19 Electric Charge

1 static electricity applications: air fresheners, xerography, painting cars, smokestack pollution control, …

2 conductors, insulators, semiconductors, superconductors

3 **Coulomb’s law**: The force of attraction or repulsion between two point charges q_1 and q_2 is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

$$F = K \cdot \frac{|q_1||q_2|}{r^2}$$

$$K = \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9 \quad \left(N\cdot m^2 / C^2\right)$$

permittivity of vacuum

$$\varepsilon_0 = 8.85\times10^{-12} \quad \left(C^2 / N\cdot m^2\right)$$

Principle of Superposition

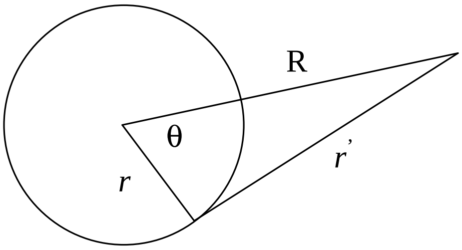
$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + ... + \vec{F}_{1n}$$

electric potential

$$V = K \cdot \frac{q_1}{r}$$

Coulomb’ s law and the **Principle of Superposition** constitute the physical input for electrostatics.

For a conducting sphere of surface charge density ρ



$$dV = K \cdot \frac{dq}{r'} \qquad dq = r \cdot r^2 \sin q \; dq \; df$$

$$V = \int dV = \iint \frac{K}{r'} r r^2 \sin q dq df$$

$$= K r r^2 \cdot 2p \int \frac{\sin q dq}{r'}$$

$$= 2pK r r^2 \int \frac{1}{r'} \cdot \frac{r'}{Rr} dr'$$

$$= \frac{2pkrr}{R} \int_{R-r}^{R+r} dr'$$

$$= \frac{4pKrr^2}{R} = \frac{K\left(4pr^2\right) \cdot r}{R} = \frac{Kq}{R}$$

$$f : 0 \rightarrow 2p$$

$$r'^2 = R^2 + r^2 - 2Rr \cos q$$

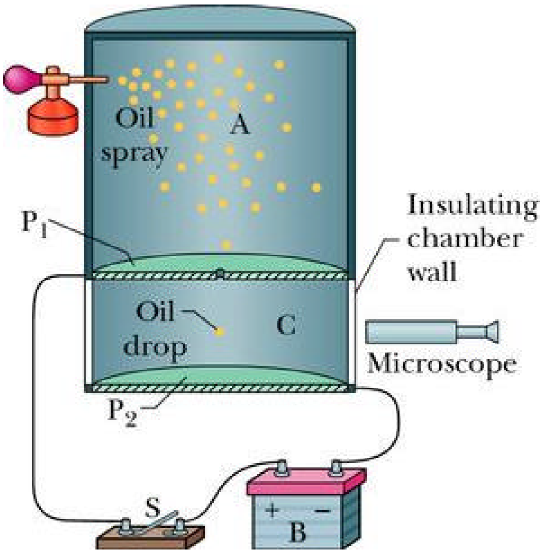
$$2r'dr' = 2Rr \sin q dq$$

$$\sin q dq = \frac{r'}{Rr} dr'$$

$$r' : R - r \rightarrow R + r$$

Charge is quantized

Millikan oil drop experiment



When the Electric field is on, and the oil drop is at rest.

$qE = mg$

When the Electric field is off, and the oil drop reaches its terminal velocity.

$mg = bu$

$qE = bu$

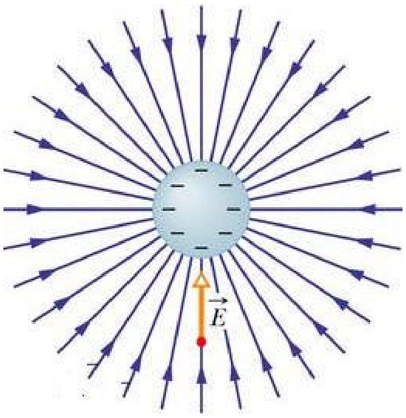
$$q = \frac{bu}{E} = ne$$

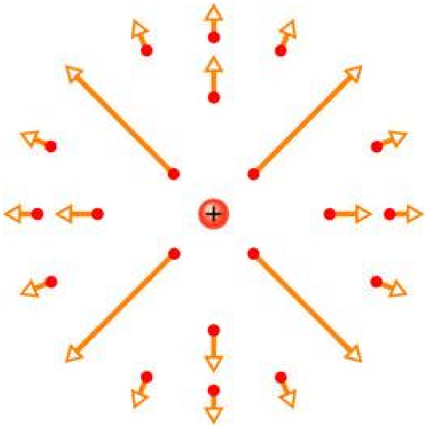
the elementary charge *e* can be obtained by the $\{n_1, n_2, \ldots n_m\}$

