How a wave packet propagates at a speed faster than the speed of light

A novel superluminal mechanism with high transmission and broad bandwidth

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Claim: The phenomena we present here do not violate the special relativity, which is a cornerstone of the modern understanding of physics for more than a century.

Outline

- Introduction (evanescent wave)
- Matter wave and electromagnetic wave
- Modal analysis (a 3D effect)
- New superluminal mechanism (propagating wave)
- Manipulating the group delay
- Conclusions
- Acknowledgement

The Fastest Person

Usain Bolt is a Jamaican sprinter widely regarded as the fastest person ever. 100 m in 9.58 s, Speed ~ 10 m/s

Top Speed of Racing Car: Formula 1

The 2005 BAR-Honda set an unofficial speed record of 413 km/h at Bonneville Speedway. Speed ~ 115 m/s

Flight Airspeed Record: SR-71 Blackbird

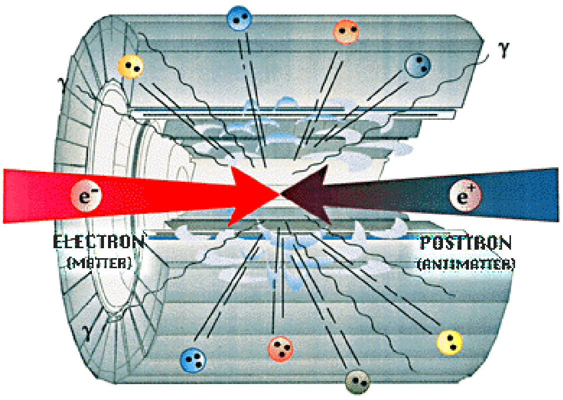
The SR-71 Blackbird is the current record-holder for a manned air breathing jet aircraft. 3530 km/h ~ 980 m/s

Controlled Flight Airspeed Record: Space Shuttle

Fastest manually controlled flight in atmosphere during atmospheric reentry of STS-2 mission is 28000 km/h ~ 7777 m/s.

Highest Particle Speed: LEP Collider

The Large Electron–Positron Collider (LEP) is one of the largest particle accelerators ever constructed. The LEP collider energy eventually topped at 209 GeV with a Lorentz factor γ over 200,000. LEP still holds the particle accelerator speed record.



$$\beta = \frac{v}{c} = \left(1 - \frac{1}{\gamma^2}\right)^{\frac{1}{2}} = 0.\underbrace{9999999999}_{10}88$$

just millimeters per second slower than *c*.

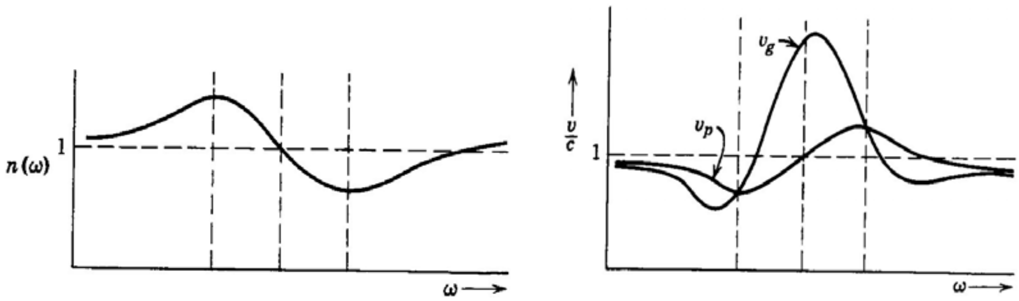
$$E = \frac{m_0}{\sqrt{1-\beta^2}} c^2$$

Matter cannot exceed the speed of light in vacuum.

How about wave?

Superluminal Mechanism: Anomalous dispersion

The index of refraction $n(\omega)$ is a function of frequency. $n(k) = \dfrac{ck}{\omega(k)}$



Phase velocity: $v_p \equiv \dfrac{\omega(k)}{k} = \dfrac{c}{n(k)}$ (7.88)

Group velocity: $v_g \equiv \dfrac{d\omega}{dk} = \dfrac{c}{n(\omega) + \omega(dn/d\omega)}$ (7.89)

Group delay: $\tau_g \equiv \dfrac{d\phi}{d\omega} \approx \dfrac{d(kL)}{d\omega} = \dfrac{L}{v_g}$

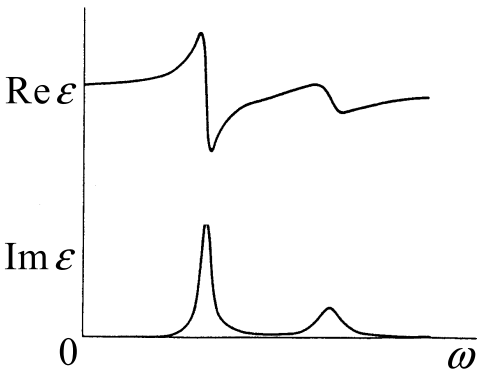
See waves in a dielectric medium [**Jackson Chap. 7**]

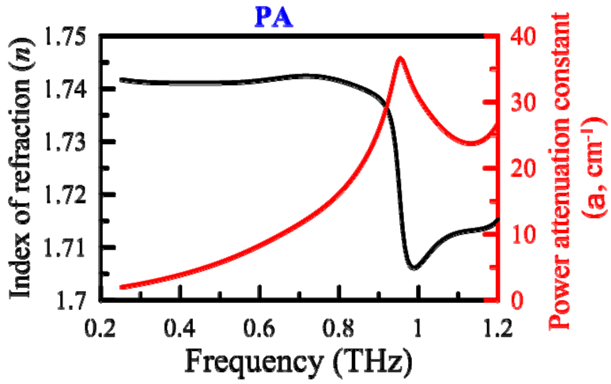
Anomalous Dispersion: Waves in a dielectric medium

$$\varepsilon=\varepsilon_0+\frac{Ne^2}{m}\sum_{j\ \text{(bound)}}\frac{f_j}{\omega_j^2-\omega^2-i\omega\gamma_j}+i\underbrace{\cancel{\frac{Ne^2 f_0}{m\omega(\gamma_0-i\omega)}}}_{\text{negligible }(\because\ f_0\,=\,0\text{ or very small})}\qquad(7.51)$$

Properties of ε:

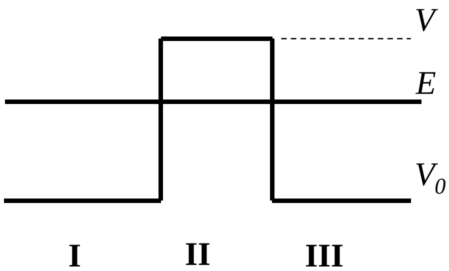
When ω is near each ω_j (binding frequency of the j^{th} group of electrons), ε exhibits resonant behavior in the form of anomalous dispersion and resonant absorption.





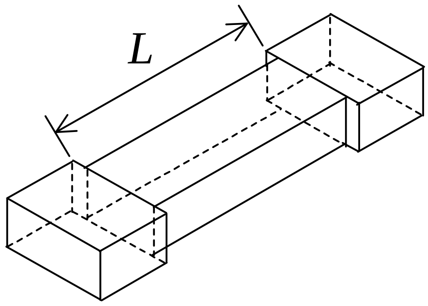
PA: Polyamides are semi-crystalline polymers.
The data was measured with a THz-TDS system.

The tunneling effect



$$v=\sqrt{\frac{2(E-V)}{m}}=?$$

The microwave propagating in a waveguide system seems to be analogous to the behavior of a one-dimensional matter wave.



Comparing with the matter wave, the electromagnetic wave is much more easier to implement in experiment.

Summary #1

- Anomalous dispersion and tunneling effect are the two major mechanisms for the superluminal phenomena.
- Both mechanisms involve evanescent waves, which means waves cannot propagate inside the region of interest.

