一. 數與座標系:

1.實數:實數座標系上的點係由有理數與無理數組成,其中有理數具有稠密性,亦即在任意兩個不相等的有理數中間至少存在一個有理數存在.

若 X、Y、Z 均爲實數,則

⑴.三一律:$X<Y$、$X=Y$、$X>Y$ 恰有一個成立.

⑵.遞移律:若 $X<Y, Y<Z$,則 $X<Z$.

⑶.$X<Y \Leftrightarrow X+Z < Y+Z$.

⑷.若 $Z>0$,則 $X<Y \Leftrightarrow X \bullet Z < Y \bullet Z$

⑸.若 $Z<0$,則 $X<Y \Leftrightarrow X \bullet Z > Y \bullet Z$

2.整數,爲有理數的一個子集,任意兩的不相等整數差的絕對值都大於或等於一,這個性質稱爲整數的離散性.

3.常見的因數、倍數判斷法:

⑴.2 的倍數$\Leftrightarrow$個位數爲偶數.

Ex : $abc = a \cdot 100 + b \cdot 10 + \underline{c} \Leftrightarrow c$ 爲偶數

⑵.3 的倍數$\Leftrightarrow$數字和爲 3 的倍數.

Ex: $abc = a \cdot 100 + b \cdot 10 + c = (a \bullet 99 + b \cdot 9) + \underline{(a+b+c)}$

⑶.4 的倍數$\Leftrightarrow$末兩位爲 4 的倍數

Ex: $abc = a \cdot 100 + \underline{(b \cdot 10 + c)}$

⑷.5 的倍數$\Leftrightarrow$末位爲 5 的倍數.

Ex: $abc = a\cdot 100+b\cdot 10+\underline{c}$

(5).8 的倍數⇔末三位爲 8 的倍數.

Ex: $abcd = a\cdot 1000+\underline{b\cdot 100+c\cdot 10+d}$

(6).9 的倍數⇔末三位爲 9 的倍數.

Ex: $abc = a\cdot 100+b\cdot 10+c = (a \bullet 99+b\cdot 9)+\underline{(a+b+c)}$

(7).11 的倍數⇔(奇位數字和)-(偶位數字和)=11 的倍數.

Ex: $abcd = a\cdot 1000+b\cdot 100+c\cdot 10+d$

$=(99+1) \bullet (11-1) \bullet a+(99+1) \bullet b+(11-1) \bullet c+d$

$=\underline{(b+d)-(a+c)}+(99 \bullet 11+11-99) \bullet a+99\cdot b+11 \bullet c$

4.因數倍數的性質:

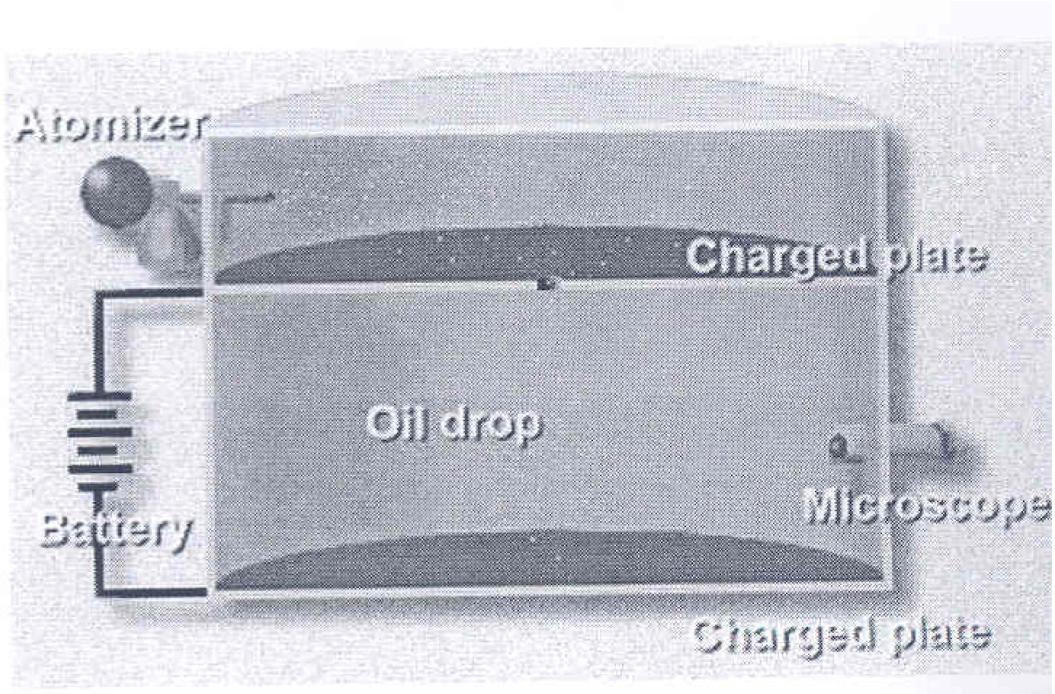
if a、b、c $\in Z$

then (1). $a|b$,$b|c$ $\Rightarrow a|c$

(2). $a|b$,$a|c$ $\Rightarrow$ $a \mid bm+cn$

(m,n) are arbitrary integers

ex:密立根油滴實驗中,藉著求得各油滴的帶電量,並求其最大公因數,即爲基本電荷.



