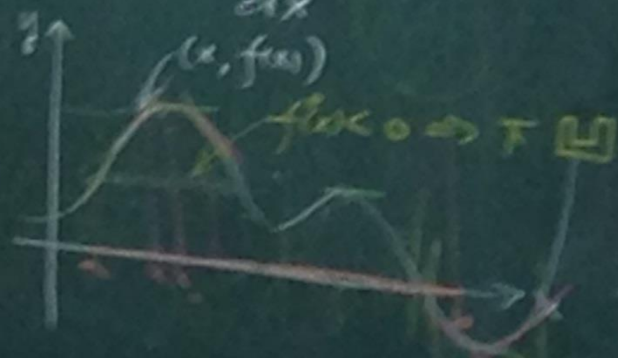
$f(x)$

$$f'(x) = \frac{df(x)}{dx}$$

$y = f(x)$



$$f''(x) = \frac{df'(x)}{dx} = \frac{d}{dx}\frac{df(x)}{dx}$$

$$\equiv \frac{d^2}{(dx)^2} f(x)$$

$$= \frac{d^2 f(x)}{dx^2}$$

$dx^2 = (dx)^2$

$|x|$

$f'(x) = 0$

$$f(x) = x^3 - x^2 - x + 1$$

$$f'(x) = 3x^2 - 2x - 1$$

$$= (3x+1)(x-1) = 0$$

$$f(x) = x^3 - x^2 - x + 1$$

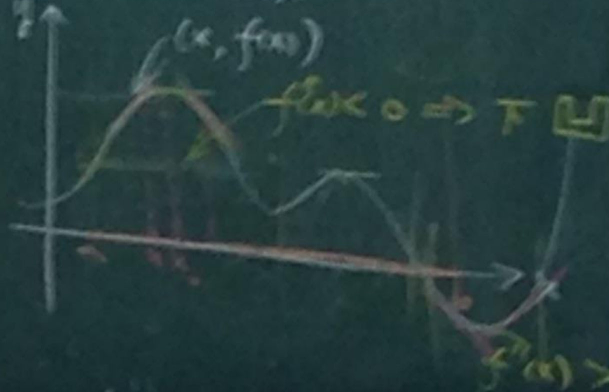
$$f'(x) = 3x^2 - 2x - 1 = (3x+1)(x-1)$$

$$f''(x) = 6x - 2 \Rightarrow f'' > 0 \text{ if } x > \frac{1}{3}$$

$f'' < 0$ …

$f'(x) = \frac{df(x)}{dx}$

$y = f(x)$



$\equiv \frac{d^2}{(dx)^2} f(x)$

$\equiv \frac{d^2 f(x)}{dx^2}$

$dx^2 = (dx)^2$

$f'(x) = 0$

