Compressive phase retrieval

George Barbastathis

University of Michigan - Shanghai Jiao Tong University Joint Institute
密西根交大学院 (academic year 2013-14)

Massachusetts Institute of Technology
Department of Mechanical Engineering

Singapore-MIT Alliance for Research and Technology (SMART) Centre

Acknowledgments

- Zhengyun Zhang, Adam Pan, Kelli Xu, Yunhui Zhou, Yi Liu, Justin Lee, Shakil Rehman
- Lei Tian, Laura Waller *UC Berkeley*
- Jon Petruccelli *SUNY Albany*
- David Brady *Duke University*
- Colin J. R. Sheppard *Italian Institute of Technology*
- Rajiv Gupta *Massachusetts General Hospital*
- Haris Kudrolli, Vivek Nagarkar *RMD Inc*
- Singapore's National Research Foundation
- US Department of Homeland Security
- Chevron Technology Company

This talk is about

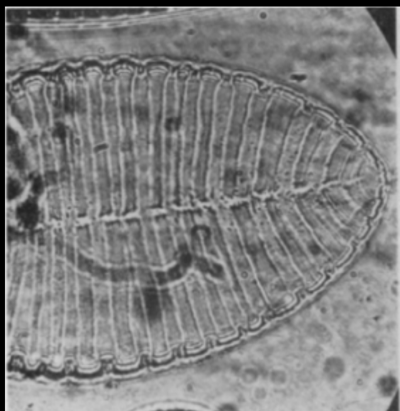
- Phase
- Phase space
- Phase space tomography
- Compressive imaging
- Phase tomography

This talk is about

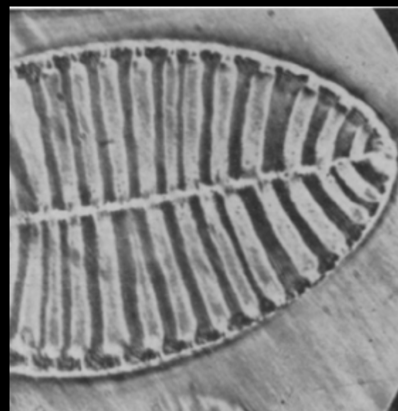
- Phase
- Phase space
- Phase space tomography
- Compressive imaging
- Phase tomography

The significance of phase

Visible



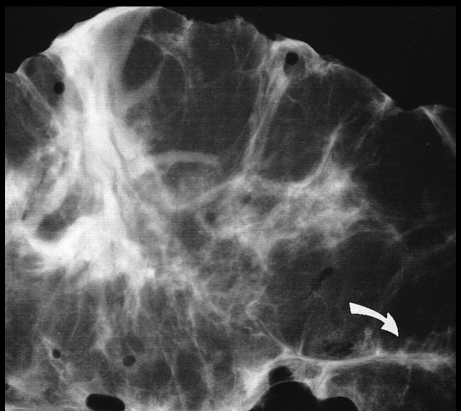
intensity image



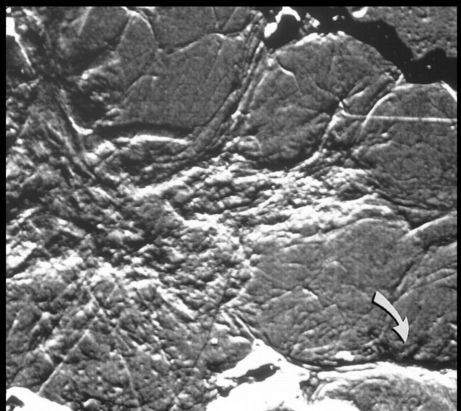
phase-contrast image

(F. Zernike, Science 121, 1955)

X-ray



attenuation image



phase-contrast image

(human breast cancer specimen)

(E. D. Pisano et al., Radiology 214, 2000)

$$\phi(x_{\mathrm{o}}) = k \int_{\Gamma} n(\mathbf{r}) \mathrm{d}l \quad \Rightarrow \quad \rho \propto \frac{n^2 - 1}{n^2 + 2}$$

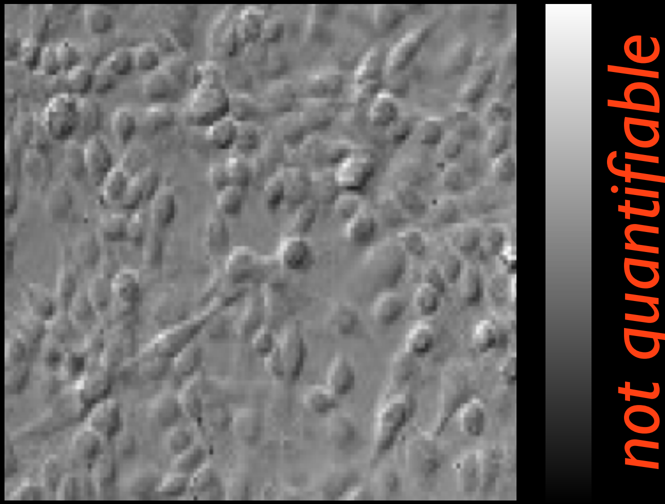
Refractive index

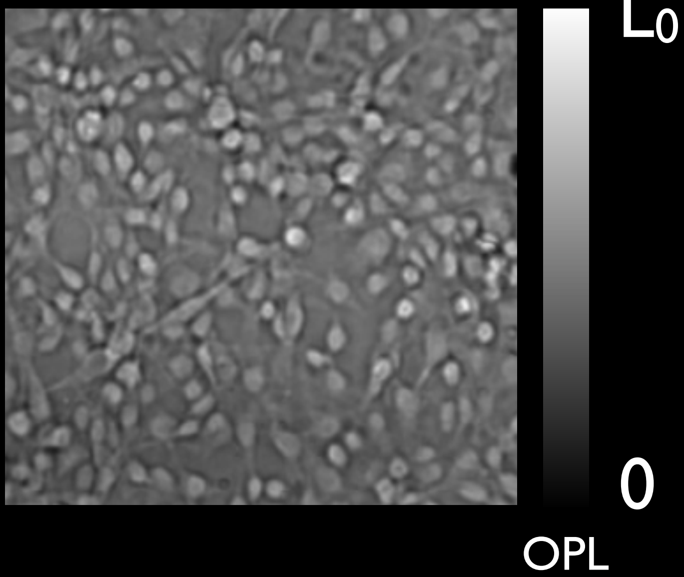
Density

$\Rightarrow \quad \rho \propto$ temperature, pressure, humidity

Phase Imaging

- Non-quantitative (phase contrast)
- Quantitative (brightness ∝ OPL)

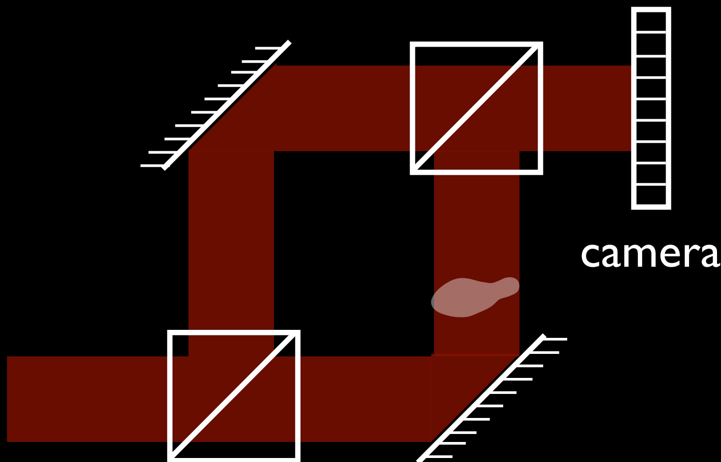




OPL = optical path length

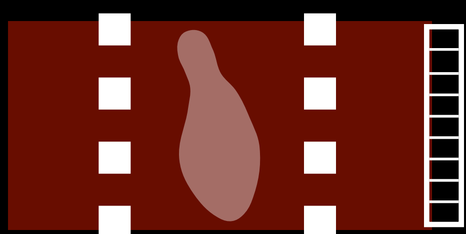
Phase Retrieval

- Interferometric

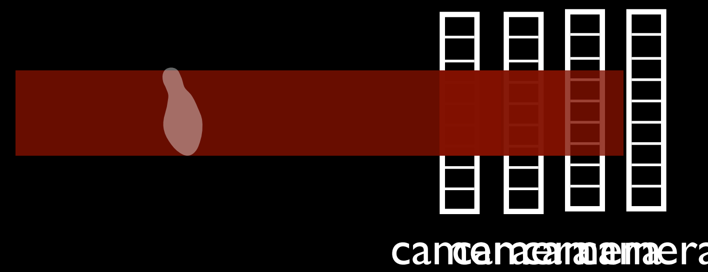


External reference

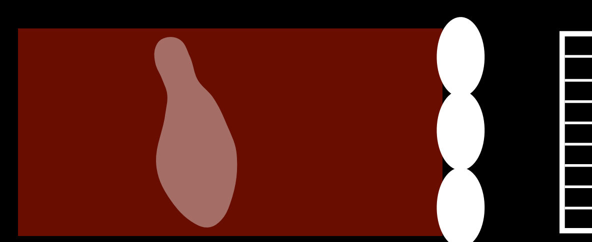
- Refractive



- Axial stack

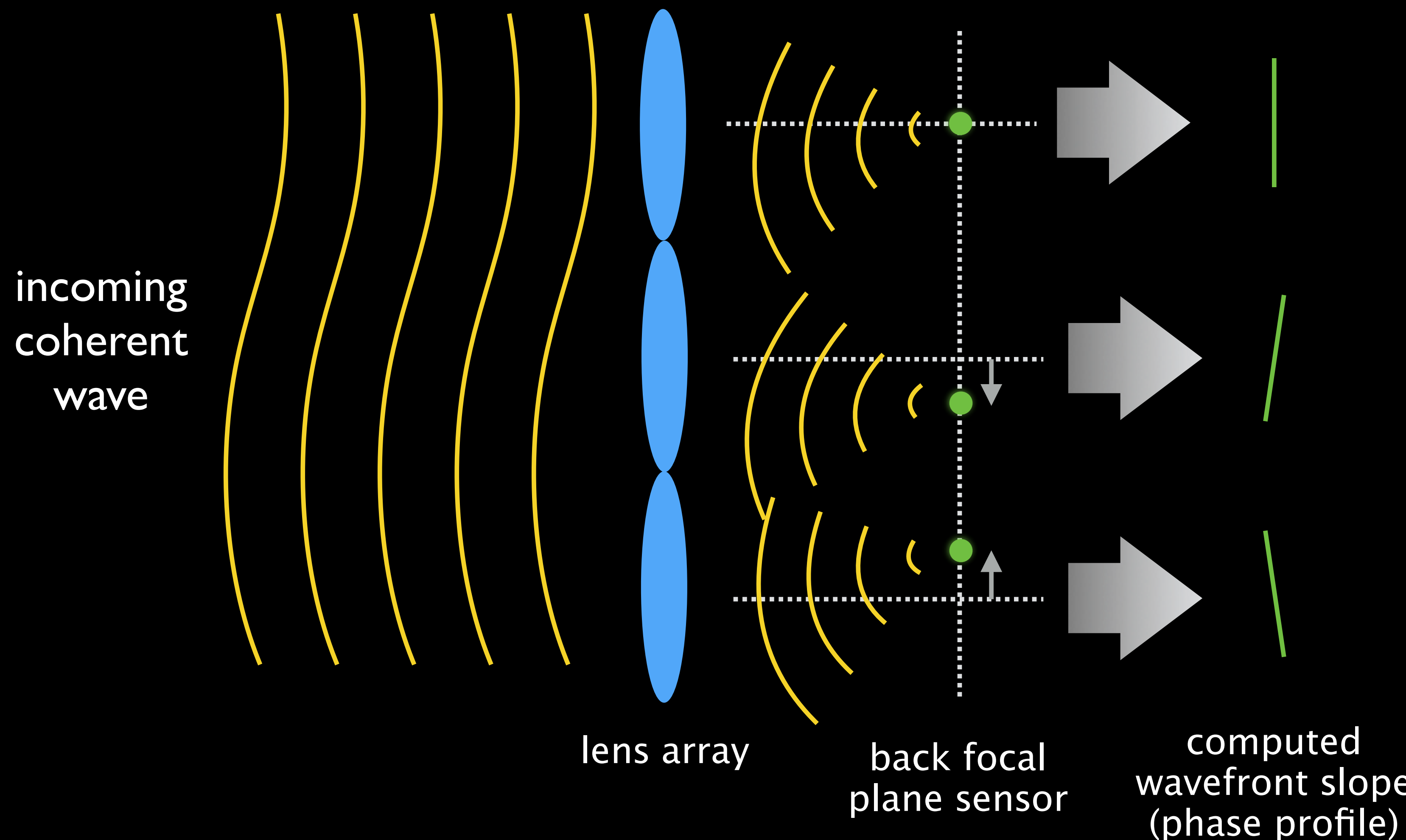


- Lippmann / Lateral stack

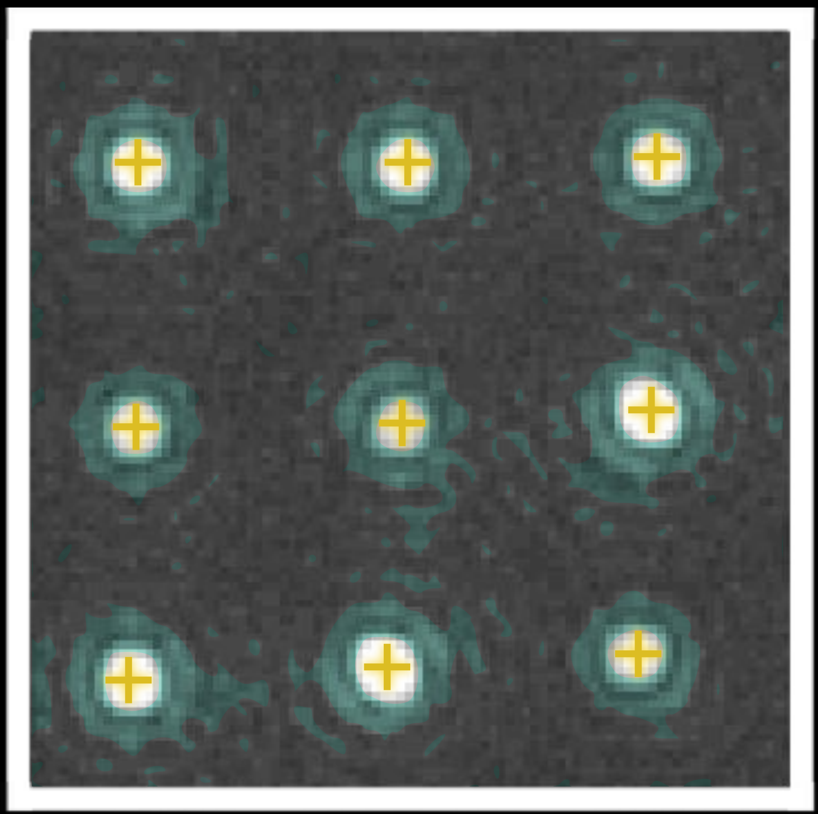


Wavefront Sensing

(Shack-Hartman)



Captured Image

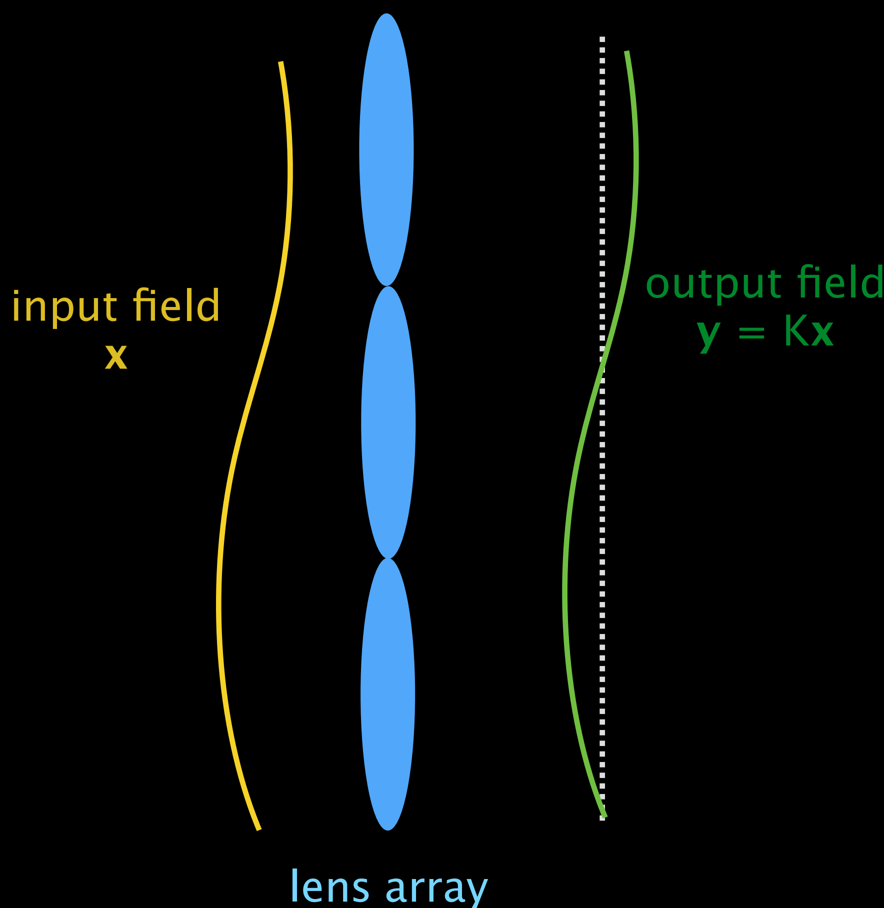


- spot location → slope estimate
- spot shape → fine details
- wavefront sensing approach
  throws away fringe info
- light field imaging approach
  assumes fringes are extra rays
- can we do better?

Z. Zhang and G. Barbastathis, *Focus on Microscopy 2014*, paper MO-AF1-PAR-D
Sydney, Australia

Factored Form Descent (FFD)

Zhang et al *Opt. Express* 21:5759 (2013)



output intensity

➔ input field

coherence retrieval

measured intensity **predicted intensity**

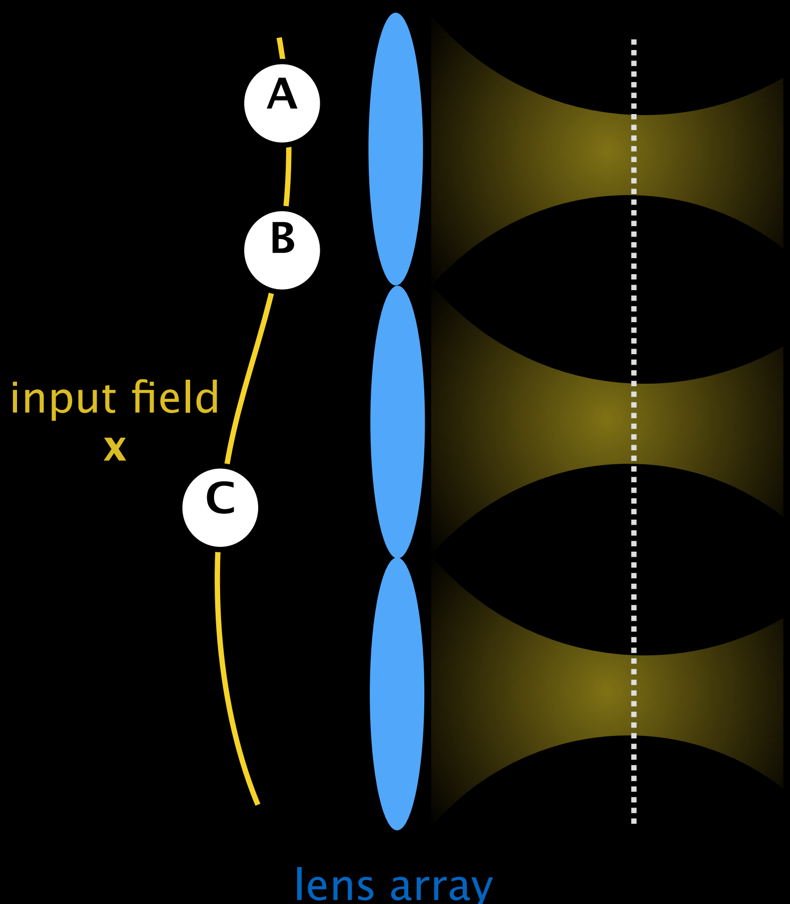
$$\operatorname{argmin}_{\mathbf{X}} \sum_{m=1}^{M} w_m^2 \left(y_m - \mathbf{k}_m \mathbf{X} \mathbf{X}^{\mathrm{T}} \mathbf{k}_m^{\mathrm{T}} \right)^2$$

coherence modes

$$\mathbf{X} \equiv \begin{pmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \ldots & \mathbf{x}_M \end{pmatrix}$$

(purely coherent) $M = 1 \Longrightarrow$ phase retrieval

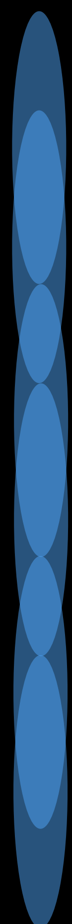
Single image not sufficient...



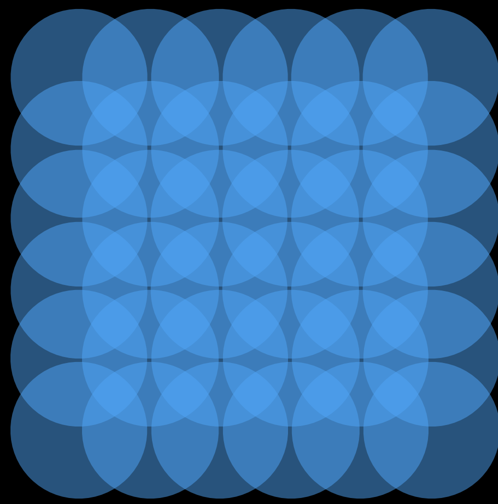
- each lens separately focuses
  - *low spatial frequency → no overlap (crosstalk)*
- phase relationship between **A** and **B** can be obtained
- phase relationship between **A** and **C** cannot be obtained

Fix with multiple images

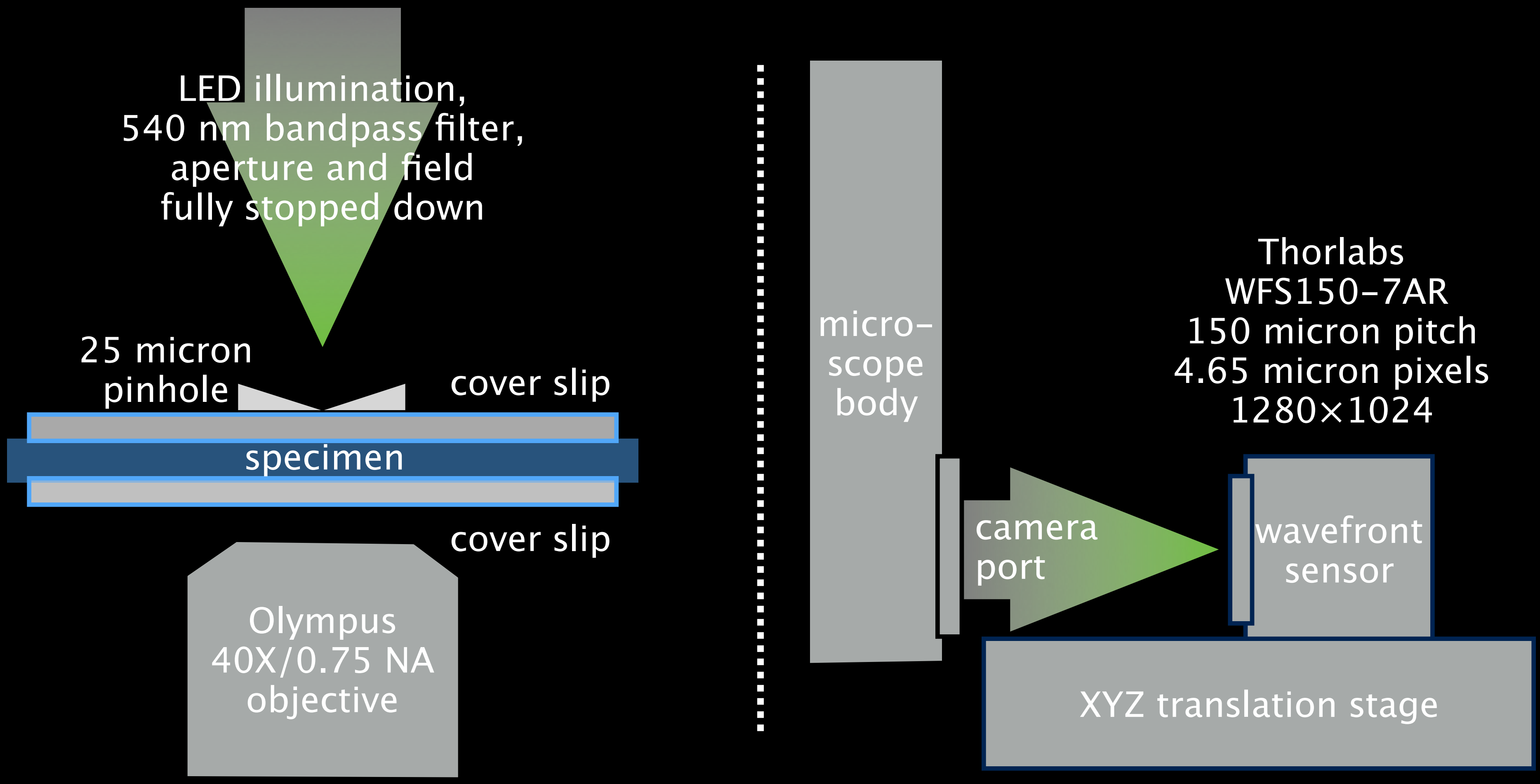
lens array
positions
(1D)



lens array
positions
(2D)

