Physics of Open Systems

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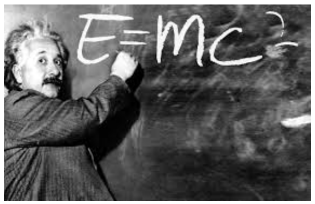
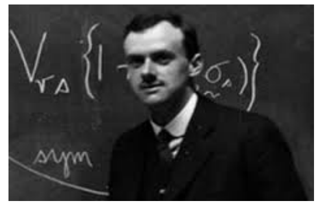
Outline

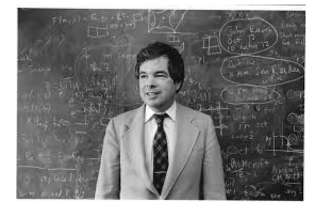
- ✧ Introduction to general open systems
- ✧ A general quantum theory of open systems
- ✧ General analytical solution of non-Markovianity
- ✧ Measurement of non-Markovianity (a new way)
- ✧ Non-Markovian nonequilibrium physics
- ✧ Non-Markovian entanglement decoherence
- ✧ Non-Markovian decoherence of Majorana qubits
- ✧ Transient quantum transport and quantum control
- ✧ Summary

Our Physical World

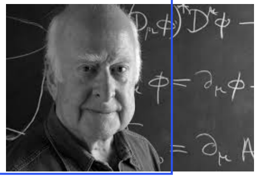
Smiling Face of Universe

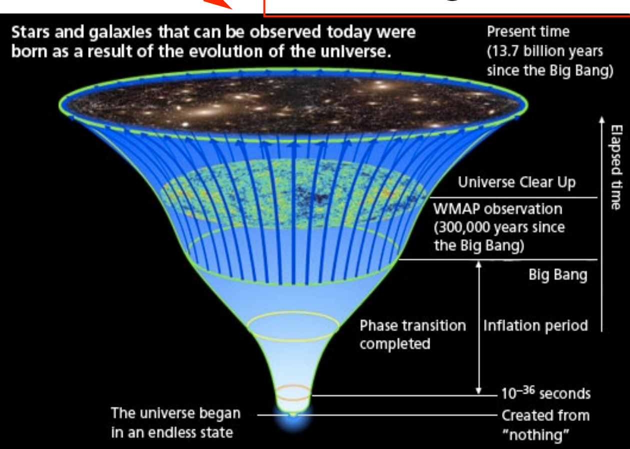
All the Physics we know



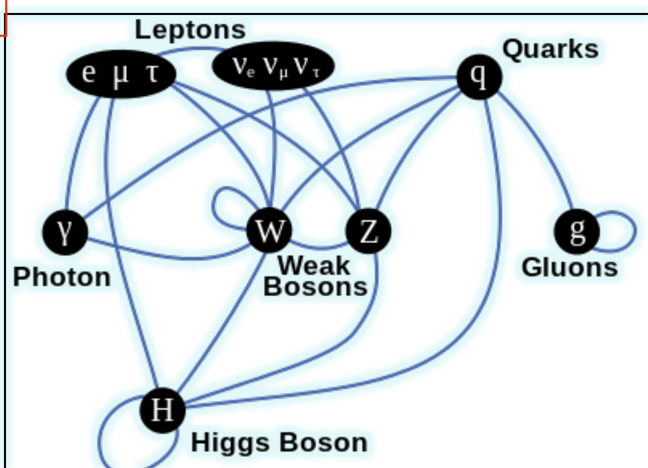
$$S=\int\Big\{\overline{\psi}(i\gamma\cdot\partial-m)\psi-g_\alpha\overline{\psi}\gamma\cdot A^\alpha\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+|(\partial_\mu+ig'A_\mu)\phi|^2-\frac{\lambda^2}{4}(|\phi|^2-v^2)^2+\frac{1}{16\pi}(R-2\Lambda)\Big\}\sqrt{-g}\,d^4x$$





All the Physics:

cosmology, gravity, particle physics, nuclear physics, condensed matter, atomic physics, molecules, optics…



That's the Physics we don't know !

Thermo-Statistics:

$$\rho = \frac{1}{Z}\exp\{-\beta(H-\mu N)\}$$

$$S = -\mathrm{Tr}[\rho \ln \rho]$$

(Information)

$$\mathrm{Tr}[\rho] = 1,$$

if $\langle H \rangle = \mathrm{const.}$

$\langle N \rangle = \mathrm{const.}$

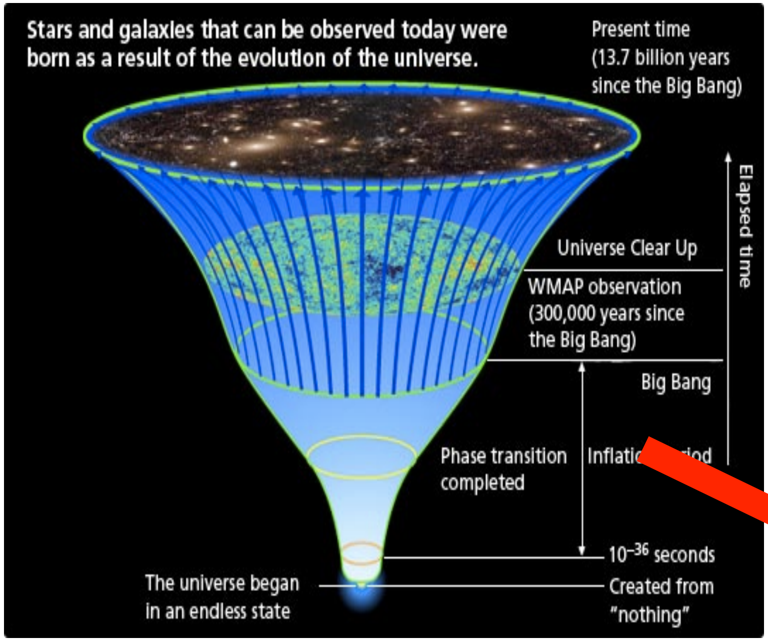
?

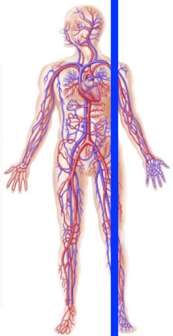
Nonequilibrium Statistics of open systems

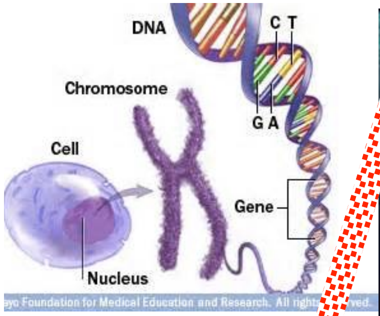
S(time)

10^{26} 10^{-6} 10^{-10} 10^{-18}

m(space)







10^{-18}

10^{-44}

How to describe information and its processing in states ?

物理學

過去	現在	將來
力 (Force)	能量 (Energy)	資訊 (Information)
$\mathbf{F} = m\mathbf{a}$	$E = mc^2$ $E = \hbar\omega$	$S = -\mathrm{tr}[\rho \ln \rho]$

Closed and open system dynamics, the essence

$$S = -\mathrm{tr}[\rho \ln \rho]$$

- For closed systems, quantum state: $|\psi(t)\rangle$ *S=0* (no information)

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$ Schrödinger equation, 1925

In terms of density matrix: $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$

$$\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[H\ ,\ \rho(t)]$$ von Neumann equation, 1927 (also Landau, 1927)

- For open systems, in general: $\rho(t) \neq |\psi(t)\rangle\langle\psi(t)|$

S≠0 (rich information)

Mixed state: loss of coherence

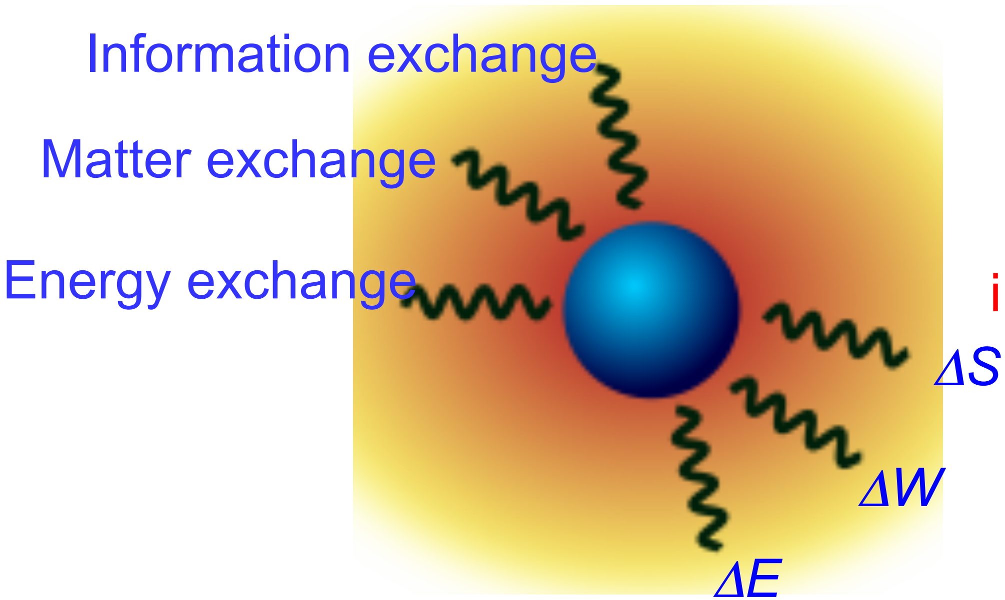
A complete description of physical systems

is

$$\rho$$

which is the density matrix that fully characterizes all possible quantum states of physical systems

Main Physics of open quantum systems



particle transport involves the phenomena of mater and energy exchanges (dissipation) and information exchange (fluctuations)

Non-equilibrium Physics

- Underlying physics of open quantum systems:
  - ✓ Dissipation and fluctuations
  - ✓ Non-Markovian memories

Physical significances of open systems

① quantum foundation: e.g. origin of probability …

② quantum information and computation: decoherence …

③ nonequilibrium physics: new physics …

④ quantum thermodynamics: mesoscopic scale

⑤ Foundation of quantum phase transition, topological orders of matter, and even quantum gravity: quantum entanglement …

⑥ Origin of the life, origina of spece and time: also entanglement ?…

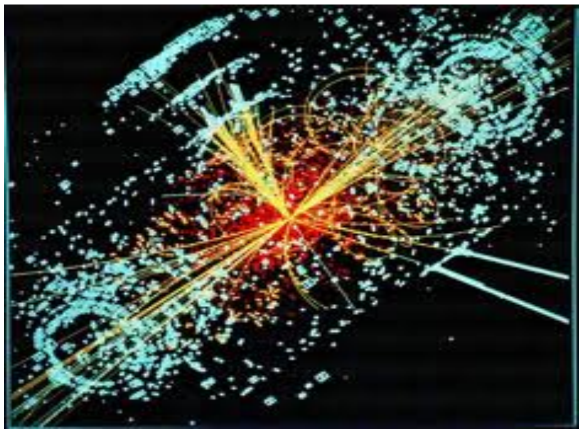
⑦ Unification of various physics at different scales …

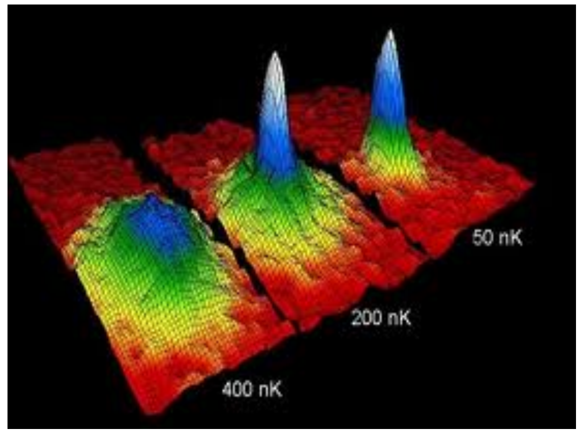
⑧ ……

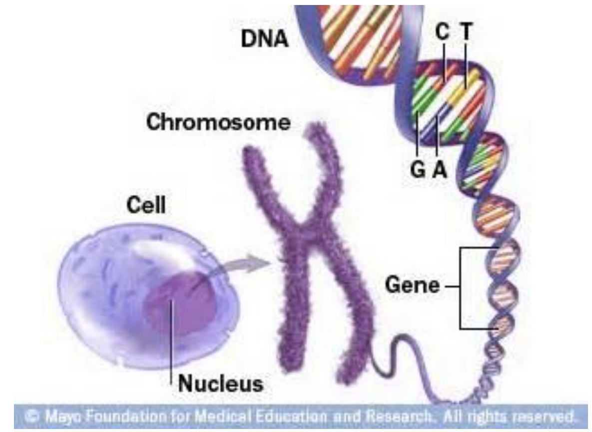
→ Open quantum systems:

cover All problems in physics !

Typical open systems:







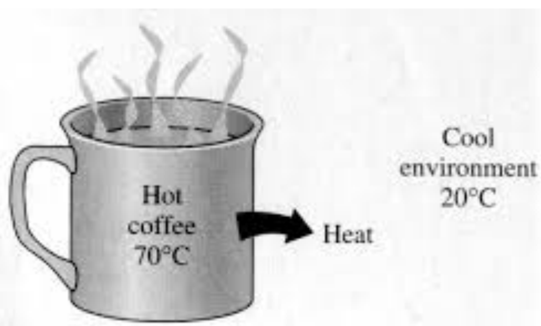
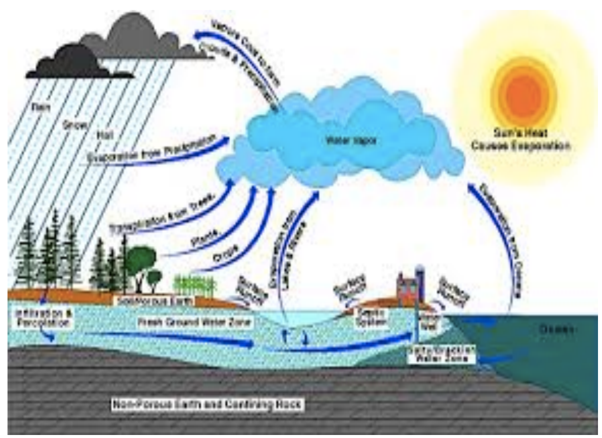
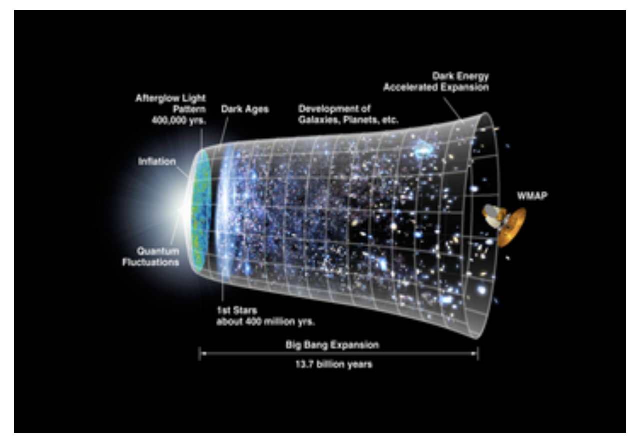
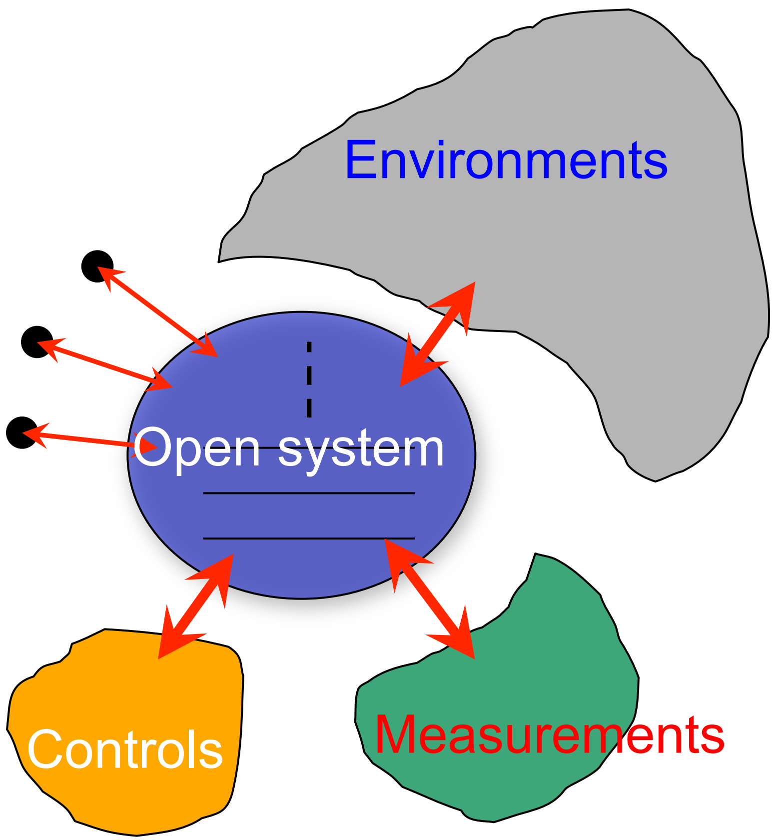


FIGURE 1–3
Heat flows in the direction of decreasing temperature.

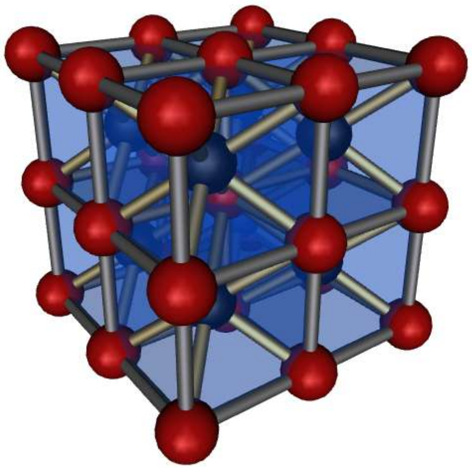




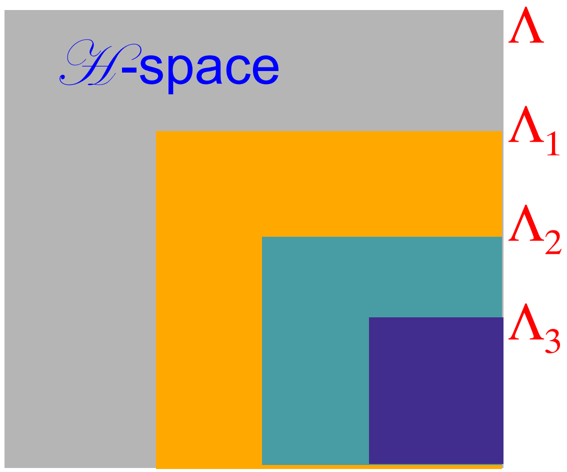
Modeling open quantum systems



- The system interacting with many other systems surrounding

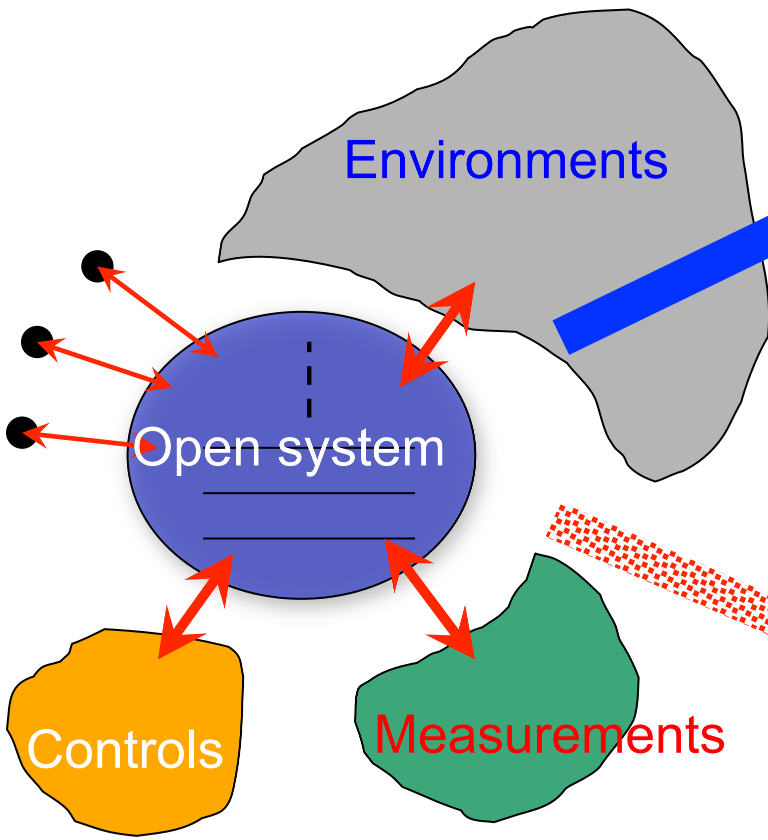


- One or a few particles interacting with other particles in many-body systems

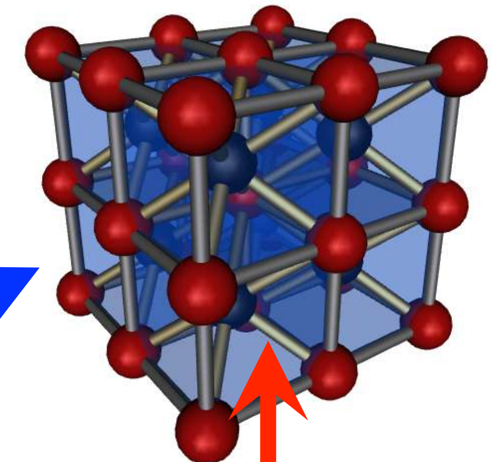


- A subspace of the Hilbert space correlating with others states in the space

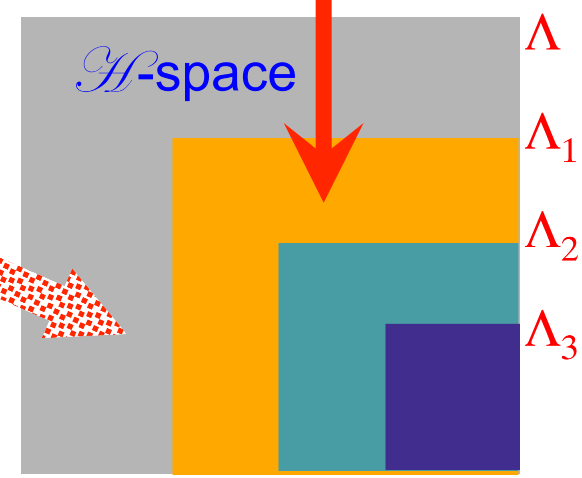
Theory of open quantum systems



- Physics is governed by master equation of the reduced density matrix



- Physics is characterized by various correlation functions in many-body systems



- Physics at different scales is related by renormalization group

Building the Theory of Open Quantum Systems:

- The theory of open quantum systems must go beyond quantum mechanics ?

- The theory of open quantum systems can be established within quantum mechanics ?

Answer: upon how you define open systems

- A1: open systems must contact with reservoirs.
  ----a definition from Statistical Mechanics (19 century)

- A2: systems of interest couple to the outside world which we are not interested.
  ----a definition by Feynman (1960's)

Theory for open quantum systems: a long-standing problem

- In general (Master Equation):

Openness

$$\boxed{\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H_S, \rho(t)]} + \underline{\int_{t_0}^{t} d\tau \mathcal{L}(t-\tau)\rho(\tau)}$$

Quantum Mechanics
J. von Neumann (1927)
L. D. Landau (1927)

Non-Markovian Memory
(Non-equilibrium Physics)

However, it has been attempted for many years without a very satisfactory answer to find the exact master equation for an arbitrary open quantum system that is tractable, since Pauli proposed the first master equation in 1928 !

