A unification of information and matter

Xiao-Gang Wen MIT/Perimeter (Jan. 6, 2015; Taiwan)

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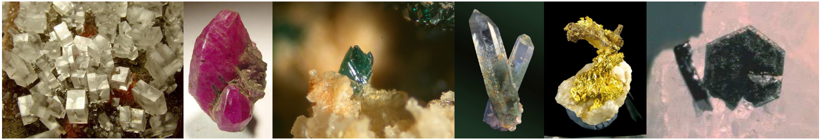
How to gain a deeper understanding of our world?



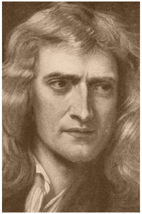
Discovery $\to$ Unification $\to$ More discovery $\to$ More unification $\to$ ...

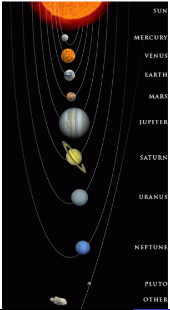
- Each unification = a revolution in physics
- Each revolution brought us into a new world
  - In the new world many fundamental concepts were changed
  - The old way of thinking and even the old language were no longer valid
  - We needed to introduce new mathematics to describe the new world.

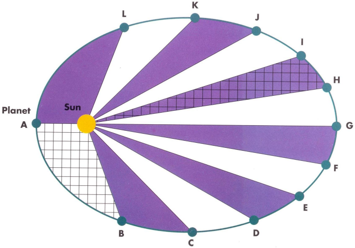
Mechanical revolution

Newton (1687)

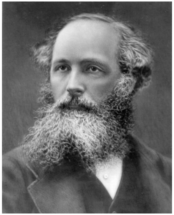
- **Unified** falling apples on earth and the planets motions in sky
- **New world view:** All matter are formed by collections of "particles", and their motion is described by the Newton's equation $\mathbf{F} = ma$.
- **New math: Calculus**







Electromagnetic revolution



Maxwell (1861)

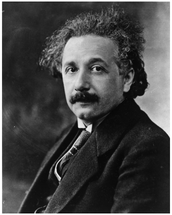
- **Unified** electricity, magnetism, and light
- **New world view:** There is a new form of matter – "wave-like" matter, which causes electromagnetic interaction between the "particle-like" matter. The motion of "wave-like" matter is described by the Maxwell equation $\dot{\mathbf{E}} - c\partial \times \mathbf{B} = \dot{\mathbf{B}} + c\partial \times \mathbf{E} = 0.$
- **New math: Fiber bundle (gauge theory)**

The first compass

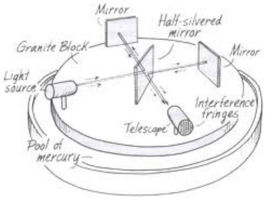
Relativity revolution

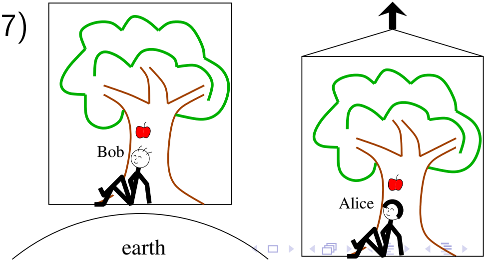
Einstein (1905,1916)

- **Unified** space, time, and gravity
- **New world view:** Even space-time is dynamical and its distortion is another "wave-like" matter. The new "wave-like" matter causes gravitational interaction between the "particle-like" matter, and satisfies the Einstein equation $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = -\frac{8\pi}{c^4}T_{\mu\nu}$
- **New math: Riemannian geometry (curved space)**

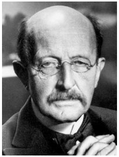
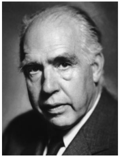


Michelson-Morley (1887)