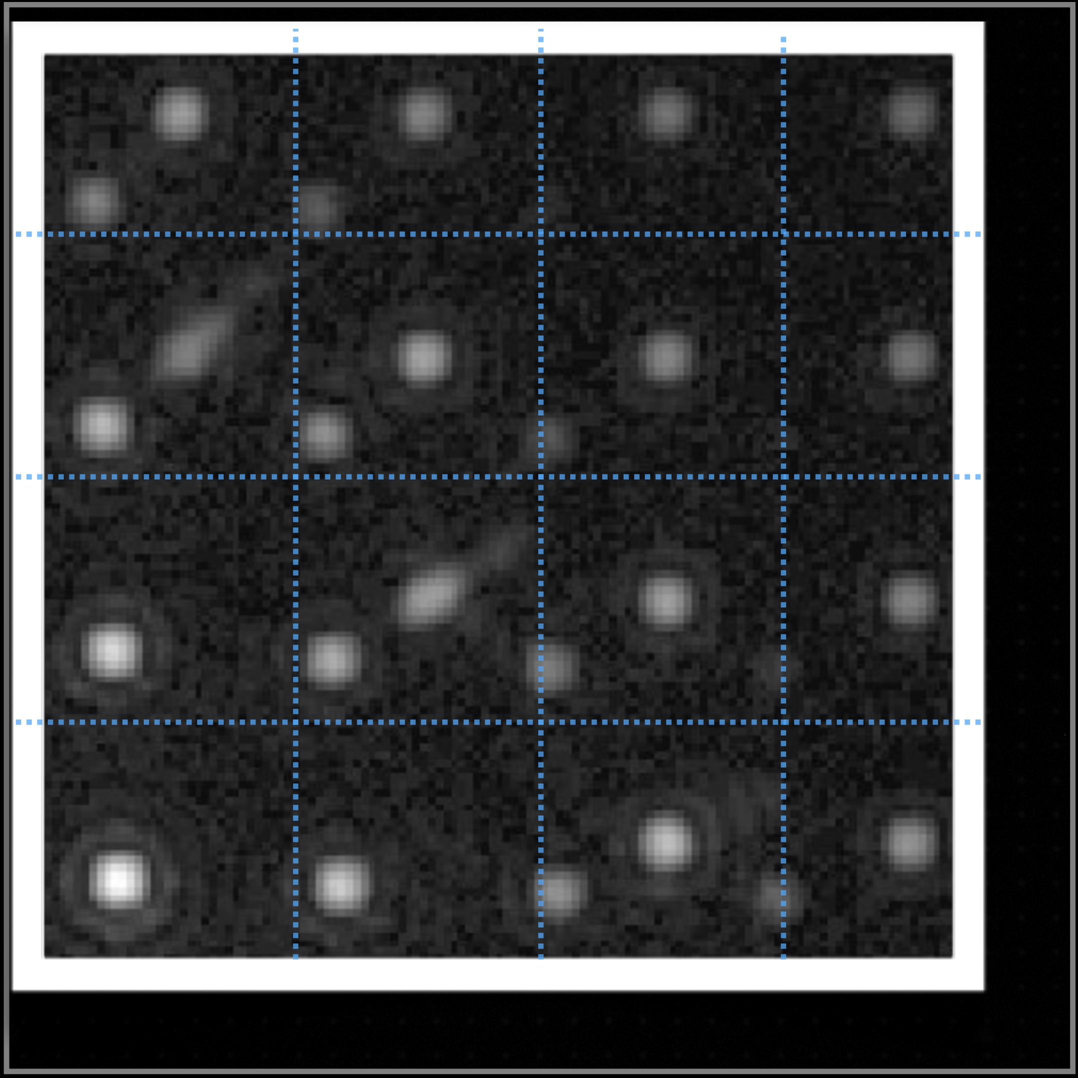


Experimental Geometry
(not to scale)



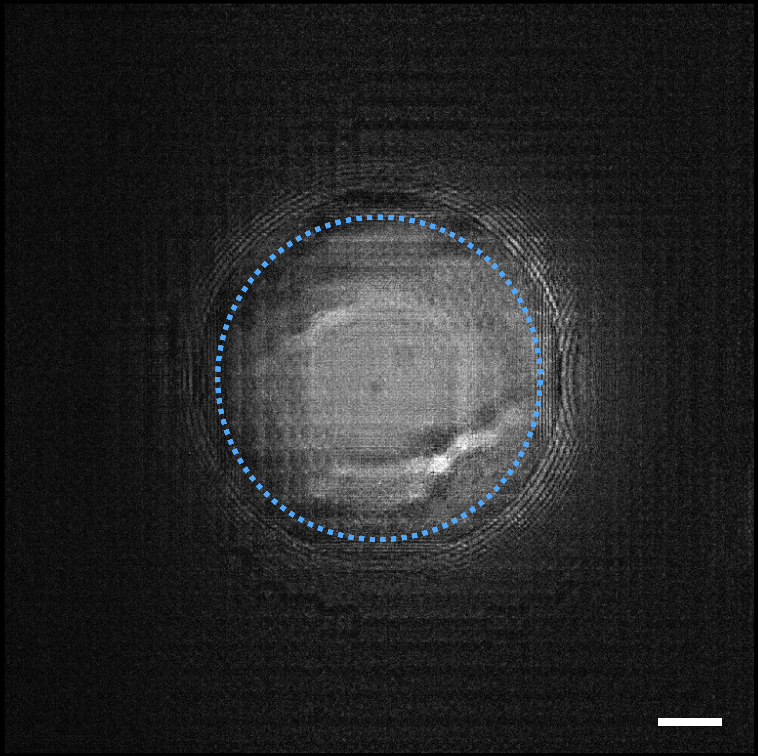
Experiment: 50µm bead

- Polysterene in ethylene glycol
- Five images:
  - four shifts
  - one background for subtraction
- repeated 16 times for noise statistics
- Reconstruction using rank-constrained FFD

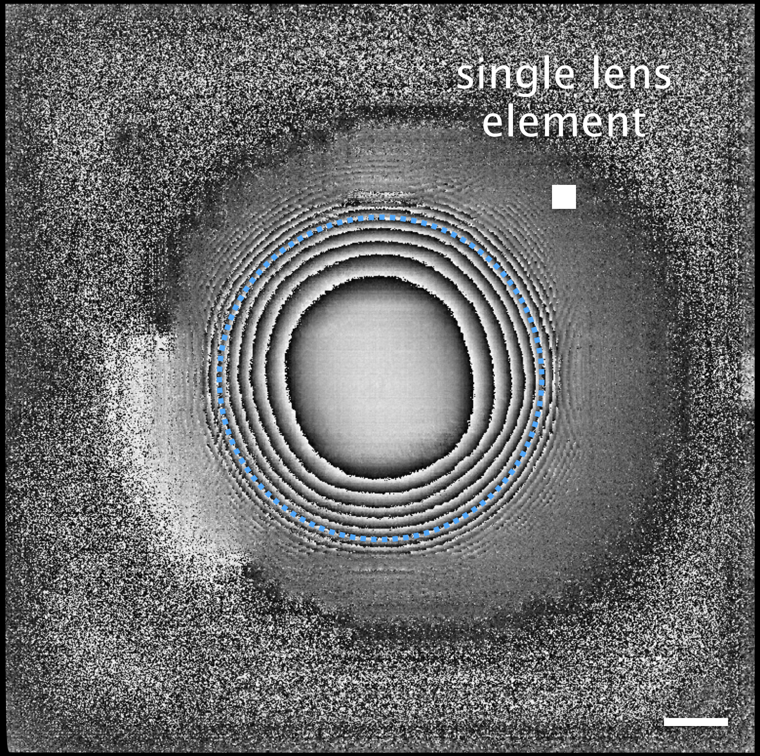


Experiment: 50µm bead

reconstruction (scalebar = 10 microns in specimen)



amplitude

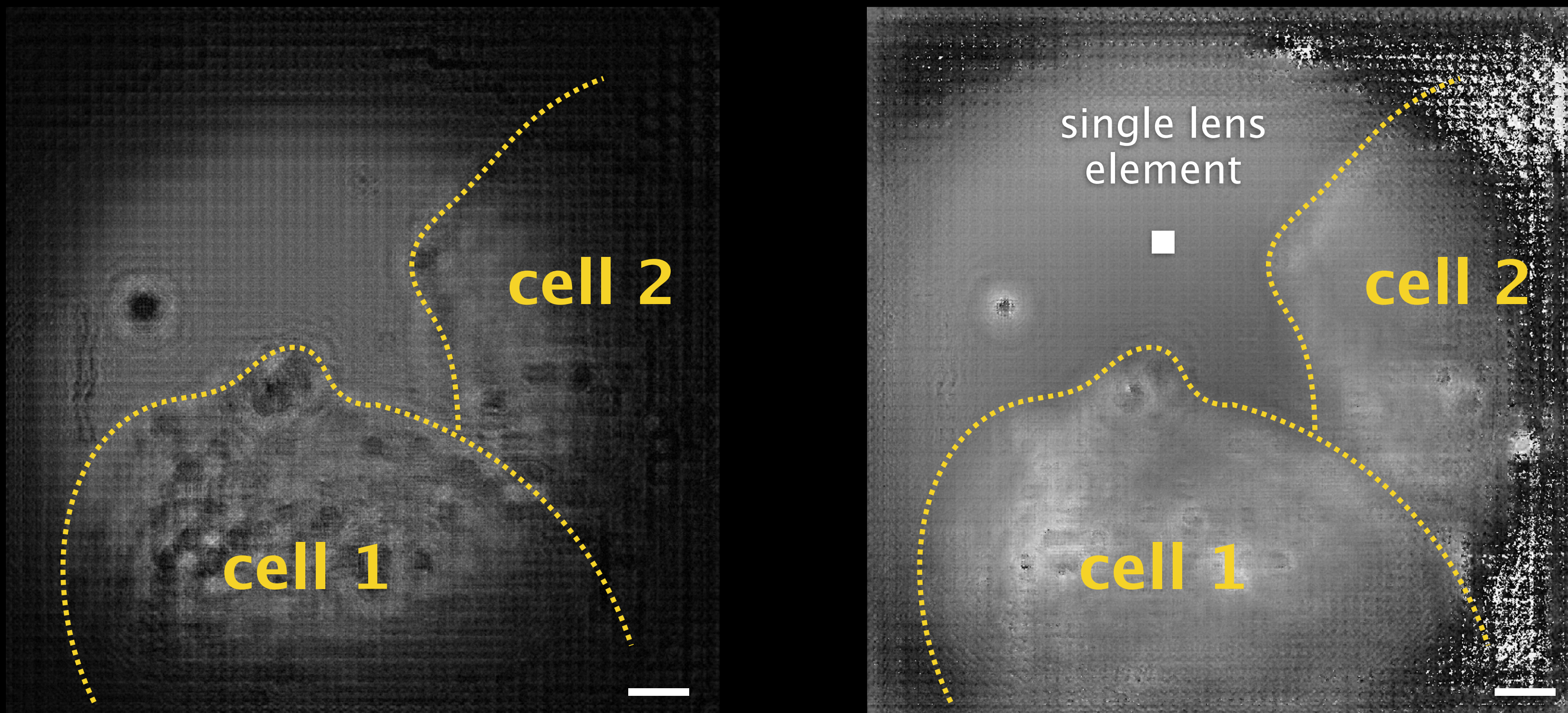


phase

Z. Zhang and G. Barbastathis, *Focus on Microscopy 2014*, paper MO-AF1-PAR-D
Sydney, Australia

Experiment: cheek cells

reconstruction (scalebar = 10 microns in specimen)



amplitude phase

Z. Zhang and G. Barbastathis, *Focus on Microscopy 2014*, paper MO-AF1-PAR-D
Sydney, Australia

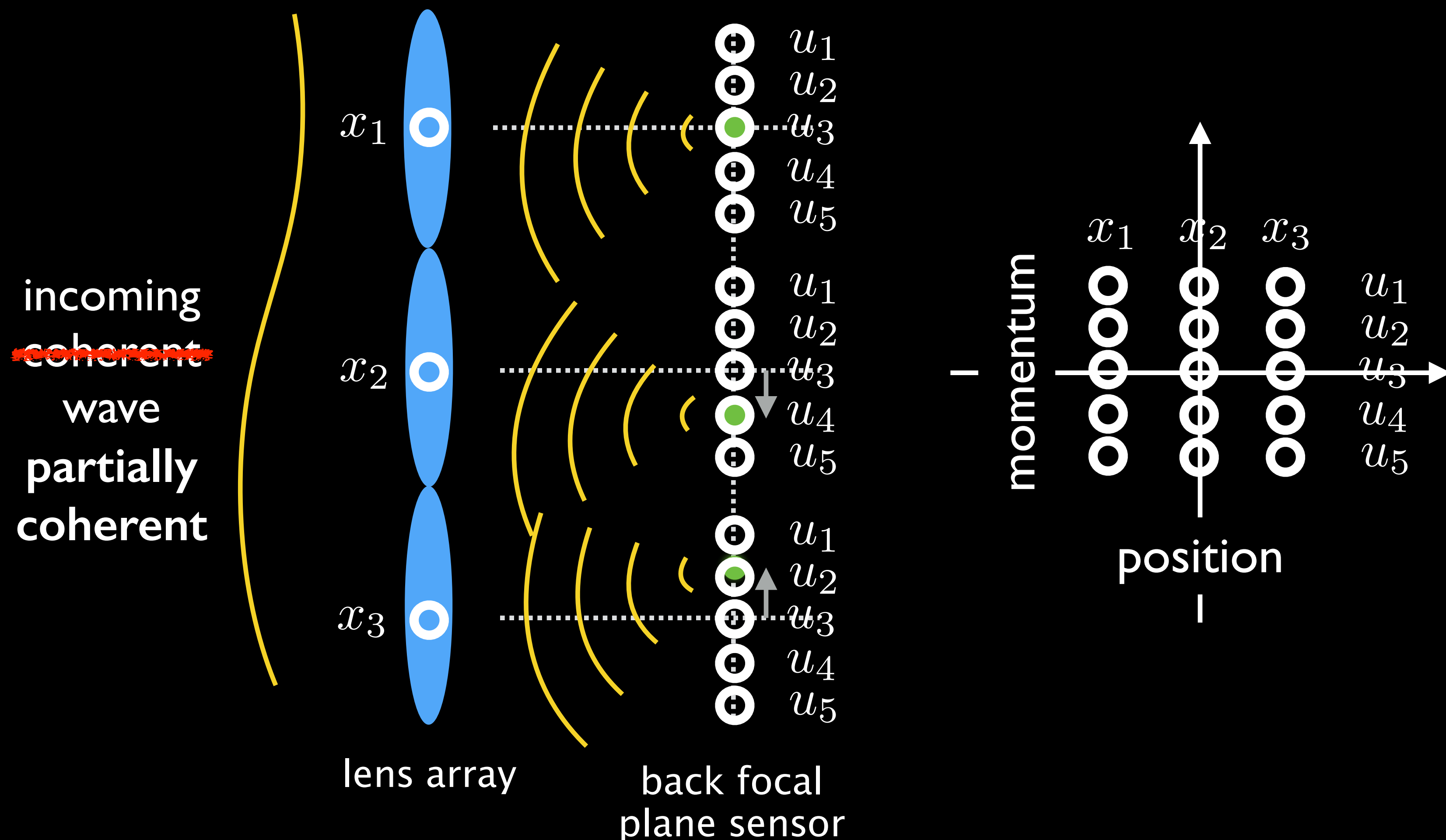
This talk is about

- Phase
- Phase space
- Phase space tomography
- Compressive imaging
- Phase tomography

Phase-space ~~Wavefront~~ Sensing

sampling position (Shack-Hartmar momentum

L. Tian et al, *Opt. Express* 21:10511, 2013

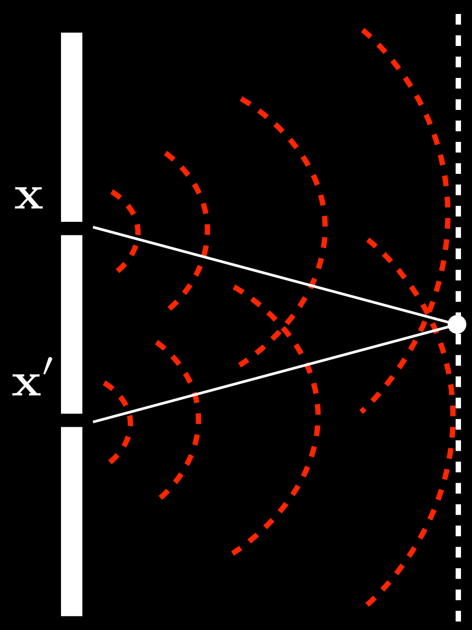


Partially coherent light

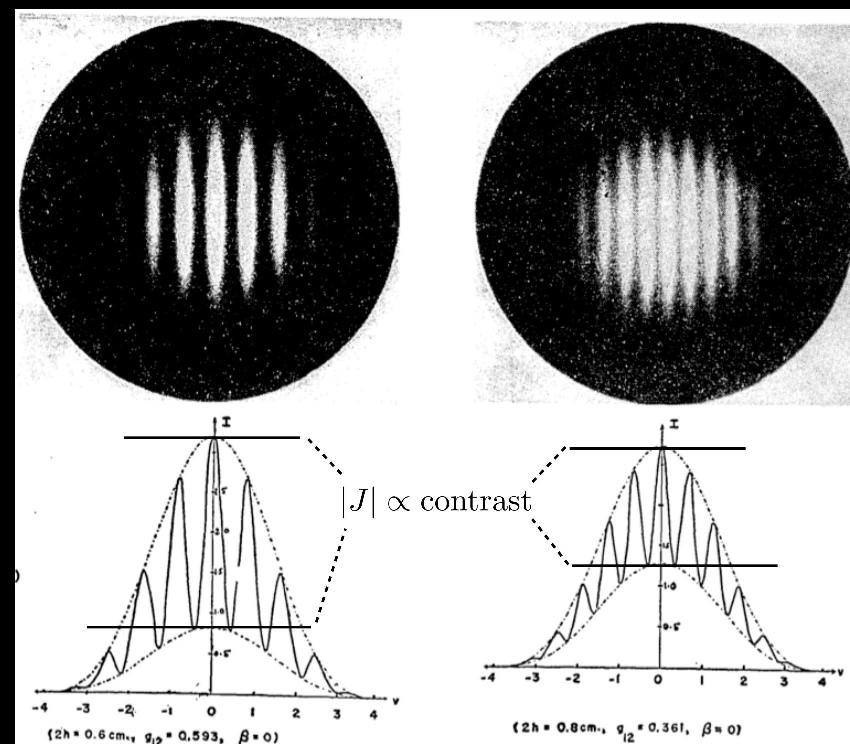
Random field $U(\mathbf{x})$

Correlation function (mutual intensity)

$$J(\mathbf{x}, \mathbf{x}') \equiv \langle U(\mathbf{x}) U^*(\mathbf{x}') \rangle$$



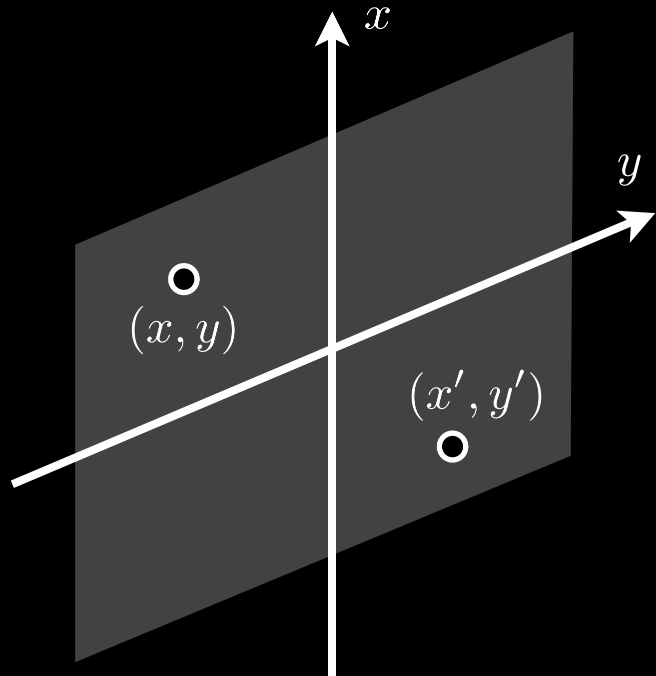
Young's two-slit experiment

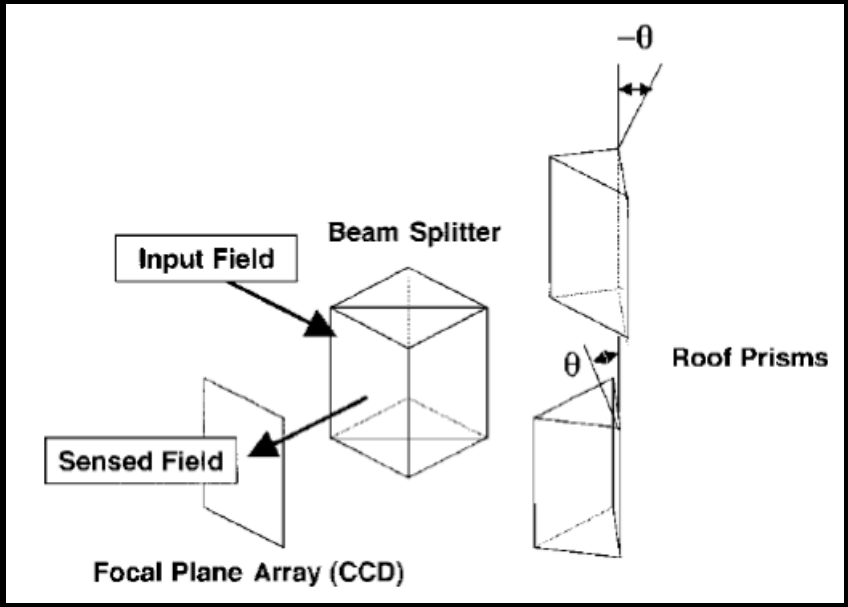


B. J. Thompson and E. Wolf, *J. Opt. Soc. Am.*, 47:895, 1957.

The mutual intensity

$$J(\mathbf{x}, \mathbf{x}') \equiv \langle U(\mathbf{x}) U^*(\mathbf{x}') \rangle$$





D. L. Marks, R. A. Stack, and D. Brady, *Appl. Opt.* 38:1332, 1999

The mutual intensity

$$J(\mathbf{x},\mathbf{x}') \equiv \langle U(\mathbf{x})U^*(\mathbf{x}')\rangle$$

- completely characterizes the (quasi-monochromatic) partially coherent field,
- in particular, the Optical Path Length (OPL);
  - J. C. Petruccelli, L. Tian, and G. Barbastathis, *Opt. Express* 21:14430, 2013
- is analogous to the density matrix in quantum mechanics;
- is semi-positive definite (eigenvalues$\geq$0);
- is a 4-dimensional quantity;
- but does it contain 4D information?
  - J. Rosen and A. Yariv, *Opt. Lett.* 21:1011, 1996
  - J. Rosen and A. Yariv, *Opt. Lett.* 21:1803, 1996
  - D. L. Marks, R. A. Stack, and D. Brady, *Appl. Opt.* 38:1332, 1999

Eugene Paul Wigner

1902 Budapest, Hungary -
1995 Princeton, New Jersey

1927 symmetries in quantum mechanics

1932 "On the quantum correction for thermodynamic equilibrium"

1960 "The unreasonable effectiveness of mathematics in the natural sciences"

1963 Nobel prize in Physics

$$W(x,u) = \int \psi\left(x + \frac{x'}{2}\right)\psi^*\left(x - \frac{x'}{2}\right) \exp\left(-i2\pi u x'\right) \mathrm{d}x'$$

http://en.wikipedia.org/wiki/E._P._Wigner

The Phase Space

- Wigner distribution function

$$W(x,u) = \int \psi\left(x+\frac{x'}{2}\right)\psi^*\left(x-\frac{x'}{2}\right)\exp\left(-i2\pi u x'\right)\mathrm{d}x'$$

<.>   <   ...   >

$$W(x,u) = \int J\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right)\exp\left(-i2\pi u x'\right)\mathrm{d}x'$$