Part II. Analogies Between Schrödinger's Equation and Maxwell's Equation

Analogies Between Schrodinger and Maxwell Equations

Time-independent Schrodinger's equation

$$[\frac{\partial^2}{\partial z^2} - \frac{2m}{\hbar^2}V(z) + \frac{2m}{\hbar^2}E]\varphi(z) = 0$$

Maxwell's wave equation for a TE waveguide mode

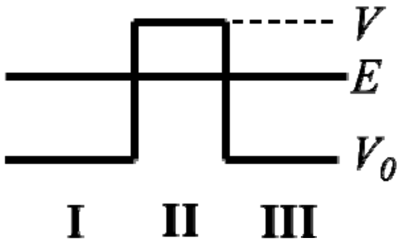
$$(\frac{\partial^2}{\partial z^2} - \mu\varepsilon\frac{\omega_c^2}{c^2}(z) + \mu\varepsilon\frac{\omega^2}{c^2})B_z = 0$$

$$\frac{2m}{\hbar^2}V(z) \longleftrightarrow \mu\varepsilon\frac{\omega_c^2}{c^2}(z)$$

$$\frac{2m}{\hbar^2}E = k_z^2 \longleftrightarrow \mu\varepsilon\frac{\omega^2}{c^2} = k_z^2$$

Anything else?
➔Transmission and reflection coefficients
➔Probability and energy velocities
➔Group and phase velocities

Transmission for a Rectangular Potential Barrier



$$E < V \qquad QM : \frac{1}{T} = 1 + \frac{1}{4}\left(\frac{(V - V_0)^2}{(V-E)(E-V_0)}\right)\sinh^2(2\kappa a), \text{ where } \kappa^2 = \frac{2m(V-E)}{\hbar^2}$$

By analogy, the transmission parameter of an electromagnetic wave can be expressed as

$$\omega < \omega_c \qquad EM : \frac{1}{T} = 1 + \frac{1}{4}\left(\frac{(\omega_c^2 - \omega_{c0}^2)^2}{(\omega_c^2 - \omega^2)(\omega^2 - \omega_{c0}^2)}\right)\sinh^2(2\kappa a), \text{ where } \kappa^2 = \frac{(\omega_c^2 - \omega^2)}{c^2}$$

Analogies Between Probability and Energy Velocities

Quantum Mechanics:
Probability velocity

$$v_{prob} = \frac{J_x}{|\psi|^2}$$

Electromagnetism:
Energy Velocity

$$v_E = \frac{P}{U}$$

$$P = \int_A (\hat{\mathbf{e}}_z \cdot \vec{S}) da$$

$$U = \frac{1}{16\pi}\int_A (\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H}) da$$

$$\sqrt{\frac{2(V-E)}{m}} \frac{2\,\mathrm{Im}(\Gamma^*)}{[e^{2\kappa x} + |\Gamma| e^{-2\kappa x} + 2\,\mathrm{Re}(\Gamma)]} \longleftrightarrow \frac{c}{\sqrt{\mu\varepsilon}} \sqrt{\frac{\omega_c^2}{\omega^2} - 1} \frac{2\,\mathrm{Im}(\Gamma^*)}{(e^{2\kappa z} + |\Gamma|^2 e^{-2\kappa z}) + 2\,\mathrm{Re}(\Gamma)}$$

$\boxed{E < V}$ $\boxed{\omega < \omega_c}$

Can we use EM wave to study a long-standing debate in QM, i.e. the tunneling time?

$\boxed{E < V}$ *QM: Tunneling Time Calculation* $\Delta t = \int_0^{2a} \frac{dx}{v_{prob}}$

$$\Delta t = \sqrt{\frac{m}{2(V-E)}} \frac{1}{2\,\mathrm{Im}(\Gamma^*)} \int_0^{2a} [(e^{2\kappa z} + |\Gamma|^2 e^{-2\kappa z}) + 2\,\mathrm{Re}(\Gamma)] dz$$

$$= \sqrt{\frac{m}{2(V-E)}} \frac{1}{2\,\mathrm{Im}(\Gamma^*)} \left[\frac{1}{2\kappa}((e^{4\kappa a} - 1) - |\Gamma|^2 (e^{-4\kappa a} - 1)) + 4a\,\mathrm{Re}(\Gamma)\right]$$

$\boxed{\omega < \omega_c}$ *EM: Tunneling Time Calculation* $\Delta t = \int_0^{2a} \frac{dx}{v_E}$

$$\Delta t = \sqrt{\frac{\mu\varepsilon\omega^2}{\omega_c^2 - \omega^2}} \frac{1}{2c\,\mathrm{Im}(\Gamma^*)} \int_0^{2a} [(e^{2\kappa z} + |\Gamma|^2 e^{-2\kappa z}) + 2\,\mathrm{Re}(\Gamma)] dz$$

$$= \sqrt{\frac{\mu\varepsilon\omega^2}{\omega_c^2 - \omega^2}} \frac{1}{2c\,\mathrm{Im}(\Gamma^*)} \left[\frac{1}{2\kappa}((e^{4\kappa a} - 1) - |\Gamma|^2 (e^{-4\kappa a} - 1)) + 4a\,\mathrm{Re}(\Gamma)\right]$$

Summary #2

- Superluminal effect is common to many wave phenomena.
- The matter wave and the electromagnetic wave share many common characteristics.

The moment of truth:

Put the idea to the test in a 3D-EM system.

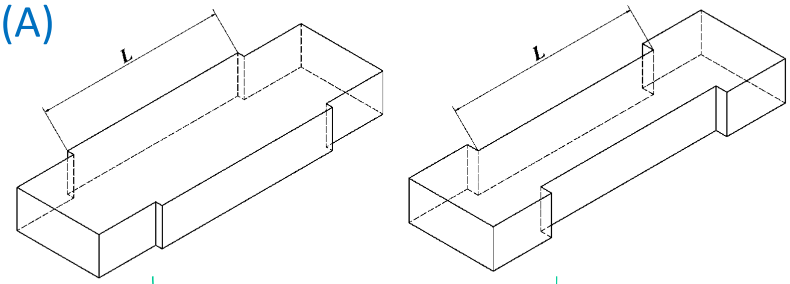
Part III. Modal Analysis:

Effect of high-order modes on tunneling characteristics

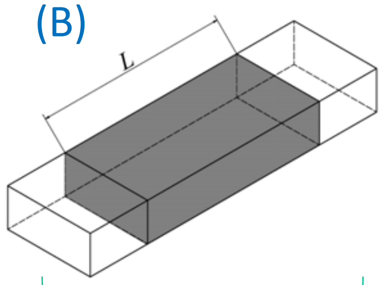
H. Y. Yao and T. H. Chang**,** **"**Effect of high-order modes on tunneling characteristics"**,** Progress In Electromagnetics Research, PIER, **101**, 291-306, 2010.

Geometric and material discontinuities

(A)



(B)



For TE_{10} mode $\quad \mu_r = \varepsilon_r = 1$ for all regions

$$k_1 = \frac{1}{c}\sqrt{\omega^2 - \omega_c^{a\,2}},\ \omega_c^a = \frac{c\pi}{a}$$

$$k_2 = \frac{1}{c}\sqrt{\omega^2 - \omega_c^{c\,2}},\ \omega_c^c = \frac{c\pi}{c}$$

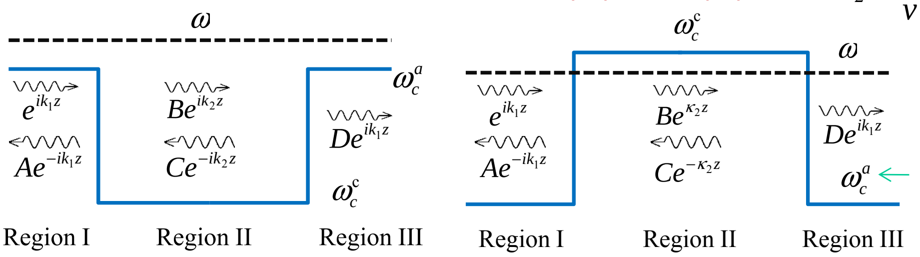
$\mu_r = 1;\ \varepsilon_r = 1$ for I and III

$$k_1 = \frac{1}{c}\sqrt{\omega^2 - \omega_c^{a\,2}},\ \omega_c^a = \frac{c\pi}{a}$$

$\mu_r = 1;\ \varepsilon_r \neq 1$ for I and III

$$k_2 = \frac{1}{v}\sqrt{\omega^2 - \left(\frac{\omega_c^a}{\sqrt{\varepsilon_r}}\right)^2}$$

What is the difference between (A) and (B)?