$f(x) = x^3 - x^2 - x + 1$

$f'(x) = 3x^2 - 2x - 1$

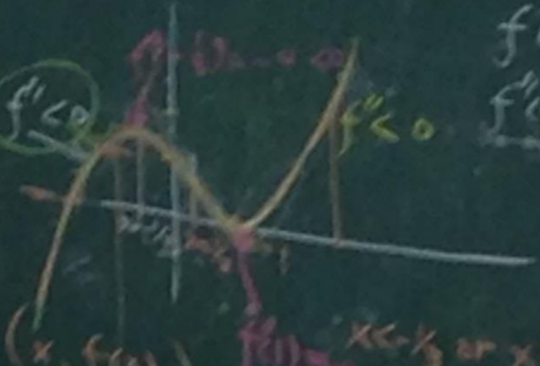
$= (3x+1)(x-1) = 0$

$f(x) = x^3 - x^2 - x + 1$

$f'(x) = 3x^2 - 2x - 1 = (3x+1)(x-1)$

$f''(x) = 6x - 2 \Rightarrow f'' > 0$ if $x > \frac{1}{3}$

$f'' < 0$ if $x < \frac{1}{3}$



$f'(x) = 0$

$f(-\frac{1}{3})$

$= -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1$

$= \frac{-1-3+9}{27} + 1 = \frac{32}{27}$

$f(1) = 0$

$x < -\frac{1}{3}$ or $x > 1 \Rightarrow f'(x) > 0$

$-\frac{1}{3} < x < 1 \Rightarrow f'(x) < 0$

$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

$f(x) = \begin{cases} \sin x \\ \cos x \end{cases}$

exponent

$f(x) = a^x$ exponential function

$=2 \to 2^x$

