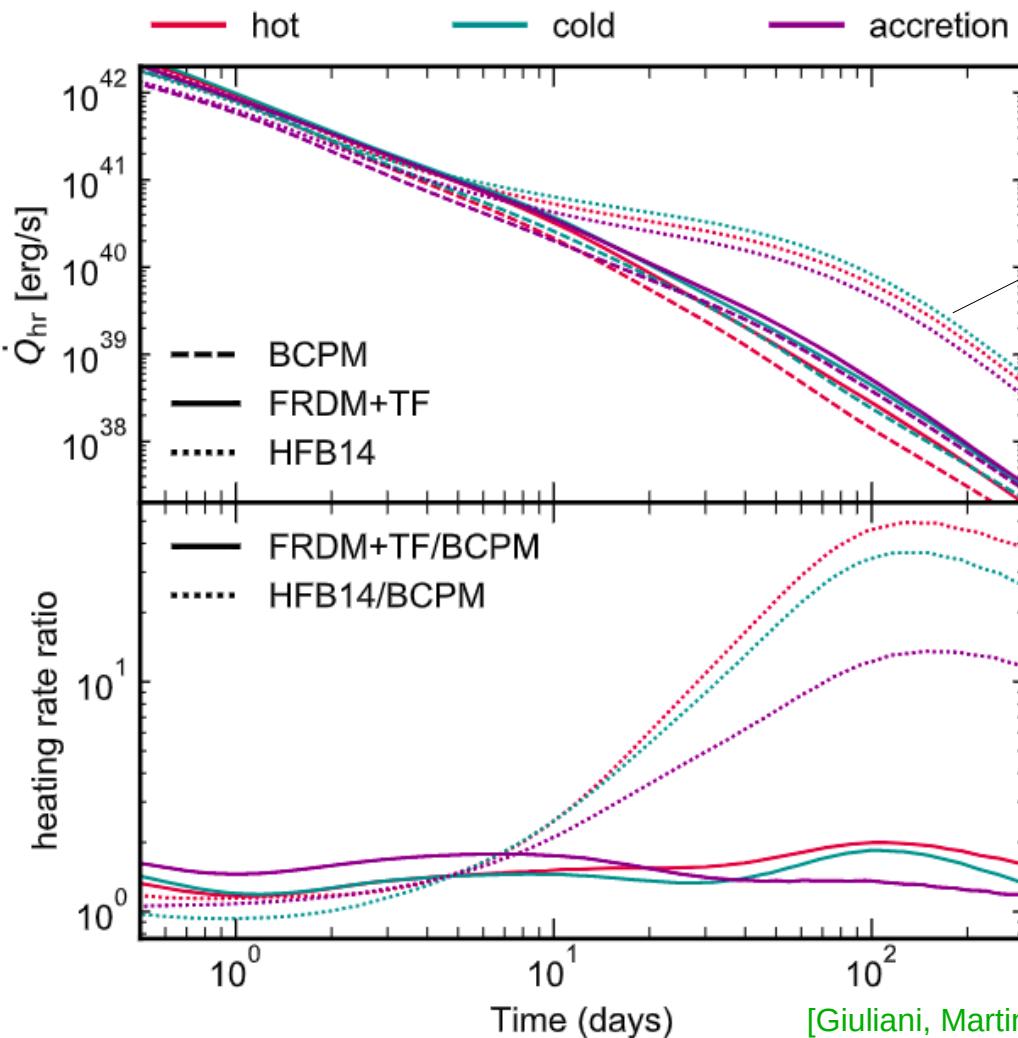
e.g., if  $\gtrsim 10^{-4}$  mass fraction of Cf gets produced



[MRW, Barnes, Martinez-Pinedo, Metzger, PRL 122 (2019) 062707]

(energy release by  $^{254}\text{Cf}$  fission)  $\gg$  (that from beta decays)

# Impact of fissions on kilonova lightcurve



models with  
high fission  
barriers and  
low fission  
branching

(see also work from  
LANL/ND/NCSU/Brussel/...)

[Giuliani, Martinez-Pinedo, MRW, Robledo, 1904.03733]

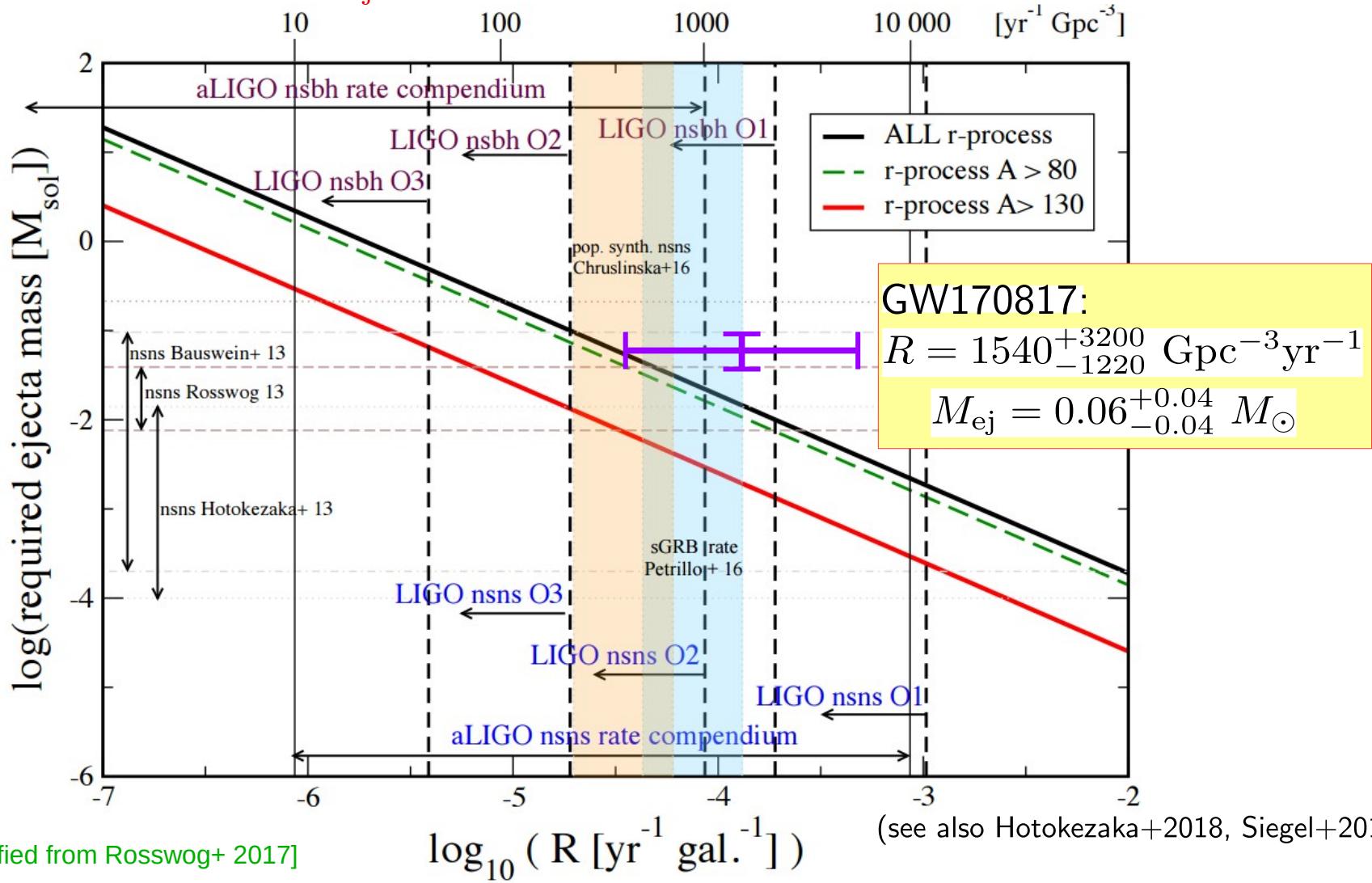
Such information may also be used to infer or constrain the properties (masses,  $\beta$ -decay, fission...) of exotic neutron-rich nuclei even beyond the reach of FRIB/FAIR/HIAF/...

Neutron star mergers as major sites of *r*-process?