5.標準分解釋的應用:

if n = $P_1^{\alpha_1} \bullet P_2^{\alpha_2} \cdots\cdot P_K^{\alpha_K}$ then

(1). n 的正整數個數爲$(+1)(\alpha_2+1)\ldots(\alpha_k+1)$

ex: n=$P_1^{\alpha_1}$， $P_1^0 \mid n$， $P_1^1 \mid n$, … $P_1^{\alpha_1} \mid n$ ⇒ (α_1+1)個

(2).n 的正因數和

$=(1+P_1+\ldots+P_1^{\alpha_1})(1+P_2+\ldots+P_2^{\alpha_2})\ldots(1+P_K+\ldots+P_K^{\alpha_K})$

$=\prod_{i=1}^{k}\left(\frac{1-P_i^{\alpha_{i+1}}}{1-P_i}\right)$ 其中 $\prod_{i=1}^{k} a_i = a_1 \bullet a_2 \bullet \cdots a_k$

(3).n 的正整數乘積:

$= n^{\frac{1}{2}(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)} = n^{\frac{1}{2}\prod_{i=1}^{k}(\alpha_i+1)}$

6.輾轉相除法:

設 a、b∈N, 若存在 q、r∈Z,使得 a=bq+r, $0 \le r \prec b$,

則最大公因數 (a , b) = (bq+r , b) = (b , r)

7.複數:

Z = a + bi , a、b 爲實數

(1).If $Z_1 = Z_2 \Rightarrow a_1 = a_2$ ， $b_1 = b_2$

(2).If $Z_1 + Z_2 = (a_1 + a_2) + (b_1 + b_2)i$,

(3).If $Z_1 \bullet Z_2 = (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i$,

(4).If $\frac{Z_1}{Z_2} = \frac{1}{a_2^2 + b_2^2}$〔$(a_1a_2 + b_1b_2) + (a_2b_1 - a_1b_2)i$〕

(5). $\overline{Z} = a - b\,\mathrm{i}$

8.一元二次方程式的解:

$ax^2 + bx + c = 0$, (a≠0 , a、b、c ϶ R)

$$a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{1}{4a^2}\left(b^2-4ac\right)$$

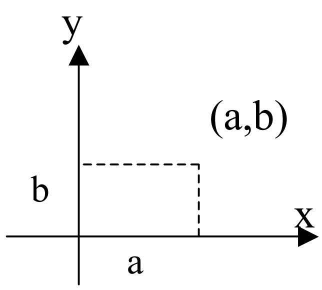
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

(1). $b^2-4ac > 0 \Rightarrow$ 相異兩實根.

(2). $b^2-4ac < 0 \Rightarrow$ 相等兩實根.

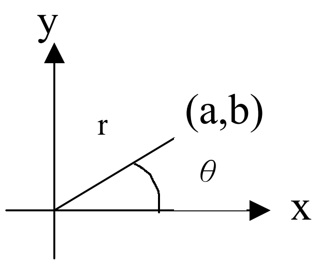
(3). $b^2-4ac = 0 \Rightarrow$ 兩共軛虛根.

9.直角座標(x,y)



Z = a + bi 可以在二維中表示.

極座標(r, θ)



(r, θ) $0 \le r = \sqrt{a^2+b^2} < \infty$, $0 \le \theta < 2\pi$

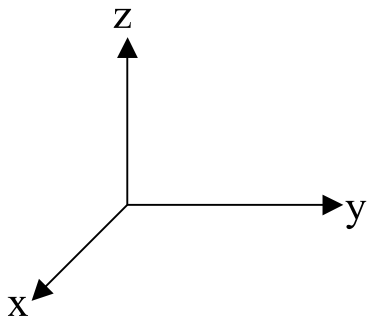
$$0 \le \sin\theta \equiv \frac{b}{r} \le 1$$

$$0 \le \cos\theta \equiv \frac{a}{r} \le 1$$

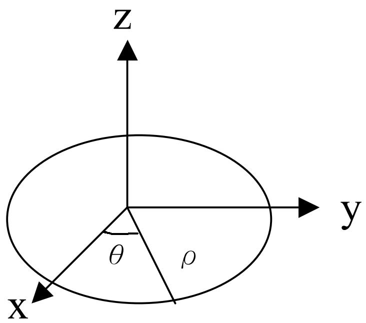
$$-\infty < \tan\theta \equiv \frac{b}{a} < \infty$$