$=3 \to 3^x$

$a^x \cdot a^y = a^{x+y}$

$(a^x)^m = a^{mx}$

$y = f(x) = a^x$

$x = \log_a y$

$\log_a(yz) = \log_a y + \log_a z$

$\log_a(b^x) = x \log_a b$

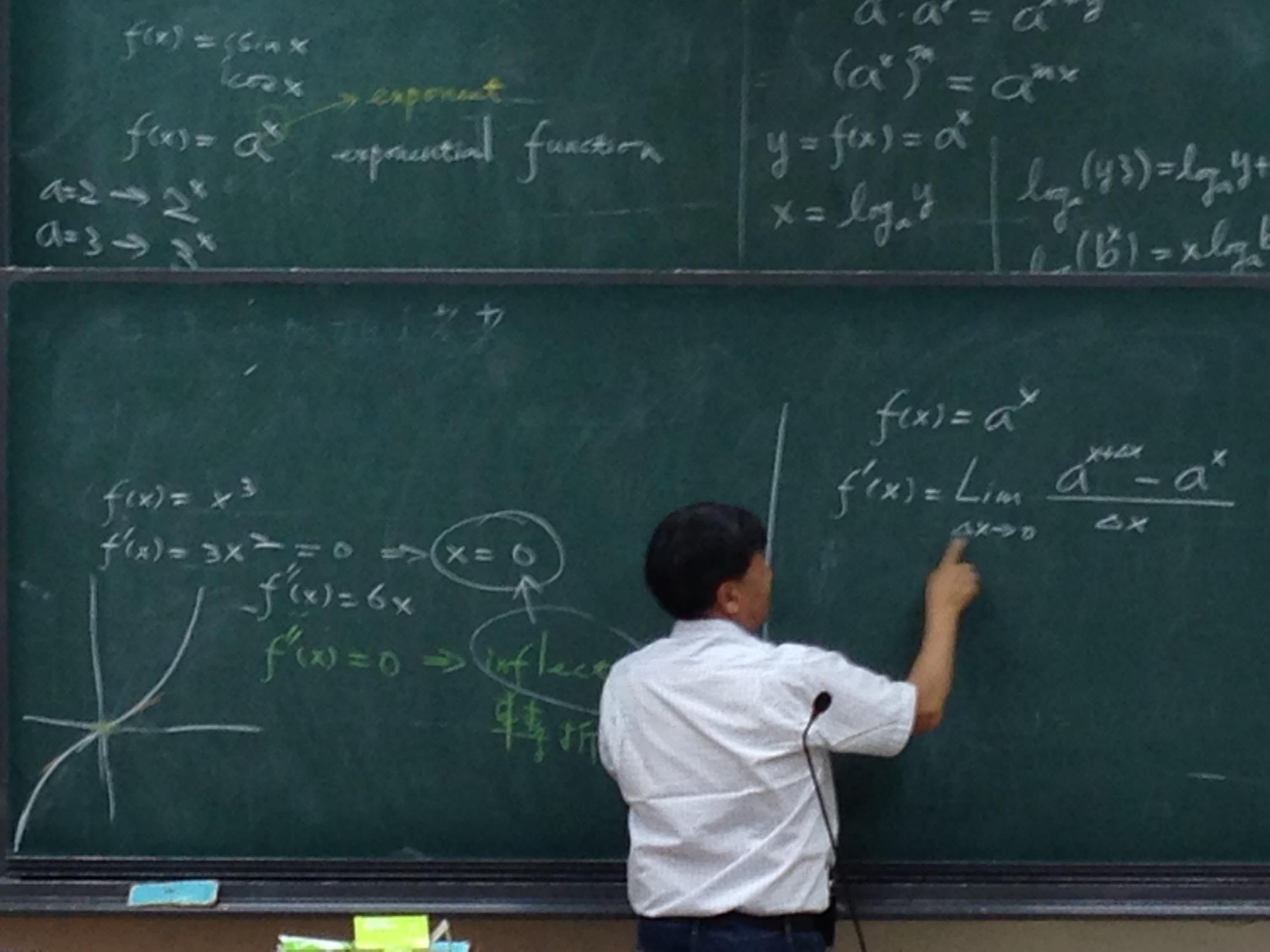
$f(x) = x^3$

$f'(x) = 3x^2 = 0 \Rightarrow x = 0$

$f''(x) = 6x$

$f(x) = a^x$

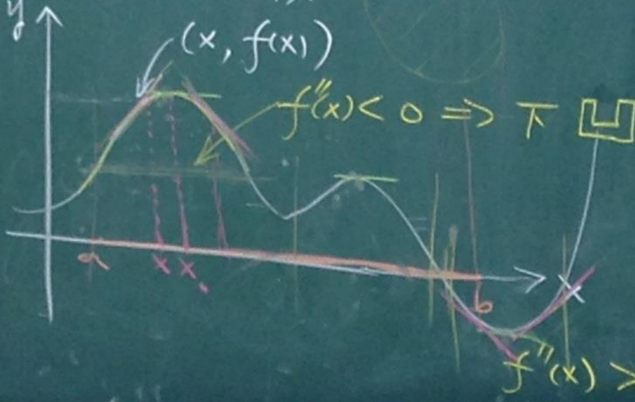
$f'(x) = \lim_{\Delta x \to 0} \dfrac{a^{x+\Delta x} - a^x}{\Delta x}$



f(x) = {sin x
cos x
exponent
f(x) = a^x
exponential function
a=2 → 2^x
a=3 → 3^x
(a^x)^m = a^{mx}
y = f(x) = a^x
x = log_a y
log_a (yz) = log_a y +
log_a (b^x) = x log_a b
f(x) = a^x
f'(x) = Lim_{Δx→0} (a^{x+Δx} − a^x)/Δx
f(x) = x^3
f'(x) = 3x^2 = 0 ⇒ x = 0
f''(x) = 6x
f''(x) = 0 ⇒ inflect