# NS mergers as the dominating source of the *r*-process?

Assuming the entire Galactic r-process elements are made in the same way:

$$M_{r,\text{tot}} \sim \tau_{\text{MW}} \times R \times M_{\text{ej}}$$

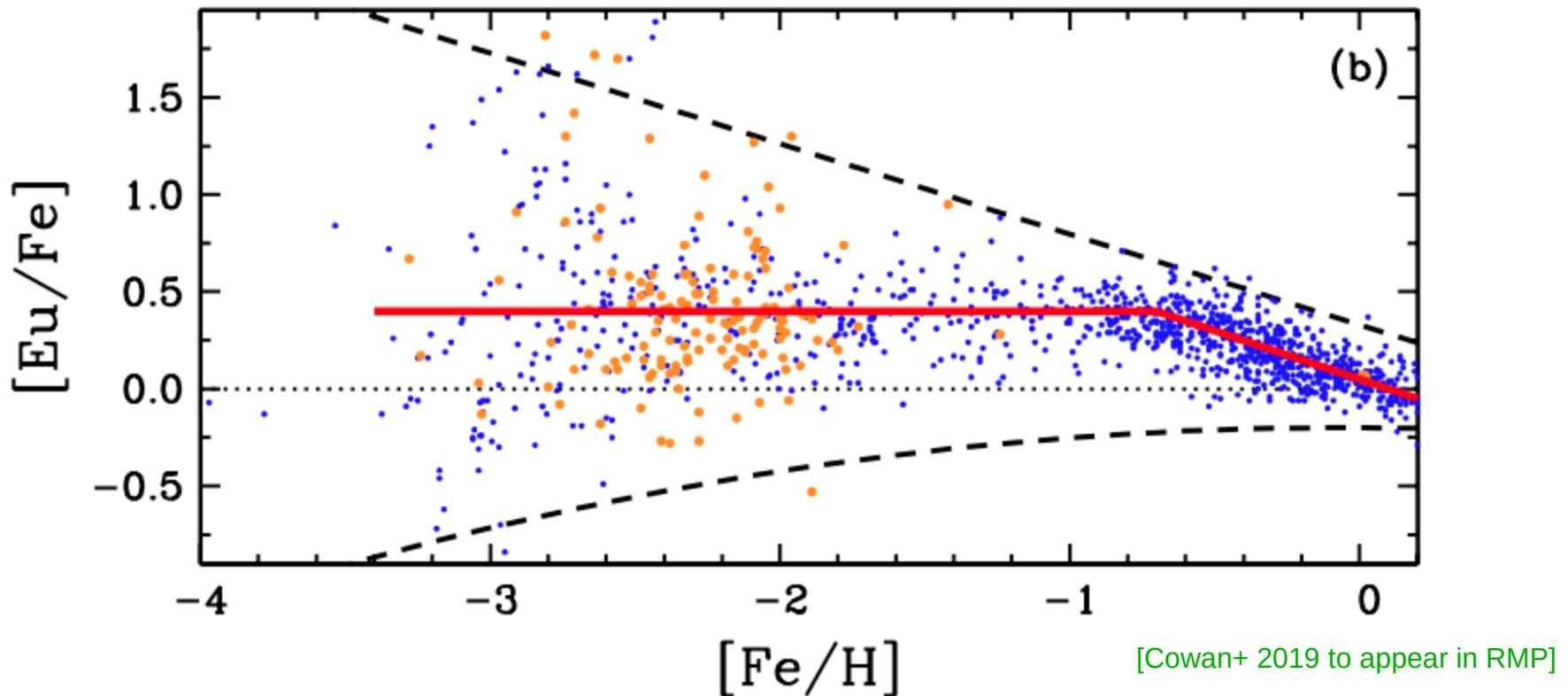


[Modified from Rosswog+ 2017]

\*\*updated rates for GW170817-like:  $760^{+1740}_{-650} \text{ Gpc}^{-3}\text{yr}^{-1}$  [LIGO & Virgo 2020]

# Tracing $r$ -process history with stellar abundances

Observation of Eu abundance for stars with different metalicity



- reach very low metalicity  $[\text{Fe}/\text{H}] < -3$  with large scatters
- $[\text{Eu}/\text{Fe}]$  decreases with  $[\text{Fe}/\text{H}] \gtrsim -0.8$

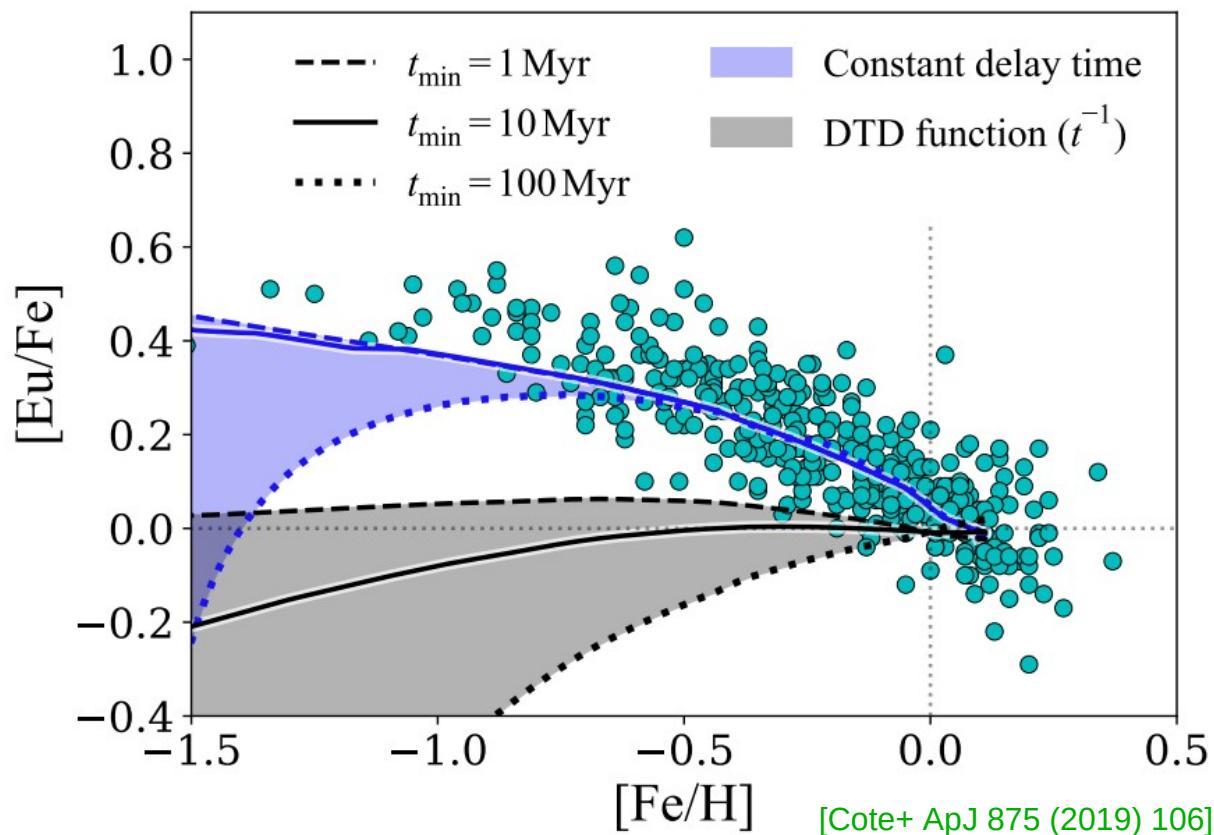
# Trend of [Eu/Fe] at high metalicity

Galactic chemical evolution simulations adopting a merger delay time distribution (DTD)  $\propto t^{-1}$ , inferred from sGRB data and population synthesis studies, result in a flat trend of [Eu/Fe] vs [Fe/H] at high metalicity

[Komiya&Shigeyama 2016, Cote+ 2017, Hotokezaka+ 2018,...]

Possible solutions:

- a steeper DTD?  
[Hotokezaka+ 2018, Cote+2018]
- subgroup of fast mergers?  
[Cote+2018]
- additional sources?  
[Cote+2018, Siegel+2019]
- different ISM enrichments  
from SNe vs NSMs?  
[Schoenrich & Weinberg 2019]



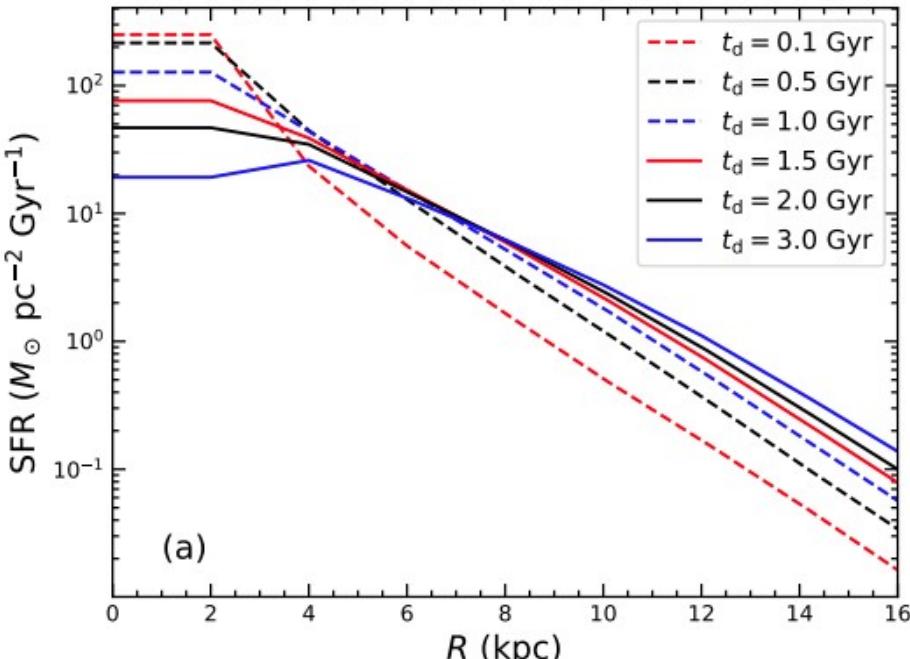
## Trend of [Eu/Fe] at high metalicity

However, no models have considered the migration of neutron star binaries in the Milky Way

$$d_{\text{travel}} \sim 10 \text{ kpc} \left( \frac{\tau_{\text{merger}}}{100 \text{ Myr}} \right) \left( \frac{v}{100 \text{ km/s}} \right)$$

