Exploring the Quantum-Classical Transition Using Optomechanical Systems

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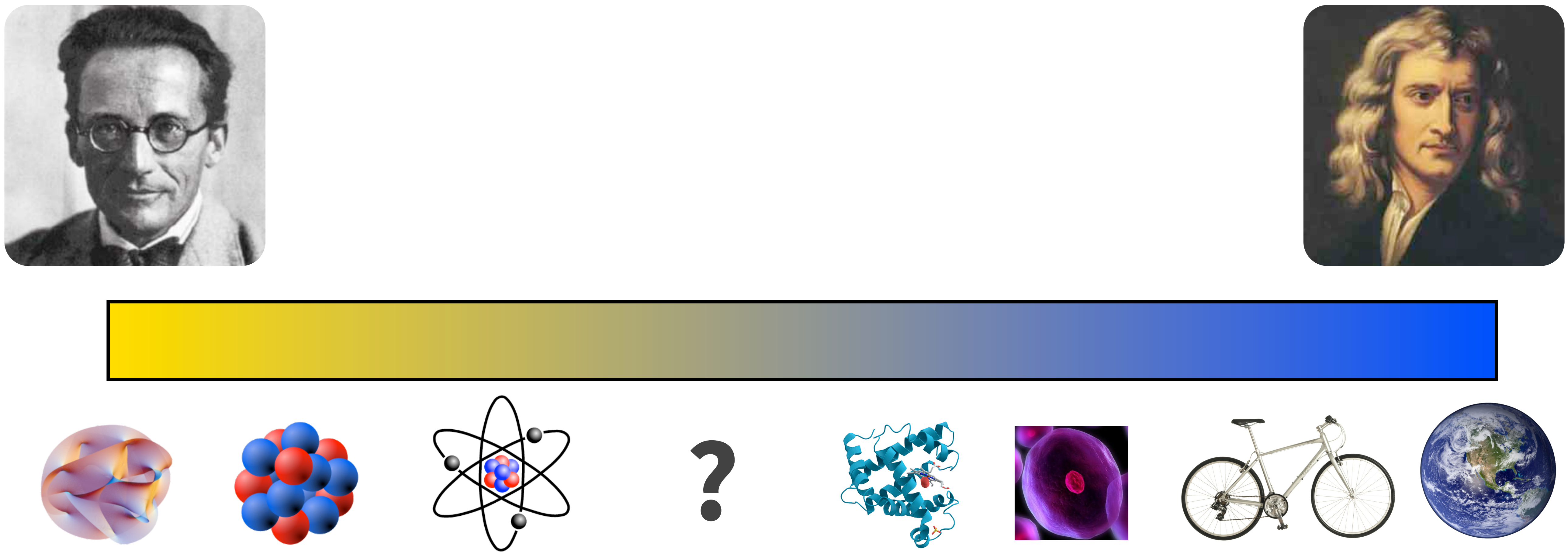
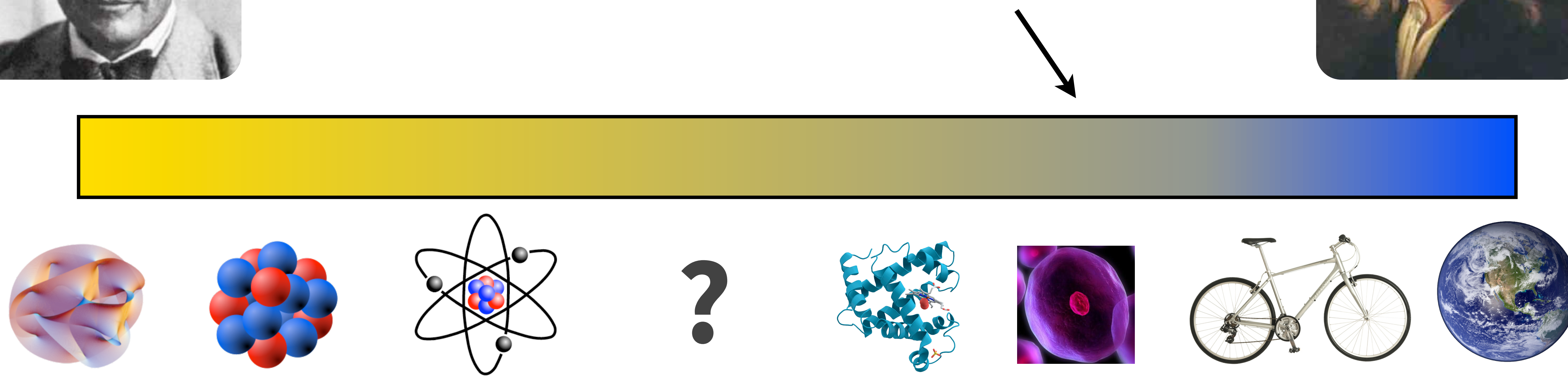
How does the classical world emerge from the underlying rules of quantum mechanics?

Quantum-Classical Crossover:



Quantum-Classical Crossover:

Can we push the boundary higher?



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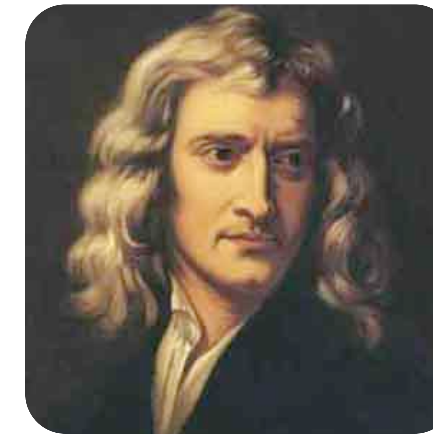

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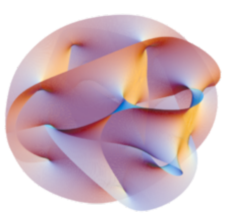
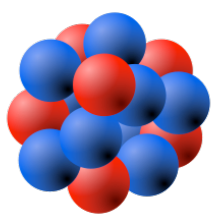
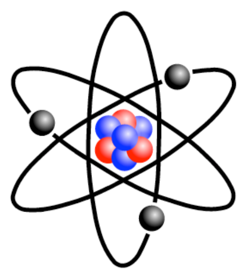
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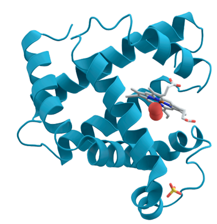
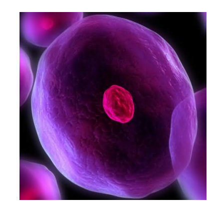
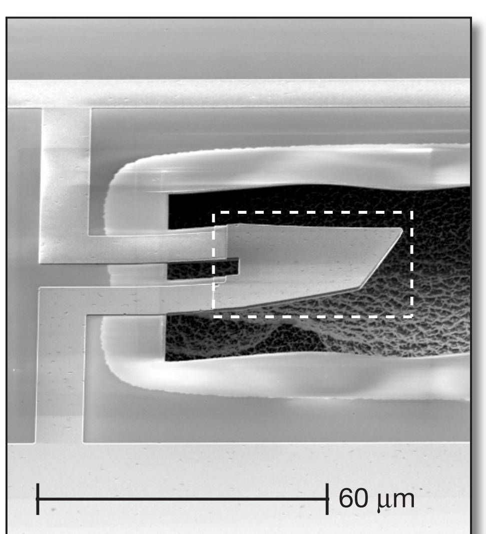
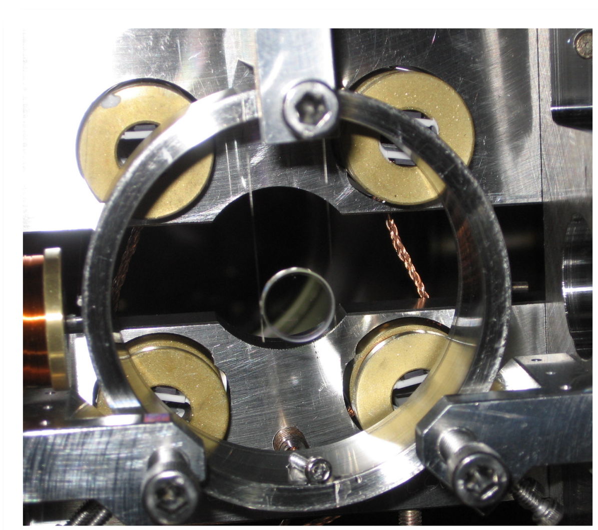



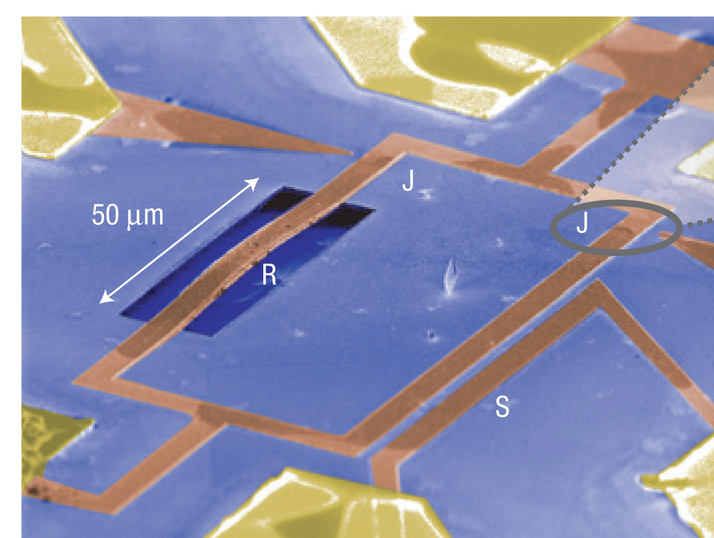
- In principle yes! One of the goals in nanomechanics:



O'Connell, Nature (2010)

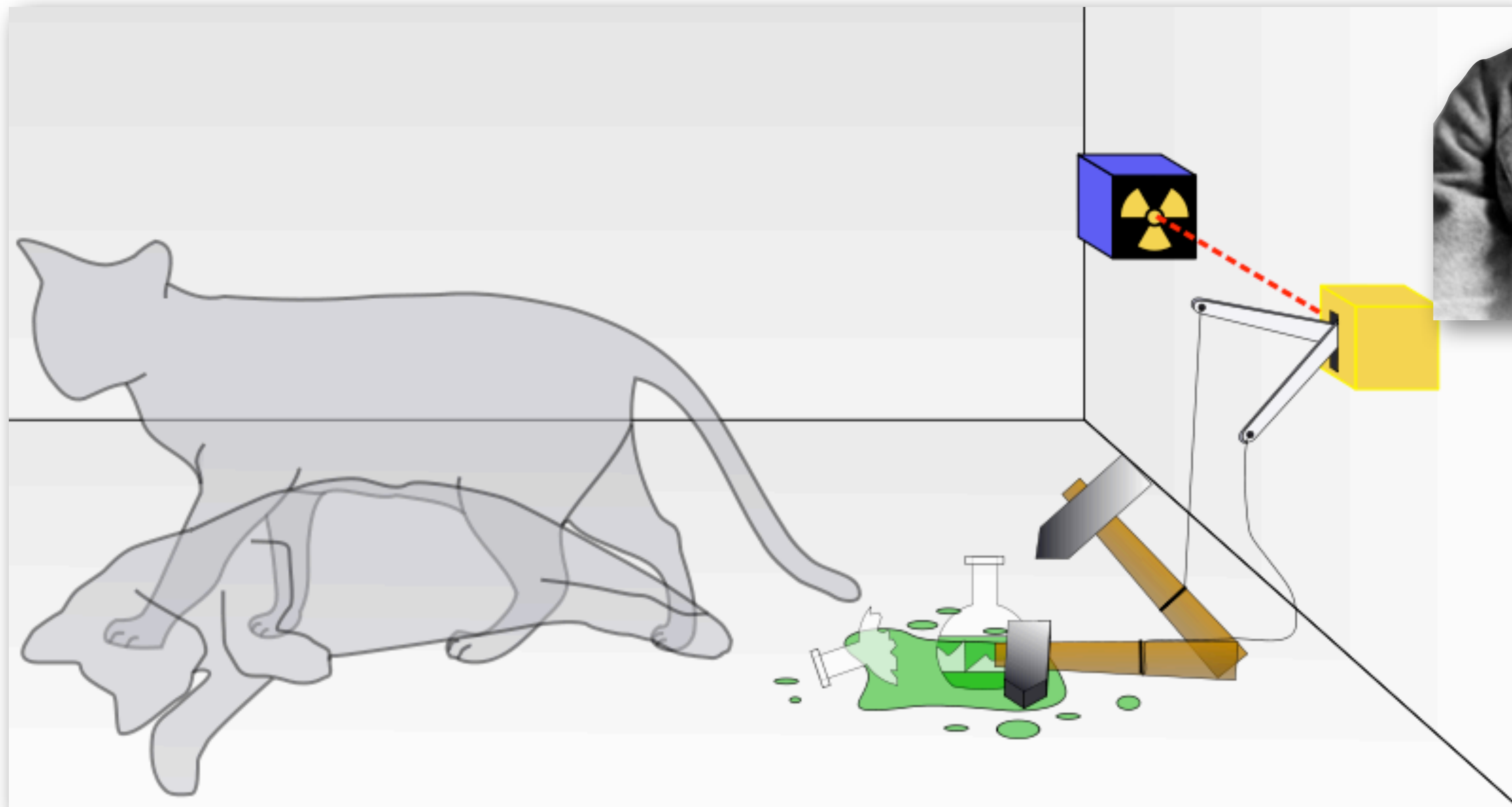


Mavalvala, MIT



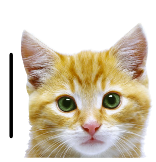
Etaki, Nature Phys. (2008)

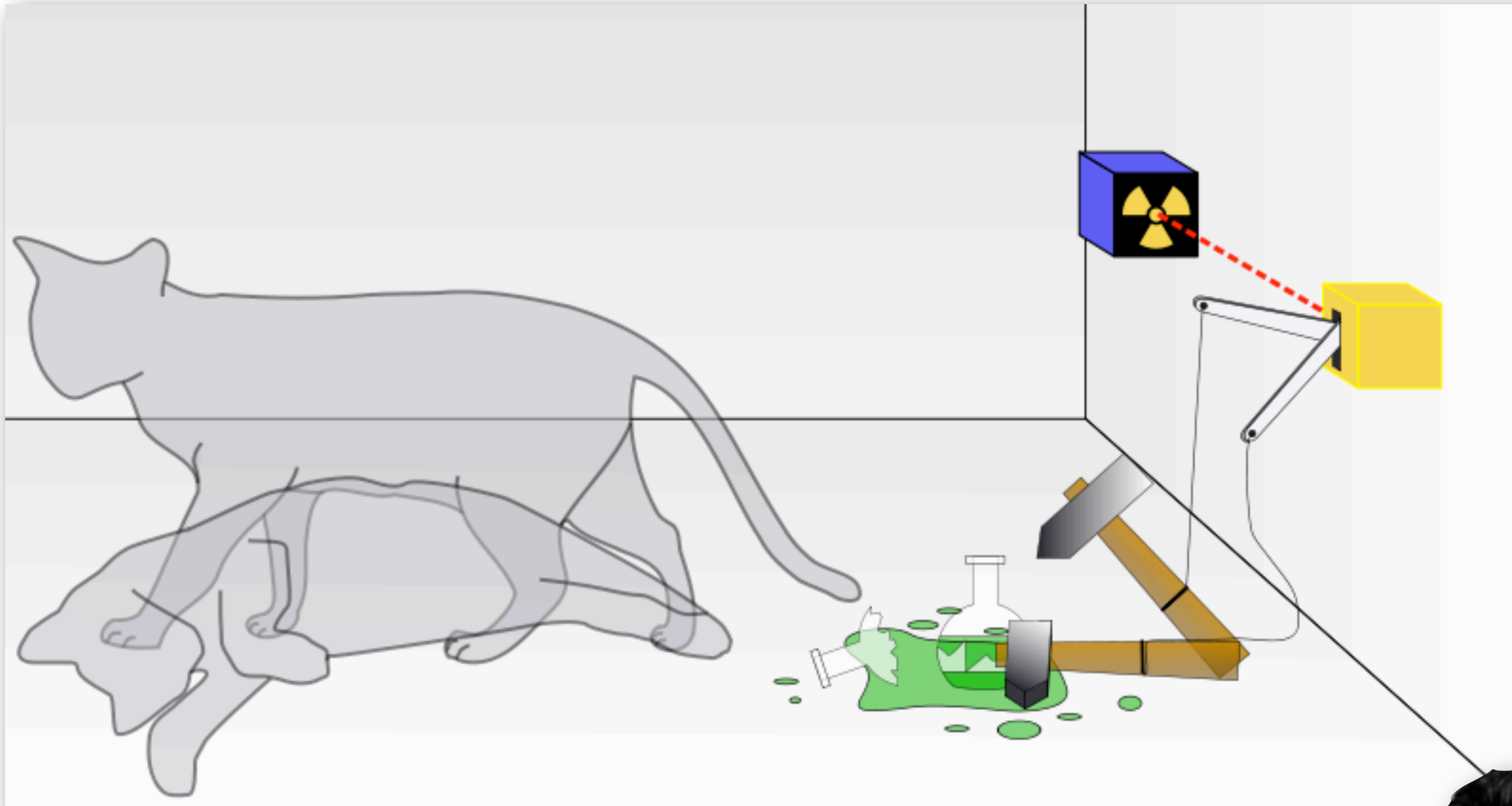
Schrödinger's Cat (1935):