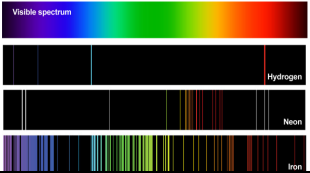


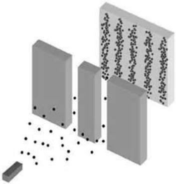


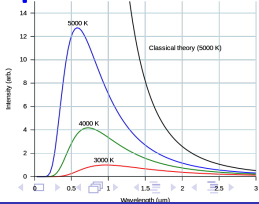
Quantum revolution

- **Unified**:Hydrogen spectra, blackbody radiation, interference
- **New world view:** "particle-like" matter and "wave-like" matter become the same. Matter is neither "particle" nor "wave", and is both "particle" and "wave".
  A new form of matter – "particle-wave-like" matter.
- **New math: linear algebra and tensor product**







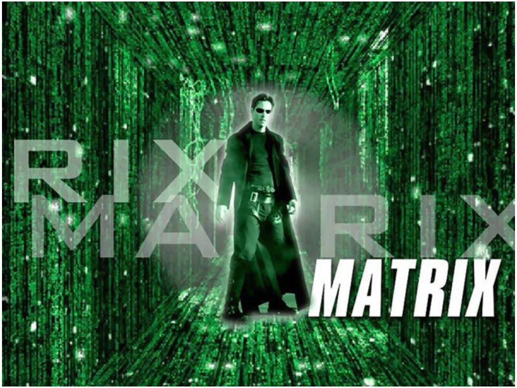
A second quantum revolution

"Matrix" is a story of two worlds:

a) A material world: everything formed by elementary particles.

b) A virtual world (in computers): everything formed by bits.

The information world is as real as the material world



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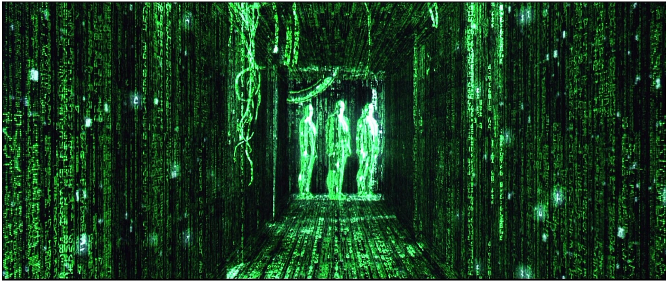

Our world is a (quantum) information world.

There is no material world

Matter = (quantum) information

We live inside a quantum computer

We used to think information and matter are two very different things: information is the attribute carried by matter.

The essence of quantum theory is a unification between **matter** and **information**

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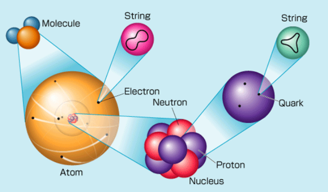
Information: Changing information (qubits) $\to$ frequency
According to quantum physics: frequency $\to$ energy
According relativity: energy $\to$ mass $\to$ **Matter**

=



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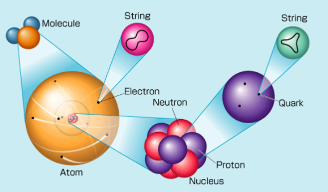
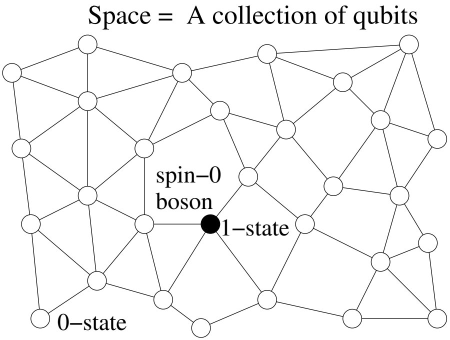


- But can simple qubits (quantum information) really produce all kinds of matter (and all the elementary particles)?

If matter was formed by spin-0 bosons, then yes

- The space = a collection of qubits.
- The 0-state = the vacuum.
- The 1-state = a spin-0 boson.



- *Ground state of the space-forming qubits = vacuum*
  Excitations above the ground state = elementary particles

But our elementary particles are very complicated

Seven wonders of our universe:

1. Identical particles
2. Spin-1 bosons with only two-components (gauge bosons)
3. Particles with Fermi statistics
4. Fractional angular momentum (spin-1/2)
5. Only left-hand fermions couple the $SU(2)$-gauge-bosons
6. Lorentz symmetry
7. Spin-2 bosons with only two-components (gravitons?)

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Can simple qubits produce the above seven wonders?