We consider the effect due to the kick of neutron star binary systems and the inside-out evolution of the MW:

SFR extracted from Minchev+2013

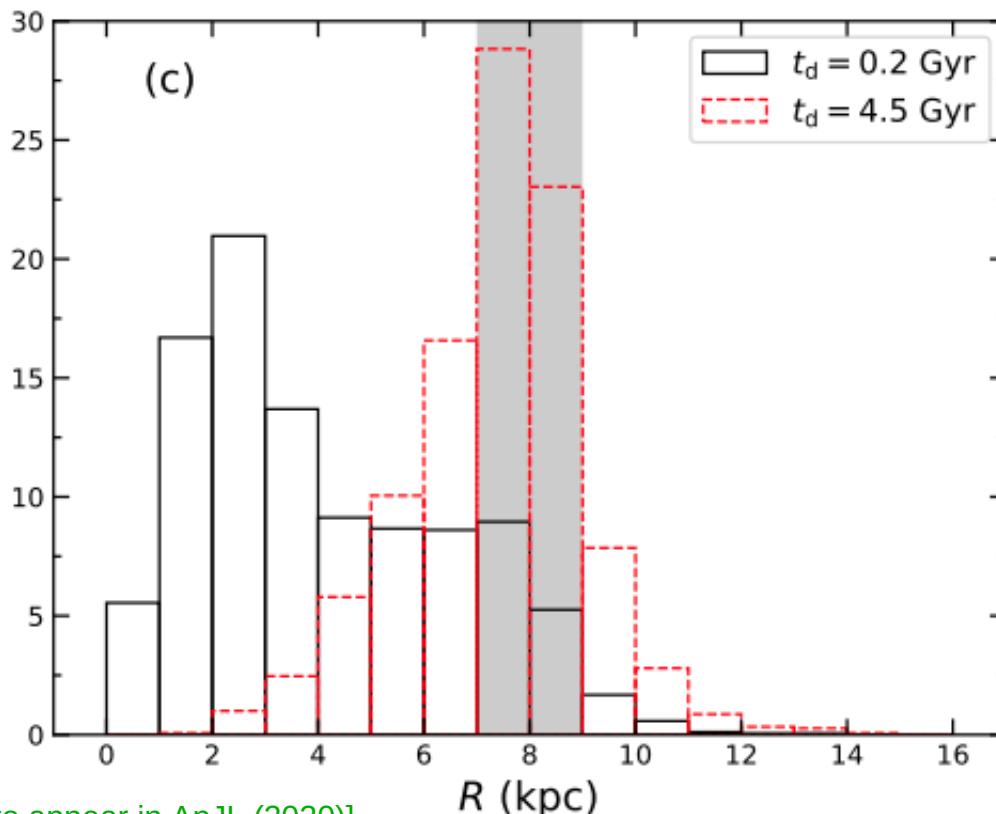


[Banerjee, MRW, Yuan, to appear in ApJL (2020)]

- SFR with inside-out evolution
- merger DTD  $\propto t^{-1}$  with different  $t_{\min}$  of 10 and 30 Myr
- kick distribution  $\propto \exp(-v/v_0)$ , with different  $v_0$  between 60 & 120 km/s
- trace the location of NSMs with GALPY using different MW potentials at different times

## Trend of [Eu/Fe] at high metalicity

normalized NSM contribution from different parts to the solar neighborhood:



[Banerjee, MRW, Yuan, to appear in ApJL (2020)]

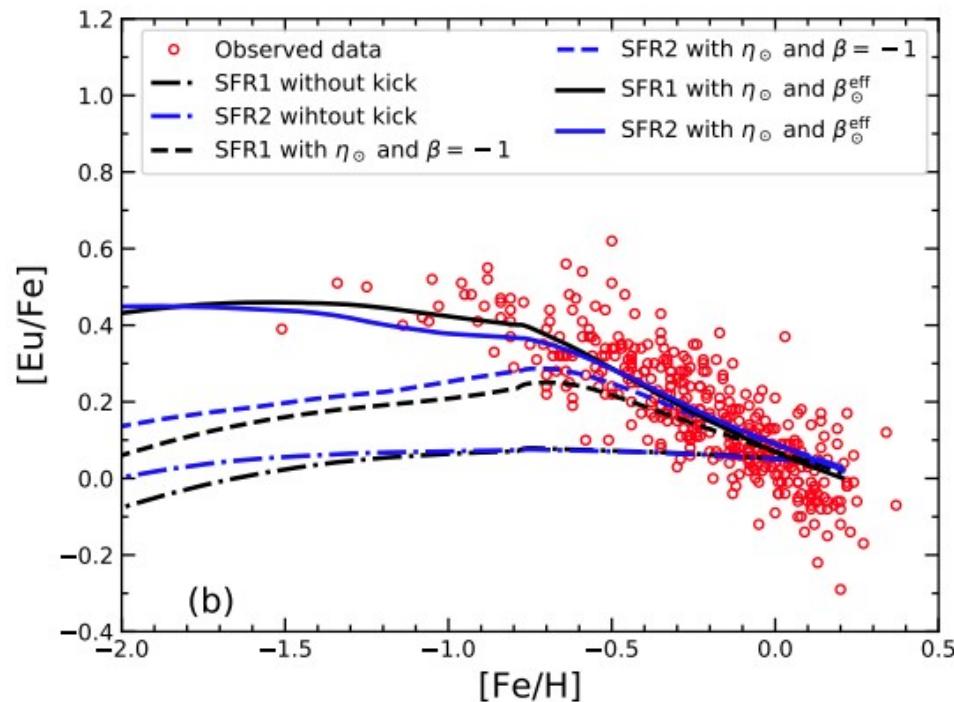
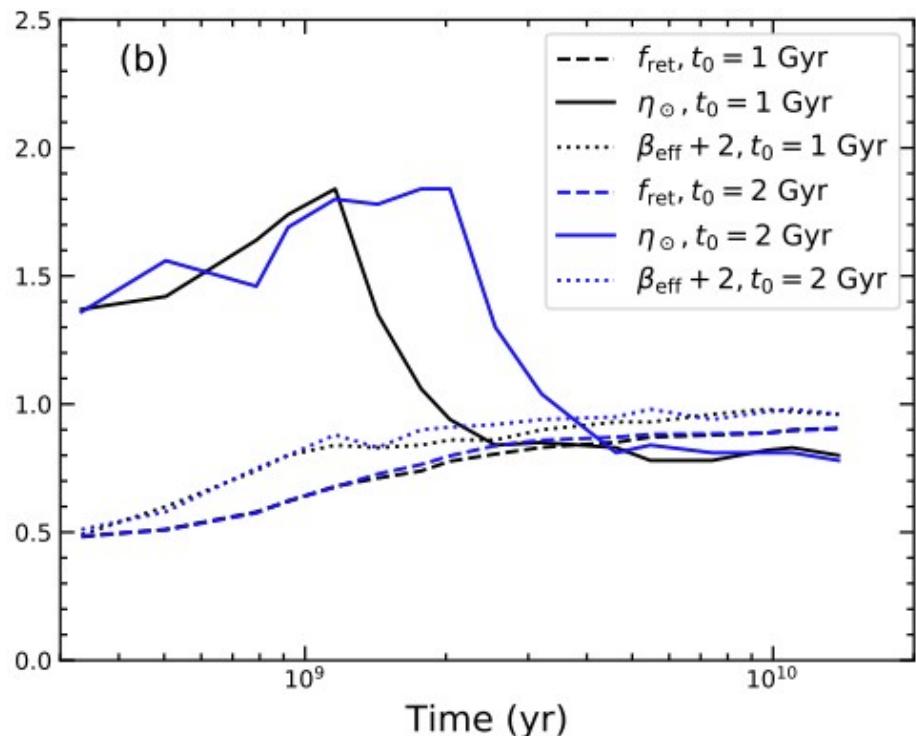
The solar neighborhood can receive a large contribution of mergers born at the inner part of MW at earlier time

## Trend of [Eu/Fe] at high metalicity

This effect boosts the effective merger rate ( $\eta_\odot$ ), and steepens the effective DTD distribution  $\propto t^{\beta, \text{eff}}$  more at earlier times