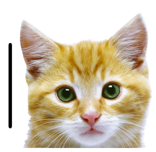


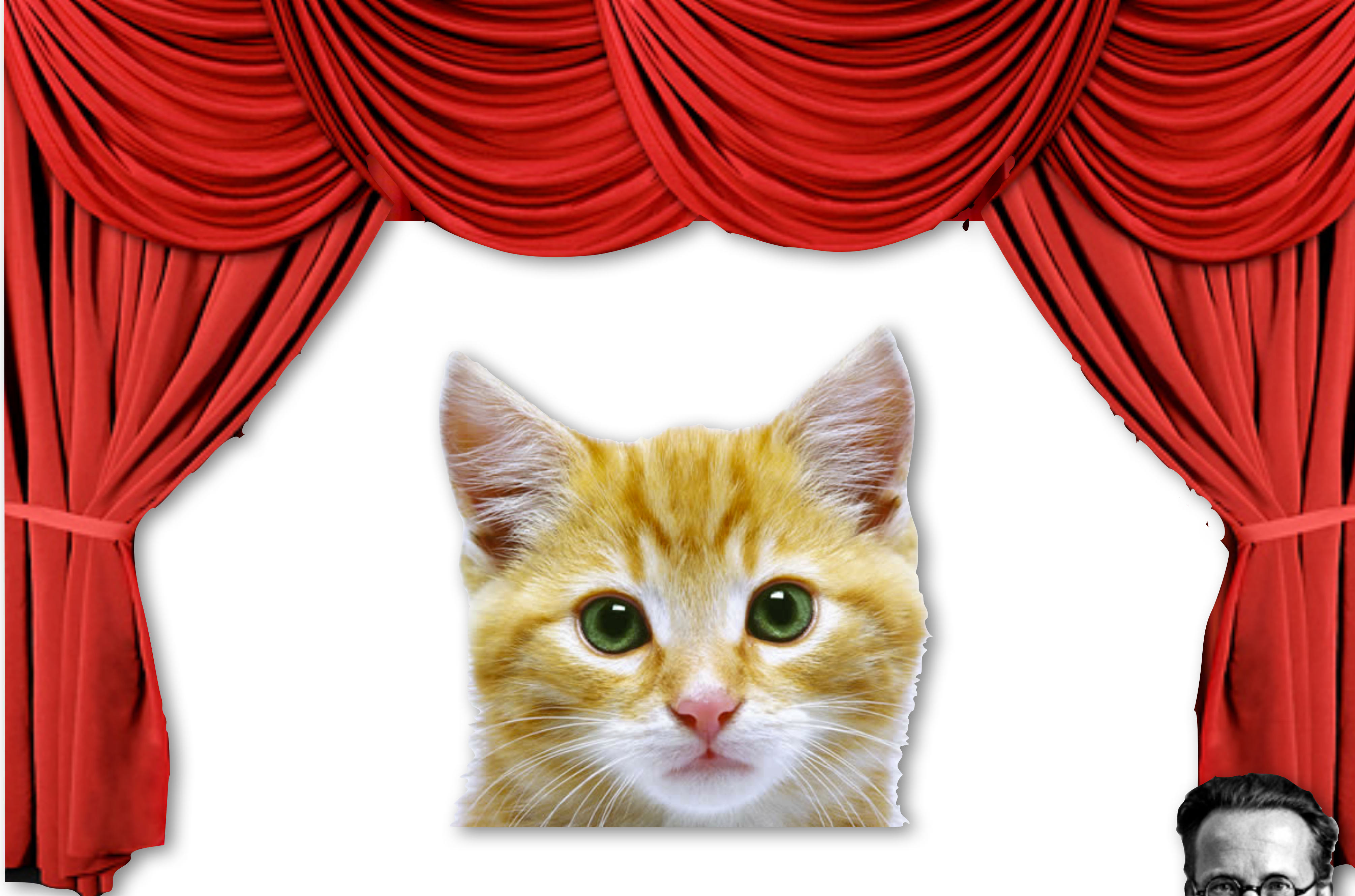
- Death of cat entangled with the quantum mechanical decay of radioactive atoms.

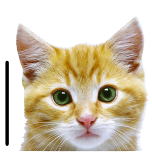
- If atom has 50% chance of decay then state of cat is:

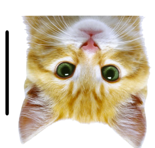
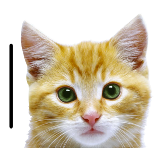
$|\psi\rangle_{\text{cat}} = \frac{1}{\sqrt{2}}\,|$  $\rangle + \frac{1}{\sqrt{2}}\,|$  $\rangle$

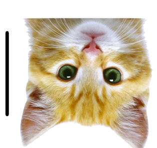


$|\psi\rangle_{\mathrm{cat}} = \frac{1}{\sqrt{2}}\,|$  $\rangle + \frac{1}{\sqrt{2}}\,|$  $\rangle$



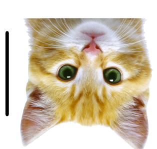
$|\psi\rangle_{\text{cat}} = |$  $\rangle$

$|\psi\rangle_{\text{cat}} = \frac{1}{\sqrt{2}}\,|$  $\rangle + \frac{1}{\sqrt{2}}\,|$  $\rangle$

$|\psi\rangle_{\text{cat}} = |$  $\rangle$

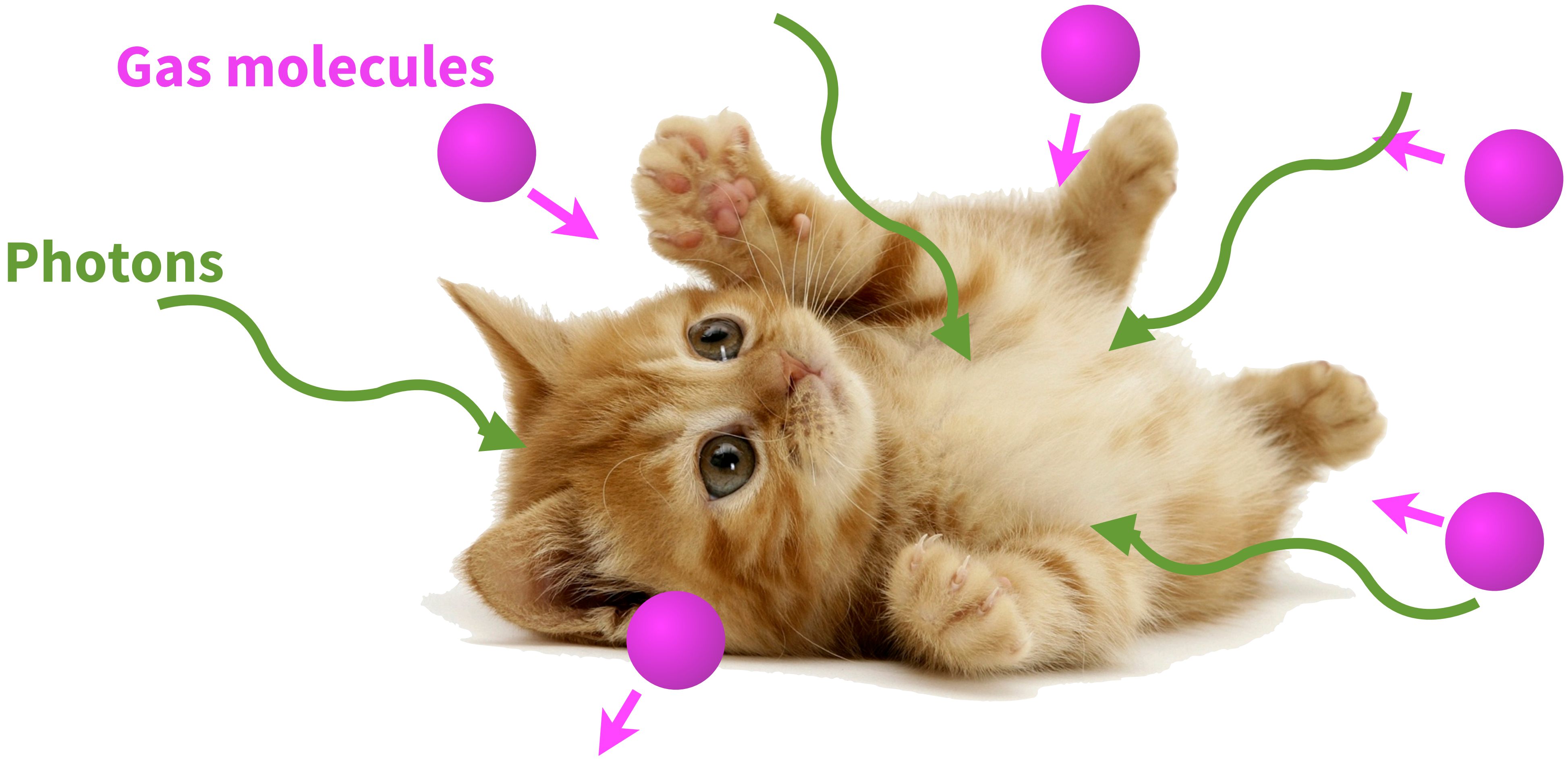
$|\psi\rangle_{\text{cat}} = |$  $\rangle$

- When Schrödinger looks he is making a **measurement**.

$$|\psi\rangle_{\mathrm{cat}} = |$$  $$\rangle$$

- When Schrödinger looks he is making a **measurement**.
- Is the cat simultaneous dead and alive before I measure?

- Absolutely not! The **Environment** is always making measurements.



- Many different environments, all too complicated to keep track of the dynamics.

- Interaction with the environment leads to classicality, (loss of entanglement, superpositions, coherence,...)

- Larger objects -> more environ. interactions.

- Can make quantum objects behave classical.

IBM, 2013.

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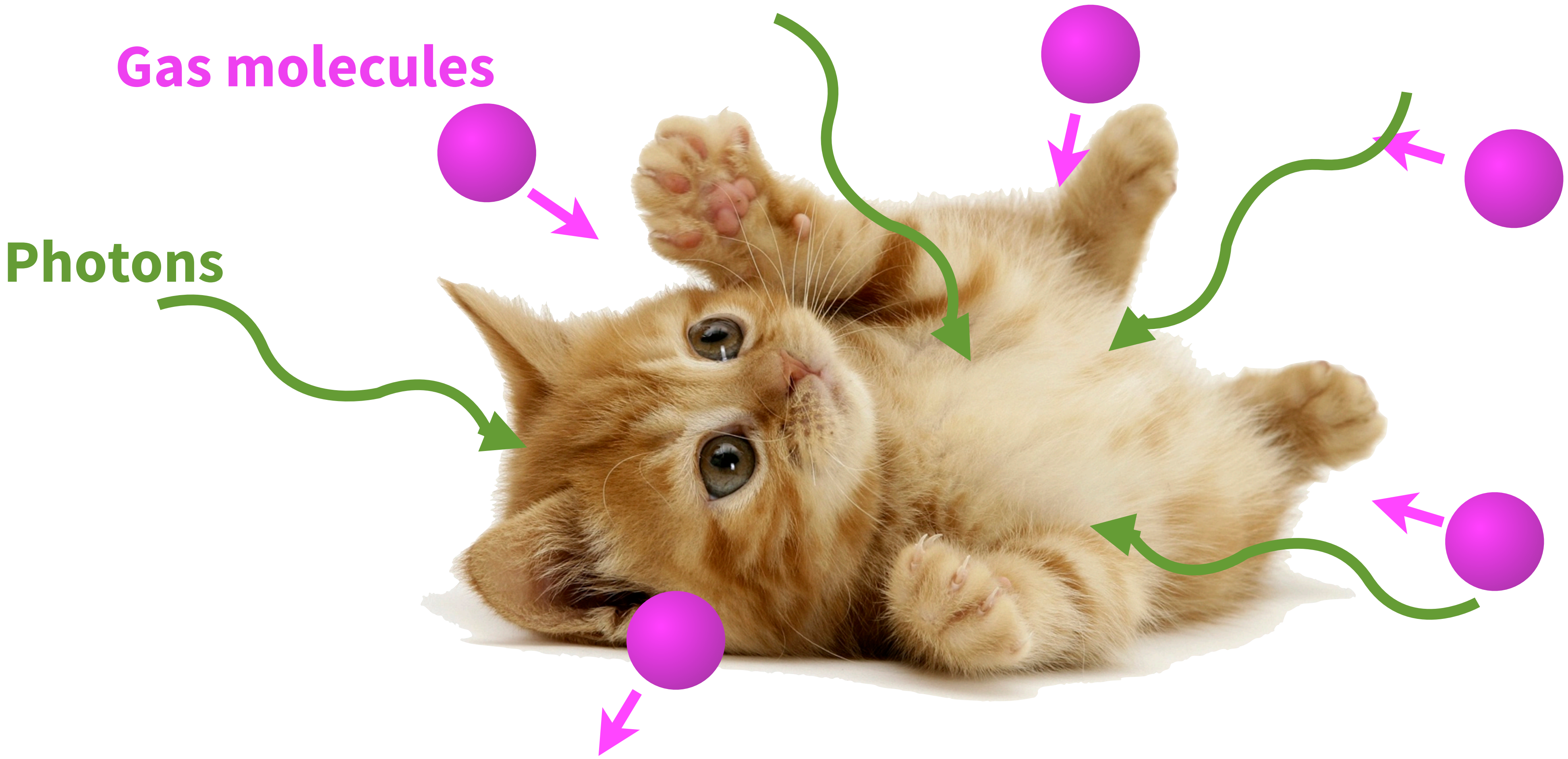

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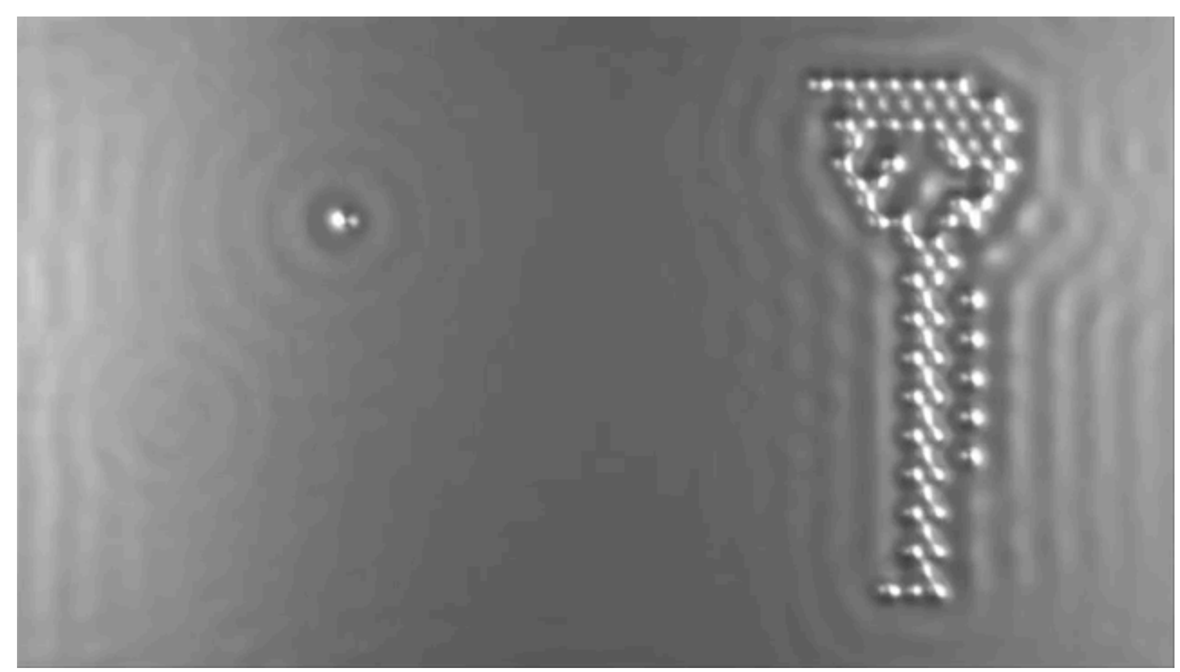


IBM, 2013.