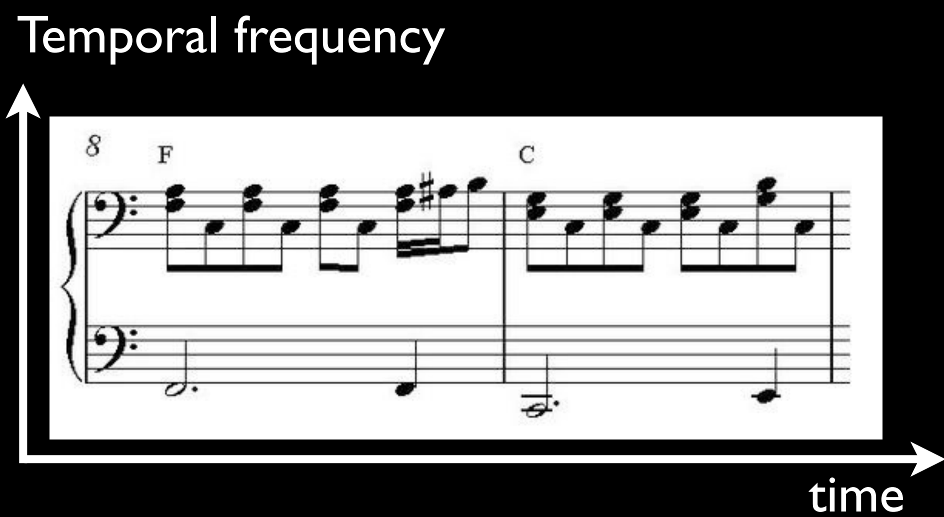
$$\Big\Updownarrow \; \mathcal{F}_{\substack{x \leftrightarrow u' \\ u \leftrightarrow x'}}$$

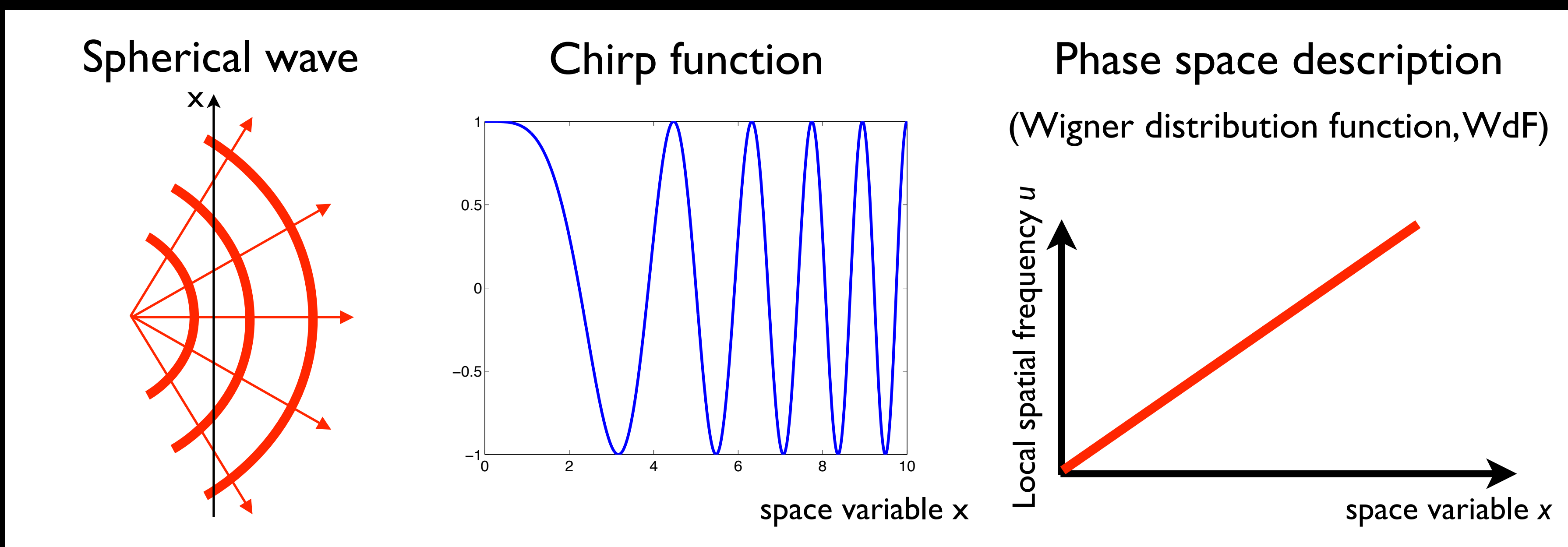
- Ambiguity function

$$A(u',x') = \int J\left(x+\frac{x'}{2}, x-\frac{x'}{2}\right)\exp\left(-i2\pi u' x\right)\mathrm{d}x$$

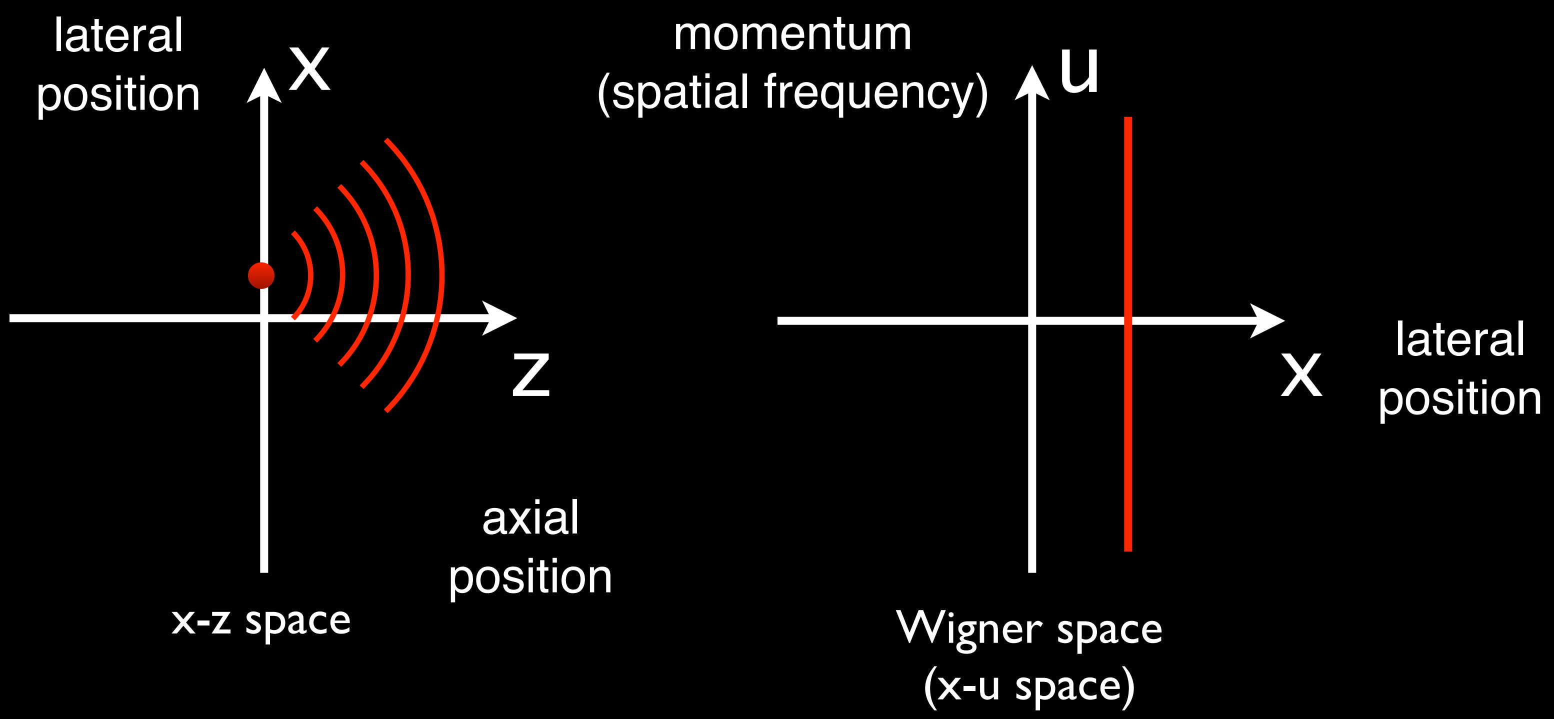
- By the way, $W(x,u)$ is real.

Phase space (Wigner space)

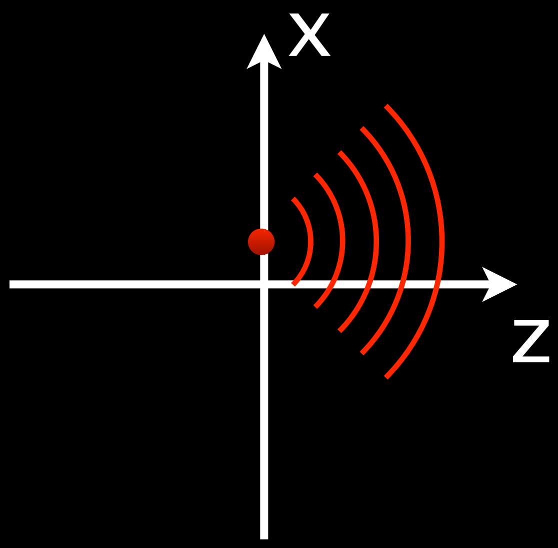




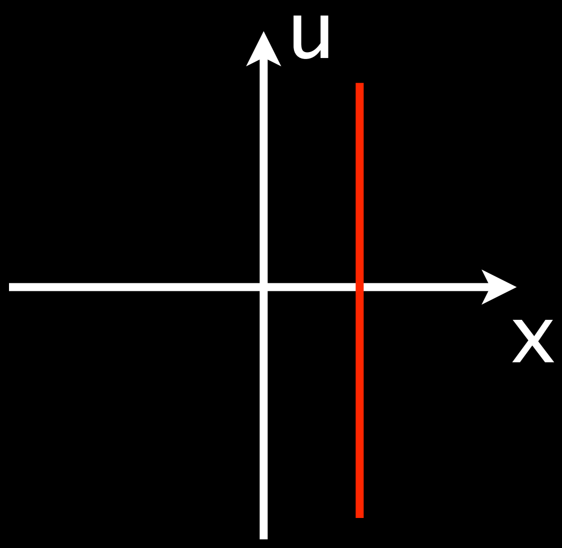
point source



spherical wave



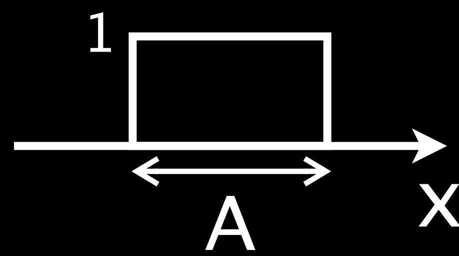
x-z space

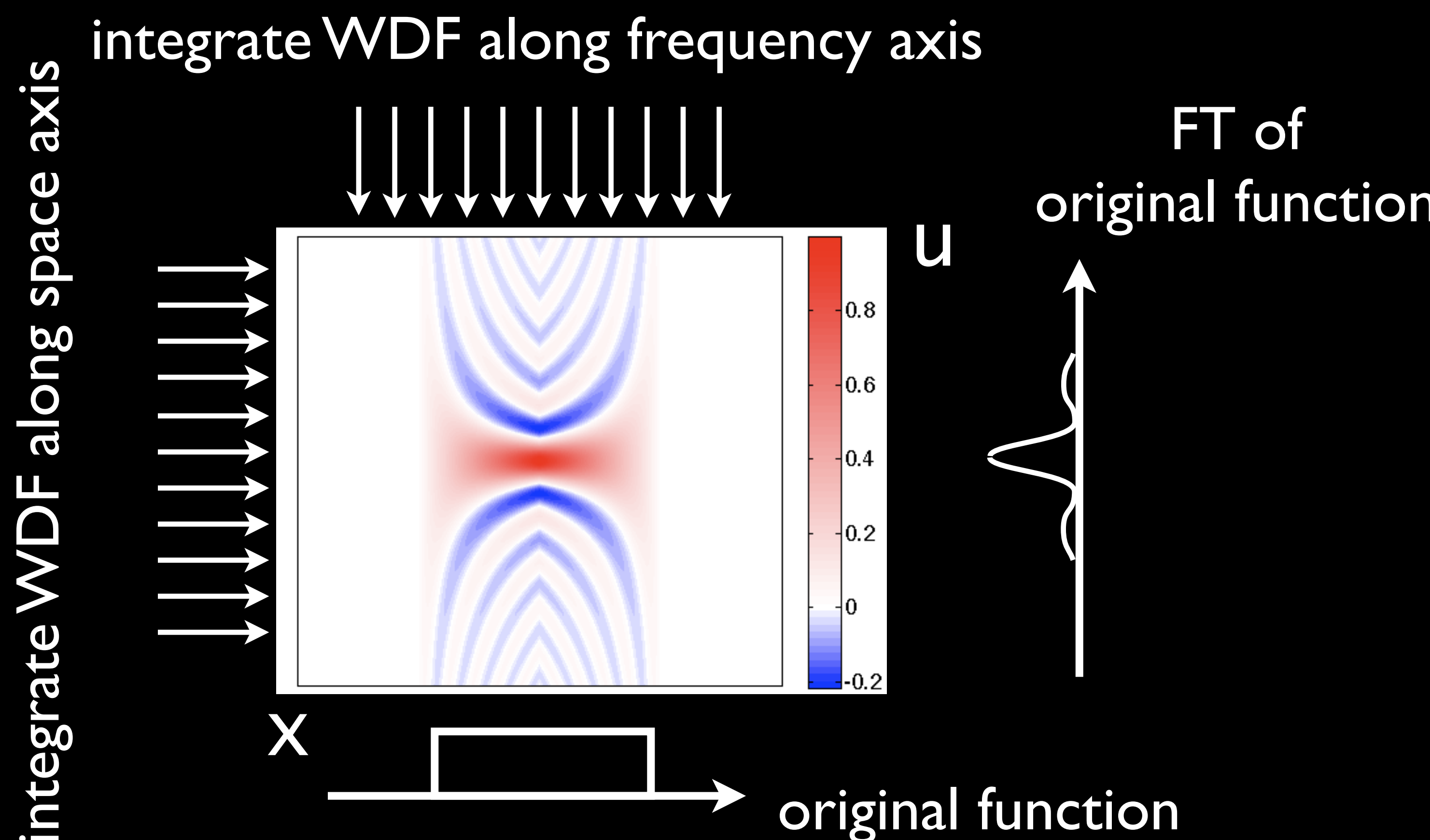


Wigner space
(x-u space)

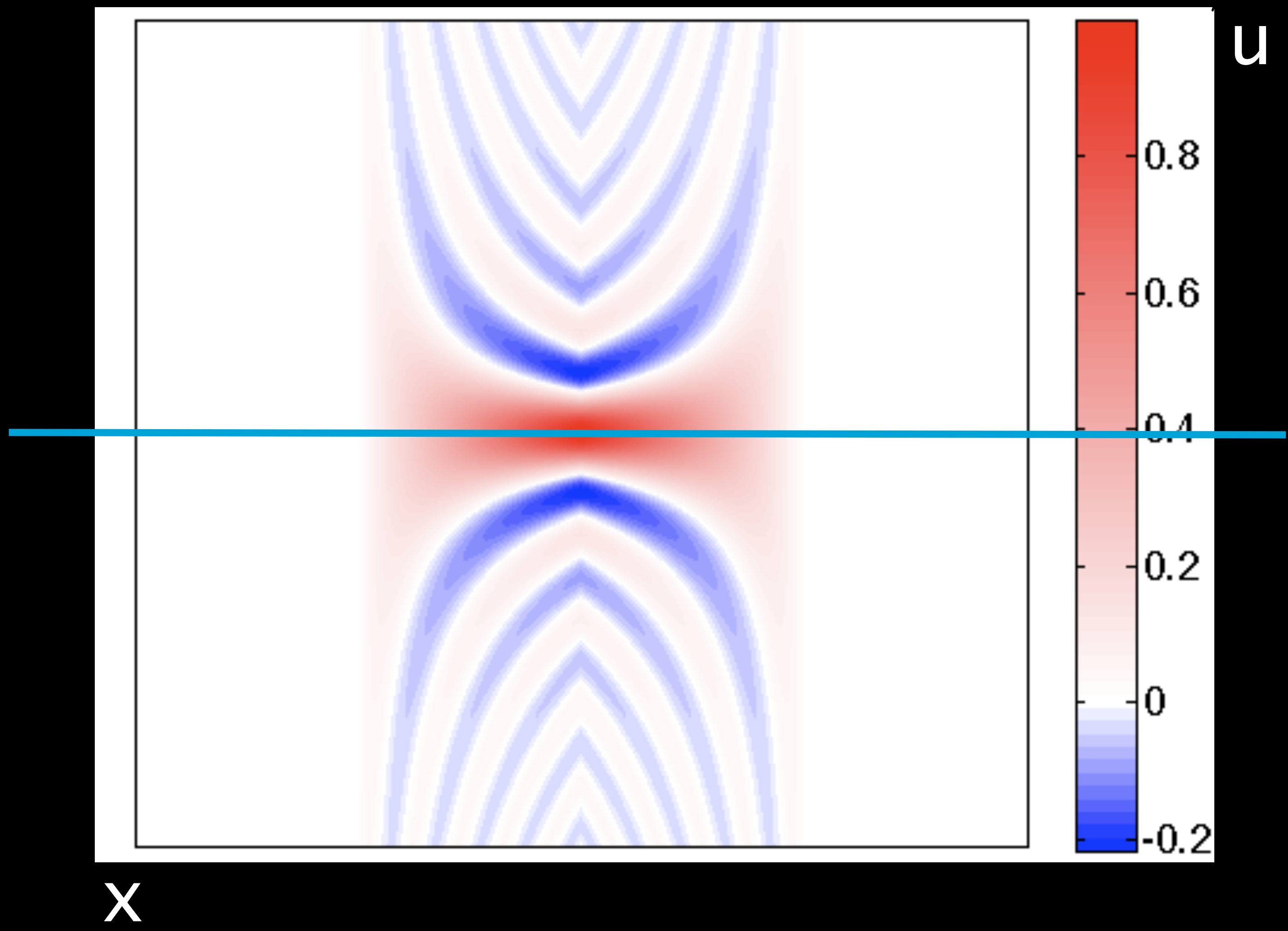
WDF ***shears/rotates*** upon propagation

boxcar ("rect") function: 1D slit

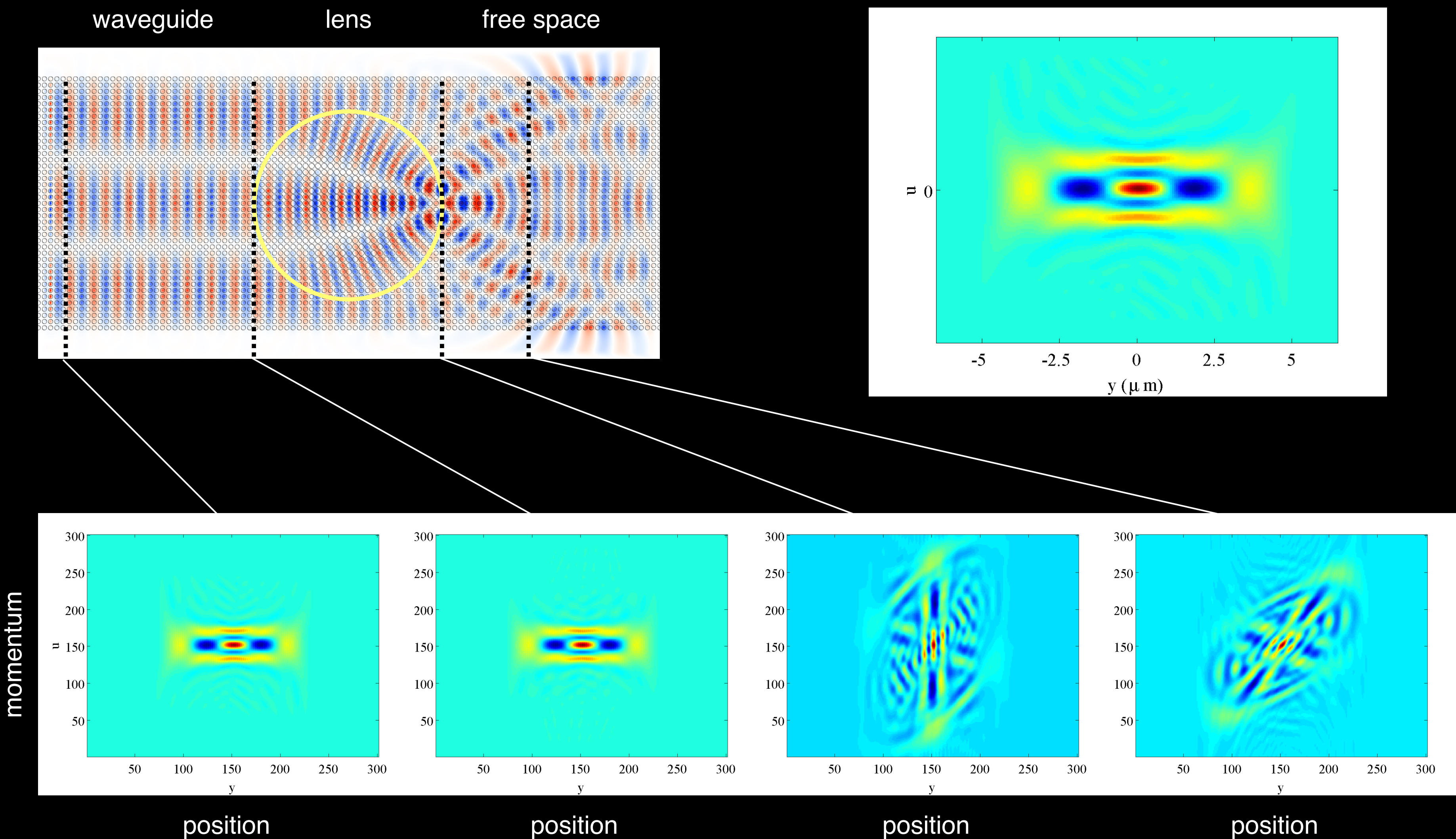




diffraction from a rectangular slit aperture



Example: waveguide (3rd mode) + lens

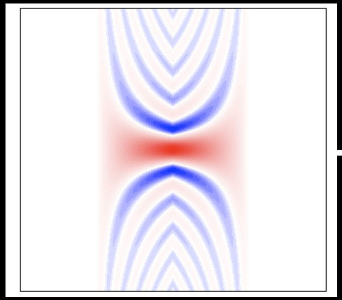


This talk is about

- Phase
- Phase space
- Phase space tomography
- Compressive imaging
- Phase tomography

Wavefunction evolution and the WDF

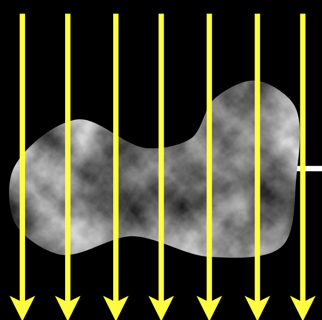
time evolution t



propagation distance z

(Fresnel propagation)

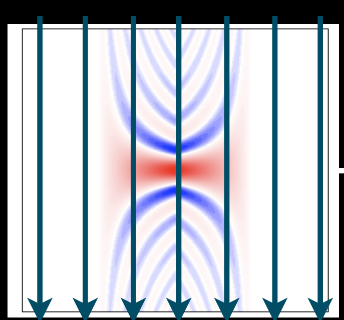
Tomographic measurement from evolution/propagation



time evolution t

ξ

measurement (quantum demolition)