- **Yes** for 1-6. Such a magic is possible if the qubits are **Long-range entangled**[Chen-Gu-Wen 10] (also refer as **topologically ordered**[Wen 89] in condensed matter systems)

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- **A great unification**: Qubits unify gauge boson and fermion
- **A new view**: Our world is made of quantum information!

Make long range entanglement (topo. order)

- The product states $|\uparrow\downarrow\downarrow\uparrow ...\rangle$ are not entangled
- To make topological order, we need to sum over many different product states, but we should not sum over everything.
  $\sum_{\text{all spin configurations}} |\uparrow\downarrow\downarrow\uparrow ...\rangle = |\rightarrow\rightarrow\rightarrow\rightarrow ...\rangle$

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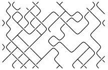
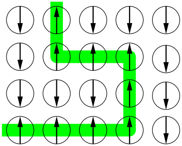
  $\sum_{\text{all spin configurations}} |\uparrow\downarrow\downarrow\uparrow\ldots\rangle = |\rightarrow\rightarrow\rightarrow\rightarrow\ldots\rangle$
- Sum over a subset of spin configurations: sum over all the "string states", where the up-spins form strings:

$|\Phi_{\text{closed string}}\rangle = \sum_{\text{all loops}} \Psi \Big|$  $\Big\rangle$

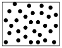
$|\Phi_{\text{string-net}}\rangle = \sum_{\text{all string-nets}} \Psi \Big|$  $\Big\rangle$

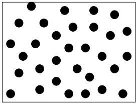


$\rightarrow$ string-net condensation [Levin-Wen 05] (string-net liquid).

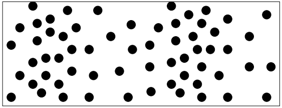
$\rightarrow$ $|\Phi_{\text{string}}\rangle$ has **long-range entanglement** and a non-trivial **topological order**

The magic of long-range entanglements $\to$ emergence of electromagnetic waves (photons)

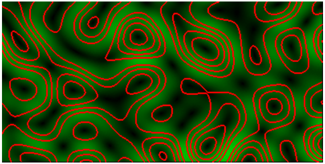
- Wave in superfluid state $|\Phi_{\text{SF}}\rangle = \sum_{\text{all position conf.}} \Big|$  $\Big\rangle$:

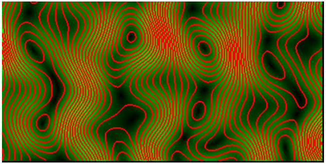


density fluctuations:
$\partial_t^2 \rho - \partial_x^2 \rho = 0$
$\to$ Longitudinal wave



- Wave in closed-string liquid $|\Phi_{\text{string}}\rangle = \sum_{\text{closed strings}} \Big|$  $\Big\rangle$:

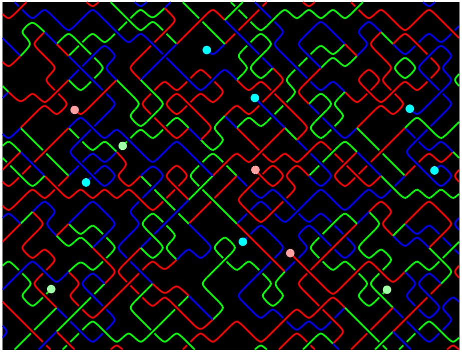




String density $\mathbf{E}(\mathbf{x})$ fluctuations $\to$ waves in string liquid.
Closed strings $\to$ $\boldsymbol{\partial} \cdot \mathbf{E} = 0$ $\to$ **only two transverse modes**.
$\to$ $\dot{\mathbf{E}} - \boldsymbol{\partial} \times \mathbf{B} = \dot{\mathbf{B}} + \boldsymbol{\partial} \times \mathbf{E} = \boldsymbol{\partial} \cdot \mathbf{B} = \boldsymbol{\partial} \cdot \mathbf{E} = 0$. ($\mathbf{E}$ electric field)

The magic of long-range entanglement → Emergence of Yang-Mills theory (gluons)

- If string has different types and can branch
  → string-net liquid → Yang-Mills theory
- Different ways that strings join → different gauge groups
- String types → representations of gauge group.