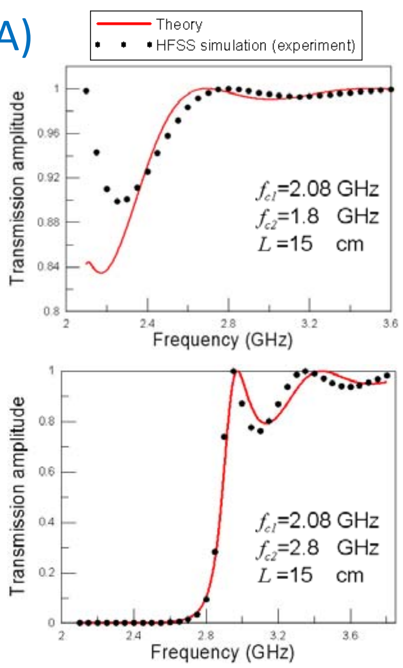
Reduce to 1-D case
Potential-like diagram

Transmission amplitude for two systems

$$D = \frac{2k_1k_2e^{-ik_1L}}{2k_1k_2\cos(k_2L) - i(k_1^{\,2} + k_2^{\,2})\sin(k_2L)} \qquad T \equiv D \times D^* \rightarrow$$

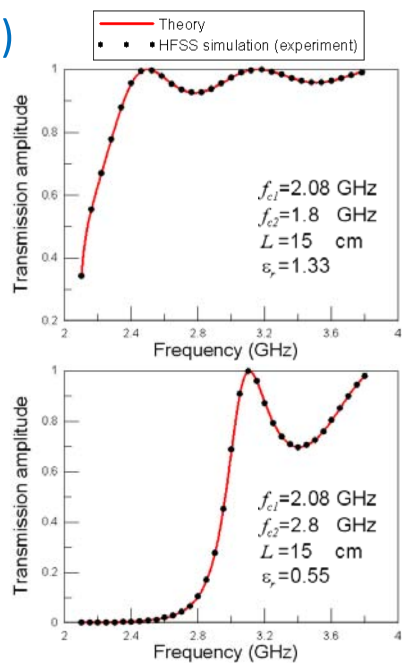
Transmission amplitude

(A)



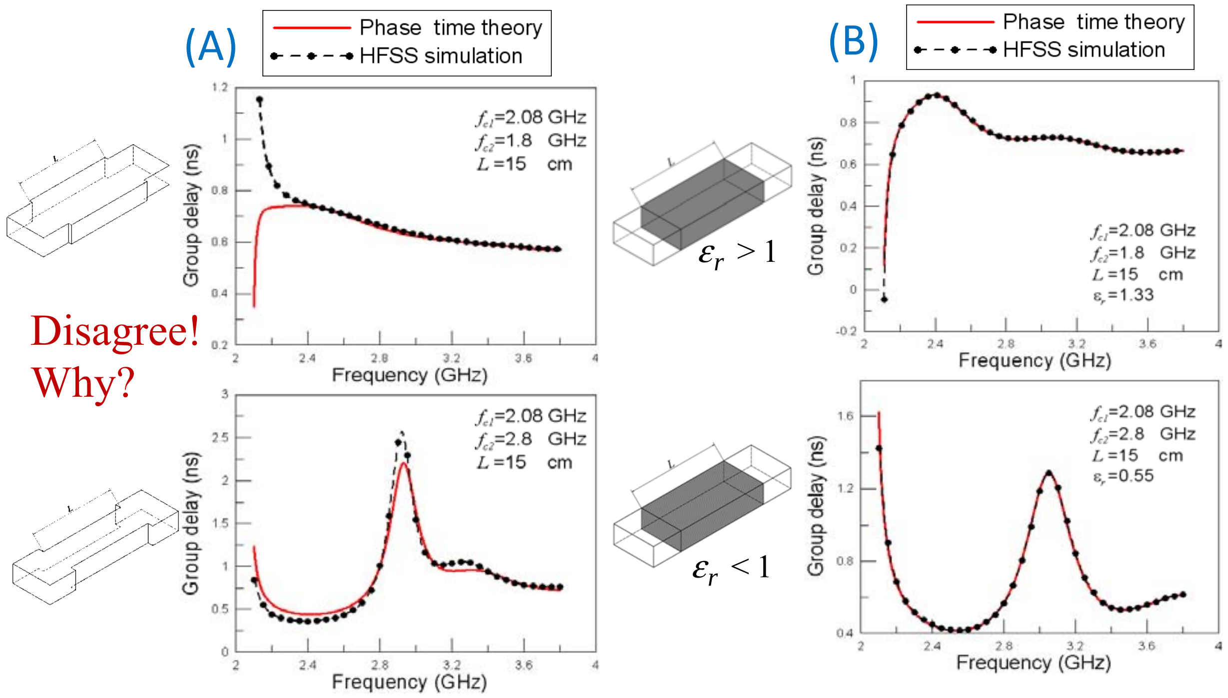
Disagree!
Why?

(B)

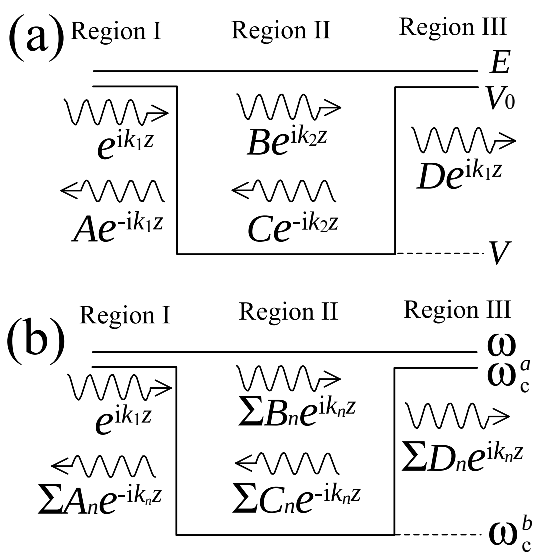


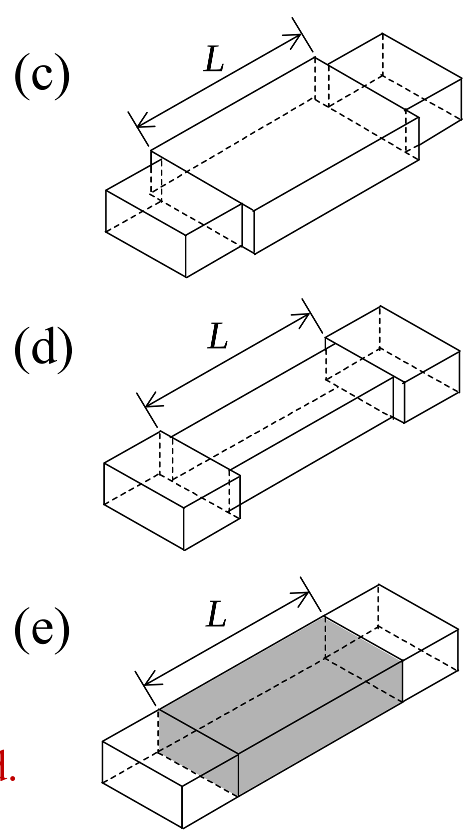
Group delay for two systems

$$\tau_g = \frac{L}{v_g} = \frac{d\delta\phi}{d\omega} \qquad \delta\phi = \tan^{-1}\left[\frac{(k_1^{\ 2}+k_2^{\ 2})\tan(k_2 L)}{2k_1 k_2}\right]$$



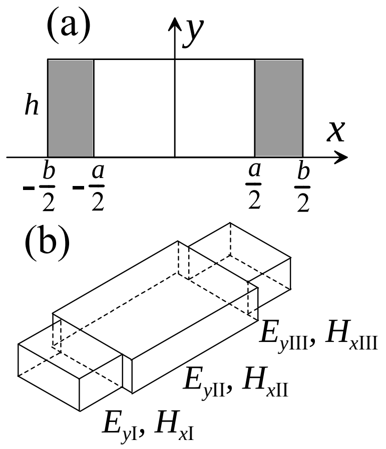
Modal Effect





It is a 3-D problem.
Modal effect should be considered.

