- Wavefunction of matter wave

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 dx = 1$$

- Quantum Mechanics: Schroedinger equation

plane wave $\psi(x,t) = A e^{i(kx-\omega t)}$ --- free particle

$k = \frac{2\pi}{\lambda}$ <--> $p = \frac{h}{\lambda} = \hbar k$

$\omega = 2\pi\nu$ <--> $E = h\nu = \hbar\omega$

$$\frac{\partial\psi(x,t)}{\partial t} = -i\omega\psi(x,t) = \frac{E}{i\hbar}\psi(x,t) \Rightarrow E\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

$$\frac{\partial\psi}{\partial x} = ik\psi = i\frac{p}{\hbar}\psi \Rightarrow p\psi(x,t) = \frac{\hbar}{i}\frac{\partial}{\partial x}\psi(x,t)$$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = -\frac{p^2}{\hbar^2}\psi \Rightarrow \frac{p^2}{2m}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi$$

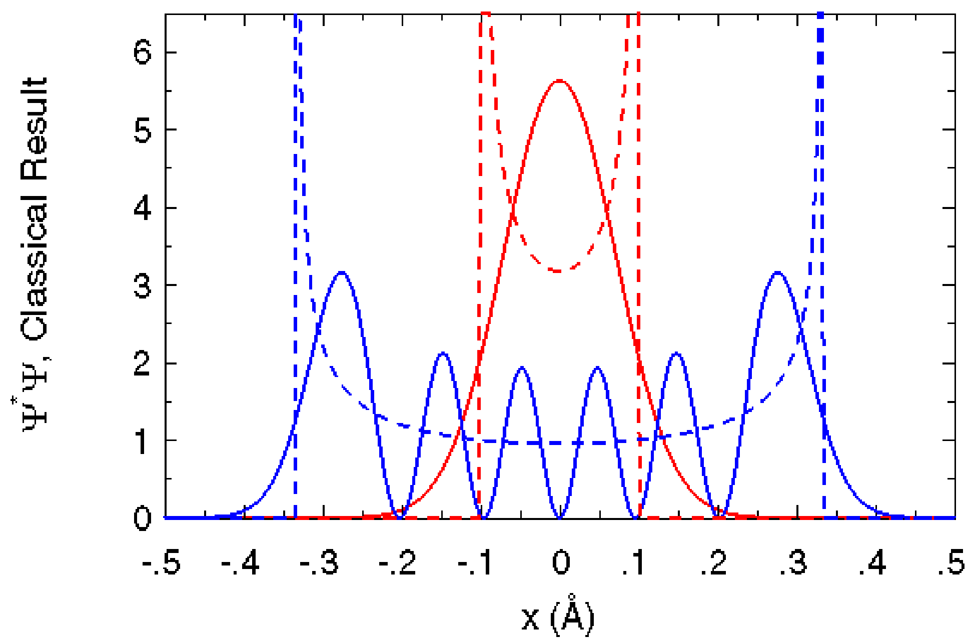
since $\frac{p^2}{2m} + V = E$ => $(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V)\ \psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$

- Dirac's relativstic Quantum Mechanics: $E^2 = p^2c^2 + m^2c^4$

- Simple Harmonic Oscillation (S.H.O.)

Planck: $E_n = h\nu = n\hbar\omega$

Heisenberg: $E_n = (n + \frac{1}{2})\hbar\omega$



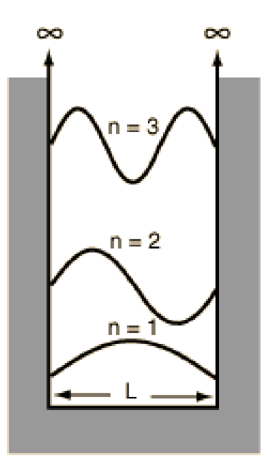
- Time-dependence: $\psi(x,t)=\psi(x)e^{-iEt/\hbar}$

- Expectation value :

$$\bar{x}=\int_{-\infty}^{+\infty}\psi^*(x,t)\,x\,\psi(x,t)\,dx \quad , \qquad \overline{x^2}=\int_{-\infty}^{+\infty}\psi^*(x,t)\,x^2\psi(x,t)\,dx$$

$$\bar{p}=\int_{-\infty}^{+\infty}\psi^*(x,t)\,p\,\psi(x,t)\,dx=\int_{-\infty}^{+\infty}\psi^*(x,t)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\psi(x,t)\,dx \quad ,$$

- Particle in a box



$$\psi(x)=0 \quad \text{for} \quad x \le \frac{-a}{2} \quad \text{or} \quad x \ge \frac{a}{2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x) \quad => \quad \frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

assume $\psi(x)=A\sin kx + B\cos kx$

Boundary condition: $\psi(x)=0 \quad at \quad x=\pm\frac{a}{2}$

Normalization: $\int_{-\frac{a}{2}}^{-\frac{a}{2}} |\psi| \, dx = 1$

=> Energy level $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

- Heisenberg uncertainty relation:

Standard deviation: $\Delta x=\sqrt{(x-\bar{x})^2}$, $\Delta p=\sqrt{(p-\bar{p})^2}$

$$\Delta x \cdot \Delta p \ge \frac{\hbar}{2}$$