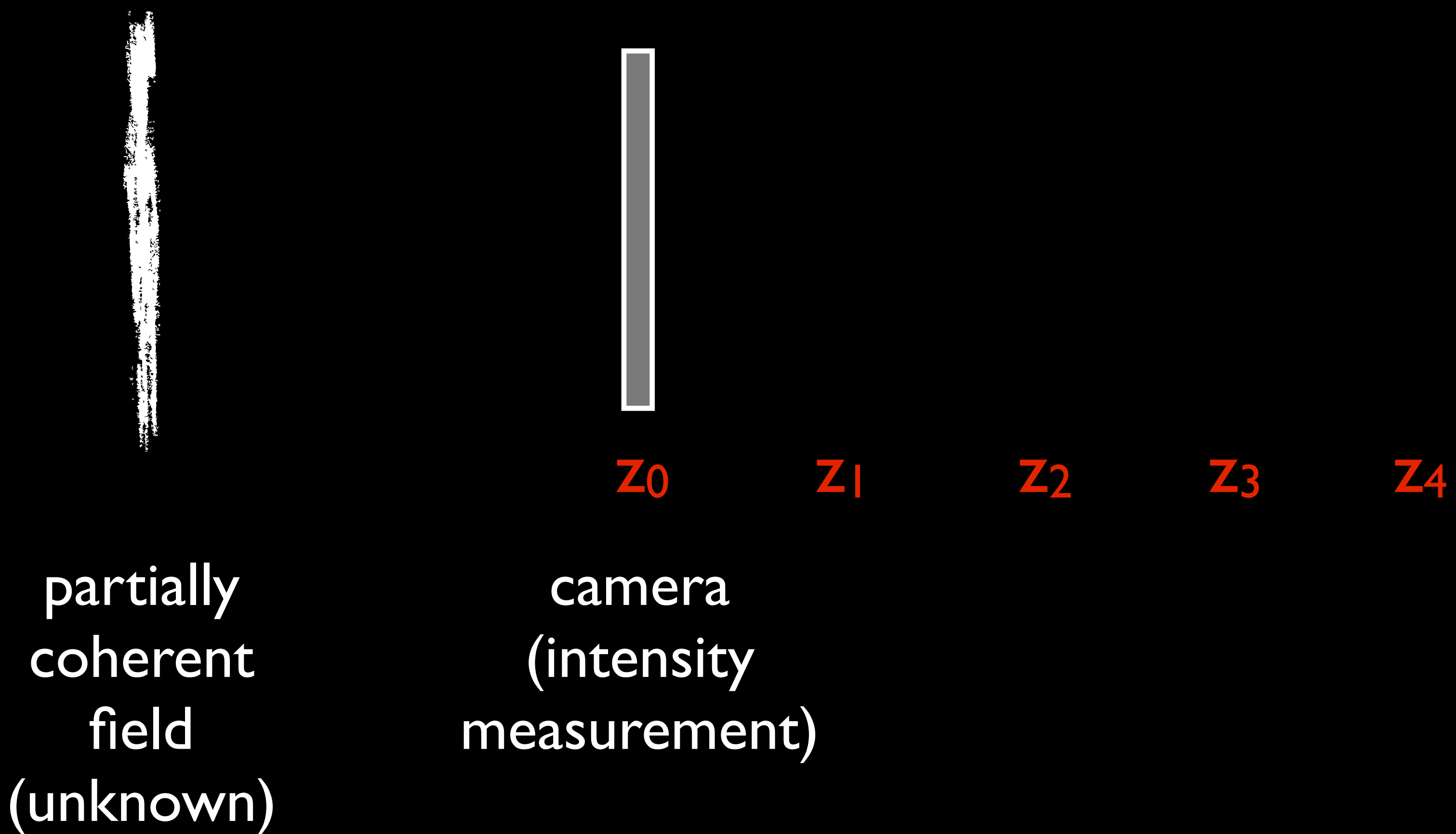
propagation distance z

$\left(\text{Fresnel propagation} \right)$

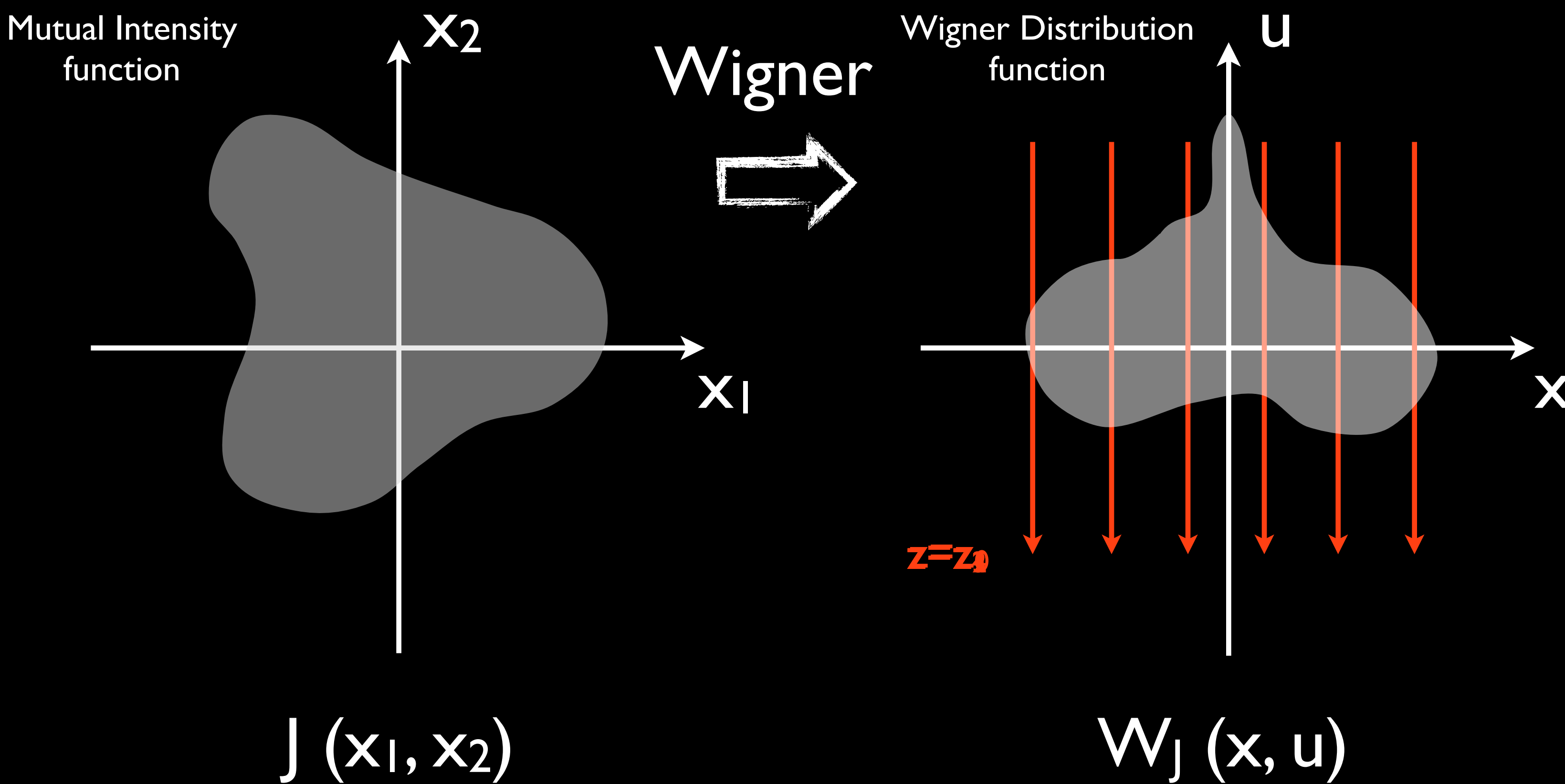
x

intensity measurement

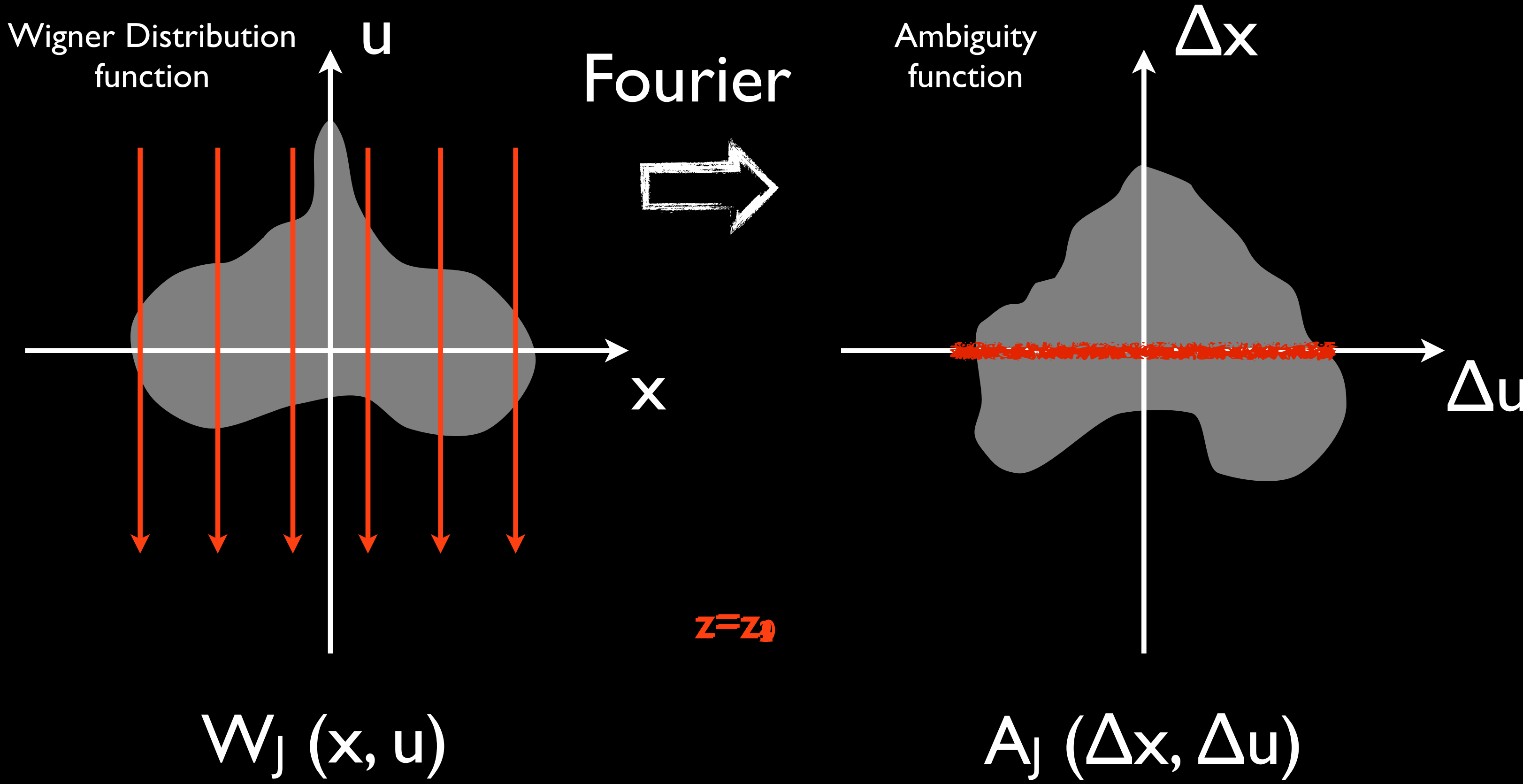
Phase-space tomography



Phase-space tomography

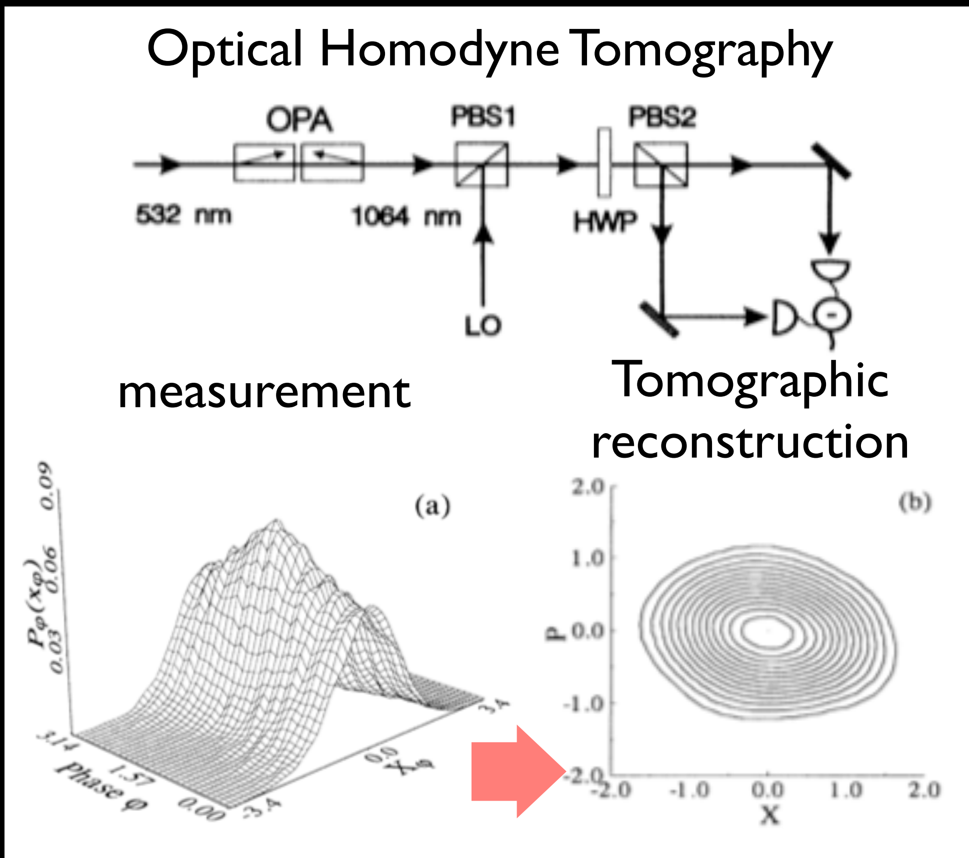


Phase-space tomography



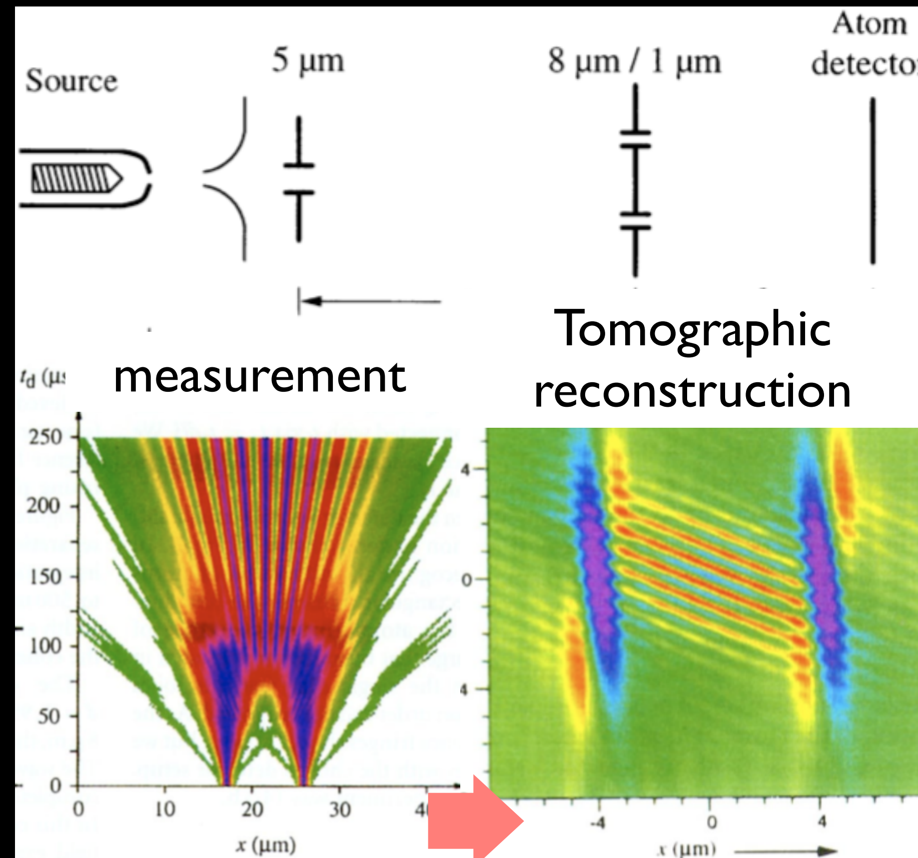
Quantum phase space tomography

Squeezed state recovery



D. Smithey, and et al, *Phys. Rev. Lett.* 1993

Matter wave interference

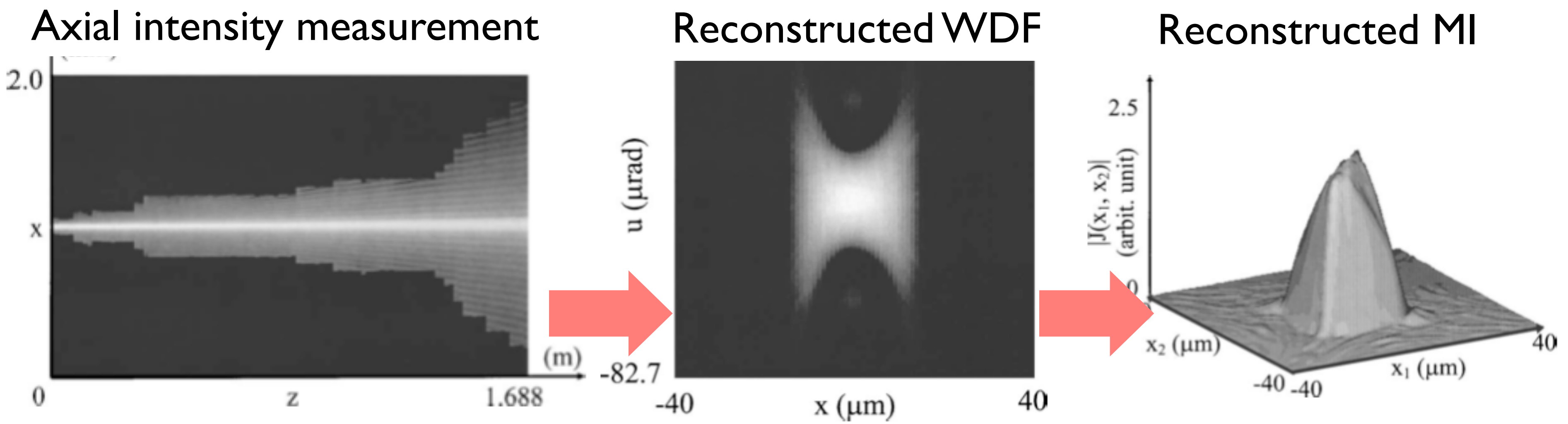


C. Kurtsiefer, and et al, *Nature*, 1997
J. Itatanl, and et al, *Nature*, 2000

Optical phase space tomography

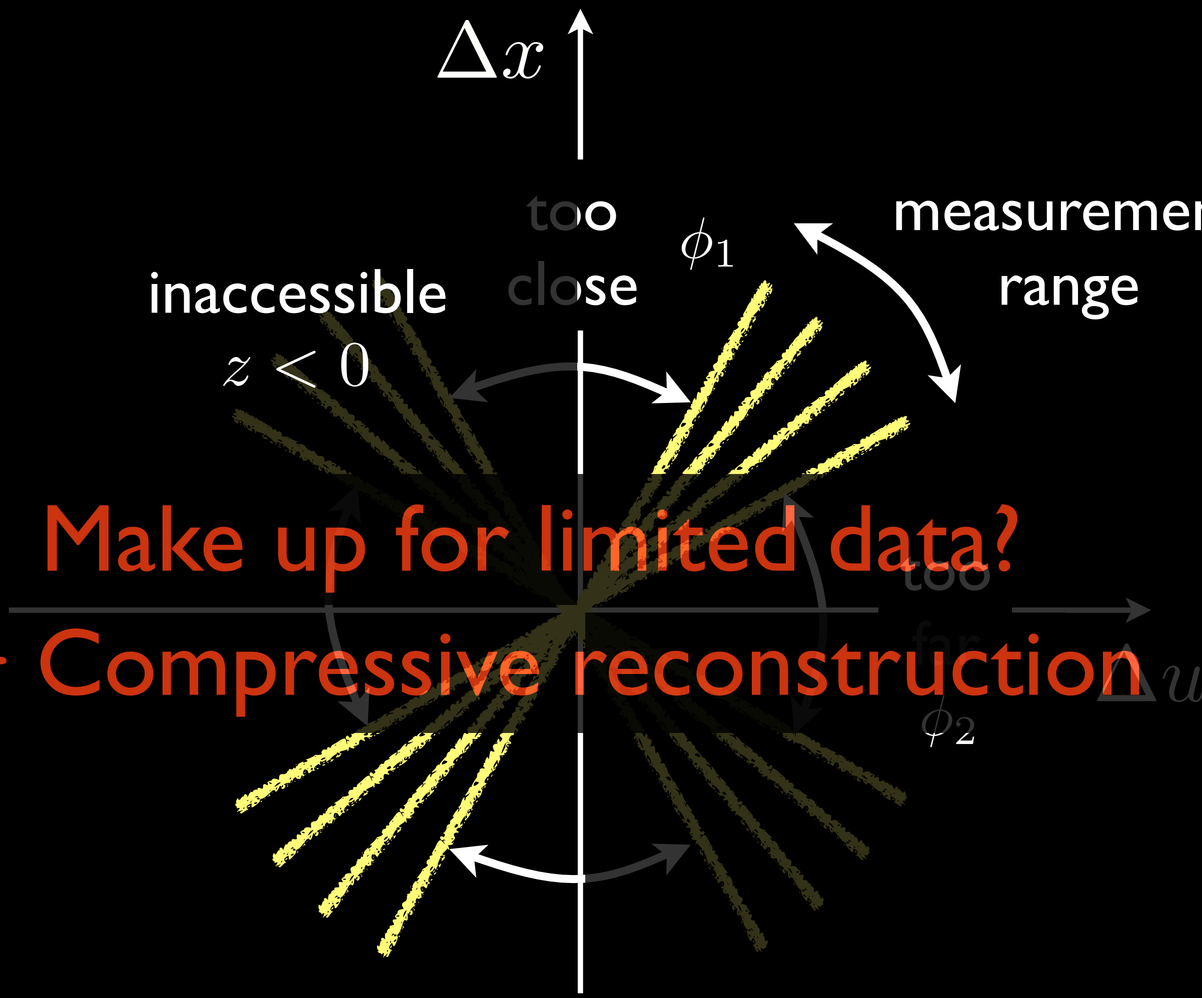
- Non-interferometric technique

Spatial coherence measurements of a 1D soft x-ray beam



C.Q.Tran, and et al, JOSA A 22, 1691-1700(2005)

The problem of limited data



Make up for limited data?

☛ Compressive reconstruction

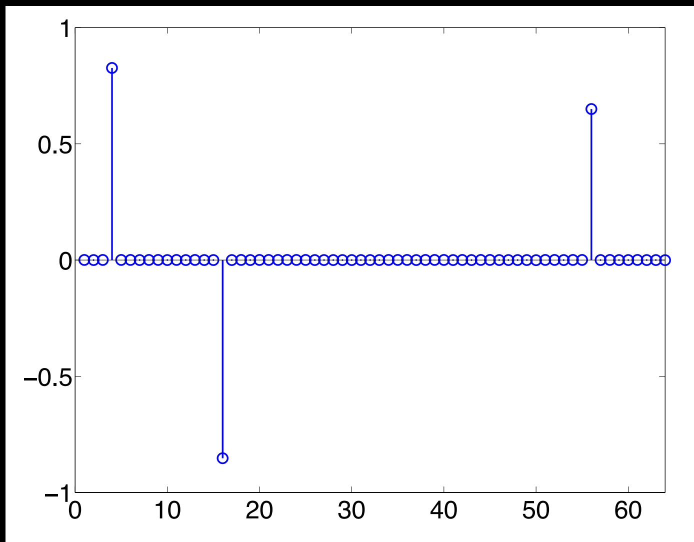
- Assume intensity symmetric about *z=0*

This talk is about

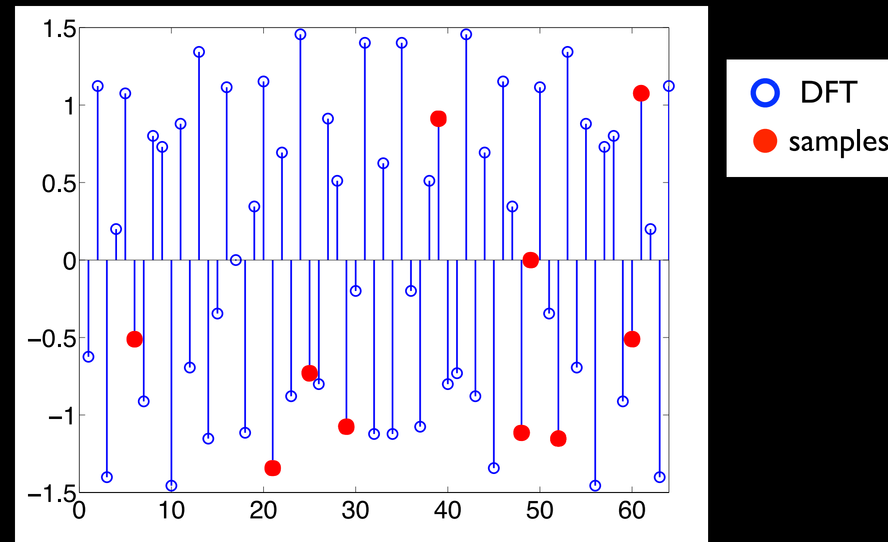
- Phase
- Phase space
- Phase space tomography
- Compressive imaging
- Phase tomography

Numerical example: 3 spikes

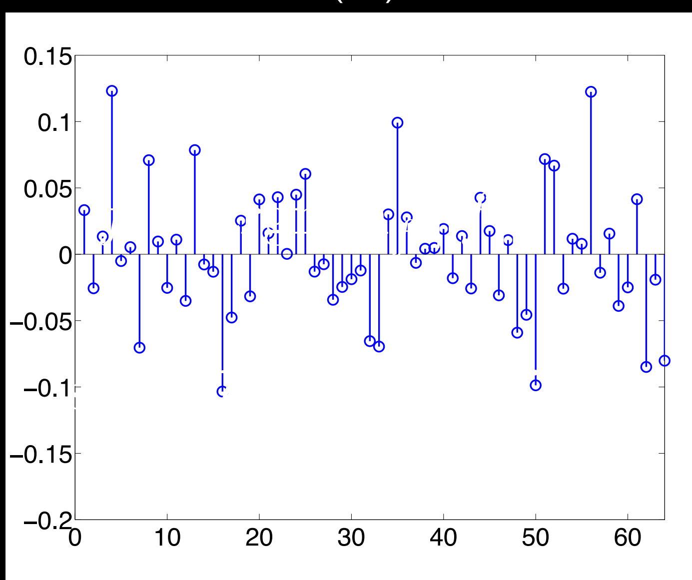
Original signal with 3 spikes (total length=64)