10.三維座標系:

⑴.直角座標系(x、y、z)



⑵.柱座標系(ρ 、 θ 、z)

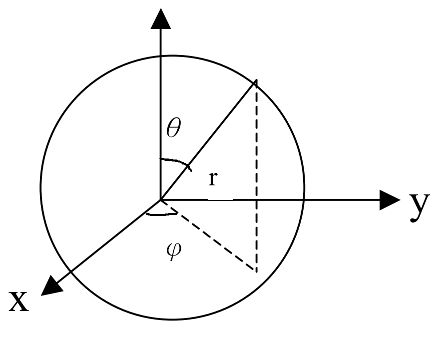


$0 \le \rho < \infty$ $x = \rho \cos\theta$

$0 \le \theta < 2\pi$ $y = \rho \sin\theta$

$-\infty < z < \infty$ $z = z$

⑶.球座標(r、 θ 、 φ)



$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

$z = r\cos\theta$

$0 \le r < \infty$

$0 \le \theta < \pi$

$0 \le \varphi < 2\pi$

11.分點公式:

設 A(x_1, y_1), B(x_2, y_2),則

⑴. $\overline{AB}$ 之中點座標$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

⑵. 若 P 介於 A，B 之間且$\overline{AP}:\overline{PB}$= m : n,則 P 為

$$\left(\frac{nx_1+mx_2}{m+n},\frac{ny_1+my_2}{m+n}\right)$$

12.質心:

⑴.若 A，B 兩點的質量均為 M,則質心為 $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

⑵.若 A，B 兩點的質量比為 m : n,則質心為$\left(\frac{mx_1+nx_2}{m+n},\frac{my_1+ny_2}{m+n}\right)$

⑶.若$P_1,P_2,\cdots P_N$的質量均為 M,則質心為

$$x_{cm}=\frac{1}{N}(x_1+x_2+\cdots x_N)=\frac{1}{N}\sum_{i=1}^{N}x_i$$

$$y_{cm}=\frac{1}{N}\sum_{i=1}^{N}y_i$$

⑷.若P_i的質量為Δm_i , for i =1,2,3…N

$$x_{cm}=\frac{1}{\sum_{i=1}^{N}\Delta m_i}\sum_{i=1}^{N}x_i\Delta m_i$$

$$y_{cm}=\frac{1}{\sum_{i=1}^{N}\Delta m_i}\sum_{i=1}^{N}y_i\Delta m_i$$

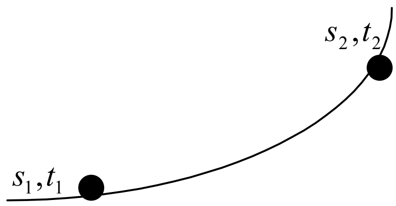
⑸. N$\rightarrow\infty$, $\Delta m_i\rightarrow dm$, $\sum_{i=1}^{N}\Delta m_i\rightarrow\int dm=M$

$$x_{cm}=\frac{1}{M}\int_{\Omega}xdm$$ Ω為其分布空間

$$y_{cm}=\frac{1}{M}\int_{\Omega}ydm$$

13.微分與積分

(1).微分:



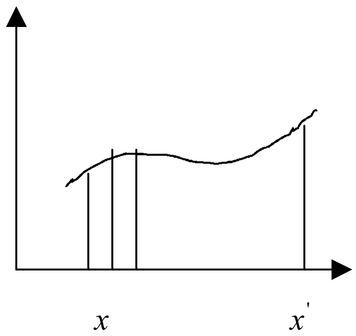
$\Delta s = s_2 - s_1$

$\Delta t = t_2 - t_1$

平均速度: $\bar{v} = \frac{\Delta s}{\Delta t}$

順時速度: $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

(2).積分:



F = F(x)

作功 $W = \int_x^{x'} F(x)dx$

$= \lim_{N \to \infty} \sum_{i=1}^{N} \overline{F}(x_i)\Delta x_i$

$\Delta x_i = \frac{x' - x}{N}$, $\overline{F}(x_i) = \frac{1}{2}(F(x_i) + F(x_{i+1}))$

14.微分運算:

f(x)

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(1). f(x) = x , $\frac{df(x)}{dx} = \frac{dx}{dx} = \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x} = 1$

