Differentiation

derivative: $$f'(x) \equiv \frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\overbrace{f(x+\Delta x) - f(x)}^{\Delta f(x)}}{\Delta x}$$

1. $(cf(x))' = cf'(x)$
2. $(f(x)+g(x))' = f'(x)+g'(x)$

$$(\alpha f(x)+\beta g(x))' = \alpha f'(x)+\beta g'(x)$$

$$(f(x)g(x))' = f'(x)g(x)+f(x)g'(x)$$

$$(f(x)/g(x))' = f'(x)\frac{1}{g(x)} + f(x)\left(\frac{1}{g(x)}\right)'$$

$$\left(\frac{1}{g(x)}\right)' = \lim_{\Delta x \to 0} \frac{\frac{1}{g(x+\Delta x)} - \frac{1}{g(x)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{-[g(x+\Delta x)-g(x)]}{\Delta x \, g(x+\Delta x)\, g(x)}$$

$$= -\frac{g'(x)}{[g(x)]^2}$$

$$\frac{dx^{\alpha}}{dx} = \alpha x^{\alpha-1}$$

$$\left(\sqrt{625-x^2}\right)' = \lim_{\Delta x \to 0} \frac{\left(\sqrt{625-(x+\Delta x)^2} - \sqrt{625-x^2}\right)}{\Delta x} \frac{\left(\sqrt{625-(x+\Delta x)^2} + \sqrt{625-x^2}\right)}{\sqrt{625-(x+\Delta x)^2} + \sqrt{625-x^2}}$$

$$-2x\Delta x - (\Delta x)^2$$

$$= \lim_{\Delta x \to 0} \frac{-(x+\Delta x)^2 + x^2}{\cancel{\Delta x}} \lim_{\Delta x \to 0} \frac{1}{\sqrt{625-(x+\Delta x)^2} + \sqrt{625-x^2}}$$

$$= \frac{-x}{\sqrt{625-x^2}}$$

$$\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$$

$$x^3\sqrt{a^2-x^2}$$

$$x^3\left(x^5+x^4+2\right)^3$$

$$-\frac{x}{\sqrt{625-x^2}}$$

$$\left(\frac{x}{\sqrt{625-x^2}}\right)' = \frac{(x)'\sqrt{625-x^2} - x\left(\sqrt{625-x^2}\right)'}{625-x^2}$$

$$= \frac{625-x^2+x^2}{\left(625-x^2\right)^{3/2}} = \frac{625}{\left(625-x^2\right)^{3/2}}$$

$$\frac{dx^\alpha}{dx} = \alpha x^{\alpha-1}$$

$$\left(\sqrt{625-x^2}\right)' = \lim_{\Delta x\to 0} \frac{\sqrt{625-(x+\Delta x)^2} - \sqrt{625-x^2}}{\Delta x} \cdot \frac{\sqrt{625-(x+\Delta x)^2} + \sqrt{625-x^2}}{\sqrt{625-(x+\Delta x)^2} + \sqrt{625-x^2}}$$

$$-2x\Delta x - (\Delta x)^2$$

$$= \lim_{\Delta x\to 0} \frac{-(x+\Delta x)^2 + x^2}{\cancel{\Delta x}} \quad \lim_{\Delta x\to 0} \frac{1}{\sqrt{625-(x+\Delta x)^2} + \sqrt{625-x^2}}$$

$$= \frac{-x}{\sqrt{625-x^2}}$$

☆ $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

☆ $$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

☆ $$\frac{df(g(x))}{dx} = \boxed{\frac{df(g(x))}{dg(x)}}\ \boxed{\frac{dg(x)}{dx}}$$

$$h(x) = \frac{1}{x}$$

$$h(g(x)) = \frac{1}{g(x)}$$

$$\frac{dh(g(x))}{dx} = \frac{dh(g(x))}{dg(x)} \frac{dg(x)}{dx}$$

$$\left(\frac{f(x)}{g(x)}\right)' = f'(x)\frac{1}{g(x)} + f(x)\left(\frac{1}{g(x)}\right)'$$

$$\to \frac{-1}{(g(x))^2} g'(x)$$

$$f'(x) \equiv \frac{df(x)}{dx}$$

$$\left(\sqrt{625-x^2}\right)' = \frac{d\sqrt{625-x^2}}{d(625-x^2)} \cdot \frac{d(625-x^2)}{dx}$$