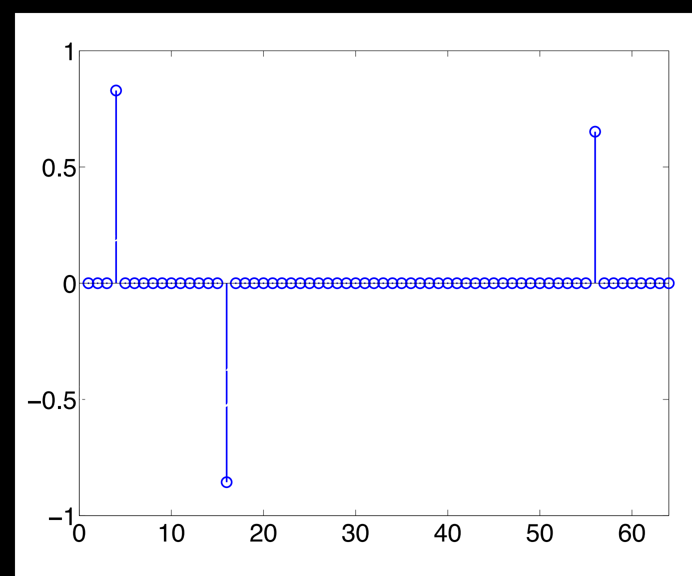
DFT measurements (# of samples=12)



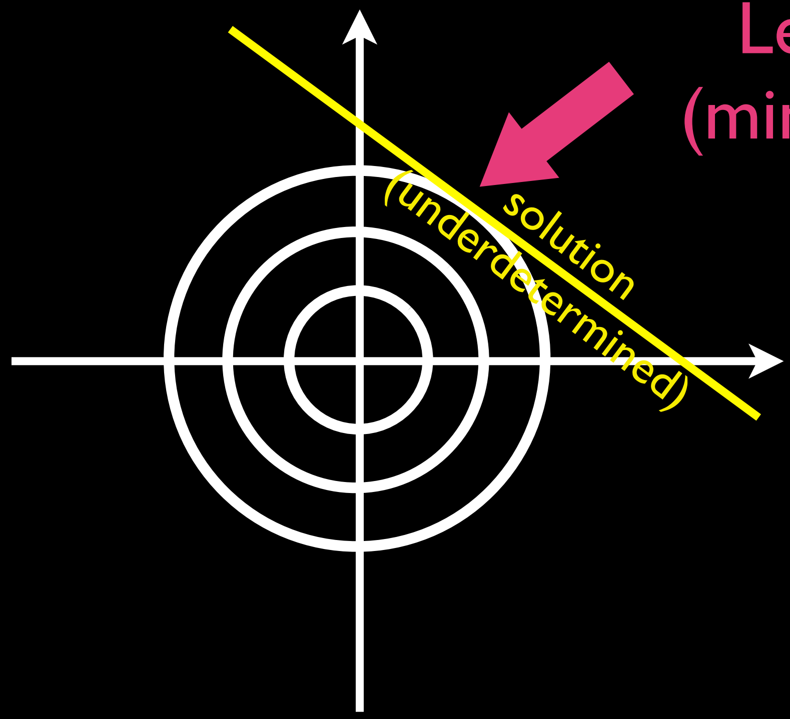
Conventional (L2) reconstruction



Compressive (L1) reconstruction



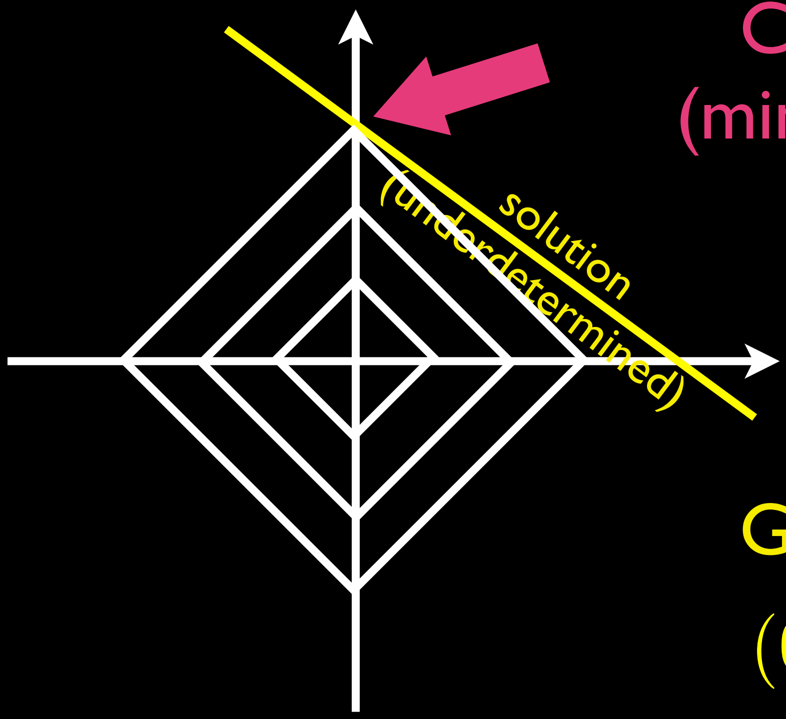
Why L1?



Least squares solution
(minimizes L2 on the line)

NOT Sparse

Why L1?



Compressive solution
(minimizes L1 on the line)

Sparse

Generally, of the form

$$(0, \ldots, 0, \xi, 0, \ldots, 0)$$

Reconstruction success is subject to sparsity

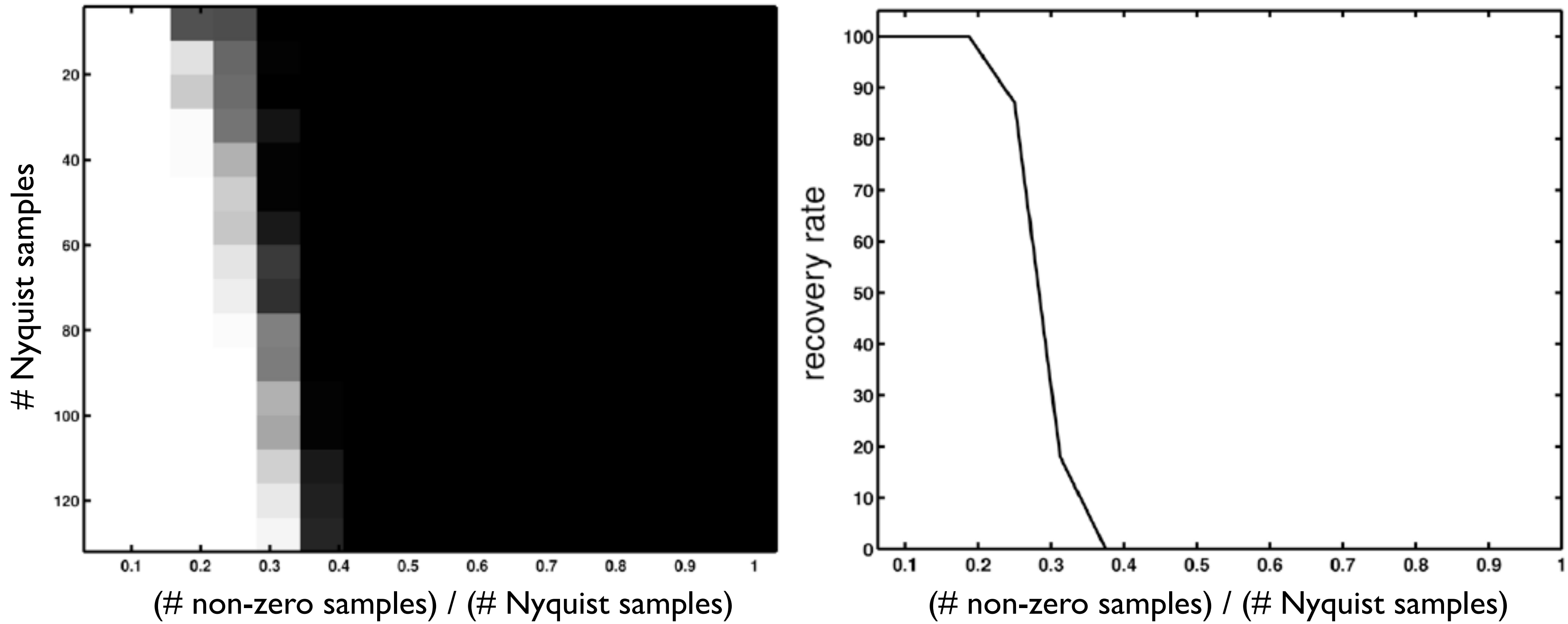
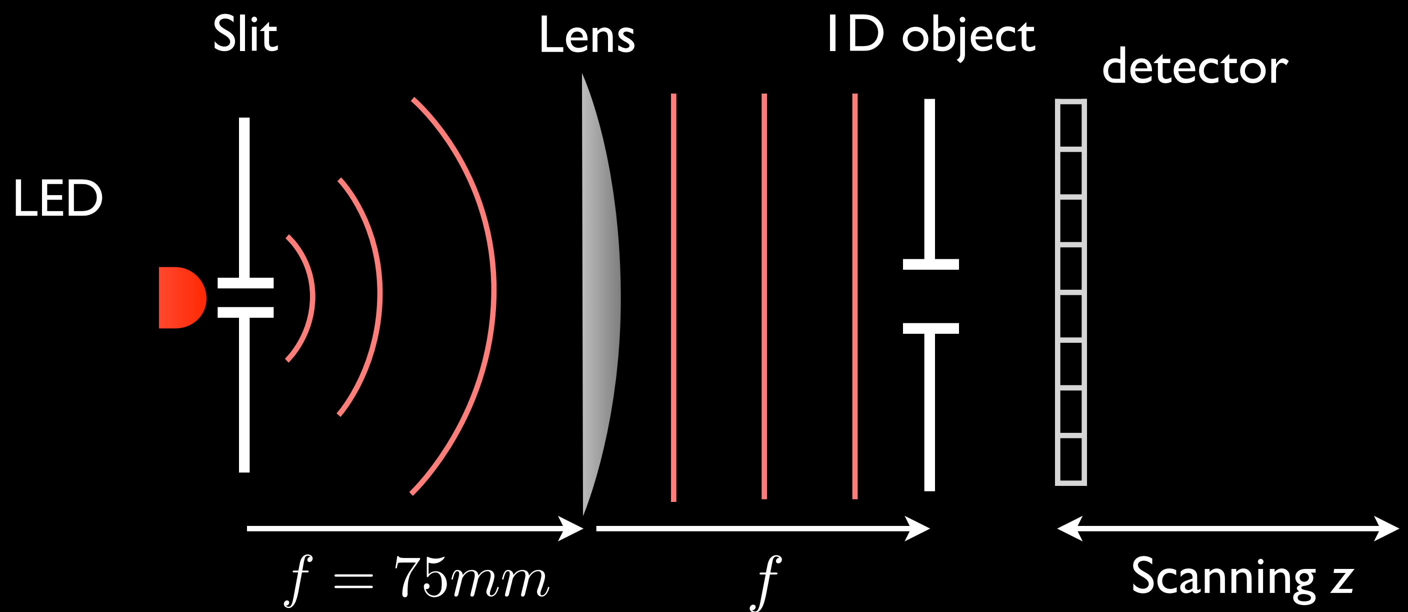


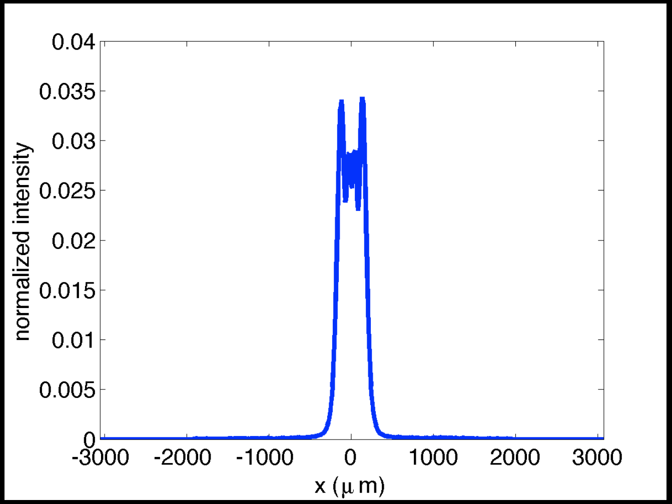
Fig. 2. Recovery experiment for $N = 512$. (a) The image intensity represents the percentage of the time solving (P_1) recovered the signal f exactly as a function of $|\Omega|$ (vertical axis) and $|T|/|\Omega|$ (horizontal axis); in white regions, the signal is recovered approximately 100% of the time, in black regions, the signal is never recovered. For each $|T|, |\Omega|$ pair, 100 experiments were run. (b) Cross section of the image in (a) at $|\Omega| = 64$. We can see that we have perfect recovery with very high probability for $|T| \leq 16$.

E. Candés, J. Romberg, and T. Tao, *IEEE Trans. Info. Th.* 52:489, 2006

Exprerimental compressive phase-space tomography

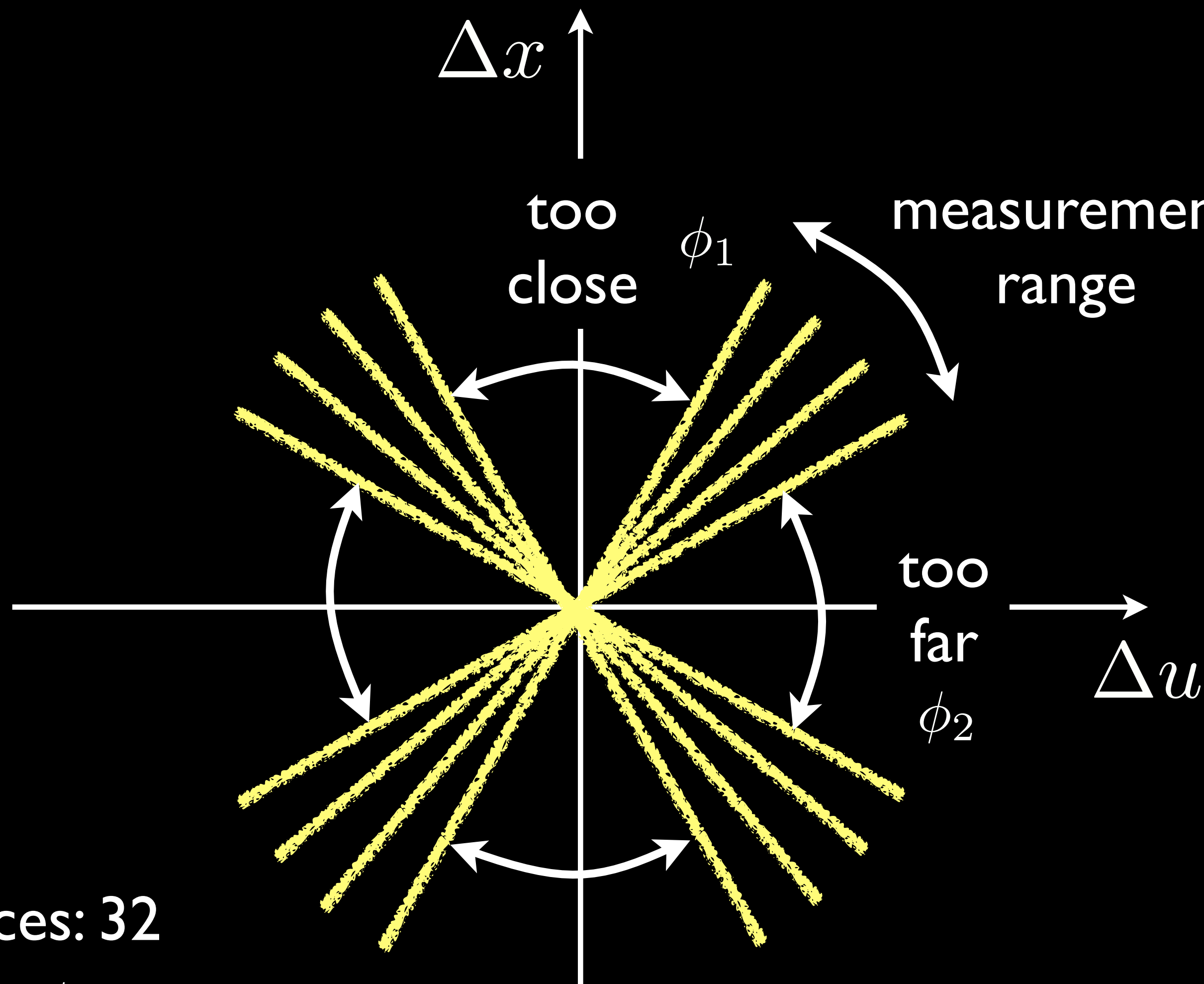


- Illumination central wavelength: 620nm; bandwith: 20nm
- Width of illumination slit: 300μm
  - Coherence length: 93μm
- Width of object slit: 400μm
- 32 measurements (axial positions)



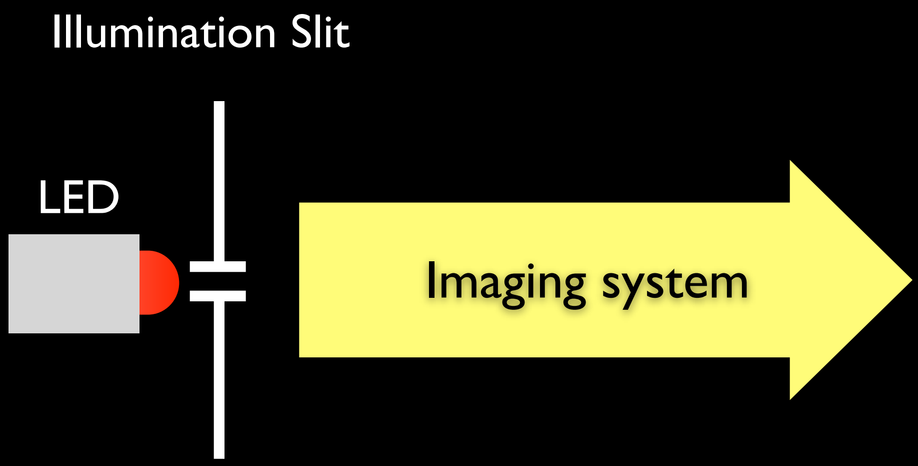
Lei Tian et al, *Opt. Expr.* 20(8):8296, 2012

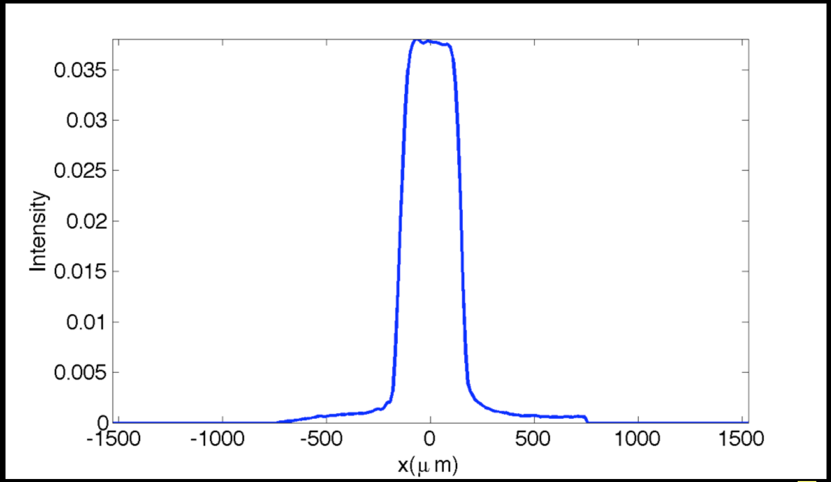
Limited data in our experiment



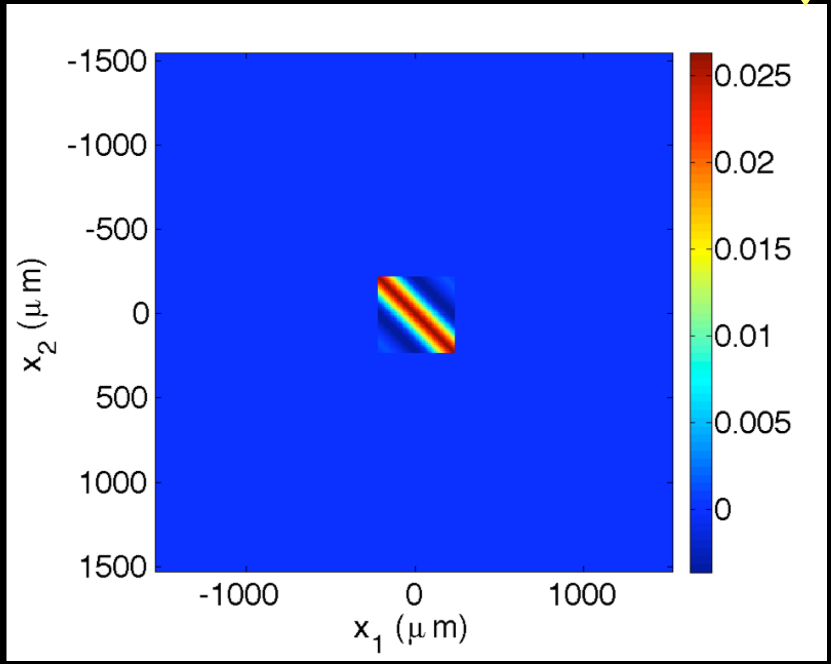
- Total # of slices: 32
- Missing angle ϕ_1: 38°
- Missing angle ϕ_2: 22°

