

4月10日 數學考試

1.  $|\vec{a}|=2$   $|\vec{b}|=3$   $|\vec{c}|=4$

$$\vec{a} + \vec{b} + \vec{c} = 0$$

求  $\vec{b} \cdot \vec{c} =$

解：  $\vec{b} + \vec{c} = -\vec{a}$   $|\vec{b} + \vec{c}|^2 = |\vec{a}|^2$

$$\Rightarrow |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |\vec{a}|^2$$

$$\Rightarrow 9 + 16 + 2\vec{b} \cdot \vec{c} = 4$$

$$\Rightarrow \vec{b} \cdot \vec{c} = \frac{-21}{2}$$

求  $|2\vec{a} + 3\vec{b} + 4\vec{c}|$

$$= |2\vec{a} + 2\vec{b} + 2\vec{a} + \vec{b} + 2\vec{c}| = |\vec{b} + 2\vec{c}|$$

$$\therefore |\vec{b} + 2\vec{c}|^2 = |\vec{b}|^2 + |2\vec{c}|^2 + 4\vec{b} \cdot \vec{c} = 9 + 64 - 42 = 31$$

$$\Rightarrow |\vec{b} + 2\vec{c}| = \sqrt{31}$$

2.  $L_1: 2x + y + 1 = 0$

$$L_2: x + 2y - 1 = 0$$

$$L_3: 2x - y - 7 = 0$$

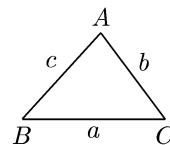
交於三點  $A(-1, 1)$   $B(\frac{3}{2}, -4)$   $C(3, -1)$

(a) 重心座標  $G$

$$\Rightarrow \therefore \vec{OG} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$$

$$= \frac{1}{3}((-1, 1) + (\frac{3}{2}, -4) + (3, -1))$$

$$= (\frac{7}{6}, \frac{-4}{3})$$



(b) 內心座標  $I$

$$\begin{aligned}
\therefore \overline{AI} &= \frac{b}{a+b+c} \overline{AB} + \frac{c}{a+b+c} \overline{AC} \\
\Rightarrow (\overline{AO} + \overline{OI}) &= \frac{b}{a+b+c} (\overline{AO} + \overline{OB}) + \frac{c}{a+b+c} (\overline{AO} + \overline{OC}) \\
\Rightarrow \overline{OI} &= \frac{a}{a+b+c} \overline{OA} + \frac{b}{a+b+c} \overline{OB} + \frac{c}{a+b+c} \overline{OC} \\
a &= \frac{3}{2}\sqrt{5}, b = 2\sqrt{5}, c = \frac{5}{2}\sqrt{5} \text{ 代入} \\
\overline{OI} &= \frac{1}{4}(-1, 1) + \frac{1}{3}\left(\frac{3}{2}, -4\right) + \frac{5}{12}(3, -1) = \left(\frac{3}{2}, \frac{-3}{2}\right)
\end{aligned}$$