電磁學 (Electromagnetism)
The combination of electrical and magnetic phenomena

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The ultimate goal of learning electromagnetism is to understand Maxwell's equations that govern the motions of charged media, effect and origins of static and dynamic electric and magnetic fields.

I. Electric Charge (電荷)

1. What is "electric charge"?

Electric charge is an intrinsic characteristic of the fundamental particles making up of objects. For example, electron has negative charge, while proton has positive charge.

Charges with same (opposite) electrical sign repel (attract) each other.

2. Application of charged particles

Inject printer, air cleaner, copy-machine, ……

3. Various conducting materials: solid-state physics

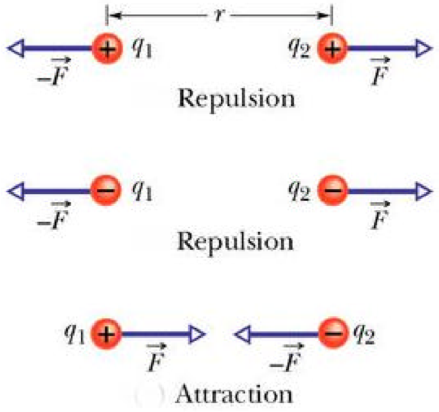
Conductor (**良導體，金屬**) and insulator (**不良導體，絕緣體**)
Semiconductor (**半導體**), superconductor (**超導體**)

4. Coulomb's law

Coulomb's law describes the **electrostatic force** between small (point) electric charges q_1 and q_2 at rest (or nearly at rest) and separated by a distance r:

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2} \quad \text{(Coulomb's law)}.$$

Here $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$ is the **permittivity constant,** and $1/4\pi\varepsilon_0 = k = 8.99 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$.



The direction of F (attractive or repulsive) depends on the relative sign of both charges.

Principle of Superposition

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n},$$

The form of Coulomb's law is the same as that of Newton's equation for the gravitational force between two particles with masses m_1 and m_2 that are separated by a distance r:

$$F = G\frac{m_1 m_2}{r^2},$$

in which G is the gravitational constant.

5. Charged is Quantized

Any positive or negative charge q that can be detected can be written as

$$q = ne, \; n = \pm 1, \pm 2, \pm 3, \ldots,$$

in which e, the **elementary charge,** has the value

$$e = 1.60 \times 10^{-19}\,\mathrm{C}.$$

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

6. Charged is Conserved

II. Electric Field (電場)

Alternative way to view the charge and electric force

1. What is "electric field"?

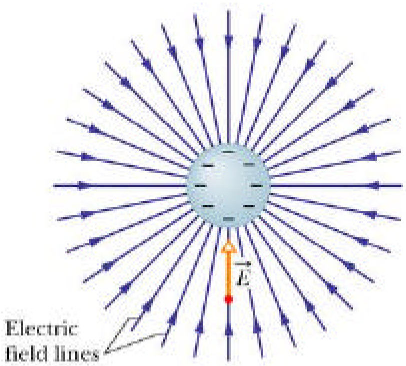
The electric field is a *vector field*; it consists of a distribution of *vectors*, one for each point in the region around a charged object, such as a charged rod. In principle, we can define the electric field at some point near the charged object.

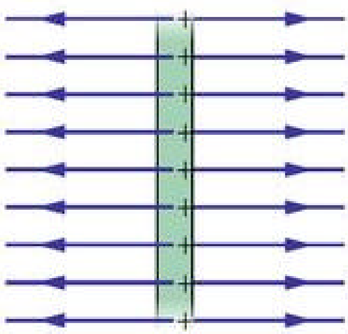
$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).}$$

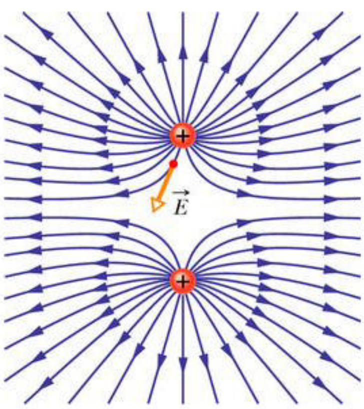
Examples:

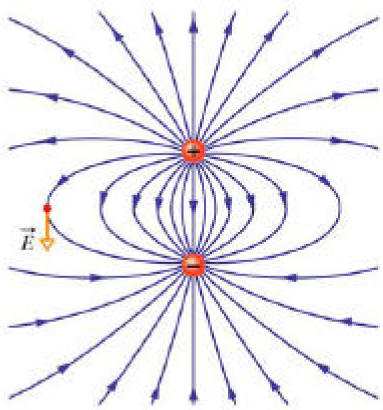
Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

2. Electric lines: a way to view the magnitude and direction of local electric field



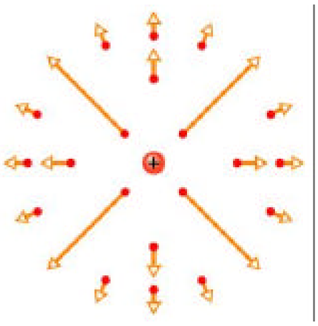






3. Important examples

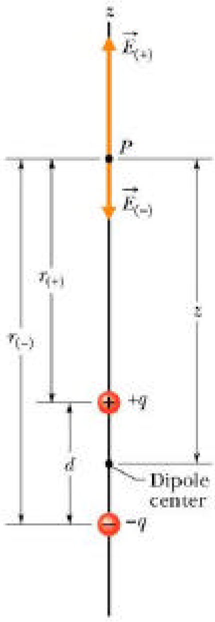
a) Electric Field due to a point charge



$$F = \frac{1}{4\pi\epsilon_0}\frac{|q||q_0|}{r^2}$$

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \quad \text{(point charge)}$$

b) Electric Field due to an electric dipole



$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\epsilon_0\left(z-\frac{1}{2}d\right)^2} - \frac{q}{4\pi\epsilon_0\left(z+\frac{1}{2}d\right)^2} \\ &= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right] \\ &= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+\frac{d}{z}+\cdots\right) - \left(1-\frac{d}{z}+\cdots\right)\right] \end{aligned}$$

