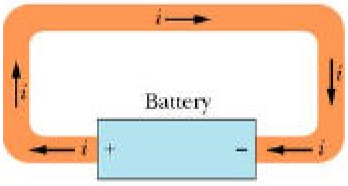
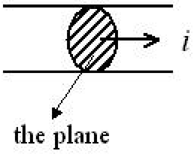
V. Electric Current and Resistance(電流與電阻)

1. Definition: Current

Electrostatics: charges are at rest.
When charges (q) start to move, they generate a current (i), and



$$i = \frac{dq}{dt} \quad \text{(definition of current)}$$

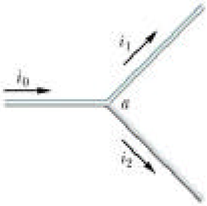


We can find the charge that passes through the plane in a time interval extending from 0 to t by integration:

$$q = \int dq = \int_0^t i\, dt,$$

The SI unit for current is the coulomb per second, also called the *ampere* (A):

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$



conservation of charge = conservation of current

$$i_0 = i_1 + i_2$$

2. Drift speed (漂移速度) and current

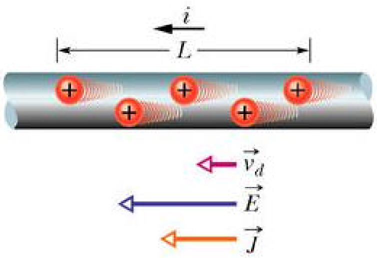
Define the current density J:

The total current through the surface is $i = \int \vec{J} \cdot d\vec{A}$.

If the current is uniform across the surface and parallel to $d\vec{A}$, then $\vec{J}$ is also uniform and parallel to $d\vec{A}$.

Then $i = \int J\, dA = J \int dA = JA$ and $J = \frac{i}{A}$

When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to *drift* with a **drift speed** v_d in the direction opposite that of the applied electric field that causes the current. The drift speed is tiny compared to the speeds in the random motion. For example, in the copper conductors of household wiring, electron drift speeds are perhaps 10^{-5} or 10^{-4} m/s, whereas the random-motion speeds are around 10^6 m/s.



The total charge within L is $q = (nAL)e$,
where n is number density of charge carriers, A is the area of the cross section, and e is the charge of each carrier.
Assume that all carriers move along the wire with v_d, it takes $t = \frac{L}{v_d}$ for all these carriers within L to pass through any cross section.

Therefore $i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$ $\qquad v_d = \frac{i}{nAe} = \frac{J}{ne}$ $\qquad$ or $\quad \vec{J} = (ne)\vec{v}_d$

3. Resistance and resistivity (電阻與電阻率)

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its electrical **resistance.** We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

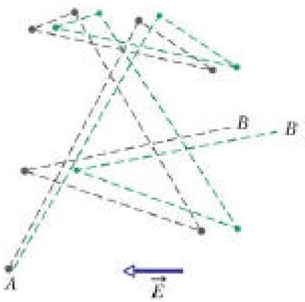
$$R = \frac{V}{i} \quad \text{(definition of } R\text{)} \qquad \text{or} \quad V = R\, i$$

Resister (-\/\/\/-)**:** A conductor whose function in a circuit is to provide a specified resistance.

Resistivity (r) of the material: $\vec{E} = \rho \vec{J}$. **This is the "Ohm's Law".**

NOTE: Resistance is a property of an object. Resistivity is a property of a material.

4. Microscopic view of Ohm's law



The gray lines show an electron moving from A to B, making six collisions en route. The green lines show what its path might be in the presence of an applied electric field $\vec{E}$. Note the steady drift in the direction of $-\vec{E}$. (Actually, the green lines should be slightly curved, to represent the parabolic paths followed by the electrons between collisions, under the influence of an electric field.)

The motion of the conduction electrons in an electric field $\vec{E}$ is thus a combination of the motion due to random collisions and that due to $\vec{E}$. When we consider all the free electrons, their random motions average to zero and make no contribution to the drift speed. Thus, the drift speed is due only to the effect of the electric field on the electrons.