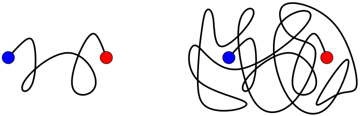


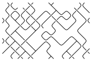
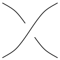
A picture of our vacuum

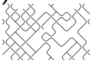
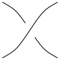
A string–net theory of light and electrons

Closed strings →
$U(1)$ **gauge theory**
String-nets → $SU(2) \times SU(3)$ **Yang-Mills gauge theory**

The magic of long-range entanglement
$\to$ Emergence of Fermi statistics



- In string liquids, the ends of string behave like point particles (gauge charges).
- **String attached to the particle does not cost energy, but can change the statistics of the particle.**
- For string liquid state $|\Phi\rangle = \sum_{\text{all conf.}} \Big|$  $\Big\rangle$,  $= +1$

  $\to$ End of strings $=$ boson (Higgs boson).
- For string liquid state $|\Phi\rangle = \sum_{\text{all conf.}} \pm \Big|$  $\Big\rangle$,  $= -1$

  $\to$ End of string $=$ fermion (electron & quark). [Levin-Wen 03]
- **A unification of gauge interactions and Fermi statistics**

Chiral fermion problem: string-net theory is wrong?

- String-net theory (or qubit theory) is a lattice theory.
- For a long time, we thought:
  Lattice theory cannot produce chiral fermion theory, such as the standard model.
  Qubit theory cannot produce an observed property: only left-hand fermions couple the SU(2)-gauge-bosons
  This seems rule out the qubit theory of our world

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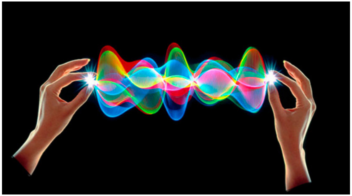
  This seems rule out the qubit theory of our world
- Recently, this long standing problem was solved:
  qubit theory can produce a modified standard model with 16 Weyl fermions per family. [Wen 13]
- Impact in lattice gauge theory: we now can have a non-perturbative definition of the modified standard model.

Long-range entangled qubits → Matter (topologically ordered qubits)

1. Identical particles
2. Spin-1 bosons with only two-components → gauge bosons
3. Fermi statistics
4. Fractional angular momentum (spin-1/2)
5. Chiral gauge coupling (Parity violation in weak interaction)
6. Lorentz symmetry (not naturally)
7. Spin-2 bosons with only two-components → gravitons?

- Space = An ocean of qubits
- Qubits form a string-net liquid
- Fluc. string-net → photon, gluon
- End of string → electron, quark



Long-range entangled qubits → Geometry?

Experimental prediction of the string-net theory (the qubit model) for elementary particles

- **All composite fermions must carry gauge charge**

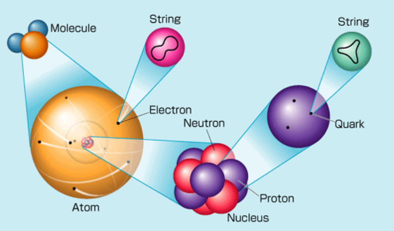
  The standard model does not have such a property,
  but the $SO(10)$ GUT has. [Levin-Wen 05]

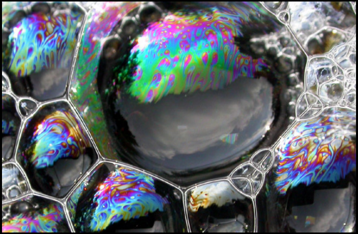
  $\to$ **Additional gauge "symmetry" in SM, such as Z_2**
  $\to$ **There are new cosmic strings** of unknown energy scale
  Experimental bound: $\sqrt{\mu} < 10^{-4} M_P = 10^{15} GeV$. [Wen 12]

- **16 Weyl fermions per family with sterile neutrino**
  as in $SO(10)$ GUT, but not 15 Weyl fermions per family
  as in $SU(5)$ GUT and in the original standard model.

  Sterile neutrino mass scale is about the same as the new cosmic string energy scale. Dark matter candidates?