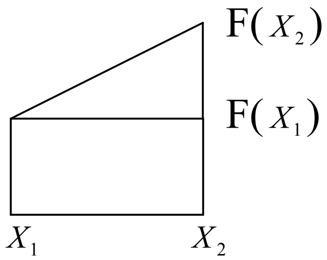
(2). f(x) $= x^2$, $\frac{df(x)}{dx} = \frac{dx^2}{dx} = \lim_{\Delta x \to 0} \frac{(x + \Delta x^2) - x^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} 2x + \Delta x = 2x$

(3). f(x)= x^n , $\frac{df(x)}{dx} = n\,x^{n-1}$

15.積分運算:

$$F(x)=x \quad , \quad \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} x dx = \frac{1}{2}x^2 \Big|_{x_1}^{x_2} = \frac{1}{2}\left(x_2^2 - x_1^2\right)$$



$$\int_{x_1}^{x_2} F(x)dx$$

$$= \frac{1}{2}(x_2 - x_1)(F(x_2) + F(x_1)) = \frac{1}{2}\left(x_2^2 - x_1^2\right)$$

$$F(x)=e^{-ax^2}$$

$$\left(\int_{-\infty}^{\infty} F(x)dx\right)^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ax^2} e^{-ay^2} dxdy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a\left(x^2+y^2\right)} dxdy \qquad x^2 + y^2 = r^2 \quad \text{,dx dy = r dr d}\theta$$

$$= \int_0^{\infty}\int_0^{2\pi} e^{-ar^2} r dr d\theta$$

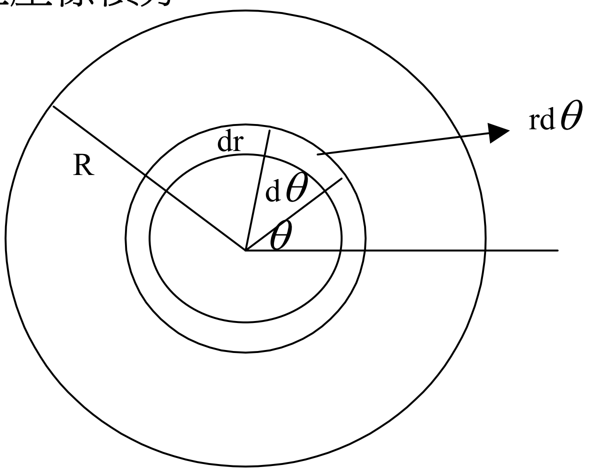
$$= \pi \int_0^{\infty} e^{-ar^2} dr^2 \qquad \text{t= a r}^2$$

$$= \frac{\pi}{a}\int_0^{\infty} e^{-t} dt$$

$$= \frac{\pi}{a}(-e^{-t})\Big|_0^{\infty} = \frac{\pi}{a}$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx^2 = \frac{\pi}{a}$$

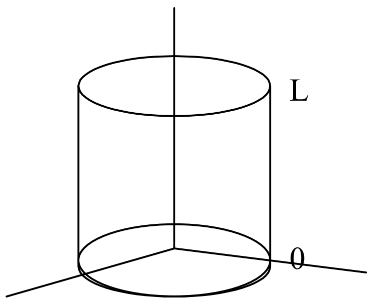
16.極座標積分



$$dA = rd\theta dr$$

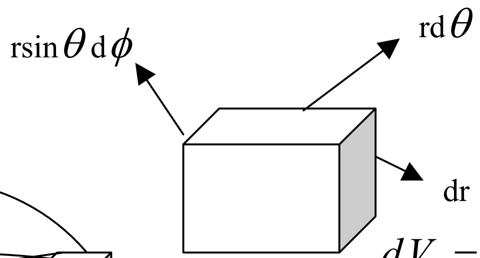
$$A = \int dA$$
$$= \int_0^R \int_0^{2\pi} rd\theta dr$$
$$= 2\pi \int_0^R rdr = 2\pi \cdot \frac{1}{2} R^2$$
$$= \pi R^2$$

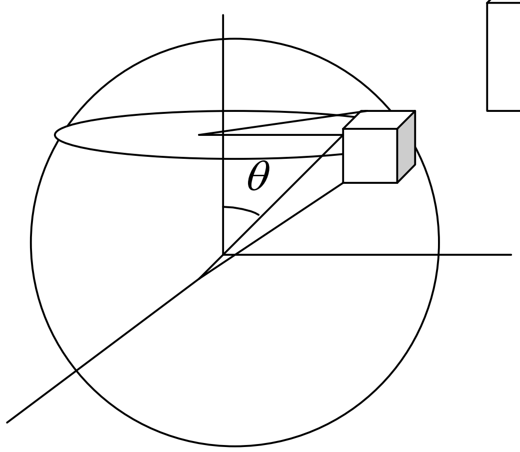
柱座標積分



$$dV = rd\theta drdz$$
$$V = \int dV$$
$$= \int_0^L \int_0^R \int_0^{2\pi} rd\theta drdz$$
$$= L\pi R^2$$