Ground Truth





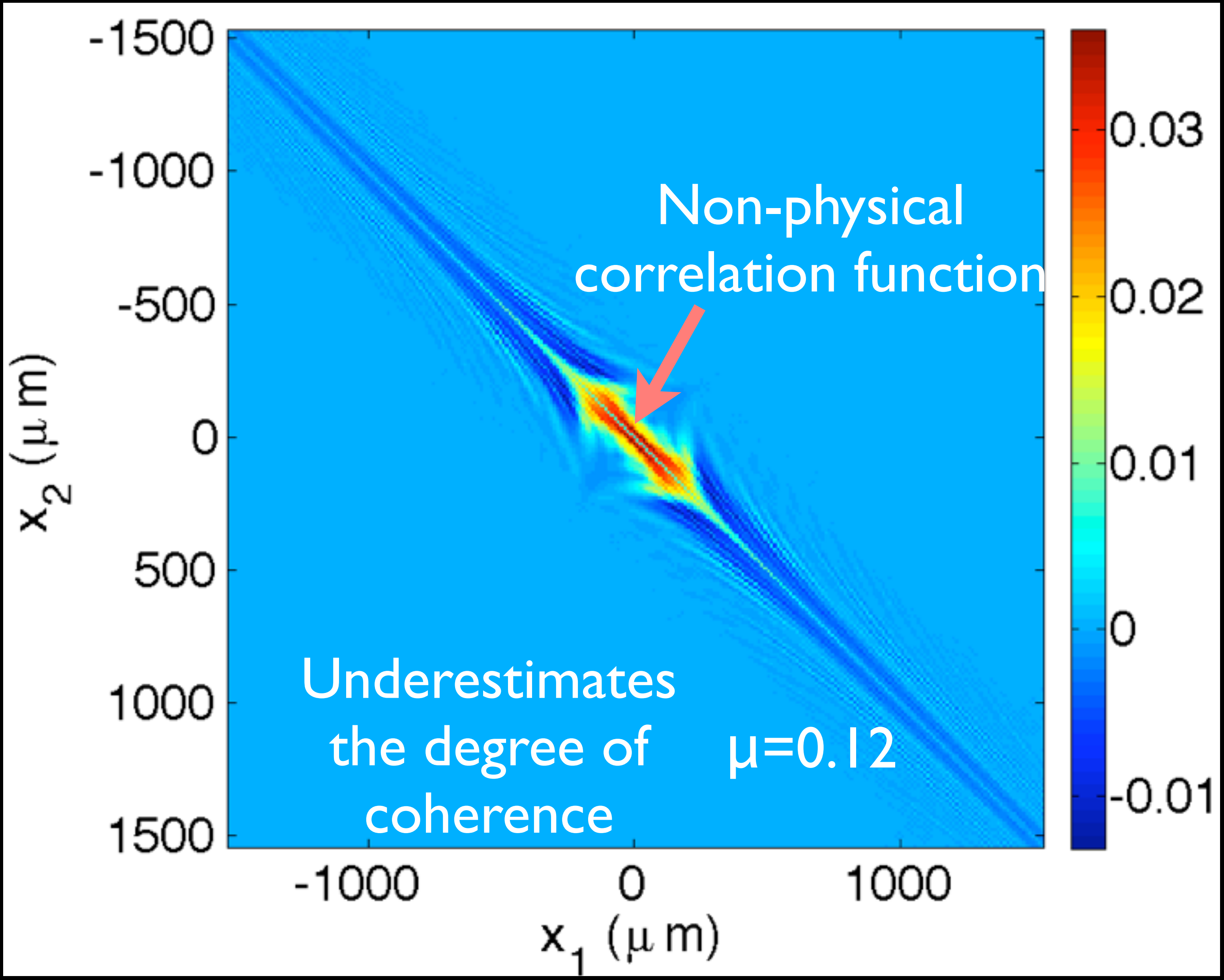
van Cittert-Zernike theorem



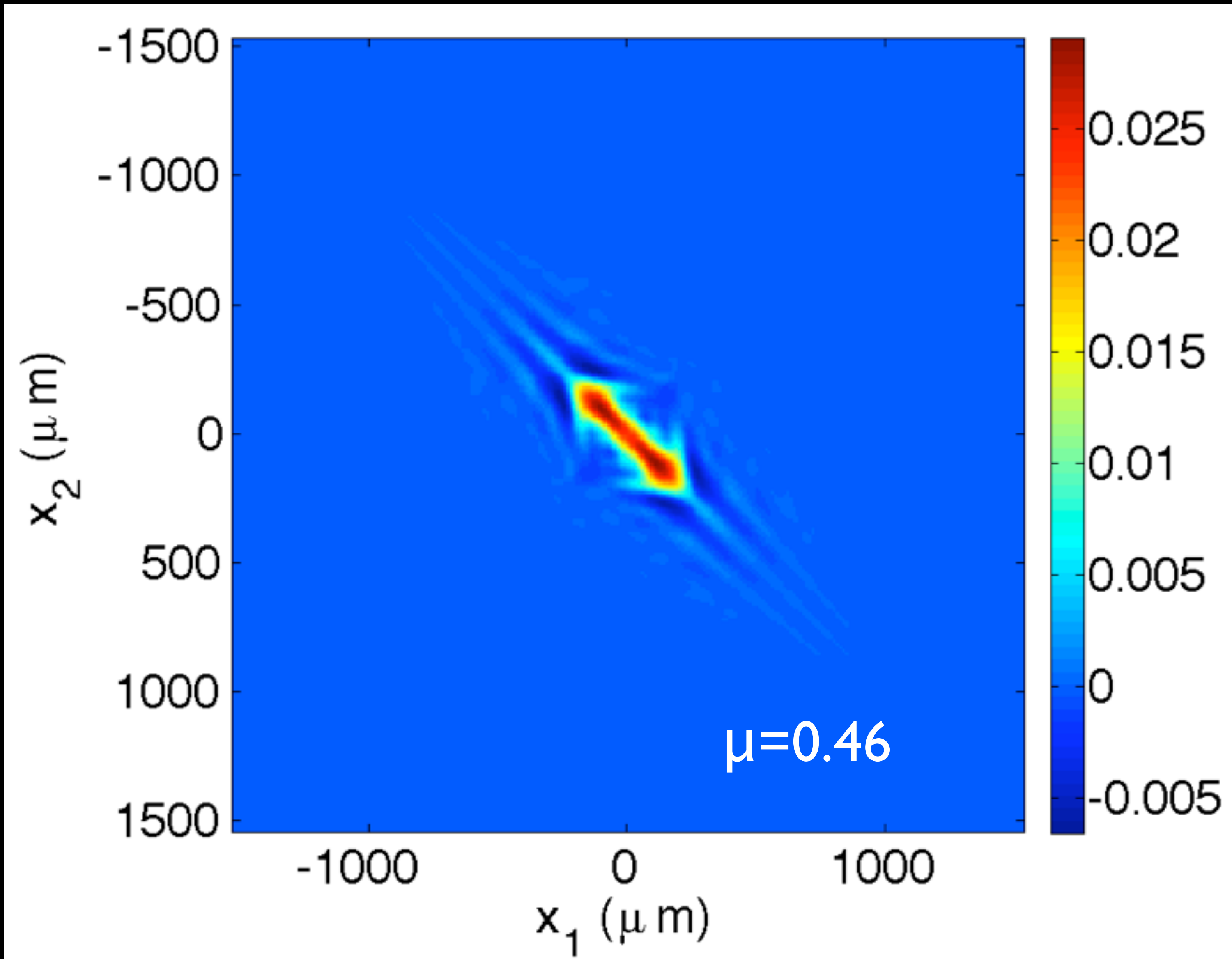
Global Degree of Coherence

$\mu=0.49$

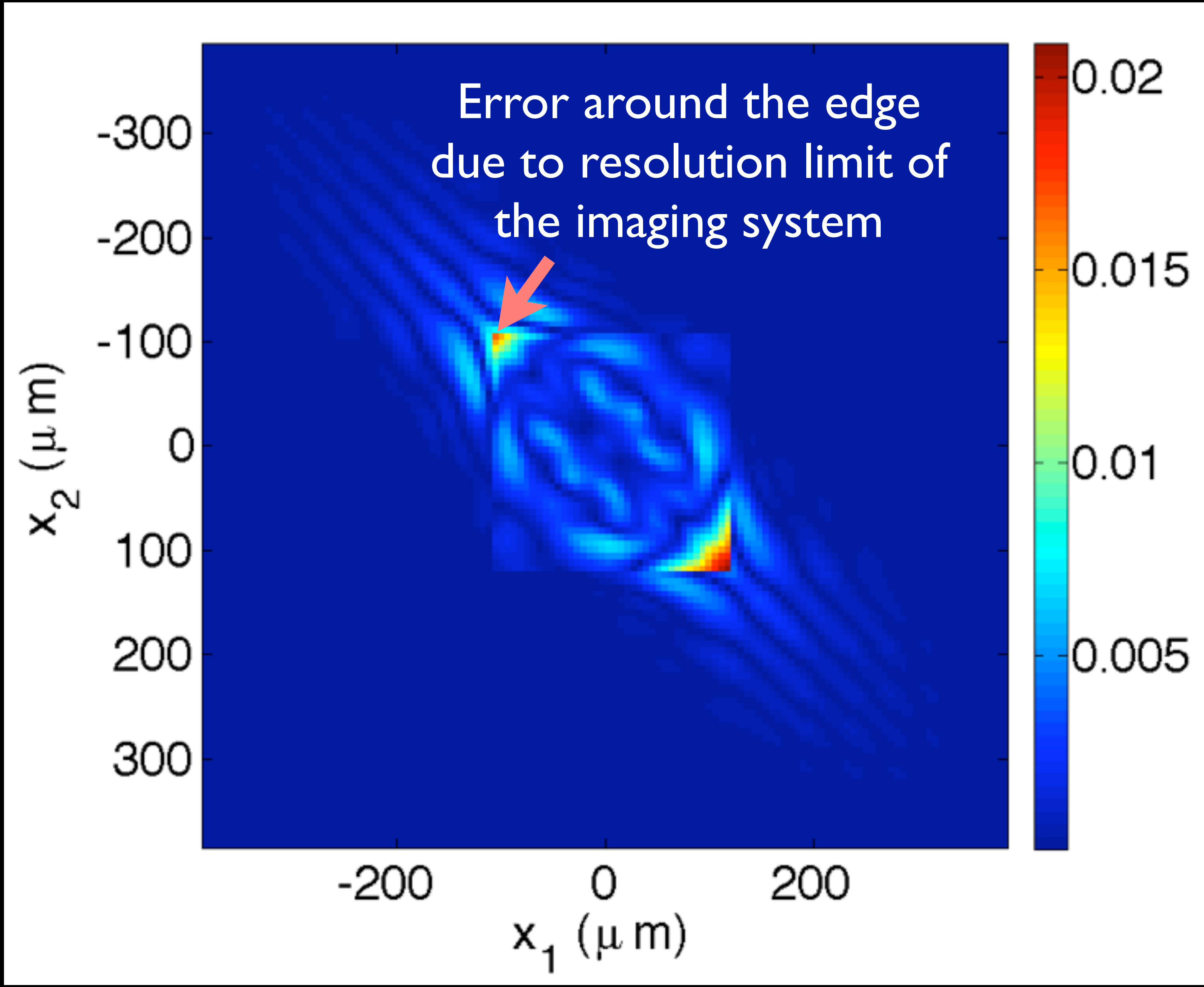
Filtered back-projection fails



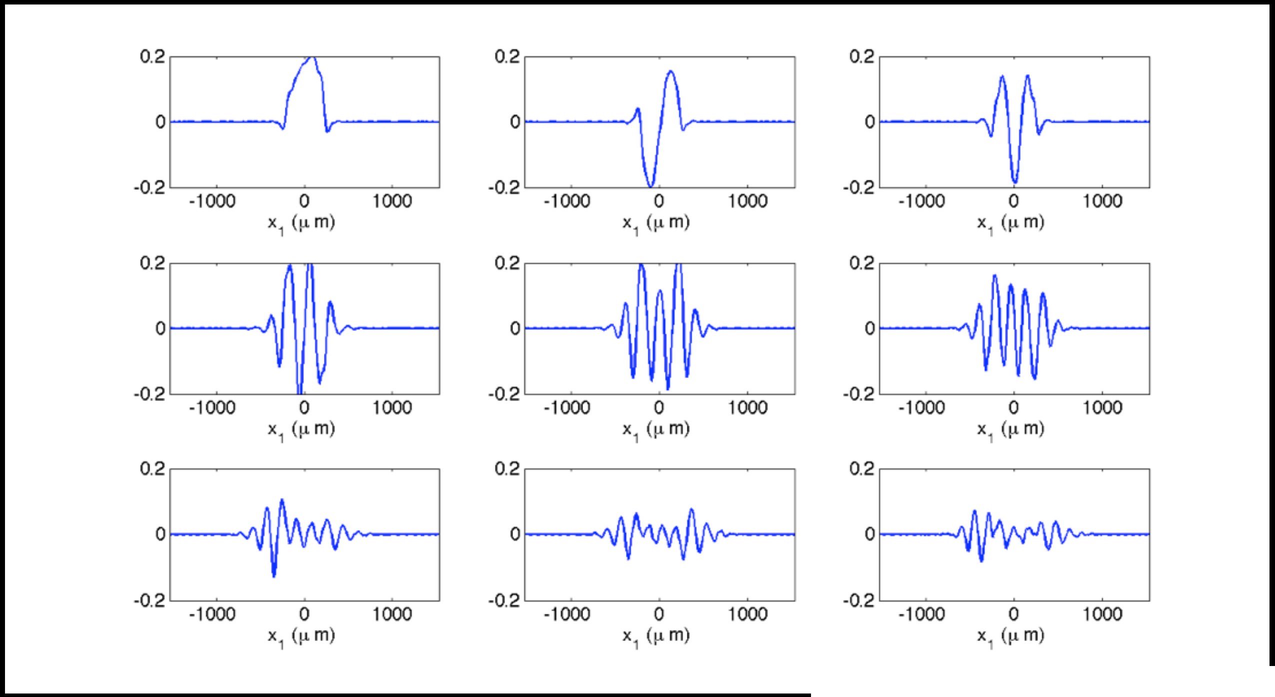
Compressive reconstruction



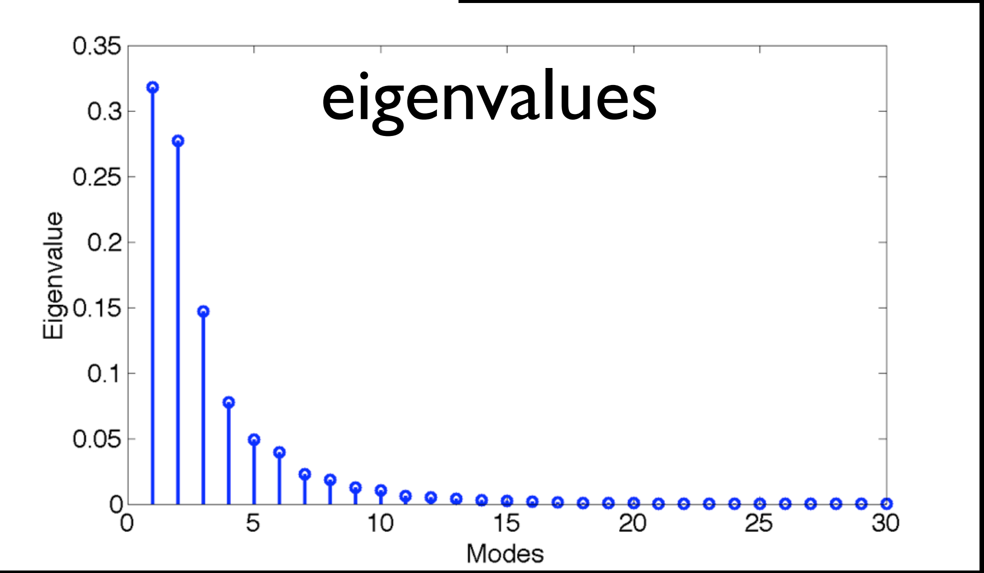
Error in compressive reconstruction
(compared to Van Cittert-Zernike)