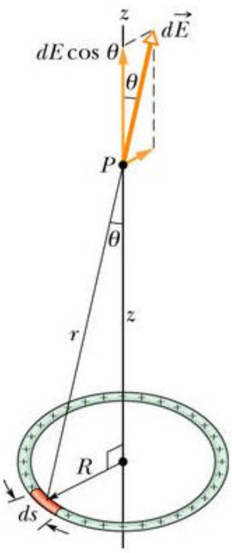
Since $d/z \ll 1$

$$E = \frac{q}{4\pi\epsilon_0 z^2}\frac{2d}{z} = \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3}$$

$$= \frac{1}{2\pi\epsilon_0}\frac{p}{z^3} \quad \text{(electric dipole)}$$

with $P = qd$ the **dipole moment.**

c) Electric Field due to a line of charge



$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{\left(z^2+R^2\right)}$$

The parallel (to z axis) component of dE is

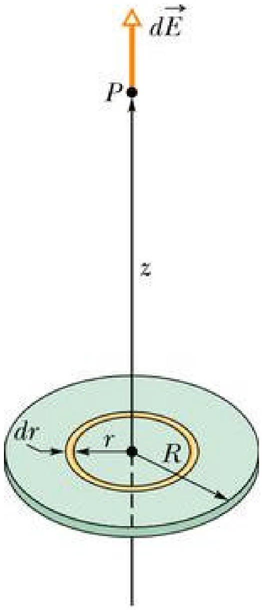
$$dE\cos\theta = \frac{z\lambda}{4\pi\epsilon_0\left(z^2+R^2\right)^{3/2}}ds$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\epsilon_0\left(z^2+R^2\right)^{3/2}}\int_0^{2\pi R} ds = \frac{z\lambda(2\pi R)}{4\pi\epsilon_0\left(z^2+R^2\right)^{3/2}}$$

$$= \frac{qz}{4\pi\epsilon_0\left(z^2+R^2\right)^{3/2}} \quad \text{(charged ring)}$$

When $z \gg R$, $\quad E = \frac{1}{4\pi\epsilon_0}\frac{q}{z^2} \quad$ (charged ring at large distance)

d) Electric Field due to a charged disk



$$dq = \sigma\, dA = \sigma(2\pi r dr),$$

$$dE = \frac{z\sigma 2\pi r dr}{4\pi\epsilon_0 (z^2 + r^2)^{3/2}} = \frac{\sigma z}{4\epsilon_0}\frac{2r dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\epsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)dr.$$

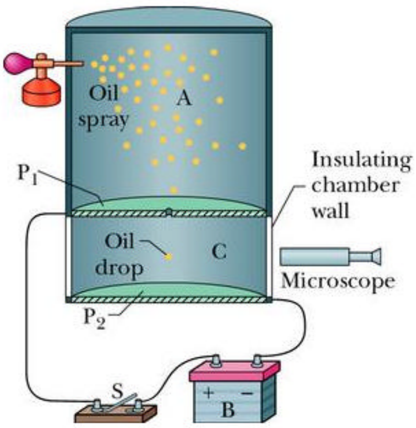
$$= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \quad \text{(charged disk)}$$

When $R \to \infty$, $E = \dfrac{\sigma}{2\epsilon_0}$ (infinite sheet)

When $z \gg R$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{z^2}$ ($q = \pi R^2\sigma$, like a point charge)

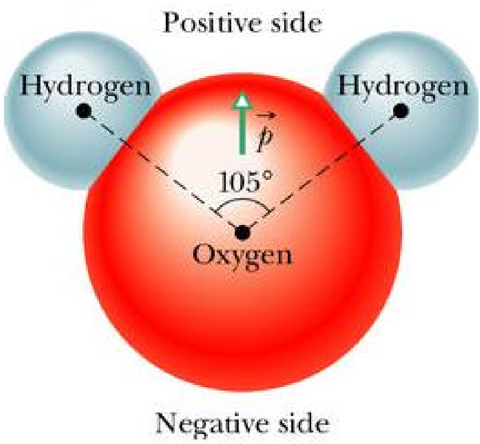
4. A point charge in an electric field

➤The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

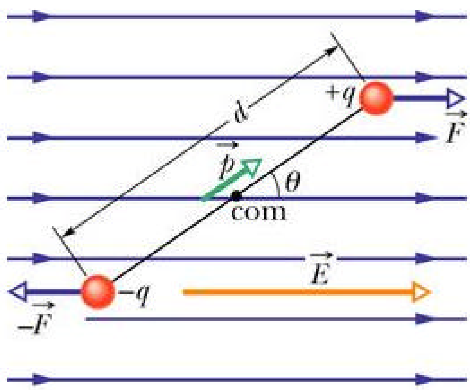


Millikan oil-drop experiment
(measuring the elementary charge)

5. A electric dipole in an electric field



H_2O



$\vec{\tau} = \vec{p} \times \vec{E}$ (torque on a dipole)

$\tau = pE\sin\theta$