球座標積分

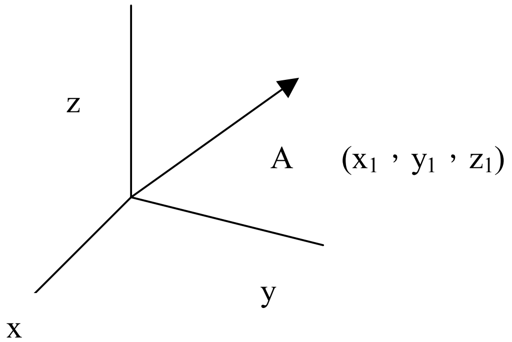




$$dV = (r\sin\theta d\phi)(rd\theta)(dr)$$
$$= r^2 \sin\theta drd\theta d\phi$$
$$V = \int dV$$
$$= \int_0^{2\pi} \int_0^{\pi} \int_0^R r^2 \sin\theta drd\theta d\phi$$
$$= 2\pi \int_0^{\pi} \int_0^R r^2 \sin\theta drd\theta$$
$$= 4\pi \int_0^R r^2 dr$$
$$= \frac{4}{3}\pi R^3$$

$$\because \int_0^{\pi} \sin\theta$$
$$= -\int_1^{-1} d\cos\theta = 2$$

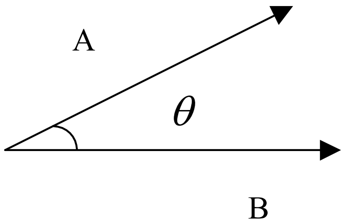
17.向量分析



$$\vec{A} = \hat{i}\, x_1 + \hat{j} y_1 + \hat{k}\, z_1$$
$$= \hat{i}\, A_x + \hat{j}\, A_y + \hat{k}\, A_z$$

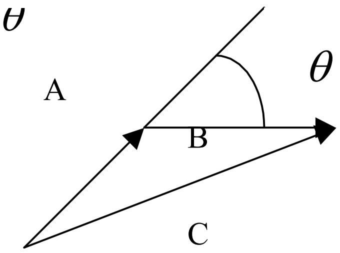
$$A_x = x_1,\ A_y = y_1,\ A_z = z_1$$

$$\vec{A} \pm \vec{B} = \vec{i}\,(A_z \pm B_z) + \vec{j}(A_y \pm B_y) + \vec{k}(A_z \pm B_z)$$
$$A_x = A\sin\theta\cos\phi$$
$$A_y = A\sin\theta\sin\phi$$
$$A_z = A\cos\theta$$

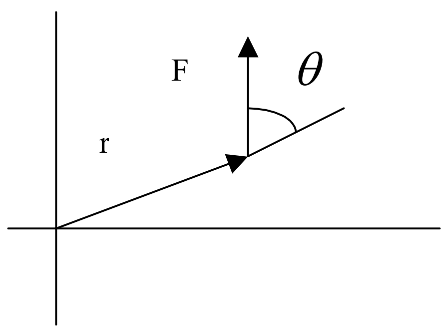


scalar product

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z = AB\cos\theta$$
$$\vec{C} = \vec{A} + \vec{B}$$
$$\vec{C} \bullet \vec{C} = (\vec{A} + \vec{B}) \bullet (\vec{A} + \vec{B})$$
$$= \vec{A} \bullet \vec{A} + \vec{B} \bullet \vec{B} + 2\,\vec{A} \bullet \vec{B}$$
$$C^2 = A^2 + B^2 + 2AB\cos\theta$$



vector product



力矩 $\vec{\tau} = \vec{r} \times \vec{F}$

$= rF\sin\theta$

$$\vec{C} = \vec{A} \times \vec{B} \equiv AB\sin\theta$$
$$= \vec{i}\ C_x + \vec{j}\ C_y + \vec{k}\ C_z$$
$$= \vec{i}\,(A_y B_z - B_y A_z) + \vec{j}\,(A_z B_x - B_z A_x) + \vec{k}\,(A_x B_y - B_x A_y)$$
$$= -\vec{B} \times \vec{A}$$