→ decreasing [Eu/Fe] at  $[\text{Fe}/\text{H}] \gtrsim -0.8$

[Banerjee, MRW, Yuan, to appear in ApJL (2020)]



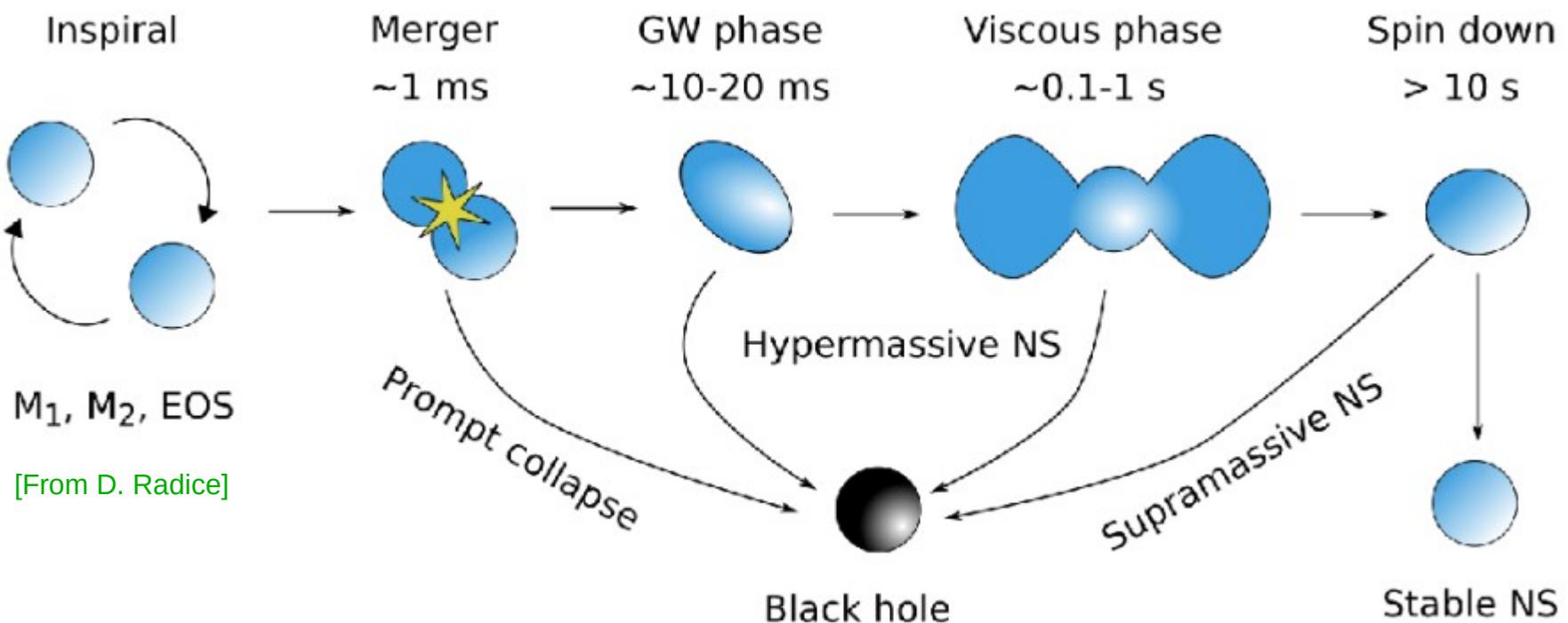
In principle this works differently at different parts of MW, can future observation possibly test it?

Neutrino flavor oscillations in dense media

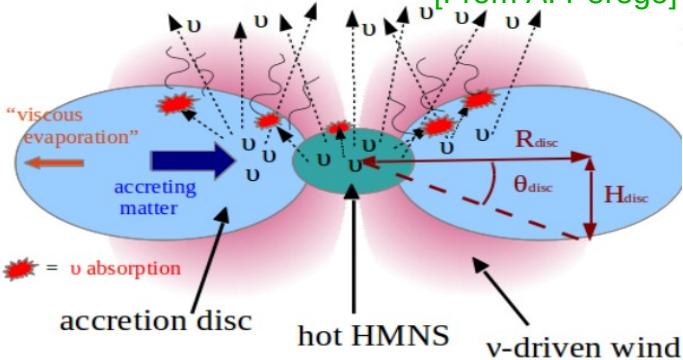
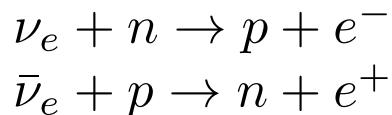
# Diversity of binary neutron star mergers

Mergers of two neutron stars can result in a variety of outcomes, depending on the initial state of the system and the yet-unknown nuclear EoS

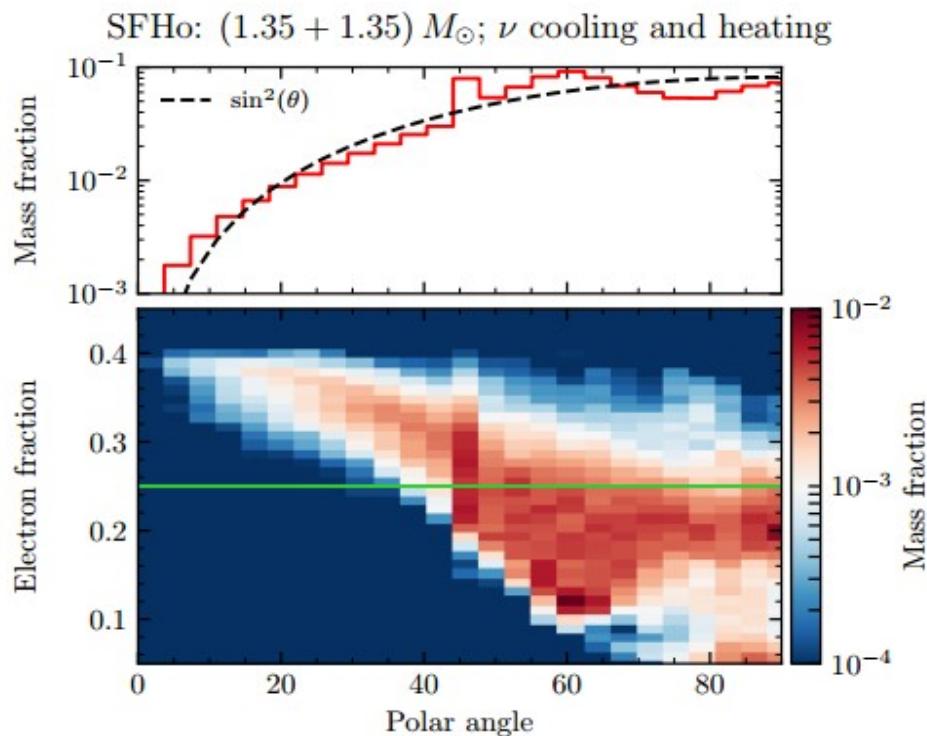
→ different post-merger GW signals & EM emissions



# Neutrinos in merger ejecta



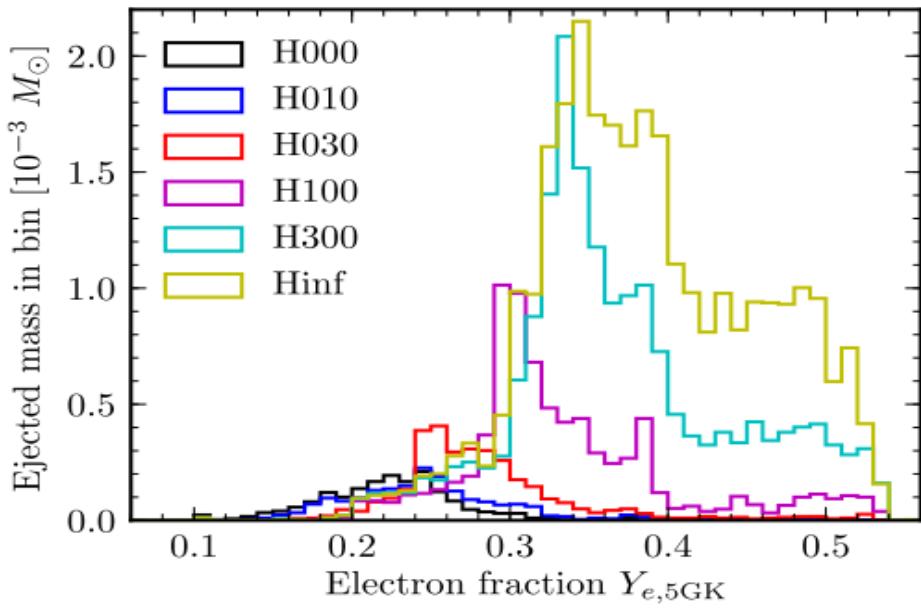
BNS dynamical ejecta: [Radice+ 2017]



(See also Wanajo+ 2015, Goriely+ 2016, Shibata+ 2017)

disk ejecta:

[Lippuner+ 2017]