- Can not get rid of all environment effects. Gravity may be ultimate environment!
- Must find balance between quantum dynamics and environmental effects.

Quantum Effects in Massive Objects:

- Must minimize the coupling to the environment.
    - Low temperatures.
    - In vacuum.
- Want quantum dynamics that are clearly distinguishable from classical motion.
- Want massive object, but simple to model theoretically.

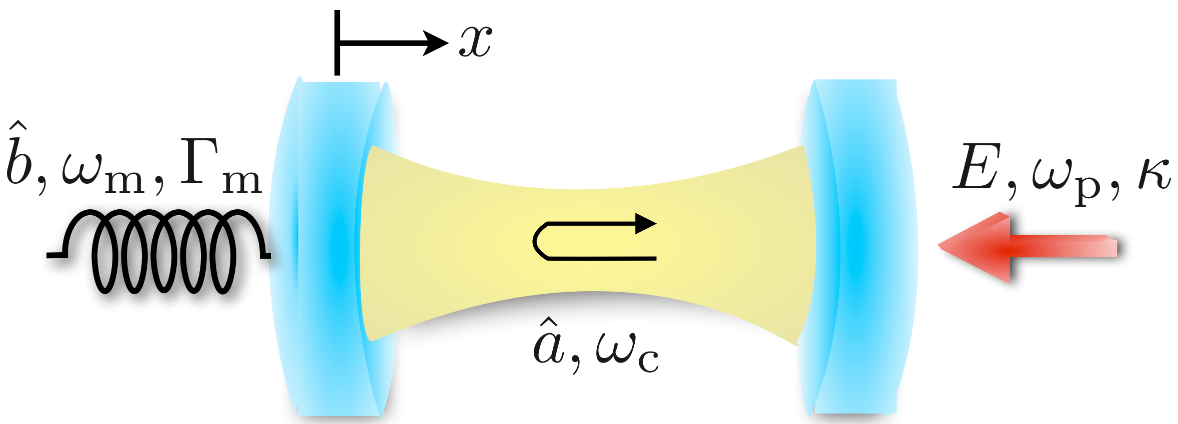
↓

Mechanical Oscillator

+

Nonlinear Interaction

Optomechanics:



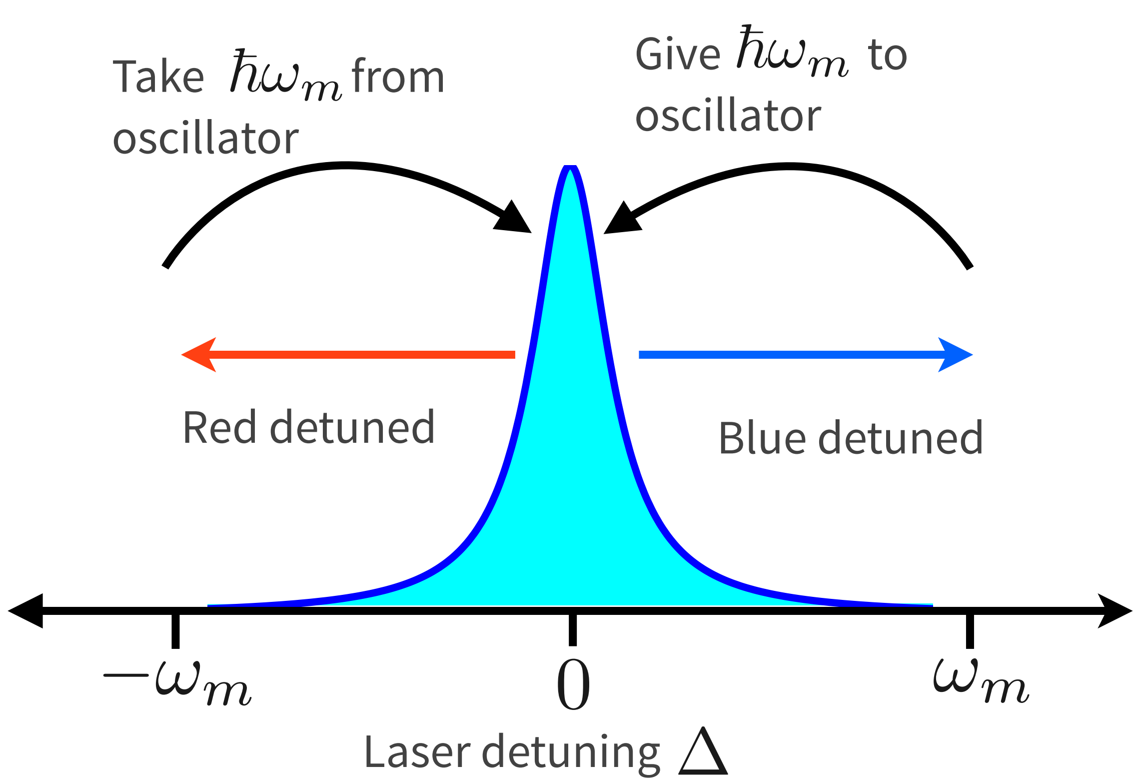
Comet “tail” due to radiation pressure of light.

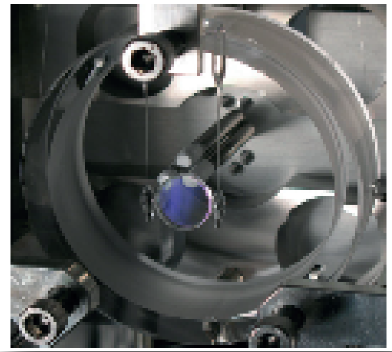
- Interaction between mechanical oscillator and optical cavity via radiation pressure generated by a laser.

- Retardation effects give rise to nonlinear interaction.

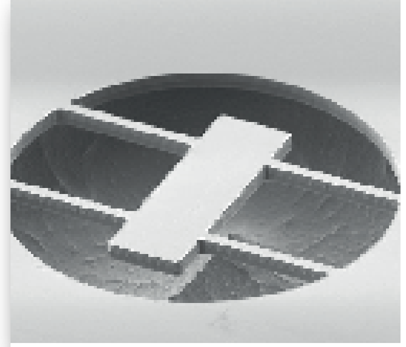
- Changing the laser frequency with respect to the optical cavity resonance frequency leads to cooling or heating of the resonator.

- Same dynamics in many quantum optics related fields.

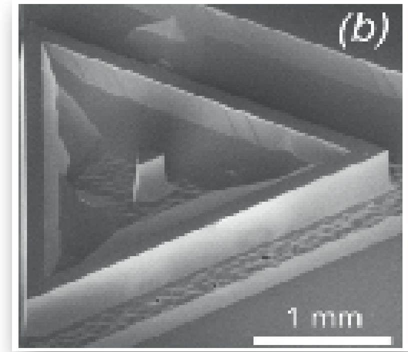




Macroscopic Mirrors



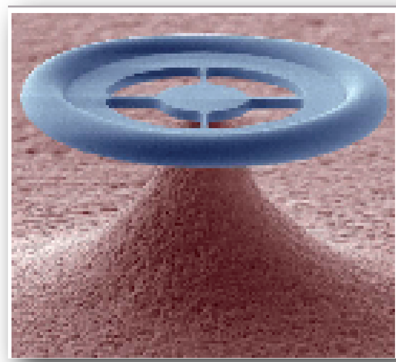
Microscopic Mirrors



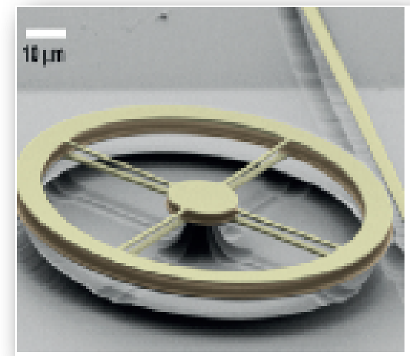
Suspended Pillars

Trampoline Resonators

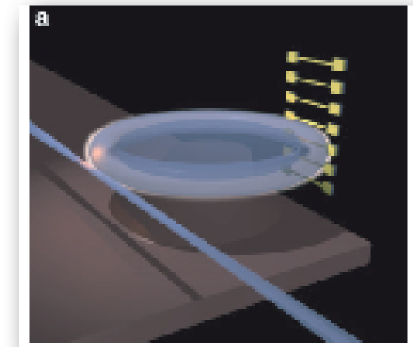
Membranes



Microtoroids

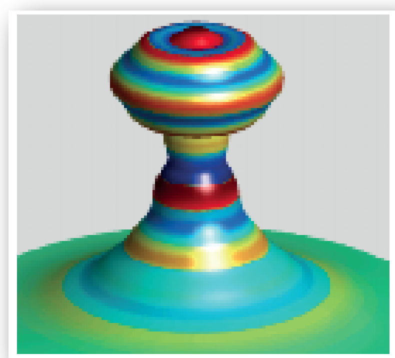


Double-disk Resonators

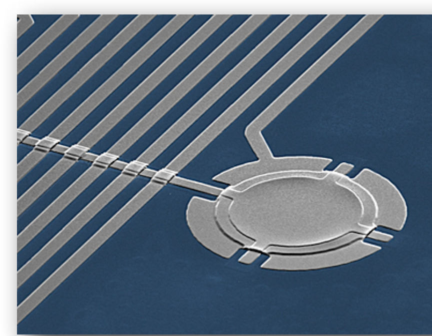


Near-field Resonators

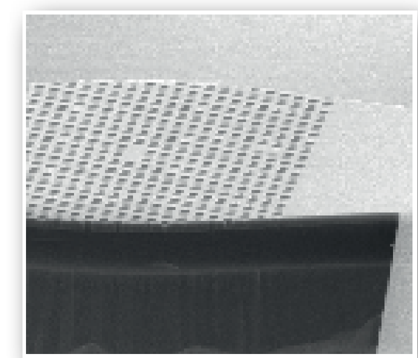
Freestanding Waveguide



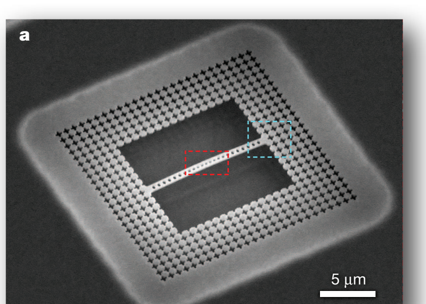
Optical Resonators



Superconducting Circuits

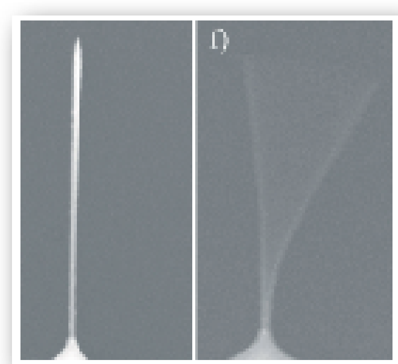


Photonic Crystals

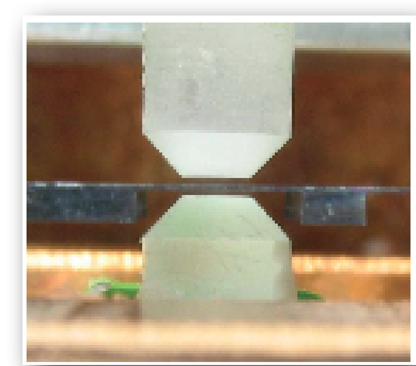


Photonic Nanobeam

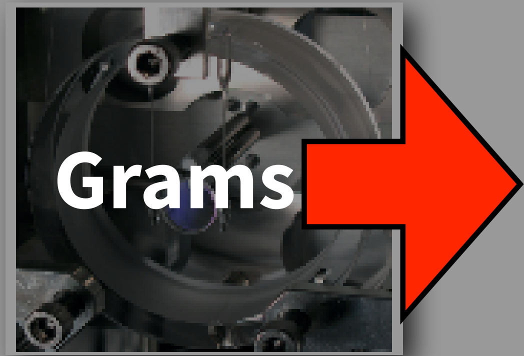
“Zipper” Cavity



Cavity Nanorods



Cold Atom Cavities



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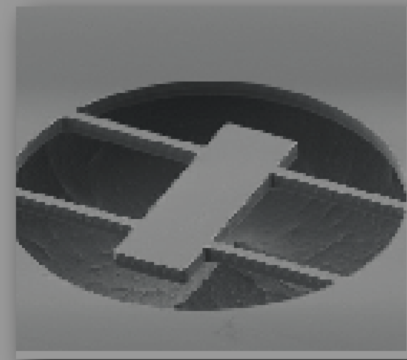
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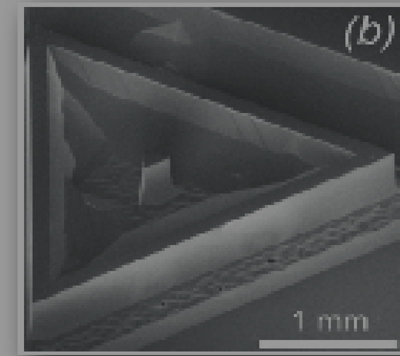

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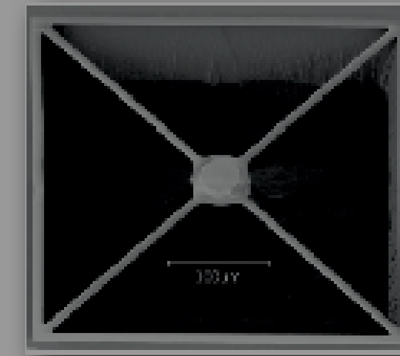
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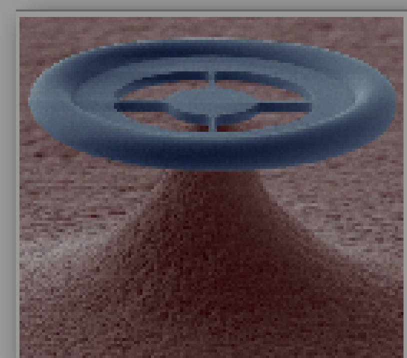
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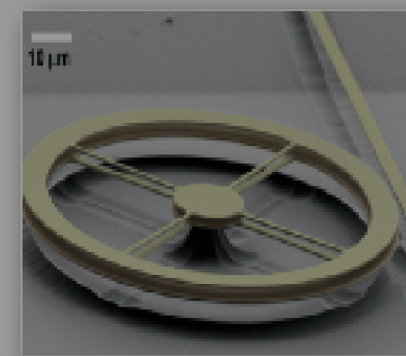
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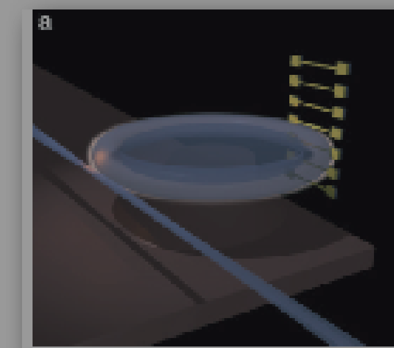
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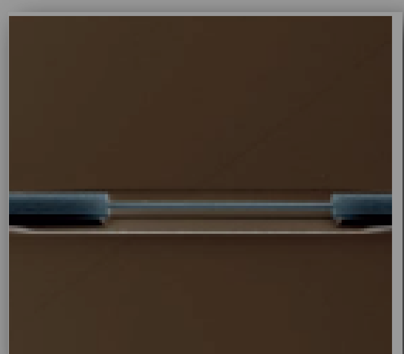
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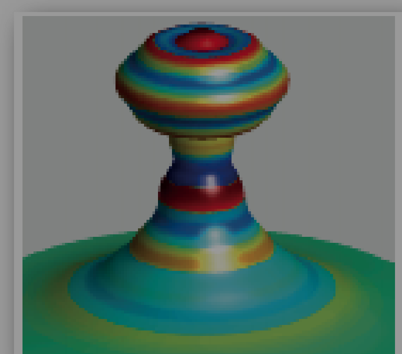
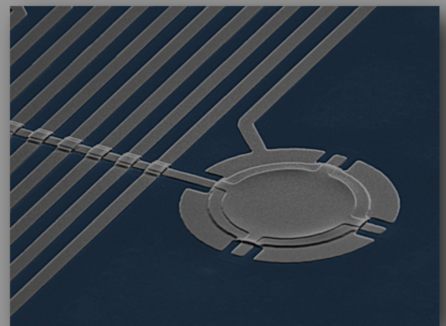