18.Triple scalar product

$$\vec{A}\bullet(\vec{B}\times\vec{C})$$

$$=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\bullet[\hat{i}(B_yC_z-C_yB_z)+\hat{j}(B_zC_x-C_zB_x)+\hat{k}(B_xC_y-C_xB_y)]$$

$$=A_x(B_yC_z-C_yB_z)+A_y(B_zC_x-C_zB_x)+A_z(B_xC_y-C_xB_y)$$

$$=\vec{B}\bullet(\vec{C}\times\vec{A})=\vec{C}\bullet(\vec{A}\times\vec{B})$$

$$=-\vec{A}\bullet(\vec{C}\times\vec{B})=-\vec{B}\bullet(\vec{A}\times\vec{C})=-\vec{C}\bullet(\vec{B}\times\vec{A})$$

triple vector product

$$\vec{A}\times(\vec{B}\times\vec{C})\equiv\vec{D}$$

$$\vec{D}\perp(\vec{B}\times\vec{C})$$

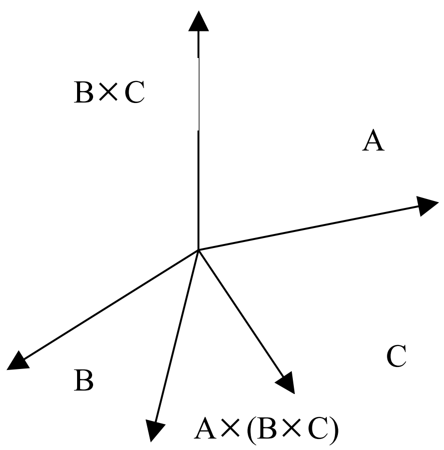
所以 $\vec{D}=\alpha\vec{B}+\beta\vec{C}$

$$\vec{A}\bullet\vec{D}=\alpha(\vec{A}\bullet\vec{B})+\beta(\vec{A}\bullet\vec{C})$$

$$\alpha(\vec{A}\bullet\vec{B})+\beta(\vec{A}\bullet\vec{C})=0$$

$\alpha=\vec{A}\bullet\vec{C}$，$\beta=-\vec{A}\bullet\vec{B}$

$$\vec{D}=\vec{B}(\vec{A}\bullet\vec{C})-\vec{C}(\vec{A}\bullet\vec{B})$$



$$\varepsilon_{ijk}=\begin{cases}1 & 123，231，312\\ -1 & 132，213，321\end{cases}$$

$\vec{A}\times\vec{B}=\varepsilon_{ijk}A_iB_j\hat{e}_k$，$i,j,k=1,2,3$

$$=A_1B_2\hat{e}_3+A_2B_3\hat{e}_1+A_3B_1\hat{e}_2$$

$$-A_1B_3\hat{e}_2-A_2B_1\hat{e}_3-A_3B_2\hat{e}_1$$

$$=(A_1B_2-A_2B_1)\hat{e}_3+(A_2B_3-A_3B_2)\hat{e}_1+(A_3B_1-A_1B_3)\hat{e}_2$$

$$\vec{A}\times(\vec{B}\times\vec{C})$$

$$=\varepsilon_{lkm}A_l\varepsilon_{ijk}B_iC_j\hat{e}_m=\varepsilon_{lkm}\varepsilon_{ijk}A_lB_iC_j\hat{e}_m$$

$$=-(\delta_{li}\delta_{mj}-\delta_{lj}\delta_{mi})A_lB_iC_j\hat{e}_m$$

$$=A_jB_iC_j\hat{e}_i-A_iB_iC_j\hat{e}_j$$

$$=(\vec{A}\bullet\vec{C})\vec{B}-(\vec{A}\bullet\vec{B})\vec{C}$$

19.Gradient，∇

$$\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$$

$$\nabla\phi=\hat{i}\frac{\partial\phi}{\partial x}+\hat{j}\frac{\partial\phi}{\partial y}+\hat{k}\frac{\partial\phi}{\partial z}$$

ex

$$r=\sqrt{x^2+y^2+z^2}$$

$$f=f(r)$$

$$\nabla f(r)=\hat{i}\frac{\partial f(r)}{\partial x}+\hat{j}\frac{\partial f(r)}{\partial y}+\hat{k}\frac{\partial f(r)}{\partial z}$$

$$\frac{\partial f(r)}{\partial x}=\frac{df(r)}{dr}*\frac{\partial r}{\partial x}$$

$$=\frac{df(r)}{dr}(\frac{1}{2})\frac{2x}{r}$$

$$=\frac{x}{r}*\frac{df(r)}{dr}$$

$f=f(r,\theta,\phi)$ chain rule

$$\frac{\partial f}{\partial x}=\frac{\partial f}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial f}{\partial\theta}\frac{\partial\theta}{\partial x}+\frac{\partial f}{\partial\phi}\frac{\partial\phi}{\partial x}$$