Historical development of Master Equation

- Pauli master equation (W. Pauli, *Festschrift zum 60. Geburtstage A. Sommerfelds* (Hirzel, Leipzig, p.30, 1928)

$$\dot{P}_k = -\gamma_k P_k + \sum_{k'} (W_{kk'} P_{k'} - W_{k'k} P_k).$$

- Nakajima-Zwanzig master equation (S. Nakajima, Prog. Theo. Phys. **20**, 948, 1958; R. Zwanzig, J. Chem. Phys. **33**, 1338, 1960):

$$\frac{d}{dt}\rho_{tot}(t) = -i[H_{tot}, \rho_{tot}(t)] \equiv \mathcal{L}(t)\rho_{tot}(t)$$

$$\rho_{tot}(t) = \mathcal{P}\rho_{tot}(t) + (1-\mathcal{P})\rho_{tot}(t)$$

$$\frac{d}{dt}\mathcal{P}\rho_{tot}(t) = \mathcal{P}\mathcal{L}(t)\mathcal{P}\rho_{tot}(t) + \mathcal{P}\mathcal{L}(t)\mathcal{G}(t,t_0)\mathcal{Q}\rho_{tot}(t_0) + \int_{t_0}^{t} d\tau \mathcal{P}\mathcal{L}(t)\mathcal{G}(t,\tau)\mathcal{Q}\mathcal{L}(\tau)\mathcal{P}\rho_{tot}(\tau)$$

- Born-Markov master equation (e.g. F. Haake, Z. Phys. **223**, 353; 364, 1969):

$$\rho_{\text{tot}}(0) = \rho(0) \otimes \rho_{\text{r}}^{\text{Eq}}$$

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H_S, \rho(t)] - \frac{1}{\hbar^2}\int_{t_0}^{t} d\tau \langle [\widetilde{H}_I(t), [\widetilde{H}_I(\tau), \widetilde{\rho}(\tau)]]^I \rangle_E$$

$\tau \rightarrow t$

Only valid in the weak system-reservoir coupling

Theory of Open Quantum Systems

- Dynamics of open quantum systems is one of biggest problems that have not been solved in physics

  - people deeply involved such research include

    - W. Pauli (1920's)
    - J. Schwinger (1960's)
    - R. Feynman (1960's)
    - A. J. Leggett (1980's)
    - ……

- It still attracts and challenges theorists to build and develop the theory of open quantum systems.

In the literature, one often uses the Lindblad-GKS form of an approximated master equation:

$$\frac{d\rho(t)}{dt} = \frac{1}{i}[\tilde{H}_S(t), \rho(t)] + \sum_{ij} \Gamma_{ij} L_{a_j, a_i^\dagger}[\rho(t)]$$

where

$$L_{a_i, a_j^\dagger}[\rho(t)] \equiv a_i \rho(t) a_j^\dagger - \frac{1}{2} a_j^\dagger a_i \rho(t) - \frac{1}{2} \rho(t) a_j^\dagger a_i$$

which only describes Markovian (memoryless) processes, the basis for quantum optics.

G. Lindblad, *Comm. Math. Phys*. **48**, 119 (1976);
V. Gorini, A. Kossakowski, E.C.G. Sudarshan, *J. Math. Phys*. **17**,821 (1976).

Goes beyond the Markovian dynamics:

- Physically, three timescales dominate the dynamics of open quantum systems

  - A typical timescale of the system.
  - A typical timescale of the environment
  - The inverse of the coupling constant between the system and environment

- The Lindblad-GKS master equation valid only for $\tau_s >> \tau_c$ in the weak system-environment coupling regime.
- Dissipation, fluctuations as well as non-Markovian memory effects strongly rely on the relations among these different timescales. However, such relationships have yet been established quantitatively !

Here I would like to show:

- The general behavior of dissipation, fluctuations and non-Markovain nonequilibrium physics of open systems is mainly determined by the energy structure or the spectral density profile of environments.

- Universal non-Markovian memory can be extracted from the exact analytical solutions, solved by connecting the exact master equation with the nonequilibirum Green functions.

- Although the exact master equations have been derived only for some class of system-environment couplings. Establishing the connection between the exact master equation and the non-equilibrium Green functions provides a general approach to explore the non-Markovian nonequilibrium physics even though the exact master equation is unknown.

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- ✧ Introduction to general open quantum systems
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- ✧ Measurement of non-Markovian decoherence
- ✧ Non-Markovian nonequilibrium physics
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- ✧ Non-Markovian decoherence of Majorana qubits
- ✧ Transient quantum transport and quantum control
- ✧ Summary

Two useful theories for nonequilibrium dynamics

- Schwinger-Keldysh's Green function technique

  J. Schwinger, *J. Math. Phys*. **2**, 407 (1961)

  L. P. Kadanoff, & G. Baym, *Quantum Statistical Mechanics* (1962)

  L. V. Keldysh, *Sov. Phys. JETP*, **20**, 1018 (1965)

- Feynman-Vernon influence functional approach

  R. P. Feynman and F.L. Vernon, *Ann. Phys*. **24**, 118 (1963)

  A. O. Caldeira and A. J. Leggett, *Physics*. A **121**, 587 (1983)

However, both theories only solve partially some problems of the nonequilibrium phy sicsof open systems,

- The former provides correlations of local operators, but lacks the full state information of the whole system.
- The later provides a nice way to treat the environmental degrees of freedom, but still far from the complete understanding of all the states of the system.

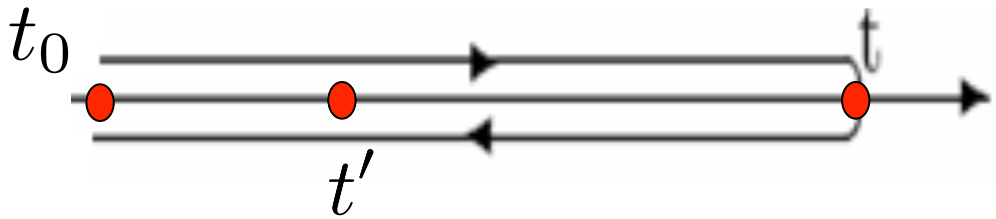
Closed-path approach for nonequilibrium dynamics:

J. Schwinger, *J. Math. Phys*. **2**, 407 (1961)
R. P. Feynman, F. L. Vernon, *Ann. Phys. (N.Y.)* **24**, 118 (1963)

- The correlation function or the evolution of density matrix

$$G(xt;x't') = -i\langle T_C[\psi(xt)\psi^\dagger(x't')]\rangle \qquad \text{if } t > t'$$

$$\rho(t) = U(t,t_0)\rho(t_0)U^\dagger(t,t_0)$$

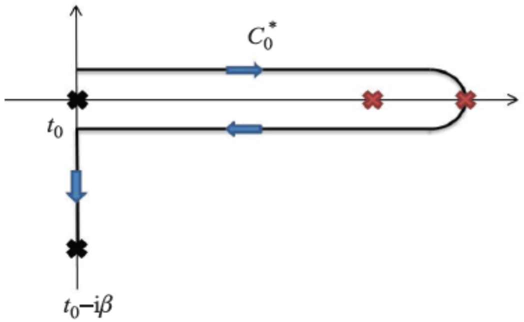


see a Review by Yang & WMZ, *Front. Phys*. **12**, 127204 (2017)

- which is very different from the scattering theory in field theory, scattering theory is in general INVALID for nonequilibrium physics !!!

Nonequilibrium Green-function approach:

J. Schwinger, *J. Math. Phys*. **2**, 407 (1961)



- Kadanoff-Baym equation (1962):

$$\left\{\mathrm{i}\frac{\partial}{\partial\tau_1} - \left[-\frac{1}{2}\nabla^2_{x_1} + U(x_1,\tau_1)\right]\right\}G(x_1,\tau_1,x_{1'},\tau_{1'})$$

$$= \delta(1-1') + \int_C \mathrm{d}\sigma \int \mathrm{d}^3y\,\Sigma(x_1,\tau_1,y,\sigma)G(y,\sigma,x_{1'},\tau_{1'})$$

where

retarded GF

$$\left\{i\frac{d}{d\tau} - \omega\right\}G^r(\tau,\tau') = \delta(\tau-\tau') + \int_{\tau'}^{\tau} \Sigma^r(\tau,\tau'')G^r(\tau'',\tau')d\tau''$$

$$G^< = (1+G^\mathrm{r}\Sigma^\mathrm{r})G_0^<(1+\Sigma^\mathrm{a}G^\mathrm{a}) + G^\mathrm{r}\Sigma^<G^\mathrm{a}$$

lesser GF, when

$$G_0^< = 0$$

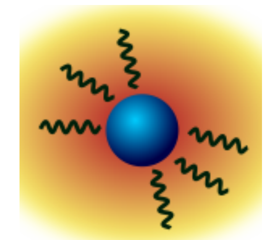
$$G^c(\tau,\tau') = \int\int G^r(\tau,\tau'')\Sigma^c(\tau'',\tau''')G^a(\tau''',\tau')d\tau''d\tau'''$$

L. V. Keldysh (1965)

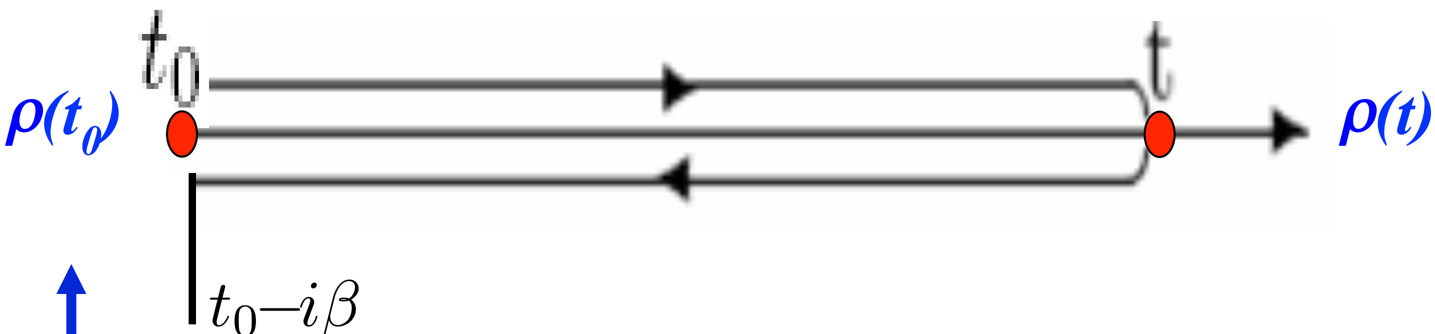
Influence functional approach

Feynman & Vernon,
Ann. Phys. **24**, 118 (1963)

- a nonperturbation way for fully covering the back-reaction of baths on the system one concerns:



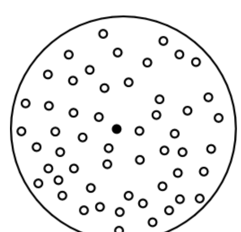
$$\rho(t) = \mathrm{tr}_{\mathrm{B}}[U(t,t_0)\rho(t_0)\otimes\rho_{\mathrm{B}}(t_0)U^{\dagger}(t,t_0)]$$



$$\mathcal{J}(t,t_0) = \int \mathcal{D}[\bar{\psi}\psi;\bar{\psi}'\psi']e^{i(S_A[\bar{\psi},\psi]-S_A^*[\bar{\psi}',\psi'])}\underline{\mathcal{F}[\bar{\psi},\psi;\bar{\psi}',\psi']}$$

propagating function

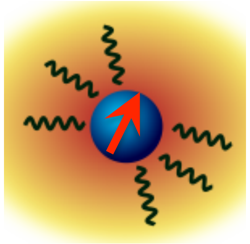
influence functional

- for quantum Brownian motion  → Caldeira & Leggett,
Physica A **121**. 587 (1983)

But, the Feynman-Vernon's influence functional does not result directly in master equation

- for example, consider a spin-boson coupling:

$$H = \frac{1}{2}\varepsilon(t)\sigma_z + \frac{1}{2}\Delta(t)\sigma_x + \sum_k \hbar\omega_k b_k^\dagger b_k + \sum_k \left[V_k(t)\sigma^+ b_k + V_k^*(t) b_k^\dagger \sigma^-\right]$$



The influence functional is given

$$\mathcal{F}(\sigma,\sigma') = \exp\Big\{ -\int_0^t d\tau \int_0^\tau d\tau' \Big[\sigma^+(\tau)g(\tau,\tau')\sigma^-(\tau') + \sigma'^+(\tau')g(\tau',\tau)\sigma'^-(\tau)\Big] + \int_0^t d\tau \int_0^t d\tau' \Big[\sigma'^+(\tau')g(\tau',\tau)\sigma^-(\tau) - [\sigma^+(\tau') - \sigma'^+(\tau')]\widetilde{g}(\tau',\tau)[\sigma^-(\tau) - \sigma'^-(\tau)]\Big]\Big\}$$

where

$$g(\tau,\tau') = \sum V_k(\tau)V_k^*(\tau')e^{-i\omega_k(\tau-\tau')}$$

$$\widetilde{g}(\tau,\tau') = \sum_k V_k(\tau)V_k^*(\tau')\frac{1}{e^{\beta\hbar\omega_k}-1}e^{-i\omega_k(\tau-\tau')}$$

➢ **None has derive the exact master equation from it !!!**

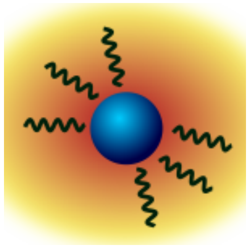
Our consideration:

General system-environment couplings:
(generalized Fano-Anderson models)

P. W. Anderson. *Phys. Rev*. **124**, 41 (1961)
U. Fano, *Phys. Rev*. **124**, 1866 (1961)

❖ The leading order system-environment couplings are dominated by particle-particle (energy and information) exchanges:

$$H_{SB} = \sum_{\alpha k i} [V_{\alpha k i} a_i^\dagger b_{\alpha k} + V_{\alpha k i}^* b_{\alpha k}^\dagger a_i]$$



- the system contains arbitrary *N* energy levels
- environment can contain many different reservoirs
- each reservoir is specified by the spectral density:

$$\boldsymbol{J}_{\alpha ij}(\omega) = 2\pi \sum_k V_{\alpha ik} V_{\alpha jk}^* \delta(\omega - \epsilon_k)$$

A. T. Leggett, *et al.*, *Rev. Mod. Phys*. **59**, 1 (1987)

Path integral approach to fermion open systems:

Tu & WMZ, Phys. Rev. B **78**, 235311 (2008)

- All matters are made by fermions

$$H = H_S + H_B + H_{SB} \quad \text{and} \quad i\partial\rho_{\rm tot}(t)/\partial t = [H, \rho_{\rm tot}(t)]$$

$$\Rightarrow \quad \boxed{\rho_{\rm tot}(t) = e^{-iH(t-t_0)}\rho_{\rm tot}(t_0)e^{iH(t-t_0)}}$$

- With the initial state $\rho_{\rm tot}(t_0) = \rho(t_0) \otimes \rho_E(t_0)$
- taking the fermionic coherent state representation

The System

$$\Rightarrow \quad \boxed{\begin{aligned}\langle\xi_f|\rho(t)|\xi'_f\rangle = \int d\mu(\xi_0)d\mu(\xi'_0)\langle\xi_0|\rho(t_0)|\xi'_0\rangle \\ \times \mathcal{J}(\bar{\xi}_f, \xi'_f, t|\xi_0, \bar{\xi}'_0, t_0)\end{aligned}}$$

The Environment

WMZ et al., Rev. Mod. Phys. 62, 867 (1990)
Tu & WMZ, Phys. Rev. B 78, 235311 (2008)

$$\mathcal{J}(\bar{\xi}_f, \xi'_f, t | \xi_0, \bar{\xi}'_0, t_0) = \int \mathcal{D}[\bar{\xi}\xi; \bar{\xi}'\xi'] e^{i(S_c[\bar{\xi},\, \xi] - S_c^*[\bar{\xi}',\, \xi'])} \mathcal{F}[\bar{\xi}\xi; \bar{\xi}'\xi'],$$

(the action of the system)

path integral over the system

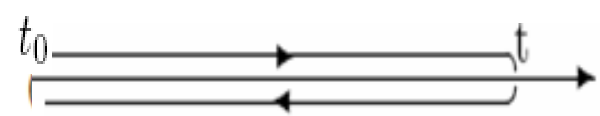
The Influence Functional

(obtained after completely integrated over all the environmental degrees of freedom)

$$\mathcal{F}[\bar{\xi}\xi; \bar{\xi}'\xi'] = \exp\Big\{ -\sum_{\alpha ij} \int_{t_0}^{t} d\tau \int_{t_0}^{\tau} d\tau' \Big(g_{\alpha ij}(\tau,\tau')\xi_i^*(\tau)\xi_j(\tau') + g^*_{\alpha ji}(\tau,\tau')\xi_i'^*(\tau')\xi_j'(\tau) \Big)$$
$$- \sum_{\alpha ij} \int_{t_0}^{t} d\tau \int_{t_0}^{t} d\tau' \Big(g_{\alpha ij}(\tau,\tau')\xi_i'^*(\tau)\xi_j(\tau') - \tilde{g}_{\alpha ij}(\tau,\tau')\big(\xi_i^*(\tau) + \xi_i'^*(\tau)\big)\big(\xi_j(\tau') + \xi_j'(\tau')\big) \Big) \Big\}.$$

where

$$g_{\alpha ij}(\tau,\tau') = \sum_k V_{i\alpha k}(\tau) V^*_{j\alpha k}(\tau') e^{-i\int_{\tau'}^{\tau} d\tau_1 \epsilon_{\alpha k}(\tau_1)},$$



$$\tilde{g}_{\alpha ij}(\tau,\tau') = \sum_k V_{i\alpha k}(\tau) V^*_{j\alpha k}(\tau') f_\alpha(\epsilon_{\alpha k}) e^{-i\int_{\tau'}^{\tau} d\tau_1 \epsilon_{\alpha k}(\tau_1)}$$

$$f_\alpha(\epsilon_{\alpha k}) = 1/(e^{\beta_\alpha(\epsilon_{\alpha k} - \mu_\alpha)} + 1)$$

Initial Fermi-Dirac distribution function

A few crucial steps toward obtaining the master equation

Tu & WMZ, PRB 78, 235311 (2008)

1. The classical action in coherent state rep.

$$S_{\rm cl}[\bar{\xi},\xi] = -\frac{i}{2}[\bar{\xi}_f\xi(t)+\bar{\xi}(t_0)\xi_0] + \int_{t_0}^{t} d\tau\Big\{\frac{i}{2}\Big[\bar{\xi}\frac{d\xi}{d\tau}-\frac{d\bar{\xi}}{d\tau}\xi\Big]-H_{\rm cl}(\bar{\xi},\xi)\Big\}$$

2. The role of the end points:

$$\mathcal{J}(\bar{\xi}_f,\xi'_f,t|\xi_0,\bar{\xi}'_0,t_0;\chi) = A(t)\exp\left\{\frac{\bar{\xi}_f\cdot\xi(t)+\bar{\xi}(t_0)\cdot\xi_i}{2}+\frac{\bar{\xi}'(t)\cdot\xi'_f+\bar{\xi}'_i\cdot\xi'(t_0)}{2}\right\}$$

3. A nontrivial transformation:

$$\boxed{\begin{aligned}&\xi(\tau)=\boldsymbol{u}(\tau)\xi_0+\boldsymbol{v}(\tau)[\xi(t)+\xi'_f],\\ &\xi(\tau)+\xi'(\tau)=\bar{\boldsymbol{u}}(\tau)[\xi(t)+\xi'_f],\end{aligned}}$$

$$\mathcal{J}(\bar{\xi}_f,\xi'_f,t|\xi_0,\bar{\xi}'_0,t_0;\chi) = \frac{1}{\det[\boldsymbol{w}(t)]}\exp\Big\{\bar{\xi}_f\boldsymbol{J}_1(t)\xi_0+\bar{\xi}_f\boldsymbol{J}_2(t)\xi'_f+\bar{\xi}'_0\boldsymbol{J}_3(t)\xi_0+\bar{\xi}'_0\boldsymbol{J}_1^\dagger(t)\xi'_f\Big\}$$

where

$$\boldsymbol{J}_1(t)=\boldsymbol{w}(t)\boldsymbol{u}(t),\ \boldsymbol{J}_2(t)=\boldsymbol{w}(t)-\boldsymbol{I}, \qquad \boldsymbol{J}_3(t)=\boldsymbol{u}^\dagger(t)\boldsymbol{w}(t)\boldsymbol{u}(t)-\boldsymbol{I},$$

$$\text{with } \boldsymbol{w}(t)=[I-\boldsymbol{v}(t)]^{-1}$$

Our Exact Master Equation:

Tu and WMZ, *Phys. Rev. B* **78**, 235311 (2008)
Jin, Tu, WMZ & Yan, *New J. Phys*. **12**, 083013 (2010)
Lei and WMZ, *Ann. Phys. (N.Y.)* **327,** 1408 (2012)
WMZ, Lo, Xiong & Nori, *Phys. Rev. Lett.* **109**, 170402 (2012)
Yang, Lin & WMZ, Phys. Rev. B **92**, 165403 (2015)
Yang & WMZ, *Frontiers of Physics* **12**, 127204 (2017)

$$
\begin{aligned}
\frac{d\rho(t)}{dt} =& \frac{1}{i}\left[\widetilde{H}_S(t), \rho(t)\right] \\
&+ \sum_{ij}\Big\{\underbrace{\gamma_{ij}(t)\left[2a_j\rho(t)a_i^\dagger - a_i^\dagger a_j\rho(t) - \rho(t)a_i^\dagger a_j\right]}_{\text{Dissipation}} \\
&\underbrace{+\widetilde{\gamma}_{ij}(t)\left[a_i^\dagger\rho(t)a_j \pm a_j\rho(t)a_i^\dagger \mp a_i^\dagger a_j\rho(t) - \rho(t)a_j a_i^\dagger\right]}_{\text{Fluctuation}}\Big\}
\end{aligned}
$$

upper sign: bosons
lower sign: fermions

- Dissipation and fluctuation coefficients in the master equation:

$$\widetilde{\varepsilon}(t) = \frac{i}{2}\left[\dot{\boldsymbol{u}}(t,t_0)\boldsymbol{u}^{-1}(t,t_0) - \text{H.c.}\right] \longleftarrow \textbf{renormalization}$$

$$\boldsymbol{\gamma}(t) = -\frac{1}{2}\left[\dot{\boldsymbol{u}}(t,t_0)\boldsymbol{u}^{-1}(t,t_0) + \text{H.c.}\right] \longleftarrow \textbf{Dissipation (relaxation)}$$

$$\widetilde{\boldsymbol{\gamma}}(t) = \dot{\boldsymbol{v}}(t,t) - \left[\dot{\boldsymbol{u}}(t,t_0)\boldsymbol{u}^{-1}(t,t_0)\boldsymbol{v}(t,t) + \text{H.c.}\right] \longleftarrow \textbf{Fluctuations (noises)}$$

Nonequilibrium Green functions *u(t,t_0)* and *v(t,t)*

- *u(t,t_0)* and *v(t,t)* are two Green basic functions in Schwinger-Keldysh's non-equilibrium theory:

$$\boldsymbol{u}_{ij}(\tau, t_0) = \langle [a_i(\tau), a_j^\dagger(t_0)] \rangle_\pm$$ ✓ Retarded GF

$$\boldsymbol{v}_{ij}(\tau, t) \sim \langle a_j^\dagger(t) a_i(\tau) \rangle$$ ✓ Correlated GF

- Kadanoff-Baym-like equations:

$$\frac{d}{d\tau}\boldsymbol{u}(\tau, t_0) + i\boldsymbol{\varepsilon}_s \boldsymbol{u}(\tau, t_0) + \int_{t_0}^{\tau} d\tau' \boldsymbol{g}(\tau, \tau')\boldsymbol{u}(\tau', t_0) = 0$$

$$\frac{d}{d\tau}\boldsymbol{v}(\tau, t) + i\boldsymbol{\varepsilon}_s \boldsymbol{v}(\tau, t) + \int_{t_0}^{\tau} d\tau' \boldsymbol{g}(\tau, \tau')\boldsymbol{v}(\tau', t) = \int_{t_0}^{t} d\tau' \widetilde{\boldsymbol{g}}(\tau, \tau')\boldsymbol{u}^\dagger(\tau', t_0)$$

subject to the conditions:

$$\boldsymbol{u}(t_0, t_0) = 0, \quad \boldsymbol{v}(t_0, t) = 0,$$
$$t_0 \le \tau \le t$$

Non-Markovian memory kernels

Tu & WMZ, Phys. Rev. B **78**, 235311 (2008)
WMZ, *et al*, *Phys. Rev. Lett.* **109**,170402 (2012)

Equation of motion for Nonequilibrium Green Function *u(t, t₀)*

$$\frac{d}{d\tau}\boldsymbol{u}(\tau,t_0)+i\boldsymbol{\varepsilon}\boldsymbol{u}(\tau,t_0)+\sum_\alpha\int_{t_0}^{\tau}d\tau'\underbrace{\boldsymbol{g}_\alpha(\tau,\tau')}_{\text{dissipation kernel}}\boldsymbol{u}(\tau',t_0)=0,$$