Optical Resonators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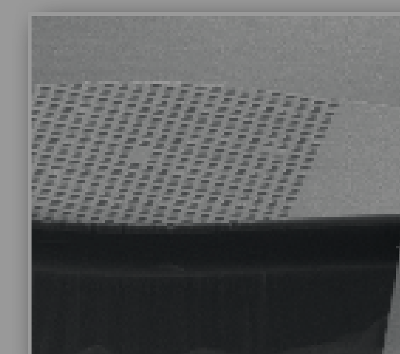
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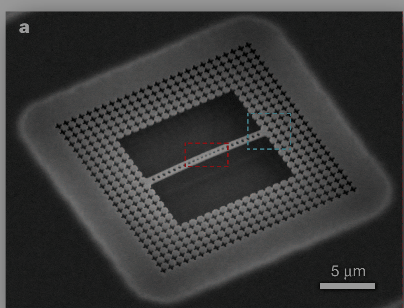

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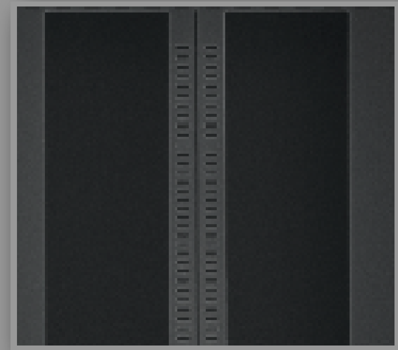
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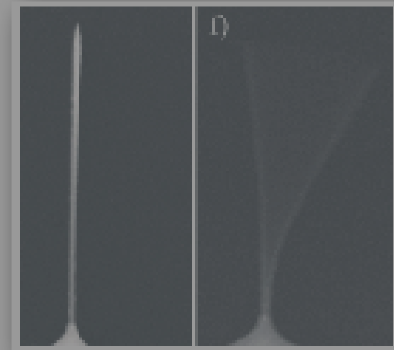
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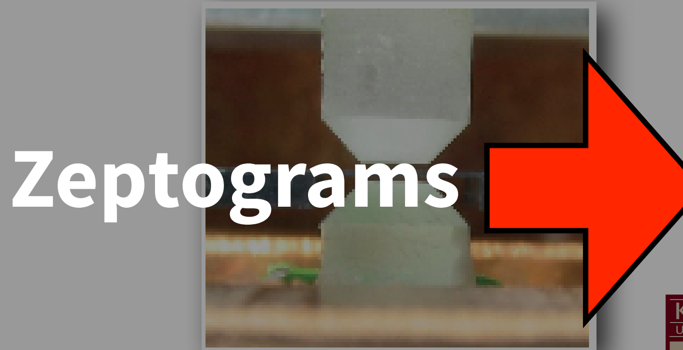

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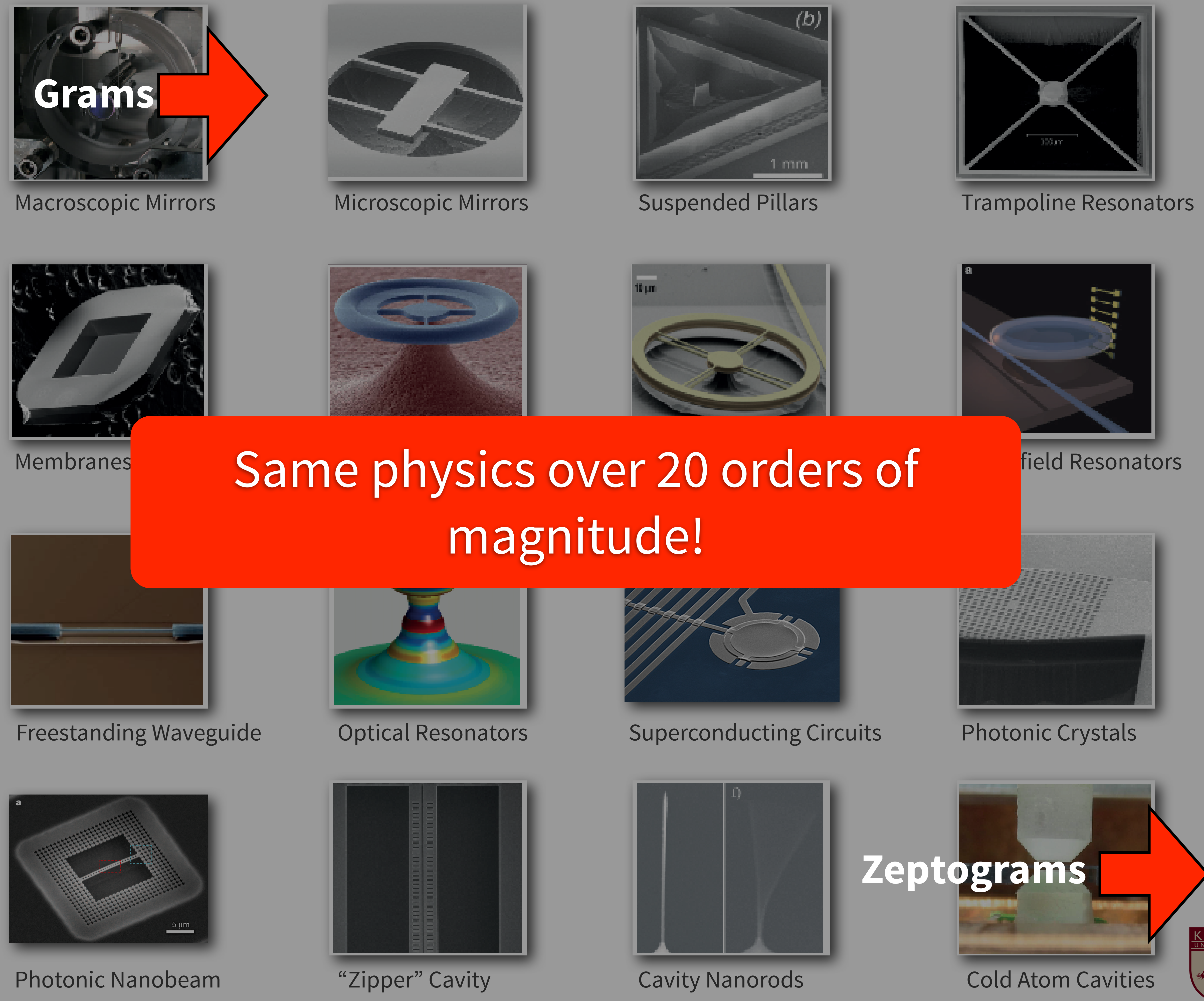

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Applications of Optomechanics:

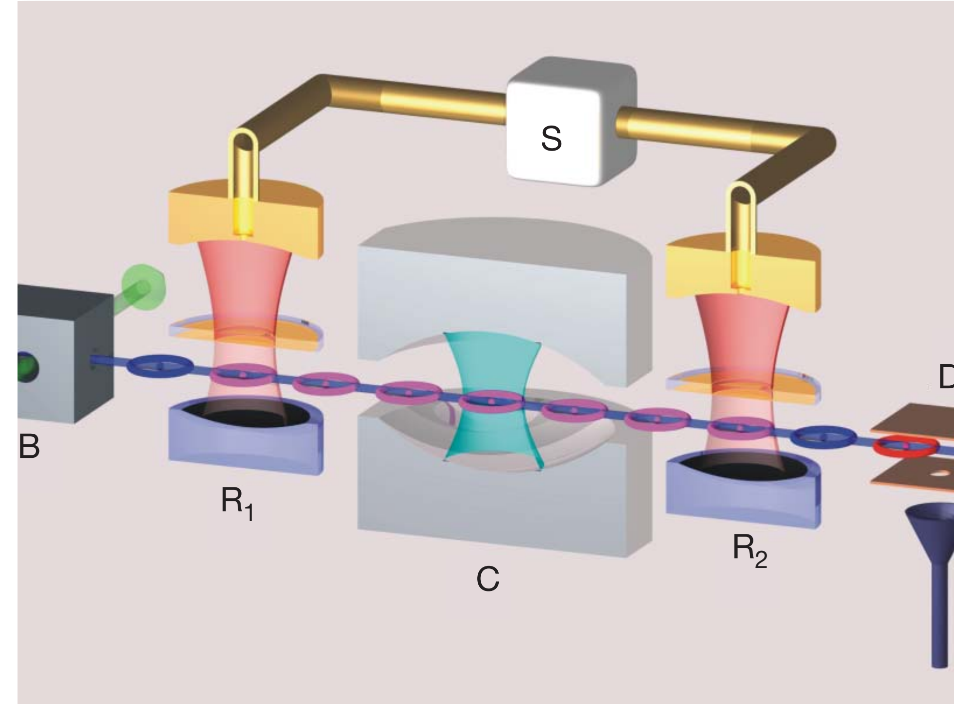
- Ground state cooling of mechanical oscillators.
- Quantum limits on continuous measurements.
- Sensitive force, mass, and position detection.
- Nonclassical states of light and matter.
- Entangled states of light and matter.
- Quantum information processing and storage.

In general,

Optomechanical Interaction $\nRightarrow$ **Nonclassical mechanical states**

- Want to find simple analogue quantum system that leads to nonclassical oscillator states?

Micromaser (single-atom laser):



Gleyzes, Nature (2007)

- Interaction between a stream of excited two-level atoms and an optical cavity.

- Only a single atom in the cavity at a given time.

- Amount of time atom spends in cavity called interaction time t_{int}.

- When cavity has a large quality factor, many interactions → Real quantum laser.

- Steady states of cavity are **sub-Poissonian**, i.e. nonclassical oscillator states.

- Crucial parameter is the "maser pump parameter": $\theta = \sqrt{N_{\text{ex}}} g t_{\text{int}}/2$

N_{ex}: # of atoms passing in cavity lifetime.

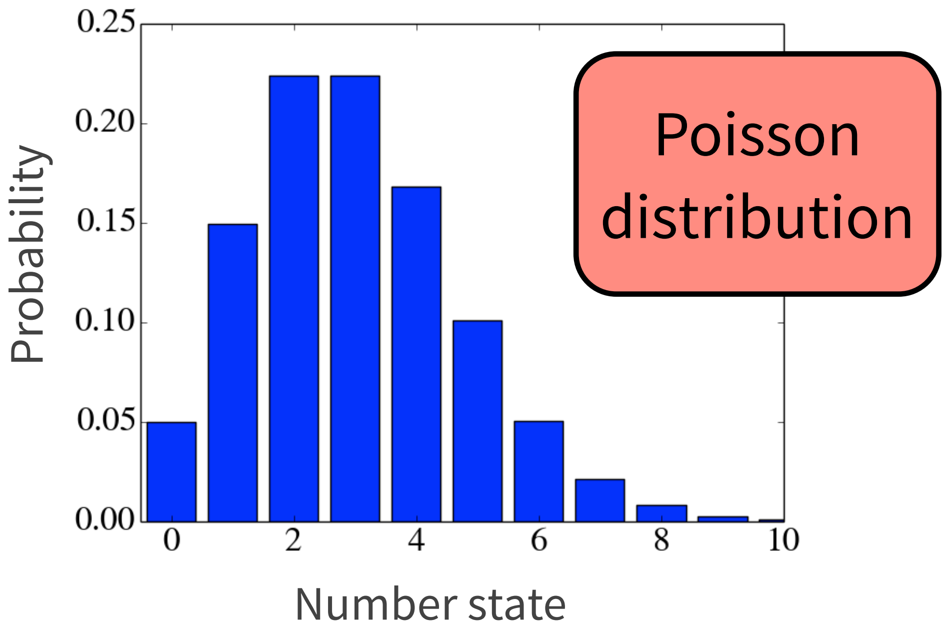
g: atom-cavity coupling

- Varying pump parameter gives oscillations in cavity photon number that can be interpreted as phase transitions: "Thumbprint of the micromaser."

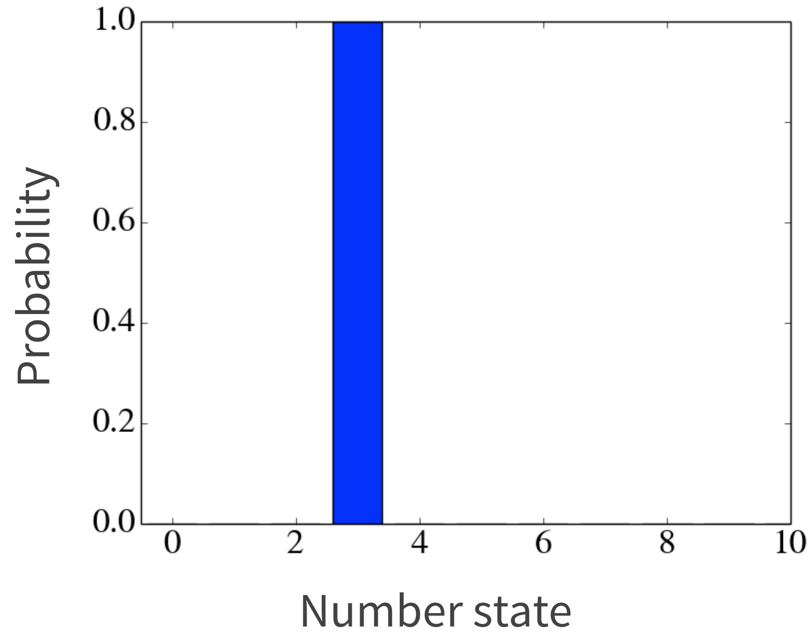
Sub-Poissonian States:

Oscillator **Fano Factor**: $F = \langle (\Delta \hat{N}_b)^2 \rangle / \langle \hat{N}_b \rangle$

Coherent (classical) state $|\alpha = \sqrt{3}\rangle$



Fock (quantum) state $|3\rangle$



- Poisson: Variance equal to average.

➡ F= 1

- Variance vanishes

➡ F= 0

- Sub-Poissonian states are quantum oscillator states with F<1.

- Strongly sub-Poissonian states characterized by negative **Wigner functions**.

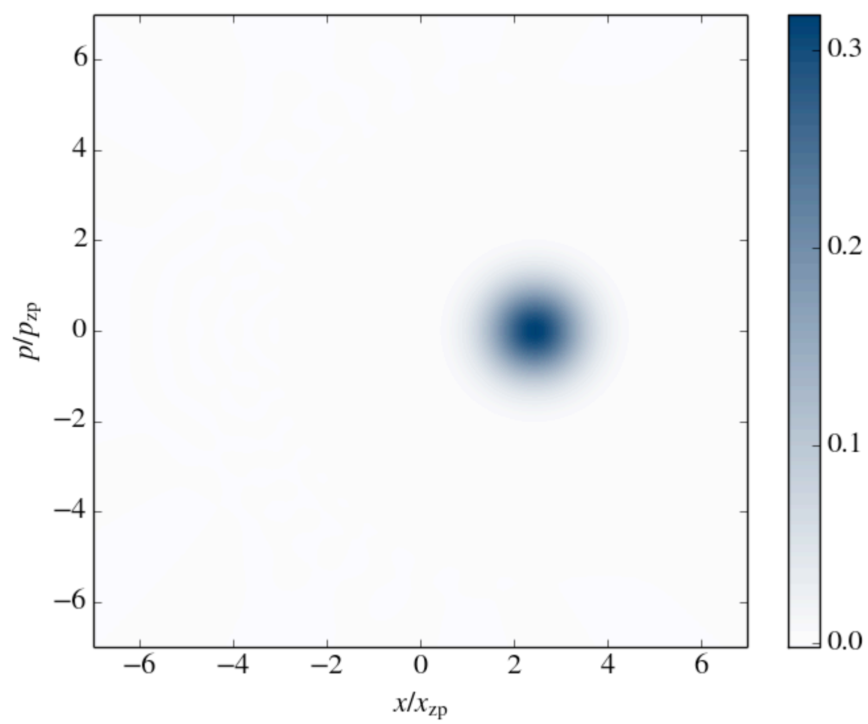
Wigner Functions:

- A quantum phase space (pseudo)probability density distribution.