(See also Perego+ 2014, Just+ 2015, MRW+ 2016)

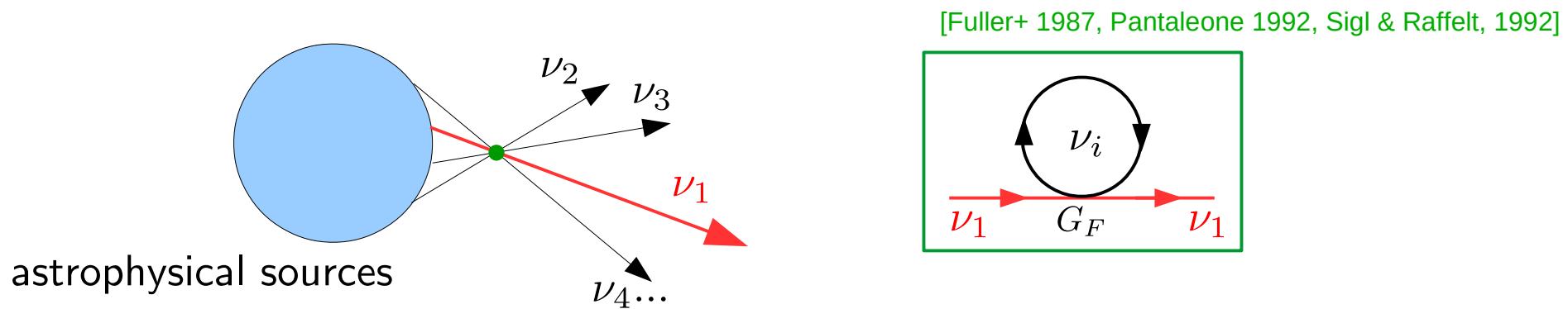
(\*\*the “blue” emission of the GW170817 kilonova also hints the role of neutrinos)

# Neutrino oscillations in neutrino-dense environment

Equation of Motion:  $(\partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}}) \varrho(\mathbf{x}, \mathbf{p}, t) = -i[H(\mathbf{x}, \mathbf{p}, t), \varrho(\mathbf{x}, \mathbf{p}, t)] + \mathcal{C}(\varrho)$

$\varrho$ : Wigner-transformed flavor density matrix,  $= \begin{pmatrix} f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{e\mu}^* & f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{e\tau}^* & \varrho_{\mu\tau}^* & f_{\nu_\tau} \end{pmatrix}$

$H(\mathbf{x}, \mathbf{p}, t) \supset \sum_{\mathbf{p}'} (\varrho(\mathbf{x}, \mathbf{p}', t) - \varrho^*(\mathbf{x}, \mathbf{p}', t))(1 - \mathbf{v} \cdot \mathbf{v}') \rightarrow$  non-linear coupling



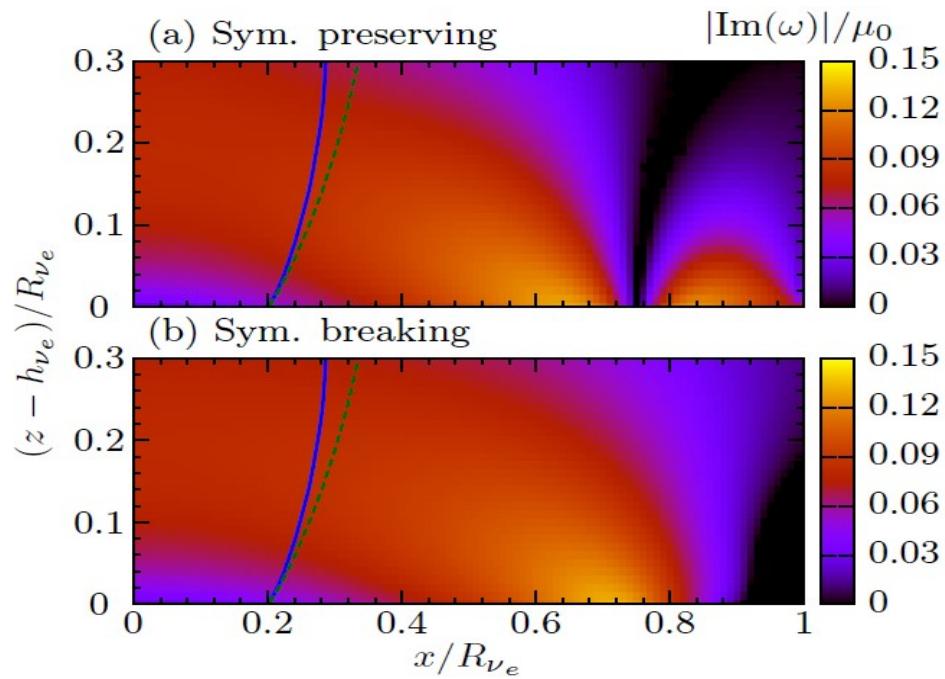
→ many-body quantum system in "strong" coupling regime ( $G_F n_\nu \gg \frac{\delta m^2}{2E_\nu}$ ), leading to "collective" oscillations, extensively studied in the context of core-collapse supernovae

[Duan+, Raffelt+, Mirizzi+, Volpe+, Balantekin+, Dasgupta+, Qian+, MRW+, Tamborra+, Lisi+, Shalgar+, Abbar+, Capozzi+, Sen+,...]

# Fast neutrino oscillations in merger remnants

In a series of (on-going) work, we show that neutrino flavor oscillations can develop in a timescale of  $\sim$ nano-seconds, or a length scale of  $\sim$  centi-meters, by means of linearized stability analysis

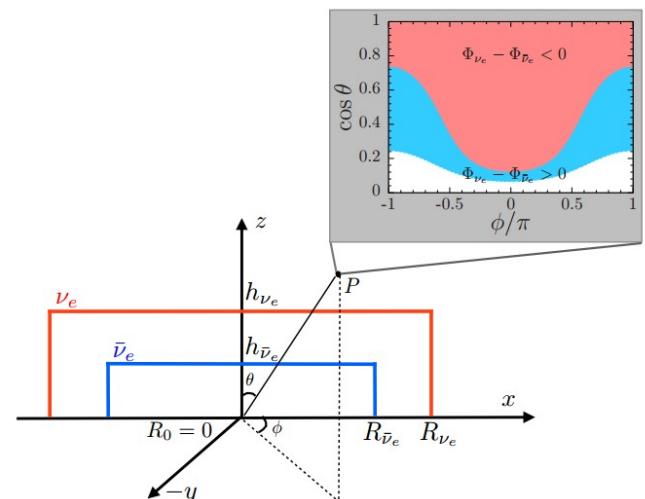
→ fast flavor conversion



[MRW & Tamborra, PRD 95, 103007 (2017)]

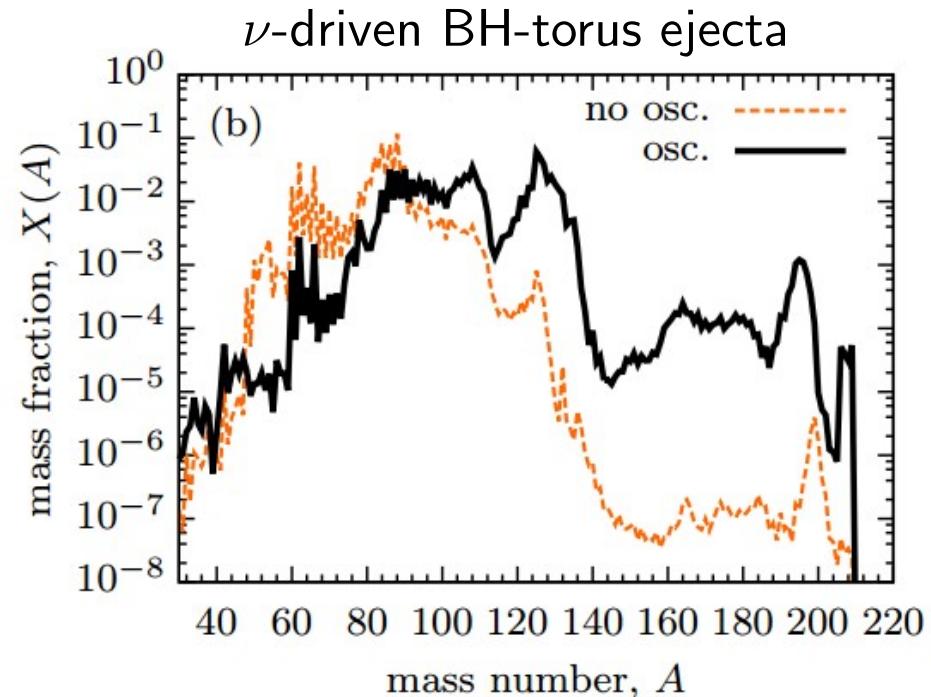
$\text{Im}(\omega)$ : growth rate of flavor mixing in the linear regime

$$\mu_0 \approx 4.25 \text{ cm}^{-1} \times \left( \frac{L_{\nu_e}}{10^{53} \text{ erg/s}} \right) \left( \frac{10 \text{ MeV}}{\langle E_{\nu_e} \rangle} \right) \left( \frac{100 \text{ km}}{R_{\nu_e}} \right)^2$$

