

$$\begin{aligned}
\frac{d}{dx} \sin x &= \lim_{\Delta x \rightarrow 0} \frac{\sin(x + \Delta x) - \sin(x)}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{\sin(x) \cos(\Delta x) + \cos(x) \sin(\Delta x) - \sin(x)}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \left[ \sin(x) \cdot \frac{\cos(\Delta x) - 1}{\Delta x} + \cos(x) \frac{\sin(\Delta x)}{\Delta x} \right] \\
&\qquad\qquad\qquad \begin{array}{c} \parallel \\ 0 \end{array} \qquad\qquad\qquad \begin{array}{c} \parallel \\ 1 \end{array} \\
&= \cos x
\end{aligned}$$

hw2: *prove* :  $\lim_{\Delta x \rightarrow 0} \frac{\cos(\Delta x) - 1}{\Delta x} = 0$

hw3: *prove* :  $\sin(x + y) = \sin x \cos y + \cos x \sin y$   
 $\cos(x - y) = \cos x \cos y - \sin x \sin y$

## 2. Euler's formule

$$e^{i\theta} = \cos \theta + i \sin \theta$$

*pf* :  $e^{i\theta} = x + iy$

$$e^{-i\theta} = x - iy$$

$$1 = e^{i\theta} \cdot e^{-i\theta} = (x + iy)(x - iy) = x^2 + y^2$$

$$x^2 + y^2 = 1$$

*Pick's + hm*

$$\begin{cases} x = \cos \theta, y = \sin \theta \\ \text{or} \\ x = \sin \theta, y = \cos \theta \end{cases}$$

$$e^{i\theta} = 1 = \cos \theta^\circ + i \sin \theta^\circ$$

so :  $e^{i\theta} = \cos \theta + i \sin \theta$