- Not a true probability distribution due to $[\hat{x}, \hat{p}] = i\hbar$.

- Can possess (nonclassical) regions where distribution is negative.

Coherent state $|\alpha = \sqrt{3}\rangle$



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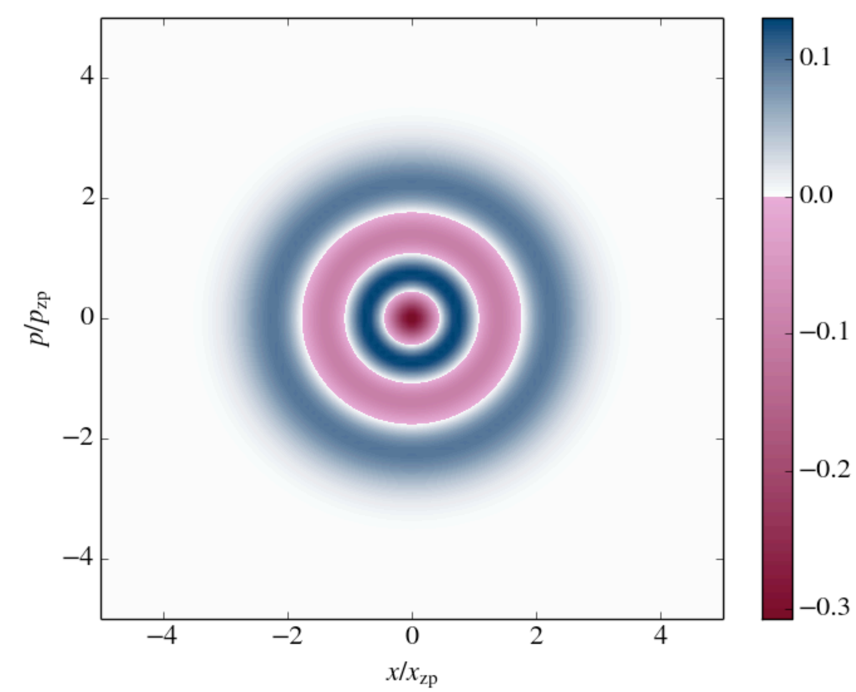


Positive Wigner func. $F \geq 1$

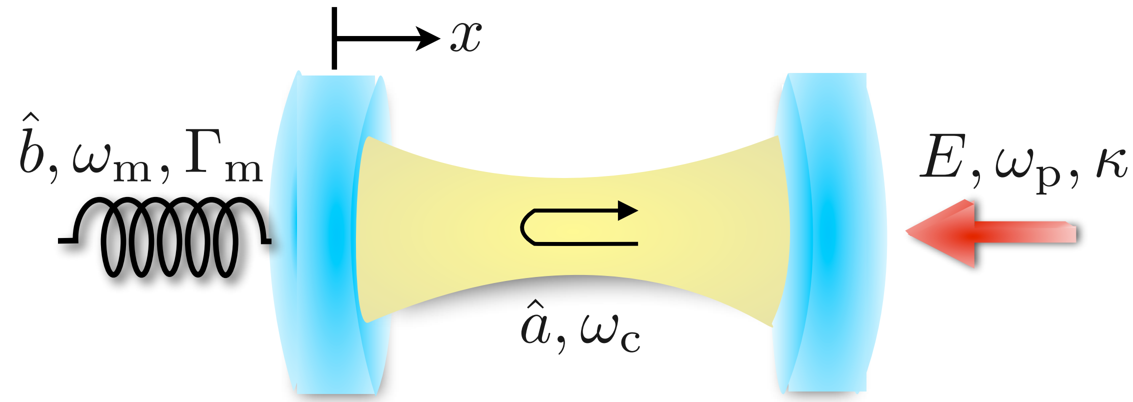
Positive Wigner func. $F < 1$

Negative Wigner func. $F \ll 1$

- Negativity of Wigner function can be used as measure of nonclassicality.

Optomechanical Setup:

- Consider a single-mode, driven optomechanical system



$$\hat{H} = -\Delta \hat{a}^{+}\hat{a} + \hat{b}^{+}\hat{b} + g_0\left(\hat{b}+\hat{b}^{+}\right)\hat{a}^{+}\hat{a} + E\left(\hat{a}+\hat{a}^{+}\right)$$

Cavity HO — Mech. HO — Radiation pressure coupling — Pumping of cavity

- All constants measured in units of the resonator frequency.

- Laser-cavity detuning given by $\Delta = \left(\omega_p - \omega_c\right)/\omega_m$

- Coupling constant g_0 measures oscillator displacement due to a single cavity photon in units of the zero-point amplitude: $x_{\rm zp} = \sqrt{\hbar/2m\omega_m}$

Key Idea: Consider high-Q resonator, $\Gamma_m = Q_m^{-1}$, and low-Q cavity, with damping rate κ, driven by weak laser.

$\Rightarrow$ $\langle \hat{N}_a \rangle \approx \langle (\Delta \hat{N}_a)^2 \rangle \ll 1$ Single-photon interaction!

Semiclassical Dynamics:

- Input-output theory gives Langevin equations of motion for Hamiltonian operators ($\tau = \omega_m t$).

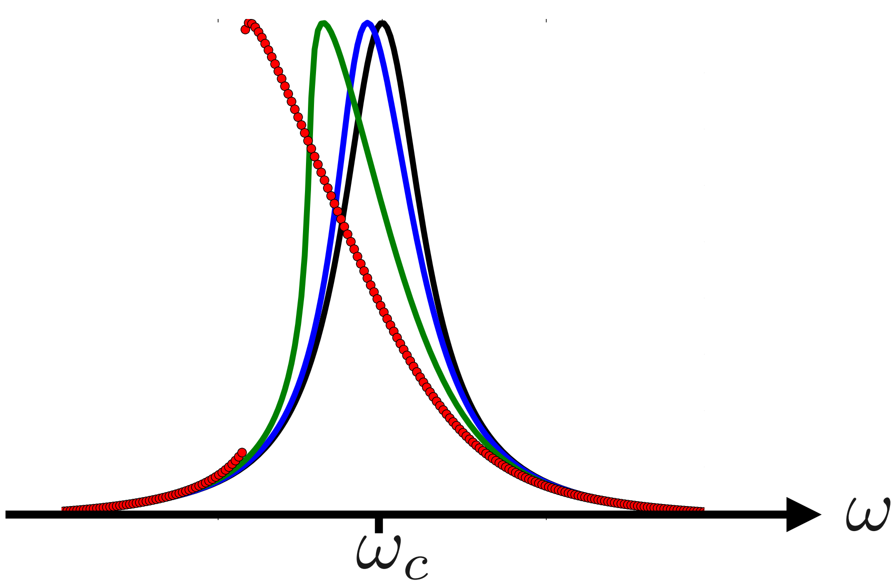
$$\frac{d\hat{a}}{d\tau} = i\Delta\hat{a} - ig_0\left(\hat{b} + \hat{b}^{+}\right)\hat{a} - \frac{\kappa}{2}\hat{a} - iE$$

$$\frac{d\hat{b}}{d\tau} = -i\hat{b} - ig_0\hat{a}^{+}\hat{a} - \frac{\Gamma_m}{2}\hat{b} - \sqrt{\Gamma}\hat{b}_{\mathrm{in}}$$

- Classical nonlinear effects can be studied in the steady state regime.

- Steady state cavity energy $\bar{N}_a$ given by:

$$E^2 = \left(\Delta^2 + \kappa^2/4\right)\bar{N}_a - 2\Delta\mathcal{K}\bar{N}_a^2 + \mathcal{K}^2\bar{N}_a^3\,; \quad \mathcal{K} = -\frac{2g_0^2}{\left(1 + \frac{\Gamma_m^2}{4}\right)}$$ (Kerr constant)



“Spring-softening”

- The renormalized cavity frequency can be defined by the detuning value at which $\bar{N}_a$ is maximized.

- The semiclassical limit-cycle dynamics of both the cavity and oscillator found by assuming oscillator undergoes sinusoidal motion (Marquardt et al. PRL 2006):

$$x(\tau) = \bar{x} + A\cos(\tau)$$

Static displacement ($\bar{x}$) Oscillation amplitude (A)

- Plug into Langevin equation for cavity amplitude $\bar{a}(\tau)$ and use Fourier series solution:

$$\bar{a}(\tau) = e^{i\varphi(\tau)} \sum_{n=-\infty}^{\infty} \alpha_n e^{in\tau} \quad \Rightarrow \quad \alpha_n = -iE \frac{J_n(g_0 A)}{i\left(n - \Delta + g_0\bar{x}\right) + \kappa/2}$$

- Time-averaged response $\overline{\langle|\bar{a}|^2\rangle} = \sum_n |\alpha_n|^2$ peaked at discrete values:

$\Delta = n + g_0\bar{x}$ n labels oscillator sidebands, i.e. $n\omega_m$

Shift due to Kerr nonlinearity (as we will see) ($g_0\bar{x}$)

- Lineshape is Lorentzian, but peak is shifted depending on g_0.

- Displacement $\bar{x}$ and amplitude A are found by self-consistently solving time averaged force balance:

$$\bar{x} = -2g_0 \sum_n |\alpha_n|^2 \quad \Rightarrow \quad g_0\bar{x} \propto \mathcal{K}$$

and power balance equations:

$$\Gamma_m A = -4g_0 \mathrm{Im} \sum_n \alpha^*_{n+1}\alpha_n$$

- In general, there are **multiple solutions to these equations**; multiple oscillator limit-cycles exist for a given set of parameters.

Quantum Dynamics:

- Here we are interested in the single-photon strong-coupling regime: $g_0^2/\kappa\omega_m \gtrsim 1$
    - Discreteness of cavity photons becomes important.
    - Radiation pressure of single-photon displaces resonator by more than its zero-point linewidth.

- Will use Master equation for full quantum dynamics to find steady state of system

$$\frac{d\hat{\rho}}{d\tau} = \mathcal{L}\hat{\rho} = -i\left[\hat{H}, \hat{\rho}\right] + \mathcal{L}_{\mathrm{cav}}[\hat{\rho}] + \mathcal{L}_{\mathrm{mech}}[\hat{\rho}] = 0$$

$$\mathcal{L}_{\mathrm{cav}} = \frac{\kappa}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right)$$

$$\mathcal{L}_{\mathrm{mech}} = \frac{\Gamma_m}{2}\left(\bar{n}_{\mathrm{th}} + 1\right)\left(2\hat{b}\hat{\rho}\hat{b}^{+} - \hat{b}^{+}\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^{+}\hat{b}\right) + \frac{\Gamma_m}{2}\bar{n}_{\mathrm{th}}\left(2\hat{b}^{+}\hat{\rho}\hat{b} - \hat{b}\hat{b}^{+}\hat{\rho} - \hat{\rho}\hat{b}\hat{b}^{+}\right)$$

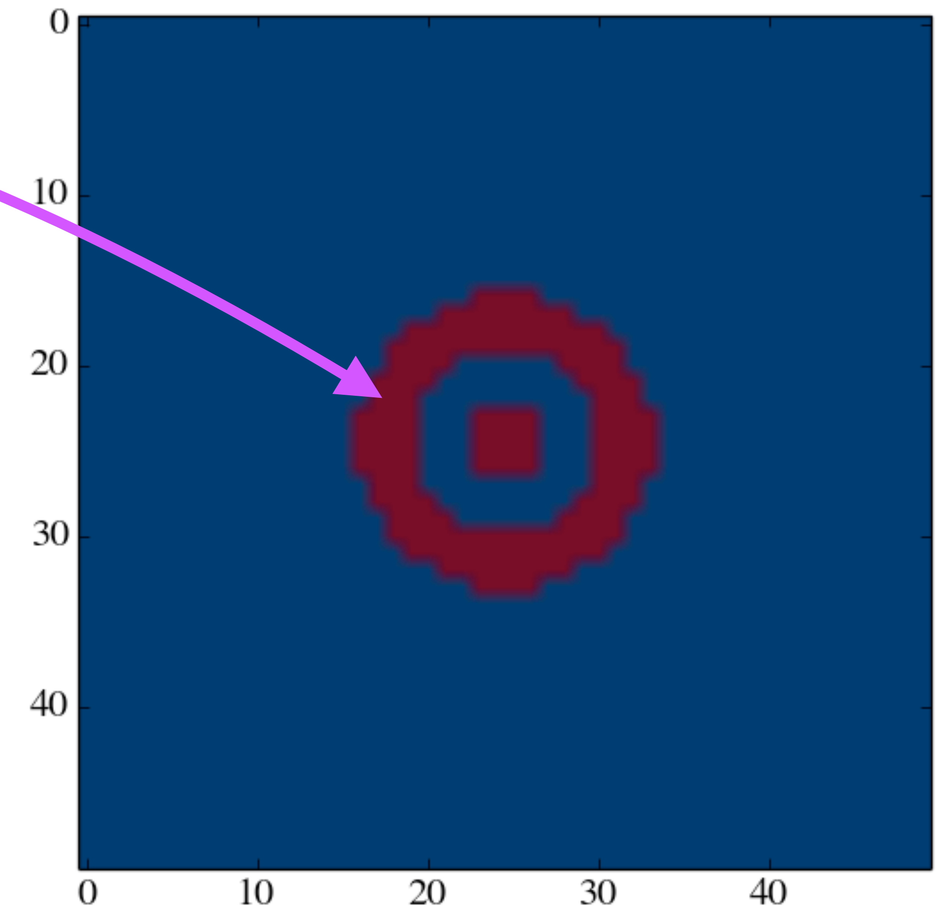
- Oscillator bath characterized by avg. excitation number:

$$\bar{n}_{\mathrm{th}} = \left[\exp(\hbar\omega_m/k_{\mathrm{B}}T) - 1\right]^{-1}$$

- To quantify the amount of "quantumness" in our oscillator states, we will take the ratio of the sum of negative Wigner densities over the positive density elements.

$$\eta = \frac{\sum_n |w_n^{(-)}|}{\sum_m w_m^{(+)}} = \frac{\sum_n |w_n^{(-)}| dxdp}{1 + \sum_n |w_n^{(-)}| dxdp}$$

"Nonclassical ratio"

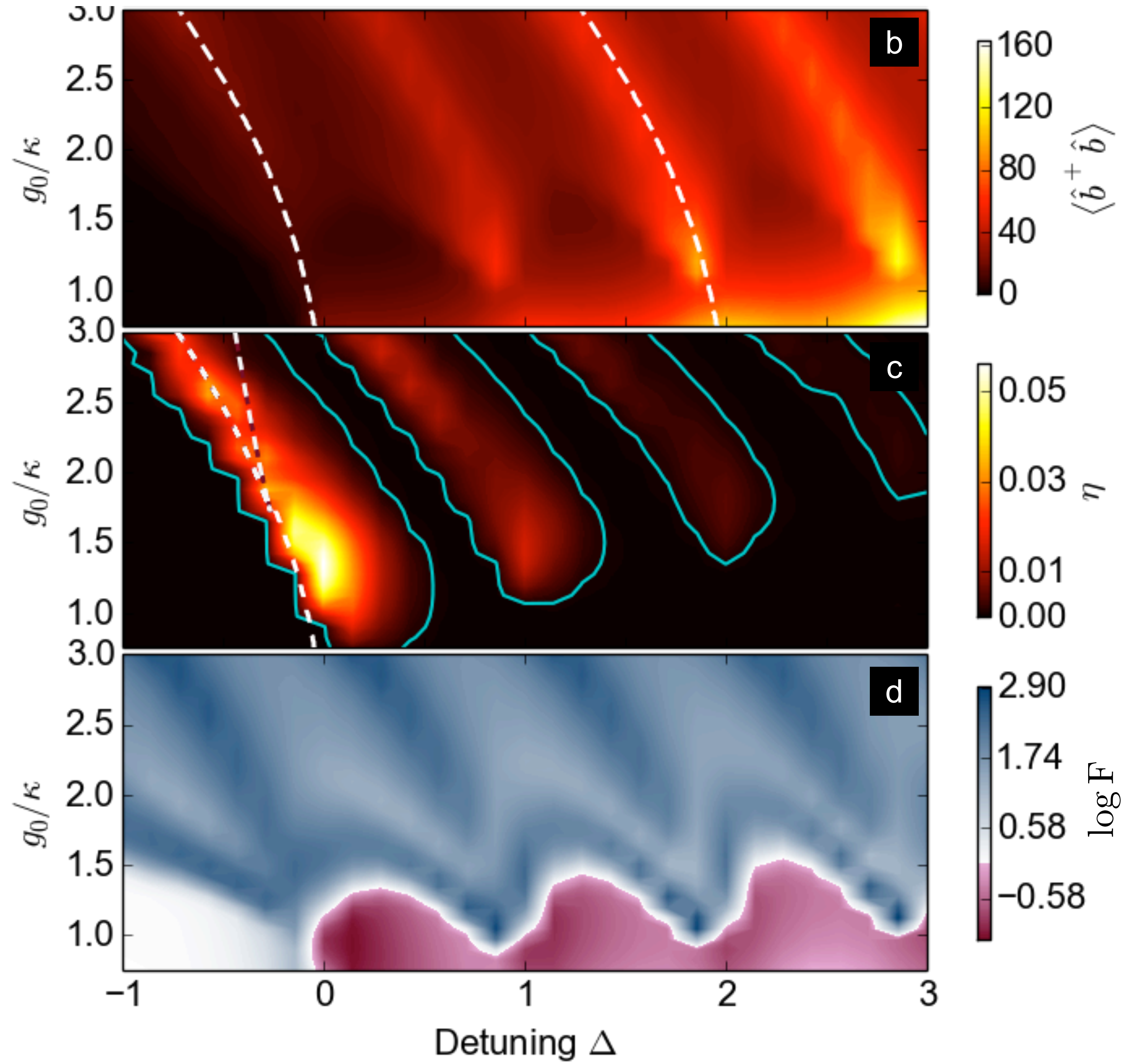


- For the states considered here, this ratio is nearly linear, a good benchmark for comparison.