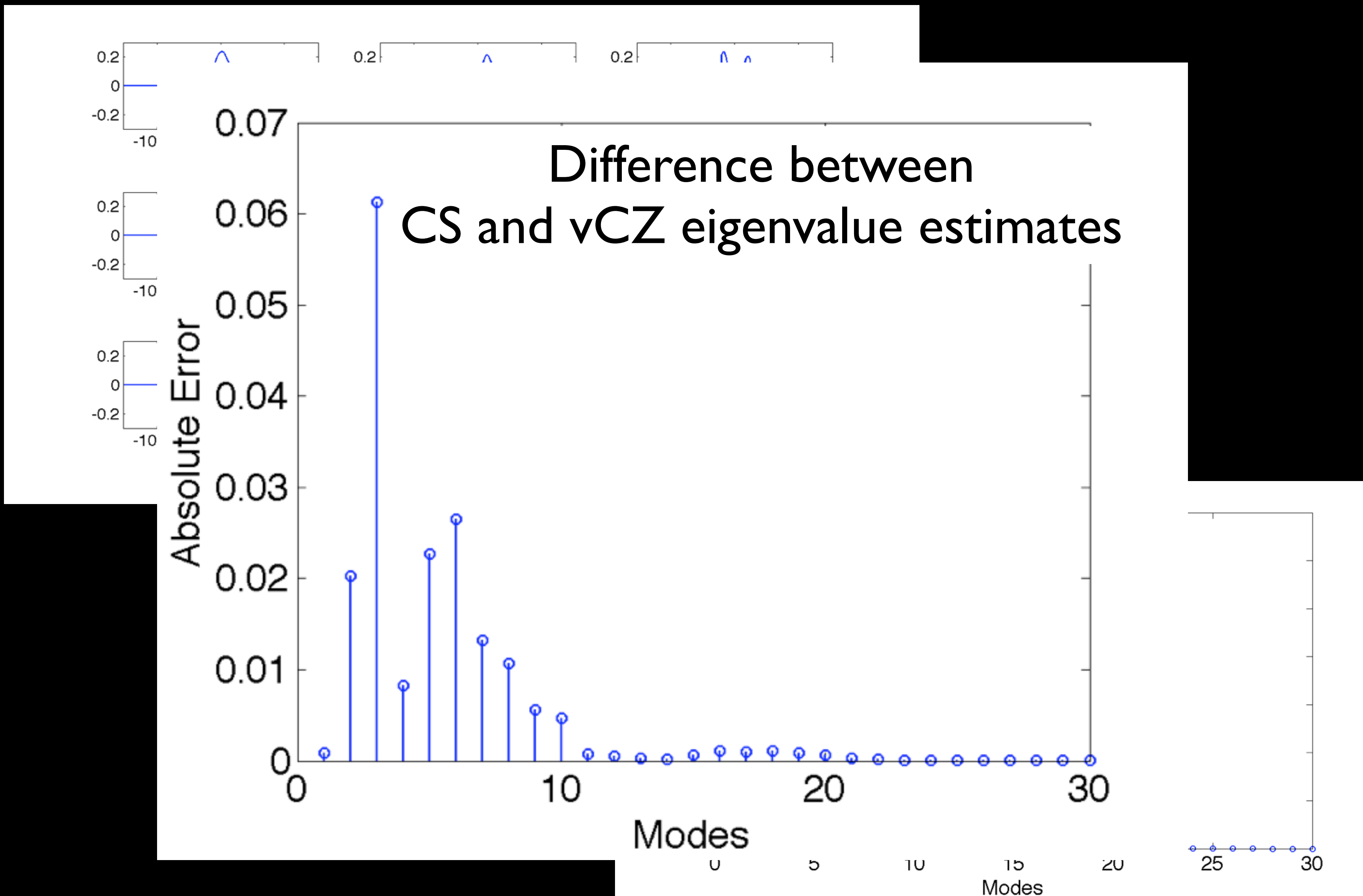
Estimate of the coherent modes



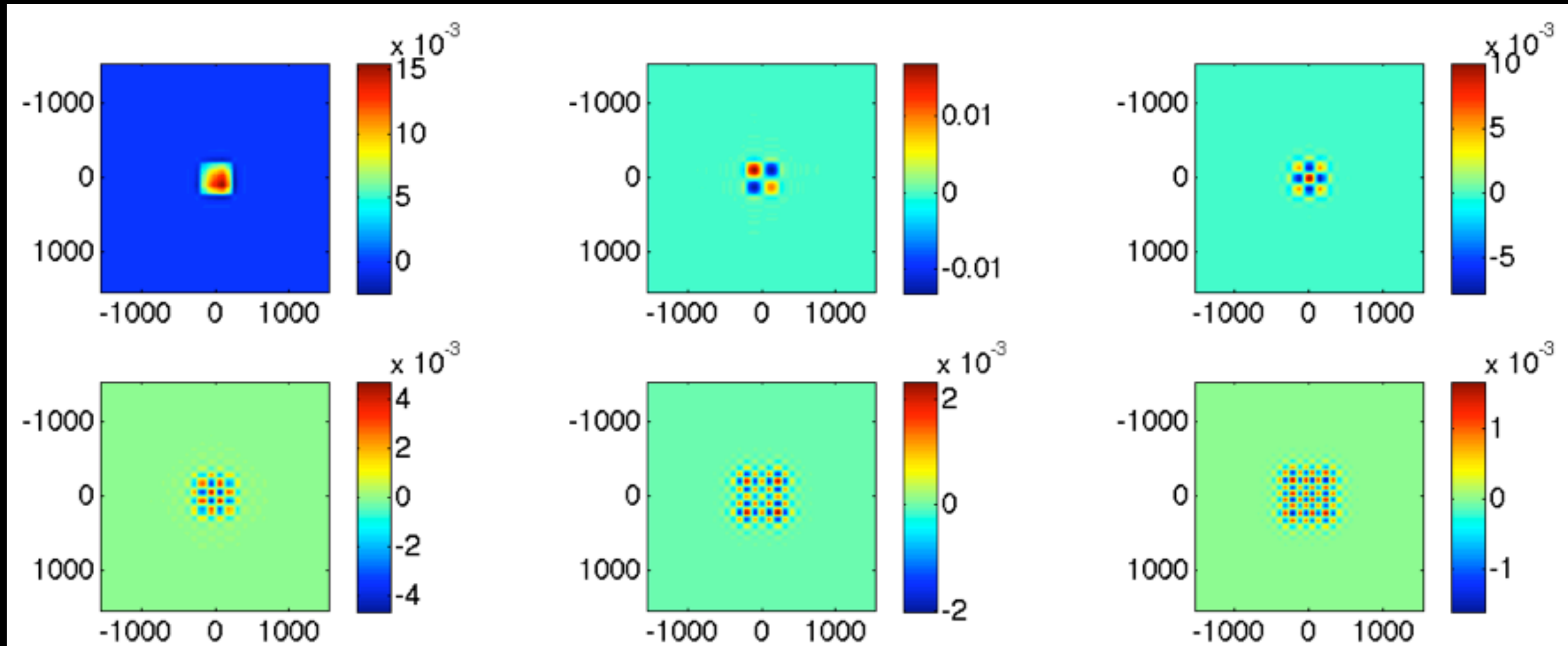
Coherent modes



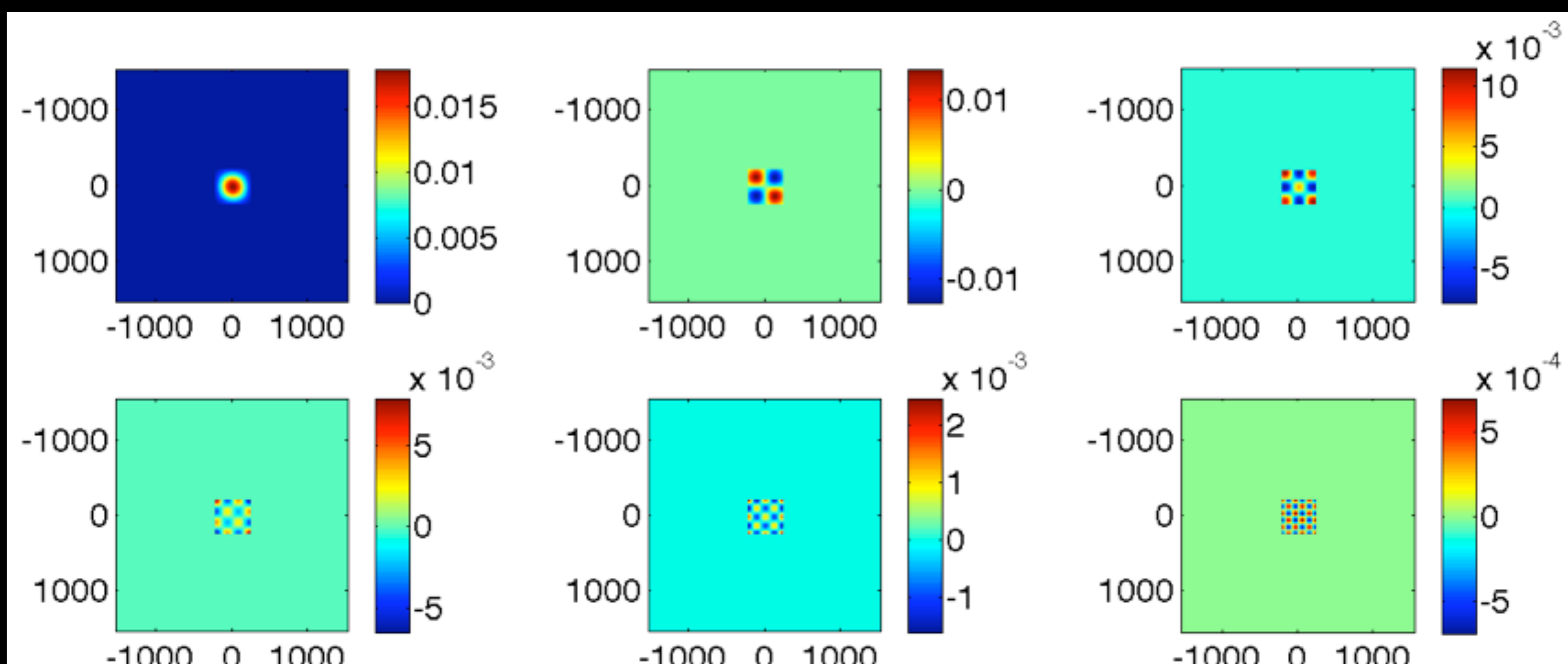
Validation: vCZ theorem



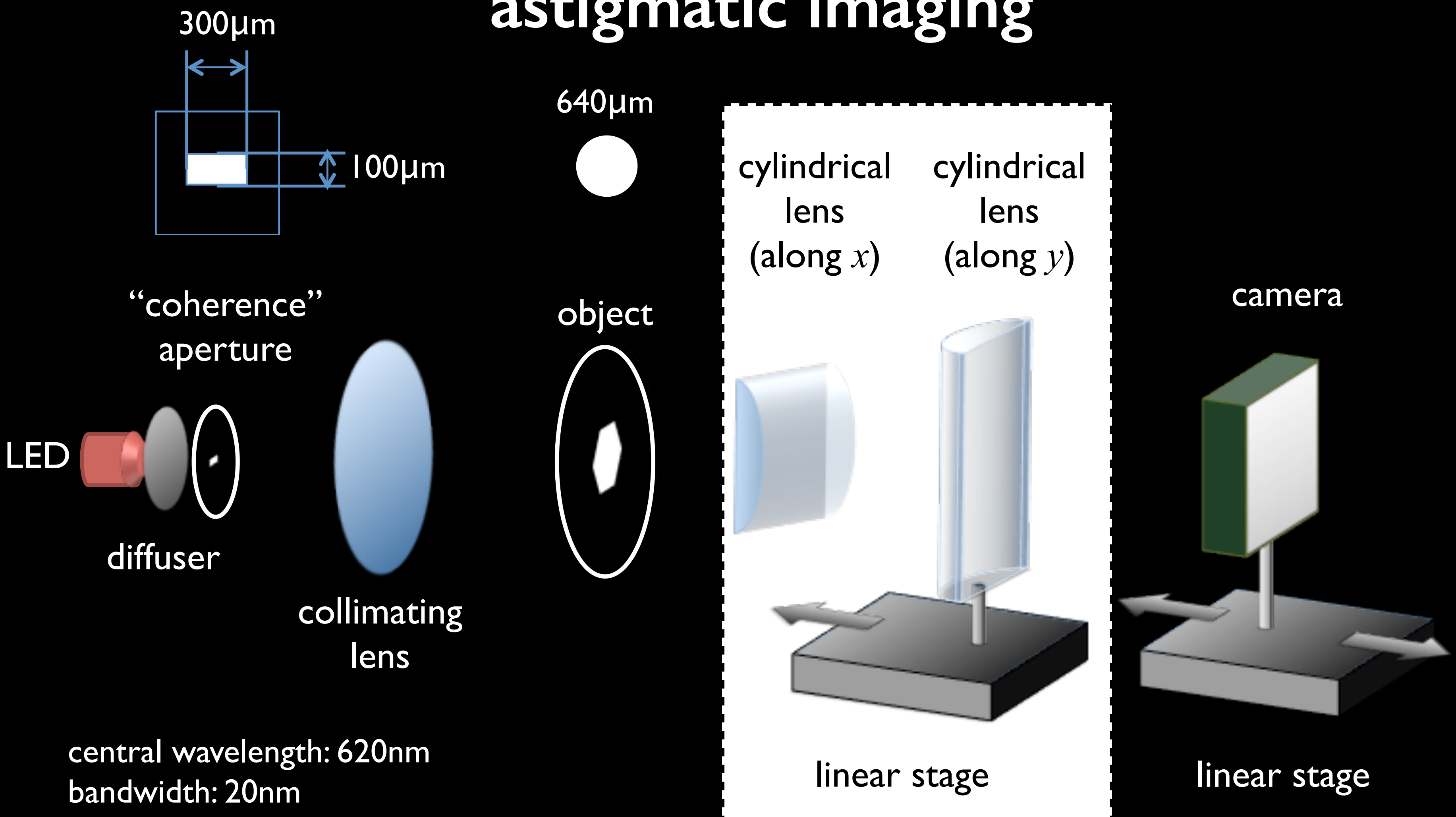
Compressive estimate



vCZ estimate



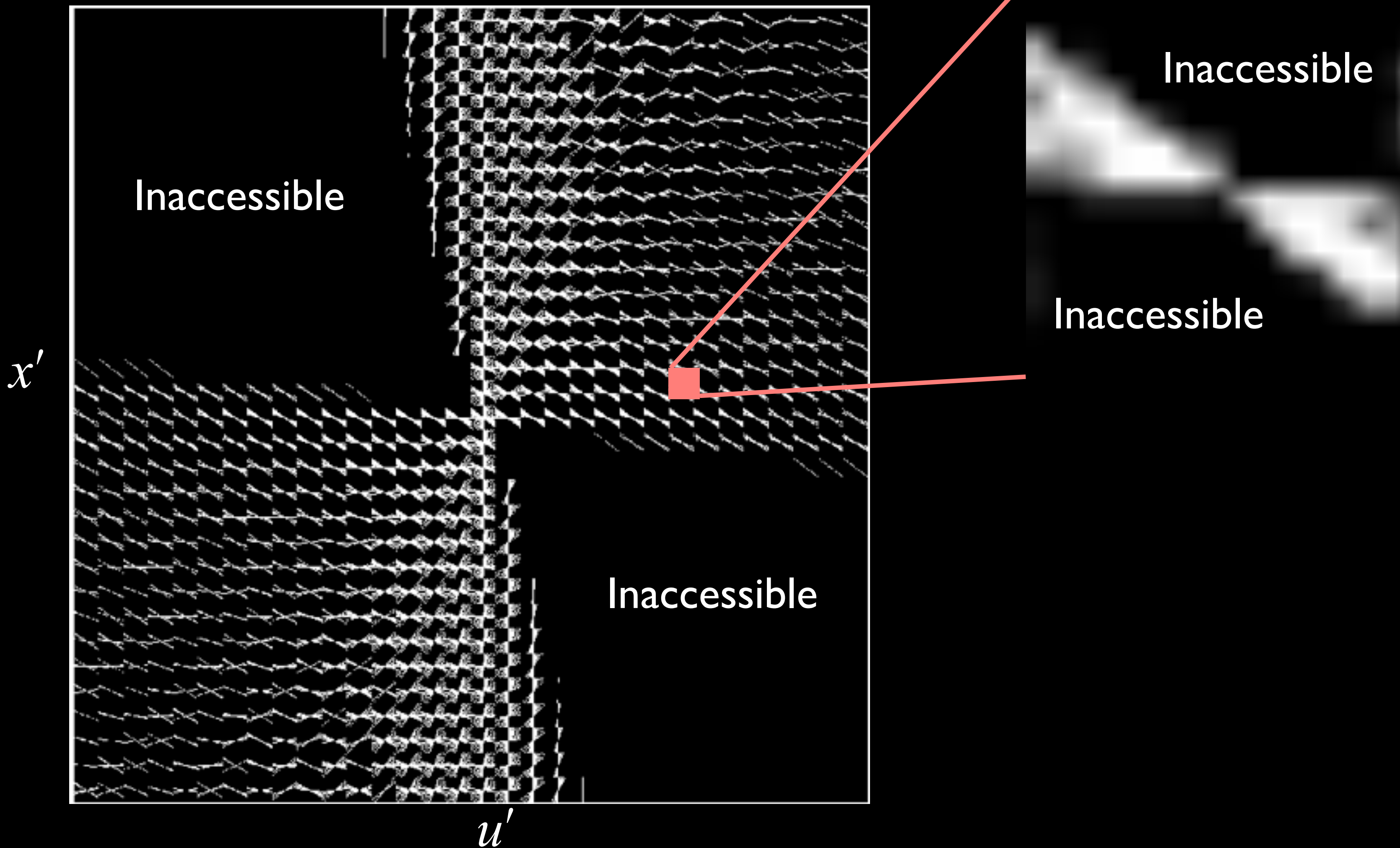
4D phase space tomography: astigmatic imaging



central wavelength: 620nm
bandwidth: 20nm

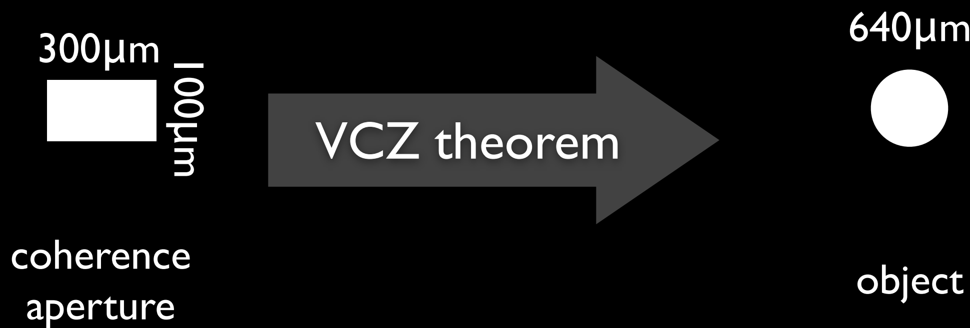
L. Tian, S. Rehman, and G. Barbastathis, in Frontiers in Optics 2012, paper FM4C.4.
M. Raymer, M. Beck and D. McAlister, Phys.Rev.Lett. 1994
D. Marks, R. Stack and D. Brady, Opt. Lett. 2000

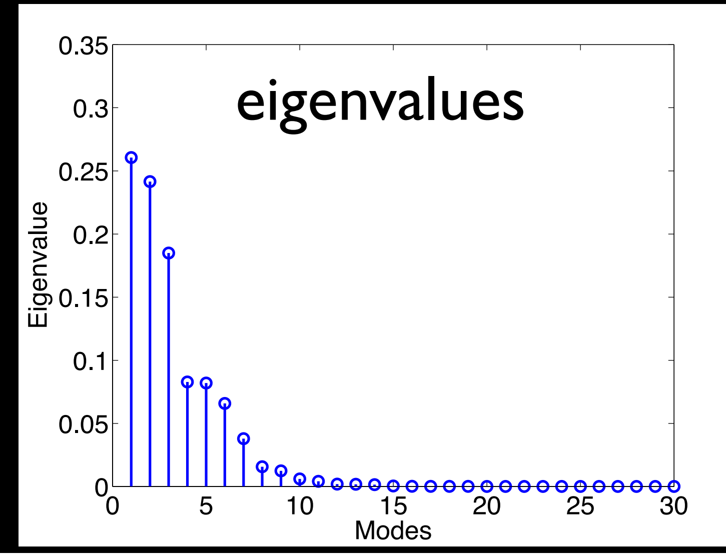
“Missing slices” in Ambiguity space

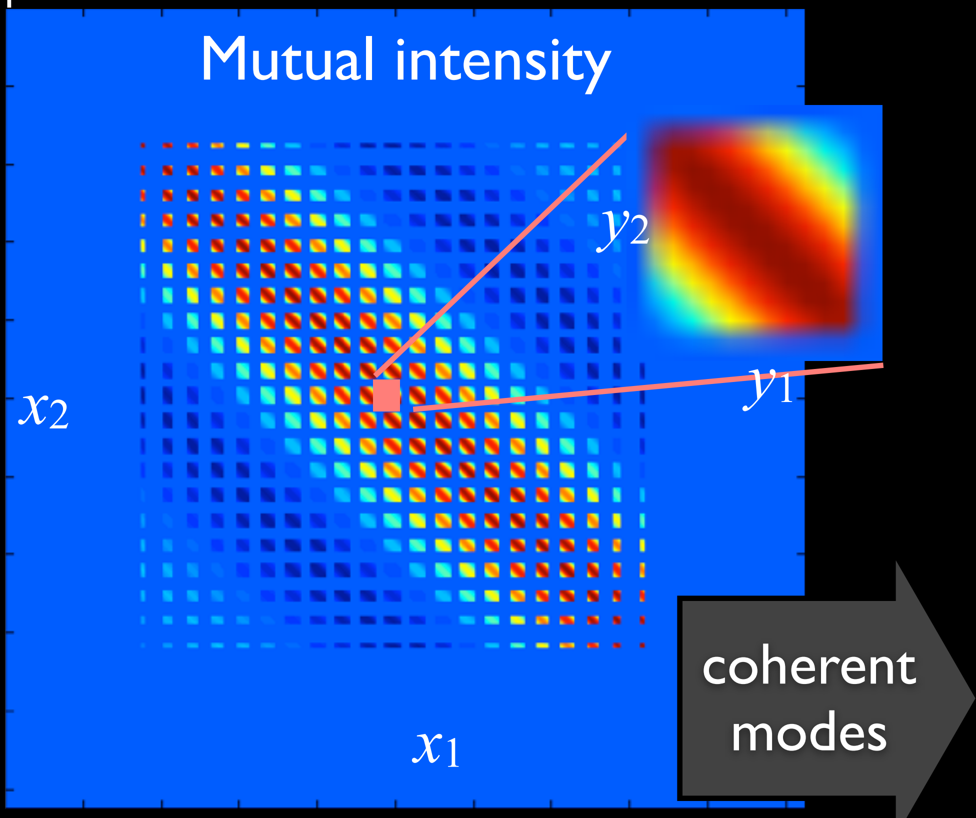


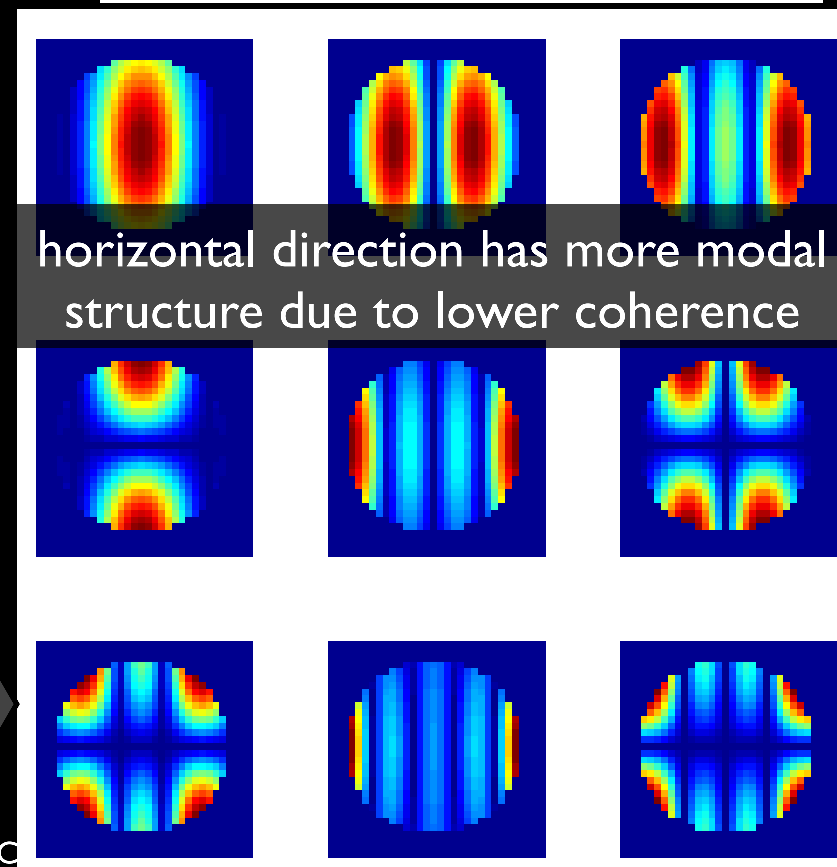
- Dimension of unknown mutual intensity: 64^4
- Total # of samples: 64^2(# of samples in each image)×20(# of planes in a focal stack)×12(# of focal stack)

Theoretical Prediction



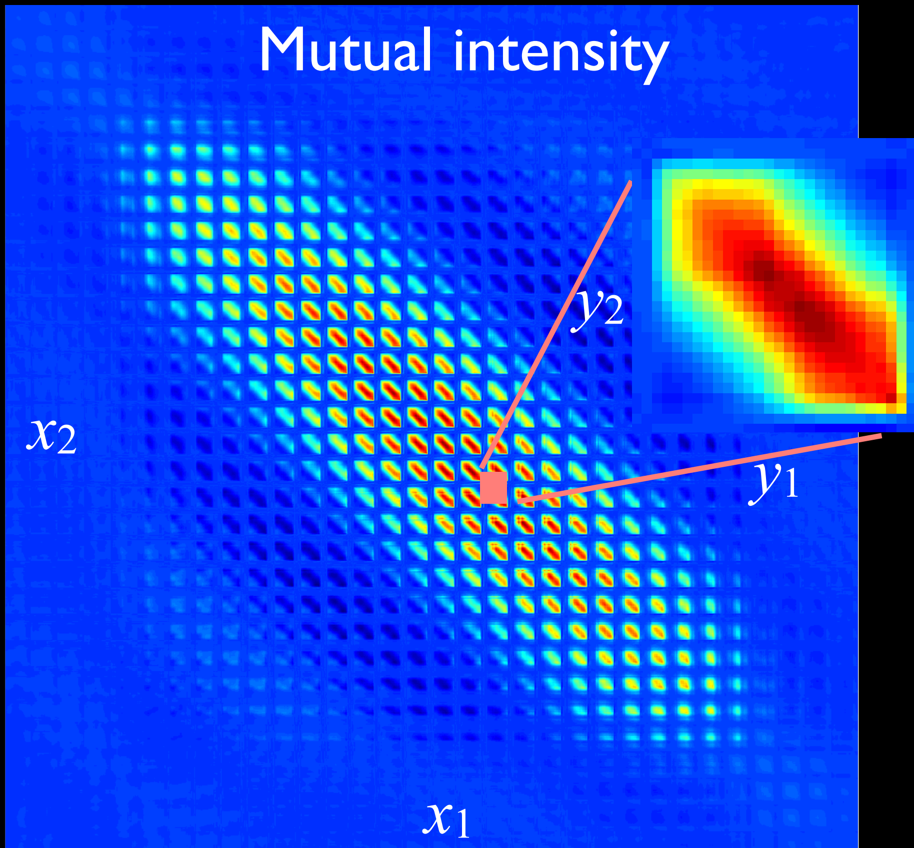


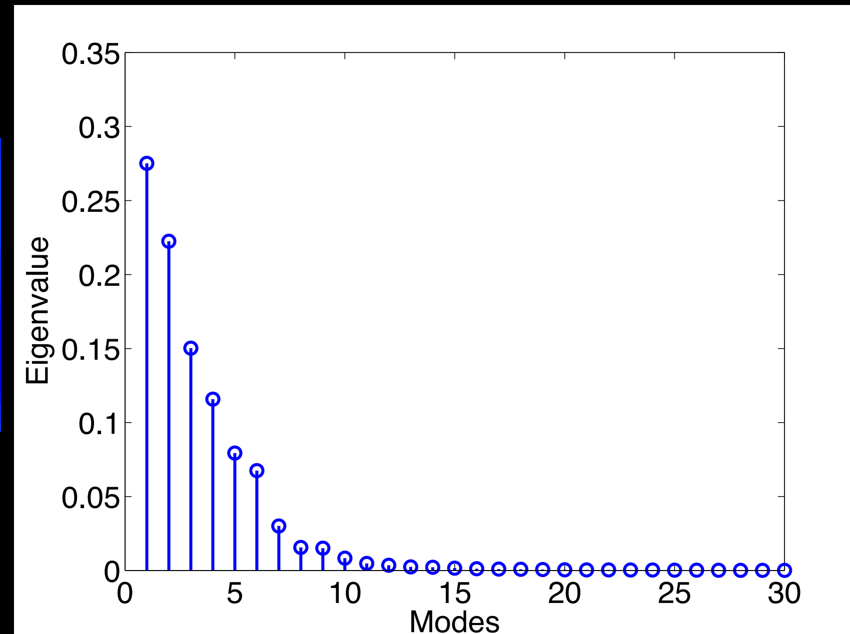


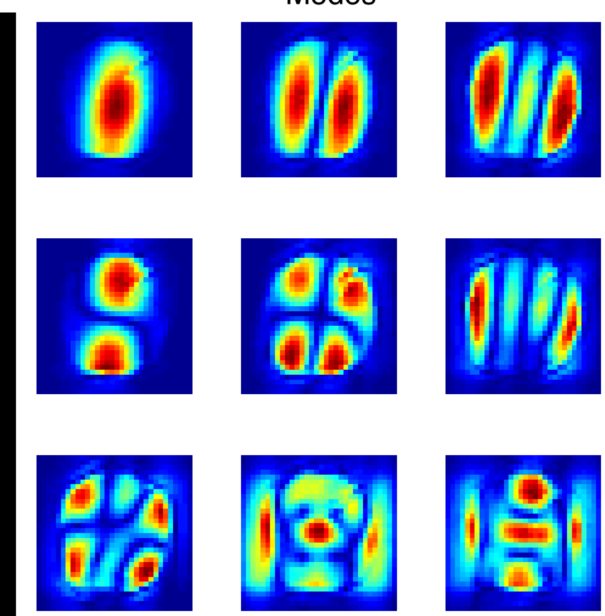


L. Tian, S. Rehman, and G. Barbastathis, in Frontiers in Optics 2012, paper FM4C

Compressive Reconstruction







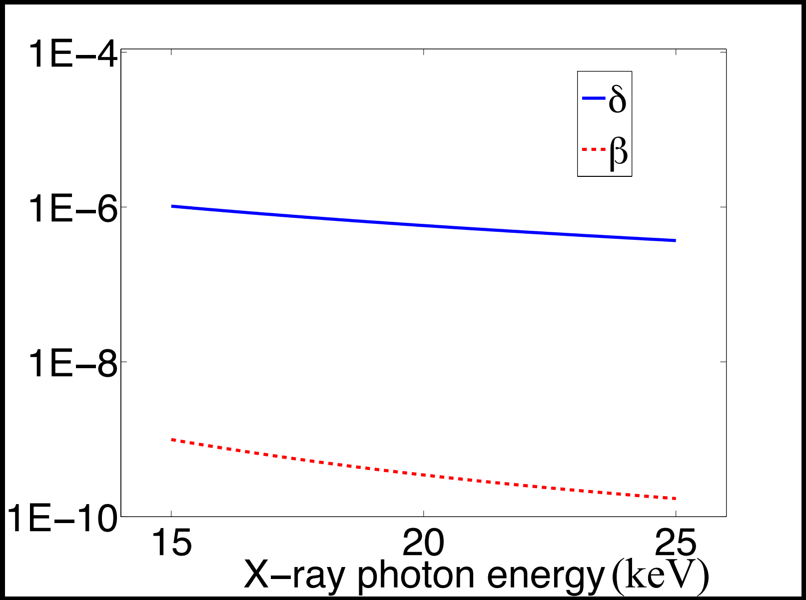
L. Tian, S. Rehman, and G. Barbastathis,
Frontiers in Optics 2012, paper FM4C.4.

This talk is about

- Phase
- Phase space
- Phase space tomography
- Compressive imaging
- Phase tomography

Why X-ray phase imaging?

http://www.presstv.ir/detail/206921.html
"X-ray not good for lung cancer screening"



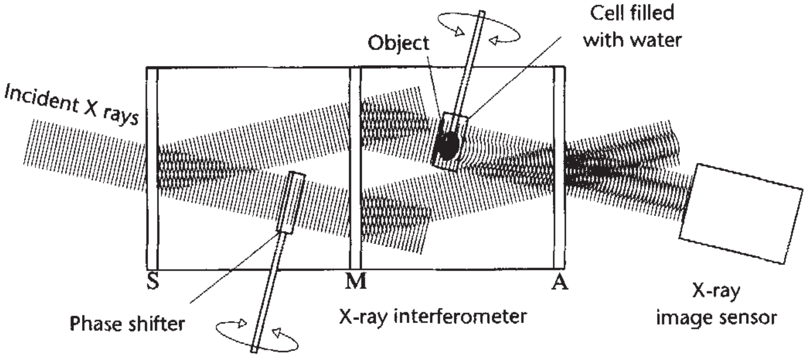
refractive index of water (n=1-δ-iβ)

- X-ray absorption images do NOT provide good contrast for soft tissues
- Call for alternative contrast mechanism
  ☛ X-ray PHASE imaging

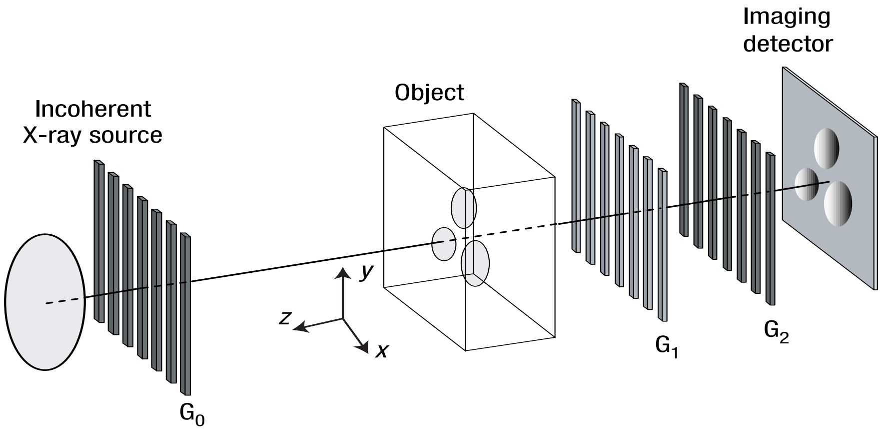
TIE for x-rays?