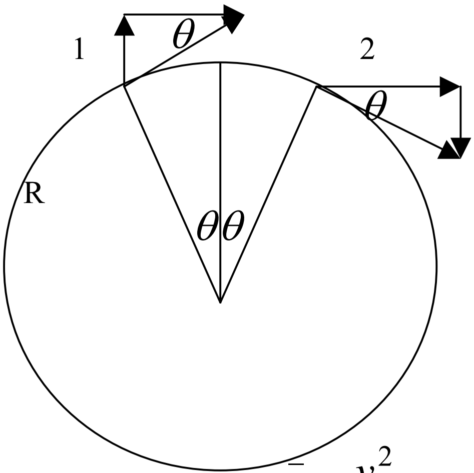
$$=\frac{df}{dr}\frac{\partial r}{\partial x}\qquad\underset{0}{\parallel}\qquad\underset{0}{\parallel}$$

所以

$$\nabla f(r)=\frac{1}{r}\frac{df(r)}{dr}(\hat{i}x+\hat{j}y+\hat{k}z)$$

$$=\hat{r}\frac{df(r)}{dr}$$

20.Circular motion



平均加速度 $\bar{a} = \frac{v_2 - v_1}{t_2 - t_1}$

$$v_2 - v_1 = 2v\sin\theta$$

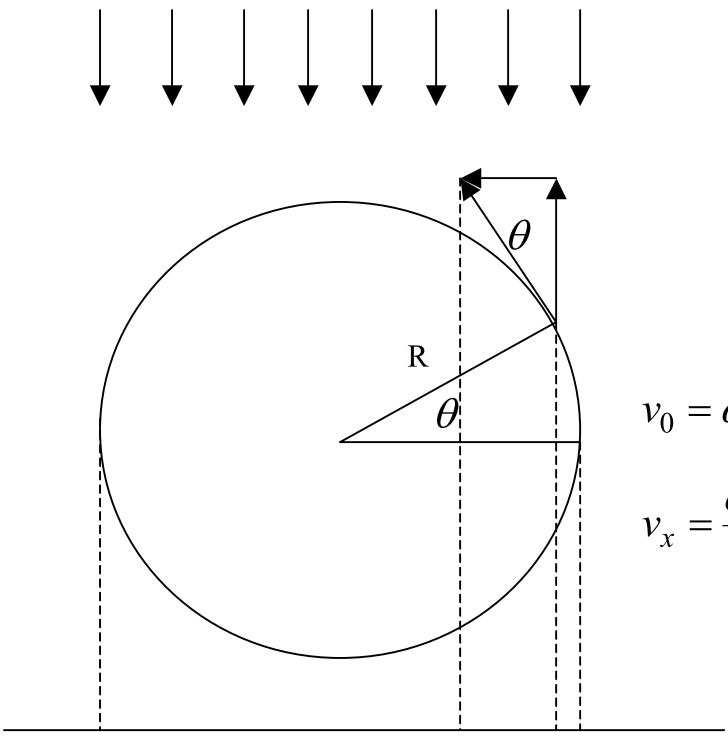
$$t_2 - t_1 = \frac{2R\theta}{v}$$

$$\bar{a} = \frac{2v\sin\theta}{2R\theta/v} = \frac{v^2}{R}\frac{\sin\theta}{\theta}$$

向心加速度$a = \lim_{\theta\to 0} \bar{a} = \frac{v^2}{R}$

hw1：prove $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

21.Simple harmonic oscillation



$$x = R\cos\theta = R\cos(\omega t)$$

$$v_x = -v_0\sin\theta = -v_0\sin(\omega t)$$

$$\theta = \omega t$$

ω：角頻率(速度) $\omega = \frac{d\theta}{dt}$

$$v_0 = \omega R = \frac{ds}{dt} = \frac{d}{dt}(R\theta) = R\frac{d\theta}{dt} = R\omega$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}R\cos(\omega t) = R\frac{d}{dt}\cos(\omega t)$$

$$\frac{d}{dx}\sin x = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x)-\sin(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin x\cos(\Delta x)+\cos x\sin(\Delta x)-\sin(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0}[\sin(x)\frac{\cos(\Delta x)-1}{\Delta x}+\cos(x)\frac{\sin(\Delta x)}{\Delta x}]$$

$$\qquad\qquad || \qquad\qquad\qquad ||$$

$$= \cos x \qquad\quad 0 \qquad\qquad\qquad 1$$

Hw2：prove $\lim_{\Delta x \to 0} \frac{\cos(\Delta x)-1}{\Delta x} = 0$

Hw3：prove
$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

22.Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

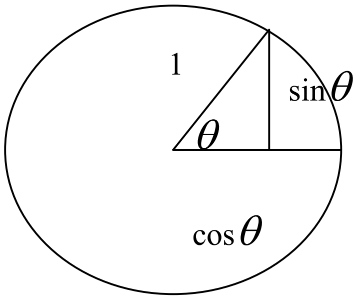
pf：

$$e^{i\theta} = x+iy$$
$$e^{-i\theta} = x-iy$$
$$1 = e^{i\theta} * e^{-i\theta} = (x+iy)(x-iy) = x^2+y^2$$
$$1 = x^2+y^2$$
$x=\cos\theta$， $y=\sin\theta$

or

$x=\sin\theta$， $y=\cos\theta$

Pick's thm



$$e^{i0} = 1 = \cos 0 + i\sin 0$$

So $e^{i\theta} = \cos\theta + i\sin\theta$

23.