If an electron of mass m is placed in an electric field of magnitude E, the electron will experience an acceleration given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m}.$$

The nature of the collisions experienced by conduction electrons is such that, after a typical collision, each electron will—so to speak—completely lose its memory of its previous drift velocity. Each electron will then start off fresh after every encounter, moving off in a random direction. In the average time τ between collisions, the average electron will acquire a drift speed of $v_d = a\tau$. Moreover, if we measure the drift speeds of all the electrons at any instant, we will find that their average drift speed is also $a\tau$. Thus, at any instant, on average, the electrons will have drift speed $v_d = a\tau$. Then

$$v_d = a\tau = \frac{eE\tau}{m}.$$

Combining this result with $\vec{J} = ne\vec{v}_d$, in magnitude form, yields

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m},$$

which we can write as

$$E = \left(\frac{m}{e^2 n\tau}\right) J.$$

Comparing this with $\vec{E} = \rho\vec{J}$, in magnitude form, leads to

$$\rho = \frac{m}{e^2 n\tau}.$$

VI. Magnetic Fields (磁場)

1. Introduction

We have discussed how a charged plastic rod produces a vector field—the electric field $\vec{E}$—at all points in the space around it. Similarly, a magnet produces a vector field—the **magnetic field** $\vec{B}$—at all points in the space around it. You get a hint of that magnetic field whenever you attach a note to a refrigerator door with a small magnet, or accidentally erase a computer disk by bringing it near a magnet. The magnet acts on the door or disk *by means of* its magnetic field.

How then are magnetic fields set up? There are two ways. (1) Moving electrically charged particles, such as a current in a wire, create magnetic fields. (2) Elementary particles such as electrons have an *intrinsic* magnetic field around them; that is, this field is a basic characteristic of the particles, just as are their mass and electric charge (or lack of charge).

Definition: Magnetic field (B) and magnetic force

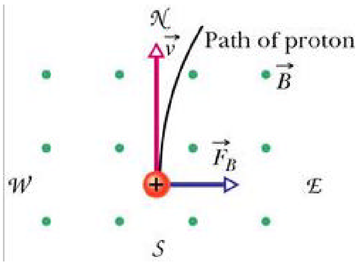
Recall the electric field is given by $\vec{E} = \frac{\vec{F}_E}{q}$. Experiment showed that $B = \frac{F_B}{|q|v}$.

And in a more detailed experiment: $\vec{F}_B = q\vec{v} \times \vec{B}$

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Sample problem:

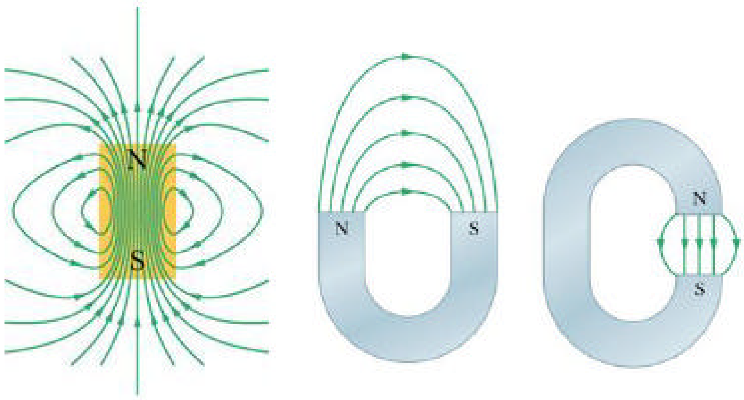
A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)



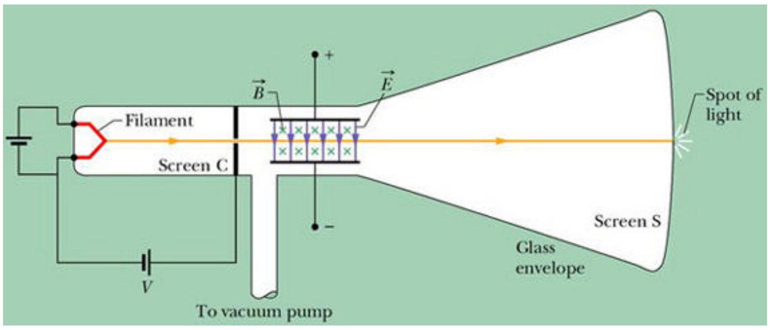
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\text{ MeV})(1.60 \times 10^{-13}\text{ J/MeV})}{1.67 \times 10^{-27}\text{ kg}}} = 3.2 \times 10^{7}\text{ m/s}$$

$$F_B = |q| vB\sin\phi$$
$$= (1.60 \times 10^{-19}\text{ C})(3.2 \times 10^{7}\text{ m/s}) \times (1.2 \times 10^{-3}\text{ T})(\sin 90°)$$
$$= 6.1 \times 10^{-15}\text{ N}.$$

2. Magnetic field lines



3. Discovery of the electron



A modern version of J. J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. The electric field $\vec{E}$ is established by connecting a battery across the deflecting-plate terminals. The magnetic field $\vec{B}$ is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of Xs (which resemble the feathered ends of arrows).