  [Wen 13, You-Xu 14]





matter = quantum information

Gauge int. and Fermi statistics come from Long-range entanglement

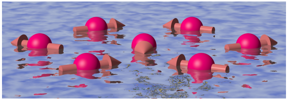

Now is the an exciting in physics, just like 1900-1930

- Physics, in particular, condensed matter physics, is a very old field. Many people are thinking that the exciting time of physics has passed. We enter the begining of the end of physics. The only important things in physics are its engineering applications, such as optical fiber and blue LED.
- However, I feel that we only see the end of the begining. The exciting time is still ahead of us. In particular, now is a very exciting time in physics, like 1900 - 1930.

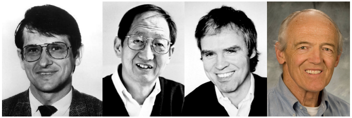
  We are seeing/making the second quantum revolution which unifies information, matter and geometry. I feel very lucky to be born in this generation.

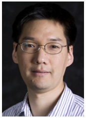
Topological order: beyond symm. breaking [Wen 89]

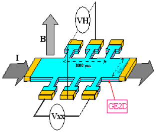
- We used to believe that **all phases and phase transitions are described by symmetry breaking**

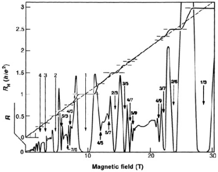


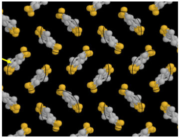
- Counter examples:
  - Quantum Hall states $\sigma_{xy} = \frac{m}{n}\frac{e^2}{h}$
  - Spin liquid states, Organics κ-$(ET)_2X$ and herbertsmithite











- FQH states and spin-liquid states have different phases with no symmetry breaking, no crystal order, no spin order, ... they have **long-range entanglement = topological order**

Entanglement through examples

- $|\uparrow\rangle \otimes |\downarrow\rangle$ = direct-product state $\to$ unentangled (classical)

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  $= (|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle) = |x\rangle \otimes |x\rangle$ $\to$ unentangled

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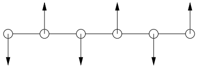
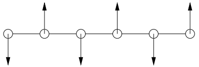
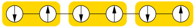
- $|\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle$ $\to$ entangled (quantum)
- $|\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle + |\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle$
  $= (|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle) = |x\rangle \otimes |x\rangle$ $\to$ unentangled
-  $= |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle ...$ $\to$ unentangled

Entanglement through examples

- $|\uparrow\rangle \otimes |\downarrow\rangle$ = direct-product state $\rightarrow$ unentangled (classical)
- $|\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle$ $\rightarrow$ entangled (quantum)
- $|\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle + |\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle$
  $= (|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle) = |x\rangle \otimes |x\rangle$ $\rightarrow$ unentangled
-  $= |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \ldots$ $\rightarrow$ unentangled
-  $= (|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle) \otimes (|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle) \otimes \ldots \rightarrow$
  short-range entangled (SRE) state

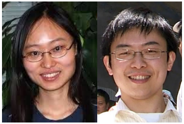
Long-range entanglement and topological order

For gapped systems with no symmetry:

- According to Landau theory, no symm. to break
  $\rightarrow$ all systems belong to one trivial phase

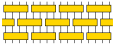
Long-range entanglement and topological order

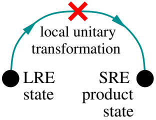
For gapped systems with no symmetry:



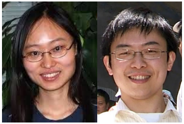
- According to Landau theory, no symm. to break
  $\to$ all systems belong to one trivial phase
- Thinking about entanglement: there are [Chen-Gu-Wen 2010]

- **long range entangled (LRE) states**
- **short range entangled (SRE) states**

$|\text{LRE}\rangle \neq$  $|\text{product state}\rangle = |\text{SRE}\rangle$



Long-range entanglement and topological order

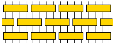


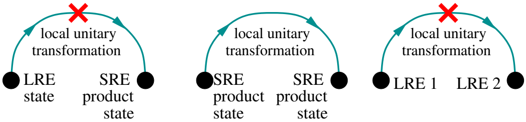
For gapped systems with no symmetry:

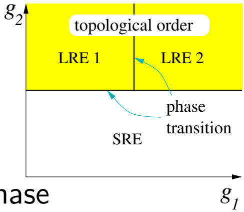
- According to Landau theory, no symm. to break
  $\to$ all systems belong to one trivial phase