- Note: You can not just count the number of negative and positive elements.

Results:

- Simulation parameters: $E = 0.1, \kappa = 0.3, Q_m = 10^4, \bar{n}_{\mathrm{th}} = 0$



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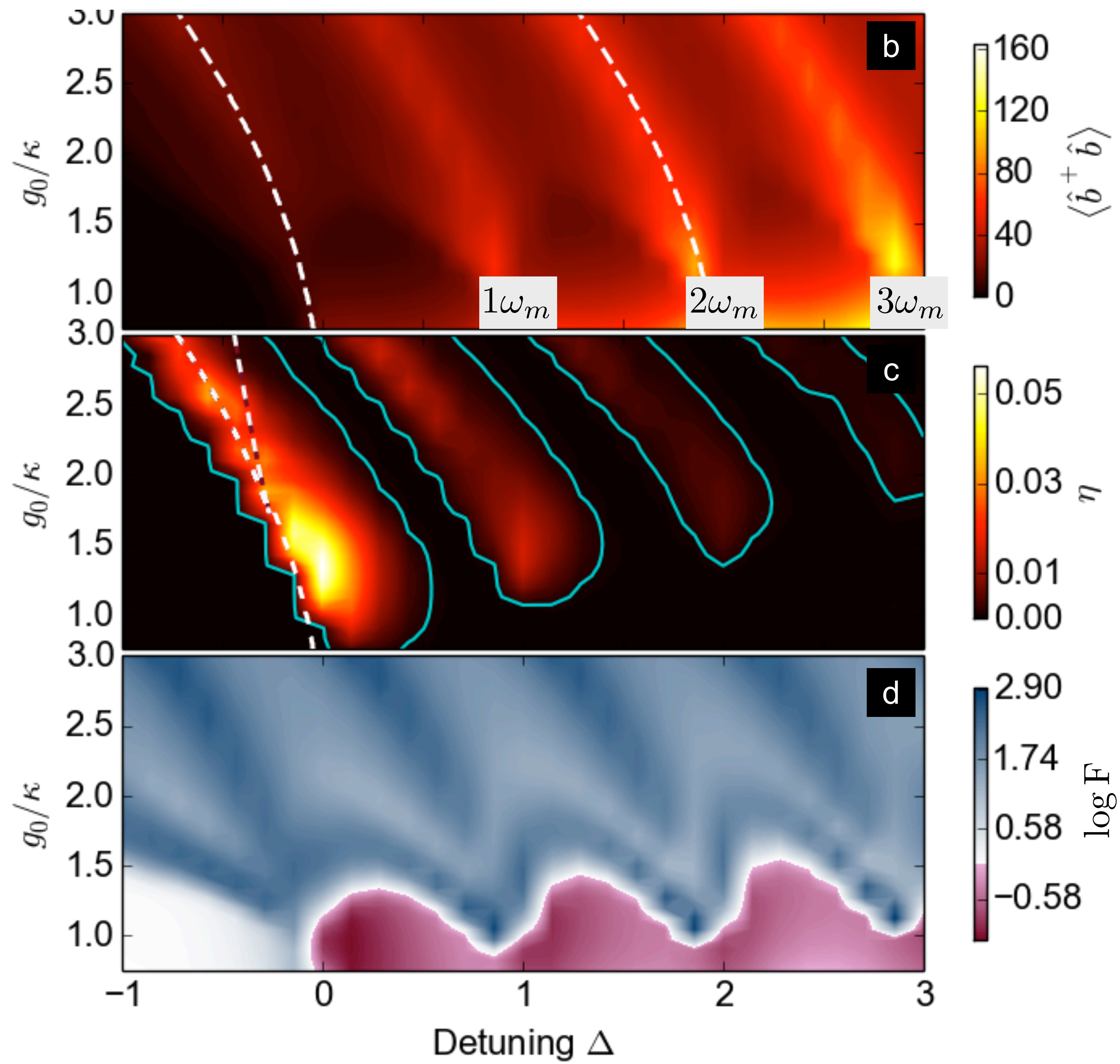
Mechanical sidebands



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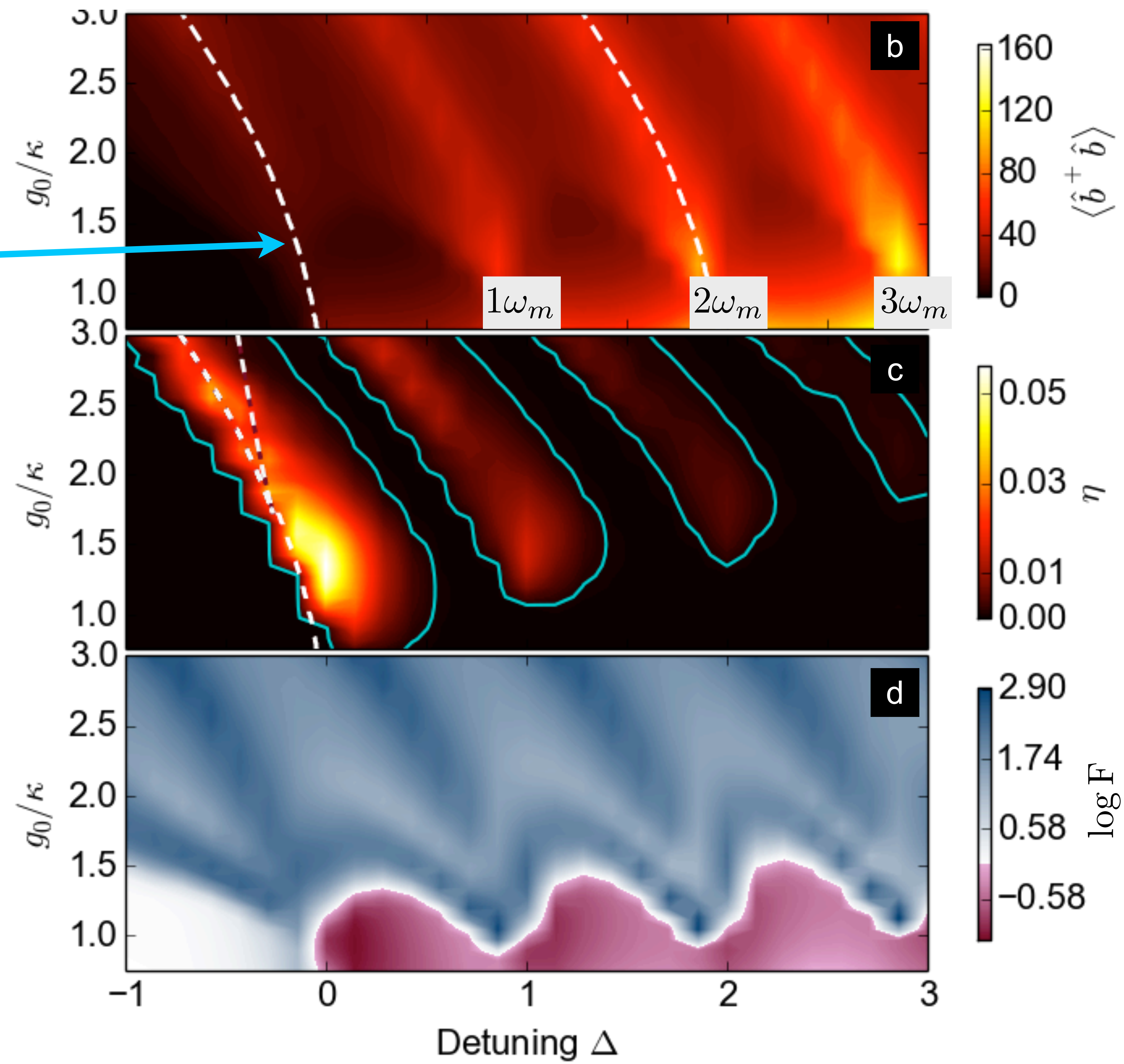
Mechanical sidebands

Nonlinear frequency-pulling



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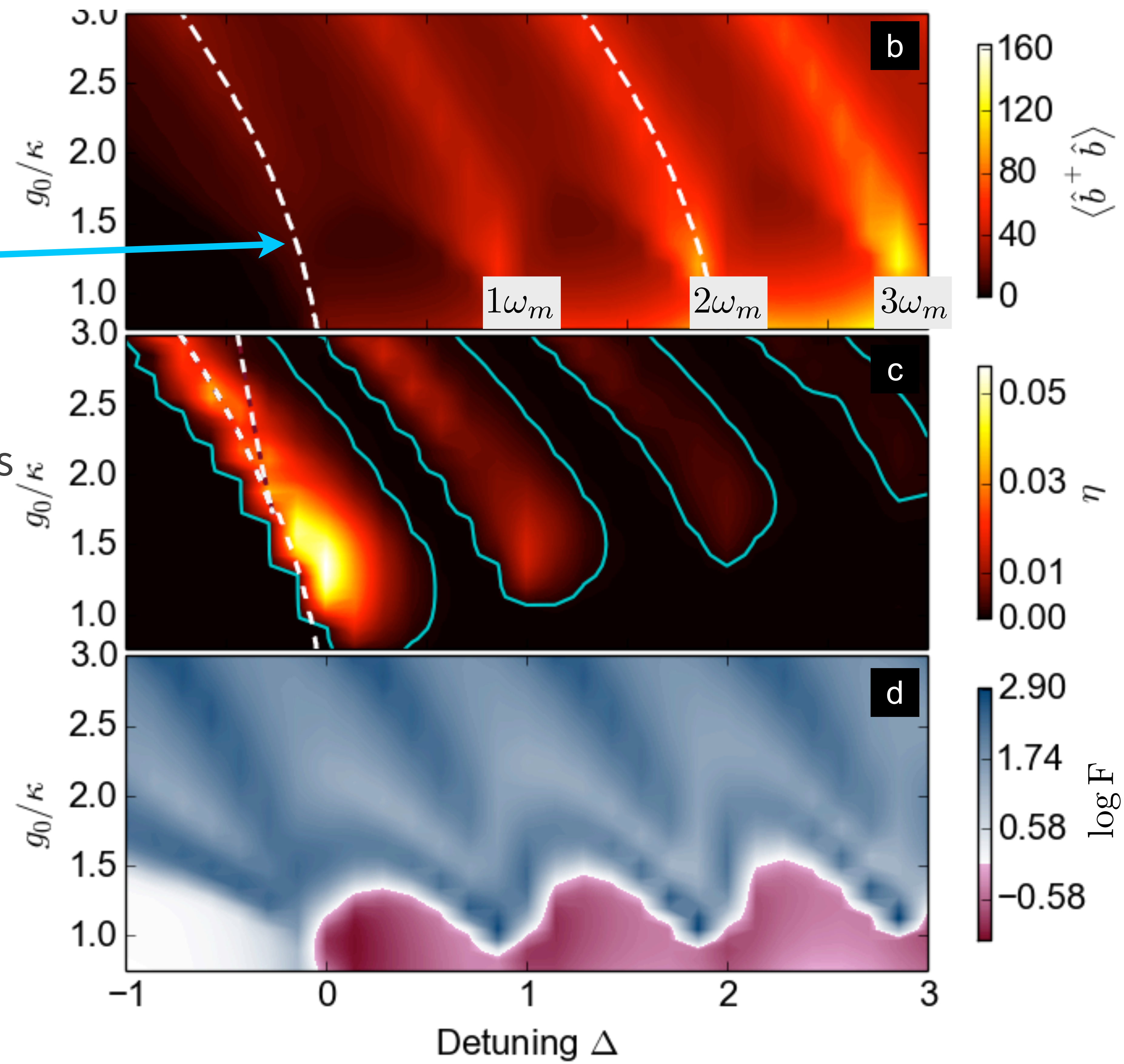
Mechanical sidebands

Nonlinear frequency-pulling

Strongest on resonance

Increasing coupling leads to decrease in quantum features.



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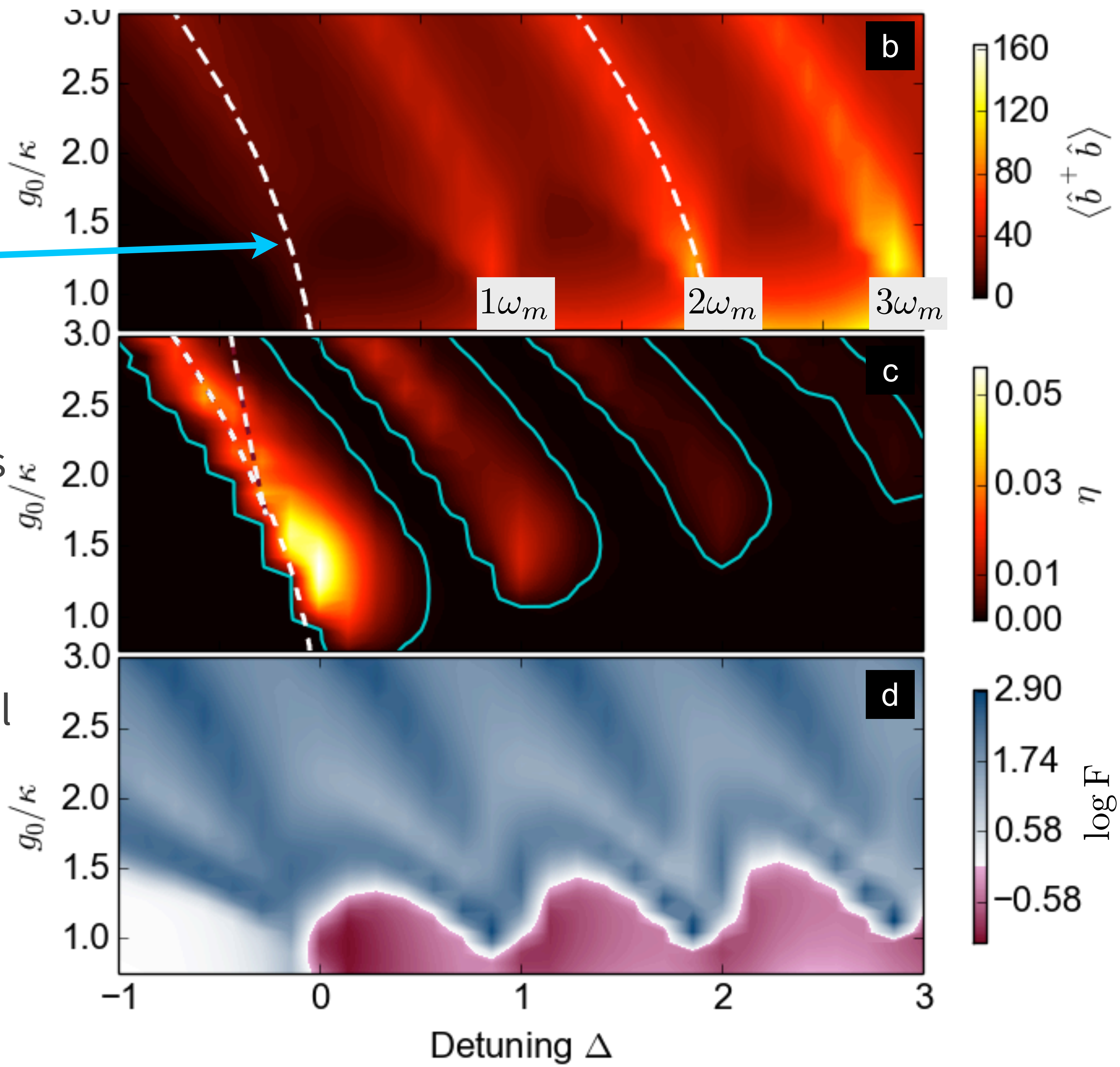
Strongest on nonclassical states occur where Fano factor is **larger** than one!