Complete wave functions and boundary conditions



1. $E_{yI}\big|_{z=0} = E_{yII}\big|_{z=0}$
2. $B_{xI}\big|_{z=0} = B_{xII}\big|_{z=0}$
3. $E_{yII}\big|_{z=L} = E_{yIII}\big|_{z=L}$
4. $B_{yII}\big|_{z=L} = B_{yIII}\big|_{z=L}$

$\boxed{0 \le x \le a}$

1. $E_{yI}\big|_{z=0} = 0$
2. $B_{xI}\big|_{z=0} = 0$
3. $E_{yIII}\big|_{z=L} = 0$
4. $B_{yIII}\big|_{z=L} = 0$

$\boxed{\begin{array}{l} \frac{a-c}{2} \le x < 0 \\ a < x \le \frac{a+c}{2} \end{array}}$

Region I
$$\begin{cases} E_{yI} = \sin\left(\frac{\pi x}{a}\right) e^{i(k_1 z - \omega t)} + \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{a}\right) e^{-i(k_n^a z + \omega t)} \\ B_{xI} = -k_1^a \sin\left(\frac{\pi x}{a}\right) e^{i(k_1^a z - \omega t)} + \sum_{n=1}^{\infty} A_n k_n^a \sin\left(\frac{n\pi x}{a}\right) e^{-i(k_n^a z + \omega t)} \end{cases}$$

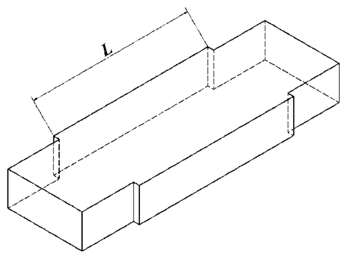
Region II
$$\begin{cases} E_{yII} = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x'}{c}\right) e^{i(k_n^c z - \omega t)} + \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi x'}{c}\right) e^{-i(k_n^c z + \omega t)} \\ B_{xII} = -\sum_{n=1}^{\infty} k_n^c B_n \sin\left(\frac{n\pi x'}{c}\right) e^{i(k_n^c z - \omega t)} + \sum_{n=1}^{\infty} k_n^c C_n \sin\left(\frac{n\pi x'}{c}\right) e^{-i(k_n^c z + \omega t)} \end{cases}$$

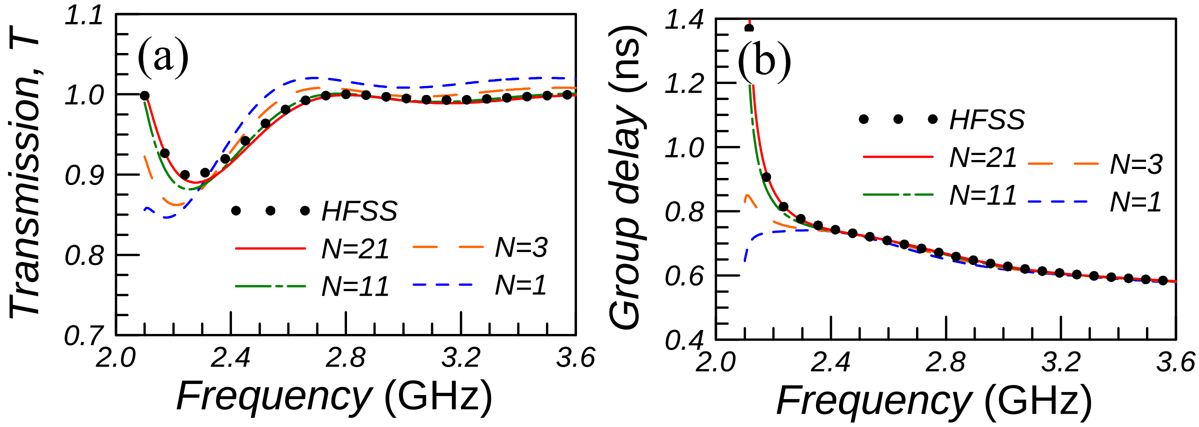
Region III
$$\begin{cases} E_{yIII} = \sum_{n=1}^{\infty} D_n \sin\left(\frac{n\pi x}{a}\right) e^{i(k_n^a z - \omega t)} \\ B_{xIII} = \underbrace{\boxed{-\sum_{n=1}^{\infty} k_n^a D_n \sin\left(\frac{n\pi x}{a}\right) e^{i(k_n^a z - \omega t)}}}_{-k_1^a D_1 \sin\left(\frac{\pi x}{a}\right) e^{i(k_1^a z - \omega t)} - i\sum_{n=2}^{\infty} \kappa_n^a D_n \sin\left(\frac{\pi x}{a}\right) e^{-k_n^a z + i\omega t}} \end{cases}$$

$$k_n^a = \frac{1}{c}\sqrt{\omega^2 - \omega_{cn}^{a\,2}} \qquad \omega_{cn}^a = \frac{n\pi c}{a}$$
$$k_n^c = \frac{1}{c}\sqrt{\omega^2 - \omega_{cn}^{c\,2}} \qquad \omega_{cn}^c = \frac{n\pi c}{c}$$

Modal Effect Corrects the Problems **(I)**

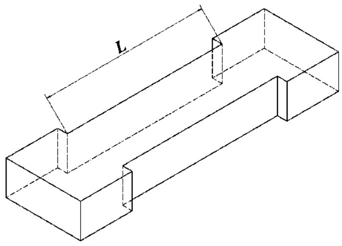
Potential well

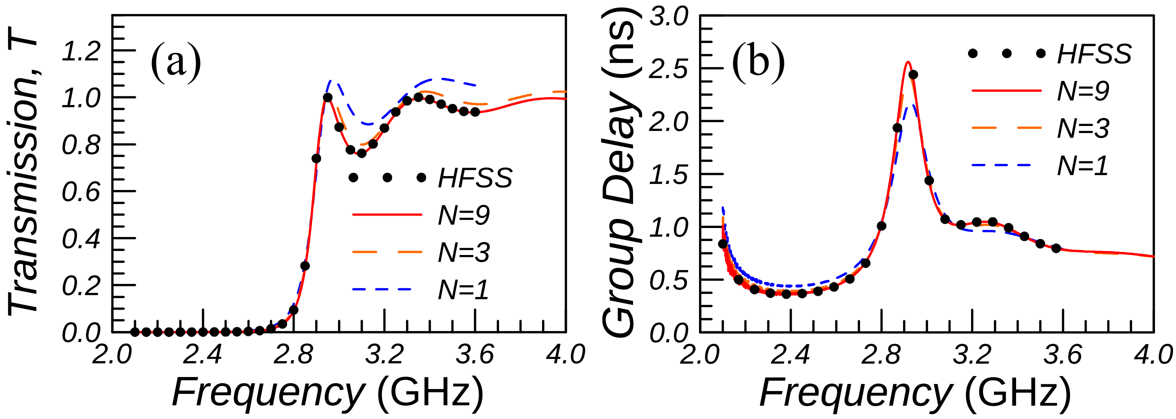




Modal Effect Corrects the Problems (II)

