• Newtonian Mechanics

1. $\vec{p} = m\vec{v}$

2. $\vec{F} = \dfrac{d\vec{p}}{dt}$

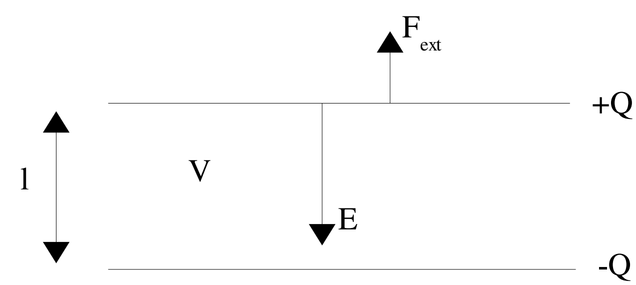
time domain → space domain

$F = F(\vec{r})$

$$\frac{p_f^2}{2m} - \frac{p_i^2}{2m} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r} = -(U(\vec{r}_f) - U(\vec{r}_i))$$, assume F is conserved

• D'Alembertian principle of virtual work

Example:



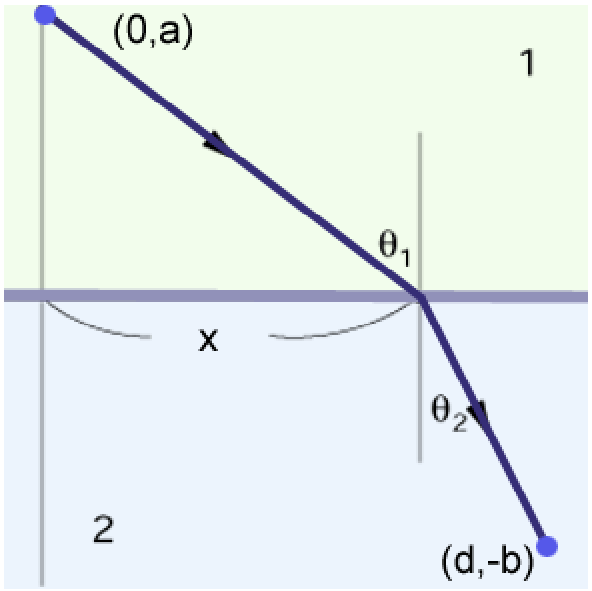
$\vec{l} \to \vec{l} + \delta\vec{l}$

work done = $\vec{F}_{con}\cdot d\vec{l} = \delta W_{con} = \frac{1}{2}Q\vec{E}\cdot d\vec{l} = -\frac{1}{2}\,Q\,\frac{V}{l}\,dl$

$$dW_{ext} = \frac{1}{2}QE\,dl = \frac{1}{2}\,Q\,\frac{Q}{A\epsilon}dl = \frac{1}{2}\epsilon E^2 A\,dl = u_E A\,dl = dU_c$$

energy density of the electric field : $u_E = \frac{1}{2}\,\epsilon\,E^2$

• Fermat's principle



the optical path is

$$t(x)=\frac{\sqrt{x^2+a^2}}{v_1}+\frac{\sqrt{(d-x)^2+b^2}}{v_2}$$

Fermat's assumption: the optical path length is *minimized:*

$$t'(x)=\frac{x}{\sqrt{x^2+a^2}}\cdot\frac{1}{v_1}-\frac{d-x}{\sqrt{(d-x)^2+b^2}}\cdot\frac{1}{v_2}\equiv 0$$

note that $\frac{x}{\sqrt{x^2+a^2}}=\sin\theta_1$, $\frac{d-x}{\sqrt{(d-x)^2+b^2}}=\sin\theta_2$

$\therefore \quad \frac{\sin\theta_1}{\sin\theta_2}=\frac{v_1}{v_2}$ --- law of reflection (Snell's law)