Nonequilibrium Fluctuation-Dissipation Theorem via *v(t,t)*

$$\boldsymbol{v}(\tau,t)=\sum_\alpha\int_{t_0}^{\tau}d\tau_1\int_{t_0}^{t}d\tau_2\boldsymbol{u}(\tau,\tau_1)\underbrace{\widetilde{\boldsymbol{g}}_\alpha(\tau_1,\tau_2)}_{\text{fluctuation kernel}}\boldsymbol{u}^\dagger(t,\tau_2)$$

Nonequilibrium Fluctuation-Dissipation Kernels

$$\boldsymbol{g}_{\alpha ij}(\tau,\tau')=\sum_k V_{i\alpha k}(\tau)V^*_{j\alpha k}(\tau')e^{-i\int_{\tau'}^{\tau}\epsilon_{\alpha k}(\tau_1)d\tau_1}$$

$$\widetilde{\boldsymbol{g}}_{\alpha ij}(\tau_1,\tau_2)=\sum_k V_{i\alpha k}(\tau_1)V^*_{j\alpha k}(\tau_2)e^{-i\int_{\tau_2}^{\tau_1}\epsilon_{\alpha k}(\tau')d\tau'}\langle b^\dagger_{\alpha k}(t_0)b_{\alpha k}(t_0)\rangle$$

Spectral Density (level-broadening function)

$$\Gamma_{\alpha ij}(\varepsilon)=2\pi\sum_k V_{i\alpha k}V^*_{j\alpha k}\delta(\varepsilon-\varepsilon_{\alpha k})$$

Tu & WMZ, PRB **78**, 235311 (2008)
Jin, Tu, WMZ & Yan, NJP **12**, 183013 (2010)
Lei & WMZ, Ann. Phys. 327, 1408 (2012)

Connection of Master Equation with Non-equilibrium Correlators for open systems:

Jin, Tu, WMZ & Yan, NJP 12, 183013 (2010)
Lei & WMZ, Ann. Phys. 327, 1408 (2012)
Yang & WMZ, Front. Phys. 12, 127204 (2017)

Relations of dissipation and fluctuation with the retarded and correlation Green's functions

$$\langle [a_i(\tau), a_j^\dagger(t_0)] \rangle_\pm = \boldsymbol{u}_{ij}(\tau, t_0) \qquad \boxed{\rightarrow iG^R(\tau, t_0)}$$

$$\langle a_j^\dagger(t) a_i(\tau) \rangle \equiv \boldsymbol{\rho}_{ij}^{(1)}(\tau, t) = \left[\boldsymbol{u}(\tau, t_0) \boldsymbol{\rho}^{(1)}(t_0) \boldsymbol{u}^\dagger(t, t_0) + \boldsymbol{v}(\tau, t) \right]_{ij}$$

$$\boxed{\rightarrow -iG^<(\tau, t)}$$

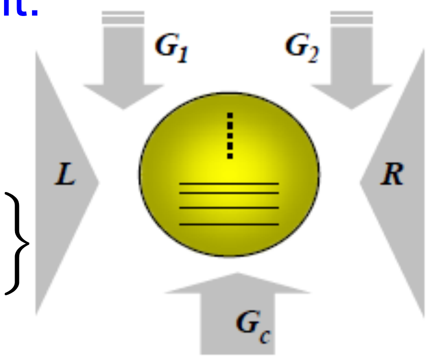
Keldysh's Green functions

L. V. Keldysh, JETP 20, 1018 (1965)

Reproduce NEGF transport theory:

Jin, Tu, WMZ & Yan, NJP 12, 183013 (2010)
Lei & WMZ, Ann. Phys. 327, 1408 (2012)
Yang & WMZ, Front. Phys. 12, 127204 (2017)

- We reproduce and further generalize the transient current:



$$I_\alpha(t) = -\frac{2e}{\hbar}\mathrm{Re}\int_{t_0}^{t} d\tau \mathrm{Tr}\Big\{\boldsymbol{g}_\alpha(t,\tau)\boldsymbol{v}(\tau,t) - \widetilde{\boldsymbol{g}}_\alpha(t,\tau)\boldsymbol{u}^\dagger(t,\tau) + \boldsymbol{g}_\alpha(t,\tau)\boldsymbol{u}(\tau,t_0)\boldsymbol{\rho}^{(1)}(t_0)\boldsymbol{u}^\dagger(t,t_0)\Big\}$$

$$= \boxed{-\frac{2e}{\hbar}\mathrm{Re}\int_{t_0}^{t} d\tau \mathrm{Tr}\Big\{\boldsymbol{\Sigma}_\alpha^R(t,\tau)\boldsymbol{G}^<(\tau,t) + \boldsymbol{\Sigma}_\alpha^<(t,\tau)\boldsymbol{G}^A(\tau,t)\Big\}}$$

Wingreen, Jauho & Meir, PRB48, 8487 (1993)

where

$$\boldsymbol{G}^<(\tau,t) = i\big[\boldsymbol{u}(\tau,t_0)\boldsymbol{\rho}^{(1)}(t_0)\boldsymbol{u}^\dagger(t,t_0) + \boldsymbol{v}(\tau,t)\big]$$
$$= \boldsymbol{G}^R(\tau,t_0)\boldsymbol{G}^<(t_0,t_0)\boldsymbol{G}^A(t_0,t)$$
$$\boxed{+\int_{t_0}^{\tau} d\tau_1 \int_{t_0}^{t} d\tau_2 \boldsymbol{G}^R(\tau,\tau_1)\boldsymbol{\Sigma}^<(\tau_1,\tau_2)\boldsymbol{G}^A(\tau_2,t).}$$

Weak coupling limit → Born-Markov Master Equation:

H. N. Xiong, WMZ, *et al. Phys. Rev* .**82**, 012105 (2010)

- Kadanoff-Baym equation can be rewritten as:

$$\dot{u}(t,t_0)u^{-1}(t,t_0) = -i\epsilon_s - \int_{t_0}^{t} d\tau g(t,\tau)u(\tau,t_0)u^{-1}(t,t_0)$$

- Take perturbation up to the 2nd-order in coupling strength:

$$\dot{u}(t,t_0)u^{-1}(t,t_0) \simeq -i\epsilon_s - \int_{t_0}^{t} d\tau \int \frac{d\omega}{2\pi} J(\omega) e^{-i(\omega-\epsilon_s)(t-\tau)}$$

$$\dot{v}(t,t) \simeq 2\int_{t_0}^{t} d\tau \int \frac{d\omega}{2\pi} J(\omega) f(\omega) \cos[(\omega-\epsilon_s)(t-\tau)]$$

- Dissipation and fluctuation in Born-Markovian master equation:

$$\varepsilon_s'(t) = -\mathrm{Im}[\dot{u}(t,t_0)u^{-1}(t,t_0)] \simeq \omega_c - \int_{t_0}^{t} d\tau \int \frac{d\omega}{2\pi} J(\omega) \sin[(\omega-\epsilon_s)(t-\tau)]$$

$$\gamma(t) = -\mathrm{Re}[\dot{u}(t,t_0)u^{-1}(t,t_0)] \simeq \int_{t_0}^{t} d\tau \int \frac{d\omega}{2\pi} J(\omega) \cos[(\omega-\epsilon_s)(t-\tau)]$$

$$\widetilde{\gamma}(t) \simeq \dot{v}(t,t) \simeq 2\int_{t_0}^{t} d\tau \int \frac{d\omega}{2\pi} J(\omega) f(\omega) \cos[(\omega-\epsilon_s)(t-\tau)]$$

Lindblad-GKS form of the Exact Master Equation:

Lei & WMZ, Ann. Phys. **327**, 1408 (2012)
WMZ *et al.*, PRL **109**, 170402 (2012)

- Our exact master equation can be rewritten as:

$$\frac{d\rho(t)}{dt} = \frac{1}{i}[\tilde{H}_S(t), \rho(t)] + \sum_{ij} \tilde{\gamma}_{ij}(t) L_{a_i^\dagger, a_j}[\rho(t)] + \sum_{ij} [2\gamma_{ij}(t) \pm \widetilde{\gamma}_{ij}(t)] L_{a_j, a_i^\dagger}[\rho(t)]$$

where

$$L_{a_i, a_j^\dagger}[\rho(t)] \equiv a_i \rho(t) a_j^\dagger - \frac{1}{2} a_j^\dagger a_i \rho(t) - \frac{1}{2}\rho(t) a_j^\dagger a_i$$

- Lindblad form of the master equation has an excellent symmetric superoperator form but it mixes the dissipation and fluctuation such that the fluctuation-dissipation theorem is not manifested.

⇒ Obtain for the first time the exact master equation with Lindblad-GKS form !!!

Goes beyond the fundamental theories for open systems:

- Schwinger-Keldysh's Green function technique

  J. Schwinger, *J. Math. Phys*. **2**, 407 (1961)
  L. P. Kadanoff, & G. Baym, *Quantum Statistical Mechanics* (1962)
  L. V. Keldysh, *Sov. Phys. JETP*, **20**, 1018 (1965)

- Feynman-Vernon influence functional approach

  R. P. Feynman and F.L. Vernon, *Ann. Phys*. **24**, 118 (1963)
  A. O. Caldeira and A. J. Leggett, *Physics*. A **121**, 587 (1983)

However, if there are initial correlations between the system and the environments,

- The Feynman-Vernon's influence functional approach is not valid, and the Schwinger-Keldysh's Green function technique is also intractable.

- We can solve the problem by the exact quantum Langevin equation and also other methods.

Tan & WMZ, Phys. Rev. A**83**, 032102 (2011)
Yang, Lin & WMZ, Phys. Rev. B**92**, 165403 (2015)

For partition-free initial states involving initial correlations :

$$\rho_{tot}(t_0) = \frac{1}{Z} e^{-\beta H_{tot}}$$

- the dissipation dynamics given by $\boldsymbol{u}(t, t_0)$ is the same, only the fluctuation $\boldsymbol{v}(\tau, t)$ is modified with

$$\boldsymbol{v}(\tau, t) = \sum_\alpha \int_{t_0}^{\tau} d\tau_1 \int_{t_0}^{t} d\tau_2 \boldsymbol{u}(\tau, \tau_1) \underline{[\boldsymbol{g}_\alpha^{ee}(\tau_1, \tau_2) + \boldsymbol{g}_\alpha^{se}(\tau_1, \tau_2)]} \boldsymbol{u}^\dagger(t, \tau_2)$$

Generalized fluctuation-dissipation theorem in time-domain

where

$$\boldsymbol{g}_{\alpha ij}^{se}(\tau_1, \tau_2) = 2i \sum_k \Big[V_{i\alpha k}^*(\tau_2) e^{i\int_{t_0}^{\tau_2} \epsilon_{\alpha k}(\tau')d\tau'} \delta(\tau_1 - t_0) \underline{\langle b_{\alpha k}^\dagger(t_0) a_j(t_0)\rangle}$$

$$- V_{i\alpha k}(\tau_1) e^{-i\int_{t_0}^{\tau_1} \epsilon_{\alpha k}(\tau')d\tau'} \delta(\tau_2 - t_0) \underline{\langle a_j^\dagger(t_0) b_{\alpha k}(t_0)\rangle} \Big]$$

$$\boldsymbol{g}_{\alpha ij}^{ee}(\tau_1, \tau_2) = \sum_{\alpha'} \sum_{kk'} V_{i\alpha k}(\tau) e^{-i\int_{t_0}^{\tau_1} \epsilon_{\alpha k}(\tau')d\tau'}$$

$$\times V_{j\alpha' k'}^*(\tau_2) e^{i\int_{t_0}^{\tau_2} \epsilon_{\alpha' k'}(\tau')d\tau'} \underline{\langle c_{\alpha' k'}^\dagger(t_0) c_{\alpha k}(t_0)\rangle}$$

Yang, Lin & WMZ, *Phys. Rev. B* **92**, 165403 (2015)

The Theory for Open Quantum Systems we have here

- The dissipation and fluctuation coefficients in the exact master equation are microscopically and nonperturbatively determined from the Kadanoff-Baym-like equations for nonequilibrium Green functions, where the environment-induced memory kernels take into account all the back-actions from the environment.

- The dissipation and fluctuation coefficients are constrained by the nonequilibrium fluctuation-dissipation theorem so that the positivity of the reduced density matrix is guaranteed during the non-Markovian time evolution. In fact, the exact master equation has the Lindblad-GKS form but it can fully address the non-Markovain physics.

- In the weak-coupling limit, the solution of the Kadanoff-Baym equations with a perturbation expansion up to second order in the coupling constant between the system and the environment reproduces the Born-Markovian master equation from the exact master equation, and then the Lindblad one in the Markov limit.

WMZ, *et al*, *Phys. Rev. Lett.* **109**,170402 (2012)

Outline

- ✧ General open quantum systems
- ✧ A general theory of open quantum systems
- ✧ General analytical solution of non-Markovianity
- ✧ Direct measurement of non-Markovianity
- ✧ Non-Markovian nonequilibrium physics
- ✧ Non-Markovian entanglement decoherence
- ✧ Non-Markovian decoherence of Majorana qubits
- ✧ Transient quantum transport and quantum control
- ✧ Summary

Non-Markovain dissipation and fluctuations

- Retarded GF and dissipation:

$$\widetilde{\boldsymbol{\varepsilon}}(t) = \frac{i}{2}\big[\dot{\boldsymbol{u}}(t,t_0)\boldsymbol{u}^{-1}(t,t_0) - \mathrm{H.c.}\big]$$

$$\boldsymbol{\gamma}(t) = -\frac{1}{2}\big[\dot{\boldsymbol{u}}(t,t_0)\boldsymbol{u}^{-1}(t,t_0) + \mathrm{H.c.}\big]$$

$$\Longrightarrow \quad \boldsymbol{u}(t,t_0) = \mathcal{T}\exp\Big\{-\int_{t_0}^{t} d\tau\big[i\widetilde{\boldsymbol{\varepsilon}}(\tau) + \boldsymbol{\gamma}(\tau)\big]\Big\}$$

The full solution of energy spectrum

- Correlated GF and fluctuation GF:

$$\widetilde{\boldsymbol{\gamma}}(t) = \dot{\boldsymbol{v}}(t,t) - \big[\dot{\boldsymbol{u}}(t,t_0)\boldsymbol{u}^{-1}(t,t_0)\boldsymbol{v}(t,t) + \mathrm{H.c.}\big]$$

$$\Longrightarrow \quad \langle a_j^{\dagger}(t)a_i(t)\rangle = \boldsymbol{u}(t,t_0)\langle a_j^{\dagger}(t_0)a_i(t_0)\rangle\boldsymbol{u}^{\dagger}(t,t_0) + \boldsymbol{v}(t,t)$$

The full solution of noise spectrum

General solution of dissipation dynamics

WMZ, *et al*, *Phys. Rev. Lett.* **109**,170402 (2012)

- Equation of motion for retarded GF:

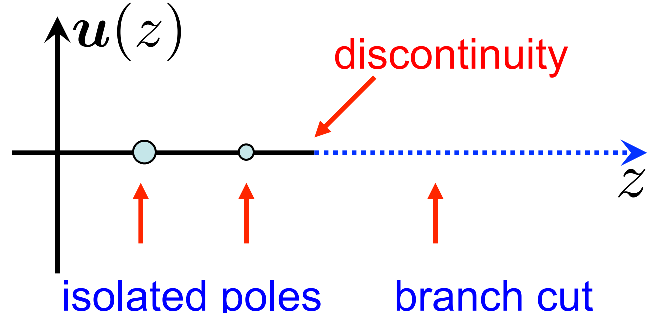
$$\frac{d}{d\tau}\boldsymbol{u}(\tau,t_0)+i\boldsymbol{\varepsilon}_s\boldsymbol{u}(\tau,t_0)+\int_{t_0}^{\tau}d\tau'\boldsymbol{g}(\tau,\tau')\boldsymbol{u}(\tau',t_0)=0$$

- Discontinuity of self-energies:

$$\boldsymbol{\Sigma}(z)=\sum_{\alpha}\int\frac{d\omega}{2\pi}\frac{\boldsymbol{J}_\alpha(\omega)}{z-\omega}$$

$$\xrightarrow{z=\omega\pm i0^+}\boldsymbol{\Delta}(\omega)\mp i\sum_{\alpha}\frac{\boldsymbol{J}_\alpha(\omega)}{2}$$

principal value discontinuity



- General solution

$$\boldsymbol{u}(t-t_0)=\sum_i\boldsymbol{\mathcal{Z}}_i e^{-i\omega_i(t-t_0)}$$
$$+\sum_k\int_{B_k}\frac{d\omega}{2\pi}\Big[\boldsymbol{u}(\omega+i0^+)-\boldsymbol{u}(\omega-i0^+)\Big]e^{-i\omega(t-t_0)}$$

dissipation

Universality of non-Markovian dissipation

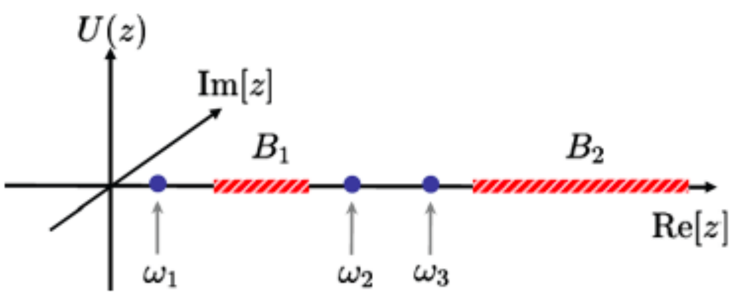
- different open systems with different environmental spectral densities have a universal structure of the non-Markovian solution:

$$u(t-t_0) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{D}(\omega) \exp\{-i\omega(t-t_0)\}$$

where the environmental-modified spectrum:

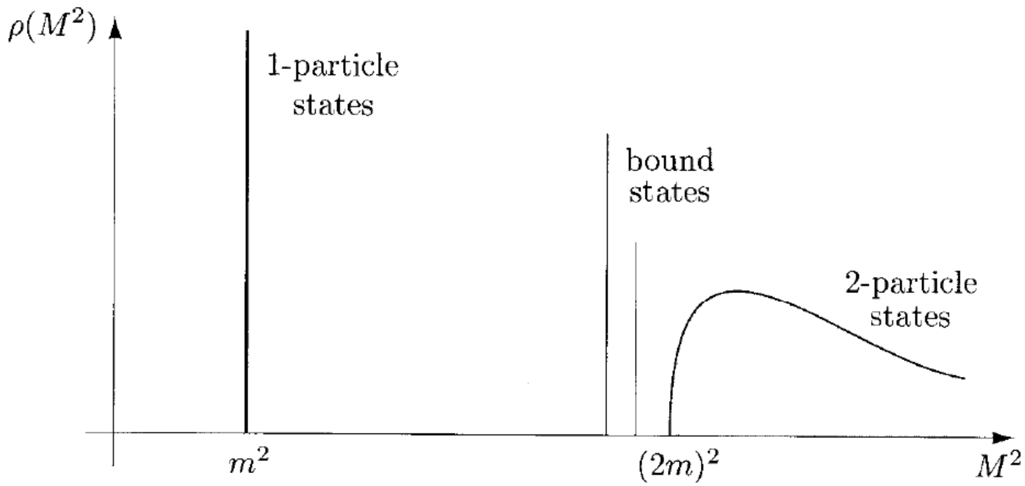
$$\mathcal{D}(\omega) = \sum_j \mathcal{Z}_j \delta(\omega - \omega_j') + \frac{J(\omega)}{[\omega - \varepsilon_s - \Delta(\omega)]^2 + J^2(\omega)/4}$$

$\varepsilon_s \longrightarrow$ localized modes   continuous spectrum part



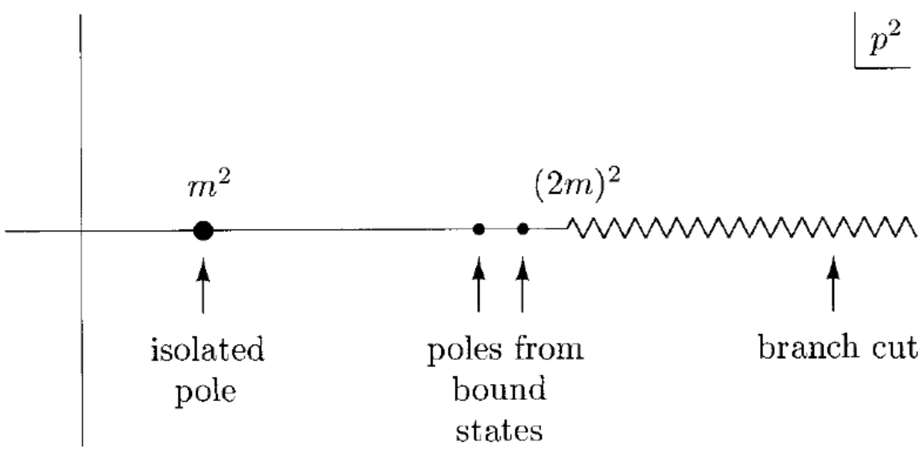
can be directly measured in experiments !!

WMZ, Lo, Xiong & Nori, *Phys. Rev. Lett.* **109**,170402 (2012)



- general structure of two-point correlation functions.

Universality of two-point Green functions



Peskin & Schroesder,

An introduction to quantum field theory, (Westview Press, 1995)

Universality of non-Markovian fluctuations

◆ generalized non-equilibrium fluctuation-dissipation theorem:

$$\boldsymbol{v}(\tau,t)=\sum_\alpha\int_{t_0}^{\tau}d\tau_1\int_{t_0}^{t}d\tau_2\boldsymbol{u}(\tau,\tau_1)\widetilde{\boldsymbol{g}}_\alpha(\tau_1,\tau_2)\boldsymbol{u}^\dagger(t,\tau_2)$$

the steady-limit: $v(t,t\to\infty)=\int\mathcal{V}(\omega)d\omega$

$$\longrightarrow\quad \mathcal{V}(\omega)=\overline{n}(\omega,T)J(\omega)\Big[\underbrace{\Big(\frac{\mathcal{Z}}{\omega-\omega_b}\Big)^2}_{\text{localized modes}}+\underbrace{\frac{1}{[\omega-\varepsilon_s-\Delta(\omega)]^2+J^2(\omega)/4}}_{\text{conventional FDT part}}\Big]$$

Lo, Xiong, WMZ, *Sci. Rep*. **5**, 9423 (2015)
Xiong, Lo, WMZ, Feng & Nori, *Sci. Rep*. **5**, 13353 (2015)

when : $\mathcal{Z}\to 0,\longrightarrow \chi''(\omega)=\overline{n}(\omega,T)\mathcal{D}_c(\omega)$

A. O. Caldeira and A. J. Leggett, *Physics*. A **121**, 587 (1983)

which reproduces the early result of FDT by, e.g. Nyquist (28) , Callen & Wilton (51), and Kubo (65), and taking high T → Einstein's FDT.

How to measure dissipation and fluctuation

WMZ, *et al.*, *Phys. Rev. Lett.* **109**, 170402 (2012)

- Environment-modified spectrum of the system

$$\mathcal{D}(\omega) = \sum_j \mathcal{Z}_j \delta(\omega - \omega_j') + \frac{J(\omega)}{[\omega - \varepsilon_s - \Delta(\omega)]^2 + J^2(\omega)/4}$$

- Environment-induced noise spectrum of the system

$$\mathcal{V}(\omega) = \overline{n}(\omega, T) J(\omega) \Big[\Big(\frac{\mathcal{Z}}{\omega - \omega_b} \Big)^2 + \frac{1}{[\omega - \varepsilon_s - \Delta(\omega)]^2 + J^2(\omega)/4} \Big]$$

◆ Physical consequences:

1. Experimental measuring $\mathcal{D}(\omega)$ and $\mathcal{V}(\omega)$ → one can determine the spectral density $J(\omega)$.
2. Knowing $J(\omega)$, one can predict the dissipation and fluctuation through $\mathcal{D}(\omega)$ and $\mathcal{V}(\omega)$.
3. One can also justify the dissipation-fluctuation theorem through the above relationship.