Thomspn's procedure:

1. Set $E = 0$ and $B = 0$ and note the position of the spot on screen S due to the undeflected beam.
2. Turn on $\vec{E}$ and measure the resulting beam deflection.

Deflection: y =1/2 $a\,\Delta t^2 = \dfrac{qEL^2}{2mv^2}$ (check it!)

3. Maintaining $\vec{E}$, now turn on $\vec{B}$ and adjust its value until the beam returns to the undeflected position. (With the forces in opposition, they can be made to cancel.)

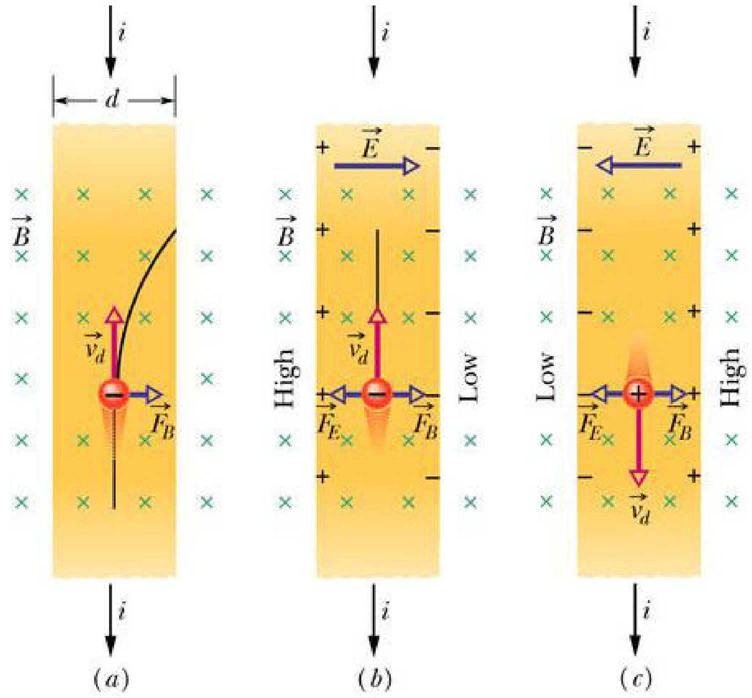
$$|q|E = |q|vB\sin(90°) = |q|vB$$

$$v = \frac{E}{B}.$$

and thus $\dfrac{m}{q} = \dfrac{B^2L^2}{2yE}$

4. Hall effect (霍爾效應)

As we just discussed, a beam of electrons in a vacuum can be deflected by a magnetic field. Can the drifting conduction electrons in a copper wire also be deflected by a magnetic field? In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can. This **Hall effect** allows us to find out whether the charge carriers in a conductor are positively or negatively charged. Beyond that, we can measure the number of such carriers per unit volume of the conductor.



A strip of copper carrying a current i is immersed in a magnetic field $\vec{B}$. (*a*) The situation immediately after the magnetic field is turned on. The curved path that will then be taken by an electron is shown. (*b*) The situation at equilibrium, which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus, the left side is at a higher potential than the right side. (*c*) For the same current direction, if the charge carriers were positively charged, *they* would pile up on the right side, and the right side would be at the higher potential.

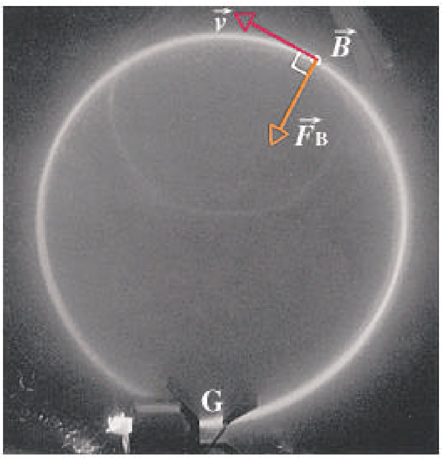
When an equilibrium is achieved (no more positive or negative charge is piling at the side), the electric potential difference V across d is stabilized ($V = E\,d$), and more importantly

$\boldsymbol{F}_E = \boldsymbol{F}_B$ and thus $eE = ev_d B$,

Because the drift speed is also given by $v_d = \frac{J}{ne} = \frac{i}{neA}$,

One can then derive he number density of carriers $n = \frac{Bi}{Vle}$, where $l = A/d$, the thickness .
This determines the **magnitude** and **sign** of carrier density.