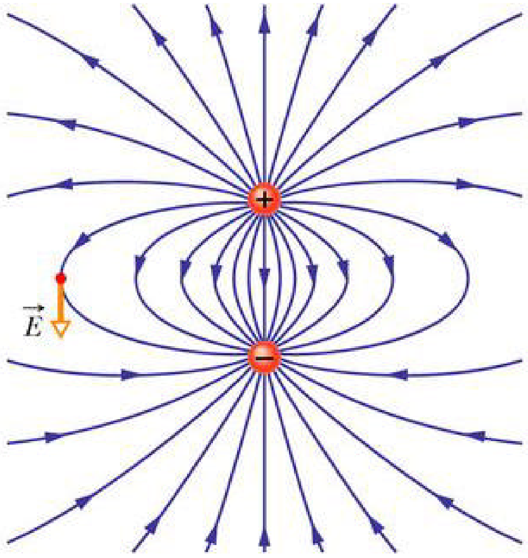
20 Electric Field

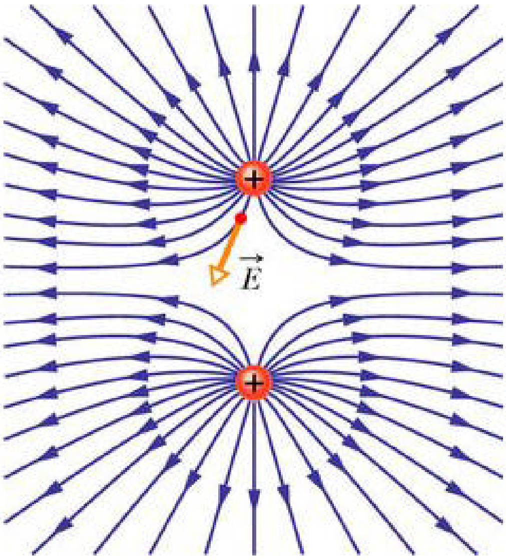
electric field

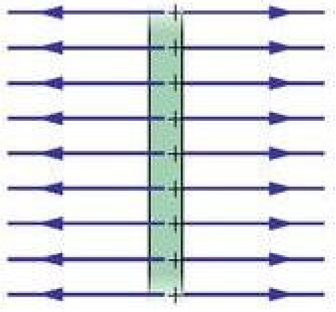
$$\vec{E} = \vec{F}/q$$







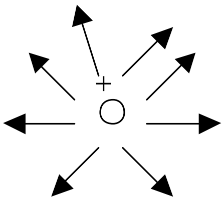




Electric Field Due to a Point Charge

$$\vec{F} = \frac{1}{4pe_0}\frac{|q||q_0|}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4pe_0}\cdot\frac{q}{r^2}\hat{r}$$



The Electric Field Due to an Electric Dipole

$$E = E_+ + E_-$$

$$= \frac{1}{4pe_0}\frac{q}{r_+^2} - \frac{1}{4pe_0}\frac{q}{r_-^2}$$

$$= \frac{q}{4pe_0}\left[\frac{1}{z^2}\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{z^2\left(1+\frac{d}{2z}\right)^2}\right]$$

$$r_+ = z - \frac{d}{2}$$

$$r_- = z + \frac{d}{2}$$

$$z \rangle\rangle d$$

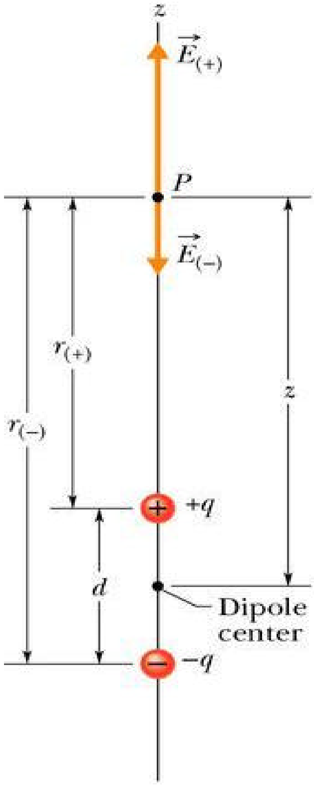
$$\cong \frac{q}{4pe_0 z^2}\left[1 + \frac{2d}{2z} - \left(1 - \frac{2d}{2z}\right)\right]$$

$$= \frac{q}{4pe_0 z^2}\frac{2d}{z} = \frac{1}{2pe_0}\cdot\frac{p}{z^3}$$

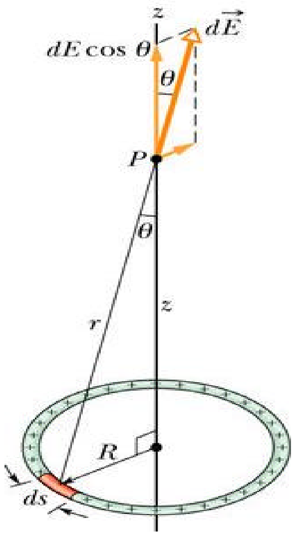
$$for \quad q \neq 0$$

$$E = \frac{1}{2pe_0}\cdot\frac{p}{r^3}\cdot f(q) \qquad f(q) = 1 \qquad for \quad q = 0$$

dipole moment $p = qd$



The Electric Field Due to a Line of Charge



λ : the charge per unit length

$$ds = R\,d\phi$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{\lambda\,ds}{r^2}$$

$$= d\vec{E}_z + d\vec{E}_\parallel$$

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,ds}{r^2}\cos\theta$$

$$dE_\parallel = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,ds}{r^2}\sin\theta \qquad \int dE_\parallel = 0$$

$$\vec{E} = \int d\vec{E} = \int d\vec{E}_z = \frac{\lambda R\cos\theta}{4\pi\epsilon_0 r^2}\int d\phi$$

$$= \frac{\lambda R\cos\theta\cdot 2\pi}{4\pi\epsilon_0\left(z^2+R^2\right)} = \frac{q\cos\theta}{4\pi\epsilon_0\left(z^2+R^2\right)}$$

$$= \frac{qz}{4\pi\epsilon_0\left(z^2+R^2\right)^{\frac{3}{2}}}\hat{z}$$

$$for \qquad z \rangle\rangle R$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 z^2}\hat{z} \qquad \text{point charge}$$

$$z = 0 \qquad \vec{E} = 0 \qquad \text{net force} = 0$$