Higgs and the Cosmos

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2013
After decades of search, the Higgs particle was discovered at CERN, in a reaction like this. In a detector like this, Higgs & Englert got the Physics Nobel Prize in 2013, for postulating the underlying Higgs field, in 1964.
The Higgs field fills the vacuum.

On microscopic scale, it gives mass to elementary particles: W, Z, quarks.

On macroscopic scale, it flows like a superfluid, due to phase variations.

On cosmic scale, it makes the universe a superfluid.
Great puzzles of our time:
• Dark energy
• Dark matter

Theme of this talk:
• Dark energy = energy of Higgs superfluid
• Dark matter = density variation of superfluid
Expanding universe

- The more distant the galaxy, the faster it moves away from us.
- Fabric of space-time expands, like balloon being blown up.
- Extrapolate backwards to “big bang”

Hubble’s law: Velocity proportional to distance

Hubble’s parameter:

\[ H = \frac{1}{a} \frac{d}{dt} = \frac{1}{15 \times 10^9 \text{ yrs}} \]
Dark energy – deviation from Hubble’s law

Accelerated expansion: Driven by “dark energy”
Dark matter

Velocity curve of Andromeda
(Rubin & Ford, 1970)
Collision of two galaxy clusters (the “bullet cluster” 2004)

- Hot gases (x-rays)
- Galaxies (visible)
- Dark-matter halo (from gravitational lensing)
Dark energy & dark matter constitute 96% of the energy in the universe.
Superfluidity

Quantum phase coherence over macroscopic distances

Order parameter: complex scalar field

\[ \phi = F e^{i\sigma} \]

Superfluid velocity = \( \kappa \nabla \sigma \)
Liquid helium below critical temperature 2.18 K becomes superfluid. It can climb over walls of containers.
Superconductivity = superfluidity arising from electron pairs in a metal

Inside a superconductor, there is a condensate of electron pairs with definite quantum phase.

Phase difference between two superconductors causes a supercurrent to flow from one to the other.

Josephson junction
The Higgs field

- is a complex scalar field that permeates all space,
- serving as order parameter for superfluidity,
- making the entire universe a superfluid.

It is a quantum field

- with momentum scale set by a cutoff momentum.
- It undergoes renormalization under a scale transformation.
Renormalization

As scale changes, one must adjust couplings so as to preserve the theory.

- The system’s appearance changes,
- But its identity is preserved.

Ignored

Hide

Cutoff $\Lambda_0$

Effective cutoff $\Lambda$

Momentum spectrum

Freeman J. Dyson
1923-

Kenneth G. Wilson
1936 - 2013
Scalar Field

Lagrangian density:
\[ \mathcal{L} = \frac{1}{2} (\partial \phi)^2 - V(\phi) \]

Potential:
\[ V(\phi) = \lambda_2 \phi^2 + \lambda_4 \phi^4 + \lambda_6 \phi^6 + \cdots \]

Equation of motion:
\[ \ddot{\phi} + V' \phi = 0 \]

High momentum cutoff = \( \Lambda \)
Length scale = \( \frac{1}{\Lambda} \)

- Renormalization makes the couplings, and thus \( V \), dependent on the length scale.
- This dependence is especially important when the scale changes rapidly, as during the big bang.
RG (renormalization group) trajectory

- The potential $V$ changes as scale changes.
- The Lagrangian traces out a trajectory in the space of all possible Lagrangians.
- Fixed points correspond to scale-invariant systems.

UV trajectory: Asymptotic freedom
IR trajectory: Triviality
The Creation

• At the big bang $\Lambda = \infty$.
• There was no interaction.
• Universe was at the **Gaussian fixed point**.
• It emerges along some direction in the space of Lagrangians, on an RG trajectory.
• Direction $\leftarrow \rightarrow$ form of the potential $V$.

