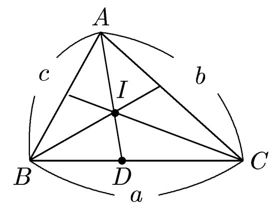
3 月 6 日 數學解答

習題 1. 求證 $\overrightarrow{AI}=\dfrac{b}{a+b+c}\overrightarrow{AB}+\dfrac{c}{a+b+c}\overrightarrow{AC}$



證： $\overline{BD}:\overline{CD}=c:b \quad \therefore \overline{BD}=\dfrac{ac}{b+c}$

$\overline{AI}:\overline{AD}=\overline{AB}:\overline{AB}+\overline{BD}=c:c+\dfrac{ac}{b+c}=b+c:a+b+c$

又 $\overrightarrow{AD}=\dfrac{c}{b+c}\overrightarrow{AC}+\dfrac{b}{b+c}\overrightarrow{AB}$

$\overrightarrow{AI}=\dfrac{b+c}{a+b+c}(\dfrac{c}{b+c}\overrightarrow{AC}+\dfrac{b}{c+C}\overrightarrow{AB})=\dfrac{c}{a+b+c}\overrightarrow{AC}+\dfrac{b}{a+b+c}\overrightarrow{AB}$

習題 2. 由柯西不等式

$(a_1b_1+a_2b_2)^2\le(a_1^2+a_2^2)(b_1^2+b_2^2)$

$(3\times\dfrac{x}{3}+4\times\dfrac{y}{4})^2\le(3^2+4^2)[(\dfrac{x}{3})^2+(\dfrac{y}{4})^2]$

$\Rightarrow (x+y)^2\le 25(\dfrac{x^2}{9}+\dfrac{y^2}{16})=25 \quad \Rightarrow -5\le x+y\le 5$

等號在$3\cdot\dfrac{y}{4}=4\cdot\dfrac{x}{3}$時成立

$\therefore x:y=9:16 \quad \Rightarrow y=\dfrac{16}{9}x$

代入$\dfrac{x^2}{9}+\dfrac{y^2}{16}=1$得

$x^2+\dfrac{16}{9}x^2=1 \quad \Rightarrow x=\pm\dfrac{9}{5}$

$\Rightarrow \begin{cases} x=\dfrac{9}{5} \\ y=\dfrac{16}{5} \end{cases} \quad \Rightarrow \begin{cases} x=\dfrac{-9}{5} \\ y=\dfrac{-16}{5} \end{cases}$

當$(x\,,\,y)=(\dfrac{9}{5},\dfrac{16}{5})$時， $x+y+2=7$爲最大值

當$(x\,,\,y)=(\dfrac{-9}{5},\dfrac{-16}{5})$時， $x+y+2=-3$爲最小值