Noise spectrum and spectral density

Noise spectrum of a system is quantum mechanically determined by the Fourier transform of two-time correlation functions

- Noise spectrum from particle-particle correlations:

$$S(\omega) = \lim_{t\to\infty}\int_{-\infty}^{\infty} e^{i\omega\tau}\langle a^\dagger(t+\tau)a(t)\rangle \, d\tau.$$
$$= \mathcal{Z}^2\delta(\omega-\omega_b)\langle a^\dagger(t_0)a(t_0)\rangle + \frac{\mathcal{Z}^2 J(\omega)\bar{n}(\omega,T)}{(\omega-\omega_b)^2}$$
$$+ \frac{J(\omega)\bar{n}(\omega,T)}{[\omega-\omega_0-\Delta(\omega)]^2+\gamma^2(\omega)}$$
$$= S_1(\omega)+S_2(\omega)+S_3(\omega)$$

Ali, Lo & WMZ, *NJP*. **16**, 103010 (2014)

- Noise spectrum from current-current correlations:

$$S_{\alpha\alpha'}(\omega) = \lim_{t\to\infty}\int_{-\infty}^{\infty} e^{i\omega\tau}\langle \delta I_\alpha(t+\tau)\delta I_{\alpha'}(t)\rangle \, d\tau.$$

with
$$\delta I_\alpha(t) = I_\alpha(t) - \langle I_\alpha(t)\rangle$$

Yang, Lin & WMZ, *PRB* **89**, 115411 (2014)

Example: bosonic open systems

- Such as nanophotonic or optomechanical resonator, etc.

- General reservoir: $J(\omega) = 2\pi\eta\omega\left(\frac{\omega}{\omega_c}\right)^{s-1}\exp\left(-\frac{\omega}{\omega_c}\right)$

A. J. Leggett, *et al*. RMP **59**, 1 (1987)

- General solution:

$$u(t) = \mathcal{Z}e^{-i\omega' t} + \frac{2}{\pi}\int_0^\infty d\omega \frac{J(\omega)e^{-i\omega t}}{4[\omega - \varepsilon_s - \Delta(\omega)]^2 + J^2(\omega)}$$

where $\Delta(\omega) = \frac{1}{2}[\Sigma(\omega + i0^+) + \Sigma(\omega - i0^+)]$ and

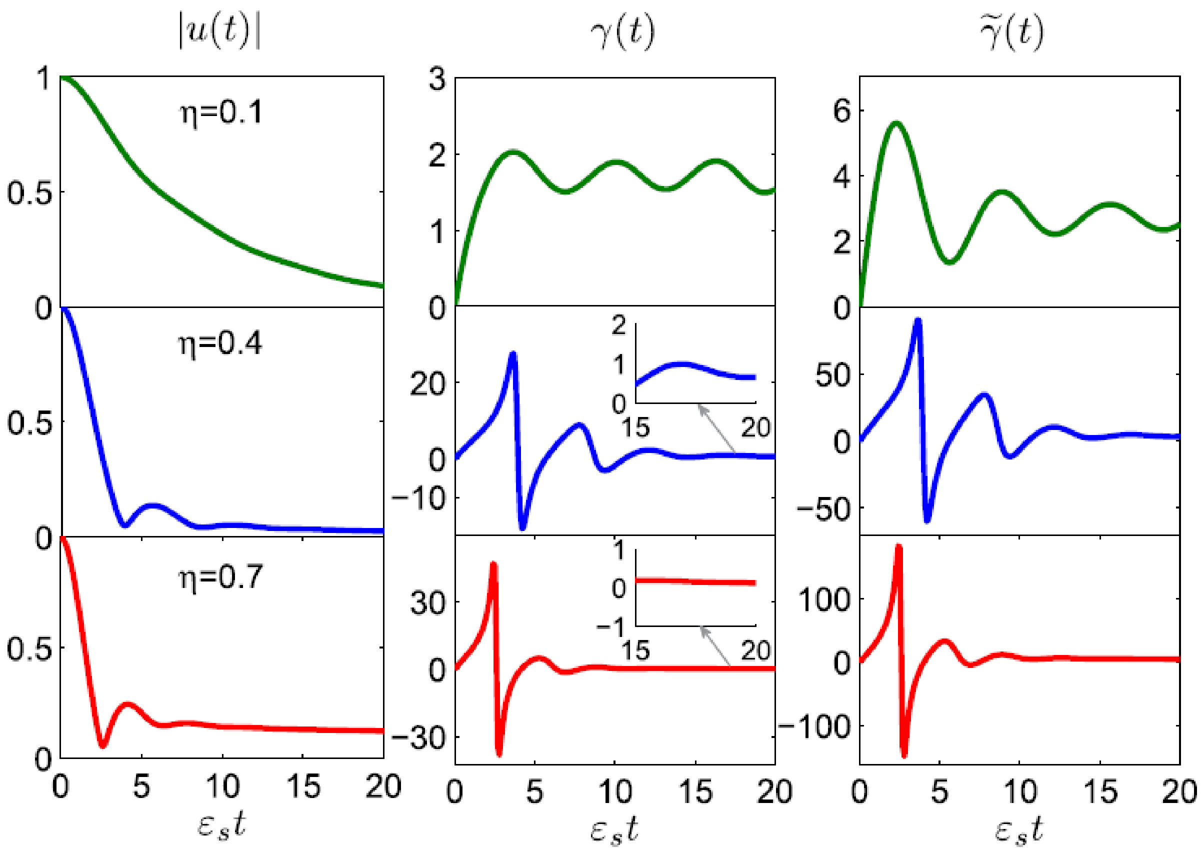
$$\Sigma(\omega) = \begin{cases} \eta\omega_c\left[\pi\sqrt{-\widetilde{\omega}}e^{-\widetilde{\omega}}\mathrm{erfc}(\sqrt{-\widetilde{\omega}}) - \sqrt{\pi}\right] & s = 1/2 \text{ sub-Ohmic} \\ \eta\omega_c\left[\widetilde{\omega}\exp(-\widetilde{\omega})\mathrm{Ei}(\widetilde{\omega}) - 1\right] & s = 1 \quad \text{Ohmic} \\ \eta\omega_c\left[\widetilde{\omega}^3 e^{-\widetilde{\omega}}\mathrm{Ei}(\widetilde{\omega}) - \widetilde{\omega}^2 - \widetilde{\omega} - 2\right] & s = 3 \quad \text{super-Ohmic} \end{cases}$$

WMZ, LO, Xiong, Tu & Nori, *Phys. Rev. Lett*. **109**, 170402 (2012).

Markovian and non-Markovian processes

Dissipation dynamics | Dissipation coefficient | Fluctuation coefficient

for sub-Ohmic bath



Markovian dynamics

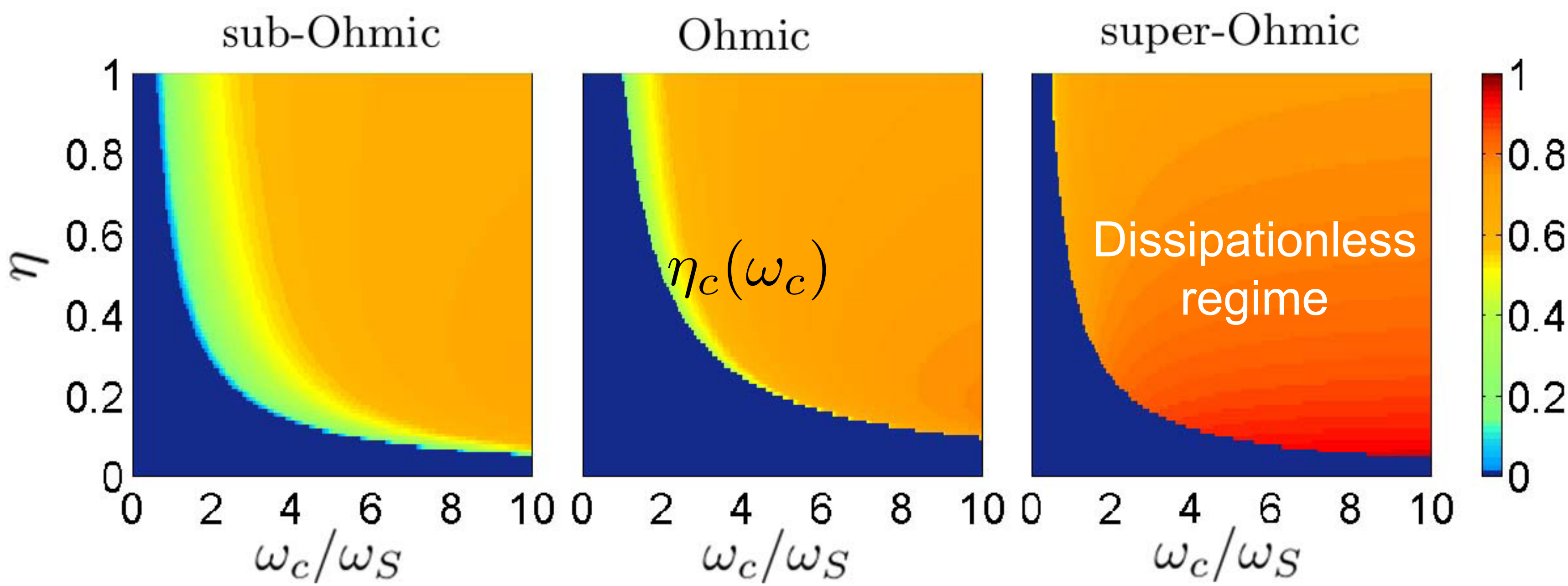
Short-time non-Markovian dynamics

Long-time non-Markovian dynamics

WMZ, LO, Xiong, Tu & Nori, *Phys. Rev. Lett*. **109**, 170402 (2012).

Dissipation to dissipationless transition for Ohmic-type spectral densities

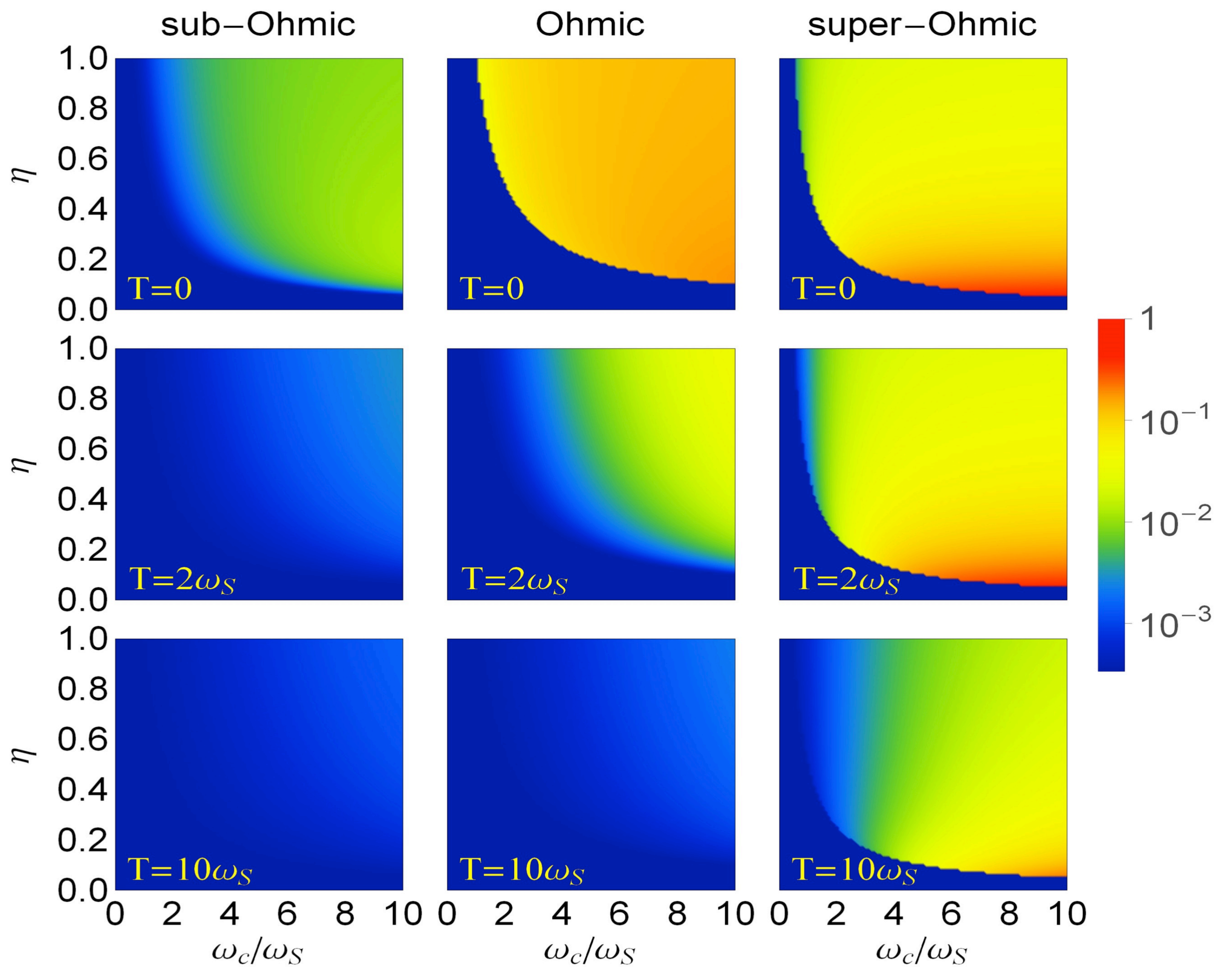
$$J(\omega) = \eta\omega(\omega/\omega_c)^{s-1}\exp(-\omega/\omega_c)$$



boundary line: $\eta_c(\omega_c) = \omega_S/[\omega_c\Gamma(s)]$

Xiong, Lo, WMZ, Feng & Nori, *Sci. Rep.* **5**, 13353 (2015)

Dissipation to dissipationless transition via temperature:



General non-Markovian dissipation and fluctuations

WMZ, *et al., Phys. Rev. Lett.* **109**, 170402 (2012)

- General solution of the spectral Green function contains two parts:
  1. Localized bound states, arisen from the band gaps of the spectral density, $J(\omega)=0$, → Dissipationless oscillations.
  2. Non-exponential decays, due to the nonanalyticity of the self-energy induced from the environment.

- In the weak coupling region:
  1. The localized bound states have little contribution.
  2. The non-exponential decays are reduced to exponential decays (mainly observed in Markovian dynamics).

Conclusions on General Theory of OQS:

- Establishing a rigorous connection of the exact master equation with the Schwinger-Keldysh's nonequilibrium Green functions → provides a new way to understand nonequilibrium physics of open quantum systems.

- Quantum dissipation and fluctuations (relaxation and dephasing) can be obtained through the computation of nonequilibrium Green functions and obey the generalized nonequilibrium fluctuation-dissipation theorem.

- Universal non-Markovianity contains localized modes plus non-exponential decays. The former comes from the band gaps of the spectral density and the latter arises from the non-analyticity of the environment-induced self-energy corrections.

- It shows that experimentally, by measuring the environment-modified spectrum of the system, one is able to understand non-Markovian effects and decoherence phenomena, and the structure of the environment.

- As long as one can compute nonequilibrium correlation Green functions in the real-time domain, one can obtain the full information about non-Markovian decoherence for more complicated open quantum systems, no need of the knowledge of master equation.

Outline

- ✧ General open quantum systems
- ✧ A general theory of open quantum systems
- ✧ General analytical solution of non-Markovianity
- ✧ Measurement of non-Markovianity (a new point of view)
- ✧ Non-Markovian nonequilibrium physics
- ✧ Non-Markovian entanglement decoherence
- ✧ Non-Markovian decoherence of Majorana qubits
- ✧ Transient quantum transport and quantum control
- ✧ Summary

Non-Markovianity measure: Memory effects

- Math of non-Markovianity: divisibility of dynamical map, monotonic decrease of distinguishability of states using trace distance, monotonic decrease of entanglement between the system and an ancilla, negative decay rate, using mutual information

  H. P. Breuer, *et al. Rev. Mod. Phys*. **88**, 021002 (2016)

- Physics of Non-Markovianity: characters of memory effect due to strong back action (memory effects), significant in the short and long transient regime, strong SE coupling, low temperature, finite size structured environment, etc.

- Two-time correlation functions measuring non-Markovianity: physically, two-time correlation functions correlating a past event with its future provide direct information about the system-environment back-action processes revealing the memory dynamics.

  M. M. Ali, P. Y. Lo, M.W. Y. Tu, WMZ, *Phys. Rev.* A **92**, 062306 (2015)

Two-time correlation functions:

$$C(t,\tau) = \frac{\langle A(t)B(t+\tau)\rangle}{\sqrt{\langle A(t)B(t)\rangle\langle A(t+\tau)B(t+\tau)\rangle}}$$

directly measurable in experiments or calculated with exact master equation

- **A quantitative measure of Non-Markovianity**:

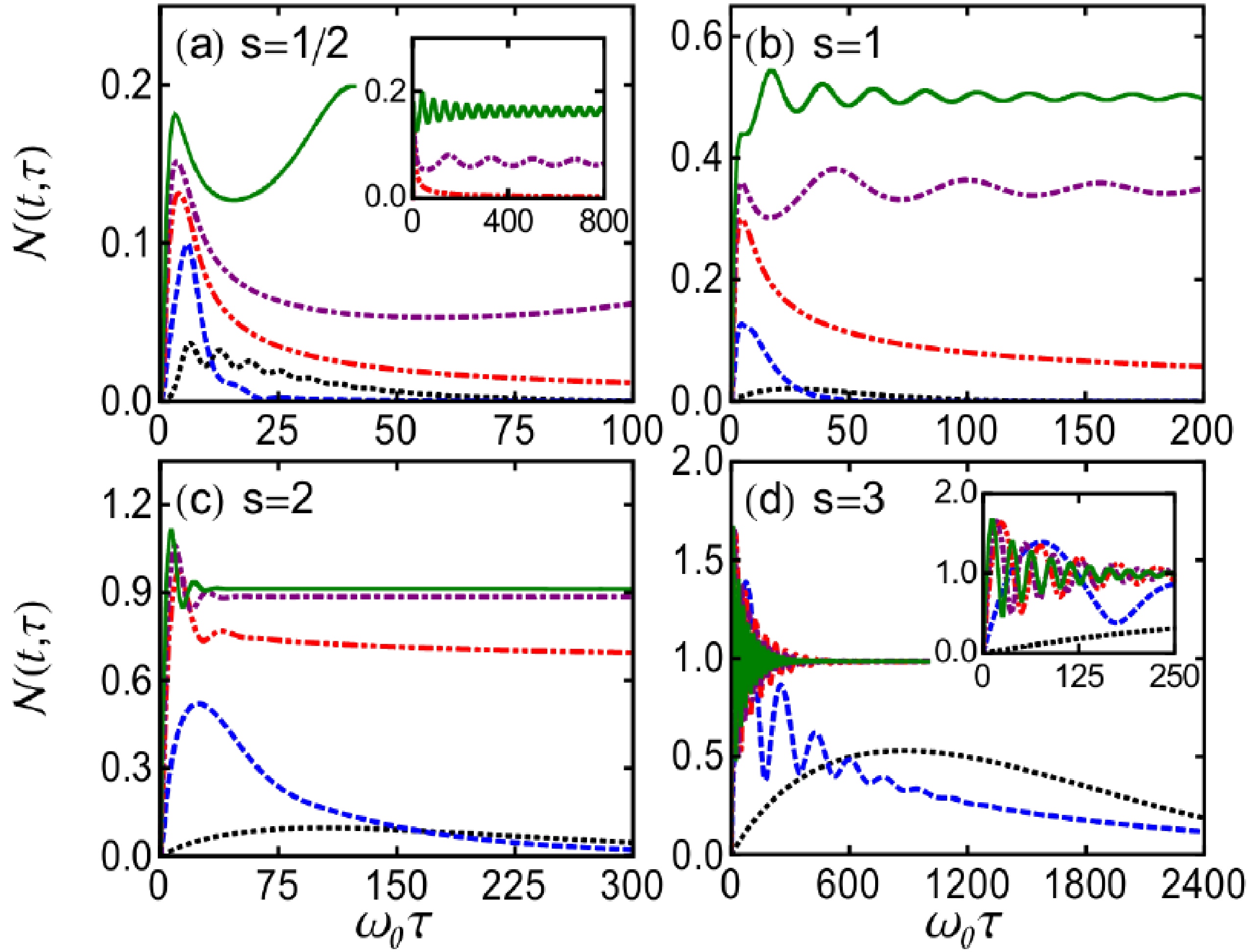
$$\mathcal{N}(t,\tau) = \left|C(t,\tau) - C_{\mathrm{B}M}(t,\tau)\right|,$$

where $C_{\mathrm{B}M}(t,\tau)$ is the correlation without memory effect, which can be calculated under Quantum Regression Theorem (QRT), and QRT is valid only under Born-Markov approximation so that it can be calculated from the Born-Markov master equation under QRT.

M. M. Ali, P. Y. Lo, M.W. Y. Tu, WMZ, *Phys. Rev.* A **92**, 062306 (2015)

A bosonic system in a thermal bath

$$\mathcal{N}(t,\tau) = |C(t,\tau) - C_{\mathrm{BM}}(t,\tau)| \qquad C(t,\tau) = \frac{\langle a^\dagger(t)a(t+\tau)\rangle}{\sqrt{\langle a^\dagger(t)a(t)\rangle\langle a^\dagger(t+\tau)a(t+\tau)\rangle}}$$