Potential barrier





Summary #3

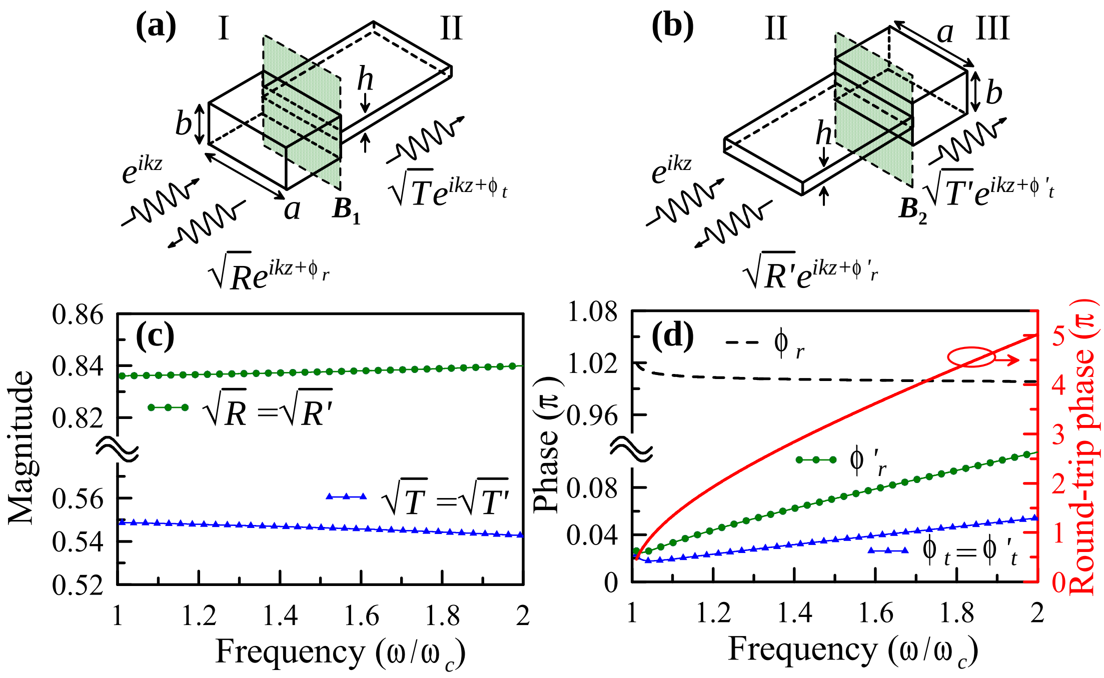
- Model effect plays an important role for a 3D discontinuity.
- To achieve a better agreement between the theory and experiment in a quantum tunneling system, the model effect should be considered.

Part IV. Superluminal Effect: Theoretical and Experimental Studies

a new mechanism

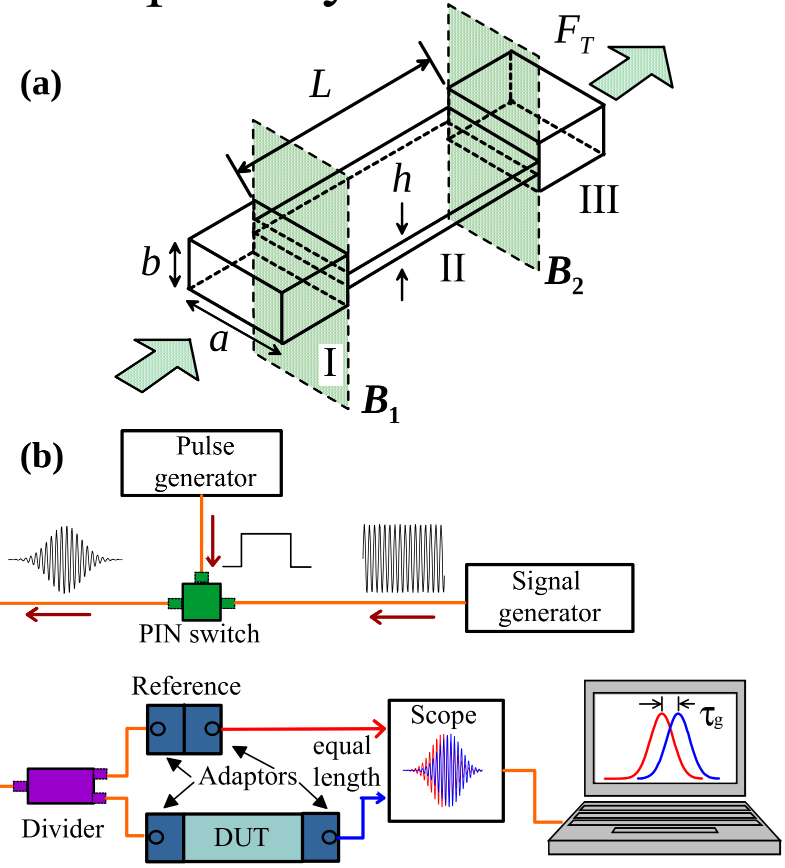
H. Y. Yao and T. H. Chang, *Progress In Electromagnetics Research, PIER* **122,** 1-13 (2012).

Transmitted/Reflected Properties due to Modal Effect



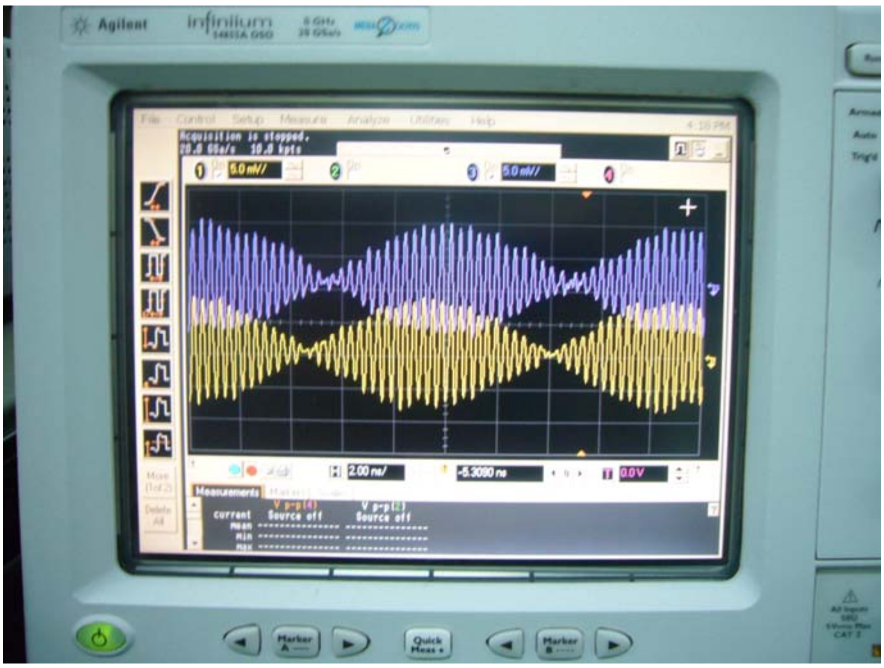
The existence of the higher order modes (evanescent waves) will modify the amplitude and phase of the dominant mode.