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\frac{de^{i\theta}}{d\theta} = -\sin\theta + i\cos\theta = i(\cos\theta + i\sin\theta)$$

$$\frac{de^{i\theta}}{d(i\theta)} = \cos\theta + i\sin\theta = e^{i\theta}$$

let $\quad i\theta = x$

$$\frac{de^{x}}{dx} = e^{x}$$

$$\frac{de^{\alpha x}}{dx} = \frac{de^{\alpha x}}{\frac{1}{\alpha}d(\alpha x)} = \alpha e^{\alpha x}$$

24.

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x) - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x+\Delta x) + f(x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[f(x+\Delta x) - f(x)]g(x+\Delta x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{[g(x+\Delta x) - g(x)]f(x)}{\Delta x}$$

$$= f'(x)g(x) + f(x)g'(x)$$

其中 $\quad f'(x) \equiv \dfrac{df(x)}{dx}$

25.Chain rule

$$h(x) = g(f(x))$$

$$\frac{dh(x)}{dx} = g'(f(x))f'(x)$$

$$\frac{dh(x)}{dx} = \lim_{\Delta x \to 0} \frac{h(x+\Delta x) - h(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{g(y+\Delta y) - g(y)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{g(y+\Delta y) - g(y)}{\Delta y} * \frac{\Delta y}{\Delta x}$$

$$= g'(y)f'(x)$$

$$= g'(f(x))f'(x)$$

let $y = f(x)$

$$h(x+\Delta x) = g(f(x+\Delta x))$$

$$= g(y+\Delta y)$$

$$h(x) = g(y)$$

$$y + \Delta y = f(x+\Delta x)$$

$$= f(x) + \Delta y$$

$$\Delta y = f(x+\Delta x) - f(x)$$

$$= \frac{f(x+\Delta x) - f(x)}{\Delta x} * \Delta x$$

$$\lim_{\Delta x \to 0} \frac{g(y+\Delta y) - g(y)}{\Delta y} = g'(y)$$

Ex：

1.

$$h(x) = \sin(\alpha x^2)$$

$$\frac{dh(x)}{dx} = \cos(\alpha x^2) * 2\alpha x$$

2.

$$h(x) = x^x = e^{\ln x^x} = e^{x \ln x}$$

$$\frac{dh(x)}{dx} = e^{x \ln x} \frac{d}{dx}(x \ln x)$$

$$= e^{x \ln x}(\ln x + 1)$$

$$= x^x(1 + \ln x)$$

Hw4： A=(x_1，y_1)，B=(x_2，y_2)，C=(x_3，y_3)， prove that

ΔABC 之面積爲 $\frac{1}{2}\left|x_1y_2+x_2y_3+x_3y_1-x_2y_1-x_3y_2-x_1y_3\right|$

Hw5： $\omega=\frac{1}{2}(-1+\sqrt{3}i)$ 求 $\omega^{51}+\omega^{52}+\cdots+\omega^{2001}=?$

Hw6：$1<x<2$

若 $\log_2 x$，$\log_2 2x$，$\log_2 x^2$ 爲一直角三角形之三邊 長

則x = ? $2^{\frac{1+\sqrt{5}}{4}}$

Hw7：

1. $\sin(\alpha+\beta)\sin(\alpha-\beta)=\sin^2\alpha-\sin^2\beta=\cos^2\beta-\cos^2\alpha$

2. $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta=\cos^2\beta-\sin^2\alpha$

Hw8：prove that

1. 二倍角公式 $\sin 2\theta=\frac{2\tan\theta}{1+\tan^2\theta}=2\sin\theta\cos\theta$

$$\cos 2\theta=\cos^2\theta-\sin^2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

2. 三倍角 $\sin 3\theta=3\sin\theta-4\sin^3\theta$ ； $\cos 3\theta=4\cos^3\theta-3\cos\theta$

半角 $\sin^2\frac{\theta}{2}=\frac{1-\cos\theta}{2}$ $\cos^2\frac{\theta}{2}=\frac{1+\cos\theta}{2}$

Hw9：求 $\frac{de^{\alpha x^2}}{dx}=?$ $\frac{d}{dx}(\sqrt{x+\sqrt{x}})=?$

Hw10： prove that $\frac{d}{dx}[\frac{f(x)}{g(x)}]=\frac{f'(x)}{g(x)}-\frac{g'(x)f(x)}{[g(x)]^2}$