Different curves for different system-bath coupling strengths

Physica **121A** (1983) 587–616 *North-Holland Publishing Co.*

PATH INTEGRAL APPROACH TO QUANTUM BROWNIAN MOTION*

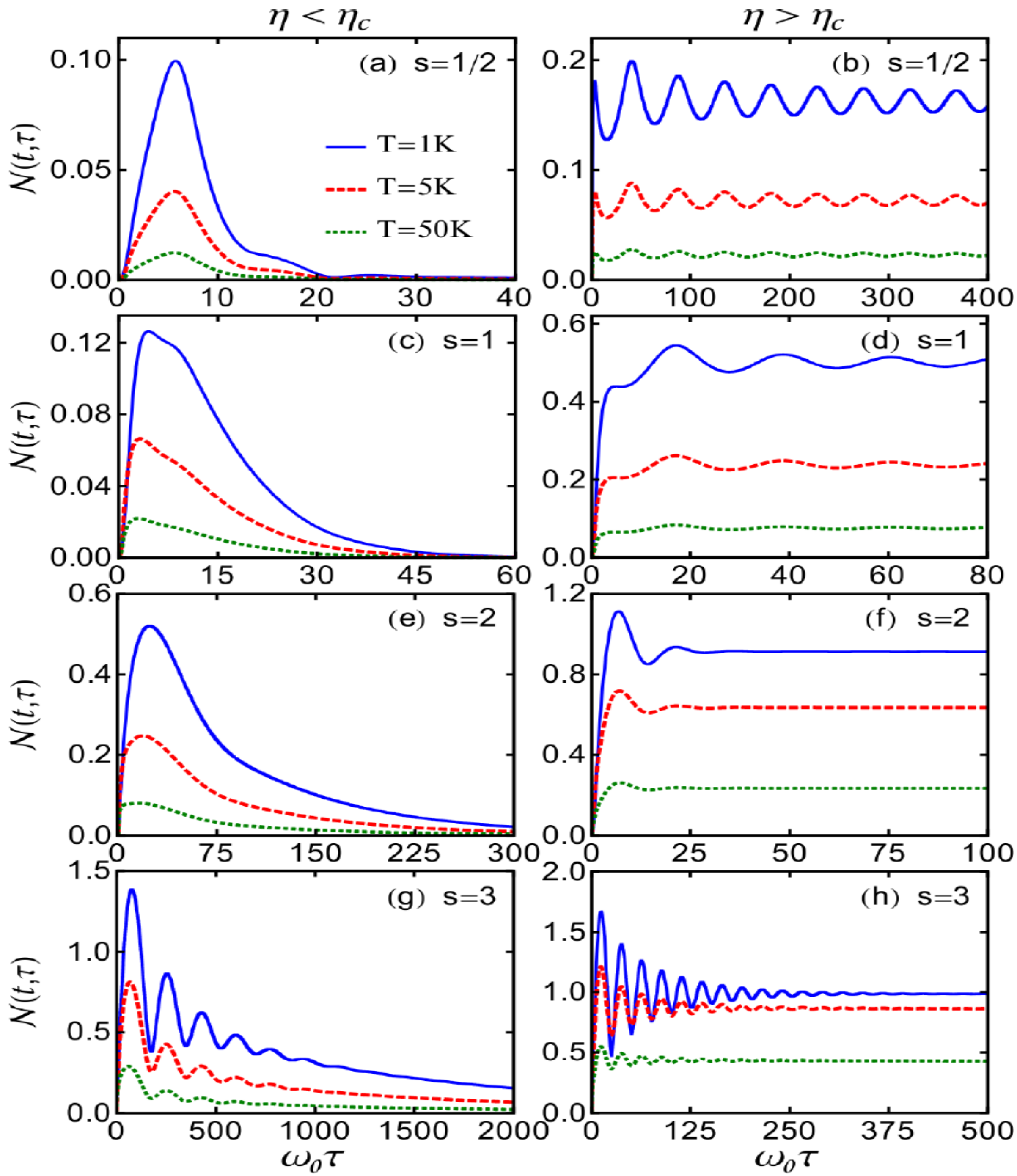
A.O. CALDEIRA AND A.J. LEGGETT

7. Conclusions

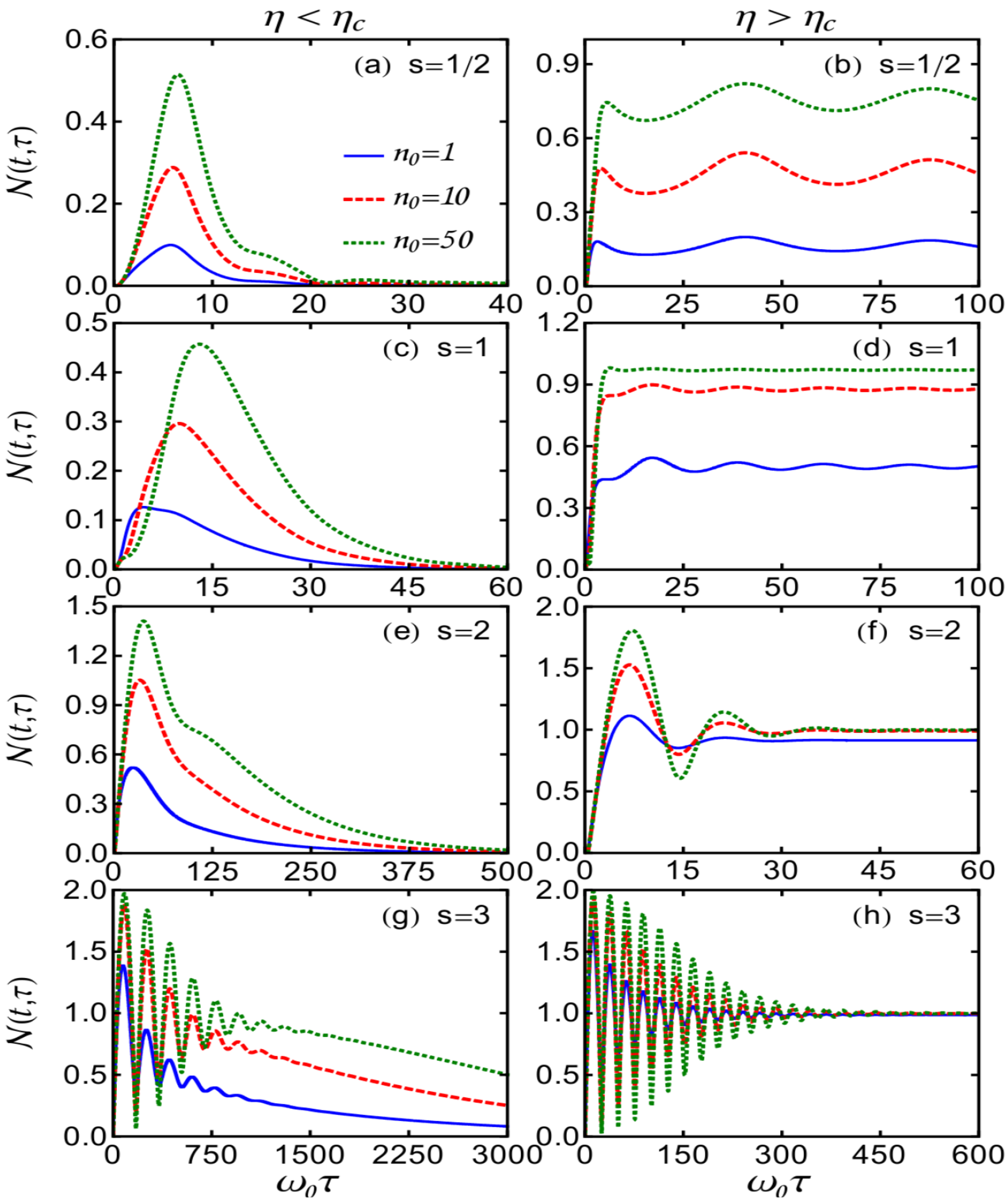
. . . .

Actually, it has been an old dream of many physicists to try to describe the relaxation to equilibrium by Markoffian equations (no time kernels involved) even in the quantum regime. However, we are a bit sceptical about the possibility of this, since as we have shown simple models can recover both equilibrium statistical mechanics and linear response theory asymptotically without being Markoffian at all. Ford et al.[16]) also achieved the same conclusion about the Markoffian assumption in the quantum limit. Notice that we are talking here only about the diffusion terms. Our drift term (the one involving γ) is always Markoffian. In order to find non-Markoffian corrections to the latter one needs to study the modifications of $\chi''(\nu)$ due to the sum rules which are important to describe the short time behaviour of the Brownian particle. incomplete in general!

Temperature dependency of the non-Markovianity



Initial state dependence of the non-Markovianity



Conclusion on non-Markovianity:

- Short-time non-Markovian memory effect, known for long time ago, A. O. Caldeira and A. J. Leggett, *Physics*. A **121**, 587 (1983)

- Long-time non-Markovian memory effect, just discovered recently in our theoretical framework,

- Non-Markovian memory effect is decreased with increasing the temperature of the reservoir.

- Non-Markovian memory effect depends sensitively on initial states, in particular, for the long-time non-Markovianity.

M. M. Ali, P. Y. Lo, M.W. Y. Tu, WMZ, *Phys. Rev.* A **92**, 062306 (2015)

Outline

- ✧ General open quantum systems
- ✧ A general theory of open quantum systems
- ✧ General analytical solution of non-Markovianity
- ✧ Measurement of non-Markovianity
- ✧ **Non-Markovian nonequilibrium physics**
- ✧ **Non-Markovian entanglement decoherence**
- ✧ **Non-Markovian decoherence of Majorana qubits**
- ✧ **Transient quantum transport and quantum control**
- ✧ **Summary**

Physica **121A** (1983) 587–616 *North-Holland Publishing Co.*

PATH INTEGRAL APPROACH TO QUANTUM BROWNIAN MOTION*

A.O. CALDEIRA AND A.J. LEGGETT

7. Conclusions

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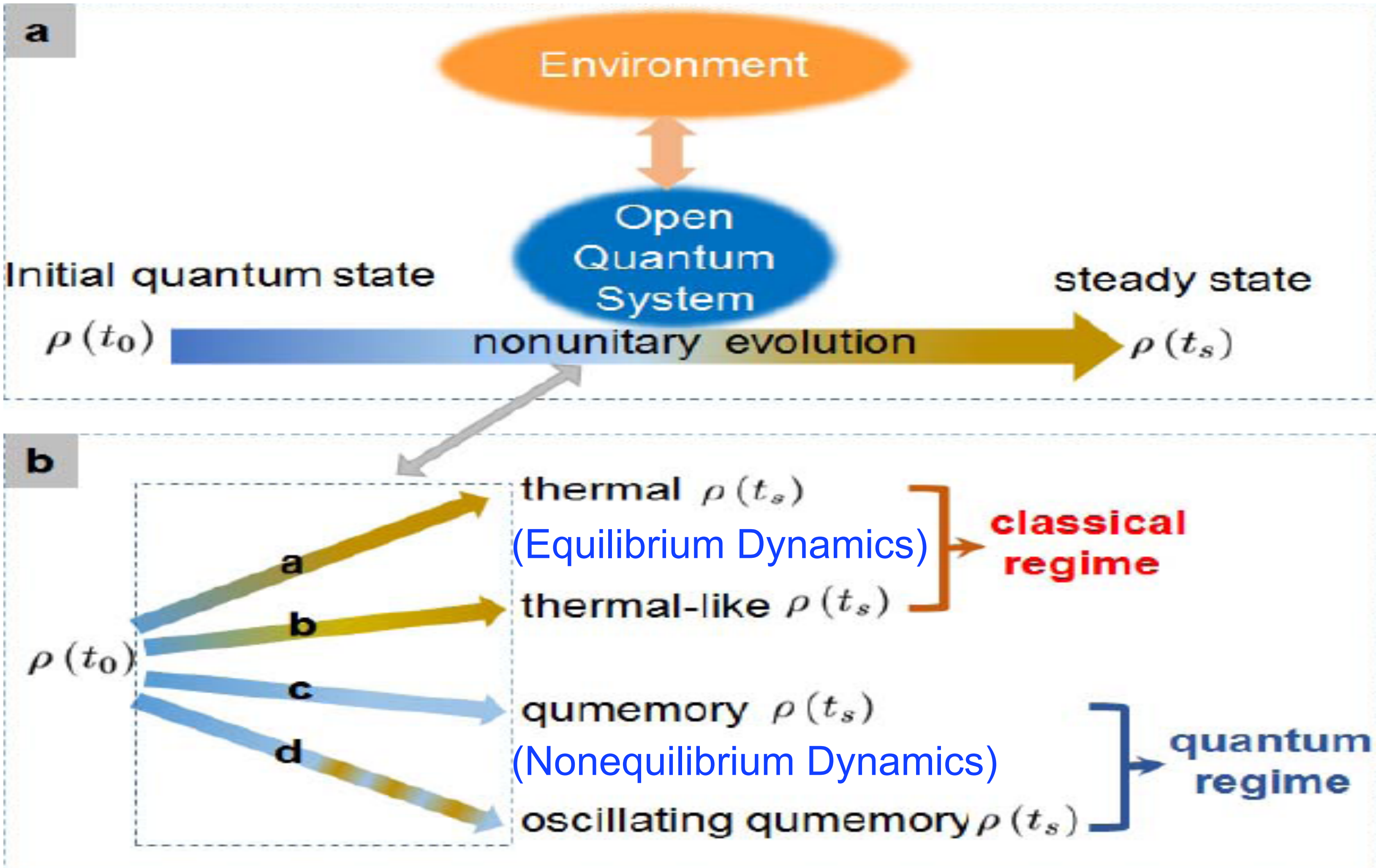
Actually, it has been an new old dream of many physicists to try to describe the relaxation to equilibrium by non-Markovian Markoffian equations (no time kernels involved) even in the quantum regime. However, we are a bit sceptical about the possibility of this, since as we have shown simple models can recover both equilibrium statistical mechanics and linear response theory asymptotically without being Markoffian at all. Ford et al.[16]) also achieved the same conclusion about the Markoffian assumption in the quantum limit. Notice that we are talking here only about the diffusion terms. Our drift term (the one involving γ) is always Markoffian. In order to find non-Markoffian corrections to the latter one needs to study the modifications of $\chi''(\nu)$ due to the sum rules which are important to describe the short time behaviour of the Brownian particle.

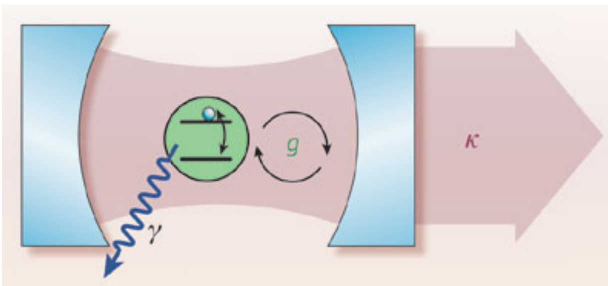
Nonequilibrium and Equilibrium Physics:

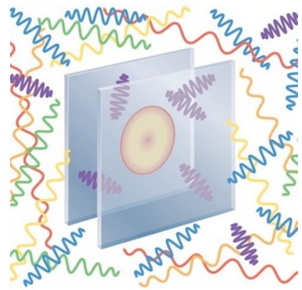
➔ Solving the exact master equation with arbitrary initial state



Xiong, Lo, WMZ, Feng and Nori, *Sci. Rep*. **5**, 13353 (2015)

Photon dynamics in cavities:





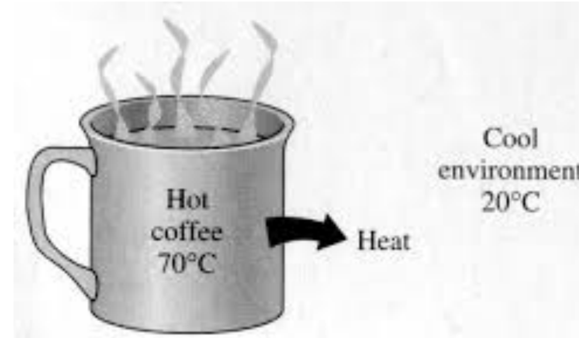


FIGURE 1–3
Heat flows in the direction of decreasing temperature.

Solving the master equation, one can find

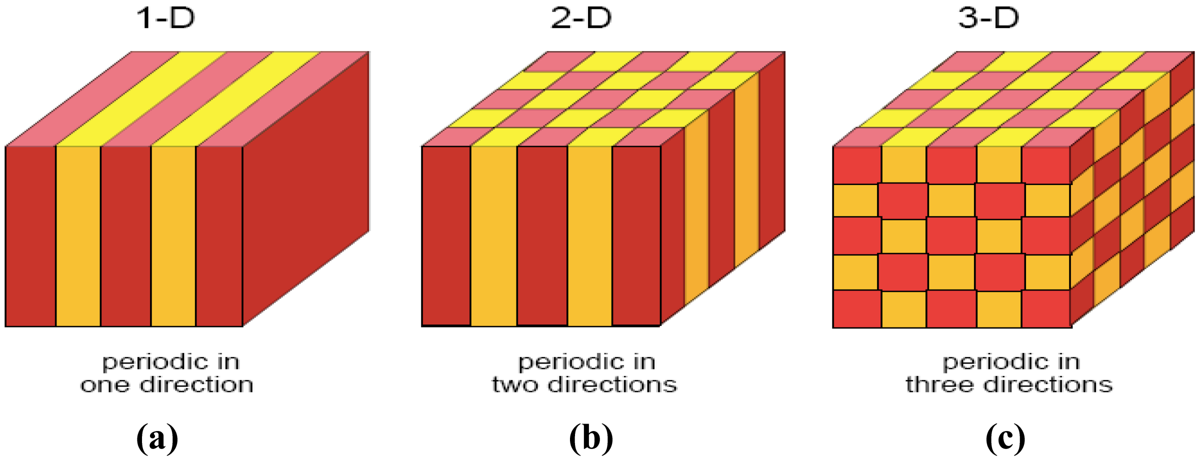
$$\langle a(t)\rangle = \langle a(t_0)\rangle e^{-(i\omega_C+\kappa)(t-t_0)} \to 0$$ ← dissipation

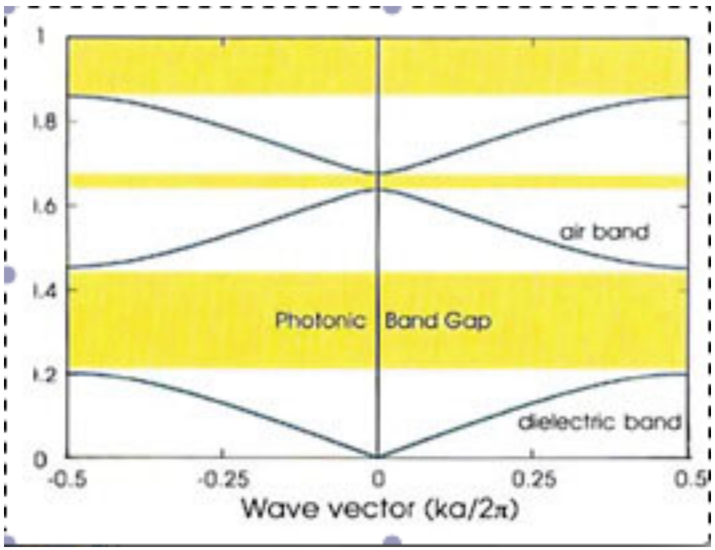
$$n(t) = n(t_0)e^{-2\kappa(t-t_0)} + \overline{n}(\omega_C, T)[1 - e^{-2\kappa(t-t_0)}]$$
$$\to \overline{n}(\omega_C, T)$$ ← fluctuations

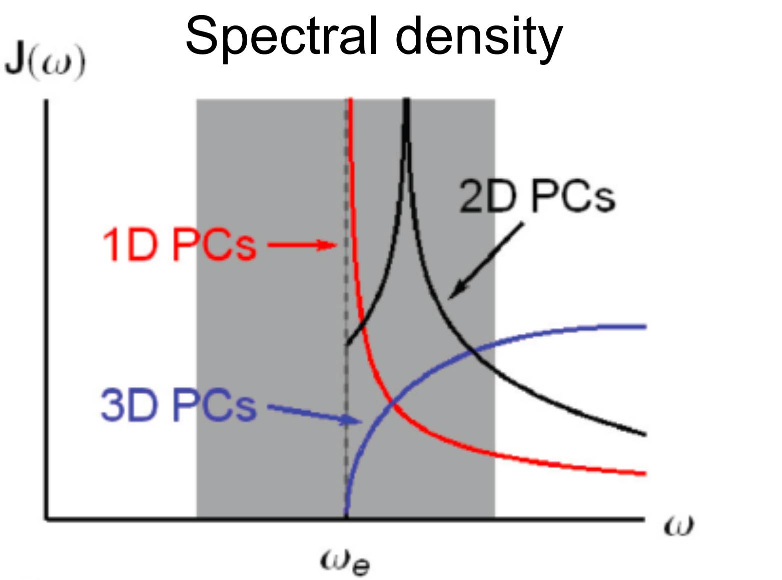
◆ Bose-Einstein distribution → thermal equilibrium

Photonic band gap structures:

J. D. Joannopoulos, et al, Photonic crystals, Modeling the Flow of Light



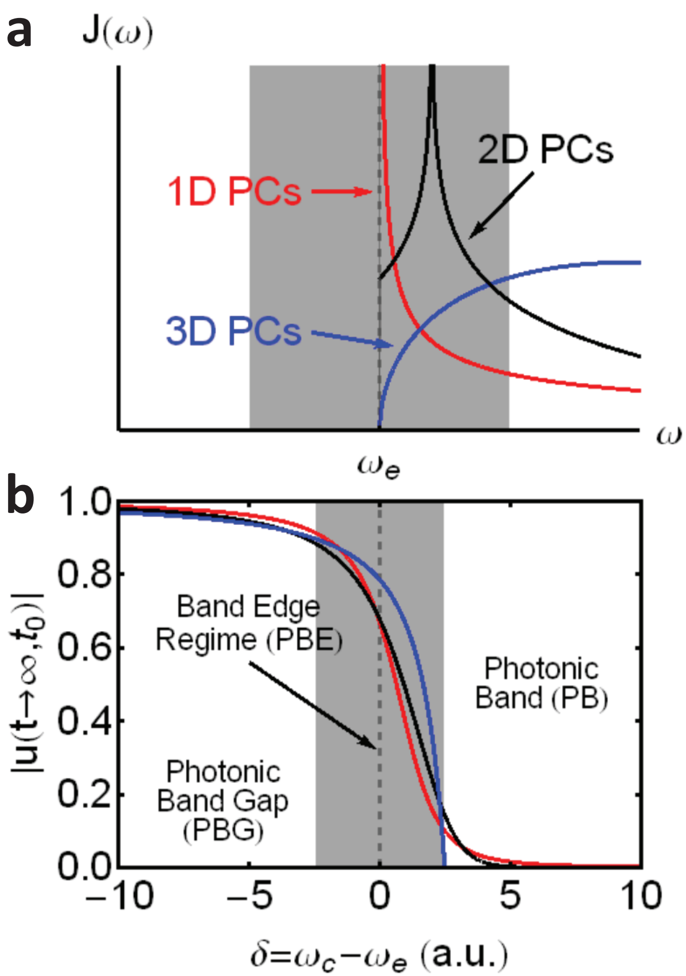


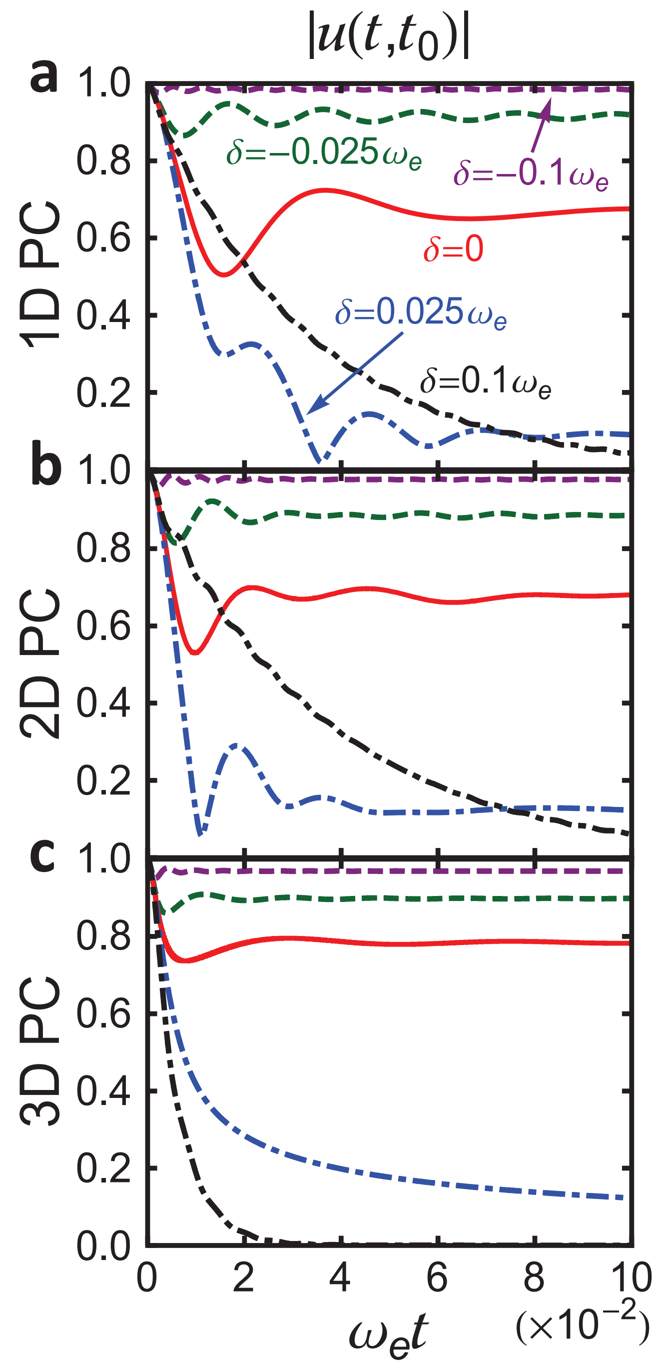


Lo, Xiong & WMZ, *Sci. Rep.* **5**, 9423 (2015)