5. Circulating charged particle



Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field $\vec{B}$, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force $\vec{F}_B$; for circular motion to occur, $\vec{F}_B$ *must* point toward the center of the circle. Use the right-hand rule for cross products to confirm that $\vec{F}_B = q\vec{v} \times \vec{B}$ gives $\vec{F}_B$ the proper direction. (Don't forget the sign of q.)

From Newton's second law ($\vec{F} = m\vec{a}$) applied to uniform circular motion

$$F = m\frac{v^2}{r},$$

we have

$$qvB = \frac{mv^2}{r}.$$

Solving for r, we find the radius of the circular path as

$$r = \frac{mv}{qB} \quad \text{(radius)}.$$

The period T (the time for one full revolution) is equal to the circumference divided by the speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB} \quad \text{(period)}.$$

The frequency f (the number of revolutions per unit time) is

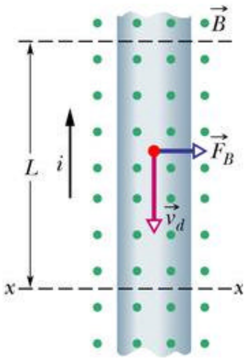
$$f = \frac{1}{T} = \frac{qB}{2\pi m} \quad \text{(frequency)}.$$

The angular frequency ω of the motion is then

$$\omega = 2\pi f = \frac{qB}{m} \quad \text{(angular frequency)}.$$

Question: What happens If the velocity of a charged particle has a component parallel to the (uniform) magnetic field?
Answer: helical path.

6. Magnetic force on a current-carrying wire



Total charge passes through any cross section within a time t is

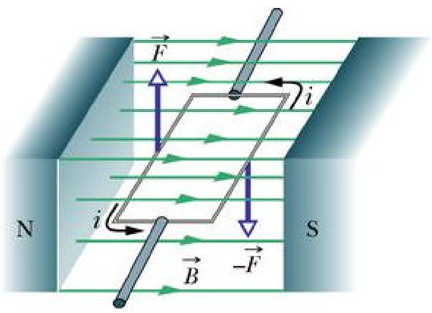
$$q = it = i\frac{L}{v_d}$$

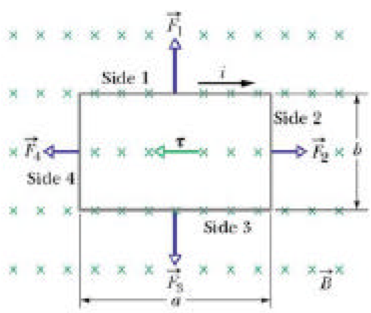
So
$$F_B = qv_d B\sin\phi = \frac{iL}{v_d}v_d B\sin 90^\circ = iLB.$$

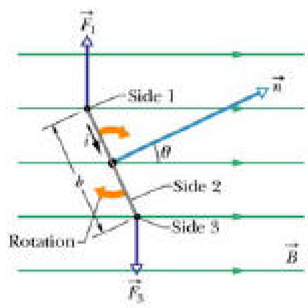
More exactly, $$\vec{F}_B = i\vec{L}\times\vec{B} \quad \text{(force on a current)}.$$

7. Magnetic dipole moment

Consider the ***torque*** on a current loop due to magnetic field







Torque: $\tau = \mathbf{r} \times \mathbf{F}$

$$\tau = \left(iaB\frac{b}{2}\sin\theta\right) + \left(iaB\frac{b}{2}\sin\theta\right) = iabB\sin\theta$$

(only F_1 and F_3 contribute; force F_2 and F_4 make no contribution because of cancellation)

More exactly

$$\tau = \mu \times \mathbf{B},$$

where the ***magnetic dipole moment*** **(磁雙極矩)**

$\mu = i\,A$ (Here $A = ab$.).

Recall the electric dipole moment (**p**) in a electric field, the corresponding torque is

$$\tau = \mathbf{p} \times \mathbf{E}.$$

A magnetic dipole in an external magnetic field has a **magnetic potential energy** that depends on the dipole's orientation in the field. For electric dipoles we have shown

$$U(\theta) = -\vec{p}\cdot\vec{E}.$$

In strict analogy, we can write for the magnetic case

$$U(\theta) = -\vec{\mu}\cdot\vec{B}.$$