$= \frac{d^2 f(x)}{dx^2}$

$y = f(x)$



$dx^2 \equiv (dx)^2$

$f'(x) = 0$

$f(x) = x^3 - x^2 - x + 1$

$f'(x) = 3x^2 - 2x - 1$

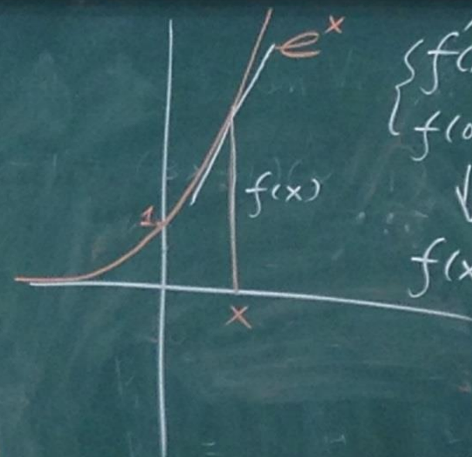
$= (3x+1)(x-1) = 0$

exponential function

$e^{-x^3+3x-1 / 2+x^2}$

$\downarrow$

$\exp\left(\frac{-x^3+3x-1}{x^2+2}\right)$

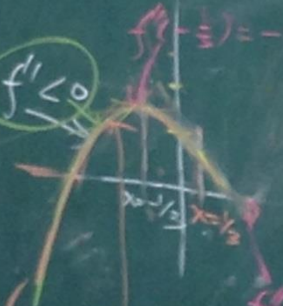


$\begin{cases} f'(x) = f(x) \\ f(0) = 1 \end{cases}$

$\Downarrow$

$f(x) = e^x$

$\equiv \exp(x)$



$f(-\frac{1}{3})$

$= -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1$

$= \frac{-1-3+9}{27} + 1 = \frac{32}{27}$

$(x, f(x))$

$f'(1) = 4 > 0$

$x < -\frac{1}{3}$ or $x > 1 \Rightarrow f'(x) >$

$-\frac{1}{3} < x < 1 \Rightarrow f'(x) < 0$

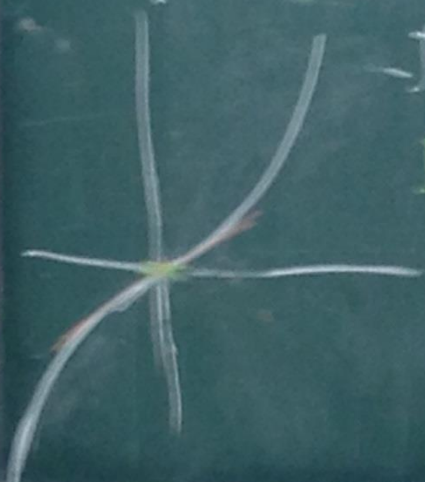
$f(x) = x^3$

$f'(x) = 3x^2 = 0 \Rightarrow x = 0$

$f''(x) = 6x$

$f''(x) = 0 \Rightarrow$ inflection point

轉折點



$f(x) = a^x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{a^{x+\Delta x} - a^x}{\Delta x}$$

$$= a^x \left\{ \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x} \right\}$$

$$\lim_{\Delta x} \frac{a^{\Delta x} - 1}{\Delta x} \simeq 0.7 \quad \text{if } a = 2$$

$$\simeq 1.1 \quad \text{if } a = 3$$

要 $\lim_{\Delta x} \frac{a^{\Delta x} - 1}{\Delta x} = 1 \Rightarrow a = 2.718281\ldots$

$a \equiv e$

$a^x \cdot a^y = a^{x+y}$

$(a^x)^m = a^{mx}$

$f(x) = \begin{cases} \sin x \\ \cos x \end{cases}$

$f(x) = a^x \rightarrow$ exponent

exponential function

$y = f(x) = a^x$

$x = \log_a y$

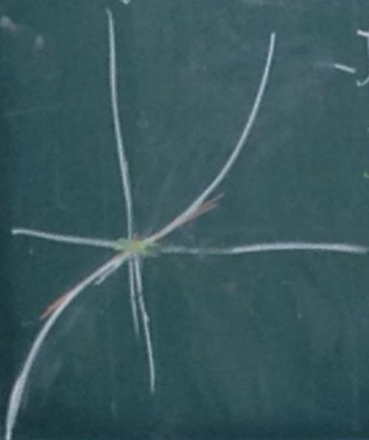
$\log_a(yz) = \log_a y + \log_a z$

$\dots = 0 \Rightarrow x = 0$

$f''(x) = 6x$

$f''(x) = 0 \Rightarrow$ inflection point

轉折點



$= a^x \left\{ \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x} \right\}$

$\lim_{\Delta x} \frac{a^{\Delta x} - 1}{\Delta x} \simeq 0.7$ if $a = 2$

$\simeq 1.1$ if $a = 3$

要 $\lim_{\Delta x} \frac{a^{\Delta x} - 1}{\Delta x} = 1 \Rightarrow a = 2.7182818$

$a \equiv e$

$f(x) = \begin{cases} \sin x \\ \cos x \end{cases}$

$f(x) = a^x$ exponential function

$x \to$ exponent

$a = 2 \to 2^x$

$a = 3 \to 3^x$

$a = e$

$f(x) = e^x$

$f'(x) = f(x) \quad \forall x \in \mathbb{R}$

$a^x \cdot a^y = a^{x+y}$

$(a^x)^m = a^{mx}$

$y = f(x) = a^x$

$x = \log_a y$

$\log_a(yz) = \log_a y + \log_a z$

$\log_a(b^x) = x \log_a b$

$$f(x) = \frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$f(x) = \frac{f(a)}{0!} + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