Photonic dissipative dynamics

$$\langle a(t)\rangle = u(t,t_0)\langle a(t_0)\rangle$$

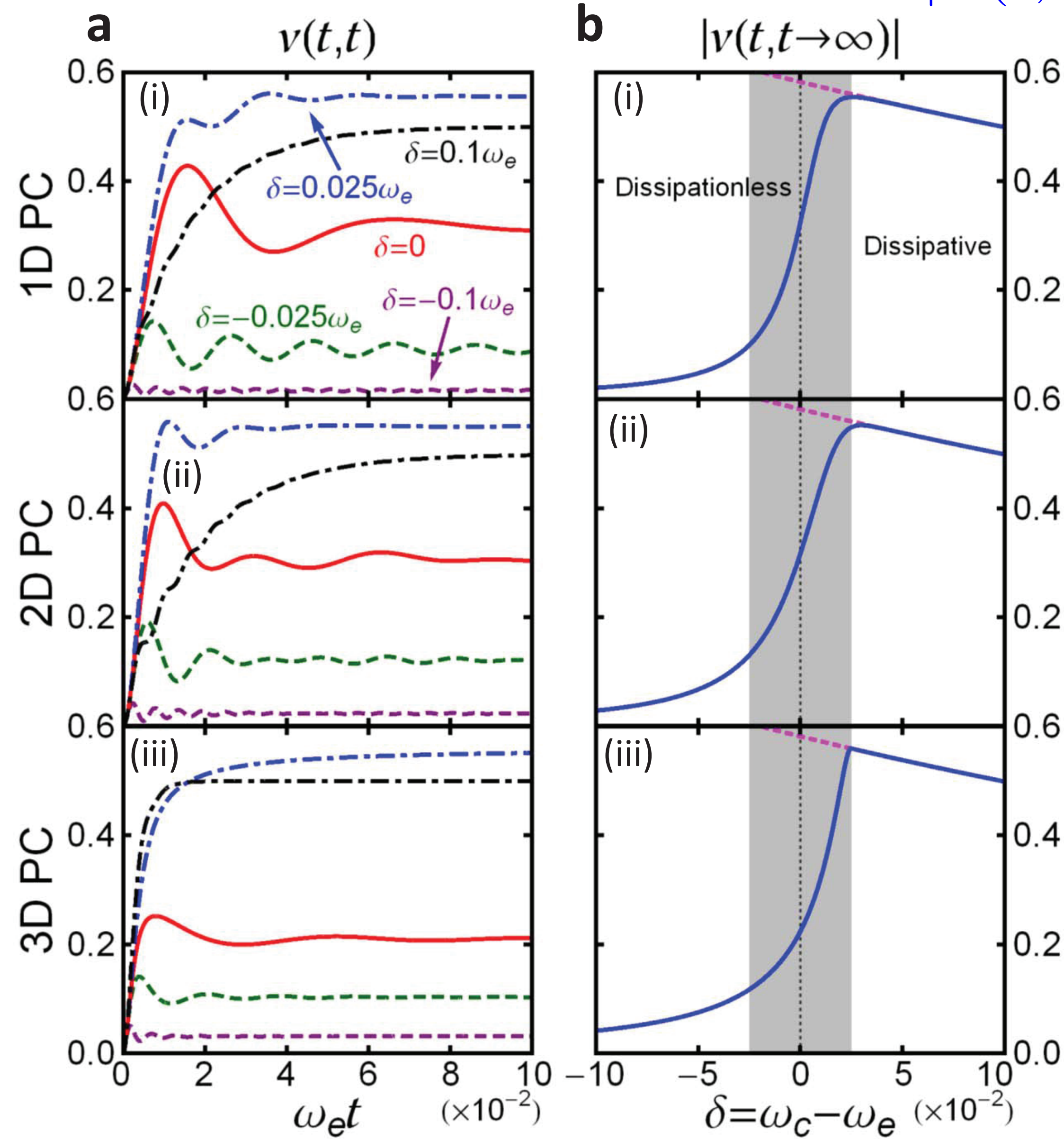




Lo, Xiong & WMZ, *Sci. Rep.* **5**, 9423 (2015)

Photonic fluctuation dynamics:

$$n(t) = \langle a^\dagger(t)\, a(t) \rangle = |u(t,t_0)|^2\, n(t_0) + v(t,t)$$

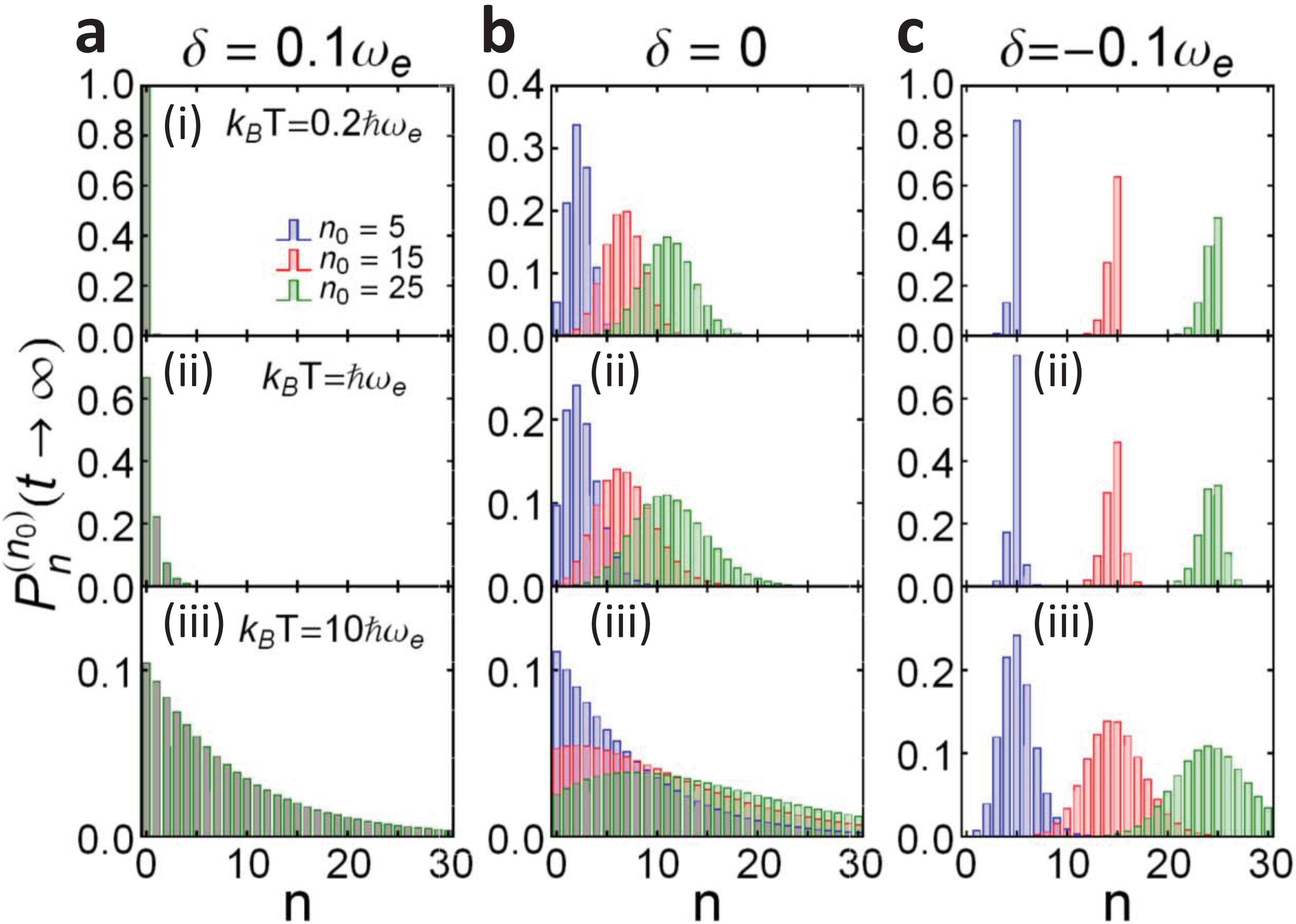


crossover

crossover

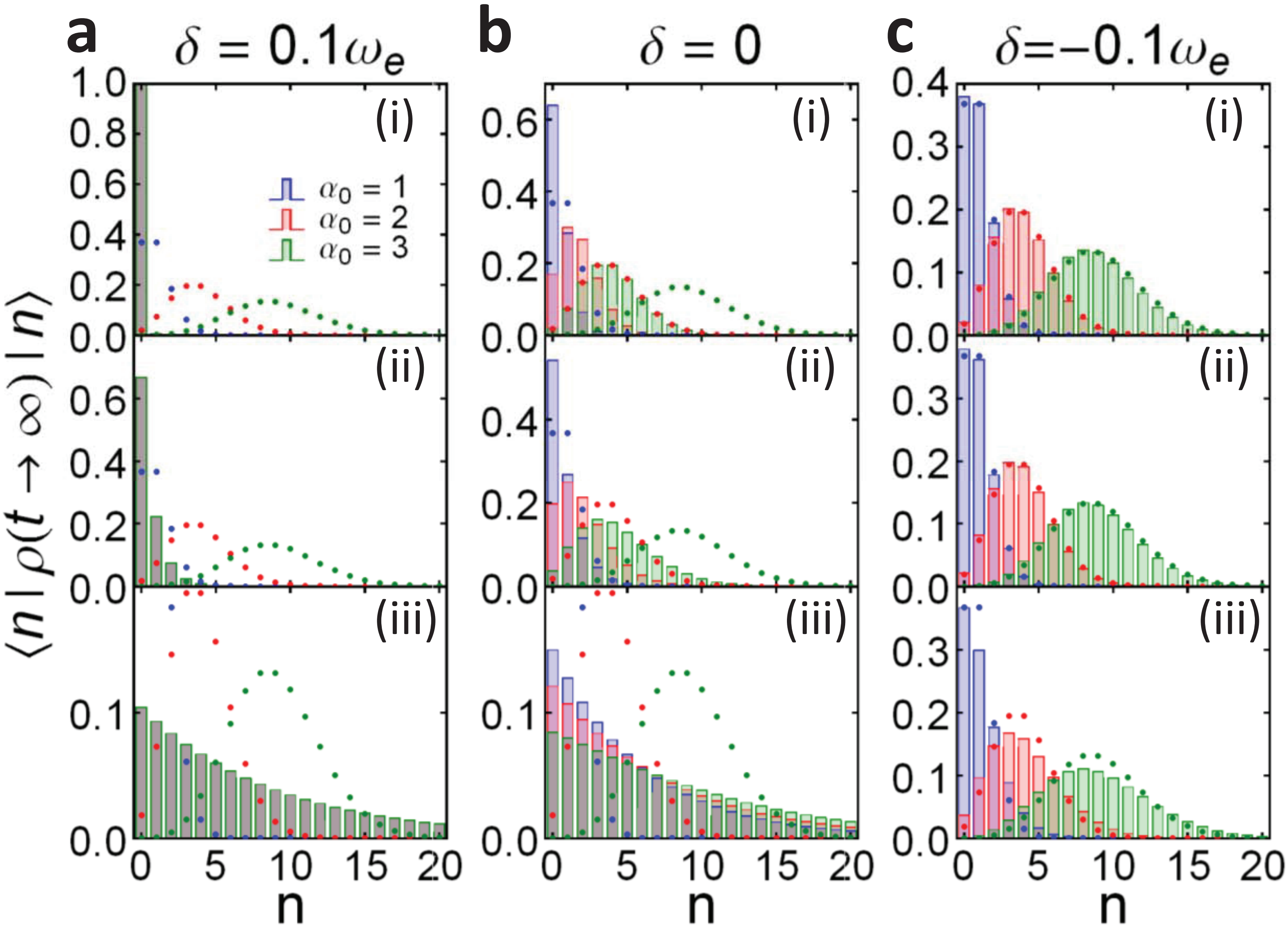
critical transition

Thermalization to Localization transition for initial Fock states :



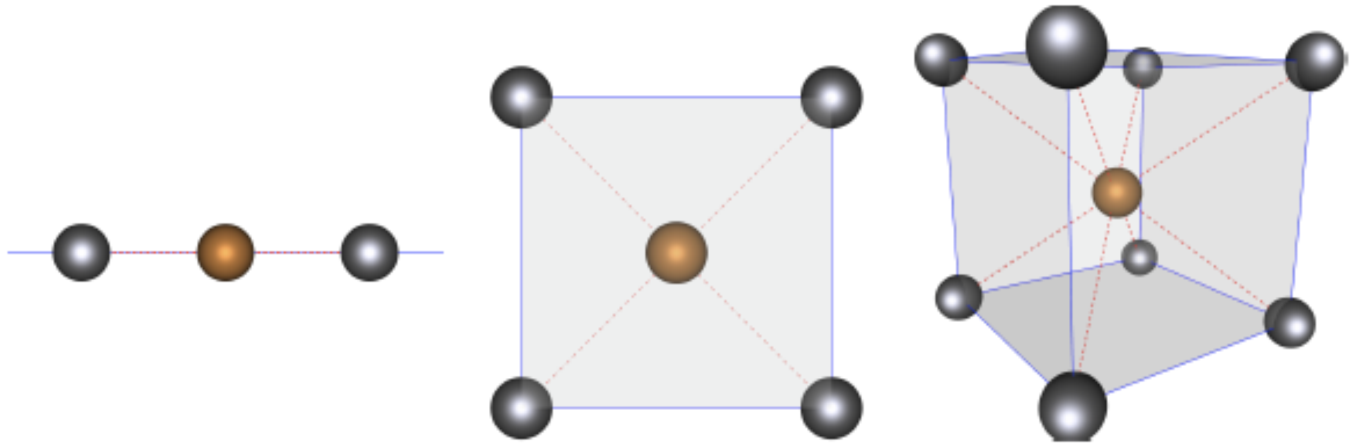
Lo, Xiong & WMZ, *Sci. Rep.* **5**, 9423 (2015)

Thermalization to Localization transition for initial coherent states :



Lo, Xiong & WMZ, *Sci. Rep.* **5**, 9423 (2015)

Defeats in lattice structures



(a) linear lattice (b) square lattice (c) simple cubic lattice

Sorry, this part is omitted because the result has not be published yet……..

Y. W. Huang and WMZ (2017)

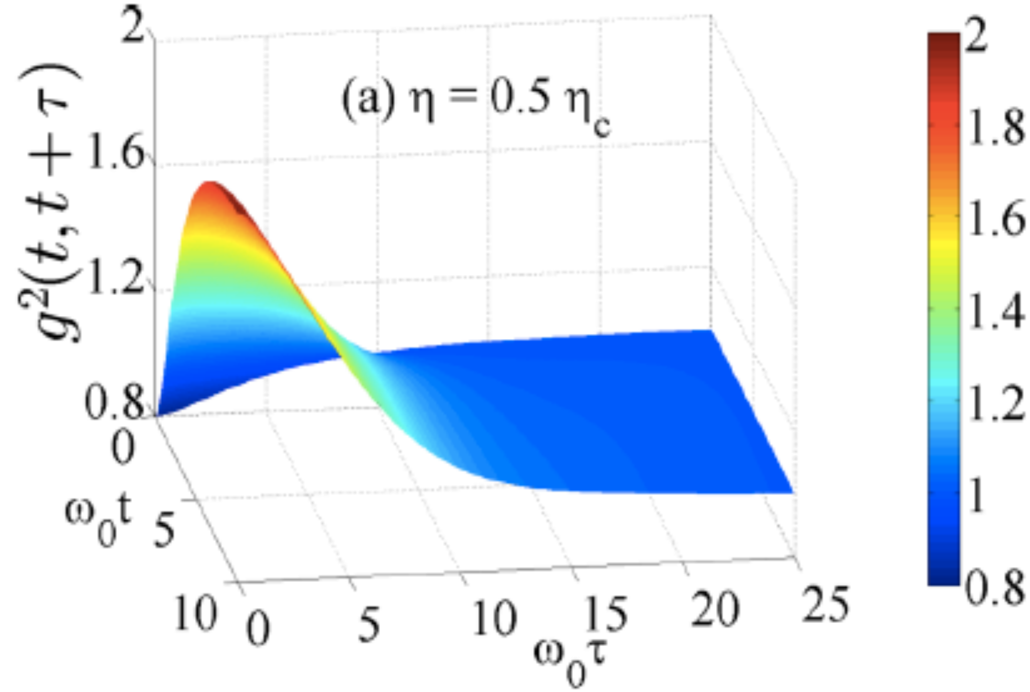
Nonequilibrium photon statistics:

Photon bunching and antibunching statistics are usually characterized by the steady-state ($t \to \infty$) second-order correlation function $g^{(2)}(t, t+\tau)$, where an increasing (decreasing) magnitude of $g^{(2)}(t, t+\tau)$ with delay-time τ demonstrate antibunching (bunching) statistics of photons.

$$g^{(2)}(t, t+\tau) = \frac{\langle a^\dagger(t) a^\dagger(t+\tau) a(t+\tau) a(t) \rangle}{\langle a^\dagger(t) a(t) \rangle \langle a^\dagger(t+\tau) a(t+\tau) \rangle}$$

We studied the exact nonequilibrium transient dynamics of photon statistics for a micro-cavity coupling with a thermal reservoir with Ohmic spectral density.

M. M. Ali and WMZ, *Phys. Rev.* A **95**, 033830 (2017)

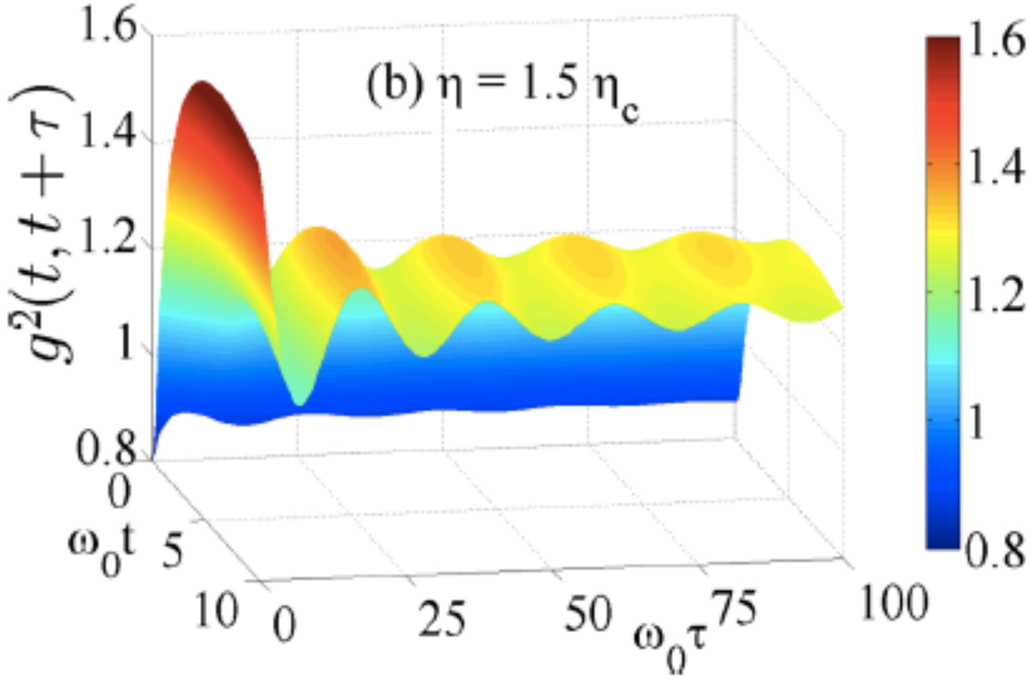


In short-time non-Markovian regime, the steady state is a thermal state:

$$\rho(t_s) = \sum_{n=0}^{\infty} \frac{[v(t_s)]^n}{[1+v(t_s)]^{n+1}} |n\rangle\langle n|,$$

M. M. Ali and WMZ,
Phys. Rev. A **95**, 033830 (2017)

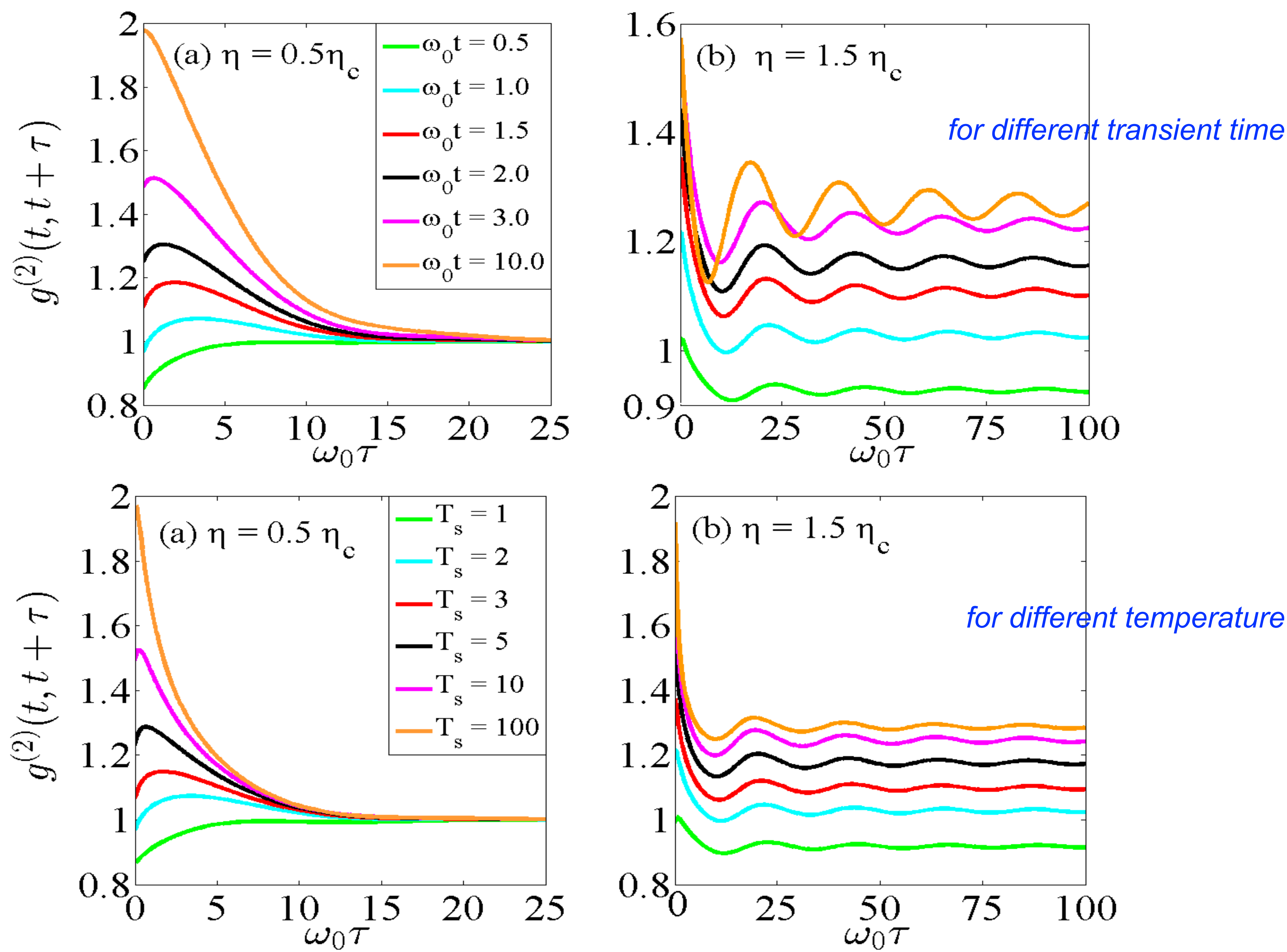
- For an initial Fock state

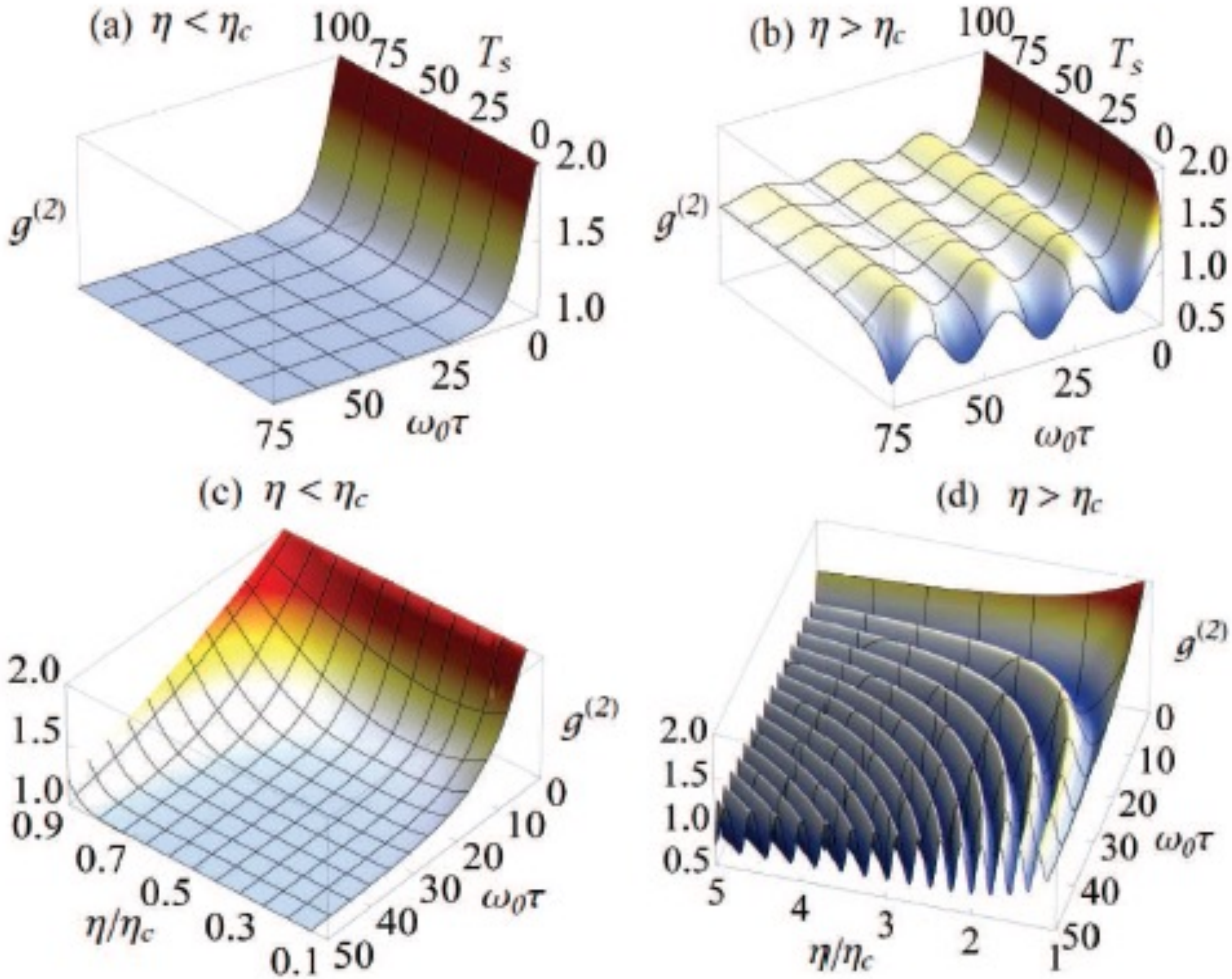


In long-time non-Markovian regime, the steady state cannot be thermalized

$$\rho(t) = \sum_{n=0}^{\infty} p_n(t_s)|n\rangle\langle n|,$$

$$p_n(t_s) = \frac{[v(t_s)]^n}{[1+v(t_s)]^{n+1}}[1-\Omega(t_s)]^{n_0} \times \sum_{k=0}^{\min\{n_0,n\}} \binom{n_0}{k}\binom{n}{k}\left[\frac{1}{v(t_s)}\frac{\Omega(t_s)}{1-\Omega(t_s)}\right]^k$$


(a) η = 0.5η_c
ω_0t = 0.5
ω_0t = 1.0
ω_0t = 1.5
ω_0t = 2.0
ω_0t = 3.0
ω_0t = 10.0
(b) η = 1.5 η_c
for different transient time
(a) η = 0.5 η_c
T_s = 1
T_s = 2
T_s = 3
T_s = 5
T_s = 10
T_s = 100
(b) η = 1.5 η_c
for different temperature
g^(2)(t, t + τ)
ω_0τ



New type of phase transition of photon statistics occurs at a critical value while passing through weak to strong system-reservoir coupling

M. M. Ali and WMZ, *Phys. Rev.* A **95**, 033830 (2017)

Outline

- ✧ General open quantum systems
- ✧ A general theory of open quantum systems
- ✧ General analytical solution of non-Markovianity
- ✧ Measurement of non-Markovianity
- ✧ Non-Markovian nonequilibrium physics
- ✧ Non-Markovian entanglement decoherence
- ✧ Non-Markovian decoherence of Majorana qubits
- ✧ Transient quantum transport and quantum control
- ✧ Summary

Nonequilibrium quantum phase transition via entanglement decoherence dynamics:

- Entanglement is not only the useful resource for quantum information processing, it is also used to characterize the many fundamental physics issues, quantum phase transition is such an issue.