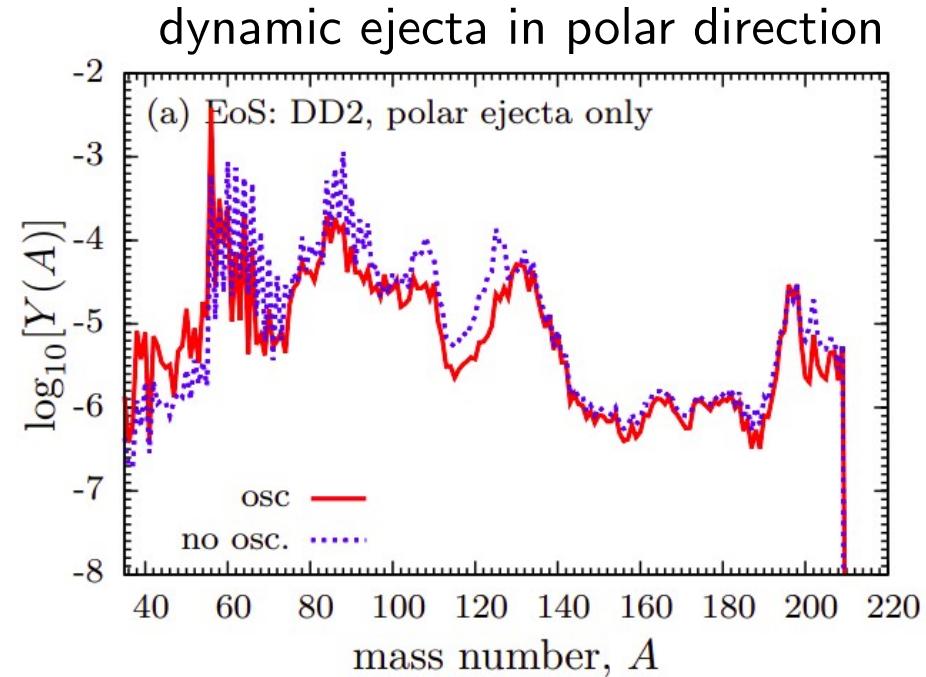


# Fast neutrino oscillations in merger remnants

Assuming flavor equipartition being the outcome of fast flavor conversions:



[MRW+ PRD 96 (2017) 123015]

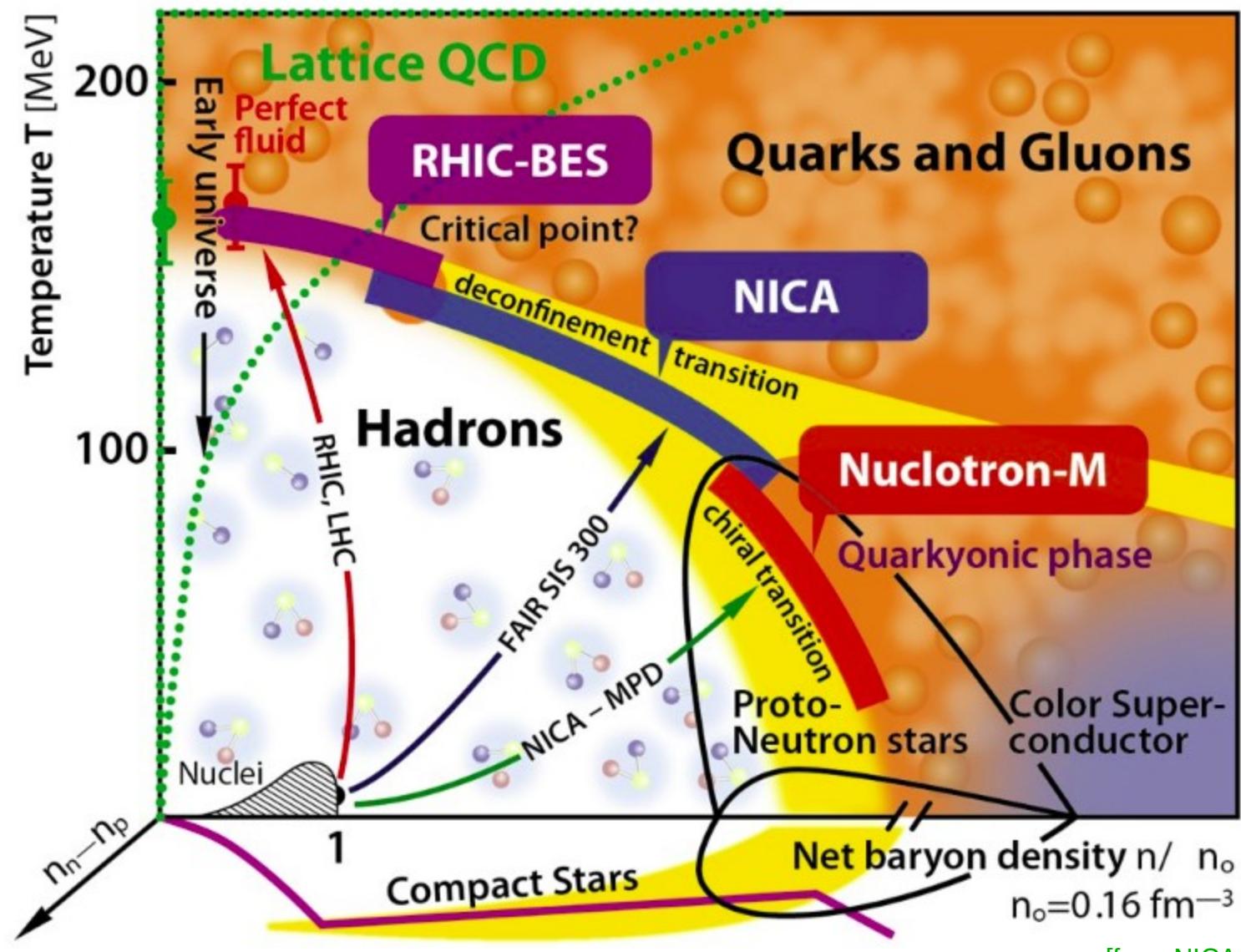


[George & MRW+, 2009.04046]

advanced numerical modeling of multi-dimensional quantum kinetic transport of neutrinos is needed

Probing dense matter with astrophysics?

# Do we have QCD matter at high density in nature?

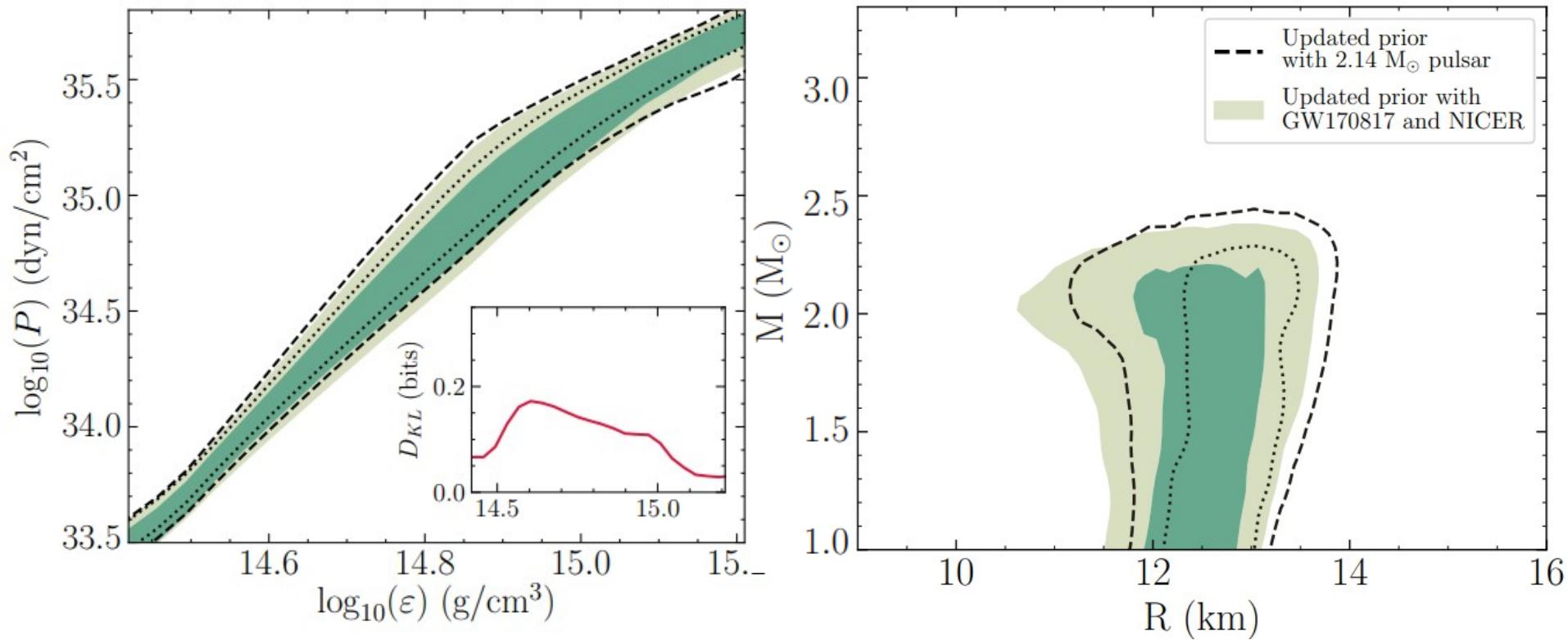


[from NICA website]