Outgoing trajectory --- Asymptotic freedom

Ingoing trajectory --- Triviality (free field)
Halpern-Huang potential

the only asymptotically free scalar potential

• Kummer function (non-polynomial)

• Exponential behavior at large fields
Cosmological equations

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \]  
\[ \partial^2 \phi + V \phi = 0 \]

(Einstein's equation)  
(Scalar field equation)

Robertson-Walker metric (spatial homogeneity)  
Gravity scale = \( a \) (radius of universe)  
Scalar field scale = \( \Lambda \) (cutoff momentum)  
Since there can be only one scale in the universe,

\[ \Lambda = \frac{\dot{\Lambda}}{a} \]

Dynamical feedback:

Gravity provides cutoff to scalar field, which generates gravitational field.
The big bang
Initial-value problem

\[ 4\pi G = c = \hbar = 1 \]

\[ \dot{a} = Ha \]
\[ \dot{H} = \frac{k}{a^2} - \dot{\phi}^2 + \frac{a}{3} \frac{\partial V}{\partial a} \]
\[ \ddot{\phi} = -3H \dot{\phi} - \frac{\partial V}{\partial \phi} \]
\[ X = H^2 + \frac{k}{a} - \frac{2}{3} \left( \frac{1}{2} \dot{\phi}^2 + V \right) = 0 \]

\( X = 0 \) is a constraint on initial values.
Equations guarantee \( \dot{X} = 0 \).

\( k \) = curvature parameter = 0, +1, -1

Trace anomaly

Constraint equation
The big bang Model starts here $O(10^{-43} \text{ s})$

- Initial condition: Vacuum field already present.
- Universe could have been created in hot "normal phase", then make phase transition to "superfluid phase".
Numerical solution

\[ H \approx t^{-p} \]
\[ a \approx \exp\left(t^{1-p}\right) \]

Dark energy without “fine-tuning” problem
Comparison of power-law prediction on galactic redshift with observations

\[ d_L = \text{luminosity distance} \]

Different exponents \( p \) only affects vertical displacement, such as A and B.

Horizontal line corresponds to Hubble’s law. Deviation indicates accelerated expansion (dark energy).

Crossover transition between two different phases B \( \rightarrow \) A (?)
Generalization to complex scalar field

New physics:
• Superfluidity
• Quantum turbulence

1. Matter creation:
   Must create enough matter for subsequent nucleogenesis before universe gets too large.

2. Decoupling of matter scale and Planck scale:
   Matter interactions governed by nuclear scale of 1 GeV.
   But equations have built-in Planck scale of $10^{18}$ GeV.
   These scales should decouple from each other.
Quantized vortex in complex scalar field

\[ \phi = F e^{i\sigma} \]

\[ \kappa \nabla \sigma = \text{superfluid velocity} \]

\[ \oint_C ds \cdot \nabla \sigma = 2\pi n \]
• Replace vortex core by tube.
• Scalar field remains uniform outside.
• Can still use RW metric,
• but space is multiply-connected.

A “worm-hole” cosmos
The vortex-tube system represent emergent degrees of freedom.
Vortex dynamics

Elementary structure is vortex ring

\[ \nu = \frac{1}{4\pi R} \ln \frac{R}{R_0} \]

Self-induced vortex motion

The smaller the radius of curvature \( R \), the faster it moves normal to \( R \).
Vortex reconnection

- The cusps spring away from each other at “infinite” speed (due to small radii), creating two jets of energy.

- Efficient way of converting potential energy to kinetic energy in very short time.
Magnetic reconnections in sun’s corona
Responsible for solar flares
Simulation of quantum turbulence
Creation of vortex tangle in presence of “counterflow”.

Fractal dimension = 1.6
Cosmology with quantum turbulence

- Scalar field has uniform modulus $F$.
- Phase dynamics manifested via vortex tangle $l$.
- Matter created in vortex tangle.

Variables

- $a =$ Radius of universe
- $F =$ Modulus of scalar field
- $\ell =$ Vortex tube density
- $\rho =$ Matter density

Equations of motion

- $\dot{a}$ from Einstein's equation with RW metric.

    \[ \dot{a} = \frac{\dot{a}}{\rho F} + T^{\mu\nu}_{\text{tot}} + T^{\mu\nu}_F + T^{\mu\nu}_\ell + T^{\mu\nu}_\rho \]

- $\dot{F}$ from field equation.

- $\dot{\ell}$ from Vinen's equation.