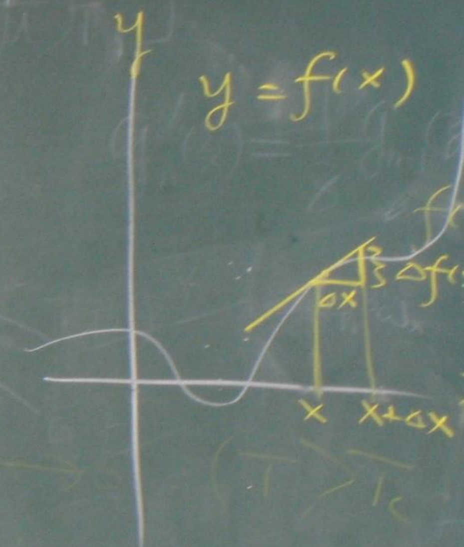
$$\frac{d\sqrt{x}}{dx} = \frac{1}{2}\frac{1}{\sqrt{x}}$$

$$= \frac{1}{2}\frac{1}{\sqrt{625-x^2}}(-2x)$$

$$= -\frac{x}{\sqrt{625-x^2}}$$

Differentiation

ve : $f'(x) \equiv \dfrac{df(x)}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{\overbrace{f(x+\Delta x) - f(x)}^{\Delta f(x)}}{\Delta x}$



1. $\left(c f(x)\right)' = c f'(x)$
2. $\left(f(x) + g(x)\right)' = f'(x) + g'(x)$

$\left(\alpha f(x) + \beta g(x)\right)' = \alpha f'(x) + \beta g'(x)$

$g(x) = a^2 - x^2 \Big\} \Rightarrow (f \circ g)(x) = f(g(x)) = \sqrt{a^2 - x^2}$

$(g \circ f)(x) = g(f(x)) = a^2 - (\sqrt{x})^2 = a^2 - x$

$= \lim\limits_{\Delta x \to 0} \dfrac{f(g(x+\Delta x)) - f(g(x))}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{f(\overset{g+\Delta g}{g(x+\Delta x)}) - f(g(x))}{\underbrace{g(x+\Delta x) - g(x)}_{\Delta g}} \lim\limits_{\Delta x \to 0} \dfrac{g(x+\Delta x) - g(x)}{\Delta x}$

$\Delta g(x) \equiv g(x+\Delta x) - g(x)$

$\to 0 \,,\ \Delta g \to 0$

$= \dfrac{df(g(x))}{dg(x)} \dfrac{dg(x)}{dx}$

1. $f(x) = \dfrac{x^2-1}{x\sqrt{x^2+1}} \Rightarrow \left(f(x)\right)' = \dfrac{(x^2-1)'\,x\sqrt{x^2+1} - (x^2-1)\left(x\sqrt{x^2+1}\right)'}{x^2(x^2+1)}$

$$\left(x\sqrt{x^2+1}\right)' = (x)'\sqrt{x^2+1} + x\left(\sqrt{x^2+1}\right)'$$

$$\left(\sqrt{x^2+1}\right)' = \frac{1}{2\sqrt{x^2+1}}\underbrace{(x^2+1)'}_{2x} = \frac{x}{\sqrt{x^2+1}}$$

2. $f(x) = \sqrt{1+\sqrt{1+\sqrt{x}}} \Rightarrow f'(x) = \dfrac{1}{2}\dfrac{1}{\sqrt{1+\sqrt{1+\sqrt{x}}}}\left(1+\sqrt{1+\sqrt{x}}\right)'$

$$\left(1+\sqrt{1+\sqrt{x}}\right)' = \left(\sqrt{1+\sqrt{x}}\right)' = \frac{1}{2\sqrt{1+\sqrt{x}}}\left(1+\sqrt{x}\right)'$$

$$\left(1+\sqrt{x}\right)' = \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{8\sqrt{x}\sqrt{1+\sqrt{x}}\sqrt{1+\sqrt{1+\sqrt{x}}}}$$

$$\int \frac{1}{\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = F(x) + C$$

$$\Downarrow$$

$$\int f(x)\,dx = F(x) + C$$

$$\frac{dF(x)}{dx} = f(x)$$

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \boxed{8\sqrt{1+\sqrt{1+\sqrt{x}}}} + C$$