Group Delay Measurement

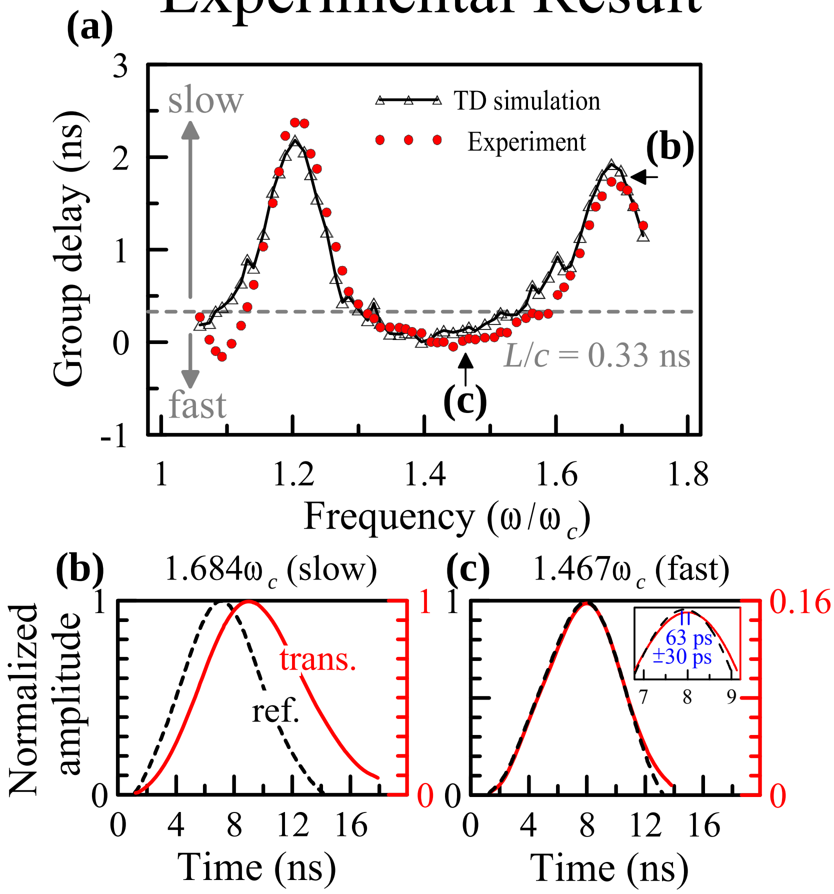


Experiment data and analysis

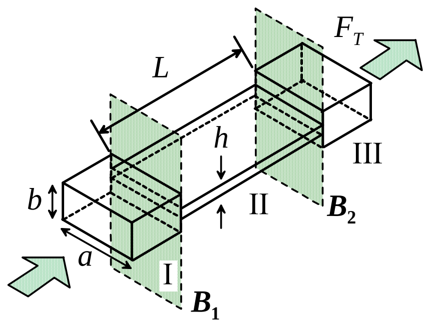
We can get the information from oscilloscope!