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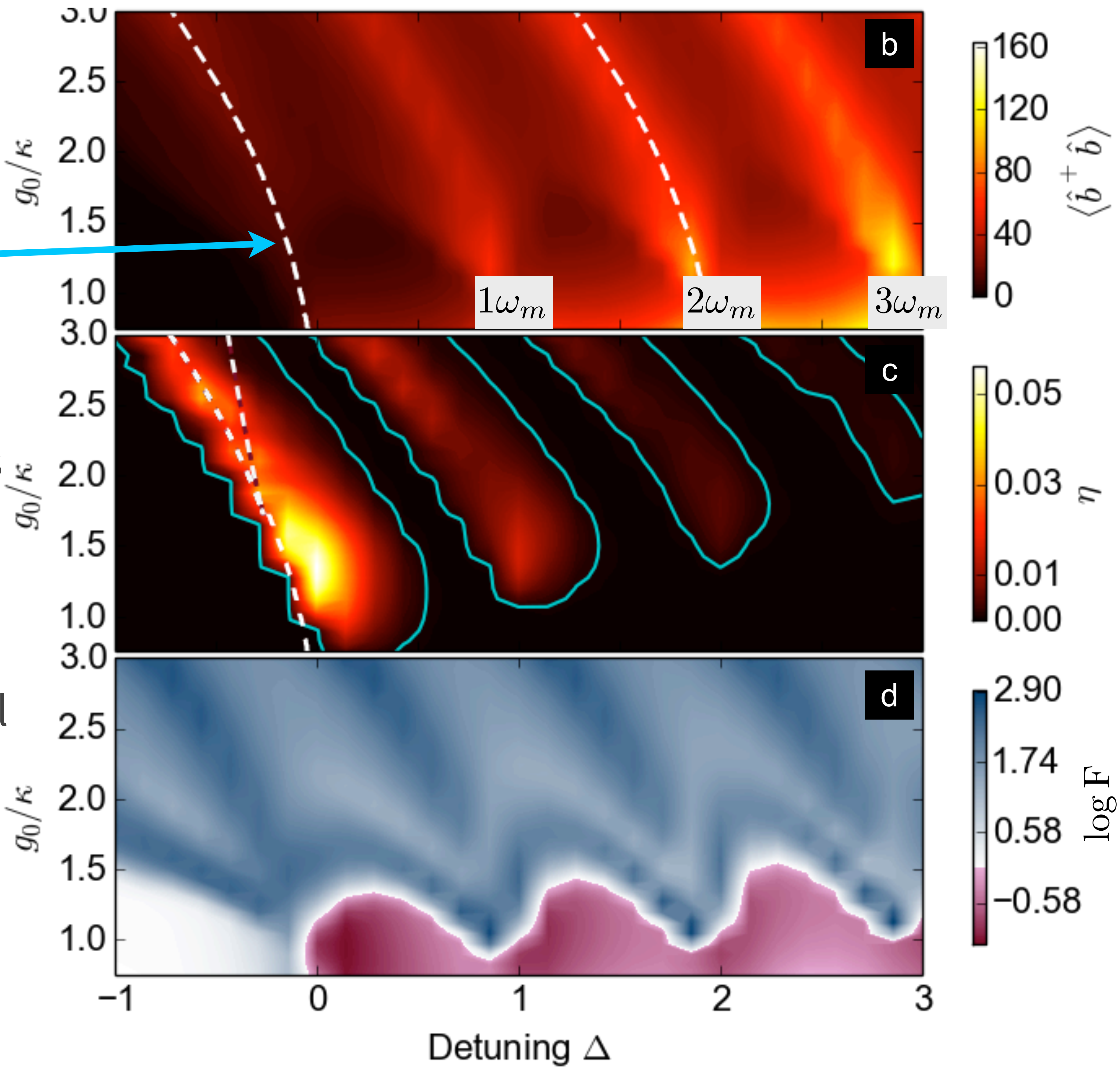

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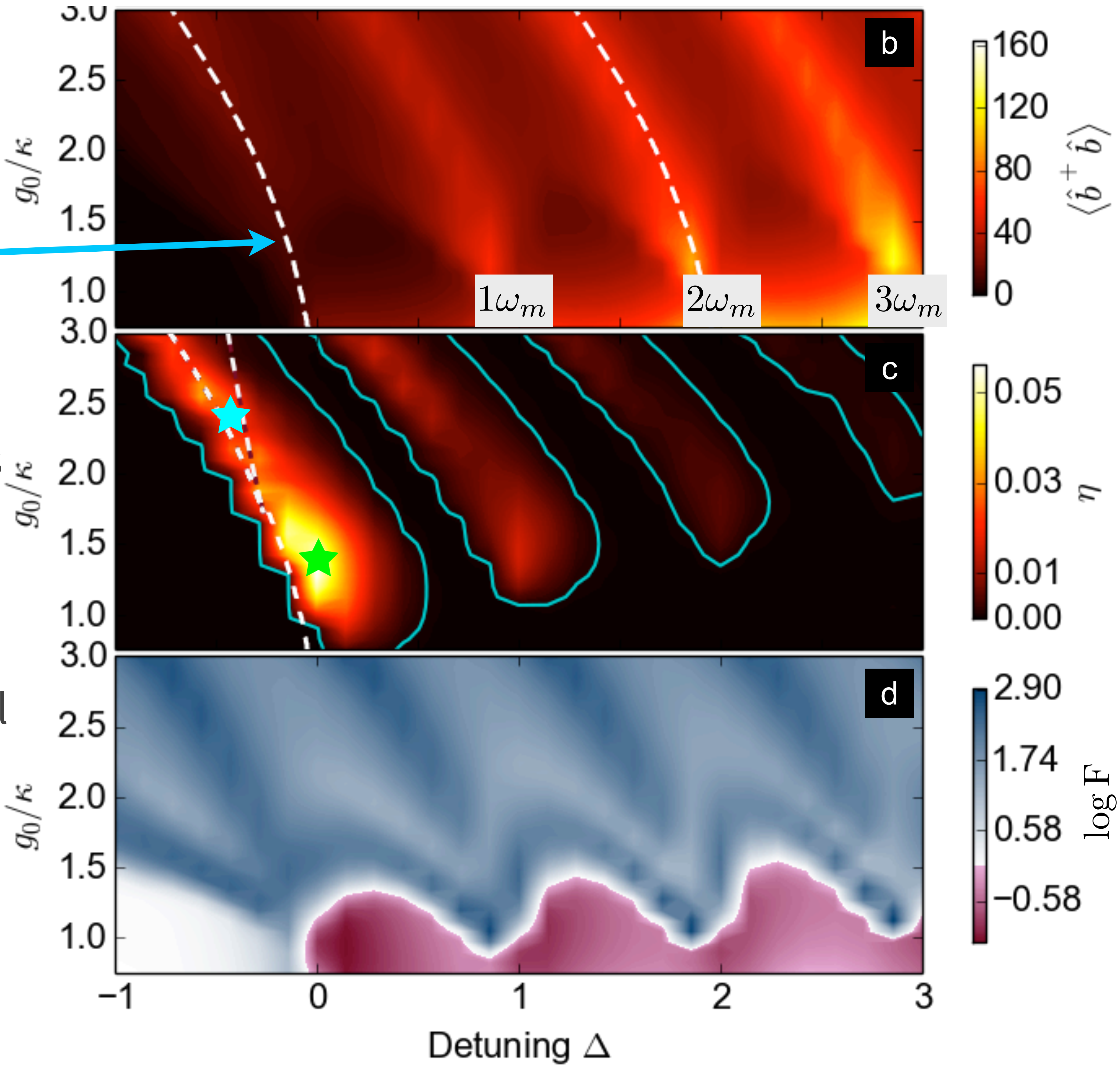
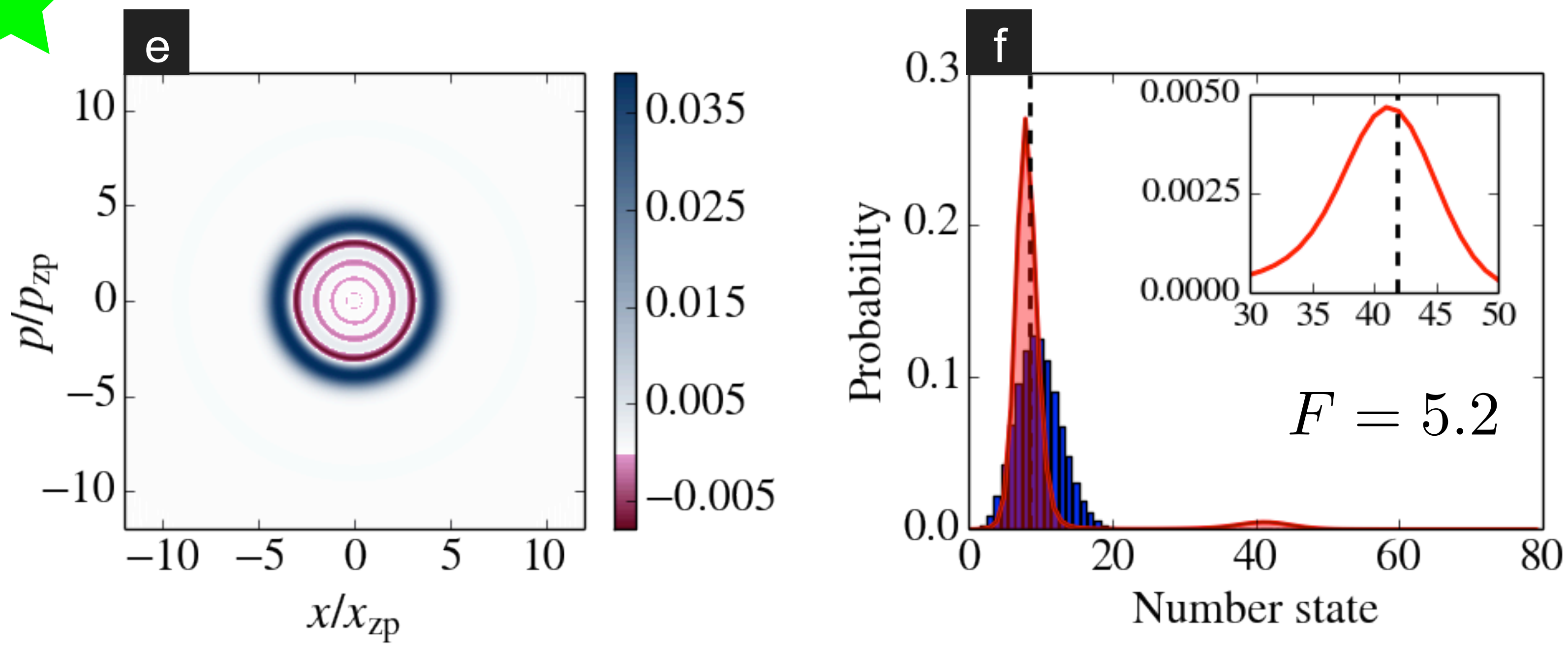


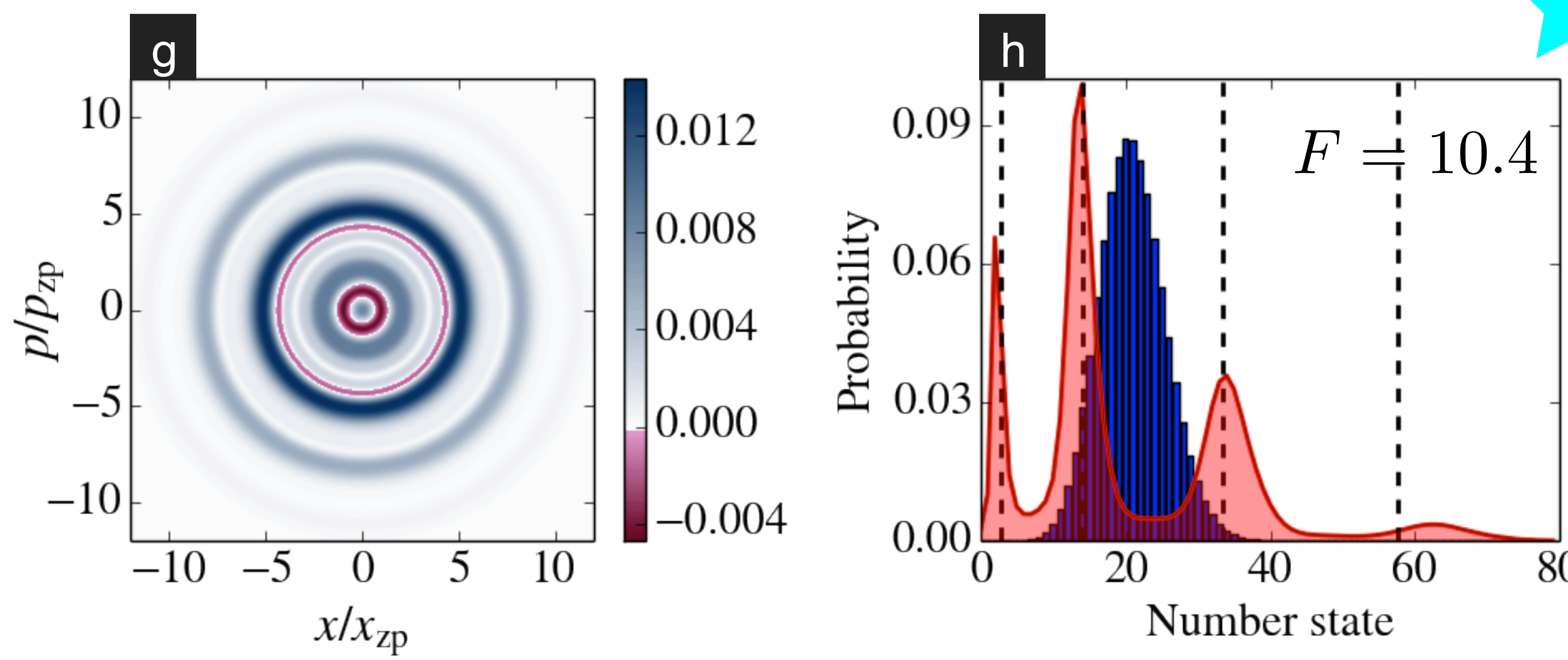


- Strongest quantum features.

- Winger functions consist of rings, one for each stable limit-cycle.

- Multiple limit-cycles means large variance

➔ $F > 1$



- Stronger the coupling g_0, and/or more phonons implies more limit-cycles exist.