- $\dot{\rho}$ from energy-momentum conservation $T^{\mu\nu}_{\text{tot};\mu} = 0$. 
Vinen’s equation for quantum turbulence

\[ \dot{\ell} = A \ell^2 - B \ell^{3/2} \]

Growth \hspace{1cm} Decay

• Vinen (1957)
• Schwarz (1988)
• Verified by experiments in superfluid helium.
Cosmological equations \((4\pi G = c = \hbar = 1)\)

Old:

\[
\dot{H} = \frac{k}{a^2} \dot{\phi} + \frac{a}{3} \frac{\partial V}{\partial a} \\
\ddot{\phi} = -3H \dot{\phi} - \frac{\partial V}{\partial \phi} \\
\text{Constraint:} \\
H^2 + \frac{k}{a^2} - \frac{2}{3} \left( \frac{1}{2} \dot{\phi} + V \right) = 0
\]

Generalized:

\[
E_v = a^3 \varepsilon_0 \ell \quad \text{(Total vortex energy)} \\
E_m = a^3 \rho \quad \text{(Total matter energy)}
\]

\[
\frac{dH}{dt} = \frac{k}{a^2} - 2 \left( \frac{dF}{dt} \right)^2 + \frac{a}{3} \frac{\partial V}{\partial a} - \frac{1}{a^3} \left( E_m + E_v \right) \\
\frac{d^2 F}{dt^2} = -3H \frac{dF}{dt} - \frac{\zeta_0 E_v}{a^3} F - \frac{1}{2} \frac{\partial V}{\partial F} \\
\frac{dE_v}{d\tau} = -E_v^2 + \gamma E_v^{3/2} \\
\frac{dE_m}{d\tau} = \left( \frac{\zeta_0}{s_1} \frac{dF^2}{dt} \right) E_v
\]

Constraint:

\[
H^2 + \frac{k}{a^2} - \frac{2}{3} \left( \dot{F}^2 + V + \frac{1+\zeta_0}{a^3} E_v + \frac{1}{a^3} E_m \right) = 0
\]

Decouples into two sets because

\[
S_1 = \frac{s}{t} = \frac{\text{Planck time scale}}{\text{Nuclear time scale}} = \frac{\text{Nuclear energy scale}}{\text{Planck energy scale}} \sim 10^{-18}
\]
Matter creation

• Vortex tangle (quantum turbulence) grows and eventually decays.

• All the matter needed for galaxy formation was created in the tangle.

• This picture replaces the usual “inflation”.

After decay of quantum turbulence, the standard hot big bang theory takes over, but the universe remains a superfluid.
Cosmic inflation:
- Radius increases by factor $10^{27}$
- in $10^{-30}$ seconds.
- Matter created = $10^{22}$ sun masses
- Eventually form galaxies outside of vortex cores.
Big bang

Quantum turbulence

Inflation

Validity of this model

$10^{-26}$ s

$10^5$ yrs

CMB formed

Standard hot big bang theory
Plus superfluidity

Time
Vortex cores have expanded with universe

Then

$10^{-26}$ sec

$10^{-10}$ cm

"Dust"

Now

1.4 billion yrs

$10^7$ light yrs

Dot = galaxy
Legacies in the post-inflation universe

Remnant vortex tubes with empty cores grow into cosmic voids in galactic distribution.

The large-scale structure of the Universe from the CfA2 galaxy survey.
The observed "stick man"

$10^7$ light years

Simulated with 3 vortex tubes
Reconnection of huge vortex tubes in the later universe will be rare but spectacular.

Gamma ray burst

- A few events per galaxy per million yrs
- Lasting ms to minutes
- Energy output in 1 s = Sun’s output in entire life (billions of years).

Jet of matter 27 light years long
“Hair” on black hole

Artist’s conception:
Rotating object in superfluid induces vortex filaments.

Observed:
“Non-thermal filaments” near center of Milky Way.
Dark matter

Vacuum field gives dark energy

Deviation from vacuum value, due to presence of galaxy, represents dark matter
Computer simulations (2D) based on phenomenological scalar field

- Nonlinear Klein-Gordon equation in curved space, with galaxy as external source.
- phi-4 scalar potential.

Response of superfluid to galaxy being dragged through it.
Two galaxies colliding headon and passing through each other
Two galaxies passing each other
Superfluid sheared into rotation by creation of vortices (black dots).
Quantized vortices generated by a rotating galaxy at center

Scalar-field modulus
The vortices are arranged in rings.

Scalar-field phase
Dark lines are “strings” across which phase jumps by 2 π.
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