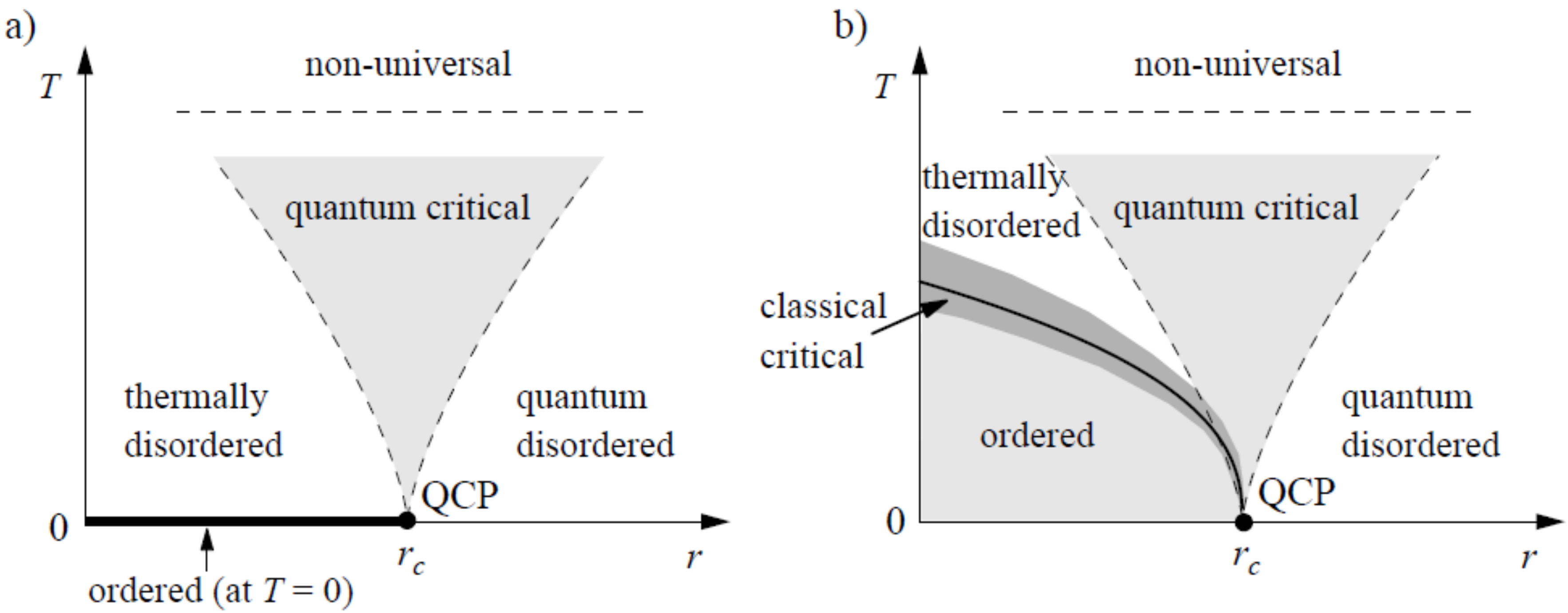
- We studied quantum phase transition in open quantum systems, which allows to explore the nonequilibrium quantum phase transition via entanglement decoherence dynamics.

- We consider a two-mode cavity system coupling to a non-Markovian reservoir, and the two mode optical fields are initially in an entangled squeezed state.

$$|\psi_s(0)\rangle = \exp\{ra_1a_2 - ra_1^\dagger a_2^\dagger\}|00\rangle$$

Y. C. Lin, P. Y. Yang and WMZ, *Scientific Reports* **6**, 34804 (2016)

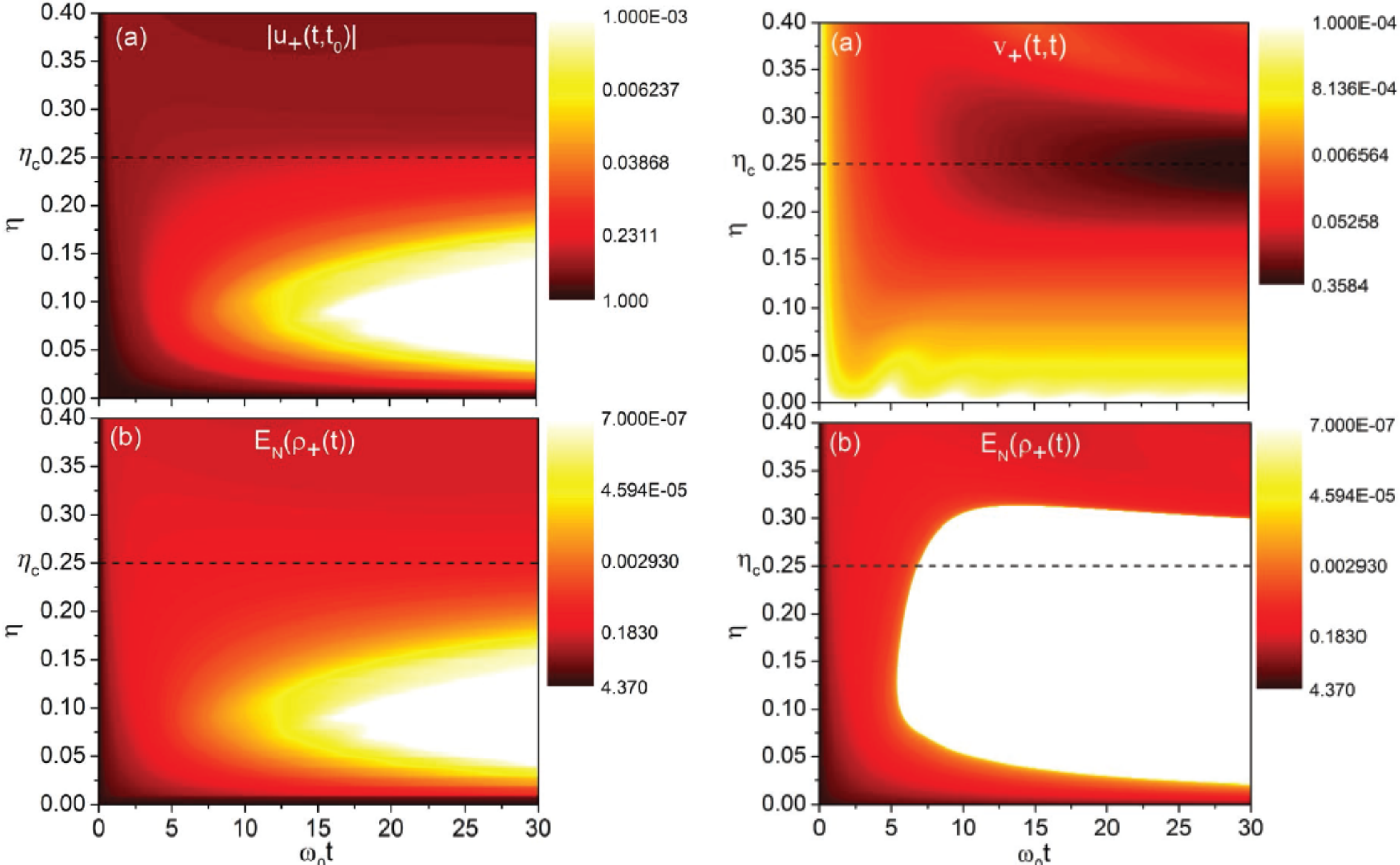
Schematic phase diagrams



M. Vojta, Rep. Prog. Phys. **66,** 2069 (2003).

Real-time entanglement decoherence dynamics

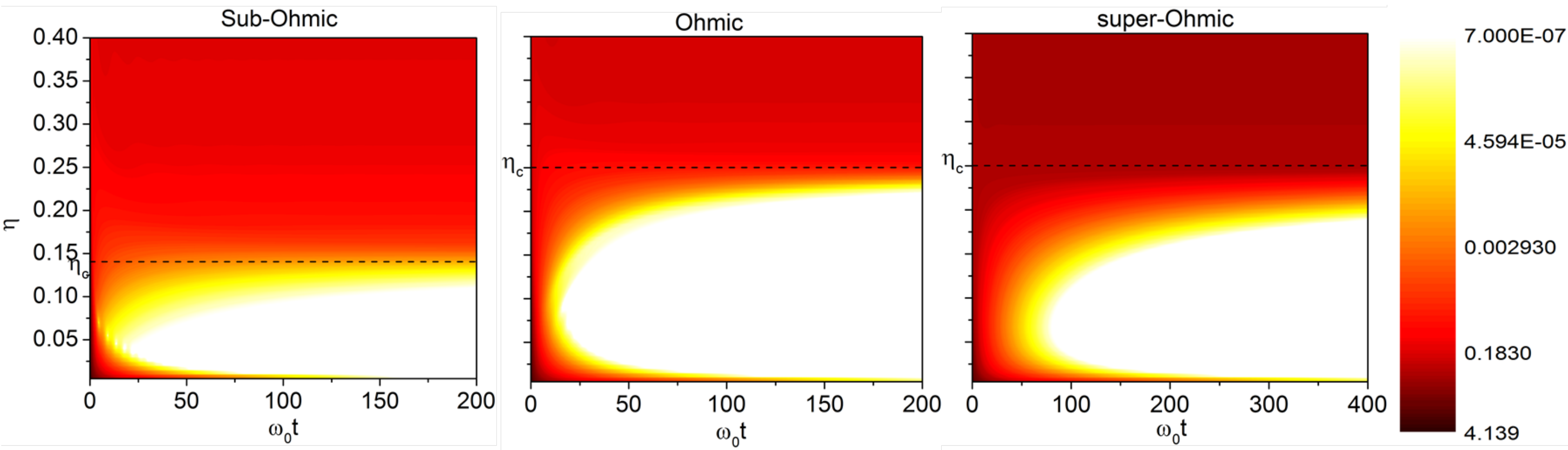
Dissipation-induced *fluctuation-induced*



$k_BT=0$ $\kappa=0.5\omega_0$, $r=3$, $s=1$, $\omega_c=3\omega_0$ $k_BT=0.1\omega_0$

Real-time entanglement decoherence dynamics for different spectral densities

T=0



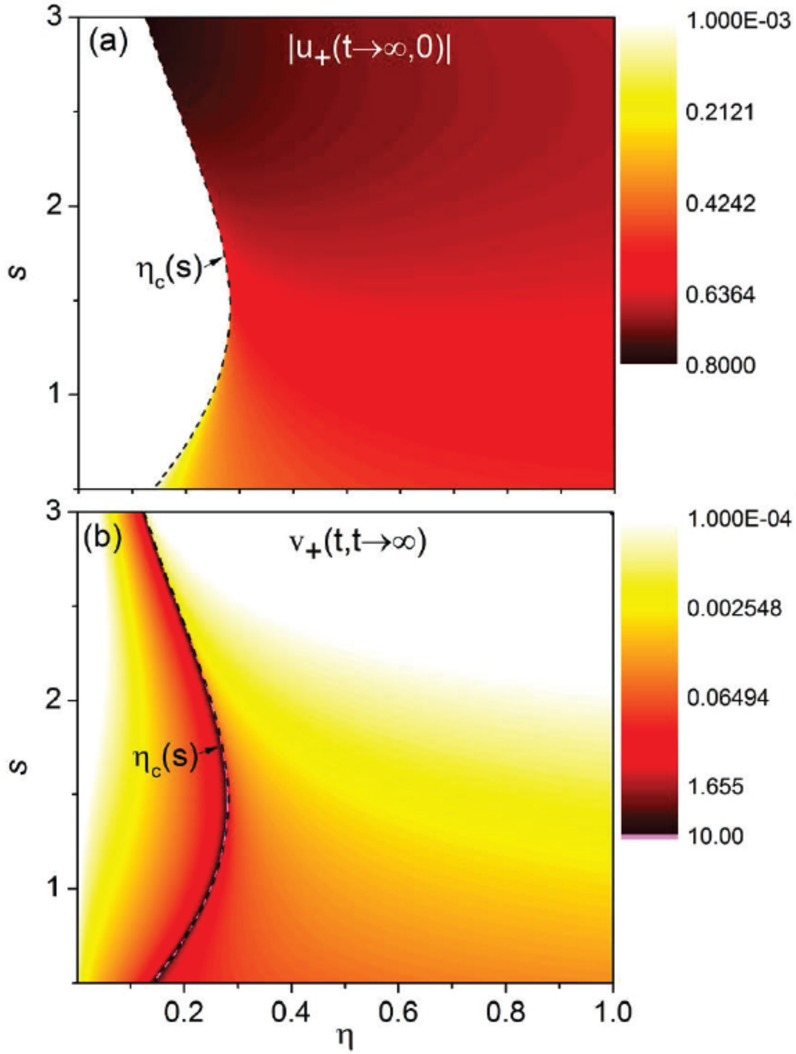
s=1/2 *s=1* *s=2*

$\kappa=0.5\omega_0$, $r=3$, $\omega_c=3\omega_0$

Steady-state non-equilibrium Green functions

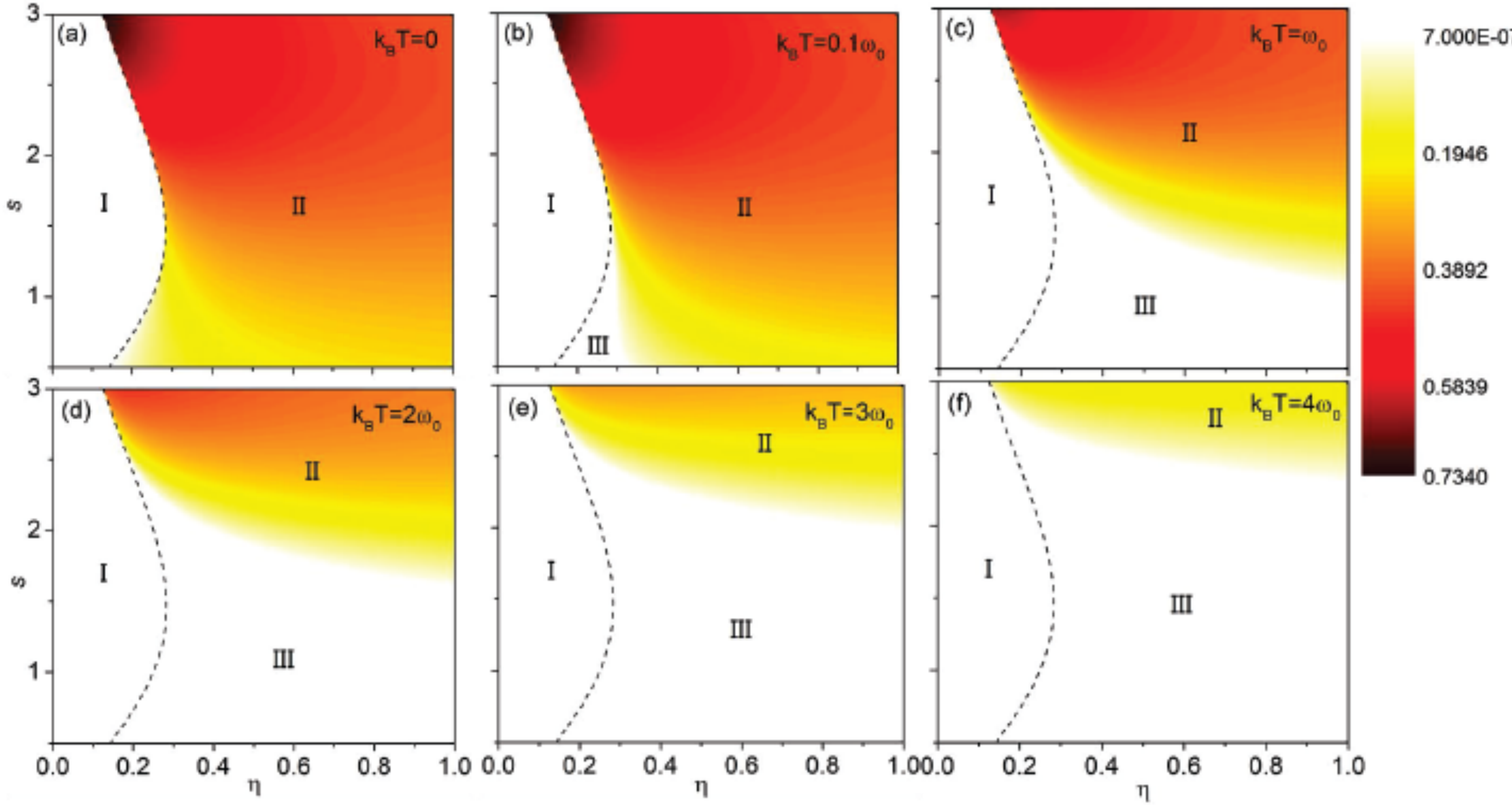
Quantum Phase Transition (T=0)

Quantum Critical Regime (T≠0)

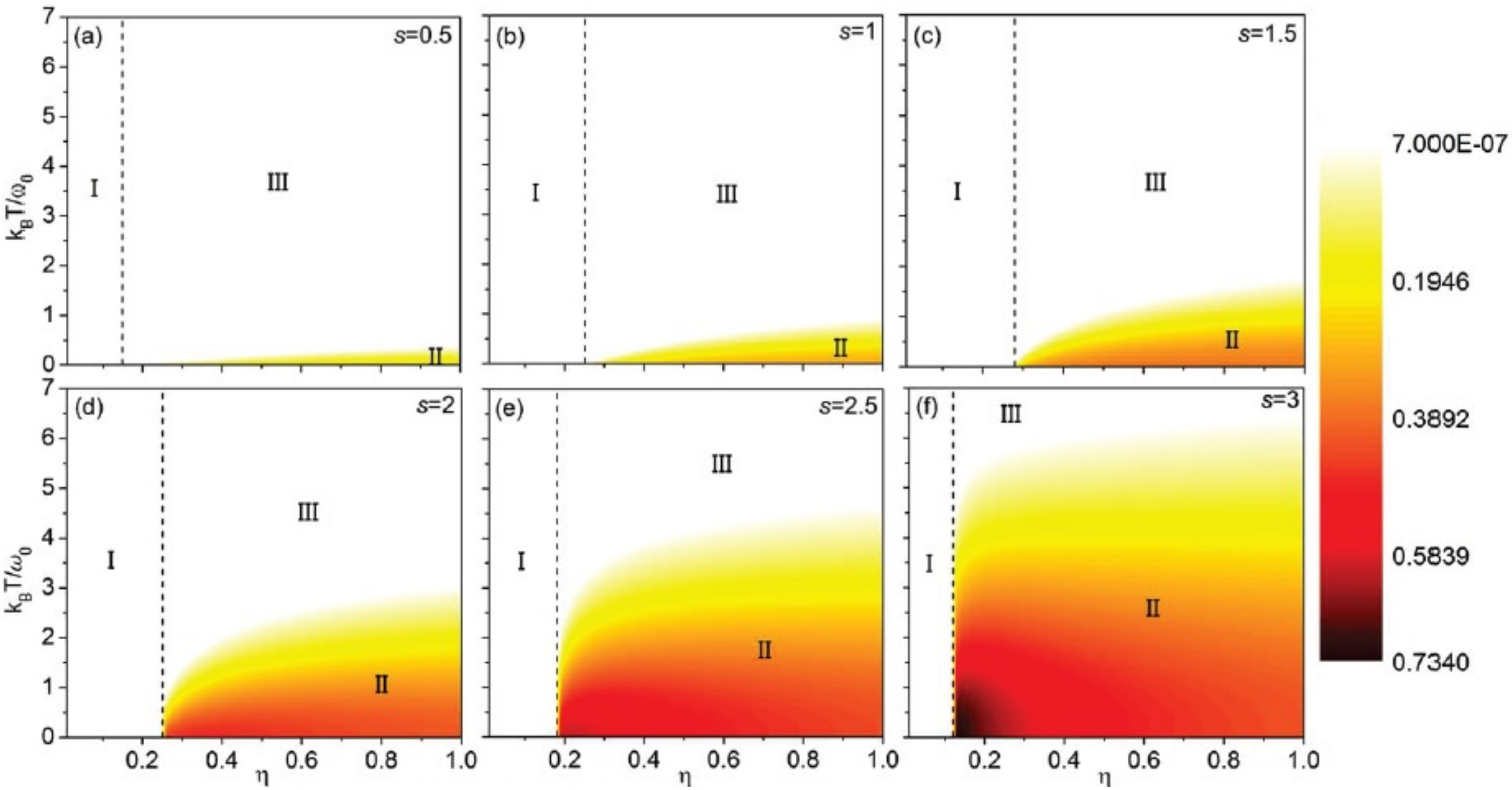


Y. C. Lin, P. Y. Yang, W. M. Zhang, Sci. Rep. **6**, 34804 (2016).

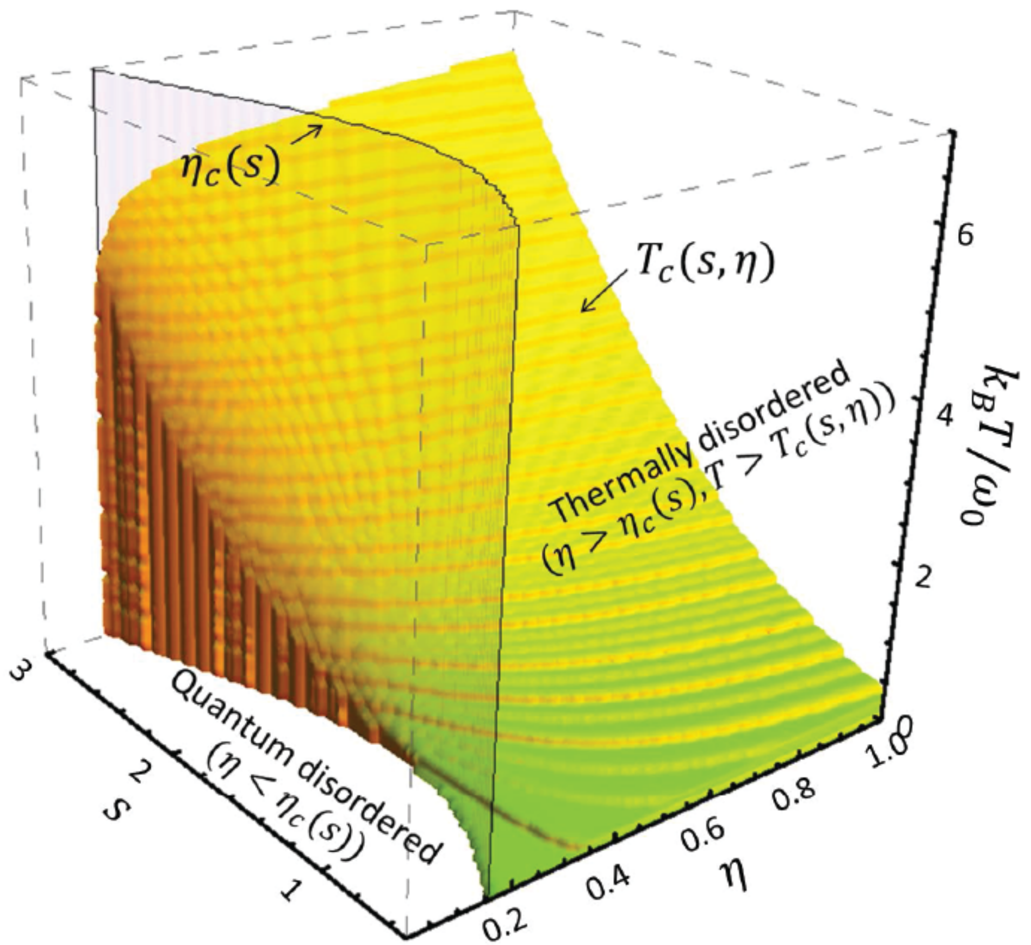
Steady-state phase diagram



Steady-state phase diagram



3-D nontrivial phase diagram



Y. C. Lin, P. Y. Yang, W. M. Zhang, Sci. Rep. **6**, 34804 (2016)

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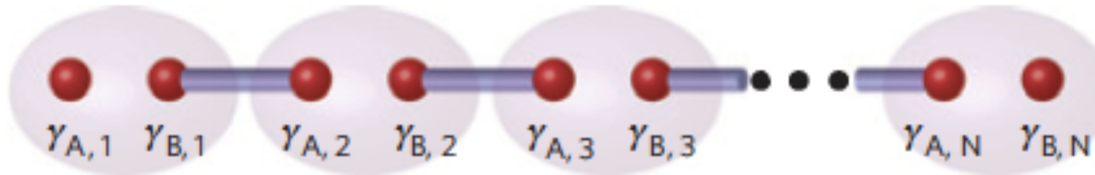
Majorana fermions in 1D spinless p-wave superconductors