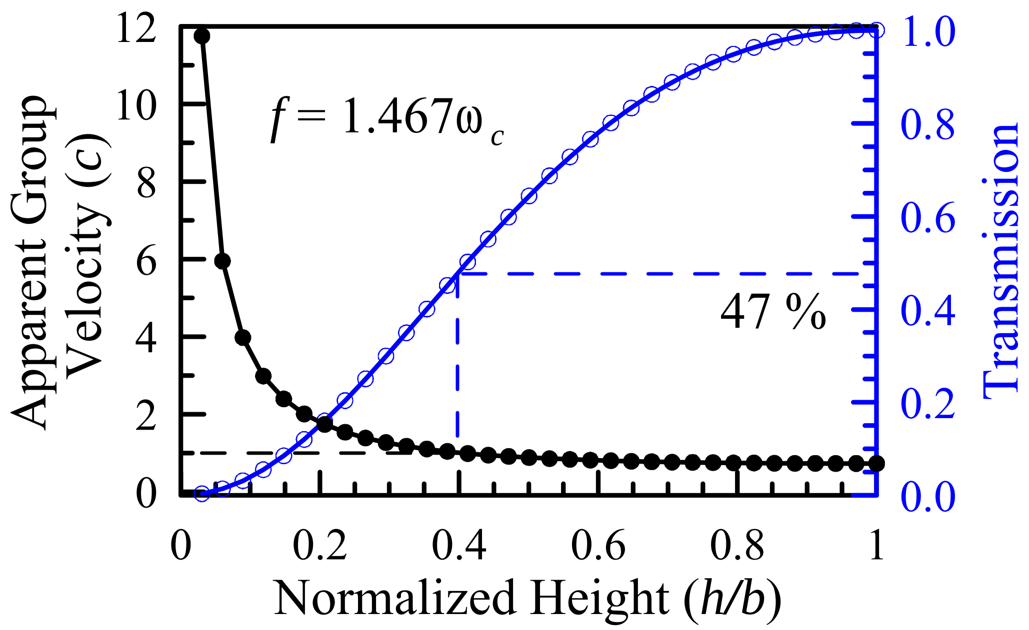


Experimental Result

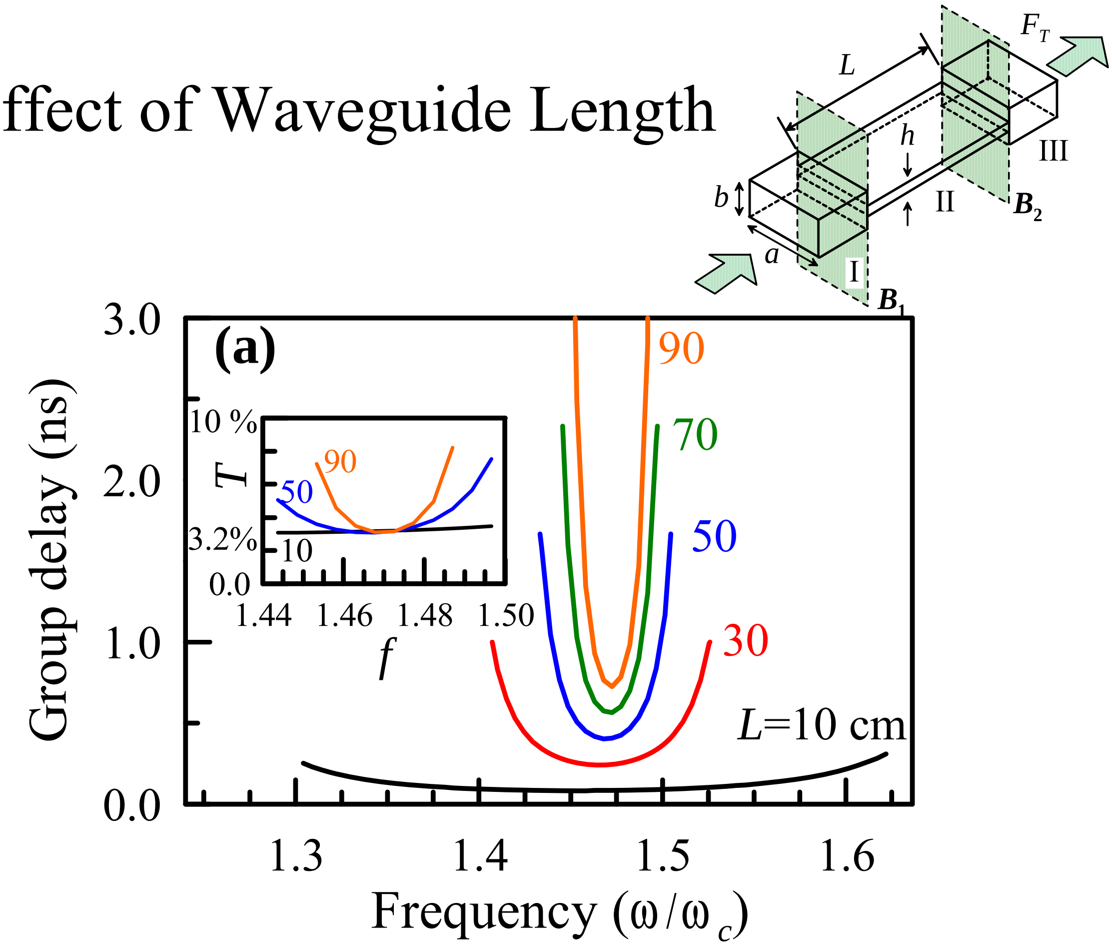


Effect of Waveguide Height





Effect of Waveguide Length



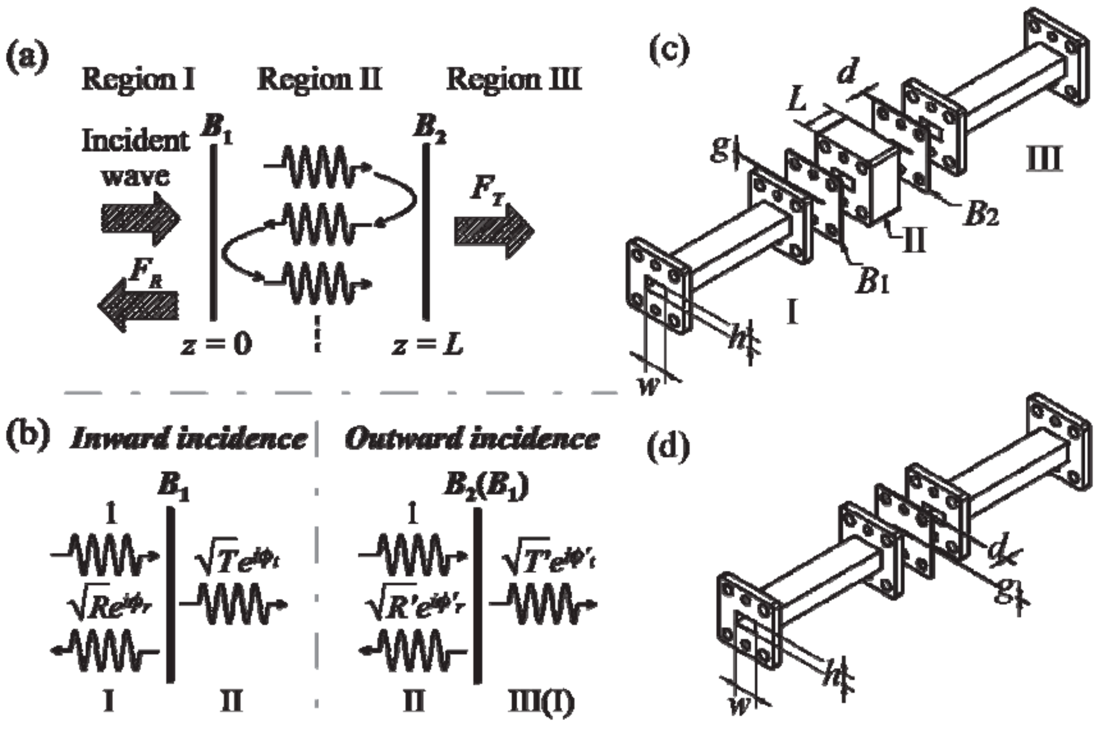
Summary #4

- A new mechanism of the superluminal effect has been theoretically analyzed and experimentally demonstrated.
- In contrast to the two traditional mechanisms which all involve evanescent waves, this mechanism employs propagating waves.
- This mechanism features high transmission and broad bandwidth.

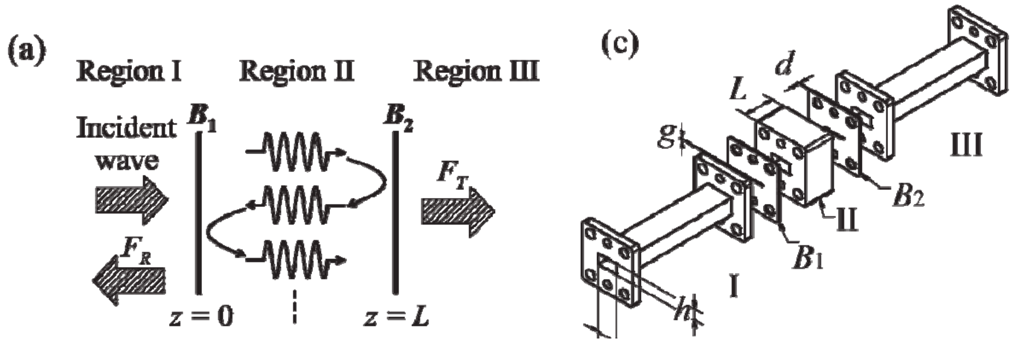
Part V. Manipulate the Group Delay

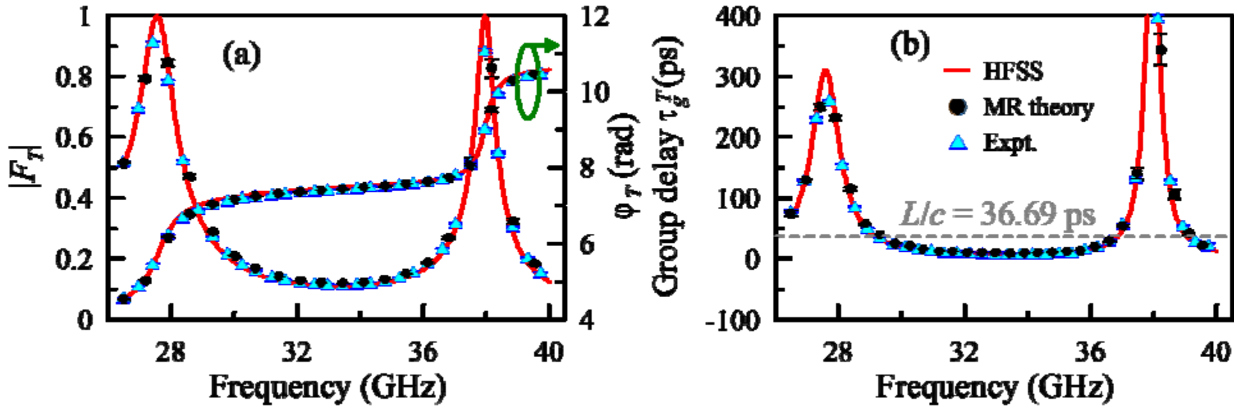
H. Y. Yao, N. C. Chen, T. H. Chang, and H. G. Winful, Phys. Rev. A **86,** 053832 (2012).

Superluminality
in a Fabry-Pérot Interferometer

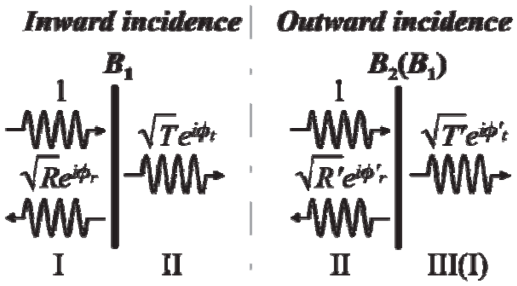


Manipulate the Group Delay





Group Delay Analysis



Multiple-reflection factor:

$$f_{MR} = \frac{R'\cos(2k_{eff}L) - R'^2}{1 - 2R'\cos(2k_{eff}L) + R'^2}$$

$$\tau_g^T = (\tau_{d0} + 2\tau_{d0} f_{MR}) + (2\tau_{\phi t} + 2\tau_{\phi r} f_{MR}) + \tau_R$$

$$\tau_{d0} = \frac{L}{v_{g\mathrm{II}}}$$

$$\tau_{\phi t} = \frac{d\phi_t}{d\omega},\ \tau_{\phi r} = \frac{d\phi'_r}{d\omega}$$

$$\tau_R = \left[\frac{\sin(2k_{eff}L)}{1 - 2R'\cos(2k_{eff}L) + R'^2}\right]\left(\frac{dR'}{d\omega}\right)$$

Group Delay Analysis II

$$\tau_g^T = \left(\tau_{d0} + 2\tau_{d0} f_{MR}\right) + \left(2\tau_{\phi t} + 2\tau_{\phi r} f_{MR}\right) + \tau_R$$

Dwell time: $\left(\tau_{d0} + 2\tau_{d0} f_{MR}\right)$

➔ Effective time for the signal staying within the system excluding boundary dispersion effect.