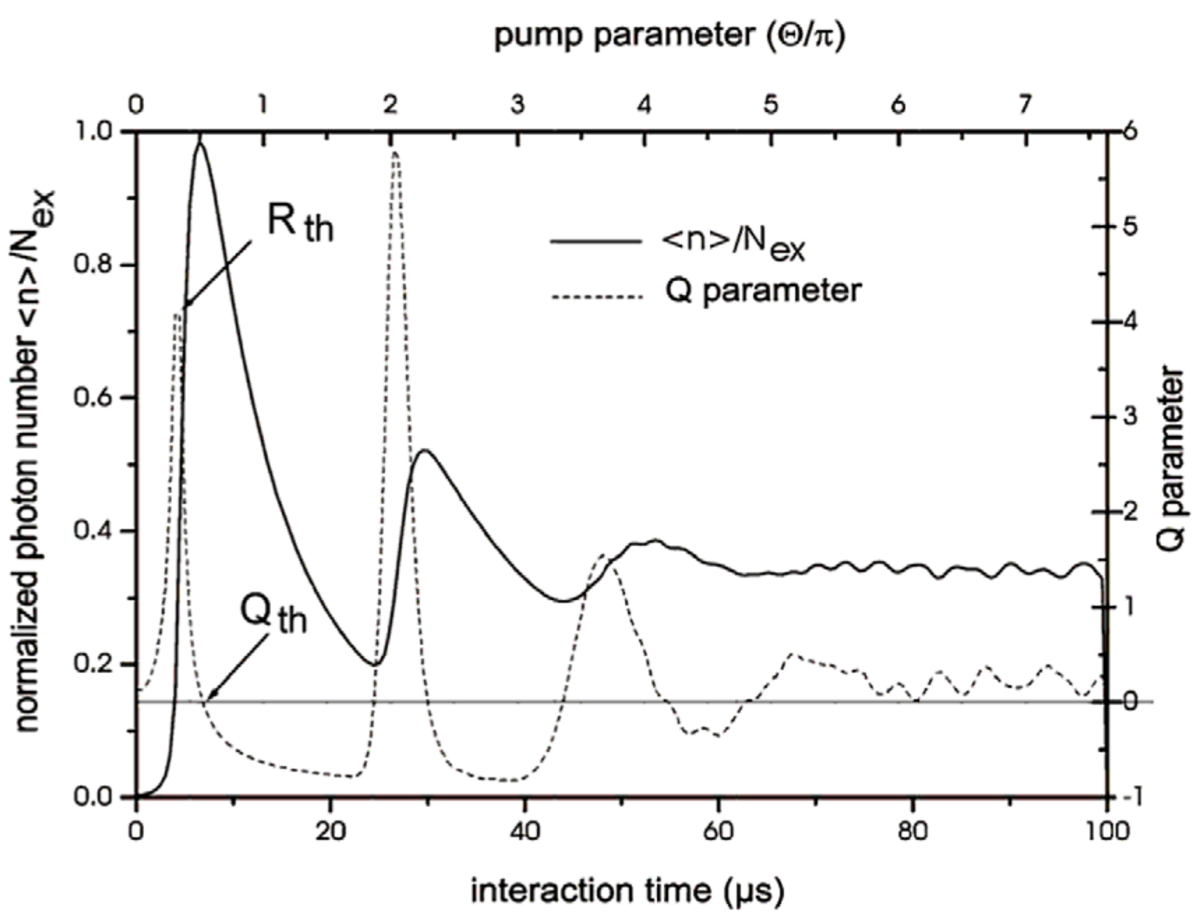
- Each limit-cycle is sub-Poissonian.

- Circular symmetry from no phase ref. ➔ density matrix is diagonal.

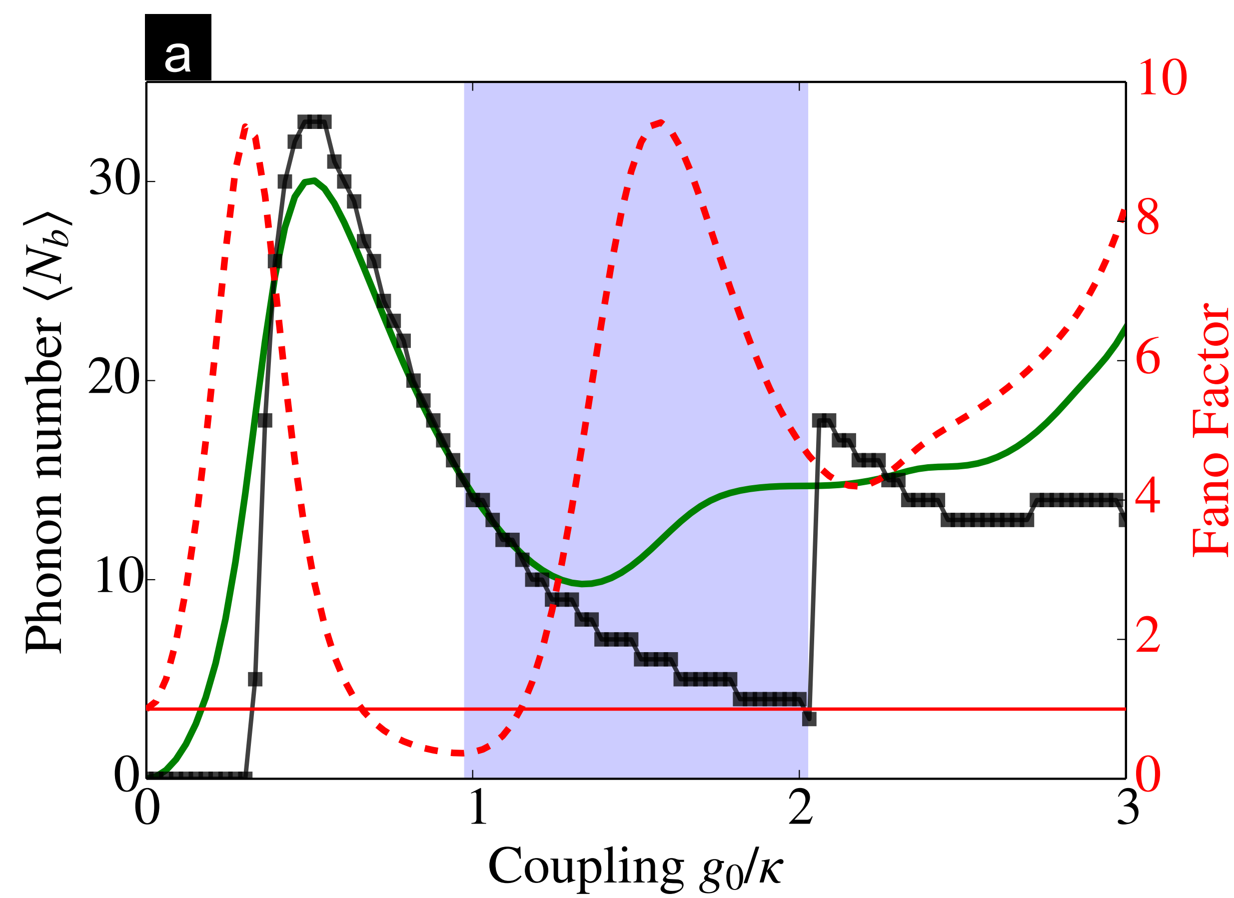
- To understand the onset, and decay, of the nonclassical oscillator properties, we fix the detuning $\Delta = 0$ and sweep the coupling strength $0 \leq g_0/\kappa \leq 3$

- The interplay between limit-cycles is measured by using the number state corresponding to the maximum probability amplitude in the density matrix as an order parameter.

- Normalized coupling strength g_0/κ corresponds to the micromaser pump parameter, $\tau_{\text{int}} = 1/\kappa$. Also proportional to resonator Q-factor.



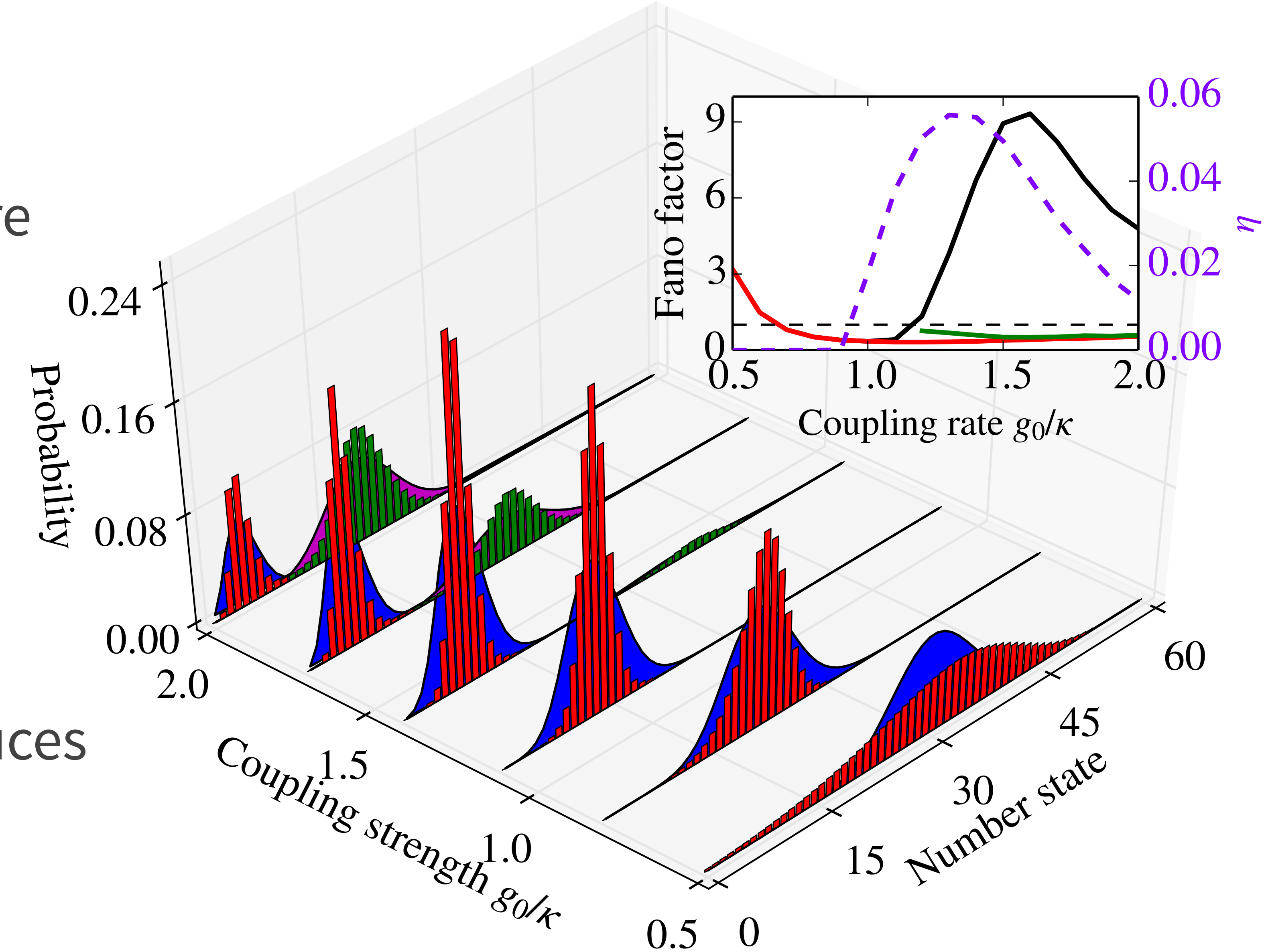
Micromaser



Optomechanical analogue

- Why do the quantum features of the states disappear at higher couplings?

- Each limit-cycle is sub-Poissonian in the regime where the nonclassical ratio is nonzero.



- The merger of limit-cycles, beginning at $g_0/\kappa \simeq 1.3$ reduces the quantum features in the mechanical states.

➡ The overall resonator distribution, which is super-Poissonian, determines the nonclassical properties.

- In general, more phonons in resonator gives overlapping limit-cycles.

➡ Smaller quantum signatures at mechanical sidebands.

Summary:

- Nonclassical states of a mechanical resonator can be generated in an analogue of the micromaser, if the cavity is sufficiently damped so as to have at most one photon at any given time.

- This system has sub-Poissonian limit-cycles, nonclassical mechanical Wigner functions, and phonon oscillations that are also features of a micromaser.

- This is the first micromaser analogue that does not have any atom-like subsystem, only harmonic oscillators!

First single-atom laser with no atom!

- Helps to understand the generation of quantum states in macroscopic mechanical systems.

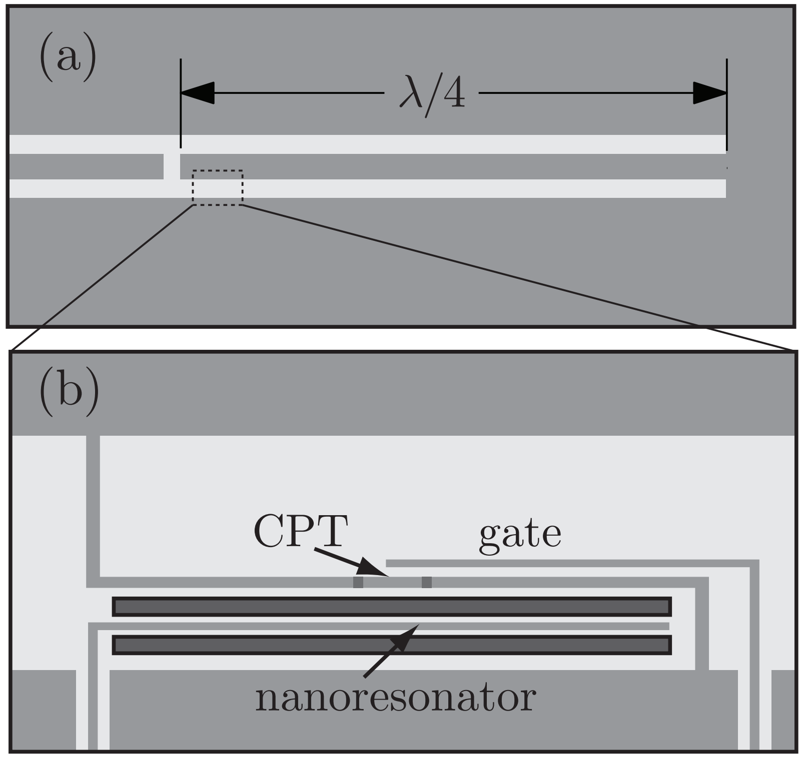
- Allows for exploring the quantum-classical transition across multiple mass scales.

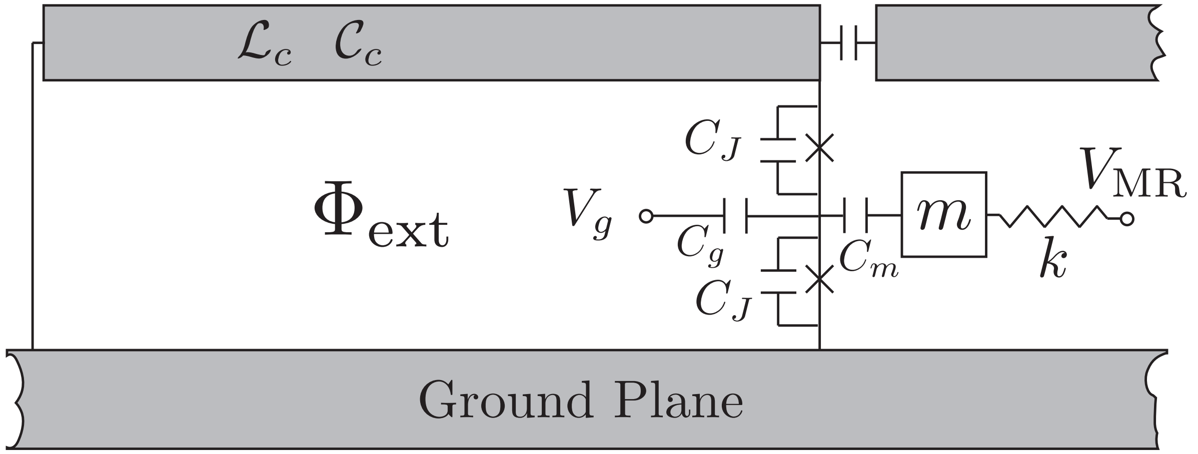
But can we build it?

Cavity-Cooper Pair Transistor:

- Most difficult part is single-photon strong coupling: $g_0^2/\kappa\omega_m \geq 1$

System	N	$\frac{\omega_m}{\kappa}$	$\frac{g_0}{\kappa}$	$\frac{g_0}{\omega_m}$	$\frac{g_0^2}{\kappa\omega_m}$
Superconducting LC oscillator [4]	1e11	60	3e−3	4e−5	1e−7
Si optomechanical crystal [5]	6e9	7	2e−3	2.5e−4	5e−7
Cold atomic gas [8]	4e4	0.06	22	340	7,500
cCPT-mechanical resonator	5e9	10	12	1.2	**14**





- Motion of mechanical resonator modulates charging energy of electrons on the Cooper-pair transistor island.

- Causes measurable frequency shift of cavity.

Thank You