Kitaev's model

Kitaev, Ann. Phys. **303**, 2 (2003)

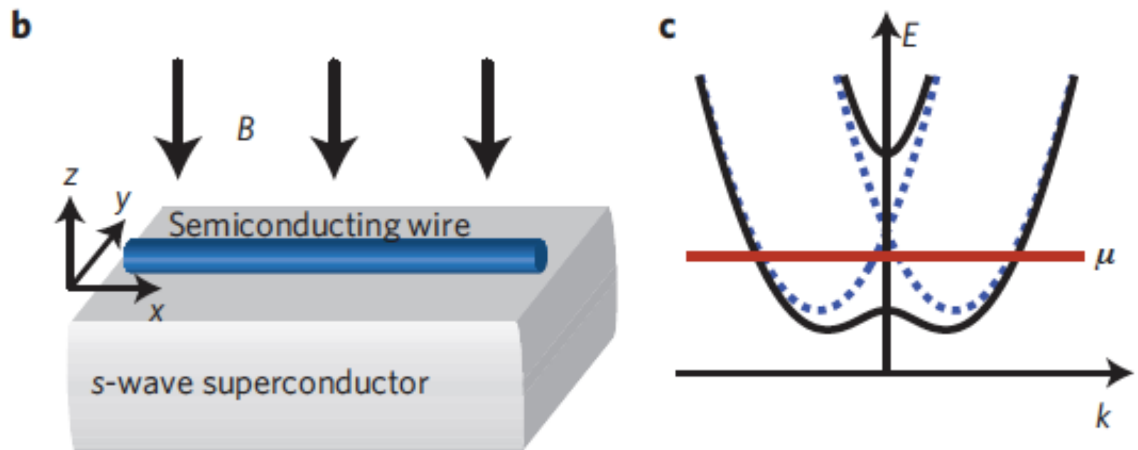


$$H = -\mu \sum_{x=1}^{N} c_x{}^{\dagger} c_x - \sum_{x=1}^{N-1} \left(t c_x{}^{\dagger} c_{x+1} + |\Delta| e^{i\phi} c_x c_{x+1} + h.c. \right)$$

$\mu = 0, t = |\Delta|$

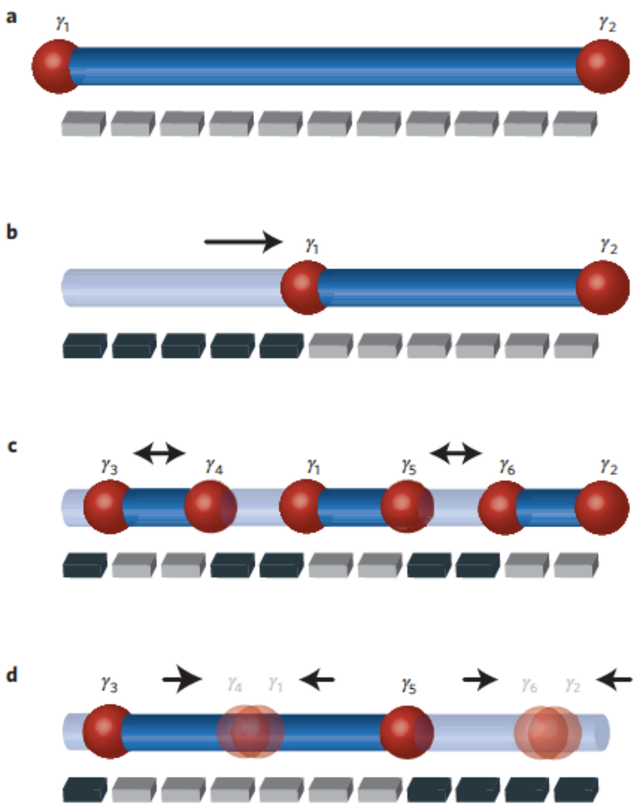
$$H = -it \sum_{x=1}^{N-1} \gamma_{B,x} \gamma_{A,x+1} \qquad c_x = \frac{1}{2} e^{-i(\phi/2)} (\gamma_{B,x} + i\gamma_{A,x})$$

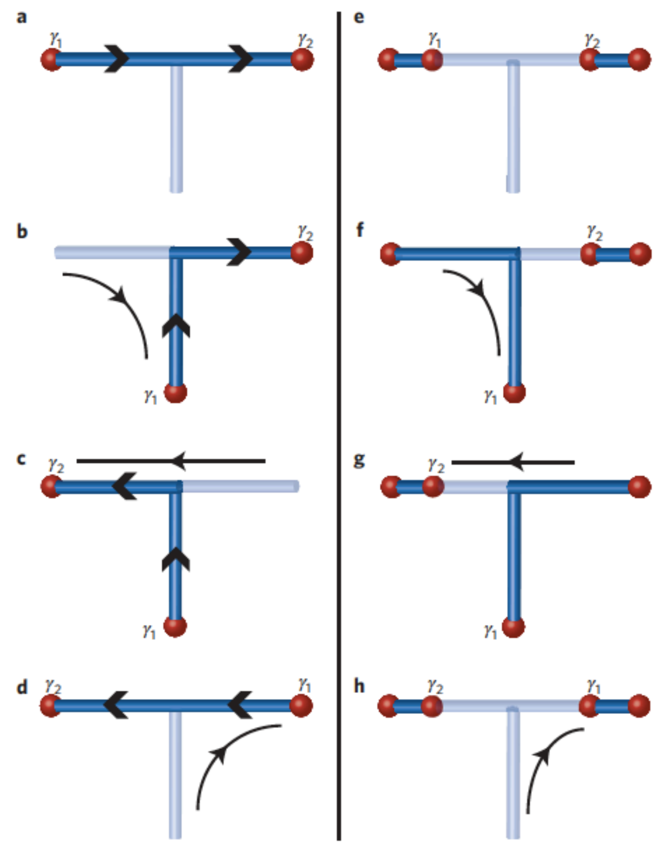
a realization with spin-orbital coupling semiconducting wire deposited in an s-wave superconductor



J. Alicea, et al. Nat. Phys. **7**, 412 (2011)

Quantum Information Processing with Majorana fermions





- use the individual tunnable gates to locally control MFs
- exchange of Majorana fermions

J. Alicea, et al. Nat. Phys. **7**, 412 (2011)

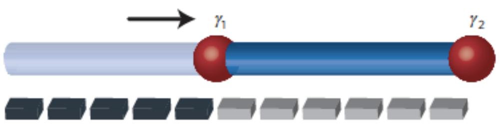
Charge fluctuation induced decoherence to Majorana fermions

- Hamiltonian of Topological Superconductor (TSC)

$$H_{TSC} = \sum_i [-\frac{\Delta}{2} c_{i+1} c_i - \frac{w}{2} c^\dagger_{i+1} c_i + H.c. - \mu_i c^\dagger_i c_i],$$

- Hamiltonian of gates as a fermion gas

$$H_G = \sum_p \epsilon_p c^\dagger_p c_p,$$



- Coupling between TSC and the gate

$$H_C = \lambda \delta Q \sum_i F_i c^\dagger_i c_i,$$

- δQ is the charge fluctuation

M. J. Schmidt, et al. Phys. Rev. **86**, 085414 (2012)

> ➢ Discussed the life-time (damping) of the Bogoliubov zero-mode (Majorana bound state) using the Fermi's golden rule (Markovian limit)

Charge fluctuation induced decoherence to Majorana fermions

- In the Bogoliubov Rep.

$$X_{0k} = X_{k0} = Y_{0k} = -Y_{k0},$$

M. J. Schmidt, et al.
Phys. Rev. **86**, 085414 (2012)

- Exact master equation

Sorry, this part is omitted because the result has not be published yet……..

H. L. Lei & WMZ (2017)

Allow us to study the non-Markovian dynamics of Majorana fermions

Outline

- ✧ General open quantum systems
- ✧ A general theory of open quantum systems
- ✧ General analytical solution of non-Markovianity
- ✧ Measurement of non-Markovianity
- ✧ Non-Markovian nonequilibrium physics
- ✧ Non-Markovian entanglement decoherence
- ✧ Non-Markovian decoherence of Majorana qubits
- ✧ **Transient quantum transport and quantum control**
- ✧ **Summary**

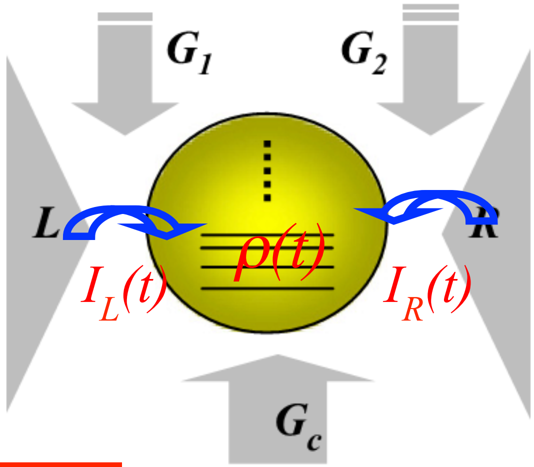
Nonequilibrium quantum transport theory based on ME:

Tu & WMZ, PRB **78**, 235311 (2008)
Jin, Tu, WMZ & Yan, NJP **12**, 183013 (2010)
Lei & WMZ, Ann. Phys. **327**, 1408 (2012)

- Quantum coherence in terms of reduced density matrix $\rho(t)$

$$\rho(t) = \mathrm{Tr}_E[\rho_{\rm tot}(t)]$$

$$\boxed{\frac{d\rho(t)}{dt} = -i[H_{\rm S}(t), \rho(t)] + \sum_\alpha [\mathcal{L}_\alpha^+(t) + \mathcal{L}_\alpha^-(t)]\rho(t),}$$



- Quantum transport in terms of transient current $I_\alpha(t)$

$$I_\alpha(t) = -e\langle \frac{d}{dt}\hat{N}_\alpha(t)\rangle$$

$$\boxed{I_\alpha(t) = \frac{e}{\hbar}\mathrm{tr_s}[\mathcal{L}_\alpha^+(t)\rho(t)] = -\frac{e}{\hbar}\mathrm{tr_s}[\mathcal{L}_\alpha^-(t)\rho(t)],}$$

Master equation and transport current:

Jin, Tu, WMZ, Yan, NJP **12**, 183013 (2010)
Lei & WMZ, Ann. Phys. **327**, 1408 (2012)

$$\boxed{\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\mathrm{i}[H_S(t), \rho(t)] + \sum_\alpha [\mathcal{L}_\alpha^+(t) + \mathcal{L}_\alpha^-(t)]\rho(t)}$$

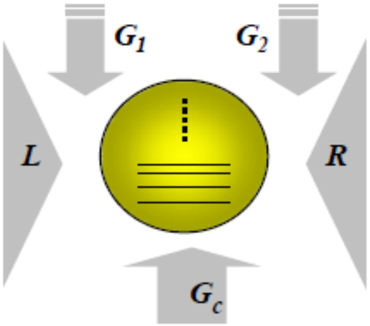
- The super-operators can be exactly expressed as:

$$\mathcal{L}_\alpha^+(t)\rho(t) = \sum_{ij}\{\lambda_{\alpha ij}(t)[a_i^\dagger\rho(t)a_j \mp a_i^\dagger a_j\rho(t)] - \kappa_{\alpha ij}(t)a_i^\dagger a_j\rho(t) + \mathrm{H.c.}\} \ ,$$

$$\mathcal{L}_\alpha^-(t)\rho(t) = \sum_{ij}\{\lambda_{\alpha ij}(t)[\pm a_j\rho(t)a_i^\dagger - \rho(t)a_j a_i^\dagger] + \kappa_{\alpha ij}(t)a_j\rho(t)a_i^\dagger + \mathrm{H.c.}\}$$

- The exact transient transport current:

$$\boxed{I_\alpha(t) = \frac{e}{\hbar}\mathrm{tr}_S[\mathcal{L}_\alpha^+(t)\rho(t)] = -\frac{e}{\hbar}\mathrm{tr}_S[\mathcal{L}_\alpha^-(t)\rho(t)]}$$



where

$$\boldsymbol{\kappa}_\alpha(t) = \int_{t_0}^{t} d\tau \mathbf{g}_\alpha(t,\tau)\boldsymbol{u}(\tau,t_0)\boldsymbol{u}^{-1}(t,t_0) \ ,$$

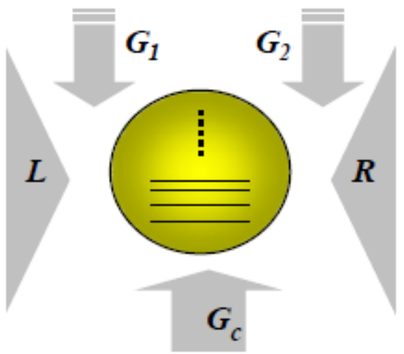
$$\boldsymbol{\lambda}_\alpha(t) = \int_{t_0}^{t} d\tau[\mathbf{g}_\alpha(t,\tau)\boldsymbol{v}(\tau,t) - \widetilde{\mathbf{g}}_\alpha(t,\tau)\bar{\boldsymbol{u}}(\tau,t)] - \boldsymbol{\kappa}_\alpha(t)\boldsymbol{v}(t,t) \ ,$$

Reproduce quantum transport of NEGF:

Jin, Tu, WMZ, Yan, NJP **12**, 183013 (2010)
Lei & WMZ, Ann. Phys. **327**, 1408 (2012)

- We reproduce and further generalize the transient current:

$$
\begin{aligned}
I_\alpha(t) &= \frac{e}{\hbar}\mathrm{tr_s}[\mathcal{L}^+_\alpha(t)\rho(t)] = -\frac{e}{\hbar}\mathrm{tr_s}[\mathcal{L}^-_\alpha(t)\rho(t)] \\
&= -\frac{2e}{\hbar}\,\mathrm{Re}\int_{t_0}^{t} d\tau\,\mathrm{Tr}\Big\{\boldsymbol{g}_\alpha(t,\tau)\boldsymbol{v}(\tau,t)-\widetilde{\boldsymbol{g}}_\alpha(t,\tau)\bar{\boldsymbol{u}}(\tau,t_0) \\
&\qquad +\underline{\boldsymbol{g}_\alpha(t,\tau)\boldsymbol{u}(\tau,t_0)\boldsymbol{\rho}^{(1)}(t_0)\boldsymbol{u}^\dagger(t,t_0)}\Big\} \\
&= -\frac{2e}{\hbar}\,\mathrm{Re}\int_{t_0}^{t} d\tau\,\mathrm{Tr}\Big\{\boldsymbol{\Sigma}^r_\alpha(t,\tau)\boldsymbol{G}^<(\tau,t) \\
&\qquad +\boldsymbol{\Sigma}^<_\alpha(t,\tau)\boldsymbol{G}^a(\tau,t)\Big\}.
\end{aligned}
$$



Wingreen, Jauho & Meir, PRB **48**, 8487 (1993)

where

$$
\boldsymbol{G}^<(\tau,t) = \underline{\boldsymbol{G}^r(\tau,t_0)\boldsymbol{G}^<(t_0,t_0)\boldsymbol{G}^a(t_0,t)} + \boxed{\int_{t_0}^{\tau} d\tau_1\int_{t_0}^{t} d\tau_2\;\boldsymbol{G}^r(\tau,\tau_1)\boldsymbol{\Sigma}^<(\tau_1,\tau_2)\boldsymbol{G}^a(\tau_2,t)}
$$

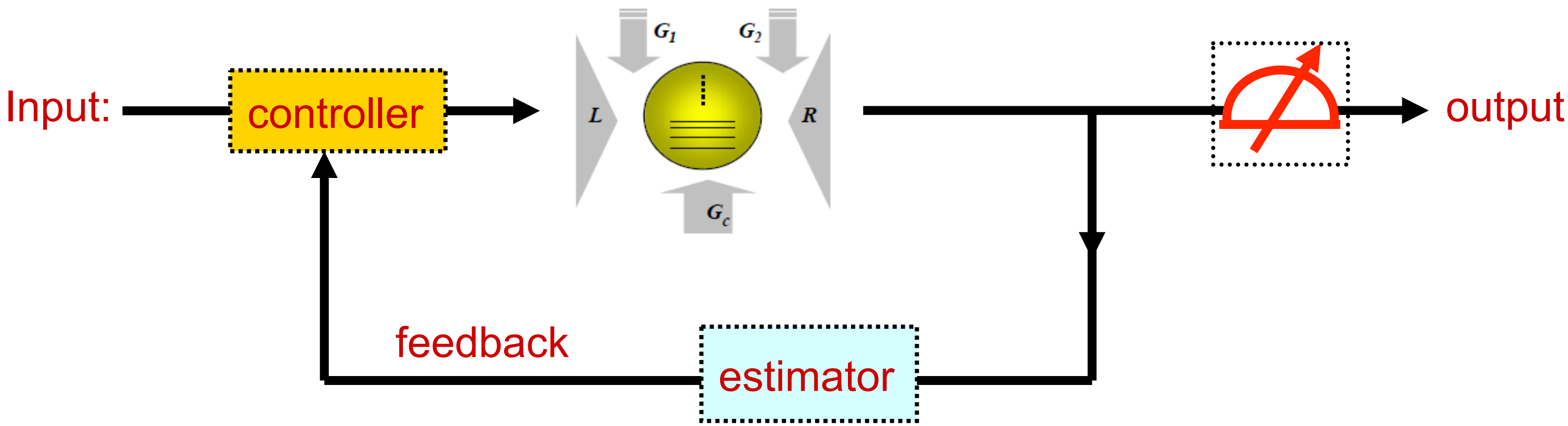
Initial-state dependent term, non-Markovian memory effect

Applications to quantum transport:

- Phase localization and coherence controls in double-dot AB interferometer.
  Tu, WMZ & Jin, *Phys. Rev*. B **83**, 115318 (2011)
  Tu, WMZ, & Nori, *Phys. Rev*. B **86**, 195403 (2012)
- Precision control of charge qubit coherence through cross-correlations. Jin, WMZ, Tu & Wang, *J. Chem. Phys*. **139**, 064706 (2013)
- Single-electron turnstile pumping mechanism
  Lin & WMZ, *Appl. Phys. Lett*. **99**, 072105 (2011)
  Lin & WMZ, arXiv:1207.0284 (2012)
- Transient quantum transport in double-dot AB interferometers
  Tu, WMZ, Jin, Entin & Aharony, *Phys. Rev*. B **86**, 115453 (2012)
  Tu, Aharony, WMZ & Entin, *Phys. Rev*. B **90**, 165422 (2014)
  Tu, Aharony, Entin, Schiller & WMZ, *Phys. Rev*. B **93**, 115437 (2016)
- Transient current-current correlation and noise spectra
  Yang, Lin & WMZ, *Phys*. Rev. B **89**, 115411 (2014)
- Initial correlation effect in transient quantum transport in nanostructures
  Yang, Lin & WMZ, *Phys. Rev*. B **92**, 165403 (2015)
- Quantum coherence of molecular states in AB interferometers
  Liu, Tu & WMZ, *Phys. Rev*. B **94**, 045403 (2016)

Theory for quantum feedback controls (to be developed......)

$$
\begin{cases}
\dfrac{d\rho(t)}{dt} = -i[H_{\mathrm{S}}(t), \rho(t)] + \sum_{\alpha}[\mathcal{L}_{\alpha}^{+}(t) + \mathcal{L}_{\alpha}^{-}(t)]\rho(t), & \text{State evolution} \\
I_{\alpha}(t) = \mathrm{tr}_{\mathrm{s}}[\mathcal{L}_{\alpha}^{+}(t)\rho(t)] = -\mathrm{tr}_{\mathrm{s}}[\mathcal{L}_{\alpha}^{-}(t)\rho(t)] & \text{Measurement}
\end{cases}
$$



- How to make Feedback control ?

Summary

- nonequilibrium physics and quantum transport can be described within the framework of open quantum systems.
- quantum dissipation and fluctuation and transport phenomena can be explicitly addressed in terms of nonequilibrium Green functions.
- non-Markovianity is physically investigated in terms of experimentally direct measurable two-time correlation functions.
- nonequilibrium transient dynamics of photon and electron statistics associated with non-Markovianity are discovered.
- nonequilibrium quantum phase transition via entanglement decoherence dynamics is explored.
- temperature and initial state dependences of non-Markovianity is also analyzed for different nonequilibrium dynamics.
- A priliminary result on topological decoherence dynamics is demonstrated.

Summary of theoretical development:

- The exact master equation was obtained for studying the non-Markovian decoherence dynamics of various nanoelectronic devices at an arbitrary temperature.
  Tu & WMZ, PRB **78**, 235311 (2008)

- The exact quantum transport theory was developed from the exact master equation, which generalizes the transport theory of Keldysh's non-equilibrium Green function technique.
  Jin, Tu, WMZ & Yan, NJP **12**, 183013 (2010)

- The exact master equation including explicitly the initial system-reservoir correlations was also obtained.
  Tan & WMZ, PRA **83**, 032102 (2011)
  Yang, Lin & WMZ, PRB **92**, 165403 (2015)

- We extended the exact master equation and the transport theory of nanophotonics with explicit external fields acting on both the system and the reservoirs
  Lei & WMZ, Ann. Phys. **327**, 1408 (2012)

- General non-Markovian dynamics of open quantum systems are developed.
  WMZ, Lo, Xiong, Tu & Nori, PRL **109**, 170402 (2012)

- Extend to topological systems for topological quantum computers (is developing) ….

THANK

YOU !