Coherent?
⇨ synchrotron

http://science.howstuffworks.com/synchrotron.htm

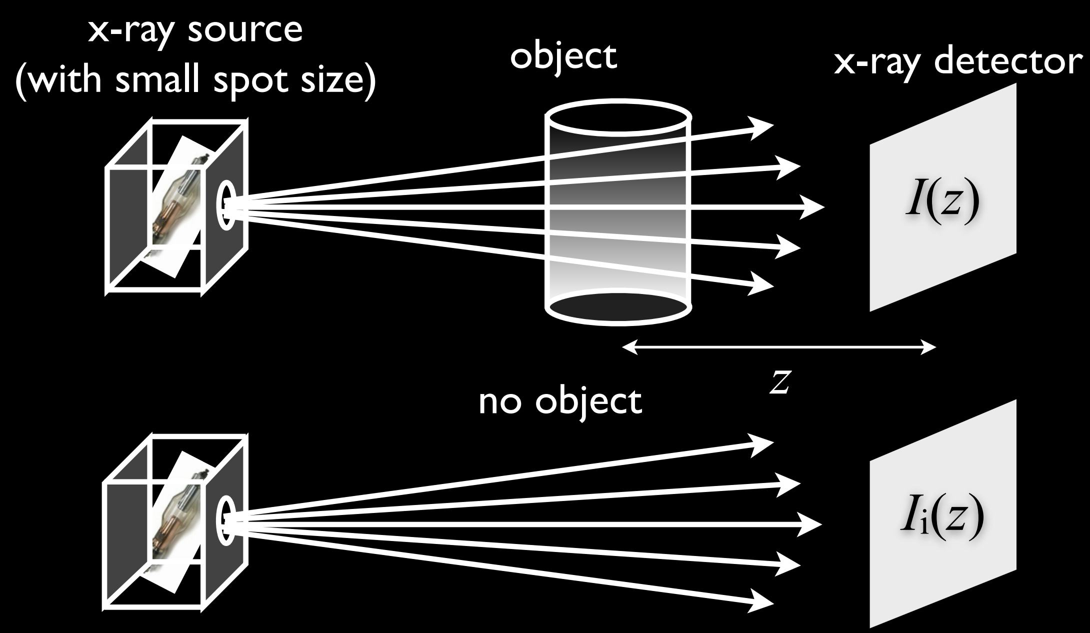


x-ray interferometer (Nat. Med. 2, 473-475)
extremely sensitive to mech. stability & alignment



Talbot interferometer (Nat. Phys. 2, 256-261)
requires 3 gratings & complicated measurement

Transport of Intensity (TIE)

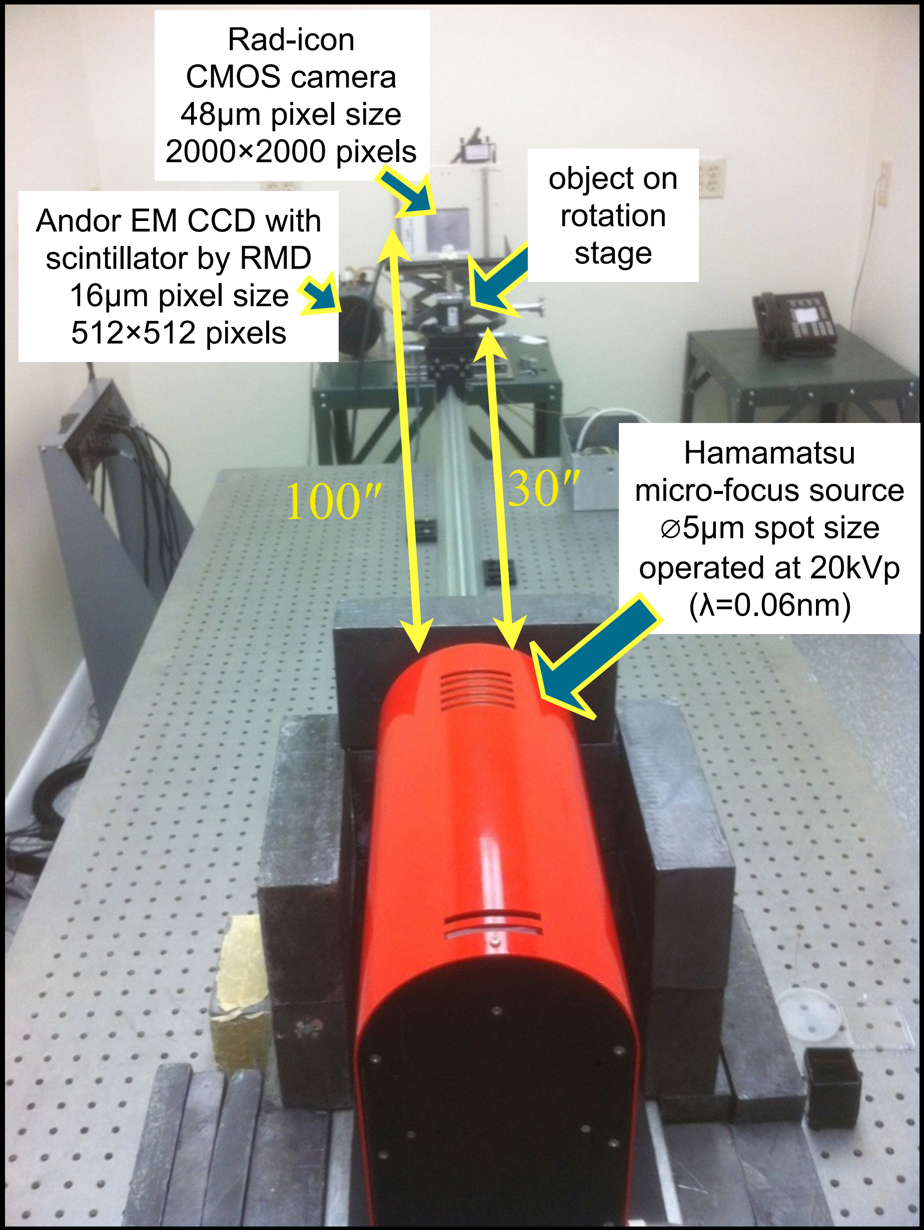


$$-k\frac{\partial I}{\partial z} = \nabla_\mathbf{x} \cdot (I \nabla_\mathbf{x} \phi)$$

$$\stackrel{I \approx 1}{\Longrightarrow} \quad -\frac{k}{z}\left(\frac{I(z)}{I_\mathrm{i}(z)} - 1\right) \approx \nabla_\mathbf{x}^2 \phi$$

- Images corrected for lateral magnification
- Attenuation neglected
- Long propagation distance (short wavelength)
  ➡ two instead of four measurements

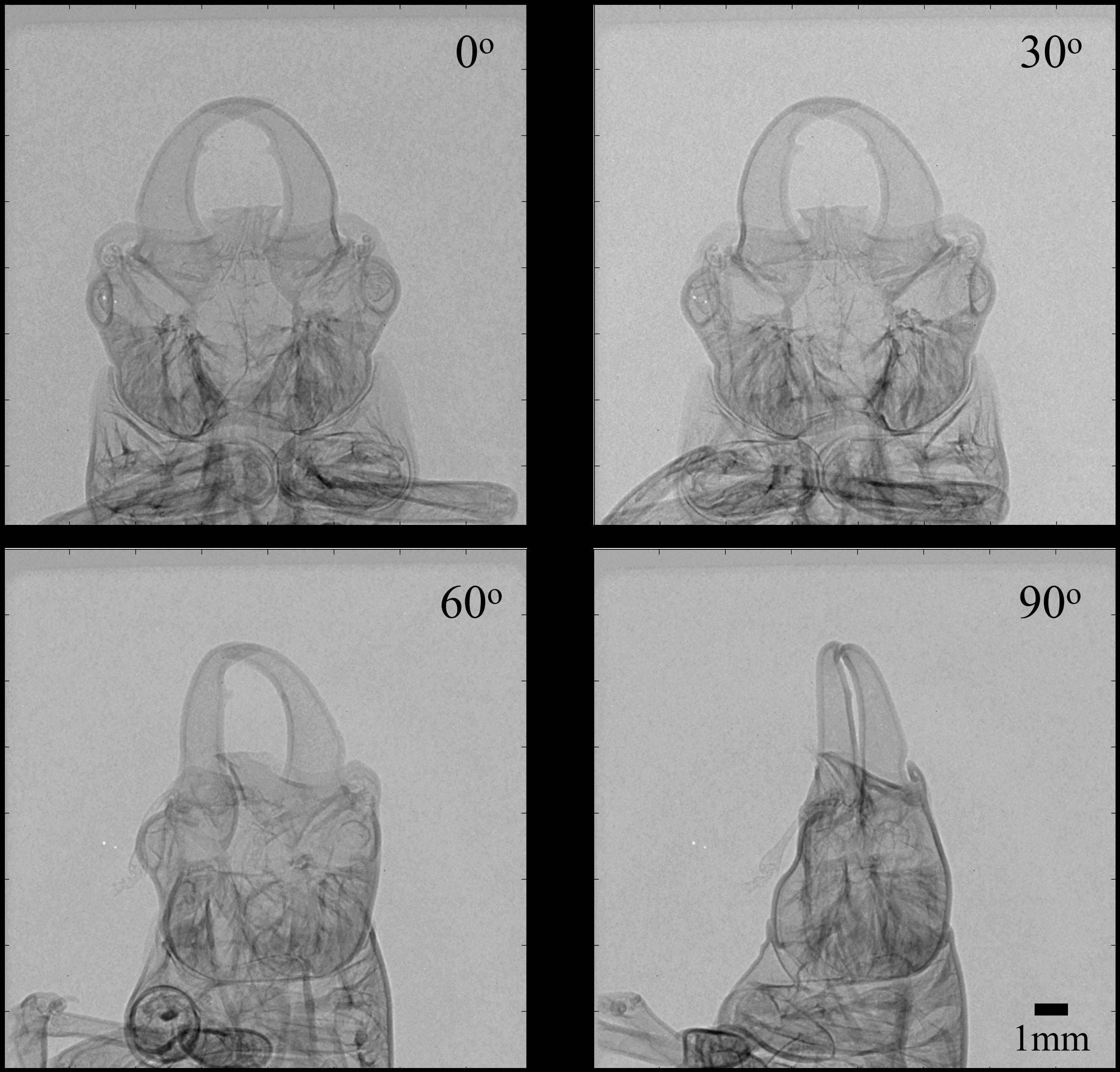
Experimental geometry



- Experimental arrangement at RMD, Inc., Watertown, Mass.
- Three experiments:
  - Polystyrene spheres on flat tape (Andor camera)
  - Polystyrene spheres taped onto drinking straw (Andor camera)
  - Beetle (Andor camera)

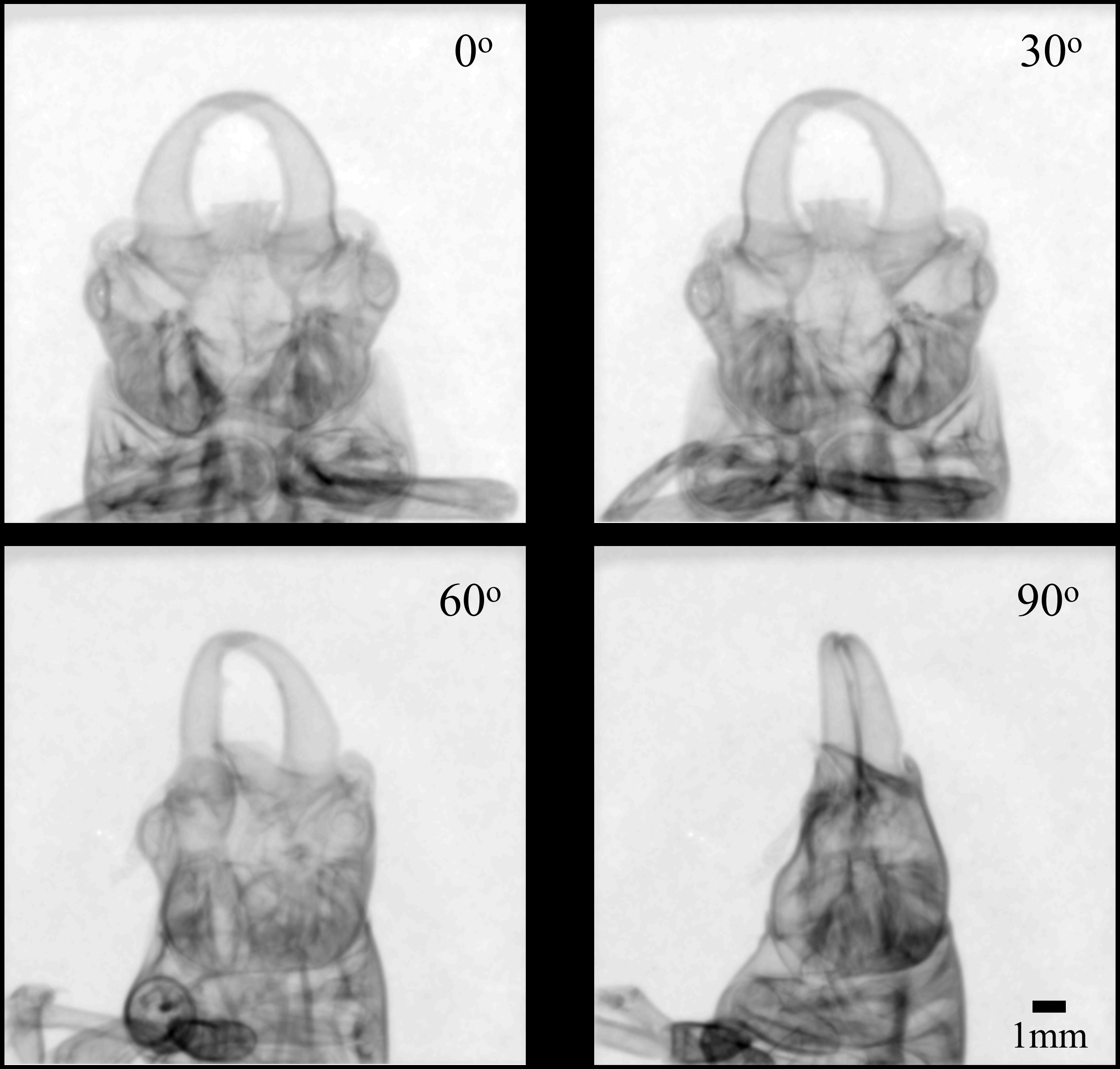
X-ray phase tomography: beetle

intensity measurements

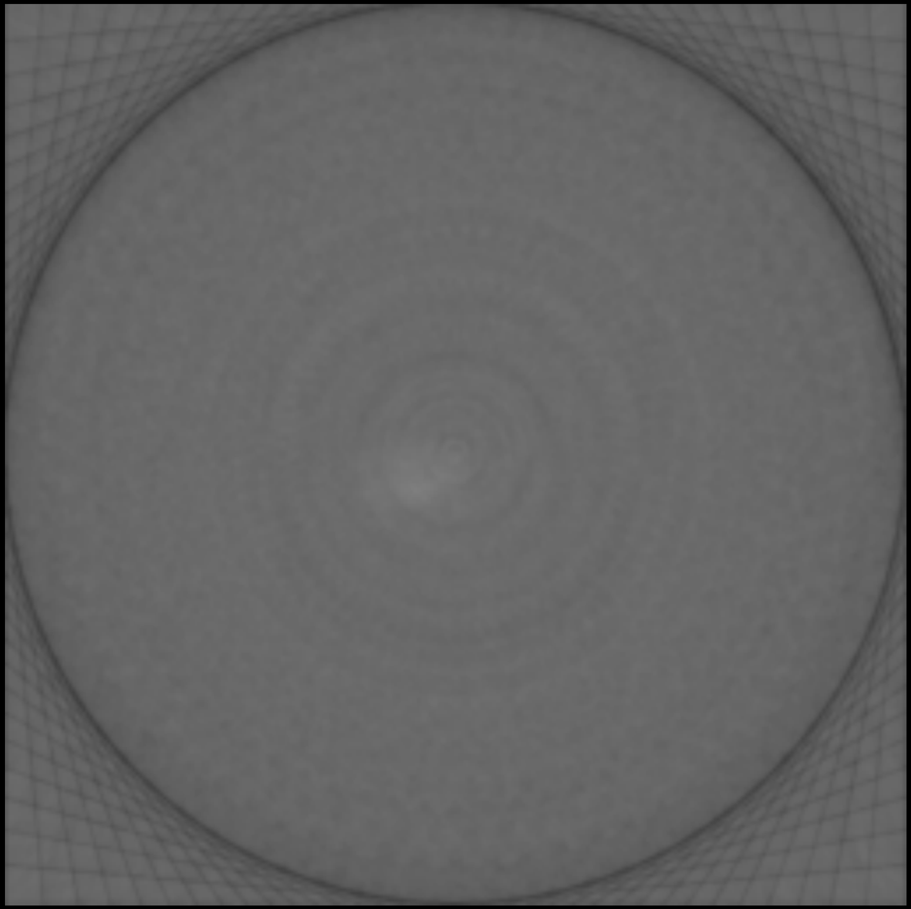


Phase projections

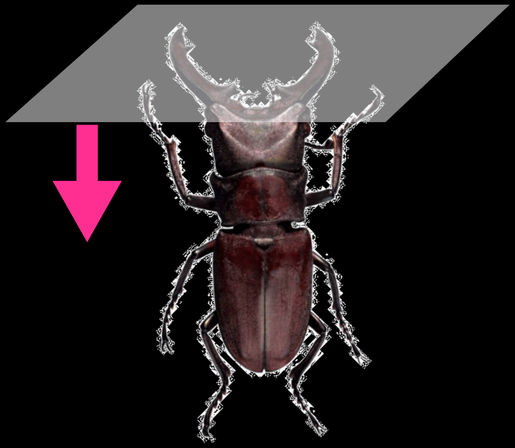
Phase projections obtained by inverting the intensity images using TIE at each angle



Filtered backprojection: reconstructed cross-sections

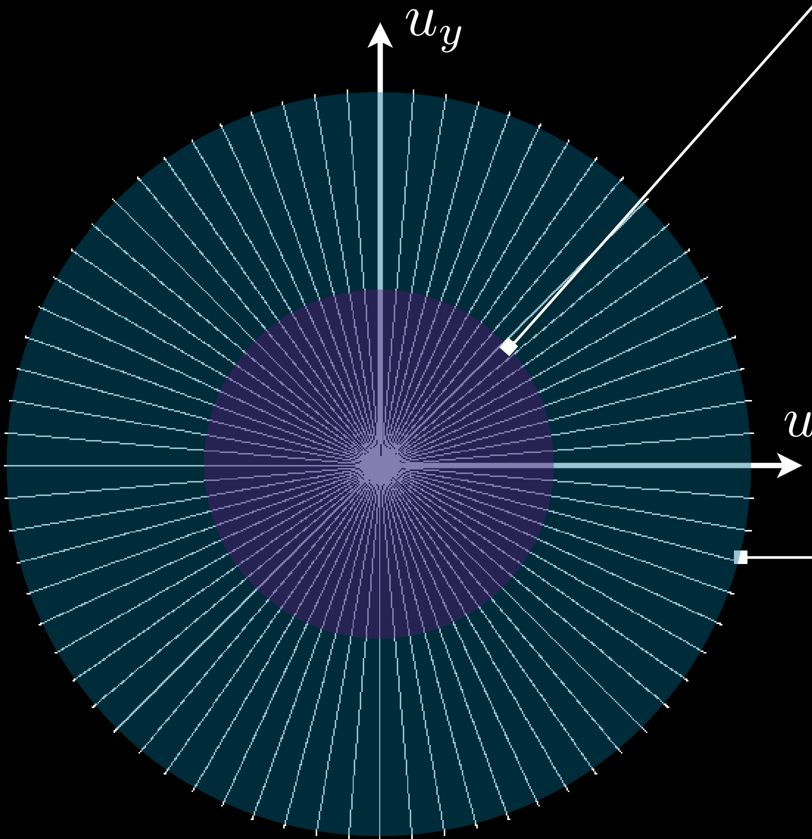


Total projections: 72 (every 5 degrees)

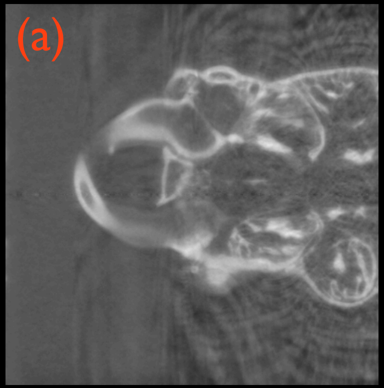


“phase CAT scan”

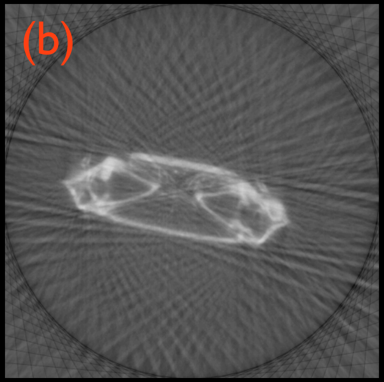
TIE tomography in the Fourier domain

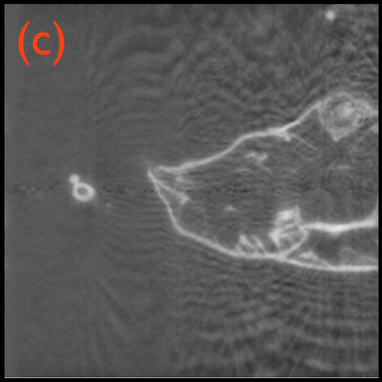


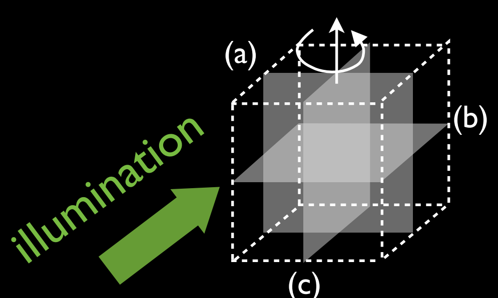
- Low frequencies
  - TIE transfer function ➡ cloud-like artifacts



- High frequencies:
  - missing Fourier slices ➡ streaking
  - finite source size ➡ blurring







Compressive reconstruction: total variation

$$\text{minimize } \frac{1}{2}\left\|g - \mathcal{F}^{-1} H_{\text{TIE}} H_{\text{proj}} \mathcal{F} n\right\|^2 + \tau \boxed{\|n\|_{\text{TV}}}$$

data fitting term

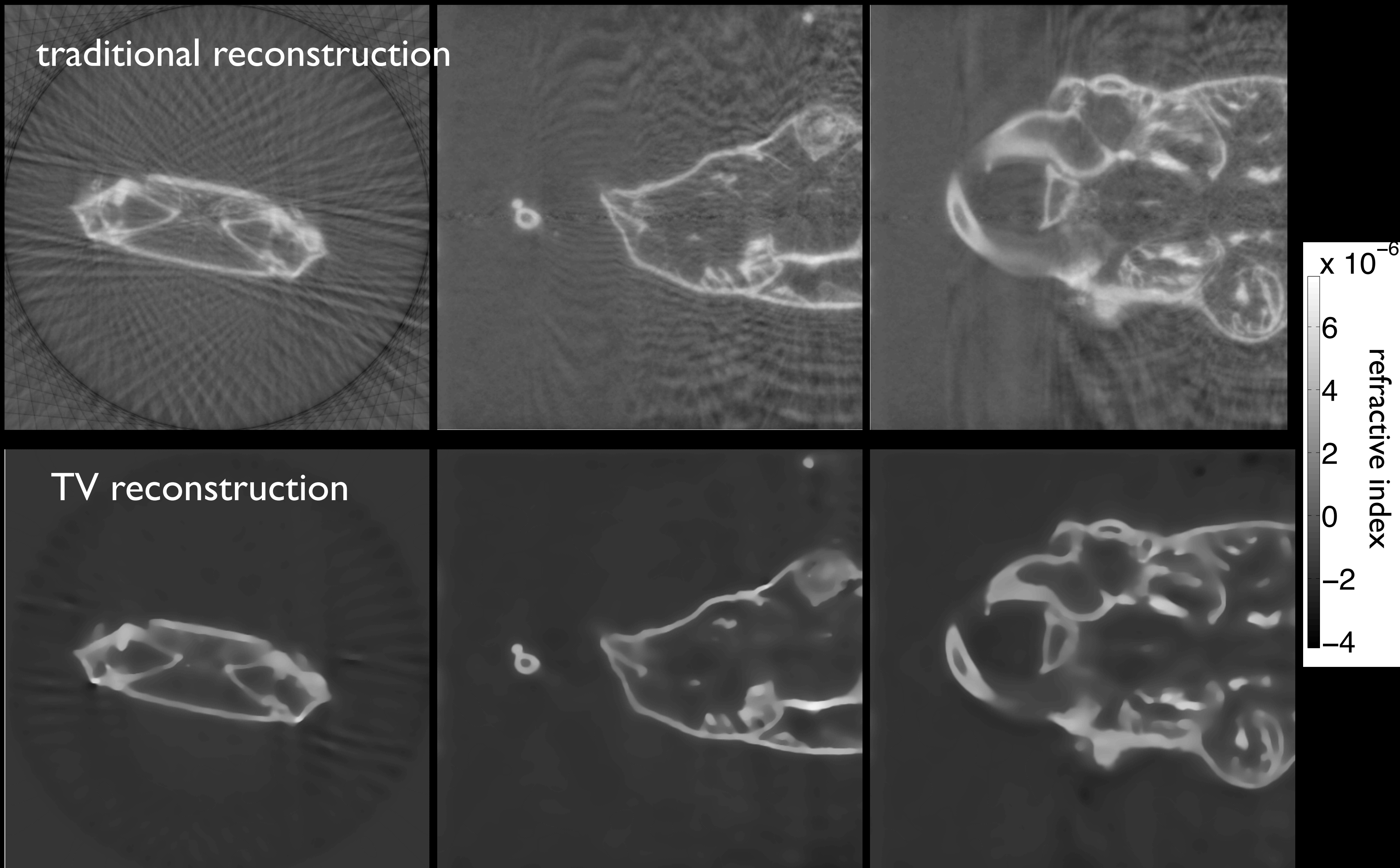
sparsity constraint

Total variation (TV) function:

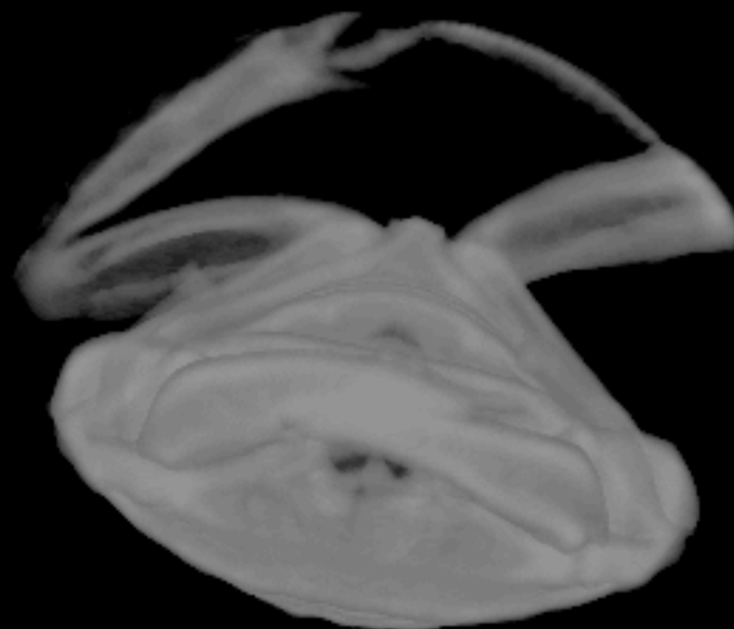
$$\|n\|_{\text{TV}} = \sum \sqrt{(\nabla_x n)^2 + (\nabla_y n)^2 + (\nabla_z n)^2}$$

- project the solution onto the *gradient* basis with *few* nonzero coefficients (which represents sharp boundaries)
- look for *piecewise constant* refractive index distribution

Compressive x-ray phase tomography



TV reconstruction



This talk was about

- Phase
  - Axial stacks with partially coherent illumination [Petruccelli *Opt. Exp.* 21:14430]
  - TIE Phase from chromatic dispersion [Waller *Opt. Exp.* 18:2287]
  - TIE Kalman filter [Waller *Opt. Exp.* 19:2805]
- Phase space
  - Wigner distribution function from lenslet arrays [Tian *Opt. Exp.* 21:10511]
  - Phase from lens let arrays [Zhang *FOM 2014*, Sydney, Australia]
- Phase space tomography
  - Compressive reconstruction of the mutual intensity [Tian *Opt. Exp.* 20:8296]
  - Factored Form Descent [Zhang *Opt. Exp.* 21:5756]
- Compressive phase tomography
  - TIE Compressive x-ray tomography [Tian *Opt. Lett.* 38:3418]
  - Nonlinear diffusion [Tian *Opt. Lett.* 37:4131]