# Tightening the constraints on dense matter

current constraint based on (extrapolation from chiral effective field theory above  $1.1 \rho_s$ ) + (a minimum  $M_{\text{NS}} = 2.14 M_{\odot}$ ) + (GW170817) + (NICER measurement)

[Raaijmakers+ ApJL 893 (2020) L21]



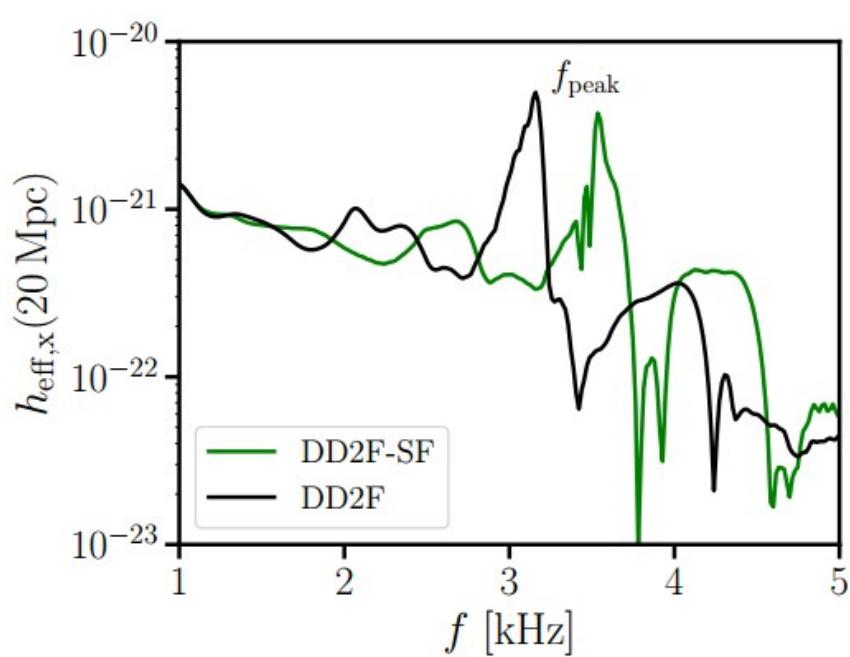
Can there be a phase transition to quark matter at high density?

[e.g. Annala+2019, Ferreira+2020, Xie+2020]

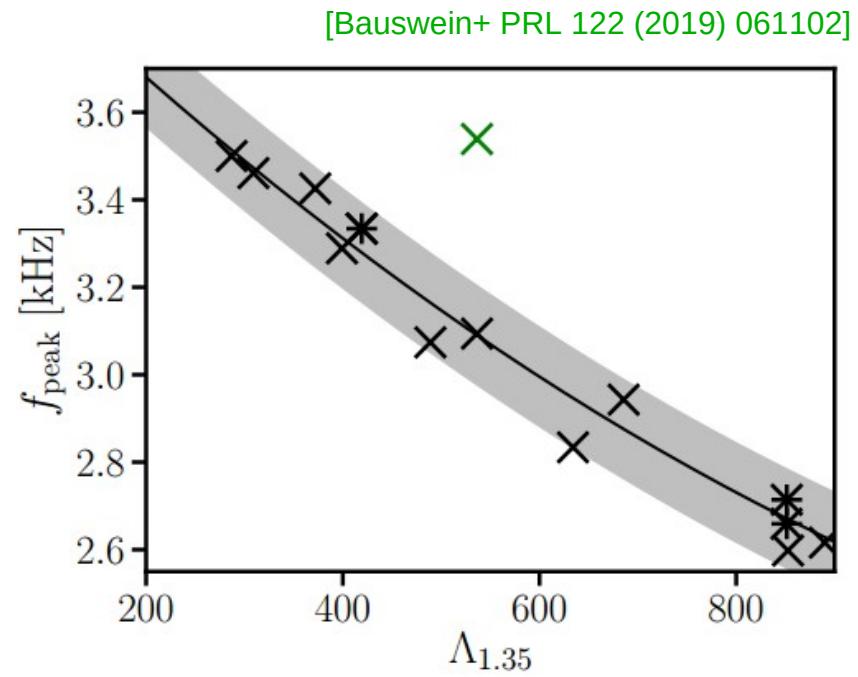
# Signaure of quark matter with neutron star mergers?

The appearance of quark matter inside the post merger remnant can make the hypermassive neutron star more compact

→ higher post-merger GW frequency that may be tested by future GW events



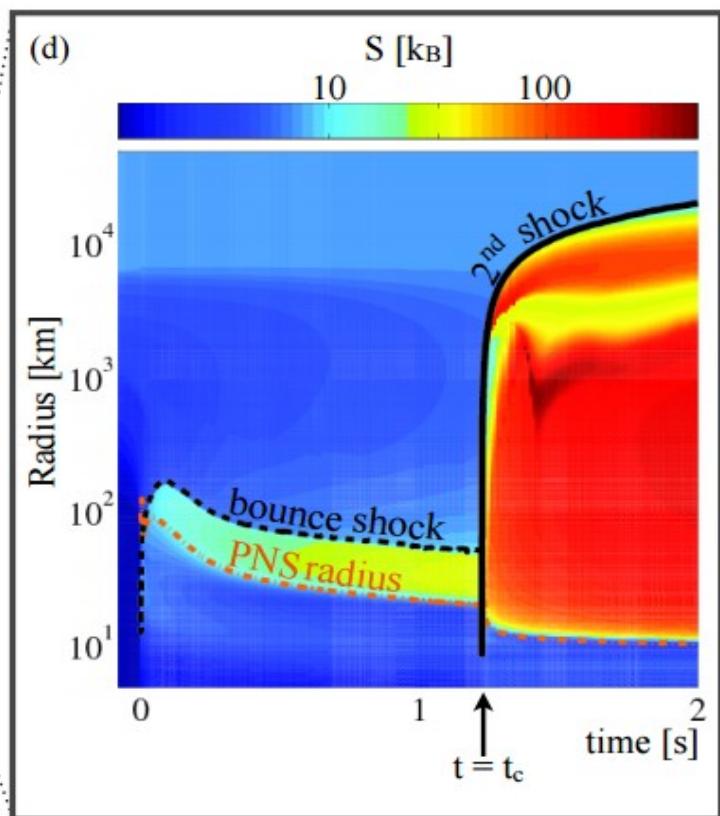
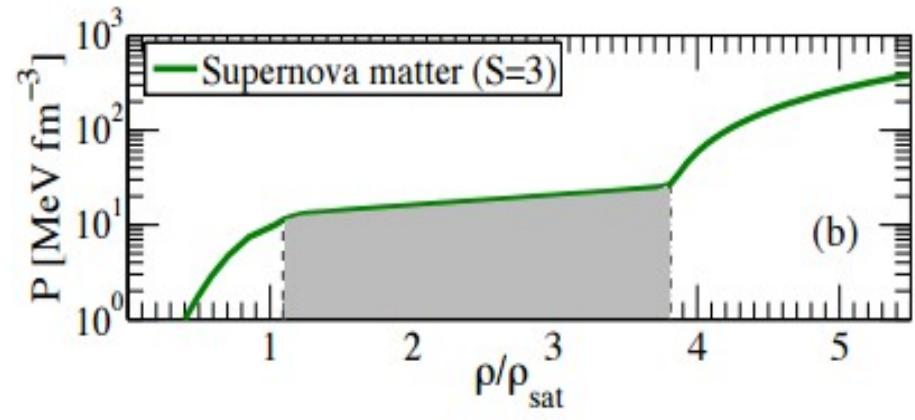
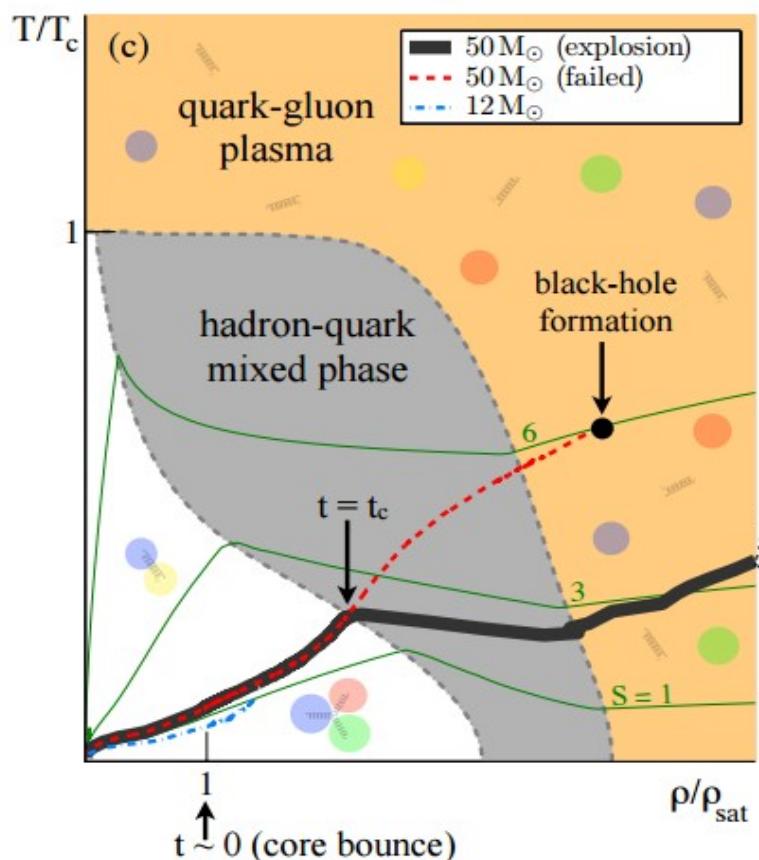
(see also e.g., Most+2019)