- Thinking about entanglement: there are [Chen-Gu-Wen 2010]

- **long range entangled (LRE) states** $\to$ many phases
- **short range entangled (SRE) states** $\to$ one phase

$|\text{LRE}\rangle \neq$  $|\text{product state}\rangle = |\text{SRE}\rangle$





- All SRE states belong to the same trivial phase
- LRE states can belong to many different phases: different patterns of long-range entanglements [defined by LU trans.]
  = different **topological orders** [Wen 1989]

Topological orders in different dimensions

- 1+1D: there is no topological order [Verstraete-Cirac-Latorre 05]
- 2+1D: Abelian topological order are classified by K-matrices [Blok-Wen 90, Read 90, Wen-Zee 92]

  2+1D: topological orders are classified by (1) modular tensor category (MTC) and (2) central charge c.

  2+1D: topo. order with gappable edge are classified by unitary fusion categories (UFC): $\mathcal{Z}_D(UFC) = MTC|_{c=0}$ [Levin-Wen 05]
- 3+1D: ???

Short-range entanglement + symm. $\to$ SPT phase

For gapped systems with a symmetry $H = U_g H U_g^\dagger,\ g \in G$

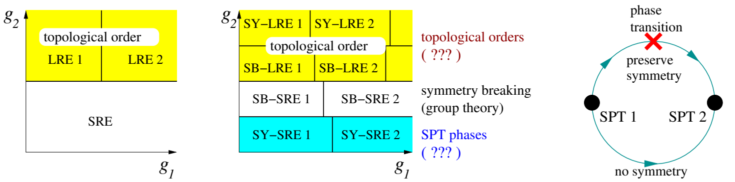
- **LRE symmetric states** $\to$ many different phases
- **SRE symmetric states** $\to$ one phase (no symm. breaking)

Short-range entanglement + symm. → SPT phase

For gapped systems with a symmetry $H = U_g H U_g^\dagger,\ g \in G$

- **LRE symmetric states** → many different phases
- **SRE symmetric states** → many different phases

We may call them **symm. protected trivial (SPT)** phase



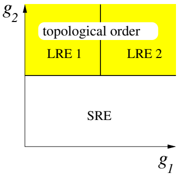
- SPT phases = equivalent class of *symmetric* LU trans.

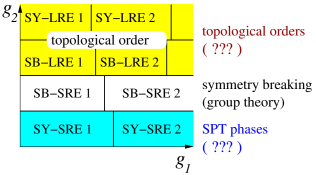
Short-range entanglement + symm. $\to$ SPT phase

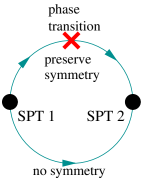
For gapped systems with a symmetry $H = U_g H U_g^\dagger,\ g \in G$

- **LRE symmetric states** $\to$ many different phases
- **SRE symmetric states** $\to$ many different phases

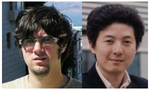
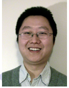
We may call them **symm. protected trivial (SPT)** phase
or **symm. protected topological (SPT)** phase





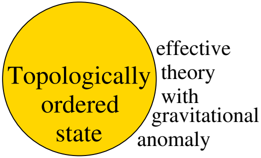


- SPT phases = equivalent class of *symmetric* LU trans.
- Examples: 1D Haldane phase[Haldane 83] 2D/3D TI[Kane-Mele 05; Bernevig-Zhang 06] [Moore-Balents 07; Fu-Kane-Mele 07]