Taylor series

Taylor Swift

$$\boxed{a \le \xi \le x}$$

$$\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$$

$$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$$

$$0! = 1$$

$$f(x) = a^x$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{a^{x+\Delta x} - a^x}{\Delta x}$$

$$= a^x \left\{ \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x} \right\}$$

$$\lim_{\Delta x} \frac{a^{\Delta x} - 1}{\Delta x} \simeq 0.7 \quad \text{if } a = 2$$

$$\simeq 1.1 \quad \text{if } a = 3$$

要 $\lim_{\Delta x} \frac{a^{\Delta x} - 1}{\Delta x} = 1 \Rightarrow a = 2.7182818$

$$a \equiv e$$

$$e^{x} = \frac{x^0}{0!} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \qquad (x = i\theta)$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^k x^{2k+1}}{(2k+1)!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^k x^{2k}}{(2k)!} + \cdots$$

$$\left\{ \begin{array}{l} f''(x) = -f(x) \\ f(0) = 1 \\ f'(0) = 0 \end{array} \right\} \Rightarrow \cos x$$

$$\left\{ \begin{array}{l} f''(x) = -f(x) \\ f(0) = 0 \\ f'(0) = 1 \end{array} \right. \Rightarrow \sin x$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \cdots & \cdots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n \times n}$$

$$|A| = \sum_P \sigma(P)\, a_{1k_1} a_{2k_2} a_{3k_3} \cdots a_{nk_n}$$

$$P = \begin{pmatrix} 1 & 2 & \cdots & n \\ k_1 & k_2 & \cdots & k_n \end{pmatrix}$$

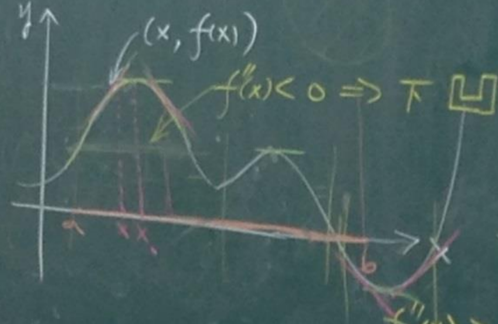
$$n!$$

$$\sigma(P) = \pm 1$$

$$f(x)$$

$$f'(x) = \frac{df(x)}{dx}$$

$y = f(x)$



$$f''(x) = \frac{df'(x)}{dx} = \frac{d}{dx}\frac{df(x)}{dx} \equiv \frac{d^2}{(dx)^2} f(x) \equiv \frac{d^2 f(x)}{dx^2}$$

$$dx^2 \equiv (dx)^2$$

$|x|$

$$f'(x) = 0$$

$$f(x) = x^3 - x^2 - x + 1$$

$$f'(x) = 3x^2 - 2x - 1 = (3x+1)(x-1) = 0$$

exponential function

Euler's formula

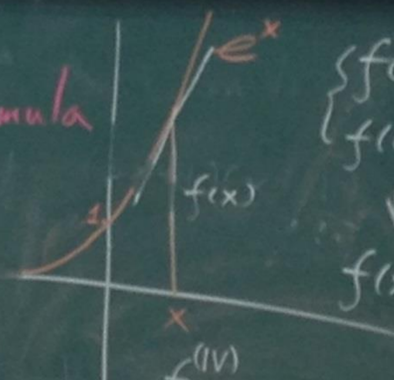
$$a_n = f^{(n)}$$

$$f(x) = \ldots$$
$$\|$$
$$e^x$$

$$f'(x) = 0 + 1 \cdot a_1 + \ldots$$

$$f''(x) = 0 + 0 + 2 \cdot 1 a_2 + 3 \cdot 2 a_3 x + \ldots$$

$$e^{i\theta} = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots\right) = \cos\theta + i\sin\theta$$



$$\begin{cases} f'(x) = f(x) \\ f(0) = 1 \end{cases}$$

$$\Downarrow$$

$$f(x) = e^x \equiv \exp(x)$$

$$f^{(IV)}(0) = 4 \cdot 3 \cdot 2 \cdot 1\, a_4$$

$$f^{(V)}$$

$$f^{(n)}(0) = n \cdot (n-1) \cdots 2 \cdot 1\, a_n$$