( $\Lambda_{1.35}$ : tidal deformability parameter of a  $1.35 M_\odot$  NS)

# Probing quark matter with supernovae?

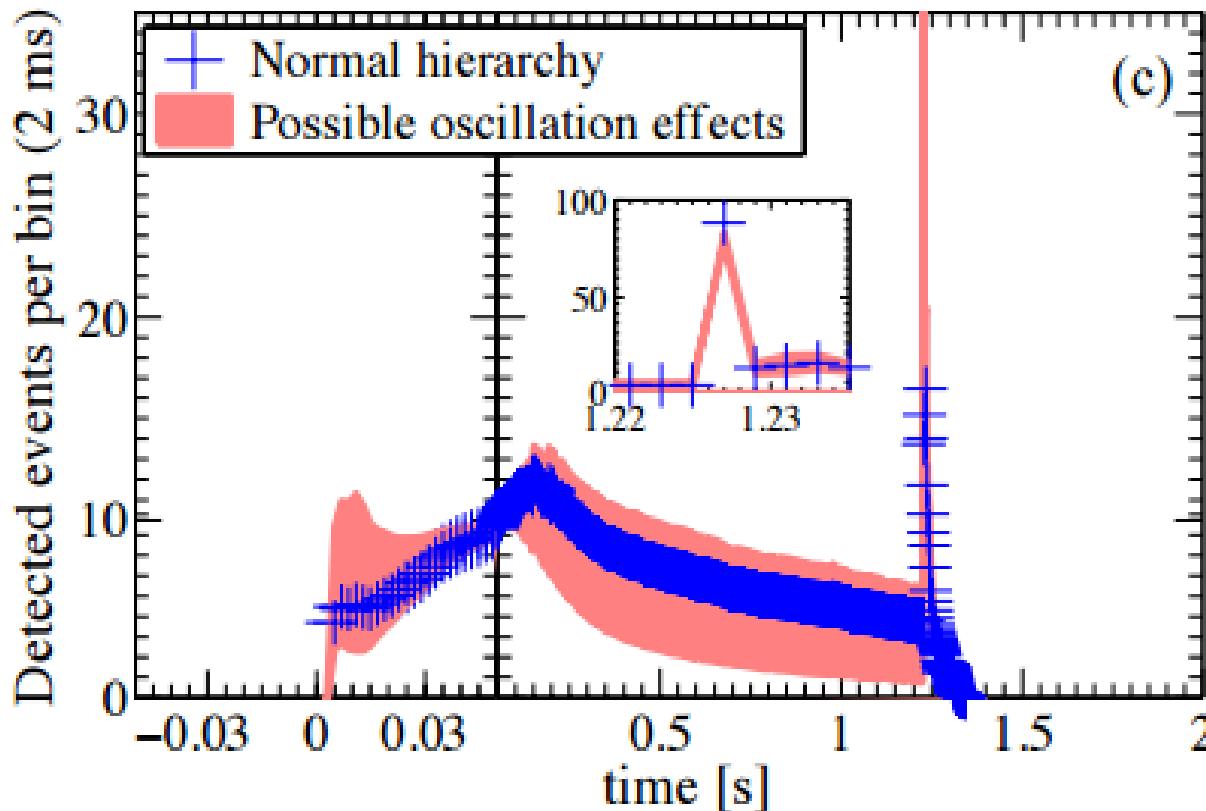
A SN explosion can also be possibly triggered by the phase-transition of hadronic matter to quark matter



# Probing quark matter with supernovae?

A SN explosion can also be possibly triggered by the phase-transition of hadronic matter to quark matter

→ a millisecond neutrino burst!

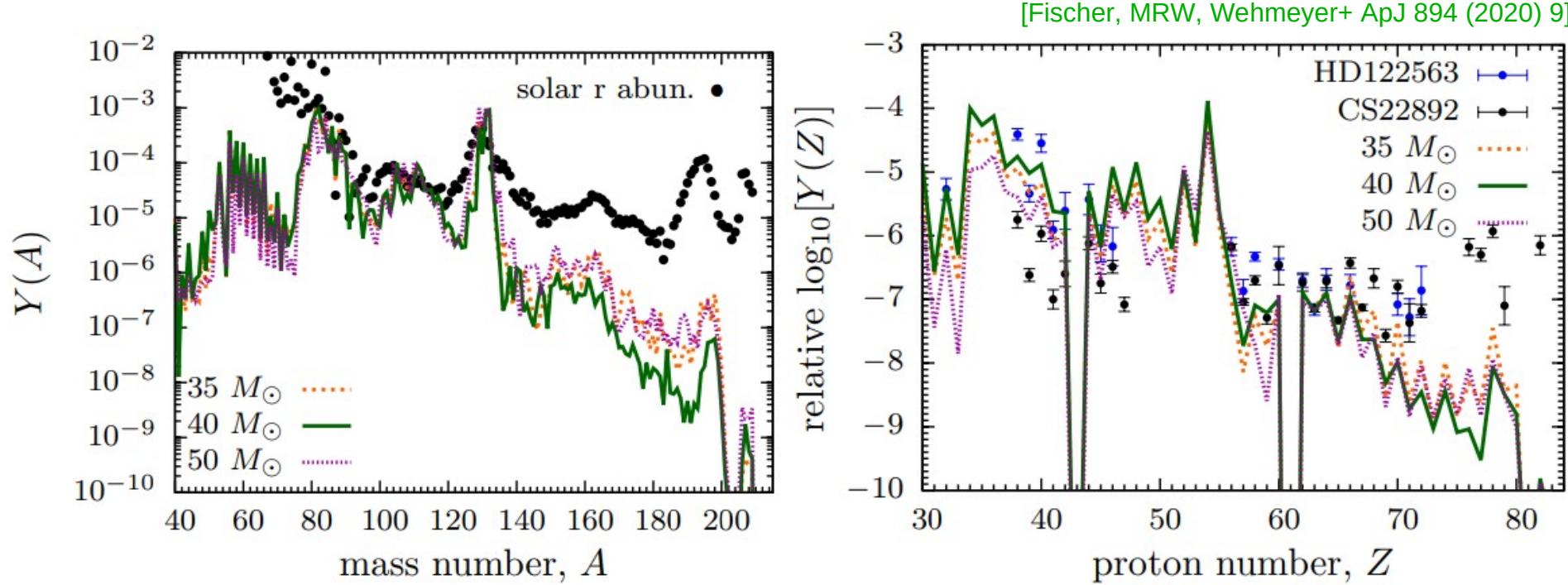


[Fischer, Bastian, MRW+ Nature Astronomy 2, 980 (2018)]

[see also e.g., Takahara & Sato (1988), Sagert, Fischer, Hempel, et. al. (2009)]

# Probing quark matter with supernovae?

The integrated nucleosynthesis yields from phase-transition supernovae:



Can we possibly constrain the fraction of massive stars undergoing phase transition with metal-poor star data?

# Summary

- Understanding the origin of heavy elements and the nature of dense matter are genuinely connected. The multimessenger detection of GW170817 has resulted in big steps forward in these directions.
- Quite a number of questions remain to be solved, e.g., what's been made in merger ejecta beyond lanthanides? do we need sites other than mergers? what are the behavior & roles of neutrinos? are quark matter present inside neutron stars?
- Synergetic efforts between theory, computation, experiments, and observations are needed to further address these questions.