⇔ Lifetime of stored field energy escaping through the both ends (B_1 and B_2) of FP cavity excluding boundary dispersion effect.

Boundary transmission times: $2\tau_{\phi t}$

➔ Effective transmission time for the signal passing through the both boundaries of FP cavity.

Boundary reflection time: $2\tau_{\phi r} f_{MR}$

➔ Effective reflection time accumulated from signal reflecting on the both boundaries of FP cavity (modified by multiple-reflection factor).

Dispersive time: τ_R

➔ due to frequency-dependent reflectivity

Slow Wave and Fast Wave Criteria

On-resonance constructive interference: Slow wave

$$\tau_g^{T(on)} = \left(\frac{1+R'}{1-R'}\right)\left(\frac{L}{v_{gII}}\right) + \frac{d\phi_t}{d\omega} + \frac{d\phi_t'}{d\omega} + \left(2\frac{d\phi_r'}{d\omega}\right)\left(\frac{R'}{1-R'}\right)$$

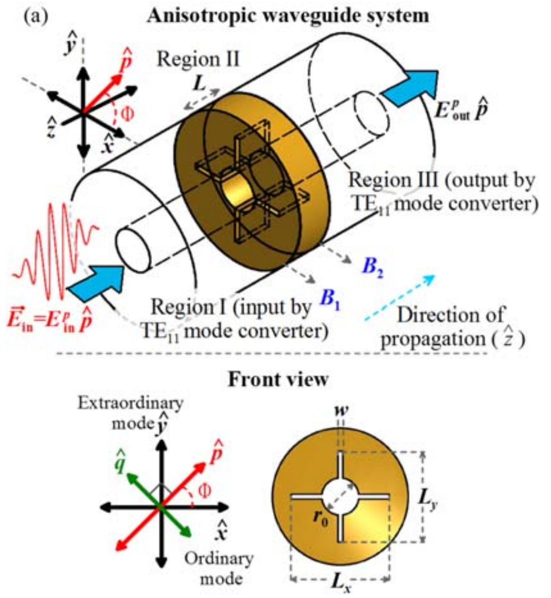
Off-resonance destructive interference: Fast wave

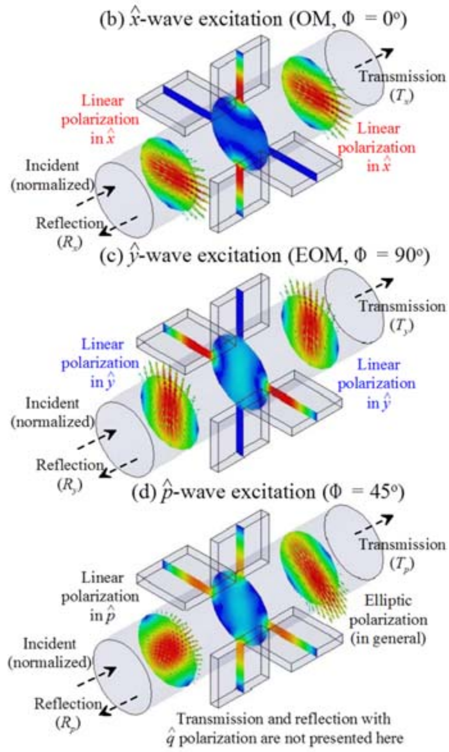
$$\tau_g^{T(off)} = \left(\frac{1-R'}{1+R'}\right)\left(\frac{L}{v_{gII}}\right) + \frac{d\phi_t}{d\omega} + \frac{d\phi_t'}{d\omega} - \left(2\frac{d\phi_r'}{d\omega}\right)\left(\frac{R'}{1+R'}\right)$$

Is it possible that the group delay becomes negative?

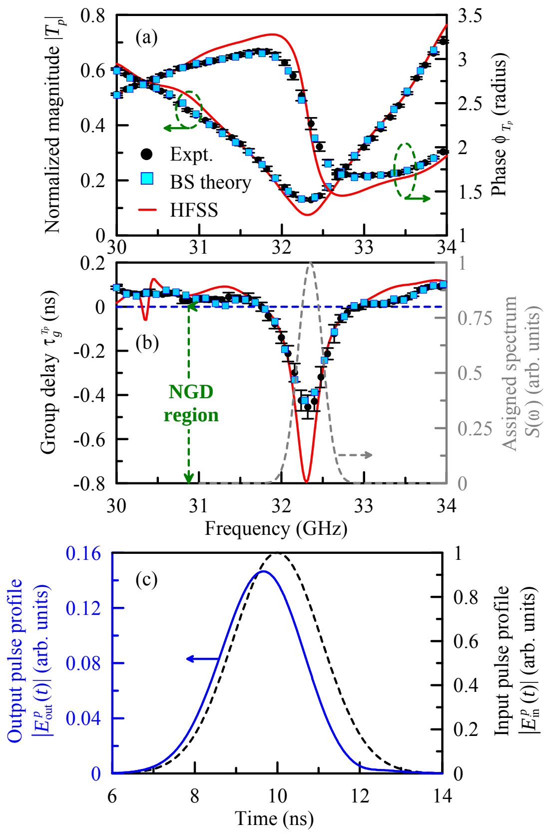
Yes, it is possible in a birefringent waveguide system.

Negative Group Delays in a Birefringent Waveguide





Negative Group Delays



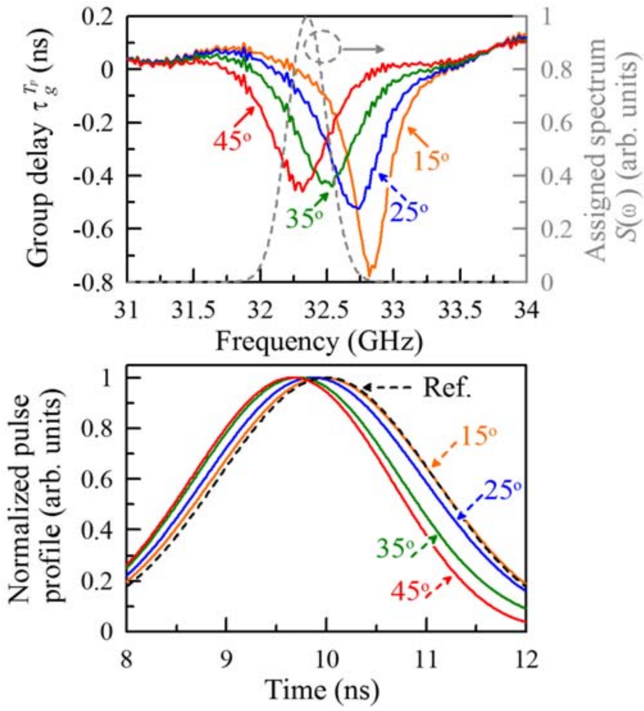
The black dots are the measured data, while the blue squares represent the theoretical results. The red curves are the simulation results.

(a) Transmission and phase

(b) Group delay when Φ= 45°

(c) The time-domain profiles of the incident and transmitted pulses.

Adjustable Group Delays & Summary #5



- We have demonstrated a negative group delay in an anisotropic waveguide system.
- This study provides a means to control the group delay by simply changing the polarization azimuth of the incident wave.

Conclusions

Phase velocity: $v_p \equiv \dfrac{\omega}{k}$

Group velocity: $v_g \equiv \dfrac{d\omega}{dk}$

Group delay: $\tau_g \equiv \dfrac{d\phi}{d\omega}$ apparent group velocity or phase time

Probability velocity: $v_{prob} \equiv \dfrac{J_x}{|\psi|^2}$

Energy velocity: $v_E \equiv \dfrac{P}{U}$

Information velocity: The speed at which information is transmitted through a particular medium.

Signal velocity: The speed at which a wave carries information.

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