Light → EM wave

$$\vec{E} = \vec{E}(\vec{r},t) \quad , \quad \vec{B} = \vec{B}(\vec{r},t)$$

wave equation: $$\nabla^2 \vec{E}(\vec{r},t) - \frac{1}{v^2}\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = 0 \quad , \quad \nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$$

In 1-dimension: $$\frac{\partial^2 E(x,t)}{\partial x^2} - \frac{1}{v^2}\frac{\partial E(x,t)}{\partial t} = 0$$

- For plane wave:

$$\vec{E}(\vec{r},t) = \hat{a}_x A\cos(\omega t - \vec{k}\cdot\vec{r} + \phi_0) \quad , \quad \hat{a}_x \text{ is the direction of polarization}$$

generalized the above formula to complex number :

$$\tilde{E}(x,t) = \hat{a}_x A e^{i(\omega t - kx + \phi_0)}$$

- Complex plane:

$$z = Ae^{i\theta} = A(\cos\theta + i\sin\theta)$$

$$Z_t = z_1 + z_2 = A_1 e^{i\theta_1} + A_2 e^{i\theta_2} \equiv A_t e^{i\theta_t}$$

for N sources:

$$A_t = (\sum_i^N Z_i)\cdot(\sum_j^N Z_j)^* = \sum_{i=1}^N |z_i|^2 + 2\sum_{i<j} A_i A_j \cos(\theta_i - \theta_j)$$

- Superposition of two EM waves (plane approximation):

$$\vec{E}_1(\vec{r},t) = A_1 \mathrm{e}^{i[\omega t - \vec{k}\cdot(\vec{r}-\vec{r}_1)+\phi_1]}$$

$$\vec{E}_2(\vec{r},t) = A_1 \mathrm{e}^{i[\omega t - \vec{k}\cdot(\vec{r}-\vec{r}_2)+\phi_2]}$$

$$\Longrightarrow \quad \vec{E}_t(\vec{r},t) = \vec{E}_1(\vec{r},t) + \vec{E}_2(\vec{r},t)$$

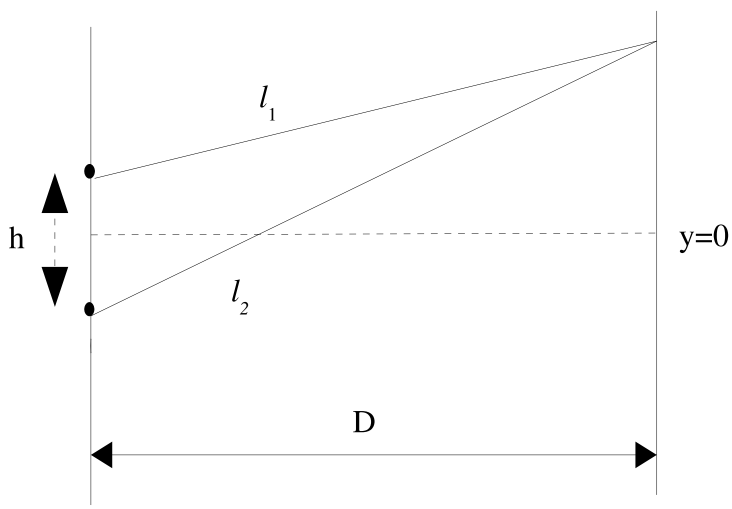
- Intensity of light

Energy density : $u_E = \frac{1}{2}\varepsilon E^2 + \frac{1}{2\mu_B}B^2 \equiv \frac{dU}{dV}$ dV is the unit volume element

Poynting vector : $S = \frac{1}{A}\frac{dU}{dt} = u_E v$

$$S = (\frac{1}{2}\varepsilon E^2 + \frac{1}{2\mu_B}B^2)\frac{1}{\sqrt{\varepsilon \mu_B}}$$

the intensity of EM wave is the time average of S : $I = \bar{S}$

- 

Assume $\phi_1 = \phi_2$, $A_1 = A_2$

$$l_i = [(y + \frac{h}{2})^2 + D^2] = D[1 + \frac{(y \pm \frac{h}{2})^2}{D^2}]^{\frac{1}{2}} \approx D[1 + \frac{1}{2}\frac{(y \pm \frac{h}{2})^2}{D^2}]$$

The optical path length $\Delta L = l_2 - l_1 = \frac{1}{2\mathrm{D}}[(y+\frac{h}{2})^2 - (y-\frac{h}{2})^2] = \frac{hy}{D}$