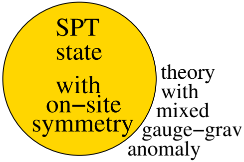
1D  2D   3D  

Boundary of topological/SPT order and anomalies

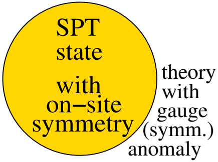
Topological orders

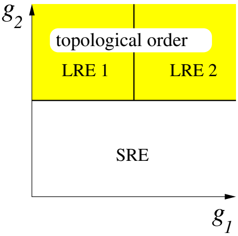


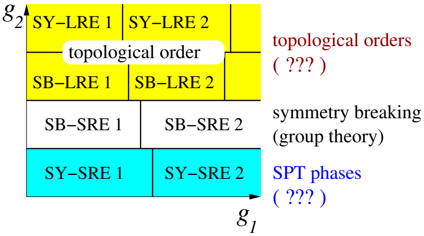
mixed SPT orders



Pure SPT orders



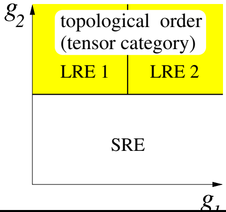


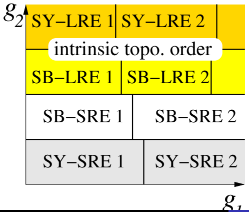


Classify bosonic SPT phases by group cohomology

- **Pure STP orders**: $\mathcal{H}^d(G, \mathbb{R}/\mathbb{Z})$ (the black entries below)
- **mixed SPT order** $\oplus_{k=1}^{d-1}\mathcal{H}^k(G, \text{iTO}^{d-k}) = \frac{\oplus\mathcal{H}^k[G, \mathcal{H}^{d-k}(SO, \mathbb{R}/\mathbb{Z})]}{\Gamma^d(G)}$
- **iTO's**: $\text{iTO}^d = \mathcal{H}^d(SO, \mathbb{R}/\mathbb{Z})/\Gamma^d$ [Chen-Gu-Liu-Wen 11, Wen 14]

$G \setminus d =$	0+1	1+1	2+1	3+1	4+1	5+1	6+1
iTO^d	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	0	$2\mathbb{Z}$
Z_n	$\mathbb{Z}_n$	0	$\mathbb{Z}_n$	0	$\mathbb{Z}_n \oplus \mathbb{Z}_n$	$\mathbb{Z}_{\langle n,2\rangle}$	$\mathbb{Z}_n \oplus \mathbb{Z}_n \oplus \mathbb{Z}_{\langle n,2\rangle}$
Z_2^T	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2 \oplus 2\mathbb{Z}_2$	$\mathbb{Z}_2$
$U(1)$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z} \oplus \mathbb{Z}$	0	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
$U(1) \rtimes Z_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}_2$	$\mathbb{Z}_2$	$2\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$2\mathbb{Z}_2 \oplus 2\mathbb{Z}_2$	$\mathbb{Z} \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z} \oplus 2\mathbb{Z}_2$
$U(1) \times Z_2^T$	0	$2\mathbb{Z}_2$	0	$3\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$4\mathbb{Z}_2 \oplus 3\mathbb{Z}_2$	$2\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$U(1) \rtimes Z_2^T$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$2\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}_2 \oplus \mathbb{Z}$	$2\mathbb{Z}_2 \oplus 2\mathbb{Z}_2$	$2\mathbb{Z}_2 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_2$

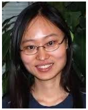




SET orders
(tensor category
w/ symmetry)

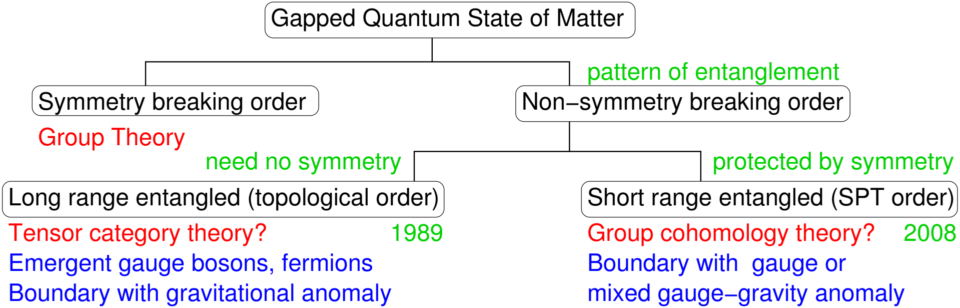
symmetry breaking
(group theory)

SPT orderes
(group cohomology
theory)



Try to classify quantum states of matter

- gapless states – Very hard beyond 1+1D. Have no clue
- gapped states – A classification maybe possible:



- Group theory classifies 230 crystal orders in 3D space.
- Group cohomology classifies SPT orders
- Higher category theory classifies topological order

Our vacuum is a particular topologically ordered quantum state of matter → all the wonders of our world